Student Number

SOLUTIONS

ST PIUS X COLLEGE
CHATSWOOD

HSC 2022 Stage 6
Year 12

Assessment Task #1

20% of School Based Assessment

MATHEMATICS ADVANCED

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this cover page

Total Marks – 35

Section I – Multiple Choice 5 marks.

- Attempt Questions 1 – 5
- Enter responses on the multiple choice answer sheet
- Allow 5 minutes for this section

Section II – 30 marks

- Attempt Questions 6 – 8
- Answer in the writing spaces provided
- Show all necessary working
- Allow 40 minutes for this section

Section I – Multiple-Choice 1 mark per question **5 Marks**

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

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1. What are the vertical asymptotes of the graph below?

$$y = \frac{1}{x^2 - 9}$$

(A) $x = 3$ and $x = -3$
(B) $x = 9$ and $x = -9$
(C) $y = 3$ and $y = -3$
(D) $y = 9$ and $y = -9$

A

2. What is the value of S_{99} for the Arithmetic Progression given by $T_n = 7n - 4$?

(A) 689
(B) 19 404
(C) 34 254
(D) 34 650

C

$T_{99} = 7(99) - 4$
$= 693 - 4$
$= 689$

$T_1 = 7(1) - 4$
$= 7 - 4$
$= 3$

$S_{99} = \frac{99}{2}(3 + 689)$
$= 49.5(692)$
$= 34254$

3.

What is the change in the amplitude and period when the function $f(x) = \frac{1}{2}\cos 4x$ is transformed into $g(x) = \cos 2x$?

(A) Amplitude is halved and period is halved

(B) Amplitude is halved and period is doubled

(C) Amplitude is doubled and period is halved

✗ (D) Amplitude is doubled and period is doubled

D

4. The first three terms of an AP are 5, 9 and 13. What is the 15th term in the series?

✗ (A) 61

(B) 66

(C) 495

(D) 585

A

$T_n = a + (n-1)d$

$= 5 + 14(4)$

$= 5 + 56$

$= 61$

5. Which of the following is the correct value S_n for the series given below?

5 + 15 + 45 + ... + 98 415

(A) 125 640

✗ (B) 147 620

(C) 155 600

(D) 162 350

B

$a = 5$ and $r = 3$

$\therefore 98415 = ar^{n-1}$

$= 5 \times 3^{n-1}$

$3^{n-1} = 19683$

$= 3^9$

$n - 1 = 9$

$n = 10$

End of Multiple-Choice Section I

$S_{10} = \frac{5(3^{10}-1)}{3-1}$

$= \frac{5}{2}(3^{10}-1)$

$= 2.5 \times 59048$

$= 147620$

Section II **30 Marks**

Attempt Questions 6 to 8.
Allow about 40 minutes for this section.

In Questions 6 to 8 your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks)

Write your solutions in the spaces provided **Marks**

(a) An arithmetic sequence has a first term of 5 and a common difference of d. The sum of the first 20 terms is 4 times the sum of the first 10 terms. Find d. **3**

$S_{20} = 4S_{10}$

$\frac{20}{2}(2(5)+19d) = 4 \times \frac{10}{2}(2(5)+9d)$ ✓

$10(10+19d) = 20(10+9d)$ ✓

$100 + 190d = 200 + 180d$

$10d = 100$

$d = 10$ ✓

(b) The second and 5th terms of a geometric progression are 750 and -6 respectively.

(i) Find the common ratio of the series. **2**

$T_2 = ar^{n-1}$ $\quad T_5 = ar^4$

$750 = ar$ $\quad -6 = ar^4$ ✓

$\therefore \frac{ar^4}{ar} = \frac{-6}{750}$

$r^3 = -\frac{1}{125}$ $\quad r = \sqrt[3]{\frac{-1}{125}}$

$= -\frac{1}{5}$ ✓

(ii) What is the limiting sum of the series? 2

$S_\infty = \frac{a}{1-r}$

$= \frac{-3750}{6/5}$

$= -3125$ ✓

$ar = 750$

$a \cdot -\frac{1}{5} = 750$

$a = -3750$ ✓

(c) Farmer Fred plants his cabbages in rows. The first row has 35 cabbages. The second row has 39 cabbages. Each subsequent row contains 4 more cabbages than the previous row, as shown in the diagram below:

(i) How many cabbages will be in the 7th row? 1

$T_n = a + (n-1)d$

$= 35 + 6(4)$ ✓

$= 59$

(ii) Which row will be the first to feature more than 80 cabbages? 2

$35 + (n-1)4 > 80$ ✓

$35 + 4n - 4 > 80$

$31 + 4n > 80$

$\frac{4n}{4} > \frac{49}{4}$ ∴ Row 13 ✓

$n > 12.25$

(iii) Farmer Fred is only going to plant 945 cabbages in total. How many rows will he need? 3

