

SOLUTIONS. * 15-6-21 E1 T3.

Start here:

1. A



2. C



3. D



4. B



$$5a) \pi \int_{-2.75}^2 x^2 dy = \pi \int_{-2.75}^2 9 - y^2 dy$$

$$= \pi \left[9y - \frac{y^3}{3} \right]_{-2.75}^2 = \pi \left(18 - \frac{8}{3} - (-24.75 + 6.9) \right)$$

$$= 33.15\pi$$

$$= 104.15$$

$$b) y = -\frac{x^2}{115.2} \tan^2 \theta + x \tan \theta - \frac{x^2}{115.2}$$

$$0 = -\frac{1600}{115.2} \tan^2 \theta + 40 \tan \theta - \frac{1600}{115.2}$$

$$0 = -13.8889 \tan^2 \theta + 40 \tan \theta - 23.8889$$

$$-40 \pm \sqrt{40^2 - 4(-13.8889)(-23.8889)}$$

$$-2 \times 13.8889$$

$$\tan \theta = -84.53584$$

$$\theta = 35^\circ 4'$$

$$2.034639$$

$$63^\circ 50'$$

c). $R = 13$

if $y = \frac{5}{13} \sin x - \frac{12}{13} \cos x$

AND $\frac{y}{13} = \sin x \cos \alpha + \cos x \sin \alpha$

$$\cos \alpha = \frac{5}{13} \quad \sin \alpha = \frac{-12}{13}$$

229(b)

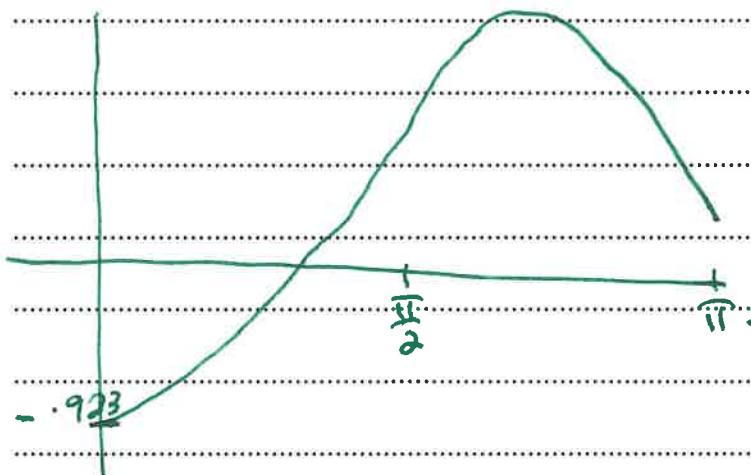
$\alpha = -1.1760^\circ$

ii) $13 \cdot \sin(x - 1.1760)$

$$x = 1.1760 = \frac{\pi}{2}$$

MAX.

$$x = 2.7468$$



d). $\int_0^{\pi} \cos^2 \frac{x}{4} dx = \int_0^{\pi} \frac{1}{2} + \cos \frac{x}{2} dx$

$$= \left[\frac{x}{2} + \sin \frac{x}{2} \right]_0^{\pi} = \frac{\pi}{2} + 1$$

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Start here:

b) i) $\sin(x + \frac{\pi}{2}) = \sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}$

$= 0 + \cos x \cdot 1$

$= \cos x$ as req'd ✓

ii) shift $\sin x$ $\frac{\pi}{2}$ steps to the left.

corresponding to the $x + \frac{\pi}{2}$ transformation ✓

b) i) $\frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x}$ u
v

$$\frac{vu' - uv'}{\sqrt{v^2}} = \frac{\cos x \cdot \cancel{\cos x} - \sin x (-\sin x)}{\cancel{\cos^2 x}}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \sec^2 x \text{ as req'd.} \checkmark$$

iii) $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \int_0^{\frac{\pi}{4}} \frac{\tan^2 x + 1}{\tan^2 x + 1} \sec^2 x - 1 \, dx.$

$$= \left[\tan x - x \right]_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4} \checkmark$$

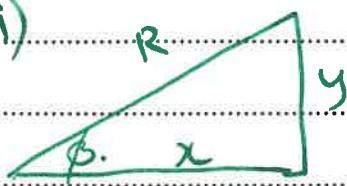
c) if $y = \sin^2 \frac{x}{\sqrt{2}}$. $\frac{dy}{dx} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{1 - \frac{x^2}{2}}} \cdot \frac{1}{2}$

$$= \frac{1}{\sqrt{2} \left(1 - \frac{x^2}{2}\right)} = \frac{1}{\sqrt{2-x^2}}$$

P4.

$$\begin{aligned} I &= \int_{\frac{\pi}{2}}^1 \frac{1}{\sqrt{2-x^2}} dx = \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_{\frac{\pi}{2}}^1 \quad \checkmark \\ &= \frac{\pi}{4} - \frac{\pi}{6} \\ &= \frac{\pi}{12}. \quad \checkmark \end{aligned}$$

d) i)



$$\frac{x}{R} = \cos \phi.$$

$$R = x \sec \phi \text{ as req'd}$$

ii) we want: $\tan \phi x = x \tan \theta - g \frac{x^2 \sec^2 \theta}{2v^2}$

$$0 = (\tan \theta - \tan \phi) - \frac{x g \sec^2 \theta}{2v^2} \quad \checkmark$$

$$x = 0 \text{ or } x = (\tan \theta - \tan \phi) \frac{2v^2}{g \sec^2 \theta} \quad \checkmark$$

$$x \sec \phi = \frac{2v^2}{g} (\tan \theta - \tan \phi) \times \cos^2 \theta \sec \phi.$$

$$\text{iii) } \frac{dR}{d\theta} = \frac{2v^2}{g} \sec \phi \left(\frac{d}{d\theta} (\tan \theta - \tan \phi) \cos^2 \theta \right) \quad \checkmark$$

$$u' = \sec^2 \theta \quad v' = -2 \sin \theta \cos \theta.$$

$$= \frac{2v^2}{g} \sec \phi \left(\cos^2 \theta \sec^2 \theta - 2 \sin \theta \cos \theta \left(\frac{\sin \theta}{\cos \theta} - \cancel{\frac{\cos \phi}{\sin \phi}} \right) \right)$$

$$= \frac{2v^2}{g} \sec \phi \left(1 - 2 \sin^2 \theta + \frac{2 \sin \theta \cos \theta \sin \phi}{\cos \phi} \right).$$

$$= \frac{2v^2}{g} \sec^2 \phi \left(\cancel{2 \sin^2 \theta} \frac{\cos^2 \theta \cos \phi}{\cos \phi} + \sin^2 \theta \sin \phi \right).$$

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$$= \frac{2v^2}{g} \sec^2 \phi \cos(2\theta - \phi) \quad \checkmark$$

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6 d) iv) we want $\frac{dR}{d\theta} = 0$.

$$\text{ie. } \cos(2\theta - \phi) = \cos \frac{\pi}{2}$$

$$\begin{aligned} 2\theta - \phi &= \frac{\pi}{2} \\ \theta &= \frac{\pi}{4} + \frac{\phi}{2} \end{aligned} \quad \checkmark$$

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You may ask for an extra Writing Booklet if you need more space