
Student Number

ST PIUS X COLLEGE
CHATSWOOD

SOLUTIONS

HSC 2020 Stage 6
Year 12

Assessment Task #1

20% of School Based Assessment

MATHEMATICS ADVANCED

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this cover page

Total Marks – 40

Section I – Multiple Choice 5 marks

- Attempt Questions 1 – 5
- Enter responses on the multiple choice answer sheet
- Allow 5 minutes for this section

Section II – 35 marks

- Attempt Questions 6 – 8
- Answer in the writing spaces provided
- Show all necessary working
- Allow 40 minutes for this section

Section I – Multiple Choice 1 mark per question **5 Marks**

Use the multiple choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

⊗ ● ○ ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

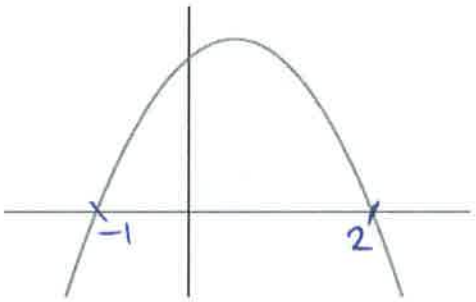
---

1. The first three terms of an arithmetic series are 3, 7 and 11. What is the 17th term?

(A) 67
(B) 71
(C) 595
(D) 666

$T_n = a + (n-1)d$
$= 3 + 4(n-1)$
$T_n = 3 + 4(16)$
$= 67$ ✓

2. The diagram below shows a parabola.



Which of the following could be the equation for this graph?

(A) $y = (2 + x)(1 - x)$
(B) $y = (2 + x)(1 + x)$
(C) $y = (1 + x)(2 - x)$
(D) $y = (1 - x)(2 - x)$ ✓

3. A student is asked to draw a graph of the function $y = \dfrac{x-5}{3x+2}$.

Their graph of the function needs to include which of the following asymptotes?

(A) $x = 5$ and $y = -\frac{2}{3}$

(B) $x = -\frac{2}{3}$ and $y = 0$

(C) $x = 5$ and $y = \frac{1}{3}$

(D) $x = -\frac{2}{3}$ and $y = \frac{1}{3}$

✓

4. If $f(x) = 4 - x^2$ and $g(x) = 2x - 1$, what would be an expression for $f \circ g(x)$?

(A) $4x^2 + 1$

(B) $3 - 4x^2$

(C) $3 + 4x - 4x^2$

(D) $5 + 4x - 4x^2$

$$f \circ g(x) = 4 - (2x-1)^2$$
$$= 4 - (4x^2 - 4x + 1)$$
$$= 4 - 4x^2 + 4x - 1$$
$$= 3 + 4x - 4x^2$$

✓

5. Consider the geometric series $a^2 + a + 1 + \frac{1}{a} + \frac{1}{a^2} + \cdots$ for $a > 1$.

Which of the following is the correct expression for the limiting sum of the series?

(A) $\dfrac{a^2}{a-1}$

(B) $\dfrac{a^3}{a-1}$

(C) $\dfrac{a^2}{1-a^2}$

(D) $\dfrac{a^3}{1-a}$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{a^2}{1-\frac{1}{a}}$$
$$= \frac{a^2}{\frac{a-1}{a}}$$
$$= \frac{a^3}{a-1}$$

✓

A C D C B

✓ 5

End of Section I

Section II **35 Marks**

Attempt Questions 6 to 8.
Allow about 50 minutes for this section.

