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Student Number

SAMPLE SOLUTIONS .

**ST PIUS X COLLEGE  
CHATSWOOD**

## **HSC 2023 Stage 6 Year 12**

### **Assessment Task #1**

20% of School Based Assessment

# **MATHEMATICS ADVANCED**

#### **General Instructions**

- Working time – 45 minutes
- Write using black or blue pen  
Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this cover page
- A multiple choice answer sheet is provided

#### **Total Marks – 35**

##### **Section I – Multiple Choice 5 marks**

- Attempt Questions 1 – 5
- Enter responses on the multiple choice answer sheet
- Allow 5 minutes for this section

##### **Section II – 30 marks**

- Attempt Questions 6 – 8
- Answer in the writing spaces provided
- Show all necessary working
- Allow 40 minutes for this section

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## Section I – Multiple-Choice

1 mark per question

5 Marks

*Use the multiple-choice answer sheet.*

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:  $2 + 4 =$       (A) 2      (B) 6      (C) 8      (D) 9  
A  B  C  D

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



- 
1. Which value of  $p$  makes the following table a discrete probability distribution?

$x$	1	2	3	4
$P(X = x)$	$p - 0.1$	$p - 0.15$	$p + 0.1$	$p + 0.55$

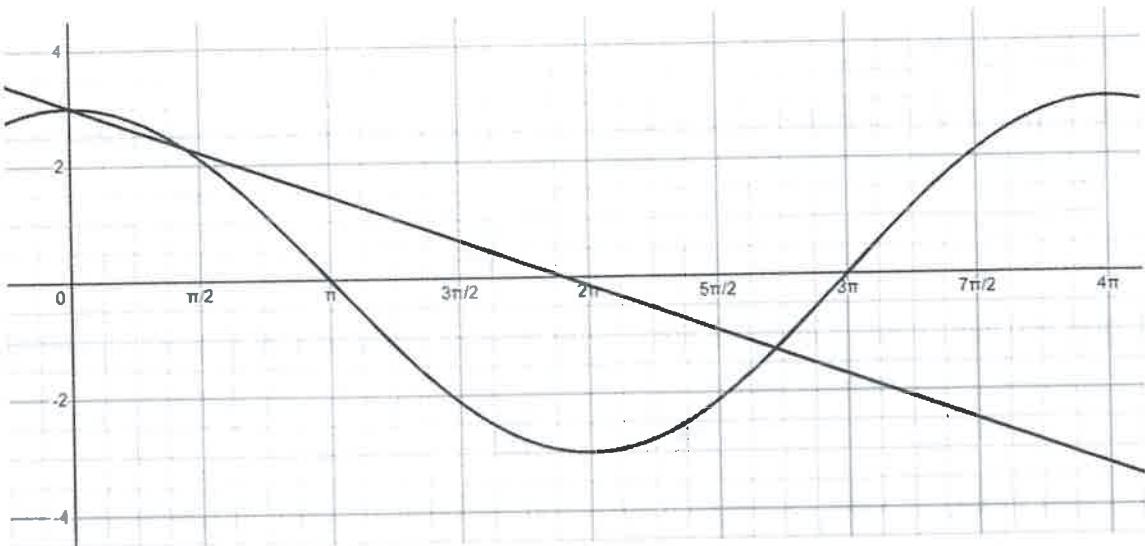
- A 0.6  
 B 0.15  
C 0.9  
D 0.1

2. Consider the curve  $y = \frac{1}{(x-2)} + 2$ .

What is the equation of the horizontal asymptote to this curve?

- A  $x = 2$   
 B  $y = 2$   
C  $y = x - 2$   
D None of the above

3. The graphs of  $f(x) = 3 \cos\left(\frac{x}{2}\right)$  and  $g(x) = 3 - \frac{x}{2}$  are shown below.



How many solutions are there to the equation  $3\cos\left(\frac{x}{2}\right) = -\frac{x}{2} + 3$  for  $0 < x < 4\pi$ ?