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$945 = \frac{n}{2}\{70 + 4n - 4\}$ ✓

$1890 = n(66 + 4n)$

$1890 = 4n^2 + 66n$

$0 = 4n^2 + 66n - 1890$ 7560

$(4n^2 - 60n) + (126n - 1890)$ ✓

$4n(n - 15) + 126(n - 15)$

$(4n + 126)(n - 15)$

n cannot be -ve

$\therefore n = 15$ rows needed ✓

Question 7 on next page

Question 7 (9 marks)

Write your solutions in the spaces provided **Marks**

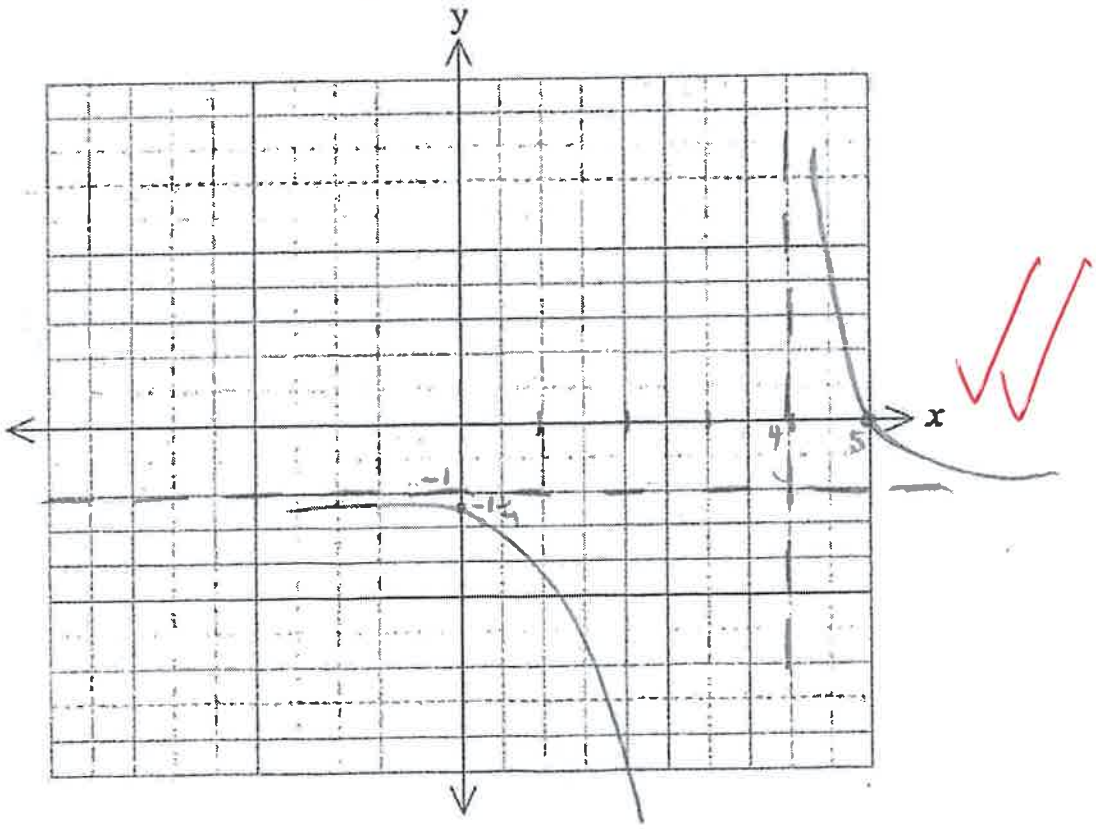
(a) For what values of x is $|3x - 2| = 4$? 2

$3x - 2 = 4$ $\quad$ $3x - 2 = -4$

$3x = 6$ ✓ $\quad$ $3x = -2$

$x = 2$ $\quad$ $x = -\frac{2}{3}$ ✓

(b) (i) Sketch the hyperbola $y = \frac{1}{x-1}$ after shifting it horizontally by 3 units to the right and 1 unit down. 2



✓✓

(ii) State the equation of the shifted hyperbola, then find all the intercepts of the shifted hyperbola with the axes and mark them on the grapy in part (i). **2**

