
Student Number

ST PIUS X COLLEGE
CHATSWOOD

HSC 2021 Stage 6
Year 12

Assessment Task #2

25% of School Based Assessment

MATHEMATICS Extension 1

SUG sols here ish

General Instructions

- Handout date: **Friday, March 26, 2021**
- Due date: **Friday, April 30, 2021**
- Read each section carefully and complete all questions.
- Each student is to submit an individual task and it must feature their own work.
- Any form of plagiarism will result in a mark of ZERO being awarded.
- NESA approved calculators may be used
- Marks may be deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total Marks – 50

Section I **40 marks**
Probability, Logarithm Function, Polynomials and Graphing, Vectors

- Attempt Questions 1 – 4
- Show all necessary working
- ***Write your solutions in the space provided***

Section II **10 marks**

Learning video and support

- Required on a thumb drive April 30
- Details to follow on p19

Marcel Qs 1 2 3.
Tom Qs 4 5.

IMPORTANT NOTES ON TASK AND SUBMISSION:

- This task is an *ALTERNATIVE* task featuring an opportunity for you to research your work and investigate concepts to a deeper level. This task also allows you adequate time (5 weeks) to appropriately answer questions and submit your work.
- Students may only ask for help or clarification of any misunderstandings during Week 10 of Term 1 (Monday, March 29 to Thursday, April 1) in 2021. Your teachers will not be responding to any enquiries across the Easter holiday break (Friday, April 2 to Monday, April 19).
- Any submissions after the due date of Friday, April 30 will NOT be accepted.
- If you use a website or other resources, you must provide a list of these resources (bibliography) at the end of your assignment.
- This task requires thorough mathematical reasoning and calculations for some questions. Marks may be deducted for not showing a clear understanding of a concept or how an answer has been obtained.
- Ensure that any graphs from Desmos, GeoGebra or any other software package are clearly presented, including the equations of any graphs.
- Additional writing space is provided at the end of this assignment. If you require further writing space to answer any questions, you must neatly staple any attached pages to the back end of this assignment.

5 10 10 5 1.

Outcomes to be assessed:

$\frac{1}{32}$.

A student:

› ME12-1 applies techniques involving proof or calculus to model and solve problems

› ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

› ME12-5 applies appropriate statistical processes to present analyse and interpret data

› ME12-6 chooses and uses appropriate technology to solve problems in a range of contexts

› ME12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Task Description Part A 40marks
Answer the following questions in the spaces provided in this paper
Question 1 **10 Marks**

You are required to perform a probability experiment 60 times. This can be done in class time during week 10.

The experiment is to drop 5 5c coins on a surface and record the number of heads in a table like this

i) You must include 3 or 4 pictures of you doing the experiment and recording your data. These should fit in the space below

Number of heads	Tally marks	Frequency	Relative Frequency
0		2	$\frac{1}{30}$
1		9	$\frac{3}{20}$.
2		19	$\frac{19}{60}$
3		22	$\frac{11}{30}$
4		8	$\frac{2}{15}$.
5		0	
		60	

sample answer shown

1 for a good job here.

Of course, Relative Frequency is the Frequency divided by the total number of trials (60)

4 photos.

= 1 mark.

1 fellow is doing a computer code to generate his numbers.

1 for photos of boy doing work.

ii) Using your relative frequencies as the probability, find the Expected Value for the number of heads. All working needs to be shown. **2**

$E(H) = 0\times\frac{1}{30} + 1\times\frac{3}{20} + 2\times\frac{19}{60} + 3\times\frac{11}{30} + 4\times\frac{2}{15}$

$= 2\frac{5}{12}$.

iii) You are also required to find the variance on the basis of your experimental results. Show that you know how to do this in 2 ways. **2**

$\text{Var } H = E(X^2) - \mu^2$.

$= 0\times\frac{1}{30} + 1\times\frac{3}{20} + 4\times\frac{19}{60} + 9\times\frac{11}{30} + 16\times\frac{2}{15} - \left(2\frac{5}{12}\right)^2$

$= \frac{137}{20} - \left(\frac{29}{12}\right)^2 = \frac{727}{720}$.

$\text{Var } H = \sum(x-\mu)^2 P(x)$

$= \frac{1}{30}\left(2\frac{5}{12}\right)^2 + \frac{3}{20}\left(1\frac{5}{12}\right)^2 + \frac{19}{60}\times\left(\frac{5}{12}\right)^2 + \frac{11}{30}\times\left(\frac{7}{12}\right)^2 + \frac{2}{15}\left(1\frac{7}{12}\right)^2$

$\frac{793}{1440} + \frac{661}{1440} = \frac{727}{720}$

iv) 1/32 5/32 10/32 10/32 5/32 1/32 **2**

Do the Expected Value and the Variance for the theoretical probabilities also.

