Student Number

ST PIUS X COLLEGE
CHATSWOOD

HSC 2021 Stage 6
Year 12

Assessment Task #2

25% of School Based Assessment

MATHEMATICS ADVANCED

General Instructions

- Handout date: **Friday, March 19, 2021**
- Due date: **Friday, April 23, 2021**
- Read each section carefully and complete all questions.
- Each student is to submit an individual task and it must feature their own work.
- Any form of plagiarism will result in a mark of ZERO being awarded.
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total Marks – 50

Section I – Graphs and Equations **20 marks**

- Attempt Questions 1 – 2
- Show all necessary working
- ***Write your solutions in the space provided***

Section II – Curve Sketching Using Derivatives **20 marks**

- Attempt Questions 3 – 4
- Show all necessary working
- ***Write your solutions in the space provided***

Section III – Investigation **10 marks**

- Attempt Question 5
- Show all necessary working
- ***Write your solutions in the space provided***

IMPORTANT NOTES ON TASK AND SUBMISSION:

- This task is an *ALTERNATIVE* task featuring an opportunity for you to research your work and investigate concepts to a deeper level. This task also allows you adequate time (5 weeks) to appropriately answer questions and submit your work.
- Students may only ask for help or clarification of any misunderstandings during Weeks 9 and 10 of Term 1 (Monday, March 22 to Thursday, April 1) in 2021. Your teachers will not be responding to any enquiries across the Easter holiday break (Friday, April 2 to Monday, April 19).
- Any submissions after the due date of Friday, April 23 will NOT be accepted.
- If you use a website or other resources, you must provide a list of these resources (bibliography) at the end of your assignment.
- This task requires thorough mathematical reasoning and calculations for some questions. Marks may be deducted for not showing a clear understanding of a concept or how an answer has been obtained.
- Ensure that any graphs from Desmos, GeoGebra or any other software package are clearly presented, including the equations of any graphs.
- Additional writing space is provided at the end of this assignment. If you require further writing space to answer any questions, you must neatly staple any attached pages to the back end of this assignment.

Outcomes to be assessed:

A student:

› MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts.

› MA12-3 applies calculus techniques to model and solve problems.

› MA12-5 applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs.

› MA12-6 applies appropriate differentiation methods to solve problems.

› MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

› MA12-9 chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use

› MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Section I

20 Marks

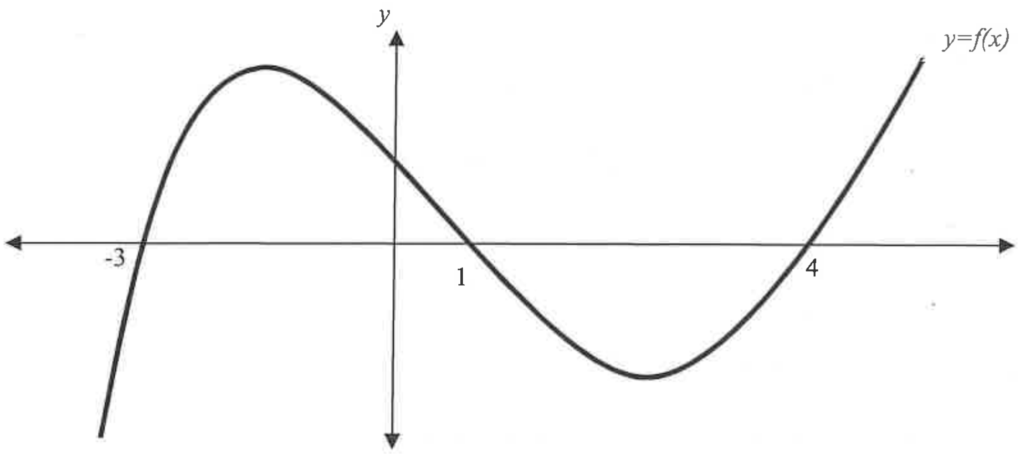
Attempt Questions 1 to 2.

