ST PIUS X COLLEGE
CHATSWOOD

35 Anderson St Chatswood

2021 HSC Task #1 weighting 20%

9 February 2021

SUG SOLS.

please check these solutions

Extension 1 Mathematics

General Instructions

- **Working time 45 minutes**
- **Write using blue or black pen**
- **Show all relevant mathematical reasoning and calculations**
- **NESA approved calculators may be used**

Total marks – 30

- **Attempt sections I and II**
- **Section I 5 marks**
- **Section II 25 marks**
- **Use the multiple choice sheet for section I and answer each of question 6 and 7 in a separate booklet**

Student Number	
Teacher's name	
Multiple Choice	/5
Question 6	/12
Question 7	/13
Total	/30

Multiple Choice Answer Sheet

Student Number	
Teacher's name	

Colour your Choice for each section

	A	B	C	D
1	○	○	●	○
2	○	●	○	○
3	○	○	○	●
4	●	○	○	●
5	○	○	○	●

6 a) $7^{2n-1}+5$ is divisible by 12.

$\boxed{A}$ If $n=1$ $7^1+5=12$ so the result holds $n=1$

$\boxed{B}$ If $k \geq 1$ is a number for which the result holds

ie $7^{2k-1}+5 = 12M$ $M \in \mathbb{N}$

Show the result follows for $n=k+1$.

ie $7^{2k+1}+5 = 12N$ $N \in \mathbb{N}$

$$LHS = 49 \times (12M-5)+5 \quad \text{by I H.}$$
$$= 588M - 240.$$
$$= 12(49M-20)$$
$$N = 49M - 20 \in \mathbb{N}.$$

$\boxed{C}$ from A and B the result follows by the principle of Mathematical Induction.

6. b) $y = x^3 + x^2 - x + 2$

$y' = 3x^2 + 2x - 1 = (3x-1)(x+1)$

$y' = 0$ if $x = \frac{1}{3}$ or $x = -1$.

$y = 1\frac{22}{27}$ $\quad y = 3.$

so $(\frac{1}{3}\ 1\frac{22}{27})$ and $(-1,3)$ are the turning points.

$y'' = 6x+2.$ $\quad f''(-1) < 0$ $\quad (1,3)$ is MAX.

$f''\ \frac{1}{3} > 0$ $\quad (\frac{1}{3}\ 1\frac{22}{27})$ is MIN.

6b) ii) $f''(x) = 6x + 2.$

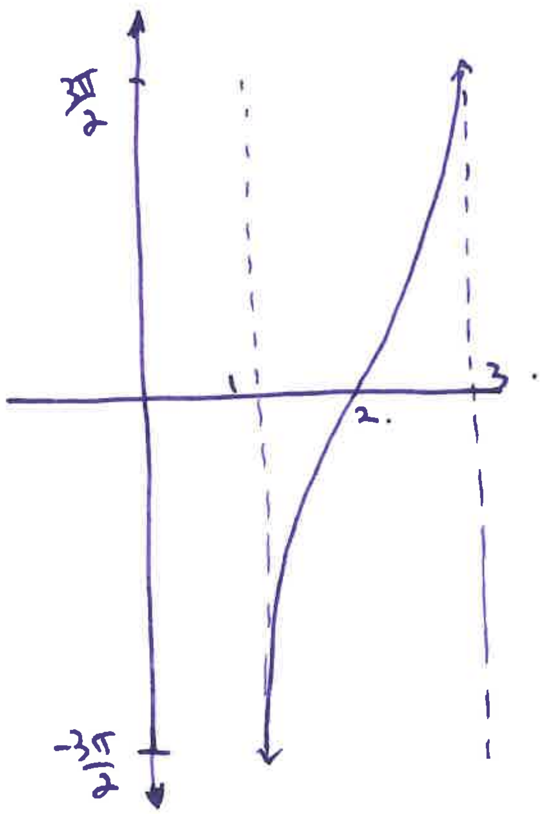
if $f''x > 0$ $x > -\frac{1}{3}.$

concave up means second derivative $> 0.$

6c) $\frac{y}{3} = \sin^{-1}(x-2)$

$-1 \le (x-2) \le 1$ $1 \le x \le 3$

$\frac{-\pi}{2} \le \frac{y}{3} \le \frac{\pi}{2}.$ $\frac{-3\pi}{2} \le y \le \frac{3\pi}{2}$



$\boxed{12}$

2021 E1 AT1 9-2-2021 $\boxed{P3}$

7a) RTP

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$

$\boxed{A}$ if $n=1$

$$LHS = \frac{1}{3} \qquad RHS = \frac{1}{2+1} = \frac{1}{3} = LHS.$$

so the result holds when $n=1$

$\boxed{B}$ Let $k \geq 1$ be a number for which the result holds.

ie $$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$

use this to prove result holds for $n=k+1$.

ie $$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}.$$

$$LHS = \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} \quad \text{by IH.}$$

$$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}.$$

$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+3} \quad \text{as required.}$$

$\boxed{C}$ It follows from parts A & B and by the principle of mathematical induction that the result holds for $n \geq 1$.

2021 E1 AT1 9-2-2021 $\boxed{P4}$

7b) (i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

$\sin(90-A) = \sin 90\cos A + \cos 90 \sin A$.

$= \cos A + 0$.

as required.

ii) $\cos(A+B) + \cos(A-B)$

$= \cos A\cos B - \sin A\sin B + \cos A\cos B + \sin A\sin B$

$= 2\cos A\cos B$

$\cos(30^\circ + 45^\circ) + \cos(45^\circ - 30^\circ) = \cos 75 + \cos 15$.

$= 2\cos 45\cos 30 = 2\times\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}$.

7c). $3\sin x - 2\cos x = 1$.

$3\times\frac{2t}{1+t^2} - \frac{2(1-t^2)}{1+t^2} = 1$.

$6t - 2 + 2t^2 = 1 + t^2$

$t^2 + 6t - 3 = 0$

$\tan\frac{x}{2} = \frac{-6 \pm \sqrt{36+12}}{2}$. $= .46410....$

OR. $-6.4641...$ not reqd.

$\frac{x}{2} = 24.896^\circ$

$x = 49^\circ 48'$

2021 E1 AT1 9-2-2021 $\boxed{P5}$

$4x + y = 100$. $y = 100 - 4x$

$V = x^2(100-4x) = 100x^2 - 4x^3$

$\frac{dV}{dx} = 200x - 12x^2 = 4x(50-3x)$.

$= 0$ if $x = 0$ or $x = \frac{50}{3}$.

$\frac{d^2V}{dx^2} = 200 - 24x < 0$ if $x = \frac{50}{3}$.

so $x = \frac{50}{3}$ yeilds a maximum volume

$V = 100\left(\frac{50}{3}\right)^2 - 4 \times \left(\frac{50}{3}\right)^3 = 9259.259\ldots \text{ cm}^3$.

$\boxed{13}$