$E(H) = 0\times\frac{1}{32} + 1\times\frac{5}{32} + 2\times\frac{10}{32} + 3\times\frac{10}{32} + 4\times\frac{5}{32} + 5\times\frac{1}{32}$

$= 2\frac{1}{2}$

$\text{Var} = 6\frac{1}{4}\times\frac{1}{32} + 2\frac{1}{4}\times\frac{5}{32} + \frac{1}{4}\times\frac{10}{32} + \frac{1}{4}\times\frac{10}{32} + 2\frac{1}{4}\times\frac{5}{32} + 6\frac{1}{4}\times\frac{1}{32}$

$= \frac{5}{4}$

OR $\text{Var} = E(x^2) - \mu^2 = \frac{1}{32}\times 0^2 + \frac{5}{32}\times 1 + \frac{10}{32}\times 4 + \frac{10}{32}\times 9 + \frac{5}{32}\times 16 + \frac{1}{32}\times 25 - 6\frac{1}{4}$.

$= \frac{5}{4}$.

v) Comparing your results with the Theoretical probabilities, make judgements about how reasonable your experimental results were. Discuss the nature of your experimental data and how the measures of Expected Value and Variance represent your data. (less than 60 words) **2**

The expected ~~stat~~ value μ for the data is $2\frac{5}{12}$ compared to $2\frac{1}{2}$ for theoretical. This is pretty close and suggests experimental data is pretty reasonable.

The Var for data is $1\frac{7}{120}$ compared to $\frac{5}{4}$ for the theoretical probability.

Again the fact that these numbers are pretty close suggest the data is to be expected.

Question 2 **10 marks**

Use Geogebra or Desmos to graph functions to help you answer these questions. When you print your responses include the equation of the function clearly shown (from the program or hand-written).

i) Graph the function $y = \log_e x$. Draw 3 tangents to this curve and clearly show how the gradient of the curve is directly related to its position. What does this demonstrate? **2**

1 mark for 3 tangents with m correctly found / shown.

1 mark for relating $m = \frac{1}{x}$.

iii) Use Geogebra or Desmos to print $y = \log_e x$ and $y = \log_e x^3$. **2**

Choose three x values including one $x < 1$.
Use the logarithm laws, and addition of ordinates to confirm the position of the x cubed curve.
Also interpret its position in terms of a dilation.

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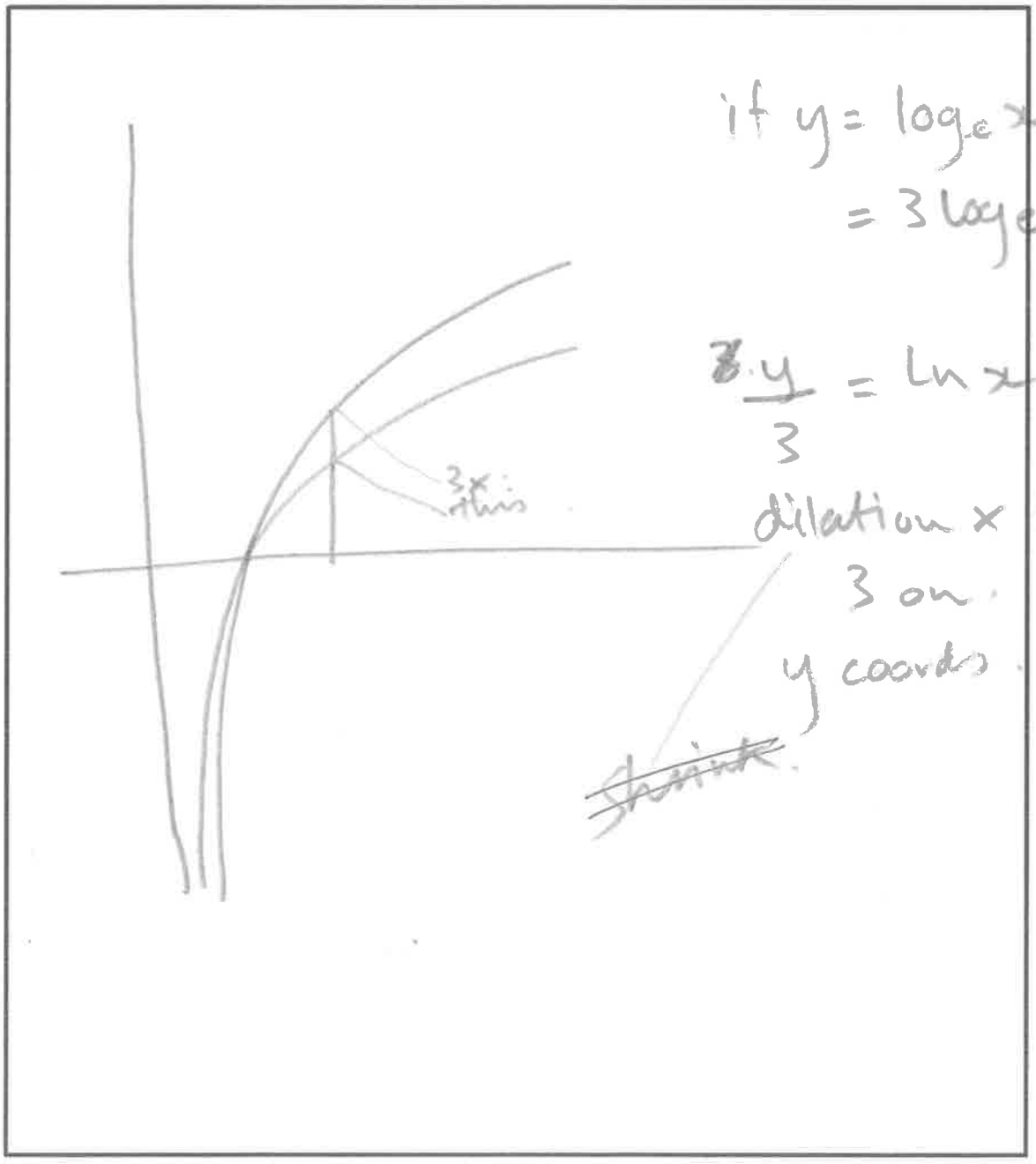

