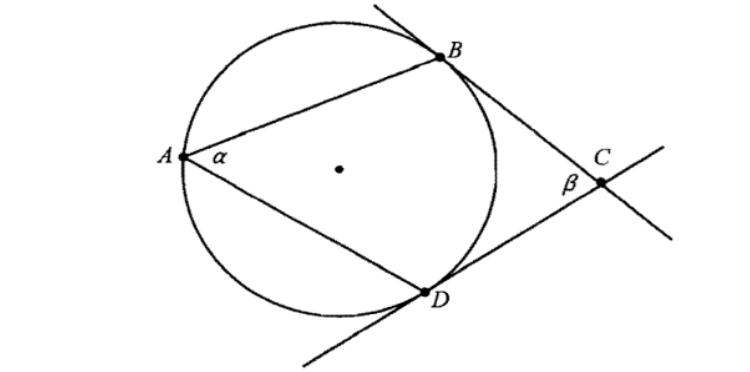
Section 1: Multiple Choice

Q1. In the diagram below BC and DC are tangents



Which of the following statements are true?

a) $\alpha + \beta = 180°$

b) $\alpha + 2\beta = 180°$

c) $2\alpha + \beta = 180°$

d) $2\alpha - \beta = 90°$

Q2. Michael made an error when trying to prove that $3^{2n} - 1$ is divisible by 8, for $n \geq 1$.

His working out is below

Step 2: Assume the result true for $n = k$

$3^{2k} - 1 = 8P$ where P *is* an integer.

Hence $3^{2k} = 8P + 1$

Step 3: To prove the result is true for $n = k + 1$

$RTP: 3^{2(k+1)} - 1 = 8Q$ where Q is an integer.	Line 1
$LHS = 3^{2k+2} - 1$	Line 2
$= 3^{2k} \times 3^2 - 1$	
$= (8P+1) \times 3^2 - 1$ (*using the assumption*)	Line 3
$= 72P + 1 - 1$	Line 4
$= 72\text{P}$	
$= 8(9\text{P})$	
$= 8Q$	
$= RHS$	

In which line did Michael make the error?

a) Line 1

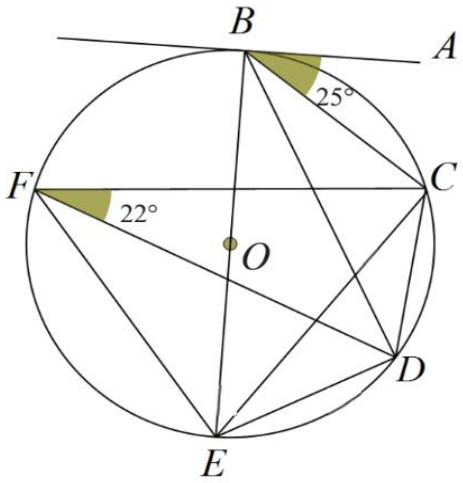
b) Line 2

c) Line 3

d) Line 4

Q3. A circle with O has a tangent AB, diameter BE, $\angle ABC = 25°$ and $\angle CFD = 22°$

NOT TO SCALE



What is the size of $\angle ECD$?

a) $25°$

b) $43°$

c) $22°$

d) $47°$

Q4. After using the substitution $= 1 - u^2$, the integral $\int \frac{x}{\sqrt{1-x}} dx$ would become which of the following?

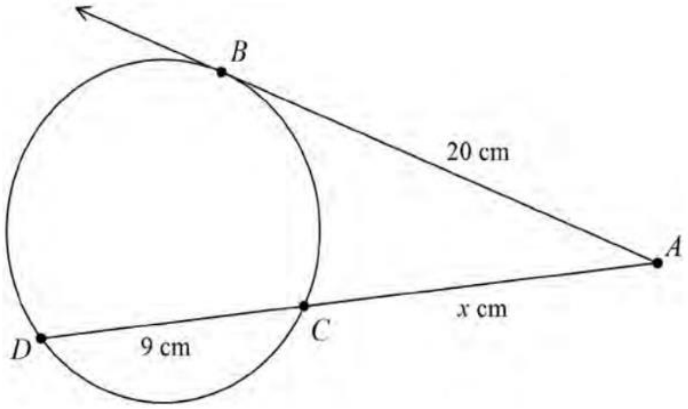
a) $-2 \int 1 - u^2 \ du$

b) $\frac{2}{3} \int u^2 - 1 \ du$

c) $\frac{1}{4} \int 1 - u^2 \ du$

d) -2$\int u^2 - 1 \ du$

Q5.