- A 1  
 B 2  
C 3  
D 4
4. The seventh term of an arithmetic sequence is 45 and the 11<sup>th</sup> term is 77.  
Find the first term ( $a$ ) and the common difference ( $d$ ).

- A  $a = -3$  and  $d = 8$   
B  $a = 3$  and  $d = 8$   
C  $a = 8$  and  $d = -3$   
D  $a = 8$  and  $d = 3$

5. The linear function  $f(x) = 5 - x$  has range  $[-4, 5]$ . The domain of this function is?

- A  $(0, 9]$   
B  $(0, 1]$   
C  $[5, -4)$   
D  $(-9, 0)$

## Section II

30 Marks

Attempt Questions 6 to 8.

Allow about 40 minutes for this section.

In Questions 6 to 8 your responses should include relevant mathematical reasoning and/or calculations.

**Question 6** (10 marks)

*Write your solutions in the spaces provided*

**Marks**

- (a) Consider the series  $\sqrt{3}, 3\sqrt{3}, 5\sqrt{3}, \dots$

2

What is the difference between the 27<sup>th</sup> and 15<sup>th</sup> terms?

$$\begin{aligned} T_2 - T_1 &= 3\sqrt{3} - \sqrt{3} \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} T_{27} &= \sqrt{3} + 26(2\sqrt{3}) \\ &= 53\sqrt{3} \end{aligned}$$

$$\begin{aligned} T_3 - T_1 &= 5\sqrt{3} - \sqrt{3} \\ &= 4\sqrt{3} \\ \therefore AP, a = \sqrt{3}, d = 2\sqrt{3}. \end{aligned}$$

$$\begin{aligned} T_{15} &= \sqrt{3} + 14\sqrt{3} \\ &= 15\sqrt{3} \end{aligned}$$

$$\begin{aligned} \therefore T_{27} - T_{15} &= 53\sqrt{3} - 15\sqrt{3} \\ &= 38\sqrt{3}. \end{aligned}$$

- (b) Consider the series  $2 - 1 + \frac{1}{2} - \frac{1}{4} + \dots$

2

Calculate the sum of the first ten terms, correct to 3 decimal places.

$$\begin{aligned} \frac{T_2}{T_1} &= -\frac{1}{2} \\ \frac{T_3}{T_2} &= \frac{1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

$$S_{10} = \frac{2 \left\{ \left(-\frac{1}{2}\right)^{10} - 1 \right\}}{\left(-\frac{1}{2} - 1\right)}$$

$$= \frac{2 \left\{ \frac{1}{1024} - 1 \right\}}{-\frac{3}{2}}$$

$$= \frac{-4 \left\{ \frac{1}{1024} - 1 \right\}}{3}$$

$$\therefore S_{10} = 1.332 (300).$$

- (c) The first four terms of a geometric series are 6,  $x$ ,  $y$ , 384.  
Calculate values for  $x$  and  $y$ .

$\frac{x}{6} = \frac{y}{x}$ $x^2 = 6y$ $y = \frac{x^2}{6} \quad \text{--- (1)}$	$\frac{x^3}{384} = 384$ $x^3 = 13824$ $x = \sqrt[3]{13824}$ $\therefore x = 24$
$\frac{384}{x} = \frac{y}{x}$ $y^2 = 384x \quad \text{--- (2)}$	$\text{Sub } x = 24 \text{ into (1)}$ $y = \frac{(24)^2}{6}$ $y = 96.$ $\therefore x = 24$ $y = 96.$
$\text{Sub (1) into (2)}$ $(\frac{x^2}{6})^2 = 384x$ $\frac{x^4}{36} = 384x$	

- (d) An Olympic hammer-thrower while training has an initial throw of 60 metres. However, on subsequent throws she is only able to achieve 95% of the previous throwing distance.

- (i) What will be the distance of her eighth throw?

2

$$T_8 = 60(0.95)^{8-1}$$

$$T_8 = 60(0.95)^8$$

$$= 60(0.95)^7$$

$$\therefore T_8 = 41.9 \text{ m (1 dp)}.$$

- (ii) At the given rate what will be the total distance thrown when the athlete is totally exhausted?