In Questions 6 to 8 your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 marks)

Write your solutions in the spaces provided **Marks**

(a) Consider the arithmetic series $40 + 46 + 52 + \cdots + 256$

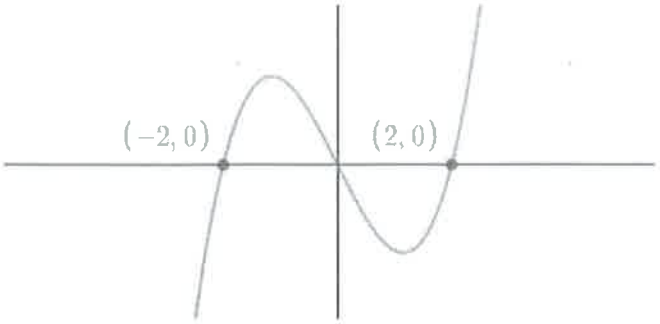
(i) How many terms are in the series? **2**

$T_n = a + (n-1)d$ ✓
$256 = 40 + 6(n-1)$
$216 = 6(n-1)$
$36 = n-1$
$n = 37$ ✓
$\therefore$ 37 terms

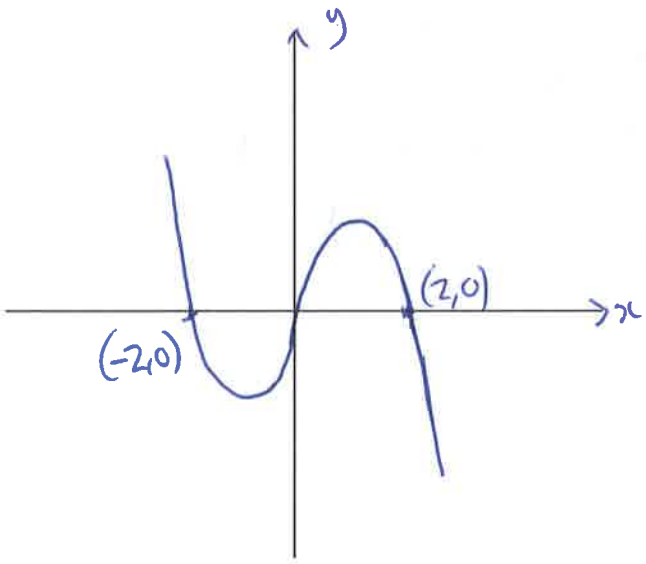
(ii) Find the sum of the series. **2**

$S_n = \frac{n}{2}(a+l)$ ✓
$S_{37} = \frac{37}{2}(40+256)$
$= \frac{37}{2}(296)$
$= 37(148)$ ✓
$= 5476$

(b) The diagram below shows the graph of a cubic function $y = f(x)$.

