STX YEAR 12 MATHEMATICS ADVANCED

2020 ASSESSMENT TASK 2.

SAMPLE SOLUTIONS.

SECTION I - MULTI-CHOICE

1. $\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3} \cdot 3x^2$

$= 3x^2 e^{x^3}$ A

2. $a = 12t + 6$

$v = \frac{12t^2}{2} + 6t + c$

$v = 6t^2 + 6t + c$

When $v = -36$, $t = 0$.

$\therefore -36 = 6(0)^2 + 6\cancel{(0)} + c$

$\therefore c = -36$

$v = 6t^2 + 6t - 36$

When $v = 0$

$6t^2 + 6t - 36 = 0$

$t^2 + t - 6 = 0$

$(t - 2)(t + 3) = 0$

$\therefore t = -3$ or 2

$\because t > 0$.

$\therefore t = 2$ seconds C

3. $a = \frac{b}{c}$ C

4. A.

SECTION II

5a (i) $y = (e^x - 3)^4$

$\frac{dy}{dx} = 4(e^x - 3)^3 \cdot e^x \cdot 1$

$\therefore \frac{dy}{dx} = 4e^x(e^x - 3)^3$

(ii) $f(x) = \tan 5x$

$f'(x) = \sec^2 5x \cdot 5$

$\therefore f'(x) = 5\sec^2 5x$

(iii) $y = \log_e(\cos x)$

$\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x$

$= \frac{-\sin x}{\cos x}$

$\therefore \frac{dy}{dx} = -\tan x$

(iv) $\frac{d}{dx}\left(\frac{\sin x}{(2x+1)}\right) = \frac{(2x+1)\cos x - \sin x \cdot 2}{(2x+1)^2}$

$= \frac{(2x+1)\cos x - 2\sin x}{(2x+1)^2}$

5b. $y = \tan x$

$\frac{dy}{dx} = \sec^2 x$

When $x = \frac{\pi}{16}$

Gradient, $\frac{dy}{dx} = \sec^2\left(\frac{\pi}{16}\right)$

$= \frac{1}{\left(\cos\left(\frac{\pi}{16}\right)\right)^2}$

$\therefore \frac{dy}{dx} = 1.04$ (3SF).

5c. $\int_0^1 (e^{3x} + 1)\,dx = \left[\frac{1}{3}e^{3x} + x\right]_0^1$

$= \left[\left(\frac{e^3}{3} + 1\right) - \left(\frac{e^0}{3} + 0\right)\right]$

$= \frac{e^3}{3} + 1 - \frac{1}{3}$

$= \frac{e^3}{3} + \frac{2}{3}$

$\therefore \int_0^1 (e^{3x} + 1)\,dx = \frac{1}{3}(e^3 + 2)$

5d. (i) After 12 seconds the particle start to slow down.

(ii) $x = (5 \times 12) + \left(\frac{1}{2} \times 5 \times 8\right)$

$= 60 + 20$

$= 80$

$\therefore$ distance travelled is 80 m.

Q6a.

$$\int_0^{\frac{\pi}{8}} \sec^2(2x)\,dx = \left[\tfrac{1}{2}\tan(2x)\right]_0^{\frac{\pi}{8}}$$

$$= \tfrac{1}{2}\left[\tan\left(\tfrac{\pi}{4}\right) - \tan(0)\right]$$

$$= \tfrac{1}{2}\left[1 - 0\right]$$

$$\therefore \int_0^{\frac{\pi}{8}} \sec^2(2x)\,dx = \tfrac{1}{2}$$

6b. $y = x \ln x$

When $x = 1$

$$y = 1 \cdot \ln(1)$$
$$= 0$$

$\therefore P(1, 0)$

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\therefore \frac{dy}{dx} = 1 + \ln x$$

When $x = 1$

$$\frac{dy}{dx} = 1 + \ln(1)$$
$$= 1$$

$\therefore$ normal at $P(1, 0)$ has gradient, $m = -1$.

$$y - 0 = -1(x - 1)$$
$$y = -x + 1$$

$\therefore$ normal at $P(1, 0)$ to $y = x \ln x$ is $y = -x + 1$.

6c. $\int \frac{6x}{x^2+6}\,dx = 3\int \frac{2x}{x^2+6}\,dx$

$= 3\log_e(x^2+6)+c$

6d. $V = 30\,000\,L$

$t = 0$ seconds.

(i) $\frac{dV}{dt} = -900 + 18t$

Water stops flowing $\frac{dV}{dt} = 0$.

$\therefore -900 + 18t = 0$

$18t = 900$

$t = \frac{900}{18}$

$\therefore t = 50$ seconds.