In Questions 1 to 2 your responses should include relevant mathematical reasoning and/or calculations in the spaces provided.

Question 1 (10 marks)

Write your solutions in the space provided **10 Marks**

(a) Consider the following diagram representing the function *f(x)*.



(i) For what values is $f(x) = 0$? **1**

$x = -3, \ x = 1, \ x = 4$

(ii) Using **bracket interval notation**, for what values is $f(x) < 0$? **1**

$[-3, 1], [4, \infty)$

(iii) Suggest a **possible** expression for the function $f(x)$. i.e. What could $f(x)$ be equal to? **1**

$f(x) = (x+3)(x-1)(x-4)$

(b) Given $f(x) = 6x - 5$ and $g(x) = x^2 + x$:

(i) Find $f \circ g(x)$ and then evaluate $f \circ g(2)$. **1**

$f \circ g(x) = 6(x^2 + x) - 5$
$= 6x^2 + 6x - 5$

$f \circ g(2) = 6(4) + 6(2) - 5$
$= 31$

(ii) What type of function is $f \circ g(x)$? **1**

A quadratic function

(iii) Find $g \circ g(x)$. Your final answer should be in polynomial form. **1**

$g \circ g(x) = (x^2+x)^2 + (x^2+x)$
$= x^4 + 2x^3 + x^2 + x^2 + x$
$= x^4 + 2x^3 + 2x^2 + x$

(c) Use Desmos, GeoGebra or any other graphing software to help you with this question.

(i) Determine the number of solutions to the equation $\ln x = 4x^3 + 6x^2 - 6$. **1**

2

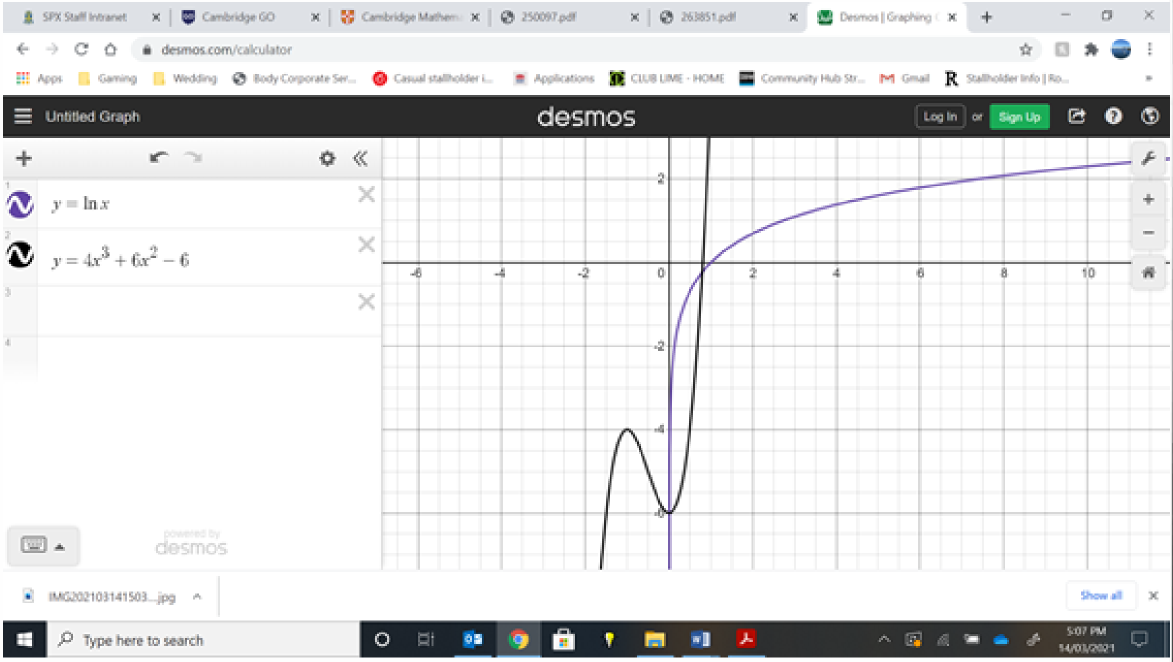
(ii) Write down the solution or solutions to the equation $\ln x = 4x^3 + 6x^2 - 6$. If required, round your answer to three decimal places. **2**

$\boxed{(0.793, -0.2319) \text{ and } (0.00248, -6)}$

$(0, -6)$ is incorrect

can not have ln 0.

