
Student Number

ST PIUS X COLLEGE
CHATSWOOD

HSC 2020 Stage 6
Year 12

ASSESSMENT TASK #1

20% of School Based Assessment

MATHEMATICS EXTENSION 1

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of all pages

Total Marks – 30

Section I – 15 marks

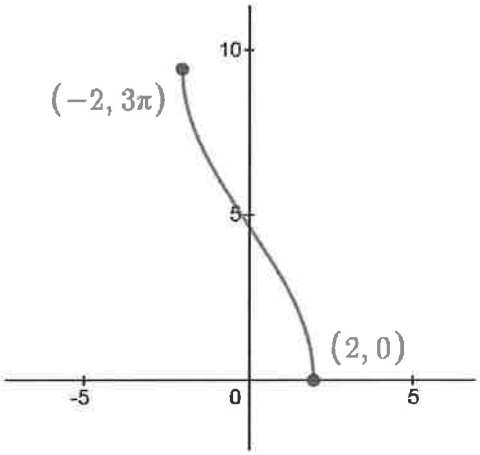
- Attempt all questions
- Show all necessary working
- ***Write solutions in space provided***
- Allow $22\frac{1}{2}$ minutes for this section

Section II – 15 marks

- Attempt all questions
- Show all necessary working
- ***Write solutions in space provided***
- Allow $22\frac{1}{2}$ minutes for this section

Section I **15 marks**

1. Consider the graph of the inverse function below written in the format $y = a\cos^{-1}(bx)$. **2**



Find the values of a and b.

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2. By considering the expansion of $\sin(A + B)$, find the exact value of $\sin 75^{\circ}$. **3**
Leave you solution in simplest surd form.

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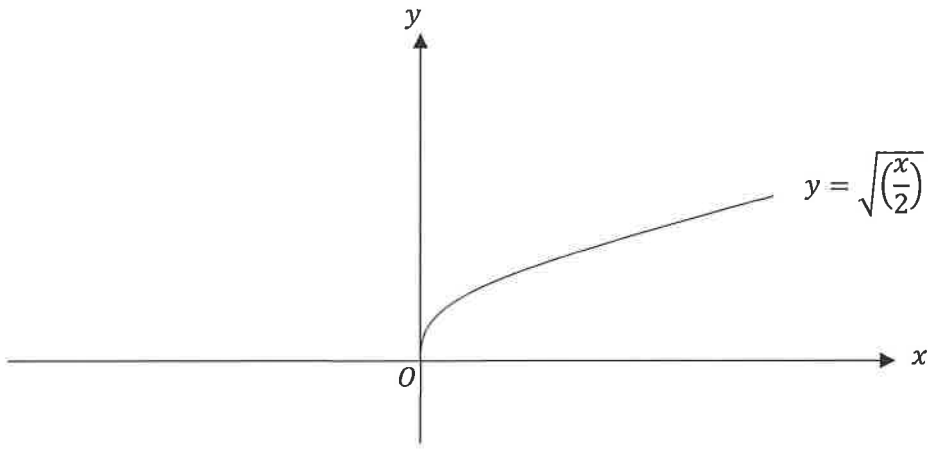
3. Prove the trigonometric identity $\frac{2\cos A}{\text{cosec}\, A - 2\sin A} = \tan 2A$. **3**

4. If $\sin\theta = \frac{3}{4}$ and $\frac{\pi}{2} < \theta < \pi$, evaluate in simplest surd form $\sec\theta$. **2**

5. Solve the equation $2\sin^2\theta = \sin 2\theta$ for $0 \leq \theta \leq 2\pi$. **3**

6. Consider the diagram.



a. Find the inverse function of $y = \sqrt{\left(\frac{x}{2}\right)}$. **1**

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b. Sketch, on the above diagram, the inverse function of $y = \sqrt{\left(\frac{x}{2}\right)}$. **1**

End of Section I

Section II **15 marks**

1. a. Use the substitution $t = \tan\frac{\theta}{2}$, to express $\frac{1}{1+\cos\theta}$ in a simplified form in terms of t. **2**

b. Hence, or otherwise, solve the equation $\frac{1}{1+\cos\theta} = 2$, for $-\pi \leq \theta \leq \pi$. **2**

2. Prove that $9^{n+2} - 4^n$ is divisible by 5 for all positive integers n. **3**

3. Prove $1 + 8 + 27 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2$ for all positive integers n. **3**

4. Consider the statement $3^n > n^2 + 20$ for $n \in \mathbb{Z}^+$.

a. Is the statement true for $n = 1$? Justify your answer. **1**

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b. Find the smallest positive integer n for which the statement is true. **1**
Show your working.

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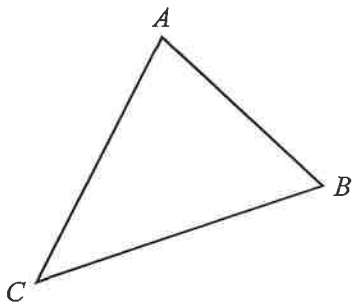
5. a. Use the compound angle results to show $\tan(\pi - \theta) = -\tan\theta$. **1**

b. Hence, or otherwise, prove that for any triangle with angles A, B and C that: **2**

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$



End of Task.

Additional working space. Ensure you identify the question number.