ii) Draw the 3 curves $y = \log_e 2x$, $y = \log_e 3x$, $y = \log_e x$. Choose two x values and draw tangents to each curve at these values. What do you notice? Justify your observation with calculus. **2**

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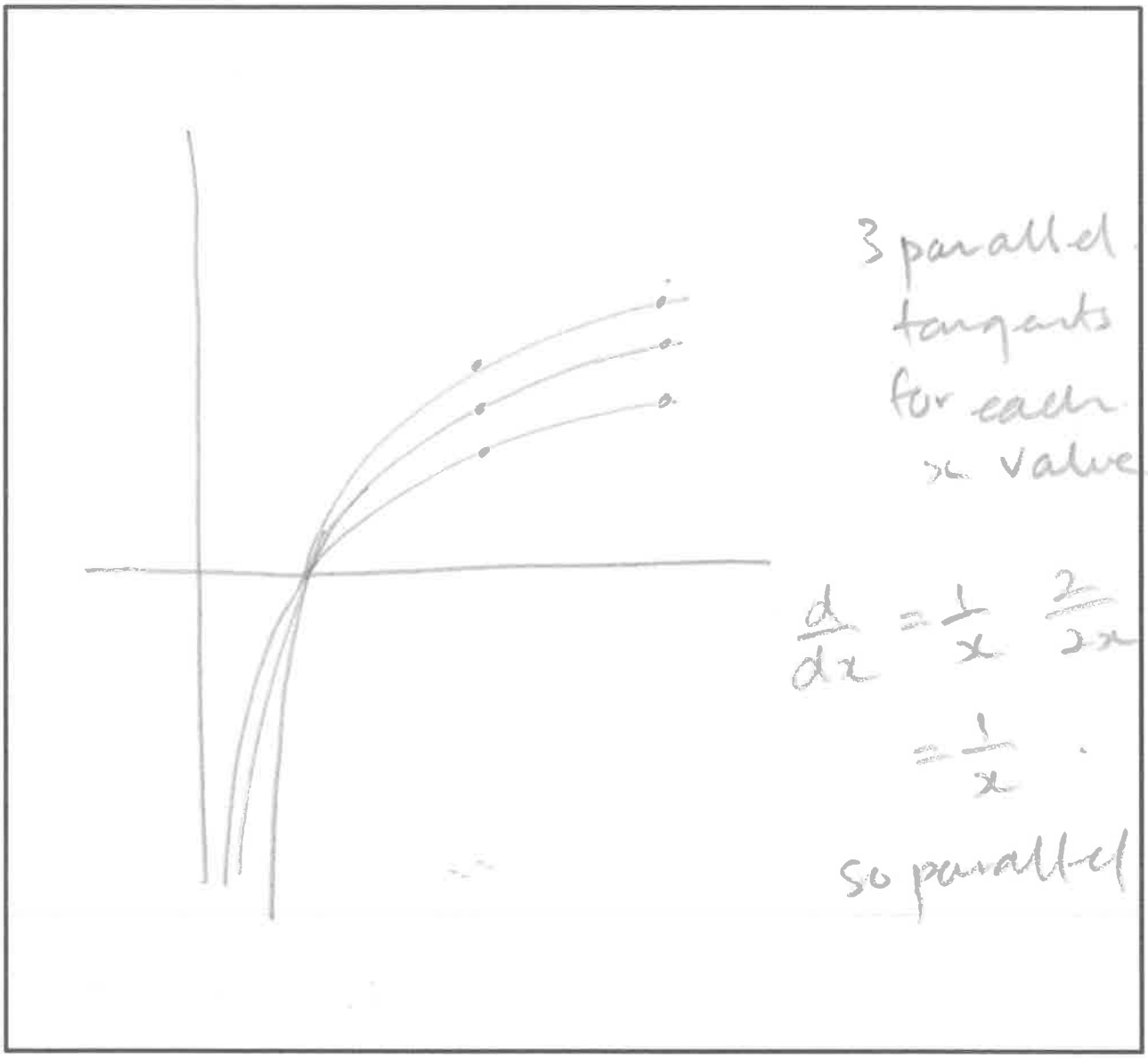