(iii) Print a copy of the diagram and paste it in the box below. You may want to mark important points and features on your diagram with a pen. **1**



Question 2 (10 marks)

Write your solutions in the space provided **10 Marks**

(a) Solve the following equations algebraically:

(i) $|3x+8|=5$ **1**

$3x+8=5$ or $3x+8=-5$

$3x=-3$ $\quad$ $3x=-13$

$x=-1$ $\quad$ $x=-\frac{13}{3}$

(ii) $|1-4x|=6x-9$ **1**

$1-4x=6x-9$ or $1-4x=-(6x-9)$

$1-10x=-9$ $\quad$ $1-4x=-6x+9$

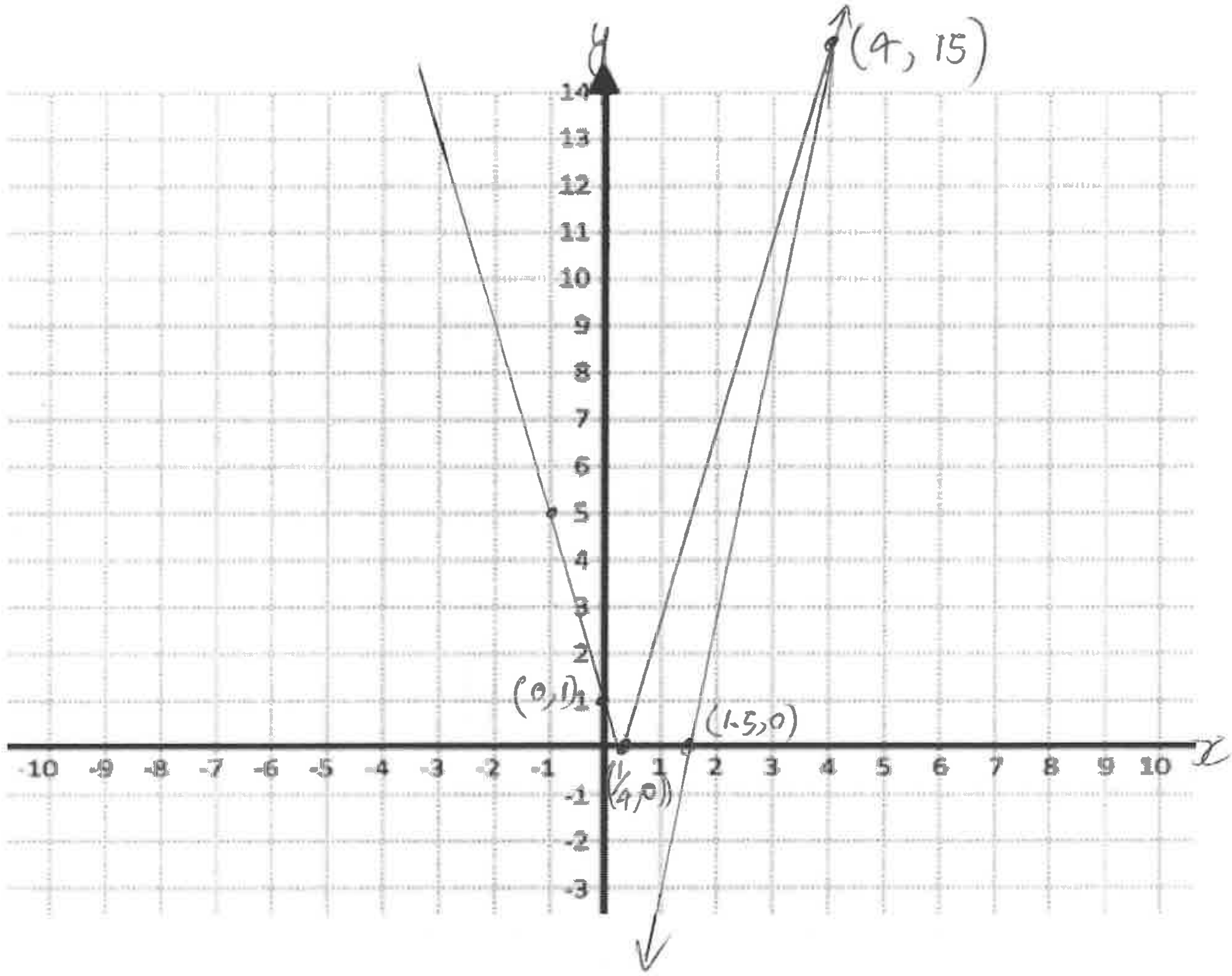
$-10x=-10$ $\quad$ $2x=8$

$x=1$ $\quad$ $x=4$

$x=-1$ is not a solution by testing.

(b) (i) Draw the graphs of $y=|1-4x|$ and $y=6x-9$ on the same set of axes by hand. **2**

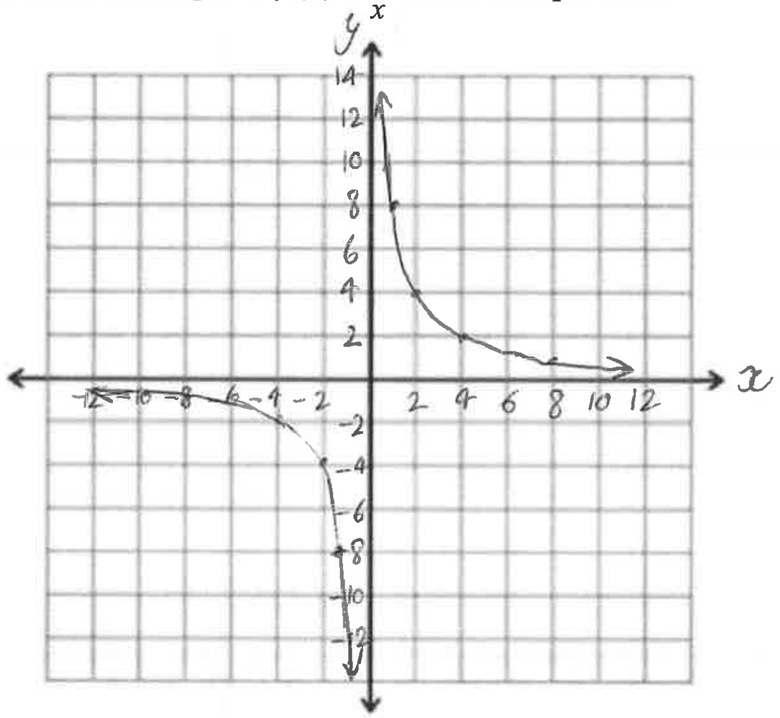
The space provided immediately below is for working. The axes provided on the next page is for your graphs.



