2022 HSC ASSESSMENT TASK 3

Mathematics Advanced

Year 12

General Instructions

- Working time – 45 minutes
- Weighting 25%
- Write using black or blue pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the end of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:

35

Section I – 5 marks

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – 30 marks

- Attempt Questions 6 – 7
- Allow about 37 minutes for this section
- Write your solutions in the space provided

Section	Marks
Section I	**/5**
Section II	**/30**
Total marks	**/35**

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Section I

5 marks

Attempt Questions 1- 5

Allow about 8 minutes to complete this section

Use the multiple-choice answer sheet for Questions 1-5.

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1 Which expression is equal to $\int tan^2x\ dx$?

A. $\frac{tan^3x}{3} + C$

B. $\tan x - x + C$

C. $\tan x + x + C$

D. $sec^2x + C$

2 The population N of a town, after t years, is given by the formula $N = N_0 e^{0.04t}$, where N_0 is the initial population.

Which expression represents the number of years it takes until the town has doubled its population?

A. $\frac{2 \log_e 100}{5}$

B. $4 \log_e 2$

C. $2 \log_e 25$

D. $25 \log_e 2$

3   The graph below shows the velocity of a particle, v metres per second, as a function of time, t seconds.

When is the acceleration of the particle equal to zero?
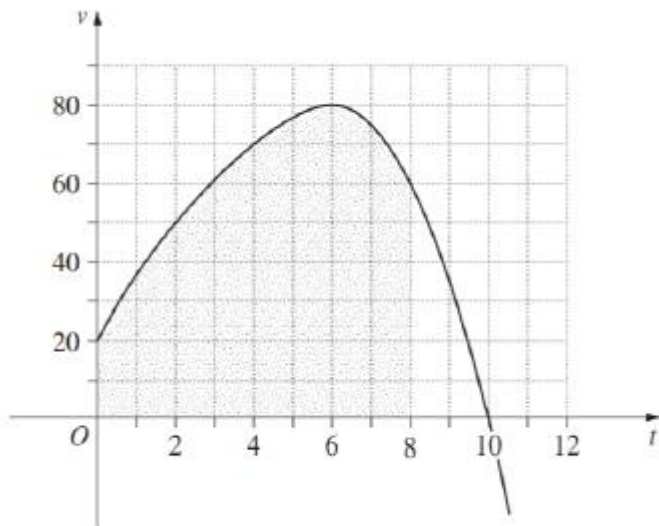
A. $t = 0$

B. $t = 10$

C. $t = 6$

D. $t = 8$

4   A particle is moving along the x axis.
The displacement of the particle at time t seconds is x metres.
At a certain time $\frac{d^2x}{dt^2} = -\frac{2m}{s^2}$ and $\frac{dx}{dt} = \frac{1m}{s^{-1}}$.

Which statement describes the motion of the particle at that time.

A. The particle is moving to the right when increasing speed.

B. The particle is moving to the left when increasing speed.

C. The particle is moving to the right when decreasing speed.

D. The particle is moving to the left when decreasing speed.

5. The derivative of $e^{x\sin 3x}$ is

   A. $e^{3x\cos 3x}$

   B. $e^{x\sin 3x}(\sin 3x + 3x\cos 3x)$

   C. $e^{x\sin 3x}$

   D. $e^{x\sin 3x}(\sin 3x - 3x\cos 3x)$

End of Section I

Section II

Attempt Questions 6 – 7
Allow about 37 minutes to complete this section

Question 6 (15 marks)

a.   Differentiate sin^2x.   **2**

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b.   Hence, calculate the definite integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} sinx cosx \, dx$.   **2**

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c.   Find $\int \cos \frac{x}{5} \, dx$.   2

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d.   Find $\int \frac{2x+2}{4x^2+8x+1} \, dx$ 1

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e. (i) Calculate the definite integral $\int_1^e \left(\frac{8}{x}\right) dx$. **1**

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(ii) Let a be a positive real number less than one. **2**

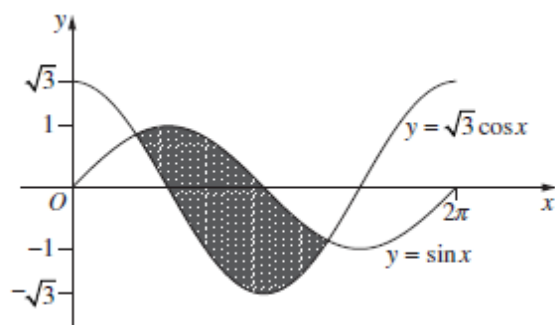Find the exact value such that $\int_a^e \left(\frac{8}{x}\right) dx = 16$.

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Question 6 continued over page…

f. Consider the diagram showing the curves $y = \sin x$ and $y = \sqrt{3} \cos x$.

(i) Show that the curves intersect at $\frac{\pi}{3}$ and $\frac{4\pi}{3}$. **2**

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(ii) Hence, find the area of the shaded region between the two curves **3**
     $y = \sin x$ and $y = \sqrt{3} \cos x$.

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Question 7 (15 marks)

a.   A particle moves in such a way that its velocity at any time is given by the equation:

$\dot{x} = 8 - 16 \sin t$

(i)   What is the initial acceleration of this particle?   **1**

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(ii)   When is the particle first at rest?   **1**

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(iii)   Given the particle is initially at the origin find an equation for the   **2**
displacement (x) at a time, t.

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b. The population of a colony is modelled by the equation $P = 1000e^{kt}$.
Where time, t is measured in weeks and k is a constant.

(i) Determine the initial population of the colony. **1**

(ii) After 5 weeks the population is 15 000. Show that $k = \frac{1}{5}\log_e 15$. **2**

(iii) Find an expression for t when the population will be exactly 2 500 000? **2**

c. The velocity of a particle is given by $\dot{x} = 1 - 2\cos t$, where x is the displacement in metres and t is the time in seconds.

Initially the particle is 3 metres to the right of the origin.

(i) Find the initial velocity of the particle. **1**

(ii) Find the maximum velocity of the particle. **1**

(iii) Find the displacement, x, of the particle in terms of t. **2**

(iv) Find the position of the particle when it is at rest for the first time.   **2**

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End of Assessment.

2022 Year 12 – Mathematics Advanced Assessment 3

Multiple Choice Answer Sheet

Student Number

Instructions for use:

• Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A)   2   (B)   6   (C)   8   (D)   9

A ⬭   B ⬤   C ⬭   D ⬭

• If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

✖   ⬤   ⬭   ⬭

• If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

✖   ✖   ⬭   ⬭

• Attempt all multiple-choice questions.

Question	**1**	A ○	B ○	C ○	D ○
	2	A ○	B ○	C ○	D ○
	3	A ○	B ○	C ○	D ○
	4	A ○	B ○	C ○	D ○
	5	A ○	B ○	C ○	D ○