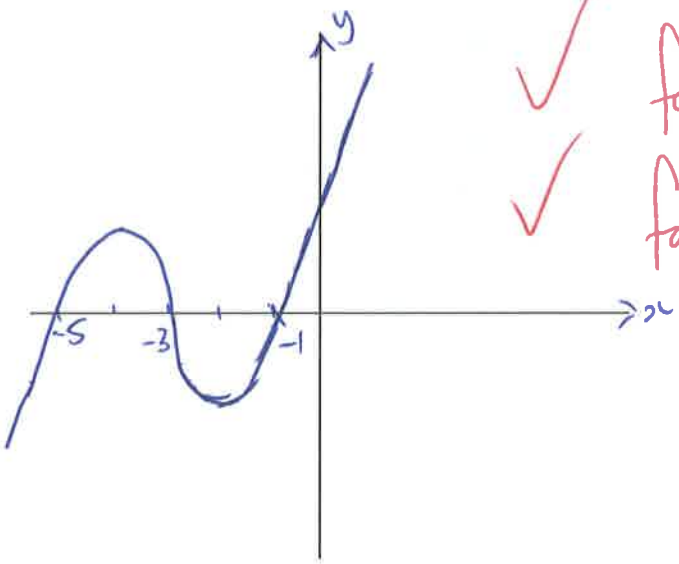


(i) In the space below, draw the graph of $y = -f(x)$ **1**



✓ for shape

(ii) In the space below, draw the graph of $y = f(x + 3)$ **2**



✓ for shape

✓ for intercepts

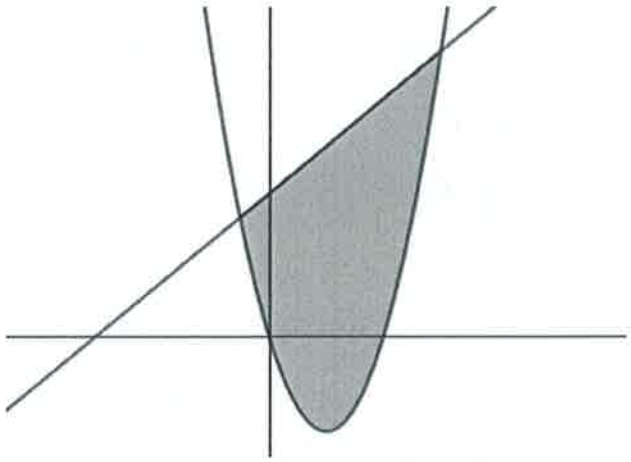
(c) Solve the inequation $|4x - 5| \leq 25$ **2**

$-25 \leq 4x - 5 \leq 25$ ✓

$+5$ $\quad -20 \leq 4x \leq 30$ ✓

$\div 4$ $\quad -5 \leq x \leq 7\frac{1}{2}$ ✓

(d) The diagram below shows the line $y = x + 6$ crossing the parabola $y = x^2 - 4x$ at two different points



Find the coordinates of the two points of intersection. **3**

$x^2 - 4x = x + 6$

$x^2 - 5x - 6 = 0$

$(x+1)(x-6) = 0$ ✓

$x = -1$ or 6

When $x = -1$, $y = 5$ ✓

$\therefore (-1, 5)$

When $x = 6$, $y = 12$ ✓

$\therefore (6, 12)$

So the two points of intersection are $(-1, 5)$ and $(6, 12)$

12

Question 7 (12 marks)

Write your solutions in the spaces provided **Marks**

(a) Braiden and Kiefer are employed by an engineering company.

Braiden accepts employment with an initial annual salary of \$50 000. In each of the following years his annual salary is increased by \$2500.

Kiefer accepts employment with an initial annual salary of \$50 000. In each of the following years his annual salary is increased by 4%.

(i) What is Braiden's annual salary in his thirteenth year? **2**

$T_n = a + (n-1)d$

$= 50\,000 + 12(2500)$

$= 50\,000 + 30\,000$

$= \$80\,000$

(ii) What is Kiefer's annual salary in his thirteenth year? **2**

$T_n = ar^{n-1}$

$= 50\,000(1{\cdot}04)^{12}$

$\doteq \$80\,051{\cdot}61$

(iii) By what amount does the total amount paid to Kiefer in his first twenty years exceed that paid to Braiden in his first 20 years? **3**

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = 10(100\,000 + 19(2500))$

$= 10(147\,500)$

$= \$1\,475\,000$

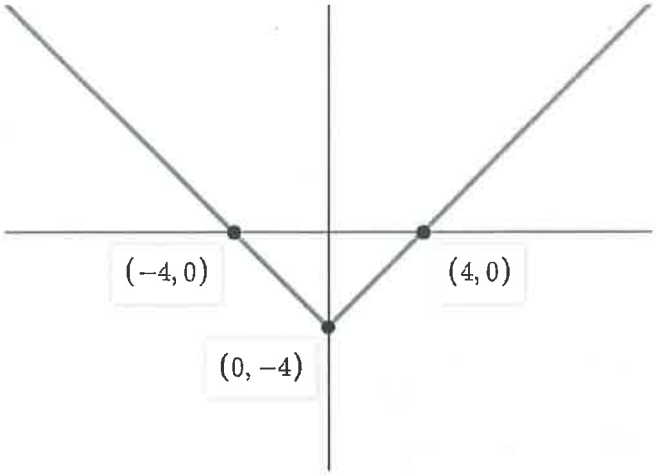
$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{50\,000(1{\cdot}04^{20} - 1)}{0{\cdot}04}$

