
Student Number

ST PIUS X COLLEGE
CHATSWOOD

HSC 2021 Stage 6
Year 12

Assessment Task #3

25% of School Based Assessment

MATHEMATICS ADVANCED

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this cover page

Total Marks – 35

Section I – Multiple Choice 5 marks

- Attempt Questions 1 – 5
- Enter responses on the multiple choice answer sheet
- Allow 5 minutes for this section

Section II – 30 marks

- Attempt Questions 6 – 8
- Answer in the writing spaces provided
- Show all necessary working
- Allow 40 minutes for this section

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Section I – Multiple-Choice 1 mark per question **5 Marks**

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

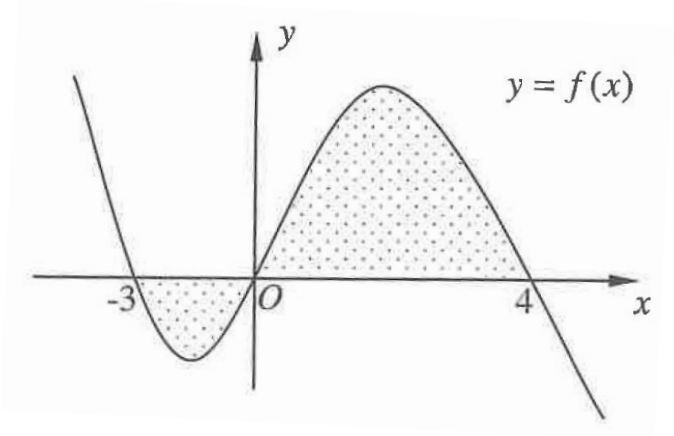
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

1. Consider the diagram below.

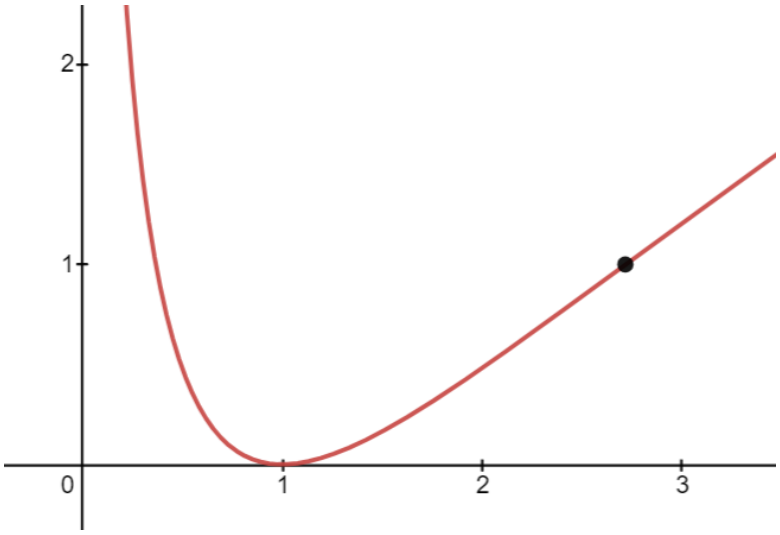


Which of the following represents the shaded area?

(A) $\int_{-3}^{4} f(x)\,dx$ (B) $2\int_{0}^{4} f(x)\,dx$

(C) $\int_{0}^{4} f(x)\,dx - \int_{-3}^{0} f(x)dx$ (D) $\int_{-3}^{0} f(x)\,dx + \int_{0}^{4} f(x)dx$

2. The diagram below shows the curve $y = (\log_e |x|)^2$.



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What would be the gradient, m, of a tangent drawn to this curve at the point $(e, 1)$?

