
Student Number

ST PIUS X COLLEGE
CHATSWOOD

HSC 2021 Stage 6
Year 12

Assessment Task #1

20% of School Based Assessment

MATHEMATICS ADVANCED

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this cover page

Total Marks – 35

Section I – Multiple Choice 5 marks

- Attempt Questions 1 – 5
- Enter responses on the multiple choice answer sheet
- Allow 5 minutes for this section

Section II – 30 marks

- Attempt Questions 6 – 8
- Answer in the writing spaces provided
- Show all necessary working
- Allow 40 minutes for this section

Section I – Multiple-Choice 1 mark per question **5 Marks**

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

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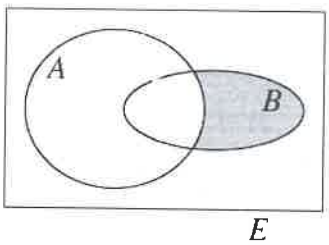
1. Using the Venn diagram below, which of the following is shown?



(A) $\bar{A}$
(B) $\bar{A} \cup B$
(C) $\bar{A} \cap B$
(D) $\bar{B}$

2. The two events A and B in the following experiments are known to be independent.

$P(A) = 0.4$ and $P(B) = 0.6$. Find $P(A \cup B)$.

(A) 0.76
(B) 0.6
(C) 0.24
(D) 0.1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.6 - (0.4 \times 0.6)$

$= 1 - 0.24$

$= 0.76$

3. State whether the following probability distribution is numeric or categorical. If it is numeric, state whether it is *discrete* or *continuous*. Note: The rainfall has NOT been rounded to the nearest *cm*.

"The daily rainfall in Chatswood on a given day in February."

(A) Categorical

(B) Numerical Discrete

(C) Numerical Continuous

(D) Cannot be determined

4. A certain arithmetic series has a first term of 15 and a common difference of –7. What is the eleventh term, T_{11}?

$a = 15 \quad d = -7 \quad n = 11$

$T_n = a + d(n-1)$

$= 15 + -7(10)$

$= 15 + -75$

$= -55$

(A) 45

(B) 55

(C) -45

(D) -55

5. Which of the following is NOT a term in the geometric series with first term $a = 4$ and a common ratio of $r = -3$?

$T_n = ar^{n-1}$

$= 4 \times (-3)^{n-1}$

CONSIDER $n = 10$

$T_{10} = 4 \times (-3)^9$

$= -78732$ ✓

$T_{11} = 4 \times (-3)^{10}$

$= 236196$ ✓

$T_{12} = 4 \times (-3)^{11}$

$= -708588$

$\therefore -236196$ is not in the GP.

(A) -78732

(B) 236196

(C) -236196

(D) -708588

End of Multiple-Choice Section 1.

Section II **30 Marks**

Attempt Questions 6 to 8.
Allow about 40 minutes for this section.

In Questions 6 to 8 your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) Suppose that the birth of boys or girls are equally likely. In a family of 3 children, determine the probability that there are:

1

(i) Three girls.

$P(\text{Girl}) = \frac{1}{2}$

$\therefore \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{8}}$

2

(ii) More boys than girls.

GBB BBG BGB BBB (1MK)

each outcome has a posibility of $\frac{1}{8}$

Thus, $4 \times \frac{1}{8} = \boxed{\frac{1}{2}}$ (1MK)

(b) Let $A = \{1, 3, 6, 8\}$ and $B = \{3, 4, 6, 7, 10\}$, and take the universal set to be the set $E = \{1, 2, 3, \ldots, 10\}$. List the members of:

(i) $A \cup B$ — ALL A AND B. **1**

$A \cup B = \{1, 3, 4, 6, 7, 8, 10\}$

(ii) $\bar{B}$ — NOT B. **1**

$\bar{B} = \{1, 2, 5, 8, 9\}$

(iii) $\overline{A \cap B}$. — NOT WITHIN $A \cap B$ **1**

$\overline{A \cap B} = \{1, 2, 4, 5, 7, 8, 9, 10\}$

(c) Use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to answer the following questions:

(i) If $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{15}$, find $P(A \cup B)$. **1**

$P(A \cup B) = \frac{1}{5} + \frac{1}{3} - \frac{1}{15}$

$= \boxed{\frac{7}{15}}$

(ii) If A and B are mutually exclusive and If $P(A) = \frac{1}{7}$ and $P(B) = \frac{4}{7}$, find $P(A \cup B)$. **1**

$P(A \cup B) = \left(\frac{1}{7} + \frac{4}{7}\right) - 0$

$= \boxed{\frac{5}{7}}$

Question 6 continues over page.

