YEAR 12 SUGGESTED SOLUTIONS

ASSESSMENT TASK #1 2019

MATHEMATICS

SECTION I M/C

Q1. C

Q2. A

Q3. B

Q4. B

Q5. C

SECTION II

Q6. a) i) $M(-3,5)$ $N(1,3)$

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-3+1}{2}, \frac{5+3}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{8}{2}\right)$$

$$\boxed{MP = (-1,4)}$$

1 MK

ii) $$m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{3-5}{1--3}$$

$$= \frac{-2}{4}$$

$$m = -\frac{1}{2}$$

1 MK

SECTION II $M(-3,5)$ $N(1,3)$

Q6. a) iii) $\sqrt{(x+3)^2+(y-5)^2} = \sqrt{(x-1)^2+(y-3)^2}$ 1MK

(square both sides)

$$\cancel{x^2}+6x+9+\cancel{y^2}-10y+25=\cancel{x^2}-2x+1+\cancel{y^2}-6y+9$$

$$24-4y=-8x$$

$(\div 4)$ $$6-y=-2x$$

$$\boxed{y=6+2x}$$ 1MK

iv) $-\frac{1}{2} \perp \frac{2}{1}$ $(m_1 \times m_2 = -1)$

Hence a Perpendicular bisector 1MK

Q6. b) i) $y^2-6y=-8x-1$

$$y^2-6y+\left(\frac{-6}{2}\right)^2=-8x-1+\left(\frac{-6}{2}\right)^2$$

$$y^2-6y+9=-8x+8$$

$$(y-3)^2=-8(x-1)$$

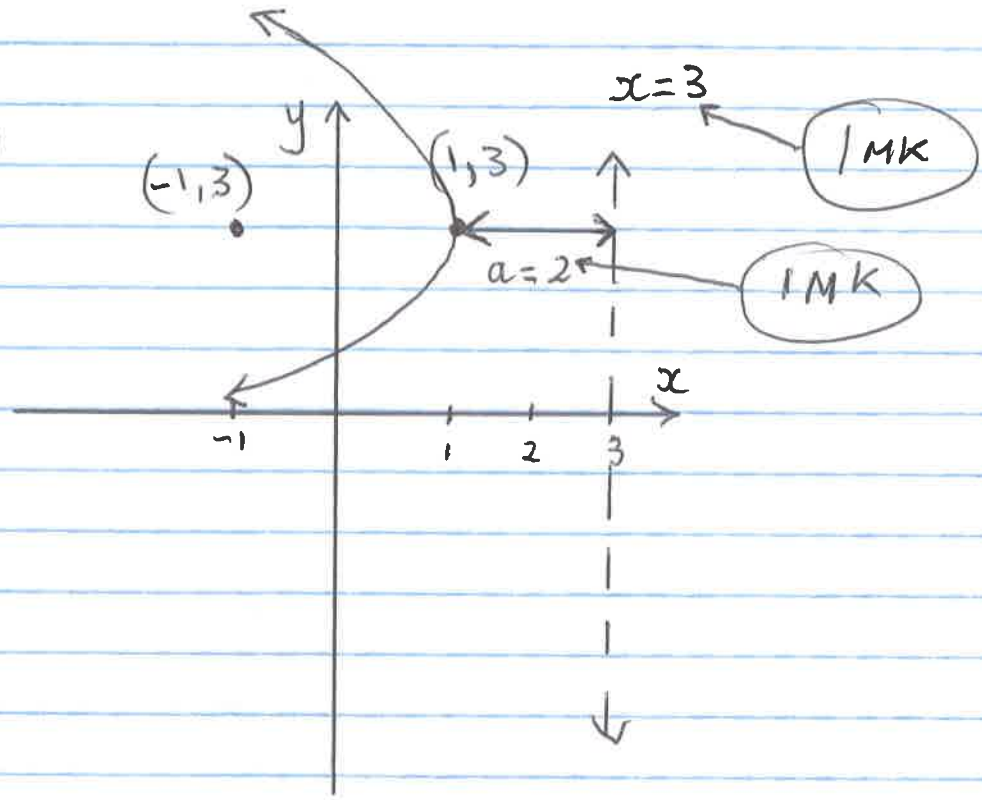
$$\boxed{(y-3)^2=-4(2)(x-1)}$$ 1MK

ii) Vertex is $(1,3)$ 1MK

$a=2$ 1MK

SECTION II

Q6 b) iii)



Q7. a) $y = \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - x$

i) y-intercept, make $x = 0$

$y = \frac{0}{4} + \frac{0}{3} - \frac{0}{2} - 0$

$\boxed{y = 0}$ 1MK

ii) $y' = x^3 + x^2 - x - 1$ 1MK

$= x^2(x+1) - (x+1)$

$= (x^2 - 1)(x+1)$

$= (x-1)(x+1)(x+1)$

$= (x-1)(x+1)^2$ 1MK

Let $y' = 0$ then $0 = x - 1$ and $0 = x + 1$

hence $x = -1$ and $x = 1$

SECTION II

$$y = \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - x$$

Q7) a) iii) Let $x = 1$

then $y = \frac{1}{4} + \frac{1}{3} - \frac{1}{2} - 1$

$y = \frac{-11}{12}$

hence, $\left(1, -\frac{11}{12}\right)$ 1 MK

Let $x = -1$

then $y = \frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1$

$y = \frac{5}{12}$

hence $\left(-1, \frac{5}{12}\right)$ 1 MK

(See over page for proofs of the nature of each stationary point.)