(A) $m = e$

(B) $m = \frac{1}{e}$

(C) $m = 2$

(D) $m = \frac{2}{e}$

3. What is the derivative of $e^{3\ln x}$?

(A) $3x^2$

(B) $3e^{3\ln x}$

(C) $(3\ln x)e^{3\ln x - 1}$

(D) $(3\ln x)e^{3\ln x} \times \frac{1}{x}$

4. What is the derivative of $\frac{x}{\cos x}$?

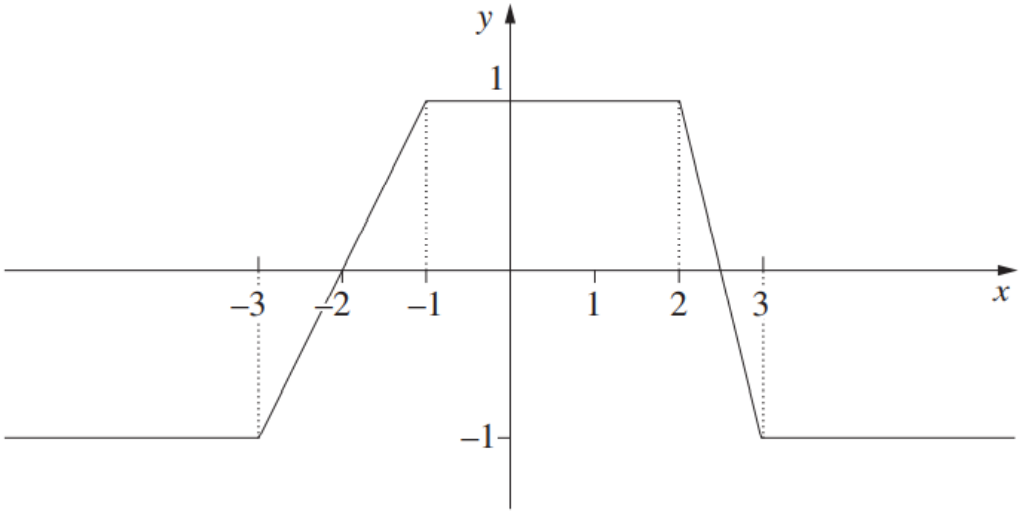
(A) $\frac{\cos x - x \sin x}{\cos^2 x}$

(B) $\frac{\cos x + x \sin x}{\cos^2 x}$

(C) $\frac{x \sin x - \cos x}{\cos^2 x}$

(D) $\frac{-x \sin x - \cos x}{\cos^2 x}$

5. The diagram shows the graph $y = f(x)$.



What value of k, where $k > 0$, would make $\int_{-k}^{k} f(x)\, dx = 0$?

(A) 3

(B) 3.5

(C) 4

(D) 4.5

End of Multiple-Choice Section 1.

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Section II **30 Marks**

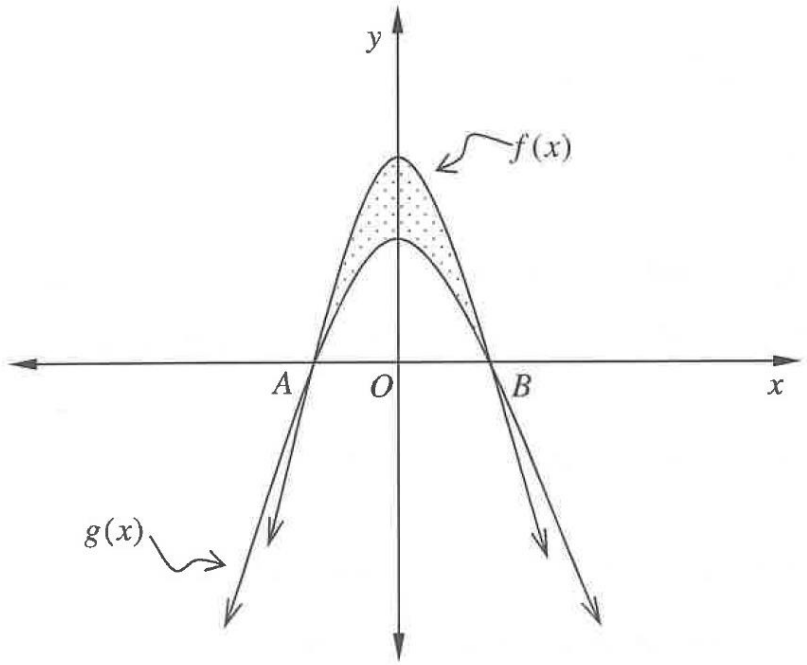
Attempt Questions 6 to 8.
Allow about 40 minutes for this section.

In Questions 6 to 8 your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) The graphs of $f(x) = (5+x)(5-x)$ and $g(x) = \frac{2}{5}(5+x)(5-x)$ intersect at points A and B, as shown in the diagram below.