Water stops flowing after 50 seconds.

(ii) $\frac{dV}{dt} = 18t - 900$

$V = \frac{18t^2}{2} - 900t + c$

$V = 9t^2 - 900t + c$

When $t = 0$, $V = 30\,000$

$\therefore 30\,000 = 9\cancel{(0)}^2 - 900\cancel{(0)} + c$

$c = 30\,000$

$\therefore V = 9t^2 - 900t + 30\,000$

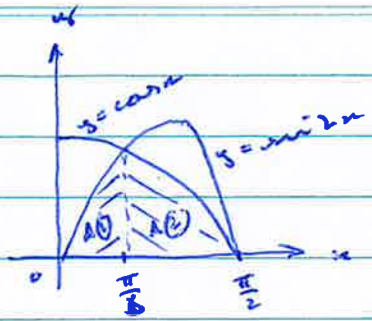
(iii) When $t = 50$

$V = 9(50)^2 - 900(50) + 30\,000$

$= 22\,500 - 45\,000 + 30\,000$

$\therefore V = 7500\,L$.

7 a.



$$A(1) = \int_0^{\frac{\pi}{6}} \sin 2x \, dx$$

$$= \left[-\frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{6}}$$

$$= -\frac{1}{2} \left[\cos\left(\frac{2\pi}{6}\right) - \cos(0) \right]$$

$$= -\frac{1}{2} \left[\cos\left(\frac{\pi}{3}\right) - \cos(0) \right]$$

$$= -\frac{1}{2} \left[\frac{1}{2} - 1 \right]$$

$$= -\frac{1}{2} \left[-\frac{1}{2} \right]$$

$$\therefore A(1) = \frac{1}{4} \, u^2$$

$$A(2) = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx$$

$$= \left[\sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right) \right]$$

$$= 1 - \frac{1}{2}$$

$$\therefore A(2) = \frac{1}{2} \, u^2$$

$\therefore$ Required Area, $A = \frac{1}{4} + \frac{1}{2}$

$$= \frac{1}{4} + \frac{2}{4}$$

$$\therefore A = \frac{3}{4} \, u^2 .$$

7b. (i) $x = t + \log_e(3t+1)$, $t \geq 0$.

$$\frac{dx}{dt} = 1 + \frac{1}{(3t+1)} \cdot 3$$

$$\therefore \frac{dx}{dt} = 1 + \frac{3}{(3t+1)}$$

As $t \rightarrow \infty+$

$$\frac{3}{(3t+1)} \rightarrow 0$$

$$\therefore \frac{dx}{dt} \rightarrow 1$$

i.e. $\frac{dx}{dt}$ can never be zero, never come to rest.

(ii) When $t = 3$

$$x_3 = 3 + \log_e(9+1)$$

$$\therefore x_3 = 3 + \log_e 10$$

i.e. $x_3 = 5.3$ cm to the right of the origin.

(iii) $$\frac{d^2x}{dt^2} = \frac{\cancel{(3t+1)} \cdot 0 - 3(3)}{(3t+1)^2}$$

$$\therefore \frac{d^2x}{dt^2} = \frac{-9}{(3t+1)^2}$$

(iv) $\frac{d^2x}{dt^2} = -1 \times \frac{9}{(3t+1)^2}$ which is always negative.

$\therefore$ particle is slowing for all ~~$t \geq$~~ $t \geq 0$ seconds.

7c. (i) $x = e^{-2t} + 3e^{-t} + 2t$

$$\frac{dx}{dt} = e^{-2t} \cdot -2 + 3e^{-t} \cdot -1 + 2$$

$$\therefore \frac{dx}{dt} = -2e^{-2t} - 3e^{-t} + 2$$

(ii) When $\frac{dx}{dt} = 0$

$$-2e^{-2t} - 3e^{-t} + 2 = 0$$

$$2e^{-2t} + 3e^{-t} - 2 = 0$$

$$2(e^{-t})^2 + 3(e^{-t}) - 2 = 0.$$

Let $k = e^{-t}$

$$\therefore 2k^2 + 3k - 2 = 0$$

$$(2k-1)(k+2) = 0.$$

$$\therefore k = (-2) \text{ or } \frac{1}{2}$$

When $e^{-t} \neq -2$ NO SOLUTION.

When $e^{-t} = \frac{1}{2}$

$$\frac{1}{e^t} = \frac{1}{2}$$

$$e^t = 2$$

$$\log_e 2 = t.$$

$\therefore$ particle comes to rest when $t = \log_e 2$ seconds.