1

$$S_{\infty} = \frac{60}{(1 - 0.95)}$$

$$= \frac{60}{0.05}$$

$$\therefore S_{\infty} = 1200 \text{ m.}$$

**Question 7 (10 marks)**

*Write your solutions in the spaces provided*

**Marks**

- (a) For what values of  $x$  is  $|4x - 7| = 11$ ?

2

$$4x - 7 = 11$$

$$-4x + 7 = 11$$

$$4x = 18$$

$$-4x = 4$$

$$x = \frac{18}{4}$$

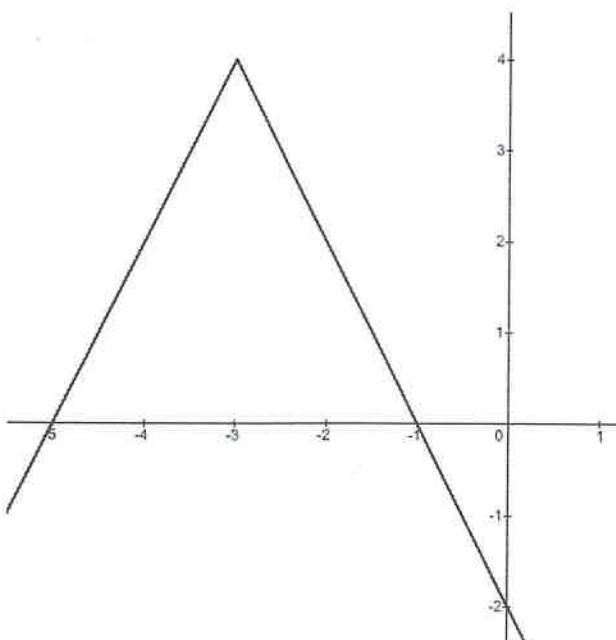
$$\therefore x = (-1)$$

$$-4x = \frac{9}{2} \approx -4\frac{1}{2}$$

$$\therefore x = (-1) \text{ or } 4\frac{1}{2}$$

- (b) The function  $F(x) = |x|$  is transformed and the new function is written in the form  $y = kF(x + b) + c$ , where  $b$ ,  $c$ , and  $k$  are integer constants.  
The transformed graph is shown below.

2



Write the equation of the transformed graph in the form  $y = kF(x + b) + c$ .

$$y = -2F(x + 3) + 4$$

- (c) The function  $f(x) = \sin x$  is transformed into  $g(x) = -2\sin 3x$ .

- (i) Describe how the amplitude and period changed in this transformation.

