ST PIUS X COLLEGE CHATSWOOD

2023 HSC ASSESSMENT TASK 3

Mathematics Extension 1 – Year 12

General Instructions

- Working time – 45 minutes
- Weighting 25%
- Write using black or blue pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 30

Section I – 5 marks

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – 25 marks

- Attempt Questions 6 – 7
- Allow about 37 minutes for this section
- Start each question in separate booklet

Multiple Choice	/5
Question 6	/13
Question 7	/12
Total	/30

BLANK PAGE

SECTION I – Multiple Choice   1 mark per question   **5 Marks**

Attempt Questions 1- 5
Allow about 8 minutes to complete this section
Use the multiple-choice answer sheet for Questions 1-5.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2   (B) 6   (C) 8   (D) 9

A ⬭   B ⬤   C ⬭   D ⬭

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

⦻   ⬤   ⬭   ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

⦻   ⦻   ⬭   ⬭

1. $3\cos\theta + 4\sin\theta$ is equivalent to

   (A) $5\cos(\theta + 0.927)$

   (B) $5\cos(\theta - 0.927)$

   (C) $5\cos(\theta + 0.644)$

   (D) $5\cos(\theta - 0.644)$

2.   What is the derivative of $\cos^{-1}(x^2)$

   (A) $\dfrac{-1}{\sqrt{1-x^4}}$

   (B) $\dfrac{1}{\sqrt{1-x^4}}$

   (C) $\dfrac{-2x}{\sqrt{1-x^4}}$

   (D) $\dfrac{2x}{\sqrt{1-x^4}}$

3. Which expression would be the correct answer to $\int \sin(5x)\sin(3x)\,dx$?

   (A) $-\frac{1}{2}\sin 2x + \frac{1}{2}\sin 8x + C$

   (B) $\frac{1}{2}\sin 2x - \frac{1}{2}\sin 8x + C$

   (C) $-\frac{1}{4}\sin 2x + \frac{1}{16}\sin 8x + C$

   (D) $\frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$

4. What is the value of k such that $\int_0^k \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{\pi}{18}$?

   (A) -3

   (B) $-\frac{1}{3}$

   (C) 3

   (D) $\frac{1}{3}$

5. An object is projected so that its position vector is given by

   $\underset{\sim}{r}(t) = (\frac{\sqrt{2}}{4}t)\,\underset{\sim}{i} + (\frac{\sqrt{6}}{12}t - 5t^2)\,\underset{\sim}{j}$. Find the initial velocity V and the angle of projection θ.

   (A) $V = \frac{1}{\sqrt{6}}$ and $\theta = 30°$

   (B) $V = \frac{1}{\sqrt{6}}$ and $\theta = 60°$

   (C) $V = \frac{1}{\sqrt{2}}$ and $\theta = 30°$

   (D) $V = \frac{1}{\sqrt{2}}$ and $\theta = 60°$

SECTION II   **25 Marks**

Attempt Questions 6 – 7
Allow about 37 minutes to complete this section
Start each question in *SEPARATE* booklet

Question 6 (13 marks)

(a) (i)   Express $\sqrt{2}\sin x + \sqrt{2}\cos x$ in the form of $R\sin(x + \alpha)$ where $R > 0$ **1**
            and $0 \le \alpha \le \dfrac{\pi}{2}$.

     (ii) What is the first positive value of x when $\sqrt{2}\sin x + \sqrt{2}\cos x$ attains   **1**
            its maximum value?

     (iii) Solve the equation of $\sqrt{2}\sin x + \sqrt{2}\cos x + \sqrt{3} = 0$ for $0 \le x \le 2\pi$.   **2**

(b) Solve the equation $\cos 2x + 2\cos x + 1 = 0$ for $0 \le x \le 2\pi$.   **2**

(c) Use the substitution $u = 1 + 3e^x$ to find the exact value of $\displaystyle\int_0^{\log_e 8} \dfrac{e^x}{\sqrt{1+3e^x}}\, dx$.   **3**

(d) A particle is projected from a point so that its position at time t is given by the position vector

$$\underline{r}(t) = (20\sqrt{3}\, t)\, \underline{i} + (-5t^2 + 20t)\, \underline{j}. \quad \textbf{[Do NOT prove this]}$$

where t is in seconds and $\underline{r}(t)$ is in metres.

     (i)   Find the horizontal distance travelled by the particle.   **2**

     (ii) Find the Cartesian equation of the path of the particle.   **2**
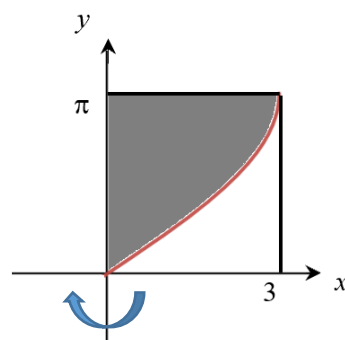
Question 7 (12 marks)   **Start in a *SEPARATE* booklet**

(a)   Find $\dfrac{d}{dx}\log_e[\sin^{-1}(x)]$ **2**

(b)   Find the equation of the curve $y = f(x)$ that has $\dfrac{dy}{dx} = \dfrac{1}{9+4x^2}$ and passes   **3**

through $(\dfrac{3}{2}, 0)$.

(c)   The shaded region bounded by the curve $y = 2\sin^{-1}\dfrac{x}{3}$, the *y*-axis and the   **3**

line $y = \pi$ is rotated about the *y*-axis as shown below.

Find the exact volume of the solid formed.

(d)   When an object is projected from a point *h* metres above the origin with initial speed *V* m/s at an angle of θ to the horizontal, its displacement vector, *t* seconds after projection is

$$\underset{\sim}{r}(t) = (V\cos\theta\, t)\,\underset{\sim}{i} +[-5t^2 + (V\sin\theta\, t) + h\,]\,\underset{\sim}{j}. \qquad [\textbf{ Do NOT prove this }]$$

Now, an object is fired at a height of 2 m from the ground with an angle of projection

given by $\theta = \tan^{-1}(\dfrac{4}{3})$. It just clears a wall that is 17 m high and 15 m away.

(i)   Find the initial velocity of the object.   **2**

(ii)   At what speed is the object travelling at the instant it clears the wall? Give   **2**
your answer correct to 2 decimal places if necessary.

End of assessment

2023 HSC ASSESSMENT TASK 3

Mathematics Extension 1 – Year 12

Multiple Choice Answer Sheet

Attempt all questions:

Question **1** A ⬭ B ⬭ C ⬭ D ⬭

2 A ⬭ B ⬭ C ⬭ D ⬭

3 A ⬭ B ⬭ C ⬭ D ⬭

4 A ⬭ B ⬭ C ⬭ D ⬭

5 A ⬭ B ⬭ C ⬭ D ⬭