$y = \frac{1}{x-1-3} - 1$

$\therefore y = \frac{1}{x-4} - 1$ ✓

at $x = 0$ $\quad y = \frac{1}{0-4} - 1$

$= -\frac{1}{4} - 1$

$y = -1\frac{1}{4}$ $\left(-\frac{5}{4}\right)$ ✓

at $y = 0$ $\quad 0 = \frac{1}{x-4} - 1$

$1 = \frac{1}{x-4}$

$x - 4 = 1$

$x = 5$

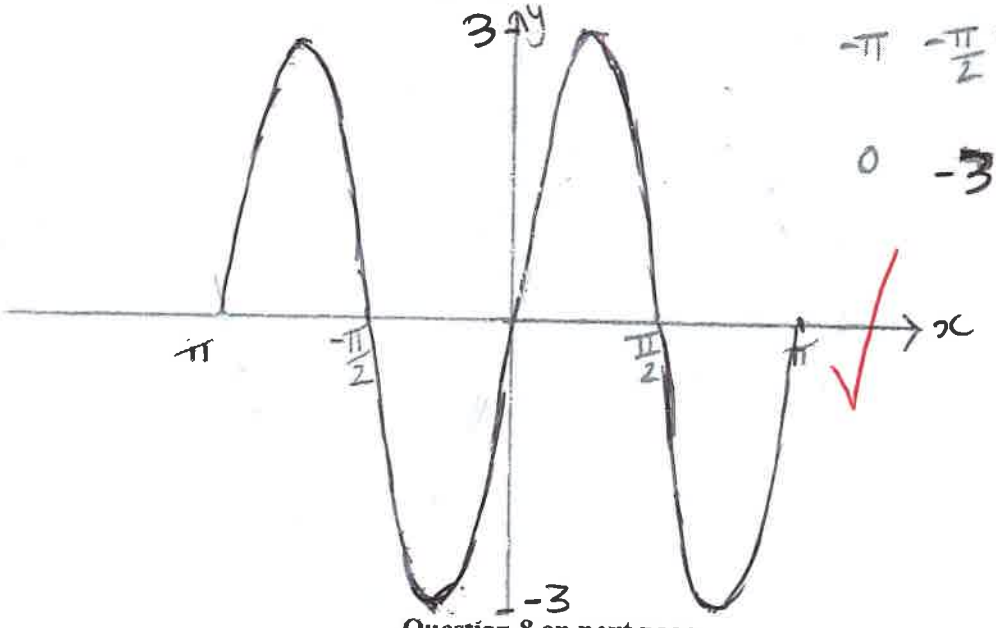
(c) (i) What is the period and amplitude of the function $y = 3\sin 2x$? **1**

amplitude = 3 ✓

period = π

(ii) Sketch the function for $-\pi \leq x \leq \pi$. **2**

$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
0	-3	0	3	0



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Question 8 on next page

Question 8 (8 marks)

Write your solutions in the spaces provided Marks

The probability of a discrete random variable X is shown below:

X	-1	0	1	2	
$P(X=x)$	$\frac{1}{4}$ t^2	$\frac{1}{4}$ t^2	$\frac{1}{8}$ $\frac{t}{4}$	$\frac{3}{8}$ $\frac{(4t+1)}{8}$	$= 1$
$xP(x)$	$-\frac{1}{4}$	0	$\frac{1}{8}$	$\frac{3}{4}$	$= 0.625$
$x^2P(x)$	$\frac{1}{4}$	0	$\frac{1}{8}$	$\frac{3}{2}$	$= \frac{15}{8}$

(a) By solving a suitable quadratic equation, show that $t = \frac{1}{2}$. 3

$t^2 + t^2 + \frac{t}{4} + \frac{4t+1}{8} = 1$ ✓

$16t^2 + 6t + 1 = 8$ ✓

$(2t-1)(8t+7) = 0$

$\therefore t = \frac{1}{2}$ or $-\frac{7}{8}$ $t > 0$

$\therefore \boxed{t = \frac{1}{2}}$ ✓

(b) Hence find the expected value, $E(X)$, of the probability distribution. 2

$\sum x P(x) = -\frac{1}{4} + 0 + \frac{1}{8} + \frac{3}{4}$ ✓

$= \frac{5}{8}$ (0.625) ✓

(c) Find the variance and standard deviation of the probability distribution. 3
Express each value correct to 2 decimal places.

$\text{Var}(X) = \sum x^2 p(x) - \mu^2$ ✓ $\mu = \frac{5}{8}$

$\therefore = \frac{15}{8} - \left(\frac{5}{8}\right)^2$

$= \frac{95}{64} \doteq 1.48$ ✓

Standard Deviation $\sigma = \sqrt{\text{Var}(X)}$

$= \sqrt{1.48}$

$\doteq 1.22$ ✓

End of Task