(d) A six-sided die is rolled twice. Using the product rule, find the probability of rolling a one and a four in any order. **2**

$= P(1 \text{ and a } 4) + P(4 \text{ and a } 1)$

$= \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right)$ ✓

$= \frac{1}{36} + \frac{1}{36}$

$= \frac{1}{18}$ ✓

Question 7 on next page.

Question 7 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) Find the expected value, E(x), for the following distribution. **2**

x_i	−3	1	2	5	6
p_i	0.1	0.3	0.2	0.3	0.1

$E.V = (-3 \times 0.1) + (1 \times 0.3) + (2 \times 0.2) + (5 \times 0.3)$ ✓
$+ (6 \times 0.1)$
$= \boxed{2.5}$ ✓

(b) A random variable X is known to have the property that $E(X) = 4$.
Use the formula: $\mathbf{E(aX + b) = aE(X) + b}$ to calculate:

$E(10 - 2X)$. **1**

$E(10-2x) = E(-2x+10)$
$= -2E(x) + 10$
$= -2(4) + 10$
$= \boxed{2}$ ✓

(c) Consider the following distribution table:

x	7	8	9	10	11	12
$p(x)$	0.25	0.1	0.25	0.3	−0.2	0.25

Give TWO reasons why this distribution cannot be considered as a valid probability distribution: **2**

- P(x) must add to 1. ✓
- You cannot have a negative probability. ✓

Question 7 continues over page.

(d) Consider the following probability distribution table below:

x	1	2	3	4	Sum
$p(x)$	0.3	0.5	0.1	0.1	1
$xp(x)$	0.3	1	0.3	0.4	2
x^2	1	4	9	16	—
$x^2p(x)$	0.3	2	0.9	1.6	4.8

(i) Complete the table by filling in the missing 15 entries: **2**

(ii) Find the variance, Var(X). **2**

$\text{Var} = E(x^2) - (E(x))^2$

$= \mu^2$

$4.8 - (2)^2 = \boxed{0.8}$

(iii) Find the standard deviation, σ, correct to 1 decimal place. **1**

$\sigma = \sqrt{0.8}$

$= \boxed{0.9}$

Question 8 on next page.

Question 8 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) Use the formula $T_n = a + (n-1)d$ to find the number of terms in the following finite sequence. **2**

2, 5, 8, . . . , 2000

$a = 2 \quad d = 3 \quad T_n = 2 + (n-1)3$

$= 2 + 3n - 3$

$T_n = 3n - 1$ ✓

then, $T_n = 2000$

$3n - 1 = 2000$

$3n = 2001$

$n = 677$ terms ✓

(b) Use the formula: $T_n = ar^{n-1}$ to find the common ratio r of a GP for which: **2**

$a = 5$ and $T_7 = 40$

$T_n = ar^{n-1}$

$40 = 5 \times r^6$

$8 = r^6$ ✓

$2^3 = r^6$

$r = \sqrt{2}$ OR $-\sqrt{2}$ ✓

Must have both for 2nd mark.

Question 8 continues over page

(c) Find the following sum by any appropriate method. **2**

$$1000 + 1001 + 1002 + \ldots + 3000.$$

Using $S_n = \frac{1}{2}n(a+l)$

$3000 = 1000 + (n-1)1$

$2000 = n - 1$

$n = 2001$

$S_{2001} = 1000.5(1000 + 3000)$

$= 1000.5(4000)$

$= 4,002,000$

(d) Consider the geometric sequence below:

$$25, 5, 1, \ldots$$

(i) Find a formula for the n^{th} term of the sequence. **1**

$\frac{T_2}{T_1} = \frac{5}{25}$ so, $a = 25$

$= \frac{1}{5}$ $r = \frac{1}{5}$

and $T_n = ar^{n-1}$ $\rightarrow$ $T_n = 25\left(\frac{1}{5}\right)^{n-1}$ ✓

(ii) From part (i) and considering logarithms, how many terms in the sequence exceed 10^{-8}? **3**

$T_n > 10^{-8}$

$25\left(\frac{1}{5}\right)^{n-1} > 10^{-8}$ ✓

$\left(\frac{1}{5}\right)^{n-1} > \frac{10^{-8}}{25}$

$\ln\left(\frac{1}{5}\right)^{n-1} \ln \frac{10^{-8}}{25}$

$n - 1 < \frac{\ln \frac{10^{-8}}{25}}{\ln\left(\frac{1}{5}\right)}$ ✓

$n - 1 < 13.445\ldots$

$n < 14.445\ldots$

$\therefore n =$ the first 14 terms. ✓

End of Task