$\doteq 1\,488\,903{\cdot}93$

$\therefore$ Difference $= \$13\,903{\cdot}93$

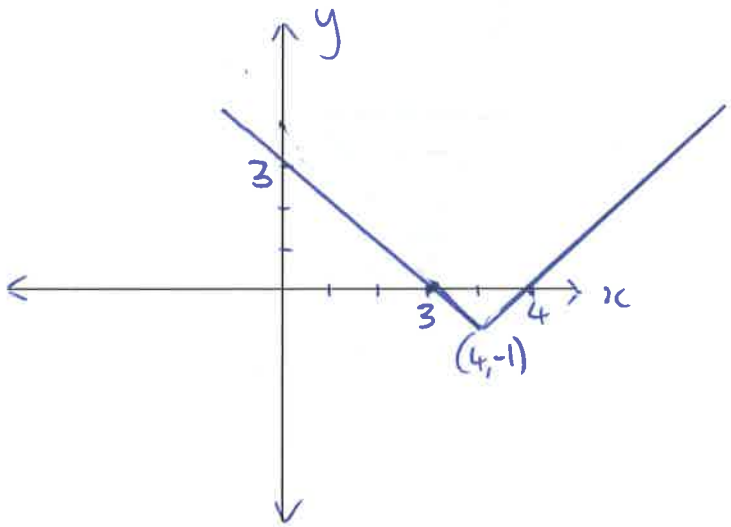
(b) The diagram below shows the graph of $f(x) = |x| - 4$.



(i) If the graph of $y = f(x)$ was translated 4 units to the right and then 3 units up, what would be the equation of the new graph? **1**

$f(x) = |x-4| - 1$

(ii) Draw this new translated graph on the coordinate axes provided below: **1**



(c) A rechargeable battery provides power for 100 hours when first purchased fully charged. After its first recharge, the battery only provides power for a further 80 hours. After its second recharge, the battery only provides power for a further 64 hours. Each subsequent recharge results in the battery providing 80% of its previous power output.

(i) How much additional time would you expect the battery to provide after its 10th recharge? Give your answer correct to the nearest minute. **2**

$100 + 80 + 64 + \ldots \quad a = 100 \quad r = 0.8$

$T_{11} = ar^{10}$

$= 100(0.8)^{10}$

$= 10.737\ldots$

$\doteq$ 10 hours 44 minutes

(ii) How many total hours would you expect the battery to provide? **1**

$S_\infty = \frac{a}{1-r}$

$= \frac{100}{1-0.8}$

$= \frac{100}{0.2}$

$= 500$ hours

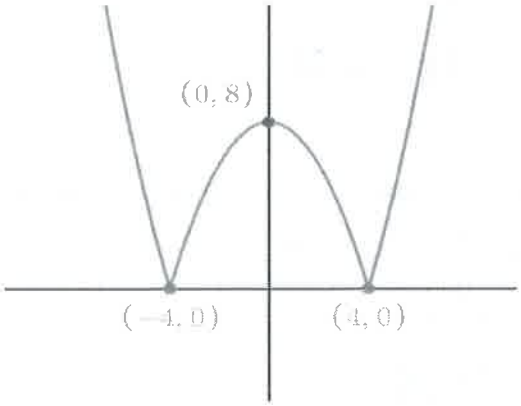
10

/12

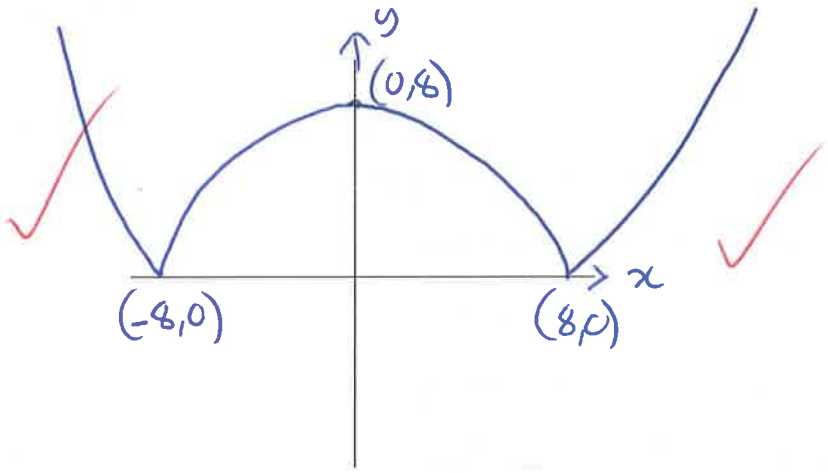
Question 8 (11 marks)