SECTION II

PROOF OF NATURE
from over page

(7) a) iii) $\frac{dy}{dx} = 0$ when $x = -1$ or 1

$\therefore \frac{dy}{dx} = x^3 + x^2 - x - 1$

$\frac{d^2y}{dx^2} = 3x^2 + 2x - 1$

when $x = -1$, $\frac{d^2y}{dx^2} = 3(-1)^2 + 2(-1) - 1$

$= 3 - 2 - 1$

$= 0$

when $x = -2$, $\frac{d^2y}{dx^2} = 3(-2)^2 + 2(-2) - 1$

$= 12 - 4 - 1$

$= 7$

when $x = 1$, $\frac{d^2y}{dx^2} = 3(0)^2 + 2(0) - 1$

$= -1$

$\therefore$ change in concavity
hence $(-1, \frac{5}{12})$ is a H.P.O.I

When $x = 1$ $\frac{d^2y}{dx^2} = 3(1)^2 + 2(1) - 1$

$= 3 + 2 - 1$

$= 4$

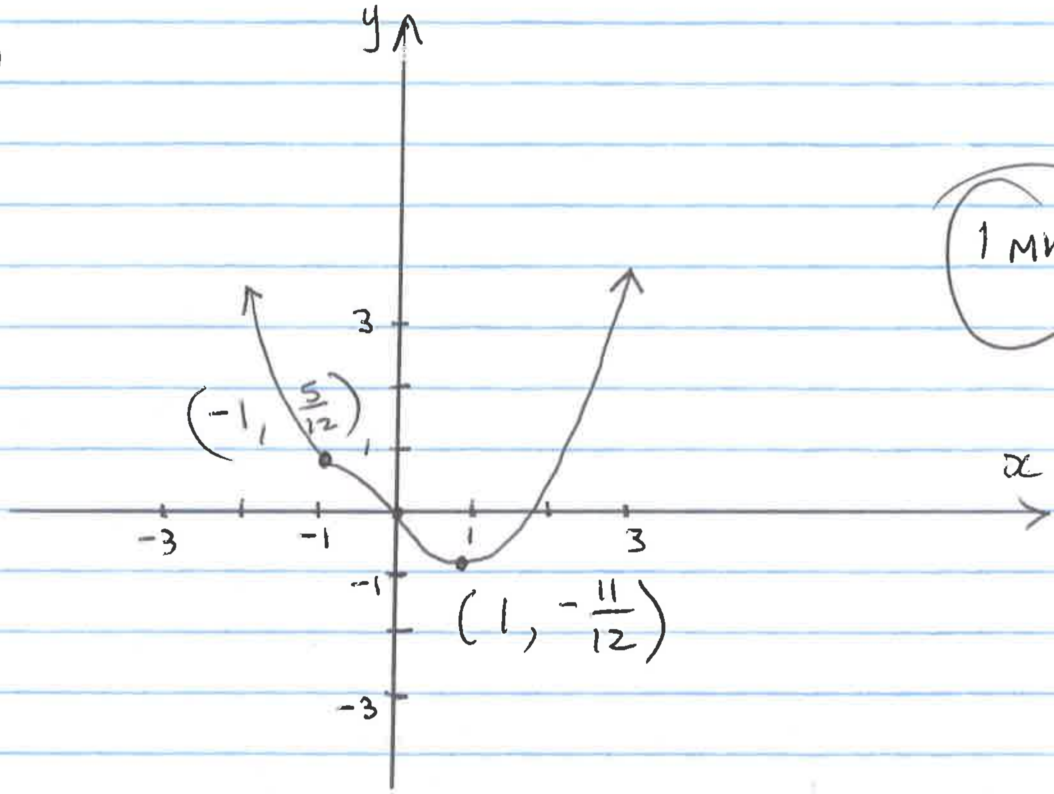
> 0

$\therefore (1, -\frac{11}{12})$ is a minimum turning point.

