



ST PIUS X COLLEGE
CHATSWOOD

2023 HSC Task #1

Weighting 20%

15th February 2023

MATHEMATICS EXTENSION 1

General Instructions

- Working time 45 minutes
- Write using blue or black pen
- Show all relevant mathematical reasoning and calculations
- NESA approved calculators may be used
- Section I – use multiple choice answer sheet
- Section II – answer each question in a new booklet

Student Number	
Teacher's name	Mr Chan / Mr Kennedy
Multiple Choice	/6
Question 7	/12
Question 8	/12
Total	/30

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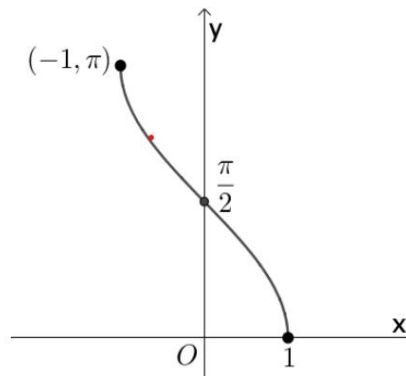
Section I – 6 Marks

MULTIPLE CHOICE one mark per question.

Answer this section on the multiple choice sheet.

1. The value of $\sin\left(\cot^{-1}\frac{24}{7}\right)$ is?
(A) $\frac{7}{25}$ (C) $\frac{25}{7}$
(B) $\frac{24}{25}$ (D) $\frac{25}{24}$
2. The horizontal asymptotes of $y = \frac{3^x - 1}{3^x + 1}$ are?
(A) $y = -1$ for $x < 0$ and $y = 1$ for $x > 0$
(B) $y = -2$ for $x < 0$ and $y = 2$ for $x > 0$
(C) $y = -3$ for $x < 0$ and $y = 3$ for $x > 0$
(D) $y = -4$ for $x < 0$ and $y = 4$ for $x > 0$
3. Mathematical induction works only in cases where,
(A) $n < 0$ and n can be both a integer and non-integer value
(B) $n > 0$ and n can be both a integer and non-integer value
(C) $n < 0$ and n can only be an integer value
(D) $n > 0$ and n can only be an integer value

4. The graph of $y = \cos^{-1}x$ is shown below



What transformation would need to take place from the above graph to create

$$y = 2\cos^{-1}(x - 1)$$

- (A) Shift one to the right and vertically dilate by a factor 2
- (B) Shift one to the left and vertically dilate by a factor of 2
- (C) Shift two to the right and horizontally dilate by a factor of 2
- (D) Shift two to the left and horizontally dilate by a factor of 2
5. $\sin(3x + x) - \sin(3x - x)$ equals which of the following ?
- (A) $-2\sin 3x \cos x$
- (B) $2\cos 3x \sin x$
- (C) $2\cos 3x \cos x$
- (D) $2\sin 3x \sin x$

6. What is the domain and range of the function $y = \frac{1}{\sqrt{a^2 - x^2}}$?

(A) $x \in [-a, a]$ and $y \in [0, \infty)$

(B) $x \in (-a, a)$ and $y \in [0, \infty)$

(C) $x \in [-a, a]$ and $y \in (0, \infty)$

(D) $x \in (-a, a)$ and $y \in (0, \infty)$

End of Section I

Section II – 24 marks

Question 7 – 12 marks

Start this question in a new booklet

- a. By considering a graphical method, or otherwise, solve the following inequality

$$|x - 2| \leq x^2 \quad 2$$

- b. Explain why the statement $2 + 4 + \dots + 2n = n(n - 1) + 2$ for $n \geq 1$, cannot be proven by mathematical induction. Support your answer with appropriate working 3

- c. Using an appropriate method, solve

$$\sin x - \cos x = 1 \text{ for } 0 \leq x \leq 2\pi \quad 3$$

- d. i. Show that $\frac{1}{(k+1)!} - \frac{k+1}{(k+2)!} = \frac{1}{(k+2)!}$ 1

- ii. Use mathematical induction to prove

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!} \quad 3$$

Question 8 – 12 marks

Start this question in a new booklet

- a. Using t results, prove,

2

$$\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

- b. Consider the function $f(x) = \frac{x^2-9}{x^2-4}$,

- i. Explain why the vertical asymptotes are $x = -2$ and 2 .

1

- ii. Show that a stationary turning point exists at $x = 0$.

2

- iii. Give a reason why a minimum turning point exists at $x = 0$.

1

- c. i. Show that $\frac{\cos 3x - \cos 11x}{\sin 9x + \sin 5x} = 2 \sin 2x$

3

- ii. Hence, or otherwise, find the four solutions closest to the origin of the following equation

$$\frac{\cos 3x - \cos 11x}{\sin 9x + \sin 5x} = 2$$

3

End of assessment

2023 Year 12 Mathematics Extension 1
Assessment #1
Multiple Choice Answer Sheet

Student Number	
Teacher's name	Mr Chan / Mr Kennedy

Colour your choice for each question.

1. A B C D
 ○ ○ ○ ○

2. A B C D
 ○ ○ ○ ○

3. A B C D
 ○ ○ ○ ○

4. A B C D
 ○ ○ ○ ○

5. A B C D
 ○ ○ ○ ○

6. A B C D
 ○ ○ ○ ○