(ii) For what values is $y = |1-4x| < 6x-9$? **1**

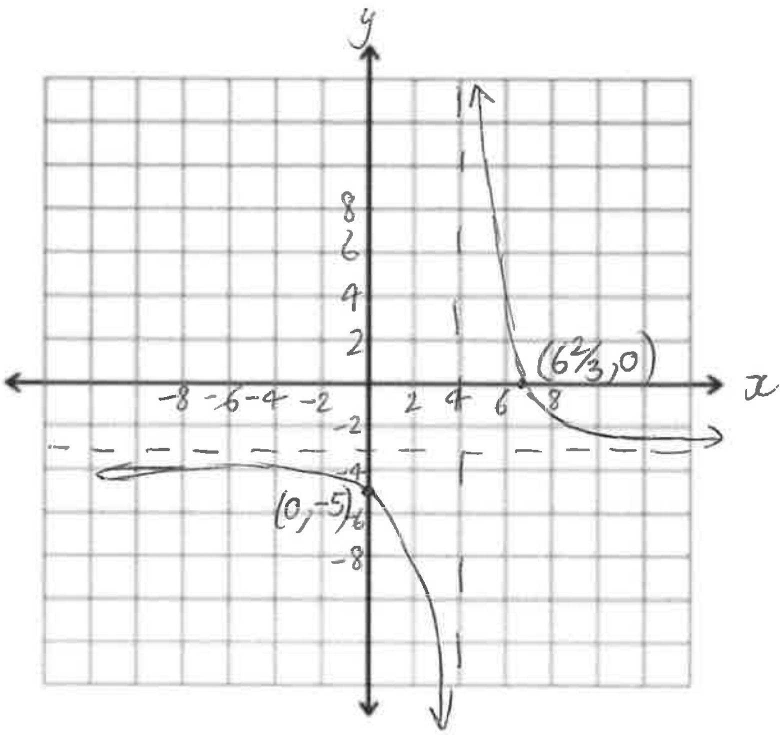
$x > 4$ or $(4, \infty)$

(c) (i) Sketch (by hand) the graph of $f(x) = \frac{8}{x}$ on the axes provided. **1**



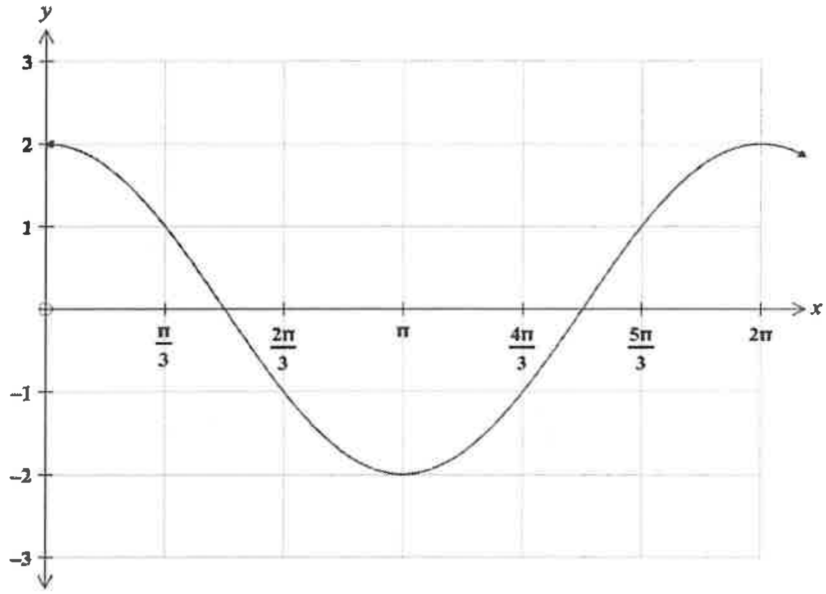
(ii) Write the equation of a new function $g(x)$ with vertical asymptote moved 4 units to the right and horizontal asymptote 3 units down then sketch (by hand) the new function below showing all important features. **2**