1 MK

SECTION II

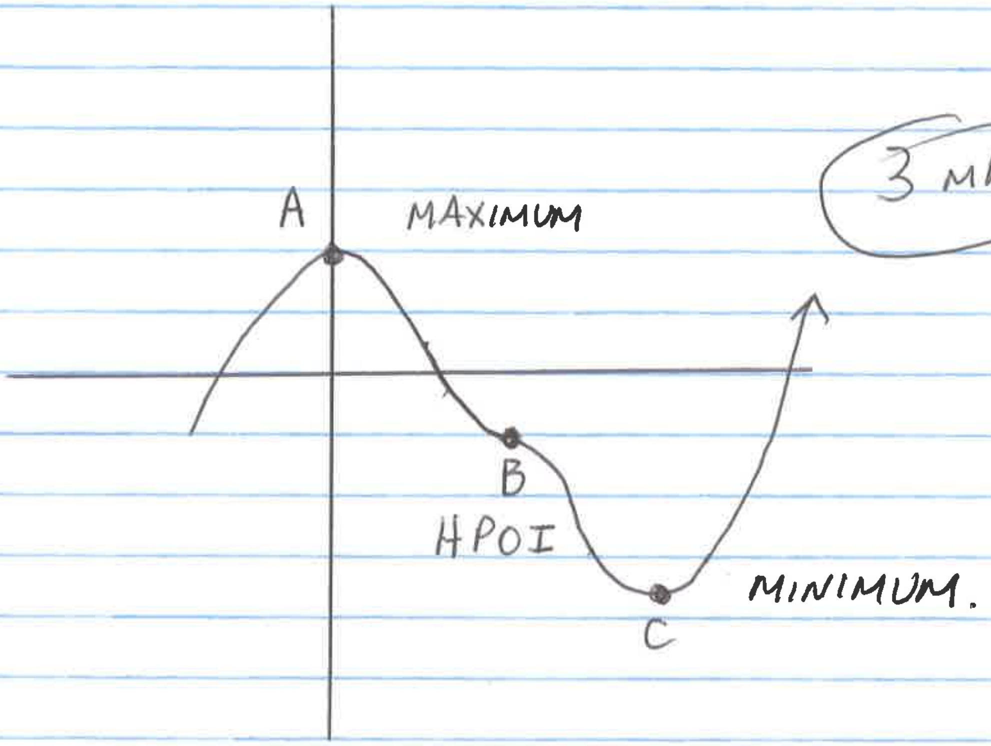
Q7) iv)



1 MK

SECTION II

Q7) b)



3 MKS

Q8. a) i) INDEFINITE

$$\int 2x - 1 \, dx$$

$$= x^2 - x + C$$

1 MK

NOTE: MUST SHOW THE C.

ii) DEFINITE

$$\int_1^4 3x^2 - x^{\frac{1}{2}} \, dx$$

$$= \left[\frac{3x^3}{3} - \frac{2}{3} x^{\frac{3}{2}} \right]_1^4$$

1 MK

$$= \left[x^3 - \frac{2x^{\frac{3}{2}}}{3} \right]_1^4$$

$$= \left(64 - \frac{2}{3}(8) \right) - \left(1 - \frac{2}{3} \right)$$

$$= 64 - \frac{16}{3} - \frac{1}{3}$$

$$\boxed{= \frac{175}{3} \quad \text{OR} \quad 58\frac{1}{3}}$$

1 MK

SECTION II

Q8. b) i) $y = x^3 - 4x$

x-intercept occurs when $y = 0$.

hence, $y = (2)^3 - 4(2)$

$= 8 - 8$

$= 0$ ✓ 1 MK

ii) $\int_1^2 x^3 - 4x \, dx = \left| \left[\frac{x^4}{4} - 2x^2 \right]_1^2 \right|$

$= \left| \left(\frac{2^4}{4} - 2(2)^2 \right) - \left(\frac{1^4}{4} - 2(1)^2 \right) \right|$

$= \left| (4 - 8) - \left(\frac{1}{4} - 2 \right) \right|$

$= \left| -4 + 1.75 \right|$

$= 2.25 \, u^2$ 1 MK

$\int_2^3 x^3 - 4x \, dx = \left[\frac{x^4}{4} - 2x^2 \right]_2^3$

$= \left(\frac{(3)^4}{4} - 2(3)^2 \right) - \left(\frac{(2)^4}{4} - 2(2)^2 \right)$

$= \left(\frac{81}{4} - 18 \right) - \left(\frac{16}{4} - 8 \right)$

$= \left(\frac{9}{4} - (-4) \right)$

$= 6.25 \, u^2$ 1 MK

Total Area is $2.25 + 6.25$

$= 8.5 \, u^2$ 1 MK

SECTION II

$$V = \pi \int_1^2 y^2 \, dx$$

(square both sides) $$= \pi \int_1^2 4 - x^2 \, dx$$ 1 MK

$$= \pi \left[4x - \frac{x^3}{3} \right]_1^2$$ 1MK

$$= \pi \left(4(2) - \frac{(2)^3}{3} \right) - \pi \left(4(1) - \frac{1}{3} \right)$$

$$= \pi \left(\frac{16}{13} \right) - \pi \left(\frac{11}{3} \right)$$

$$\boxed{= \frac{5\pi}{3} \text{ units}^3}$$ 1 MK