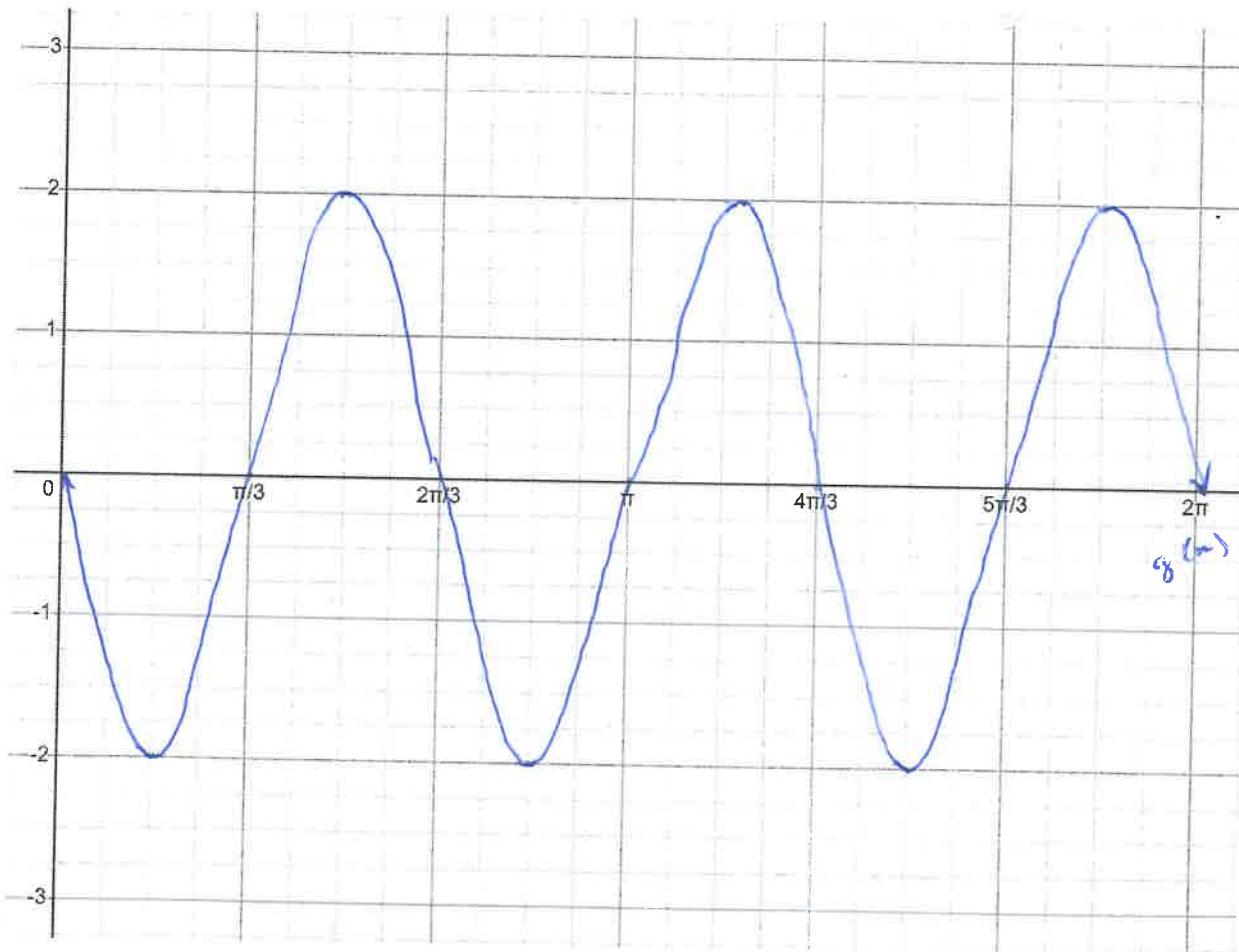
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AMPLITUDE IS DOUBLED.

PERIOD IS DIVIDED BY 3.

