Student Number

2022 HSC ASSESSMENT TASK 3

Mathematics Extension 1

Year 12

General Instructions

- Working time – 45 minutes
- Weighting 25%
- Write using black or blue pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the end of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:

35

Section I – 5 marks

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – 30 marks

- Attempt Questions 6 – 8
- Allow about 37 minutes for this section
- Start each question in separate booklet

Section	Marks
Section I	**/5**
Section II	**/30**
Total marks	**/35**

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Section I

5 marks

Attempt Questions 1- 5

Allow about 8 minutes to complete this section

Use the multiple-choice answer sheet for Questions 1-5.

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1 If $\sin\alpha = \frac{3}{5}$ and $\sin\beta = \frac{5}{13}$, and α and β are acute then $\cos(\alpha+\beta)$ =

A. $\frac{14}{65}$

B. $\frac{64}{65}$

C. $\frac{33}{65}$

D. $\frac{60}{65}$

2 If $t = \tan\frac{x}{2}$, then $\sec x - \tan x =$

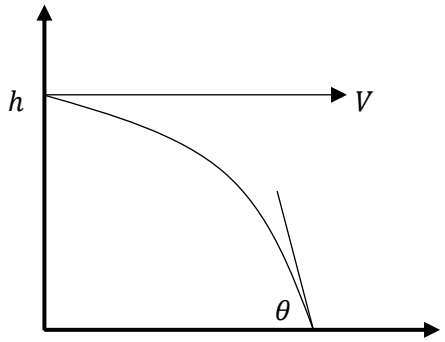
A. $\frac{1+t}{1-t}$

B. $\frac{1-t}{1+t}$

C. $\frac{(1-t)^2}{(1+t)^2}$

D. $\frac{(1+t)^2}{(1-t)^2}$

3 The diagram shows the path of a projectile launched with a horizontal velocity V from a cliff of height h.



Which of the following pairs of values of V and h give the greatest value of θ?

A. $V = 10, h = 30$

B. $V = 30, h = 50$

C. $V = 50, h = 10$

D. $V = 10, h = 50$

4. What is $\frac{d}{dx}[cos(\ln x)]$?

A. $-\sin(\ln x)$

B. $\frac{\cos(\ln x)}{x}$

C. $\sin(\ln x)$

D. $\frac{-\sin(\ln x)}{x}$

5. The derivative of $\sin^{-1}\left(x\sqrt{x}\right)$ is

A. $\frac{3\sqrt{x}}{2\sqrt{1-x^3}}$

B. $\cos^{-1}\left(x\sqrt{x}\right)$

C. $\frac{-3\sqrt{x}}{\sqrt{1+x^3}}$

D. None of these

End of Section I

Section II

30 marks

Attempt Questions 6 – 8
Allow about 37 minutes to complete this section
Start each question in *SEPARATE* booklet

Question 6 (10 marks)

a. Solve $4\cos A = \sec A$ for $0° \leq A \leq 360°$. **2**

b. Find the derivative of $y = \tan^{-1}\left(\frac{1}{2}x + 1\right)$. **2**

c. Consider the function $y = 3\sin x - 2\cos x$.

i. Show that the function $y = 3\sin x - 2\cos x$ can be expressed in the form $\sqrt{13}\sin(x - 0.588)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. **2**

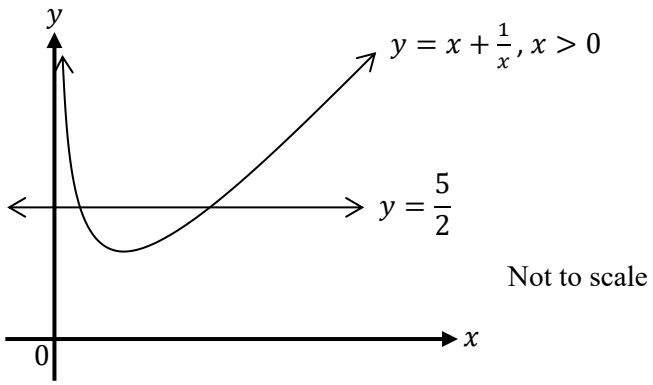
ii. Hence solve the equation $3\sin x - 2\cos x = 1$ for $0 \leq x \leq \frac{\pi}{2}$, giving solution(s) correct to 3 significant figures. **2**

d. Find $\int \cos^2\left(\frac{x}{3}\right) dx$. **2**

Question 7 (10 marks) Start in a *SEPARATE* booklet

a. Evaluate the definite integral $\int_e^{e^2} \frac{1\,dx}{x(\log_e x)^2}$, by using the substitution $u = \log_e x$. **3**

b. The diagram shows the graph of $y = x + \frac{1}{x}$ for $x > 0$.