v) Graph $y = \log_e x$ and $y = \log_e \frac{1}{x}$. Note an observation and justify it both in terms of transformations and the logarithm laws. **2**

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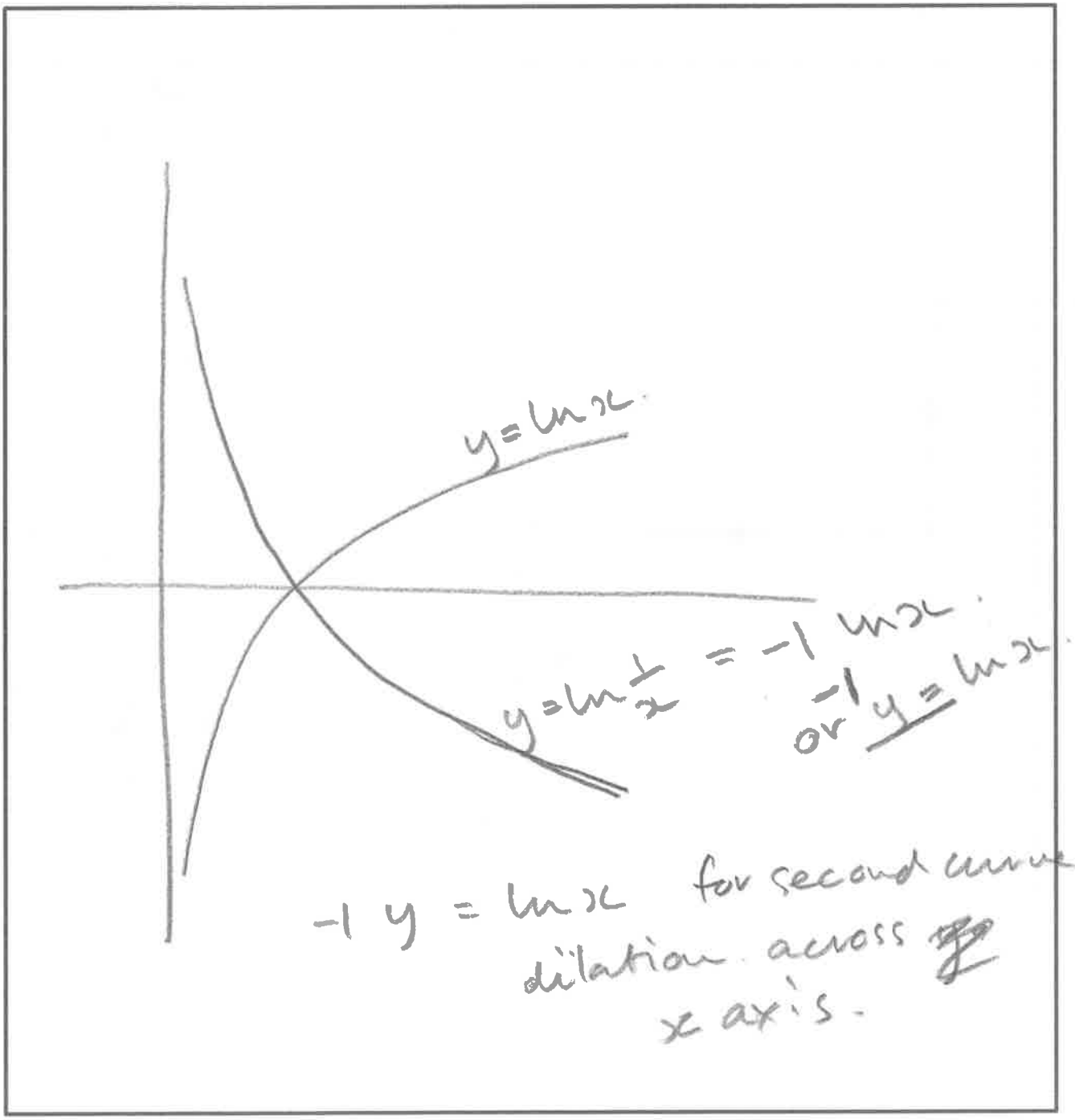
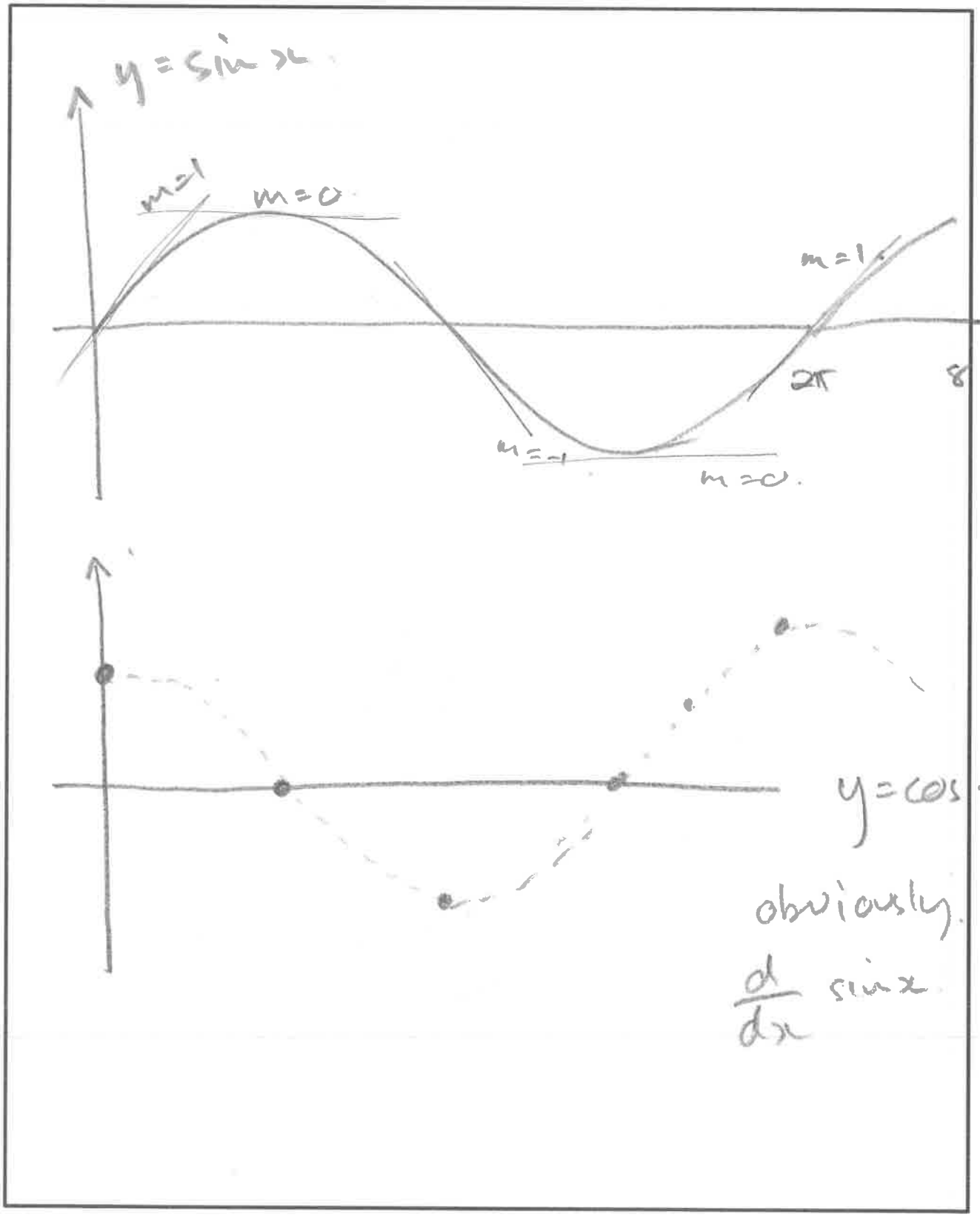


iv) Print $y = \sin x$ for the domain $0 \leq x \leq 8$ radians. Draw 6 tangents to the sine curve and measure their gradients. Print a blank number plane for the same domain and glue it in below the first. Plot the tangent gradients as the y value for each x value on the plane below. What do you observe? **2**