$y = \frac{8}{x-4} - 3$



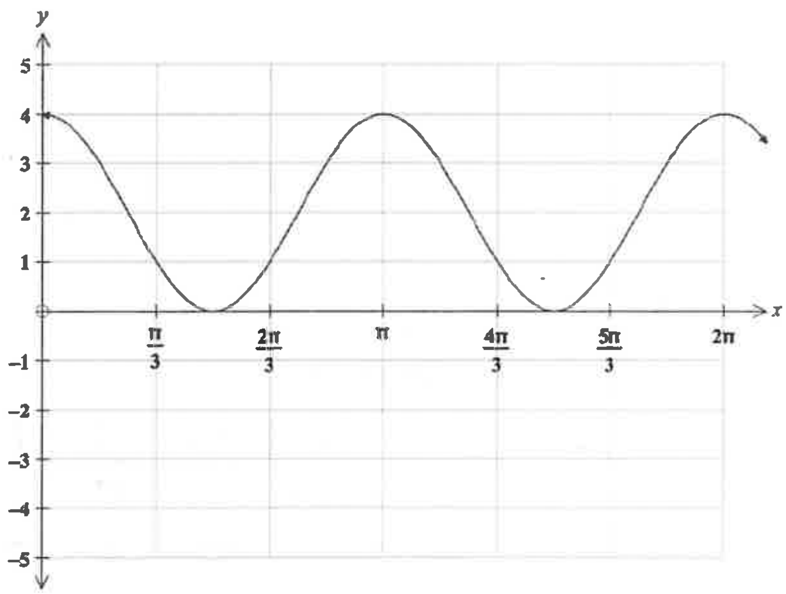
(d) The diagrams below show cosine functions which have been manipulated and graphed in the domain $0 \le x \le 2\pi$. Consider each graph and determine the equation of each graph using your knowledge of shifting, amplitude and period:

(i) **1**



$y = 2\cos x$

(ii) **1**



$y = 2\cos(2x) + 2$

Section II **20 Marks**

Attempt Questions 3 to 4.

In Questions 3 to 4 your responses should include relevant mathematical reasoning and/or calculations in the spaces provided.

Question 3 (10 marks)

Write your solutions in the space provided **10 Marks**

(a) What kind of point exists (maximum turning, minimum turning, point of inflection, stationary point of inflection) at $x = a$:

(i) if $f'(a) = 0$ and $f''(a) < 0$? **1**

A maximum turning point.

(ii) if $f'(a) = 0$ and $f''(a) = 0$ and there is a sign change on either side of $x = a$? **1**

stationary/ point of inflection
Horizontal

(b) (i) Find the first and second derivatives of $f(x) = 3x^4 - 8x^3 + 2$. **2**

$f'(x) = 12x^3 - 24x^2$

$f''(x) = 36x^2 - 48x$

(ii) Find any turning points of $f(x) = 3x^4 - 8x^3 + 2$ and describe them. **2**
You may want to draw a sign table.

When $f'(x) = 0$
$12x^2(x-2) = 0$
$\therefore x = 0$ or 2

x	-1	0	1	2	3
$f'(x)$	-36	0	-12	0	108
sign	$-$ \	* —	$-$ \	* —	$+$ /

At $(0, 2)$ we have a ~~minimum turning point~~ possible point of inflection
At $(2, -14)$ we have a ~~possible point of inflection~~ minimum turning point

(iii) Find any points of inflection and the gradients of the inflectional tangents and describe the concavity. **2**

You may want to draw a concavity table.

when $f''(x)=0$

$12x(3x-4)=0$

$x=0$ or $x=1\frac{1}{3}$

x	-1	0	1	$1\frac{1}{3}$	2
$f'(x)$	-36	0	-12	$-14\frac{2}{9}$	0
$f''(x)$	84	0	-12	0	48
	$\cup$	$-$	$\cap$	$-$	$\cup$

