



--	--	--	--	--	--	--	--	--

Student Number

**ST PIUS X COLLEGE  
CHATSWOOD**

**HSC 2020 Stage 6  
Year 12**

**ASSESSMENT TASK #2**

30% of School Based Assessment

# MATHEMATICS EXTENSION 1

## General Instructions

- Working time – 1 hour
- Write using black or blue pen  
Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of all pages

## Total Marks – 30

### Section I – Multiple Choice 4 marks

- Attempt Questions 1 – 4
- Enter responses on the multiple choice answer sheet
- Allow about 10 minutes for this section

### Section II – 26 marks

- Attempt Questions 5 – 6
- Show all necessary working
- **Start each question in a SEPARATE booklet**
- Allow about 50 minutes for this section

B L A N K   P A G E

Use the multiple choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:  $2 + 4 =$  (A) 2 (B) 6 (C) 8 (D) 9  
A ☐ B ☒ C ☐ D ☐

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

☒ ☒ ☐ ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

☒ ☒ ☐ ☐  
correct

1) If  $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$ , which of the following is  $f'(x)$ ?

(A)  $\frac{x^2}{1+x^2}$

(B)  $-\frac{1}{1+x^2}$

(C)  $\frac{1}{1-x^2}$

(D)  $-\frac{x^2}{1-x^2}$

2) Which of the following is equivalent to  $\sqrt{3} \cos \theta - \sin \theta$ ?

(A)  $2 \cos\left(\theta + \frac{\pi}{3}\right)$

(B)  $2 \cos\left(\theta - \frac{\pi}{3}\right)$

(C)  $2 \cos\left(\theta + \frac{\pi}{6}\right)$

(D)  $2 \cos\left(\theta - \frac{\pi}{6}\right)$

3) Which expression is equivalent to  $\int \sin^2 3x \, dx$  ?

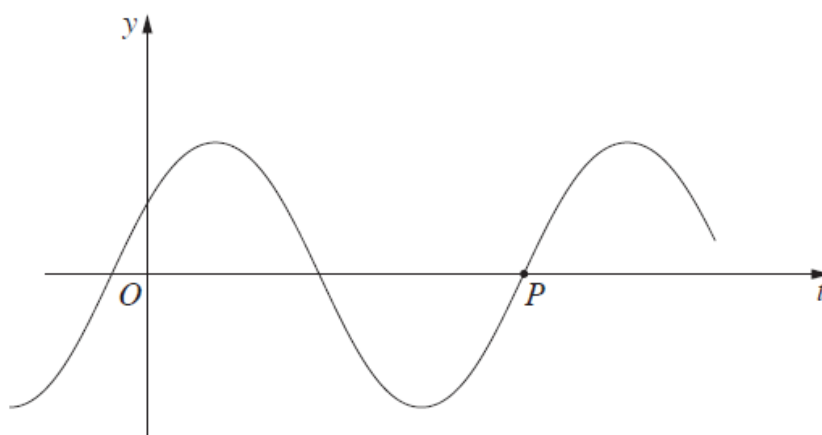
(A)  $\frac{1}{2}\left(x + \frac{1}{3} \sin 3x\right) + C$

(B)  $\frac{1}{2}\left(x - \frac{1}{3} \sin 3x\right) + C$

(C)  $\frac{1}{2}\left(x + \frac{1}{6} \sin 6x\right) + C$

(D)  $\frac{1}{2}\left(x - \frac{1}{6} \sin 6x\right) + C$

4) The graph of the function  $y = \cos\left(2t - \frac{\pi}{3}\right)$  is shown below.



What are the coordinates of the point  $P$ ?