Question 3 **10 marks**

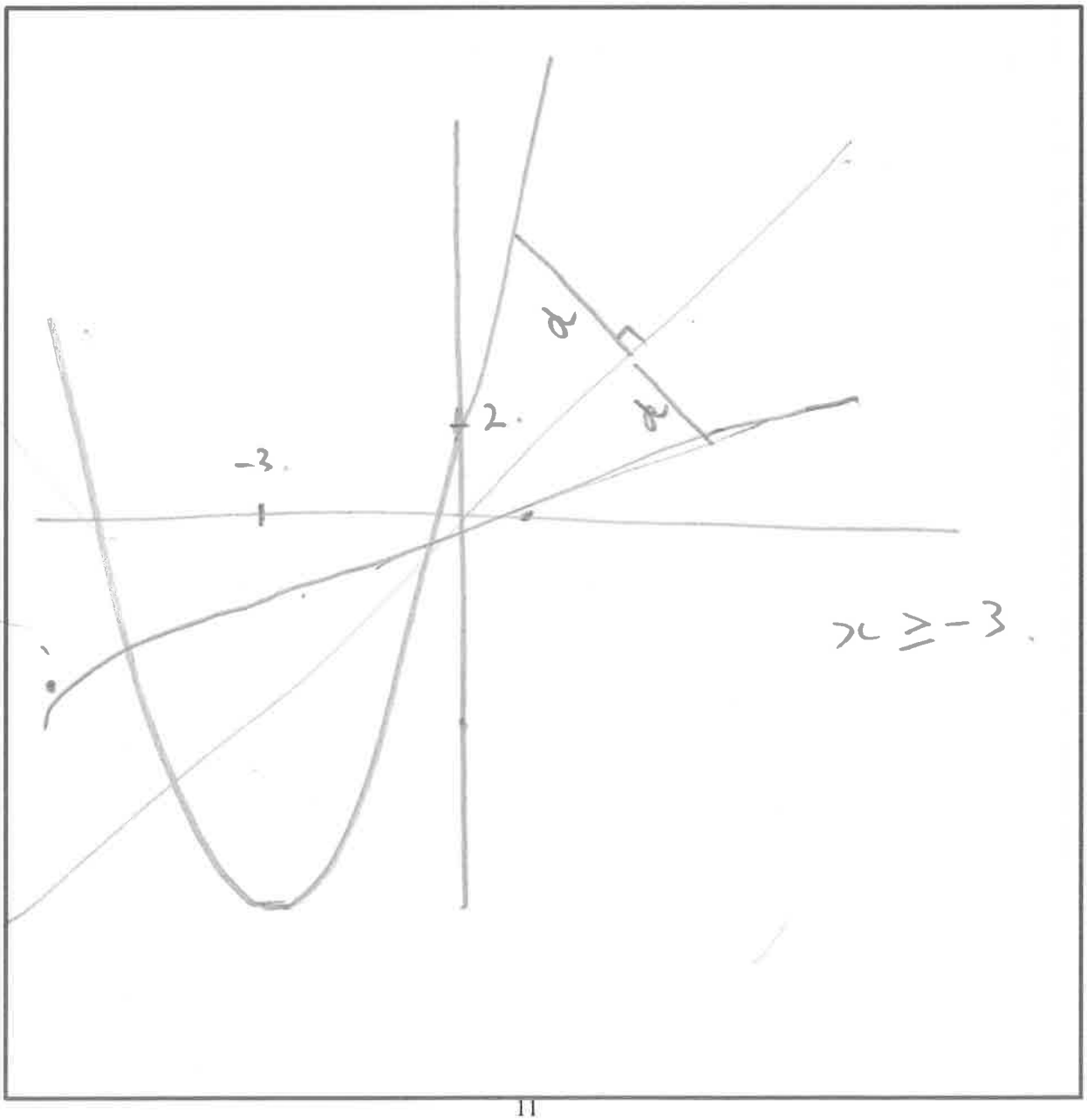
i) Graph the curve $y=(x+3)^2-k$ where k is the number of letters in your middle name. Specify a domain including the origin for which the function has an inverse.

a. Give the equation of this inverse and graph it also. **1**

b. Graph the line $y=x$, together with the original and the inverse **1**
and demonstrate for 3 points how the inverse curve comes from a geometrical transformation of the original.

me VAUGHAN -7.

want boys to show reflection means
~~the same~~ go ⊥ to $y=x$ and
the same distance from $y=x$.