Write your solutions in the spaces provided **Marks**

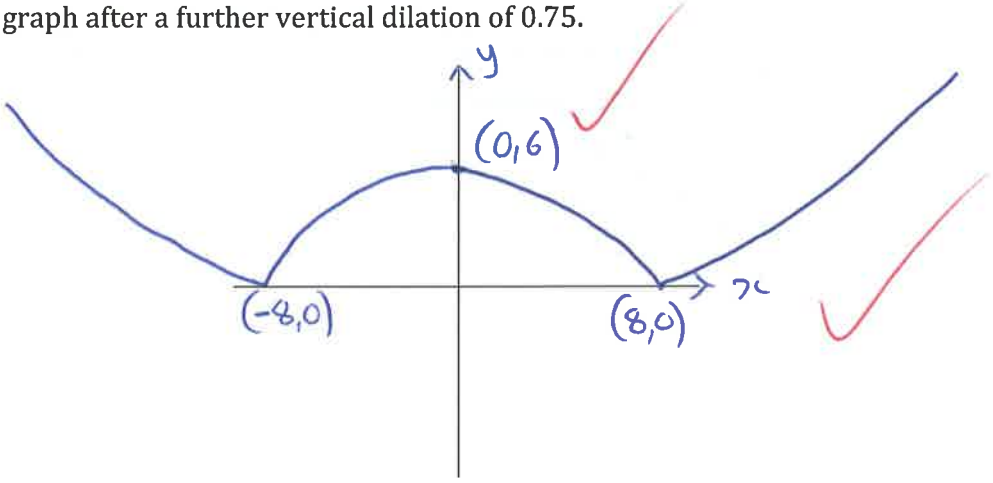
(a) The diagram below shows a function $y = f(x)$ with x intercepts at $(-4, 0)$ and $(4,0)$ and a y intercept at $(0,8)$.



(i) On the coordinate axes provided below, draw a new graph after dilating the function y = f(x) horizontally by a factor of 2. **2**



(ii) In the space provided below, take your dilation from part (i) and draw the graph after a further vertical dilation of 0.75. **2**



(b) Consider the series $101+92+83+74+\cdots$

(i) Find a simplified expression for the sum of the first n terms. **2**

$T_n = a+(n-1)d$

$= 101+(n-1)(-9)$

$= 101-9n+9$

$= 110-9n$

(ii) ~~What is the maximum number of terms for which the sum remains positive?~~ **2**

We need $T_n < 0$

$\therefore 110-9n < 0$

$9n > 110$

$n > 12\frac{2}{9}$

$\therefore n = 13$

$T_{13} = 110-9(13)$

$= 110-117$

$= -7$

So -7 is the first negative term

(iii) Find the number of terms that add together to generate a sum of -725. **3**

$S_n = \frac{n}{2}[2a+(n-1)d]$

$-725 = \frac{n}{2}[202-9(n-1)]$

$-1450 = n(202-9n+9)$

$-1450 = n(211-9n)$

$-1450 = 211n-9n^2$

$9n^2-211n-1450 = 0$

$n = \frac{211 \pm \sqrt{(-211)^2-4(9)(-1450)}}{2(9)}$

$= \frac{211 \pm \sqrt{96721}}{18}$

$= \frac{211 \pm 311}{18}$

$n = -\frac{100}{18}$ or $\frac{522}{18}$

$= -\frac{50}{9}$ or 29

But $n > 0$

$\therefore n = 29$ terms

The sum of the first 29 terms is -725

11

End of Task

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student Number

Mathematics Extension 2 – Multiple Choice Questions Answer Sheet

Attempt all questions:

Question	**1**	A ⬭	B ⬭	C ⬭	D ⬭
	2	A ⬭	B ⬭	C ⬭	D ⬭
	3	A ⬭	B ⬭	C ⬭	D ⬭
	4	A ⬭	B ⬭	C ⬭	D ⬭
	5	A ⬭	B ⬭	C ⬭	D ⬭