In the diagram above, $AB = 20\ cm,\ CD = 9\ cm$ and $AC = x\ cm$. What is the value of x?

a) *16*

b) *11*

c) *5*

d) *6*

End of Section 1: Multiple Choice

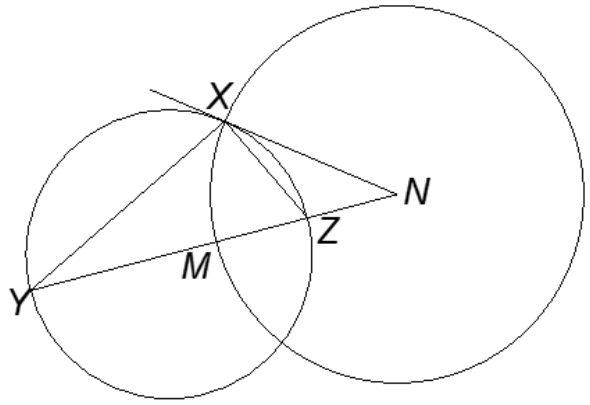
Assessment continues over the page

Section 2

Q6. **13 Marks**

a) Using the substitution $u = 1 + x^2$ to integrate $\int 2x\sqrt{1+x^2}\ dx$ **3**

b) The diagram below shows a tangent to a circle at x and a secant YZ in the same circle. The tangent and secant intersect at N. The circle with the radius NX intersects YZ at M. **3**



By letting $\angle ZXN = \alpha$ and $\angle ZXM = \beta$, prove that MX bisects $\angle YXZ$.

c) Prove by Mathematical Induction that for all positive integers n

$(1 \times 4) + (2 \times 5) + (3 \times 6) + \cdots + [n \times (n \times 3)] = \frac{1}{3}n(n+1)(n+5)$ **3**

d) Using the substitution $u = 1 + \sqrt{x}$ to evaluate $\int_1^4 \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$ **4**

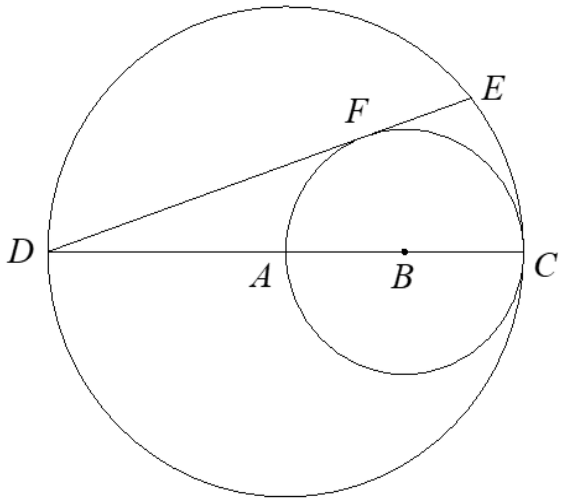
Q7. **12 Marks**

a) Prove by Mathematical Induction that

$(n+1)(n+2)\ldots\ldots(2n-1)(2n) = 2^n[1\times 3\times 5\ldots\ldots\ldots(2n-1)]$ **3**

for integers $n \geq 1$

b) In the diagram below, DC is a diameter of the larger circle centred at A and AC is a diameter of the smaller circle centred at B. The chord DE is a tangent to the smaller circle at F, and $DC = 12cm$.



(i) Prove $\Delta\, BDF \,|||\, \Delta CDE$ **2**

(ii) Calculate the exact area of $\Delta\, CDE$ **3**

c) Evaluate $\int_0^2 x(x-1)^c dx$,

using the substitution $u = x - 1$, where c is a positive integer **4**

Note: Your evaluation needs to consider c as an odd and even integer.

End of Task