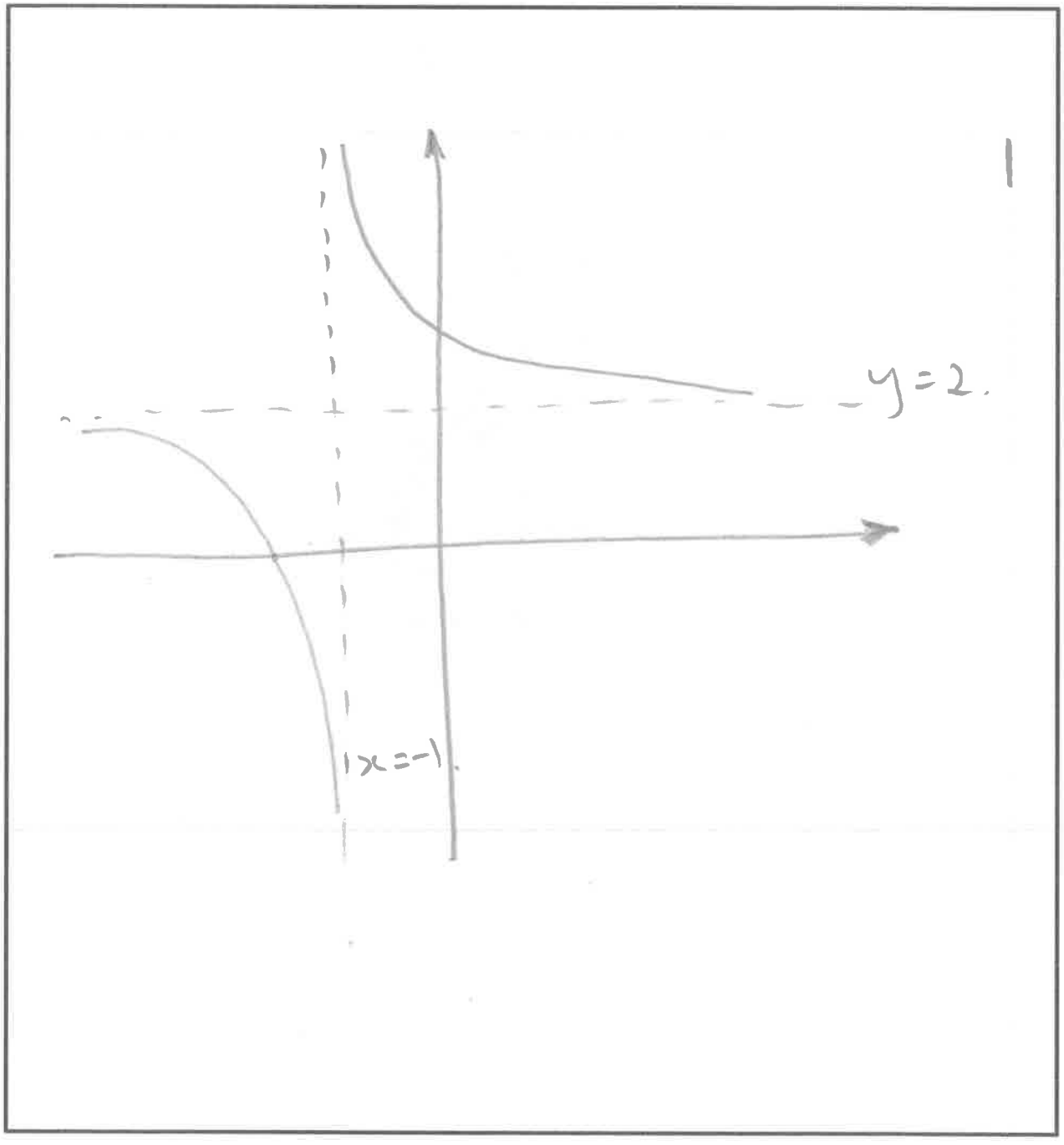


ii) Choose integer values for a, $0 < a < 6$ and b, $-4 < b < 4$. **2**
Create a function with horizontal asymptote $y = a$ and vertical asymptote $x = b$.
Graph your curve on Geogebra or Desmos.

$b = -1$

$a = 2$

$y = \dfrac{2x+3}{x+1}$ 1



1

iv) Any polynomial can be written as a product of linear factors and quadratic factors which have no real roots

i) In terms of the quadratic formula, what needs to be true if a quadratic is to have no real roots. **1**

$b^2 - 4ac < 0$.

ii) Create 2 quadratic factors, with no real roots. $x^2 + 2$ $x^2 + 1$. **1**

They can have a coefficient of x^2 which is 1.

Create 2 linear factors which have integer roots. Also include a quadratic with 2 roots which are surds.

$(x-1)(x+2)$

$x^2 - 3x + 1$

roots $\frac{3 \pm \sqrt{5}}{2}$

iii) The product of these factors will be a degree 8 polynomial. **1**

Find the sum of the roots by addition. Expand your degree 8 polynomial and show how the sum of the roots can be found from the coefficients.