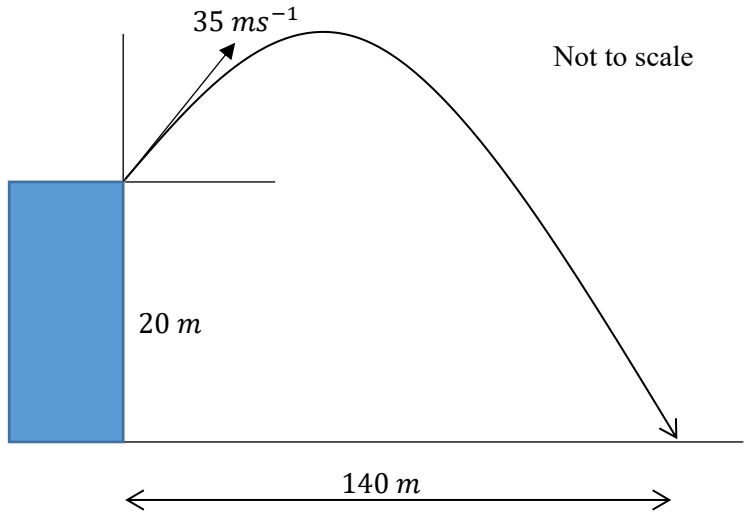
i. Show the x-component of the points where the line $y = \frac{5}{2}$ intersects with this graph are $\frac{1}{2}$ and 2. **1**

ii. Find the exact volume of the solid generated when the region between the curve and the line $y = \frac{5}{2}$ is rotated about the x-axis. **3**

c. Find the exact value of $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1\,dx}{\sqrt{4-x^2}}$. **3**

Question 8 (10 marks) Start in a *SEPARATE* booklet

a. The diagram shows the path of a ball thrown from the top of a $20\ m$ tall building with an initial velocity of $35\ ms^{-1}$. The ball lands on a target $140\ m$ horizontally from the base of the building. Consider gravity to be $10\ ms^{-2}$.



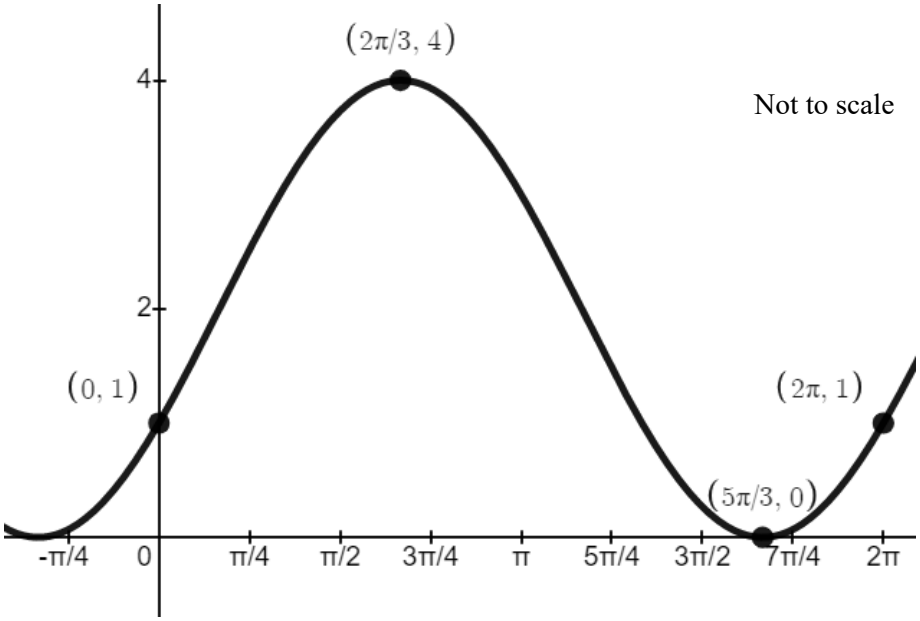
i. If $x = 35t\cos\theta$ and $y = -5t^2 + 35t\sin\theta + 20$ are the equations of motion for this ball (do not prove these results); write the Cartesian equation for path of the ball in the simplest form. **3**

ii. Hence find two possible angles, correct to the nearest degree, of projection for the ball so that it lands on the target. **3**

b. Show that $\tan^{-1}\left(\frac{3}{4}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$. **2**

Question 8 continued over page…

c. Consider the diagram.



Not to scale

Write the equation of this function in the form $y = Rsin(x - \theta) + C$. **2**

End of Assessment

2022 Year 12 – Mathematics Extension 1 Assessment 3

Multiple Choice Answer Sheet

Student Number

Instructions for use:

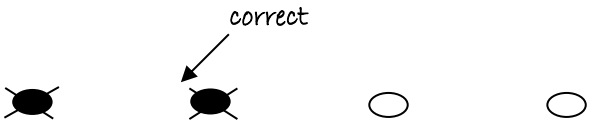
• Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ⬬ C ⬭ D ⬭

• If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

• If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



• Attempt all multiple-choice questions.

Question	**1**	A ○	B ○	C ○	D ○
	2	A ○	B ○	C ○	D ○
	3	A ○	B ○	C ○	D ○
	4	A ○	B ○	C ○	D ○
	5	A ○	B ○	C ○	D ○