



ST PIUS X COLLEGE
CHATSWOOD

2022 HSC Task #1

Weighting 20%

14 February 2022

MATHEMATICS EXTENSION 1

General Instructions

- Working time 45 minutes
- Write using blue or black pen
- Show all relevant mathematical reasoning and calculations
- NESA approved calculators may be used
- Section I – use multiple choice answer sheet
- Section II – answer each question in a new booklet

Student Number	
Teacher's name	Mr Wall / Mr Reay
Multiple Choice	/6
Question 7	/12
Question 8	/12
Total	/30

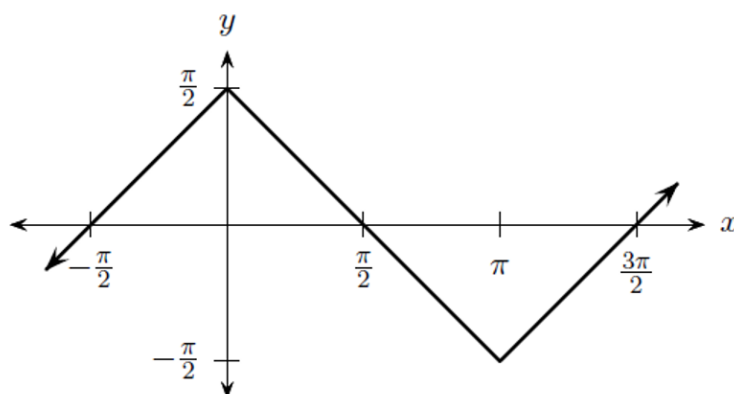
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Section I – 6 Marks

MULTIPLE CHOICE one mark per question.

Answer this section on the multiple choice sheet.

1. Which of the following equations is shown in the sketch below?



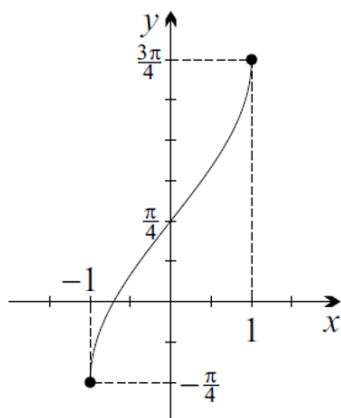
- (A) $y = \sin^{-1}(\sin x)$ (C) $y = \cos(\sin^{-1} x)$
(B) $y = \sin^{-1}(\cos x)$ (D) $y = \sin(\sin^{-1} x)$
2. Which of the following is a *true* statement?

- (A) $\sin 3x \sin 4y = \frac{1}{2} [\cos(3x - 4y) - \cos(3x + 4y)]$
(B) $\sin 3x \sin 4y = \frac{1}{2} [\cos(3x + 4y) - \sin(3x - 4y)]$
(C) $\sin 3x \sin 4y = \frac{1}{2} [\cos(3x + 4y) - \cos(3x - 4y)]$
(D) $\sin 3x \sin 4y = \frac{1}{2} [\cos(3x - 4y) - \sin(3x + 4y)]$

3. The value of $\cot\left(\sin^{-1} \frac{5}{13}\right)$ is?

- (A) $\frac{5}{12}$ (C) $\frac{12}{13}$
(B) $\frac{13}{12}$ (D) $\frac{12}{5}$

4. Which of the following equations is represented by the graph shown below?



- (A) $y = -\cos^{-1} x - \frac{\pi}{4}$
 (B) $y = \sin^{-1} x + \frac{\pi}{4}$
 (C) $y = \tan x + \frac{\pi}{4}$
 (D) $y = \cos^{-1} x - \frac{\pi}{4}$

5. A function is defined by the rule $f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x + 2 & \text{for } x \geq 1 \end{cases}$

Which statement is *incorrect*?

- (A) The value of $f(-2)$ is 1.
 (B) The graph is not continuous at $x = 1$.
 (C) The domain is all real values of x .
 (D) The range is $f(x) \geq 1$.
6. What is the domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 9}}$?

- (A) $(-\infty, -3) \cup (3, \infty)$
 (B) $(-\infty, -3)$
 (C) $[-\infty, -3] \cup [3, \infty]$
 (D) $(3, -3)$

End of Section I

Section II – 24 marks

Question 7 – 12 marks

Start this question in a new booklet

- a. It is given that $(\sin \alpha - \cos \alpha)^2 = 1 - \sin 2\alpha$. 2

What is the exact value of $\sin 75^\circ - \cos 75^\circ$? Leave your answer with a rational denominator.

- b. Use the principle of mathematical induction to show for all integers $n \geq 1$, 3

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}$$

- c. Consider the function $y = 2 \cos^{-1}(x - 1)$.

i. Write the domain of this function. 1

ii. Write the range of this function. 1

iii. Sketch the function $y = 2 \cos^{-1}(x - 1)$. 2

- d. Prove by mathematical induction that $4^n + 14$ is divisible by 6, for all integers $n \geq 1$. 3

Question 8 – 12 marks

Start this question in a new booklet

- a. Prove the trigonometric identity $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$. **3**
- b. Consider the curve $y = 4x^2 - 2x^3$.
- i. Find the stationary points of the curve $y = 4x^2 - 2x^3$ and determine their nature. **3**
- ii. It is known there is ONE point of inflection on this cubic. **1**
Find the coordinates of the point of inflection.
- iii. Hence, sketch the graph of the curve $y = 4x^2 - 2x^3$. **2**
Clearly label the stationary points, the point of inflection and any intercepts with the axes.
- c. Solve the equation $\tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3} = \sin^{-1} x$. **3**

End of assessment

2022 Year 12 Mathematics Extension 1
Assessment #1
Multiple Choice Answer Sheet

Student Number	
Teacher's name	Mr Wall / Mr Reay

Colour your choice for each question.

1. A B C D
 ○ ○ ○ ○

2. A B C D
 ○ ○ ○ ○

3. A B C D
 ○ ○ ○ ○

4. A B C D
 ○ ○ ○ ○

5. A B C D
 ○ ○ ○ ○

6. A B C D
 ○ ○ ○ ○