(A)  $\left(\frac{5\pi}{12}, 0\right)$

(B)  $\left(\frac{2\pi}{3}, 0\right)$

(C)  $\left(\frac{11\pi}{12}, 0\right)$

(D)  $\left(\frac{7\pi}{6}, 0\right)$

**End of Section I**

## SECTION II

### Question 5 (13 marks)

*Start this question in a SEPARATE answer booklet*

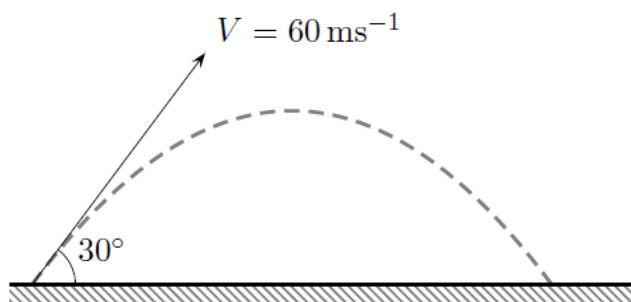
**Marks**

- a) If  $t = \tan \frac{\theta}{2}$ , write an expression in terms of  $t$  for  $\sec \theta + 2 \tan \theta$ . **1**

- b) Find the following: **1**

$$\int \frac{1}{\sqrt{9-x^2}} dx$$

- c) A projectile is launched from the origin, across a level plain at  $30^\circ$  to the horizontal and at an initial speed of 60 metres/second.



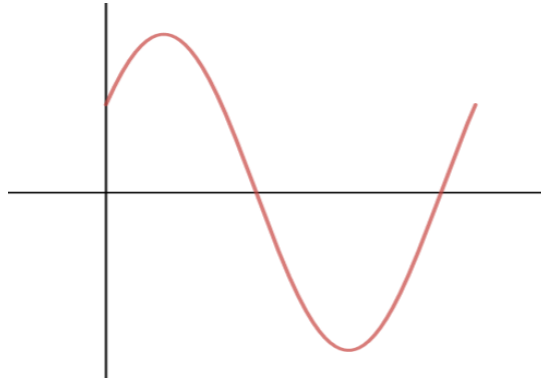
With  $g = 10 \text{ ms}^{-2}$ , the displacement equations of motion are given by:

$$x = 30\sqrt{3}t \quad \text{and} \quad y = 30t - 5t^2 \quad (\text{Do NOT prove this.})$$

- (i) Find the maximum height of the particle. **2**
- (ii) Find the speed of the particle one second after it is launched. **2**
- d) Use the substitution  $u = x + 1$  to evaluate the following integral: **3**

$$\int_0^1 \frac{x}{\sqrt{x+1}} dx$$

e) Consider the graph  $y = 3 \sin x + 2 \cos x$  for  $0^\circ \leq x \leq 360^\circ$  shown below.



- (i) Write  $y = 3 \sin x + 2 \cos x$  in the form  $y = R \sin(x + \beta)$ , where  $R > 0$  and  $0^\circ < \beta < 90^\circ$ . **2**

Give  $R$  as an exact value and  $\beta$  correct to the nearest degree.

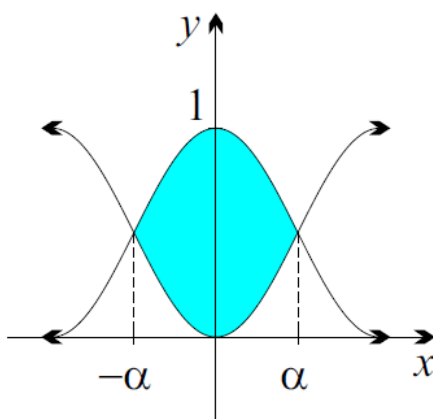
- (ii) Hence solve the equation  $3 \sin x + 2 \cos x = 1$  for  $0^\circ \leq x \leq 360^\circ$ , giving your answers correct to the nearest degree. **2**