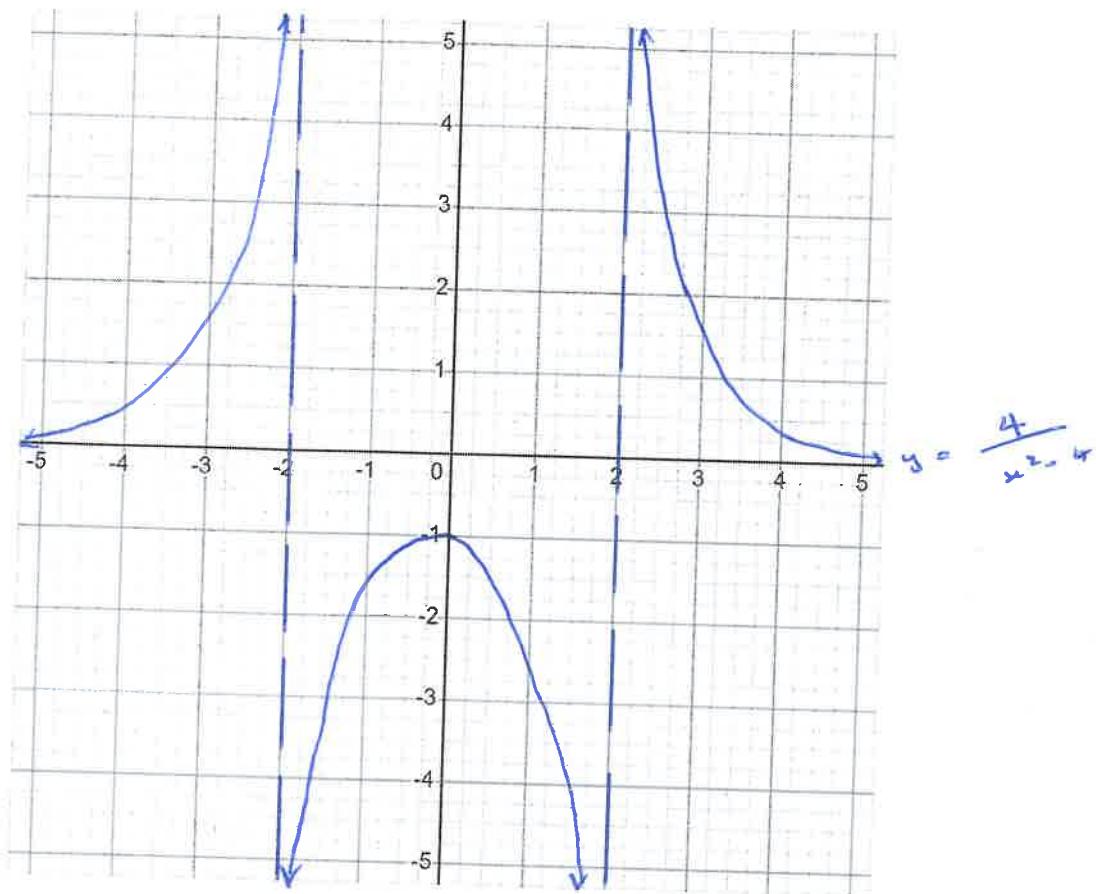
- (ii) In the space provided, sketch  $g(x) = -2\sin 3x$ .

2



- (d) Sketch  $y = \frac{4}{x^2 - 4}$ , in the space provided, showing asymptotes and intercepts.

2



**Question 8 (10 marks)**

Write your solutions in the spaces provided

Marks

- (a) (i) Complete this table of values for a discrete random variable  $x$ .

2

$x$	0	1	2	3	4	Sum
$p(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$	1
$xp(x)$	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	$\frac{32}{16}$
$x^2 p(x)$	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$	$\frac{80}{16}$

- (ii) Use the formula  $Var(x) = \Sigma x^2 p(x) - \mu^2$  to calculate the variance.

1

$$5 - 4 = 1$$

- (b) In a game a six-sided die is rolled twice. The difference between the two results is recorded.

- (i) A table of results has been created with some missing results.  
Complete the table.

1

		Second Roll					
		1	2	3	4	5	6
First roll	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	D	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

- (ii) What is the value of  $P(1)$ ?

1

$$P(1) = \frac{5}{36}$$

- (iii) A probability distribution table is drawn to summarise the results.  
Complete the table.

2

$x$	0	1	2	3	4	5
$P(X=x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

- (iv) Find the expected value,  $E(X)$ , and the standard deviation  $\sigma$ .

3

$$\begin{aligned} E(x) &= 0 + \frac{5}{18} + \frac{4}{9} + \frac{3}{6} + \frac{2}{9} + \frac{5}{18} \\ &= \frac{35}{18} \approx 1.9444 \text{ (4dp).} \end{aligned}$$

$$\begin{aligned} E(x^2) &= 0 + \frac{25}{18} + \frac{16}{9} + \frac{9}{6} + \frac{16}{9} + \frac{25}{18} \\ &= \frac{105}{18} \approx 5.8333 \text{ (4dp).} \end{aligned}$$

$$\therefore E(x) = \frac{35}{18} \quad E(x^2) = \frac{105}{18}$$

$$\begin{aligned} \text{var}(x) &= E(x^2) - E(x)^2 \\ &= \frac{105}{18} - \left(\frac{35}{18}\right)^2 \\ \therefore \text{var}(x) &= \frac{665}{324} \approx 2.0284 \text{ (4dp).} \end{aligned}$$

$$\therefore \sigma = \sqrt{\frac{665}{324}}$$

$$\sigma = 1.4326 \text{ (4dp).}$$

End of Assessment.