1. $\boxed{B}$

$3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $R > 0$

$R = \sqrt{3^2 + 4^2} = 5$ $0 < \alpha < \frac{\pi}{2}$

$\tan\alpha = \frac{4}{3}$ $\therefore \alpha = \tan^{-1}\frac{4}{3} = 0.927$

$\therefore 3\cos\theta + 4\sin\theta = 5\cos(\theta - 0.927)$

2. $\boxed{C}$ $y = \cos^{-1}(x^2)$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot \frac{d(x^2)}{dx} = \frac{-2x}{\sqrt{1-x^4}}$

3. $\boxed{D}$ $\int \sin 5x \cdot \sin 3x \cdot dx$

$= \int \frac{1}{2}[\cos(5x-3x) - \cos(5x+3x)]\,dx$

$= \frac{1}{2}\int(\cos 2x - \cos 8x)\,dx$

$= \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right) + c = \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$

4. $\boxed{D}$ $\int_0^k \frac{1}{\sqrt{4-9x^2}}dx = \frac{\pi}{18}$ $\therefore \int_0^k \frac{1}{\sqrt{2^2-(3x)^2}}dx = \frac{\pi}{18}$

$\therefore \left[\frac{1}{3}\sin^{-1}\frac{3x}{2}\right]_0^k = \frac{\pi}{18}$ $\therefore \frac{1}{3}\sin^{-1}\frac{3k}{2} = \frac{\pi}{18}$

$\therefore \sin^{-1}\frac{3k}{2} = \frac{\pi}{6}$ $\therefore \frac{3k}{2} = \sin\frac{\pi}{6} = \frac{1}{2}$

$\therefore 3k = 1$ $\therefore k = \frac{1}{3}$

5. $\boxed{A}$ $x = \frac{\sqrt{2}}{4}t$ $\therefore \dot{x} = \frac{\sqrt{2}}{4}$

$y = \frac{\sqrt{6}}{2}t - 5t^2$ $\therefore \dot{y} = \frac{\sqrt{6}}{2} - 10t$

At $t=0$: $\dot{y} = \frac{\sqrt{6}}{2}$

$\therefore v^2 = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(\frac{\sqrt{2}}{4})^2 + (\frac{\sqrt{6}}{12})^2} = \sqrt{\frac{1}{6}} = \frac{1}{\sqrt{6}}$

$\tan\theta = \frac{\dot{y}}{\dot{x}} = \frac{(\frac{\sqrt{6}}{12})}{(\frac{\sqrt{2}}{4})} = \frac{1}{\sqrt{3}}$ $\therefore \theta = 30°$

Question 6

a) i) $\sqrt{2}\sin x + \sqrt{2}\cos x = R\sin(x+\alpha)$

(1) $R = \sqrt{\sqrt{2}^2 + \sqrt{2}^2} = 2$, $\tan\alpha = \frac{\sqrt{2}}{\sqrt{2}} = 1$ $\therefore \alpha = \frac{\pi}{4}$ (1)

$\therefore \sqrt{2}\sin x + \sqrt{2}\cos x = 2\sin(x+\frac{\pi}{4})$

ii) $\sqrt{2}\sin x + \sqrt{2}\cos x = 2\sin(x+\frac{\pi}{4})$

(1) is maximum when $x + \frac{\pi}{4} = \frac{\pi}{2}$ $\therefore x = \frac{\pi}{4}$ (1)

iii) $\sqrt{2}\sin x + \sqrt{2}\cos x + \sqrt{3} = 0$

(2) $2\sin(x+\frac{\pi}{4}) + \sqrt{3} = 0$

$2\sin(x+\frac{\pi}{4}) = -\sqrt{3}$

$\sin(x+\frac{\pi}{4}) = -\frac{\sqrt{3}}{2}$

$\therefore x + \frac{\pi}{4} = \pi + \frac{\pi}{3}$ or $x + \frac{\pi}{4} = 2\pi - \frac{\pi}{3}$ (2)

$\therefore x = \frac{13\pi}{12}$ or $x = \frac{17\pi}{12}$

b) $\cos 2x + 2\cos x = 1$

(2) $2\cos^2 x - 1 + 2\cos x + 1 = 0$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$\cos x = 0$ or $\cos x = -1$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $x = \pi$ (2)

$\therefore x = \frac{\pi}{2}, \pi$ or $\frac{3\pi}{2}$

Question 6

(c) Let $u = 1 + 3e^x$ $\therefore du = 3e^x dx$ $\therefore e^x dx = \frac{1}{3} du$

when $x = 0$, $u = 1 + 3e^0 = 4$

$x = \ln 8$, $u = 1 + 3e^{\log_e 8} = 1 + 3 \times 8 = 25$

$\therefore \int_0^{\log_e 8} \frac{e^x}{\sqrt{1 + 3e^x}} dx = \int_4^{25} \frac{\frac{1}{3} du}{\sqrt{u}} = \frac{1}{3} \int_4^{25} u^{-\frac{1}{2}} du$

$= \frac{1}{3} \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^{25} = \frac{2}{3} \left[\sqrt{25} - \sqrt{4} \right] = 2$ ③

(d) $\underset{\sim}{r}(t) = 20\sqrt{3}\, t\, \underset{\sim}{i} + (-5t^2 + 20t) \underset{\sim}{j}$

Let $x = 20\sqrt{3}\, t$

$y = -5t^2 + 20t$

i) when $y = 0$: $-5t^2 + 20t = 0$

$-5t(t - 4) = 0$

$\therefore t = 0$ or $t = 4$ ②

$\therefore$ the particle reaches the ground after 4 seconds

Sub $t = 4$ into x $\therefore$ Range $= 20\sqrt{3} \times 4 = 80\sqrt{3}$ m

(or 138.56 m)

ii) $\because x = 20\sqrt{3}\, t$ $\therefore t = \frac{x}{20\sqrt{3}}$ sub into y

$\therefore y = -5 \left(\frac{x}{20\sqrt{3}} \right)^2 + 20 \left(\frac{x}{20\sqrt{3}} \right) = -5 \left(\frac{x^2}{1200} \right) + \frac{x}{\sqrt{3}}$ ②

$\therefore y = \frac{-x^2}{240} + \frac{x}{\sqrt{3}}$ is the Cartesian Equation

(or $y = \frac{-x^2}{240} + \frac{\sqrt{3}x}{3}$ or $y = \frac{-x^2 + 80\sqrt{3}x}{240}$)

Question 7

a) $\dfrac{d}{dx} \log_e (\sin^{-1}x) = \dfrac{1}{\sin^{-1}x} \cdot \dfrac{d \sin^{-1}x}{dx} = \dfrac{1}{\sin^{-1}x} \cdot \dfrac{1}{\sqrt{1-x^2}}$

(2)　　　　(2)

b) $y = \displaystyle\int \dfrac{1}{9+4x^2}\, dx = \int \dfrac{1}{3^2+(2x)^2}\, dx = \dfrac{1}{2}\int \dfrac{2}{3^2+(2x)^2}\, dx$

(3)

$= \dfrac{1}{2} \times \left[\dfrac{1}{3} \tan^{-1} \dfrac{2x}{3}\right] + C = \dfrac{1}{6}\tan^{-1}\dfrac{2x}{3} + C$　　　(1)

Sub $x = \dfrac{3}{2}$, $y = 0$　　　$\therefore 0 = \dfrac{1}{6}\tan^{-1}\dfrac{2\left(\frac{3}{2}\right)}{3} + C$　　(1)

$\therefore 0 = \dfrac{1}{6}\tan^{-1}1 + C$　　$\therefore 0 = \dfrac{1}{6}\times\dfrac{\pi}{4} + C$

$\therefore C = -\dfrac{\pi}{24}$　　$\therefore y = \dfrac{1}{6}\tan^{-1}\dfrac{2x}{3} - \dfrac{\pi}{24}$　　(1)

c) $y = 2\sin^{-1}\dfrac{x}{3}$　　　　$\therefore \dfrac{y}{2} = \sin^{-1}\dfrac{x}{3}$

(3)

$\therefore \sin\dfrac{y}{2} = \dfrac{x}{3}$　　　$\therefore x = 3\sin\dfrac{y}{2}$　　　(1)

Volume $= \displaystyle\int_0^\pi \pi x^2\, dy = \int_0^\pi \pi \left(3\sin\dfrac{y}{2}\right)^2 dy$　　(1)

$= \pi \displaystyle\int_0^\pi 9\sin^2\dfrac{y}{2}\, dy = 9\pi \int_0^\pi \dfrac{1}{2}(1-\cos y)\, dy$

$= \dfrac{9\pi}{2}\left[y - \sin y\right]_0^\pi$

$= \dfrac{9\pi}{2}\left[(\pi - \sin\pi) - (0 - \sin 0)\right]$

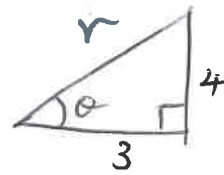$= \dfrac{9\pi}{2}\times\pi = \dfrac{9\pi^2}{2}$ sq units　　(1)

Question 7

(d) i) ②

$$\because \theta = \tan^{-1} \frac{4}{3} \qquad \therefore \tan\theta = \frac{4}{3}$$

$$r = \sqrt{3^2 + 4^2} = 5 \qquad \therefore \cos\theta = \frac{3}{5}$$

$$\& \sin\theta = \frac{4}{5}$$

$$\therefore \underline{r}(t) = (V\cos\theta \, t)\,\underline{i} + (-5t^2 + V\sin\theta \, t + h)\,\underline{j}$$

$$= \left[V\left(\frac{3}{5}\right)t \right]\underline{i} + \left[-5t^2 + V\left(\frac{4}{5}\right)t + 2 \right]\underline{j}$$

$$= \left(\frac{3V}{5}t\right)\underline{i} + \left(-5t^2 + \frac{4}{5}Vt + 2\right)\underline{j}$$

let $x = \frac{3V}{5}t \qquad \& \quad y = -5t^2 + \frac{4}{5}Vt + 2$ ①

when $x = 15$, $y = 17$

$$\therefore 15 = \frac{3V}{5}t \qquad \therefore t = \frac{25}{V} \quad \text{Sub into } y$$

$$\therefore 17 = -5\left(\frac{25}{V}\right)^2 + \frac{4}{5}V\left(\frac{25}{V}\right) + 2$$

$$17 = -5\left(\frac{625}{V^2}\right) + 20 + 2$$

$$\therefore 5\left(\frac{625}{V^2}\right) = 5 \qquad \therefore \frac{625}{V^2} = 1$$

$$\therefore V^2 = 625 \qquad \therefore V = \sqrt{625} = 25$$

$$\therefore \text{initial velocity} = 25 \text{ ms}^{-1} \qquad ①$$

Question 7

(d) ii) At the instant it clears the wall

(2) $x = \frac{3v}{5}t$ with $x=15$ & $v=25$

$\therefore 15 = \frac{3 \times 25}{5}t$ $\therefore t = 1$ (1)

$\because x = \frac{3v}{5}t = \frac{3 \times 25}{5}t = 15t$ $\therefore \dot{x} = 15$

& $y = -5t^2 + \frac{4}{5}vt + 2 = -5t^2 + \frac{4}{5} \times 25 + 2$

$\therefore y = -5t^2 + 20t + 2$

$\therefore \dot{y} = -5(2t) + 20$

$= -10t + 20$

when $t = 1$

$\dot{x} = 15$ & $\dot{y} = -10(1) + 20 = 10$

$\therefore$ speed $= \sqrt{15^2 + 10^2} = \sqrt{325}$

$= 18.03 \ ms^{-1}$ (1)