**Question 6** (13 marks)

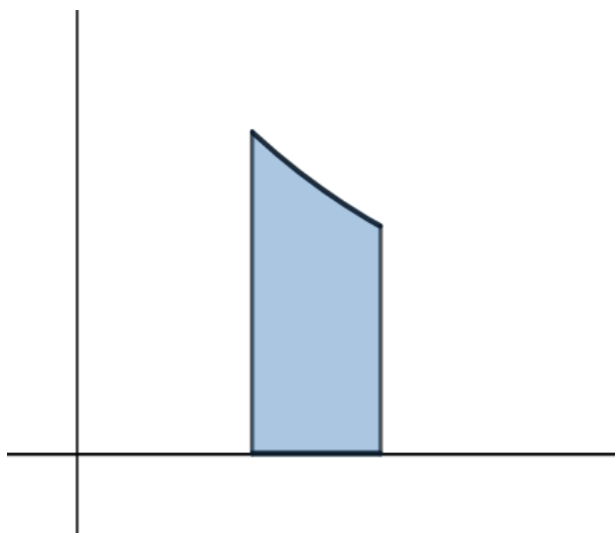
*Start this question in a SEPARATE answer booklet*

**Marks**

- a) The graph below shows the curves  $y = \cos^2 x$  and  $y = \sin^2 x$ .



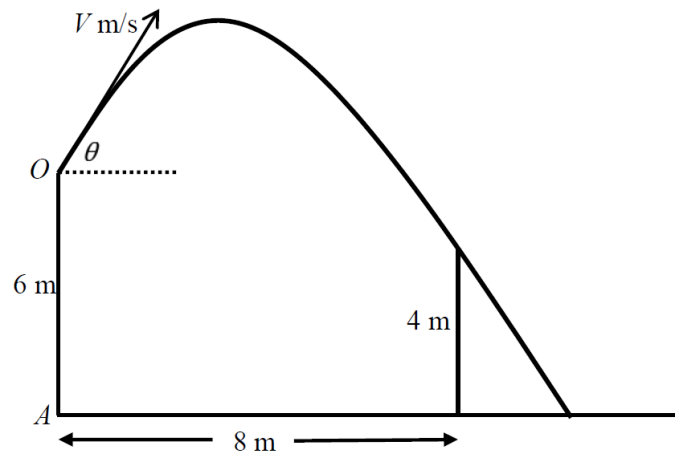
- (i) Find the value of  $\alpha$ . 1
- (ii) Find the area of the shaded region in the graph. 2
- b) The region between the curve  $y = \frac{3}{\sqrt{1+4x^2}}$  and the  $x$ -axis between  $x = \frac{1}{2}$  and  $x = \frac{\sqrt{3}}{2}$  is shown below. 3



Find the exact volume of the solid of revolution formed when the shaded area is rotated about the  $x$ -axis.

- c) A projectile is fired from a point  $O$ , which is 6 metres above horizontal ground, with initial velocity  $V$  m/s, at an angle of  $\theta$  to the horizontal.

There is a thin vertical post 4 metres high and 8 metres horizontally away from point  $A$ , directly below  $O$ , as shown in the diagram below.



The equations of motion are given by:

$$x = Vt \cos \theta$$

$$y = Vt \sin \theta - 4.9t^2$$

Do NOT prove this.

- |       |  |          |
|-------|--|----------|
| (i)   | If the projectile just passes over the top of the vertical post after exactly 2 seconds of motion, show that $\tan \theta = 2.2$       | <b>2</b> |
| (ii)  | Show that the projectile hits the ground approximately 0.3 seconds after it passes over the vertical post.                             | <b>3</b> |
| (iii) | Find the angle that the projectile makes with the ground when it strikes the ground, giving your answer correct to the nearest degree. | <b>2</b> |

**End of Task**



--	--	--	--	--	--	--	--	--

Student Number

## Mathematics Extension 1 – Multiple Choice Questions Answer Sheet

Attempt all questions:

- |          |   |   |                       |   |                       |   |                       |   |                       |
|----------|---|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|
| Question | 1 | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|          | 2 | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|          | 3 | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|          | 4 | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |