

# Mathematics Extension 1 Task 2 2020

## SOLUTIONS

Section 1:

1	2	3	4
B	C	D	C

①  $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$

$$f'(x) = -\frac{1}{x^2} \times \frac{1}{1 + \left(\frac{1}{x}\right)^2}$$

$$= -\frac{1}{x^2} \times \frac{1}{1 + \frac{1}{x^2}}$$

$$= -\frac{1}{x^2} \times \frac{x^2}{x^2 + 1}$$

$$= -\frac{1}{1 + x^2}$$

B



②  $\sqrt{3} \cos \theta - \sin \theta = 2 \cos(\theta + \alpha)$

$$\sqrt{3} \cos \theta - \sin \theta = 2 [\cos \theta \cos \alpha - \sin \theta \sin \alpha]$$

Equate coefficients:

$\cos \theta : \sqrt{3} = 2 \cos \alpha$  (1)

$\sin \theta : -1 = -2 \sin \alpha$  (2)

(2)  $\div$  (1)  $-\frac{1}{\sqrt{3}} = -\tan \alpha$

$$\tan \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\pi}{6} \therefore 2 \cos\left(\theta + \frac{\pi}{6}\right)$$

C



$$\textcircled{3} \quad \cos 6x = \cos^2 3x - \sin^2 3x \\ = 1 - 2\sin^2 3x$$

$$2\sin^2 3x = 1 - \cos 6x$$

$$\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$$

$$\int \sin^2 3x \, dx = \frac{1}{2} \int 1 - \cos 6x \, dx \\ = \frac{1}{2} \left( x - \frac{1}{6} \sin 6x \right) + C$$

✓  
D

$$\textcircled{4} \quad y = \cos \left( 2t - \frac{\pi}{3} \right)$$

$$0 = \cos \left( 2t - \frac{\pi}{3} \right)$$

$$2t - \frac{\pi}{3} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$2t = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \dots$$

$$t = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \dots$$

$P$  is the second positive horizontal intercept here

$$\text{so } t = \frac{11\pi}{12}$$

$$\therefore P \text{ is } \left( \frac{11\pi}{12}, 0 \right)$$

✓  
C

## Section II:

$$\begin{aligned} (5) (a) \quad \sec \theta + 2 \tan \theta &= \frac{1}{\cos \theta} + 2 \tan \theta \\ &= \frac{1+t^2}{1-t^2} + 2 \times \frac{2t}{1-t^2} \\ &= \frac{1+4t+t^2}{1-t^2} \quad \checkmark \end{aligned}$$

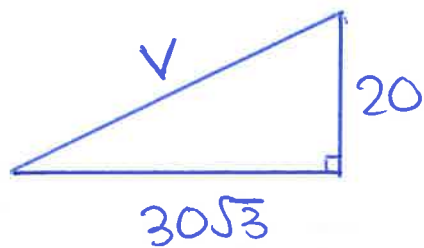
$$(b) \quad \int \frac{1}{\sqrt{9-x^2}} dx = \sin^{-1}\left(\frac{x}{3}\right) + C \quad \checkmark$$

$$\begin{aligned} (c) (i) \quad y &= 30t - 5t^2 \\ \dot{y} &= 0 \quad \text{at maximum height} \\ \dot{y} &= 30 - 10t \\ 0 &= 30 - 10t \\ t &= 3 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{When } t=3, \quad y &= 30(3) - 5(3)^2 \\ &= 90 - 45 \\ &= 45 \text{ metres} \end{aligned}$$

$\therefore$  Maximum height is 45 metres  $\checkmark$

$$\begin{aligned} (ii) \quad x &= 30\sqrt{3}t & y &= 30t - 5t^2 \\ \dot{x} &= 30\sqrt{3} & \dot{y} &= 30 - 10t \\ \text{When } t=1, \quad \dot{x} &= 30\sqrt{3} & \dot{y} &= 30 - 10(1) \\ & & &= 20 \quad \checkmark \end{aligned}$$



$$\begin{aligned} v^2 &= (30\sqrt{3})^2 + 20^2 \\ &= 2700 + 400 \\ &= 3100 \end{aligned}$$

$10\sqrt{31}$  or 55.7 are acceptable here

$$\begin{aligned} v &= \sqrt{3100} \\ &= 10\sqrt{31} \text{ m/s} \\ &\doteq 55.7 \text{ m/s} \end{aligned}$$

$\therefore$  The speed is  $10\sqrt{31}$  m/s after 1 second of motion

(d)  $u = x+1$   
 $du = dx$   
 $x = u-1$

When  $x = 1$ ,  $u = 2$   
 When  $x = 0$ ,  $u = 1$  ✓

$$\therefore \int_0^1 \frac{x}{\sqrt{x+1}} dx = \int_1^2 \frac{u-1}{\sqrt{u}} du$$

$$= \int_1^2 \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}} du$$

$$= \int_1^2 u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$$

$$= \left[ \frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]_1^2$$

$$= \left( \frac{2}{3} \times 2\sqrt{2} - 2\sqrt{2} \right) - \left( \frac{2}{3} - 2 \right)$$

$$= -\frac{2}{3}\sqrt{2} + \frac{4}{3}$$

$$= \frac{2}{3}(2-\sqrt{2})$$

Students can give an approximation here as their answer BUT the exact value is preferred

$$\frac{2}{3}(2-\sqrt{2}) \doteq 0.39$$

(e) (i)  $y = 3 \sin x + 2 \cos x$

$$3 \sin x + 2 \cos x = R \sin(x + B)$$

$$3 \sin x + 2 \cos x = R [\sin x \cos B + \cos x \sin B]$$

Equate coefficients:

$\sin x$ :  $3 = R \cos B$  (1)

$\cos x$ :  $2 = R \sin B$  (2)

$$\begin{aligned} (1)^2 + (2)^2: \quad 13 &= R^2 \sin^2 B + R^2 \cos^2 B \\ &= R^2 (\sin^2 B + \cos^2 B) \\ &= R^2 \end{aligned}$$

$$\therefore R = \sqrt{13} \quad \text{since } R > 0$$

$$(2) \div (1) \quad \frac{2}{3} = \tan B$$

$$B \doteq 34^\circ \quad (\text{nearest degree})$$

$$\therefore y = \sqrt{13} \sin(x + 34^\circ)$$

(ii)  $3 \sin x + 2 \cos x = 1$  for  $0^\circ \leq x \leq 360^\circ$

$$\sqrt{13} \sin(x + 34^\circ) = 1 \quad \text{for } 34^\circ \leq x + 34^\circ \leq 394^\circ$$

$$\sin(x + 34^\circ) = \frac{1}{\sqrt{13}}$$

$$x + 34^\circ = 164^\circ \quad \text{or } 376^\circ$$

$$x = 130^\circ \quad \text{or } 342^\circ$$

⑥ (a) (i)

$$\cos^2 x = \sin^2 x$$

$$1 = \tan^2 x$$

$$x = -\frac{\pi}{4} \text{ or } \frac{\pi}{4} \text{ on the graph}$$

$$\therefore \alpha = \frac{\pi}{4}$$

(ii) Area =  $\int_a^b f(x) - g(x) dx$  "top curve - bottom curve"

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x - \sin^2 x dx$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x dx$$

$$= 2 \int_0^{\frac{\pi}{4}} \cos 2x dx \text{ because } \cos 2x \text{ is EVEN}$$

$$= 2 \left[ \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{4}}$$

$$= [\sin 2x]_0^{\frac{\pi}{4}}$$

$$= \sin \frac{\pi}{2} - \sin 0$$

$$= 1 - 0$$

$$= 1 \text{ unit}^2 \text{ (1 square unit)}$$

$$(b) \quad y = \frac{3}{\sqrt{1+4x^2}}$$

$$y^2 = \frac{9}{1+4x^2}$$

$$\text{Volume} = \pi \int_a^b y^2 dx$$

$$= \pi \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{9}{1+4x^2} dx$$

$$= 9\pi \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{4(\frac{1}{4}+x^2)} dx$$

$$= \frac{9\pi}{4} \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{(\frac{1}{2})^2 + x^2} dx$$

$$= \frac{9\pi}{4} \times \left[ \frac{1}{\frac{1}{2}} \times \tan^{-1}\left(\frac{x}{\frac{1}{2}}\right) \right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{9\pi}{2} \left[ \tan^{-1}(2x) \right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{9\pi}{2} \left( \tan^{-1}\sqrt{3} - \tan^{-1}1 \right)$$

$$= \frac{9\pi}{2} \left( \frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= \frac{9\pi}{2} \times \frac{\pi}{12}$$

$$= \frac{3\pi^2}{8} \text{ units}^3$$

Answer must be given as an EXACT value in its simplest form for full marks

(c) (i) When  $t = 2$ ,  $x = 8$  and  $y = -2$  ✓

$$\therefore 8 = 2V \cos \theta \quad \text{and} \quad -2 = 2V \sin \theta - 19.6$$

$$V \cos \theta = 4 \quad (1)$$

$$V \sin \theta = 8.8 \quad (2)$$

$$(2) \div (1) \quad \frac{V \sin \theta}{V \cos \theta} = \frac{8.8}{4}$$

$$\therefore \tan \theta = 2.2 \quad \checkmark$$

I mark here if  
you recognise  $y = -2$   
when  $t = 2$

(ii) When  $y = -6$  the projectile hits the ground ✓

$$-6 = V t \sin \theta - 4.9 t^2 \quad \checkmark$$

$$-6 = 8.8 t - 4.9 t^2 \quad \text{since } V \sin \theta = 8.8$$

$$4.9 t^2 - 8.8 t - 6 = 0$$

$$t = \frac{8.8 \pm \sqrt{(-8.8)^2 - 4(4.9)(-6)}}{2(4.9)}$$

$$= \frac{8.8 \pm \sqrt{195.04}}{9.8} \quad \checkmark$$

$$\doteq -0.527 \text{ or } 2.323$$

But  $t > 0$  here

So the projectile hits the ground approximately 2.3 seconds after it is launched and 0.3 seconds after it clears the vertical post ✓

$$(iii) \quad x = Vt \cos \theta$$

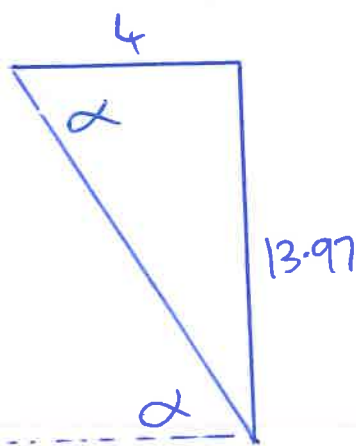
$$\dot{x} = V \cos \theta \\ = 4$$

$$y = Vt \sin \theta - 4.9t^2$$

$$\dot{y} = V \sin \theta - 9.8t \\ = 8.8 - 9.8t$$

using part (i)

$$\text{When } t \doteq 2.323, \quad \dot{x} = 4 \quad \text{and} \quad \dot{y} = 8.8 - 9.8(2.323) \\ \doteq -13.97 \quad \checkmark$$



Let  $\alpha$  be the angle at which the projectile strikes the ground

$$\tan \alpha = \frac{13.97}{4}$$

$$\alpha = \tan^{-1} \left( \frac{13.97}{4} \right)$$

$$\doteq 74^\circ \quad \checkmark$$

So the projectile strikes the ground at an angle of  $74^\circ$  with the horizontal