Horizontal point of inflection at $x=0$ or $(0,2)$

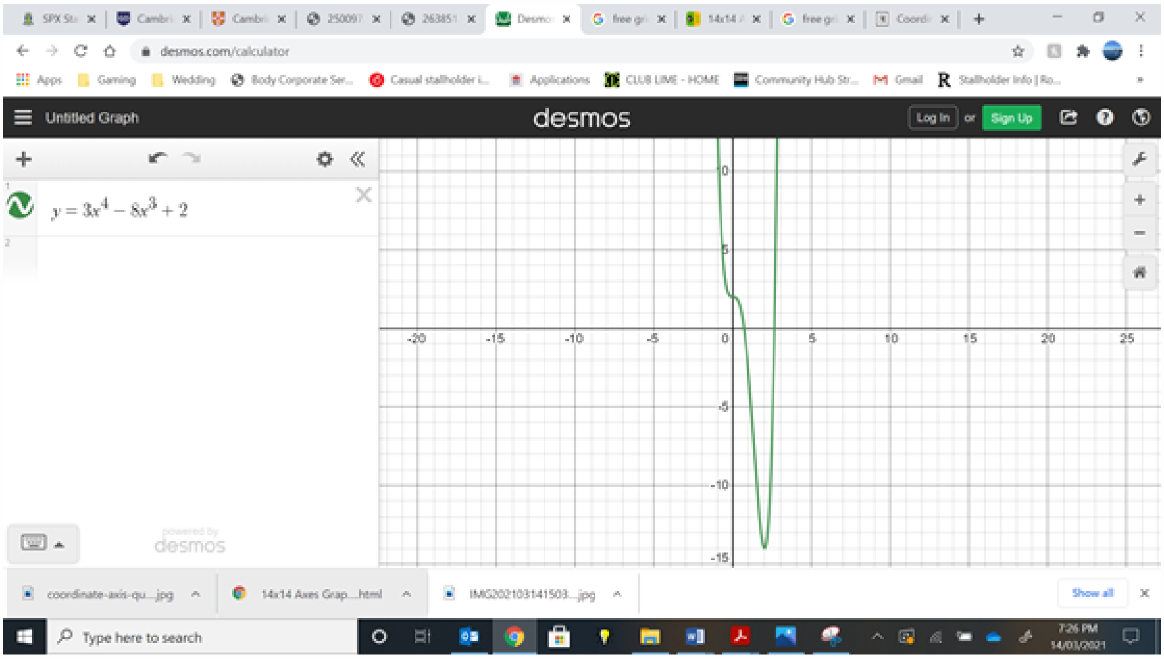
Point of inflection at $\left(1\frac{1}{3}, -7\frac{13}{27}\right)$

At $x=1\frac{1}{3}$, $f'(x)=m=-14\frac{2}{9}$

$f(x)$ is decreasing for $x \leq 2$, increasing for $x \geq 2$

(iv) Print a copy of the diagram of $f(x)=3x^4-8x^3+2$ and paste it in the box below. **1**

You may want to mark important points and features on your diagram with a pen.



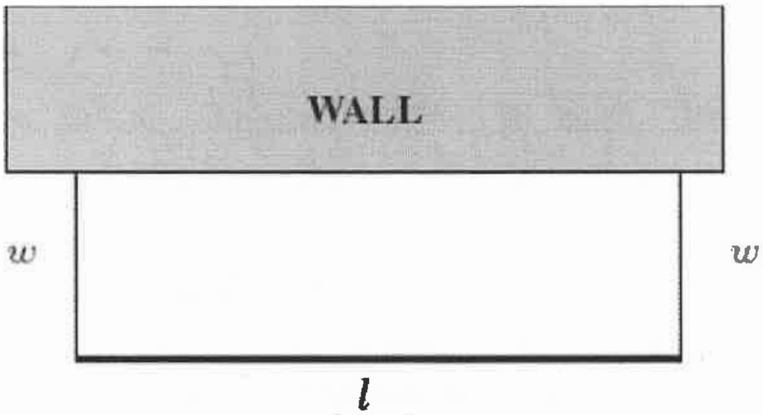
(v) For what values is $f(x)=3x^4-8x^3+2$ decreasing? **1**

$x \leq 2$

Question 4 (10 marks)

Write your solutions in the space provided **10 Marks**

(a) A farmer decides to fence a rectangular area for storage, using part of the rear wall of his barn as one boundary. He needs an area of $800\,m^2$. Fencing will cost \$14 per metre.