(i) Show that the area of the shaded region is given by $A = \frac{6}{5}\int_0^5 (25 - x^2)\ dx$. **1**

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(ii) Hence evaluate the area of the shaded region. **2**

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(b) Find a primitive for $\frac{5}{x^2} - 8x$. **2**

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(c) Find $\int \frac{2}{\sqrt{3x-1}}\,dx$ **2**

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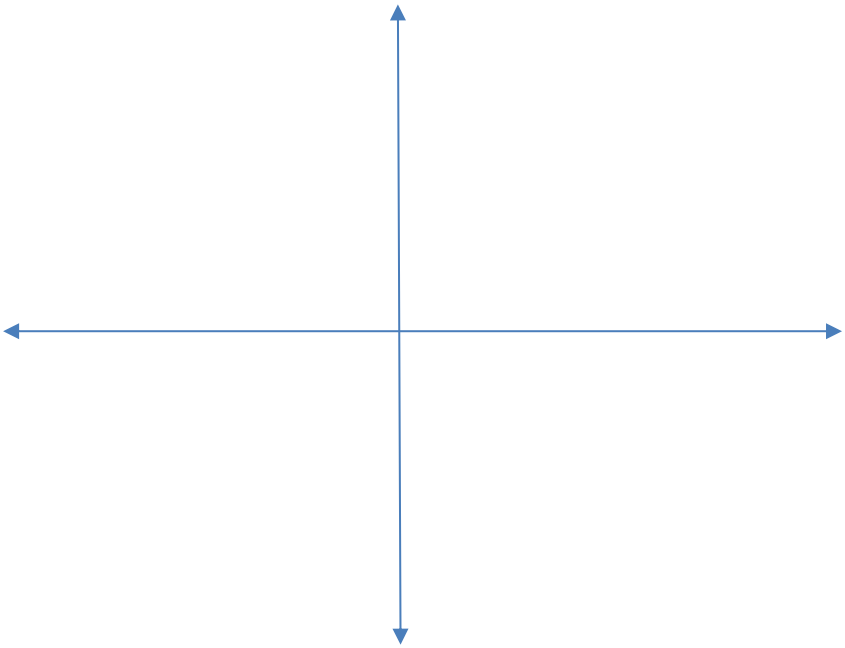
(d) (i) Sketch the curve $y = x^2 - 3x - 10$. **1**



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(ii) Hence find the area between the curve and the x-axis. **2**

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Question 7 on next page.

Question 7 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) Differentiate with respect to x:

(i) $x\log_e 2x$ **2**

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(ii) $4e^{2x}$ **1**

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(b) Find $\int \frac{x}{x^2-7}\,dx$ **1**

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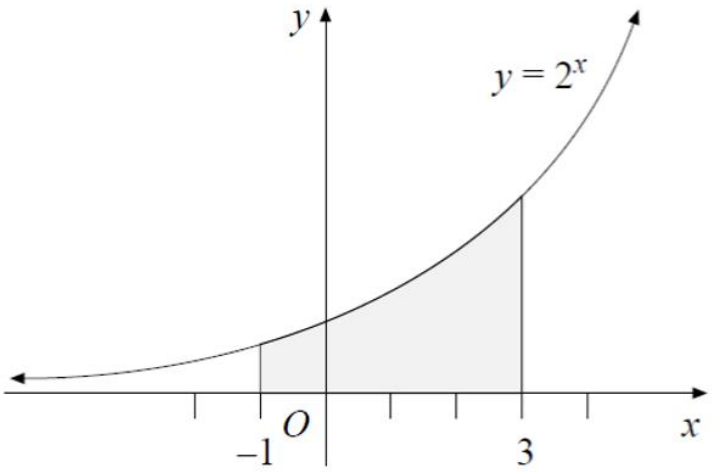
(c) Consider the function $y = 2^x$ shown below.



(i) Complete the following tables of values for $y = 2^x$: **1**

x	−1	0	1	2	3
2^x					

(ii) Use the Trapezoidal rule with these five function values to find an estimate for the area of the shaded region in the diagram. **2**

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(iii) Find the EXACT area by evaluating the integral $\int_{-1}^{3} 2^x \, dx$. **2**

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(iv) By what percentage does the Trapezoidal rule in part (ii) overestimate the true area bounded by the curve and the x-axis between $x = -1$ and $x = 3$? **1**

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Question 8 on next page.

