Yr 12 Ext 1 Ass #1

Aides to solutions

Multiple choice section 1

Q1 C

Q2 D

Q3 B

Q4 A

Q5 A

Section 2

Q6 a) $\int 2x\sqrt{1+x^2}\,dx$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$du = 2x \cdot dx$ — 1

$\therefore \int \sqrt{u}\,du$

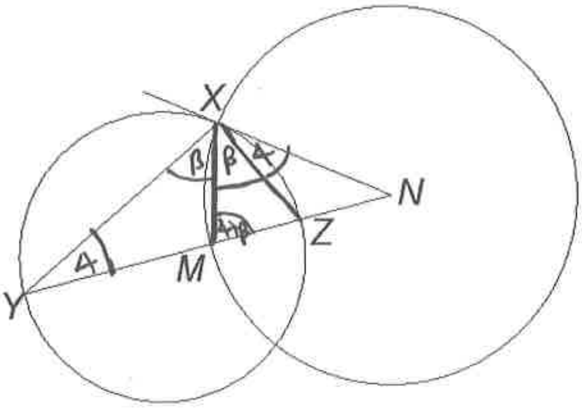
$= \left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$

$\frac{2u^{\frac{3}{2}}}{3} + C$ — 1

$= \frac{2\sqrt{(1+x^2)^3}}{3} + C$ — 1

②

Q6 b)



$\angle XYM = 4$ (Angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment). —①

$\angle NMX = 4 + \beta$ (Base angles of isosceles triangle NMX are equal) —①

$\therefore$ $\angle MXY = \beta$ (external angle of a triangle is equal to the opposite two internal angles)

$\therefore$ MX bisects $\angle ZXY$ —①

c) $(1\times4) + (2\times5) + (3\times6) + \ldots + [n(n\times3)] = \frac{1}{3}n(n+1)(n+5)$

Prove true for $n=1$

L.H.S $1\times4 = 4$ R.H.S $= \frac{1}{3}\times1\times2\times6 = 4$ $\therefore$ true for $n = 1$

① —

3

Q6 c) Assume true for $n=k$, where k is a positive integer

$$(1\times4)+(2\times5)+(3\times6)+\dots+(k(k+3)=\frac{1}{3}k(k+1)(k+5)$$

Prove true for $n=k+1$

$$(1\times4)+\dots(k+1)(k+4)=\frac{1}{3}(k+1)(k+2)(k+6)$$

now $(1\times4)\times(2\times5)+\dots k(k+3)=\frac{1}{3}k(k+1)(k+5)$ from assumption. → (1)

L.H.S

∴ $\frac{1}{3}k(k+1)(k+5)+(k+1)(k+4)$

$= k+1\left[\frac{1}{3}k(k+5)+(k+4)\right]$

$= \frac{1}{3}(k+1)\left[k(k+5)+3(k+4)\right]$

$= \frac{1}{3}(k+1)\left[k^2+5k+3k+12\right]$

$= \frac{1}{3}(k+1)\left[k^2+8k+12\right]$

$= \frac{1}{3}(k+1)(k+2)(k+6)$ = R.H.S

∴ proved true for $n=k+1$, $n=1,2$, and holds true for all positive integers, proved by Mathematical Induction.

4

Q6) d) $\displaystyle\int_1^4 \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$ using $u = 1+\sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2.du = \frac{dx}{\sqrt{x}}$ — 1

when $x=4$, $u = 1+\sqrt{4}$

$= 3$ 1 —

when $x=1$ $u = 1+\sqrt{1}$

$= 2$

$\therefore \displaystyle\int_2^3 \frac{2.du}{u^2}$

1 — $\left[\frac{u^{-2+1}}{-2+1}\right]_2^3$

$= \left[\frac{-1}{u}\right]_2^3$

$= \left(\frac{-1}{3}\right) - \left(\frac{-1}{2}\right)$

$= -\frac{1}{3} + \frac{1}{2}$

$= \frac{1}{6}$ — 1

Q7 a) $(n+1)(n+2) \ldots (2n-1)(2n) = 2^n (1\times3\times5\times \ldots 2n-1)$

for $n \geq 1$

Prove true for $n=1$

L.H.S $(1+1) = 2$ R.H.S $= 2^1 = 2$ — 1

$\therefore$ true for $n=1$

5 Q1a) Assume true for $n=k$ $\quad k \geq 1$

$$(k+1)(k+2) \cdots (2k-1)(2k) = 2^k[1 \times 3 \times 5 \times \cdots 2k-1]$$

so [redacted] $(k+2) \cdots (2k-1)(2k)$ can be replaced by

$$\frac{2^k(1 \times 3 \times 5 \times \cdots 2k-1)}{(k+1)}$$

Prove true for $n=k+1$

$$\underbrace{(k+2)(k+3) \cdots (2k-1)(2k)}(2k+1)(2k+2) = 2^{k+1}[1 \times 3 \times \cdots 2k+1]$$