(i) Determine the dimensions of the fence so that the cost of the fence will be Minimised. **3**

$A = wl$ let $w = x$, $l = y$ $\therefore A = xy$

$800 = xy$

$y = \frac{800}{x}$

$P = 2w + L$ or $P = 2x + y$

$C = (2w + L)14$ or $C = 14(2x + y)$

$= 28x + 14y$

$= 28x + 14\left(\frac{800}{x}\right)$

$= 28x + 11200x^{-1}$

$C' = 28 - 11200x^{-2}$

When $C' = 0$

$28 = \frac{11200}{x^2}$

$x^2 = \frac{11200}{28}$

$= 400$

$\therefore x = \pm 20$ but $x > 0$

$C'' = 22400x^{-3}$, when $C = 20$

$C'' = 22400(20)^{-3}$

$> 0 \therefore$ Min

$\therefore$ when $x = 20$, $y = \frac{800}{20}$

$= 40$

i.e. Minimum cost when $w = 20$, $L = 40$

(ii) What is the cost of fencing this enclosure? **1**

When $x = 20$, $C = 14\left(\frac{800}{20}\right) + 28 \times 20$

$= \$1120$

(b) For the function $f(x)=\frac{2}{x^2}$:

(i) Use calculus to show there are no turning points or points of inflection. **1**

$f'(x) = -4x^{-3}$ $\quad$ $f''(x) = 12x^{-4}$

$= -\frac{4}{x^3}$ $\quad$ $= \frac{12}{x^4}$

$\therefore f'(x) \neq 0$ $\quad$ $\therefore f''(x) \neq 0$

(ii) Determine algebraically whether the function odd, even or neither. **2**

$f(-x) = \frac{2}{(-x)^2}$

$= \frac{2}{x^2}$

$= f(x)$

$\therefore$ Function is even

(iii) As $x \to +\infty$, and $x \to -\infty$ $f(x) \to ?$ **1**

0

(iv) As $x \to 0$, $f(x) \to ?$ **1**

∞

(v) An asymptote occurs when? **1**

$x = 0$

Section III

10 Marks

Attempt Question 5.

In Question 5 your responses should include relevant mathematical reasoning and/or calculations in the spaces provided.

Question 5 (10 marks)

Write your solutions in the space provided **10 Marks**

Graphing websites like Desmos can be used for the following task.

Fill in your full name in the area below.

FIRST NAME:	BILLY	Number of letters: 5	α
MIDDLE NAME:	SAMUEL	Number of letters: 6	β
SURNAME:	HENDRICK-JONES	Number of letters: 13	γ

The number of letters in your name will become the constants (α , β and γ) in the function below. If you have no middle name $\beta = 0$.

$$y = (x - \alpha)(x + \beta)(x - \gamma)$$

- α is the number of letters in their first name.
- β is the number of letters in their middle name.
- γ is the number of letters in their surname.

For example, Billy Samuel Hendrick-Jones would have $\alpha = 5$, $\beta = 6$, and $\gamma = 13$.

His equation will be $y=(x-5)(x+6)(x-13)$.

(a) Create your unique equation below and expand and simplify your equation. **2**
(Eg: Billy Samuel Hendrick-Jones will have equation $y=x^3-12x^2-43x+390$)

....

....

....

....

....

....

....

(b) Find the first derivative of your unique equation and determine any stationary points. **2**

$\frac{dy}{dx} = 3x^2 - 24x - 43$ When $\frac{dy}{dx} = 0$

$x = \frac{24 \pm \sqrt{(-24)^2 - (4 \times 3 \times -43)}}{6}$

$= \frac{24 \pm \sqrt{1092}}{6}$ Stationary points at

$= 9.51$ or -1.51 ~~(9.51, 0)~~ and ~~(-1.51, 0)~~

(9.51, -244.13) (-1.51, 424.13)

(c) Find the second derivative of your unique equation and determine any points of inflection. **2**

$\frac{d^2y}{dx^2} = 6x - 24$

When $f''(x) = 0$

$6x - 24 = 0$

$6x = 24$

$x = 4$

(d) What effect does making the coefficient of x^3 to 2 have on the graph? Compare and contrast. **2**

It narrows the graph maintaining the same shape. The new graph increases faster as $x \to \infty$ and decreases faster as $x \to -\infty$. The local maxima and minima are smaller in absolute value.

(e) Cut and paste in two diagrams of your original unique function and the modified function in (d) below. **2**

Diagram 1:

Diagram 2:

End of Task

Section II extra writing space

If you use this space, clearly indicate which question you are answering.