Question 8 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) Differentiate the following with respect to x:

(i) $5x + \sin 5x$ **1**

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(ii) $\cos(x^2 - 3)$ **1**

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(b) Find $\int_0^{\frac{\pi}{4}} \sec^2 3x \; dx$. **2**

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(c)

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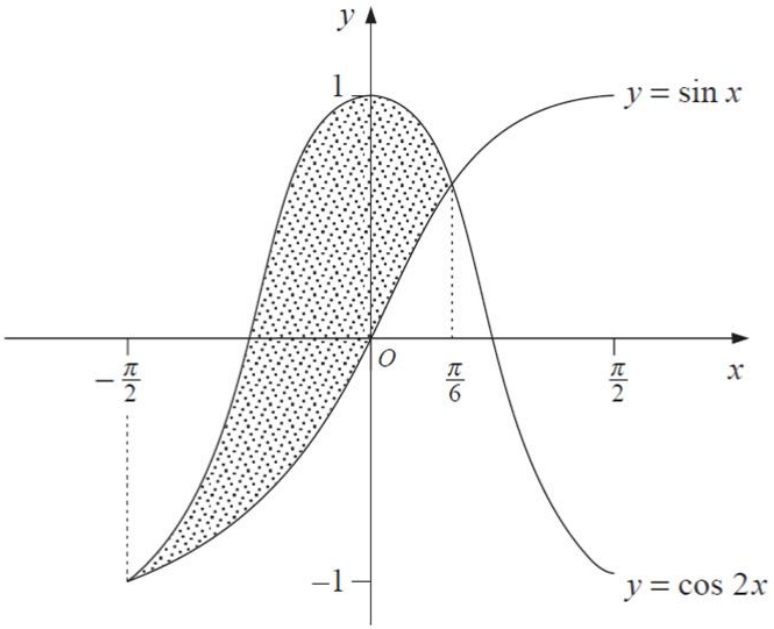

The diagram above shows the graphs of the functions $y = \cos 2x$ and $y = \sin x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. The two graphs intersect at $x = \frac{\pi}{6}$ and $x = -\frac{\pi}{2}$.

Calculate the EXACT area of the shaded region.

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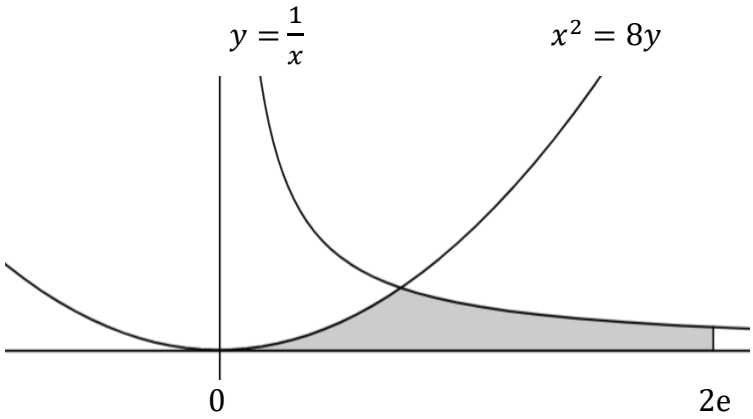
(d)



The graph above shows the parabola $y = \frac{x^2}{8}$ and the hyperbola $y = \frac{1}{x}$.

The curves intersect at a point in the 1st quadrant. The region between the x-axis and the curves from $x = 0$ to $x = 2e$ has been shaded.

(i) By solving simultaneously, show that the curves meet at the point $(2, \frac{1}{2})$. **1**

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(ii) Show that the area of the shaded region is $1\frac{1}{3}$ square units. **2**

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End of Task

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Number

Mathematics Advanced – Multiple Choice Answer Sheet

Attempt all questions:

Question	**1**	A ⬭	B ⬭	C ⬭	D ⬭
	2	A ⬭	B ⬭	C ⬭	D ⬭
	3	A ⬭	B ⬭	C ⬭	D ⬭
	4	A ⬭	B ⬭	C ⬭	D ⬭
	5	A ⬭	B ⬭	C ⬭	D ⬭