L.H.S $\quad \dfrac{2^k(1 \times 3 \times 5 \times \cdots 2k-1)}{k+1} \cdot (2k+1)(2k+2)$

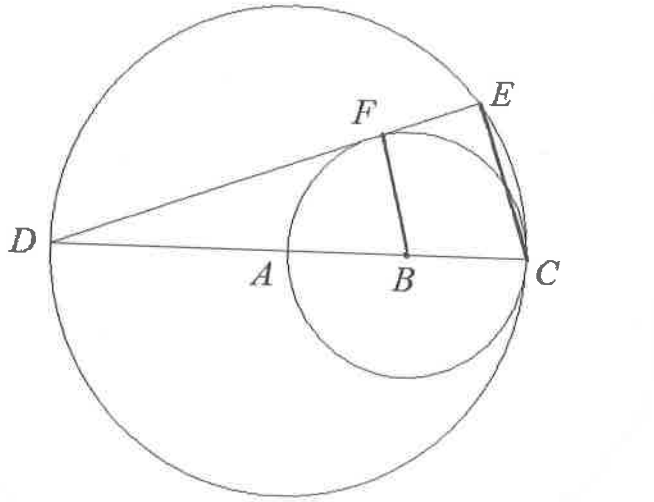
$$\frac{2^k(1 \times 3 \times 5 \times \cdots 2k-1)\, 2 . \cancel{(k+1)}(2k+1)}{\cancel{(k+1)}} \quad -(1)$$

$$2^{k+1}(1 \times 3 \times 5 \times \cdots 2k-1)(2k+1) = \text{R.H.S}$$

$\therefore$ prove true for $n=k+1$, and all positive integers, proved by Mathematical Induction.

6

Q7(b)



i) In Δ's BDF and CDE

∠D is common angle

∠DFB = 90° (The tangent to a circle is perpendicular to the radius drawn at the point of contact

as is

∠DEC = 90° (Angle at the centre is twice the angle at the circumference standing on the same arc)

∴ Δ BDF ||| Δ CDE

ii) DA = 6cm

AB = 3cm

BC = 3cm

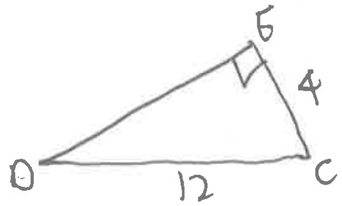
FB = 3cm

∴ Ratio of Δ BDF to CDE is

3 : 4 —(1)

so if FB = 3cm

EC = 4cm



and using Pythagoras Theorem DE = $8\sqrt{2}$ —(1)

∴ Area = $\frac{1}{2} \times 4 \times 8\sqrt{2} = 16\sqrt{2}$ u^2 —(1)

7 Q7 c)

$$\int_0^2 x(x-1)^c \, dx$$

$u = x-1$ $\qquad$ $u = 2-1$ $\qquad$ $u = 0-1$ $\qquad$ also

$du = dx$ $\qquad$ $u = 1$ $\qquad$ $u = -1$

$x = u+1$

$$\therefore \quad \therefore \int_{-1}^{1} (u+1)\, u^c \, du$$

$$\int_{-1}^{1} u^{c+1} + u^c \, du$$

$$\left[\frac{u^{c+2}}{c+2} + \frac{u^{c+1}}{c+1} \right]_{-1}^{1}$$

$$\left(\frac{1^{c+2}}{c+2} + \frac{1^{c+1}}{c+1} \right) - \left(\frac{(-1)^{c+2}}{c+2} + \frac{(-1)^{c+1}}{c+1} \right) \quad — 2$$

now if c is an even integer

$$\left(\frac{1}{c+2} + \frac{1}{c+1} \right) - \left(\frac{1}{c+2} - \frac{1}{c+1} \right)$$

$$\therefore \frac{2}{c+1} \text{ if } c \text{ is even} \quad — 1$$

if c is an odd integer

$$\left(\frac{1}{c+2} + \frac{1}{c+1} \right) - \left(\frac{-1}{c+2} + \frac{1}{c+1} \right)$$

$$= \frac{2}{c+2} \text{ if } c \text{ is odd} \quad — 1$$