$1 + -2$ $\frac{3+\sqrt{5}}{2}$ $\frac{3-\sqrt{5}}{2}$

$\Sigma = -1 + \frac{3}{2} + \frac{3}{2} = 2$

$(x^2 + x - 2)(x^2 - 3x + 1)(x^2 + 1)(x^2 + 2)$

$(x^4 - 2x^3 - 4x^2 + 7x - 2)(x^4 + 3x^2 + 2)$

$x^8 - 2x^7 - x^6 \ldots$

$-\frac{b}{a} = 2$

ONLY LUCK

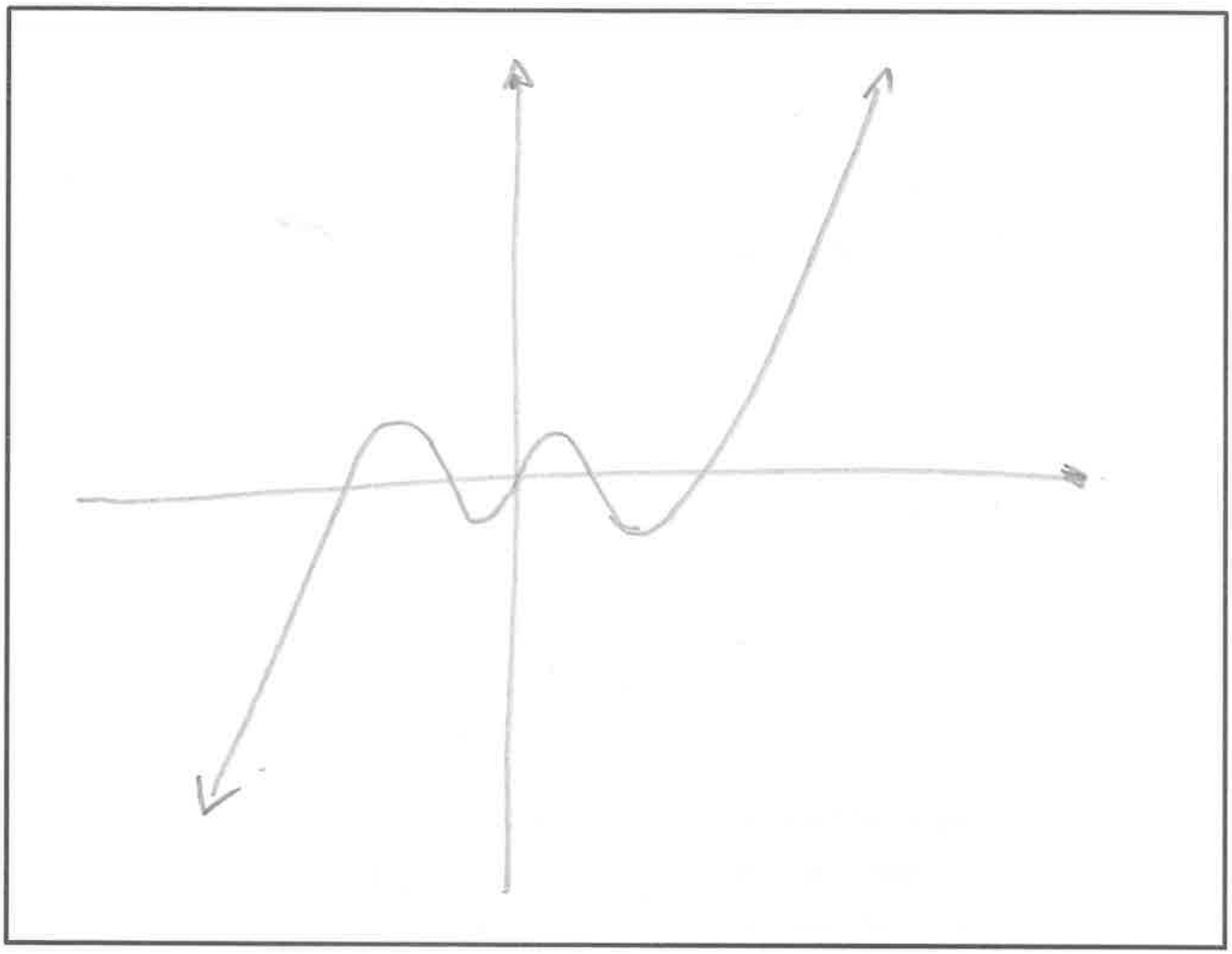
COMPLEX Roo

strictly

imaginary

iii) Josh proclaims that any polynomial with only odd numbered powers is odd. **1**

a. Create a degree 7 polynomial with 5 terms and integer coefficients. Only the coefficients of the odd powers are non-zero.
Draw it on Geogebra or Desmos to demonstrate it is odd.



b. Consider a degree 7 polynomial with pronumeral coefficients a to h and prove that if it is odd, it will have zero coefficients for its even powers. **2**

$y = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$

if it is odd $f(-x) = -f(x)$ ie $f(-x) + f(x) = 0$

$f(-x) = -ax^7 + bx^6 - cx^5 + dx^4 - ex^3 + fx^2 - gx + h$

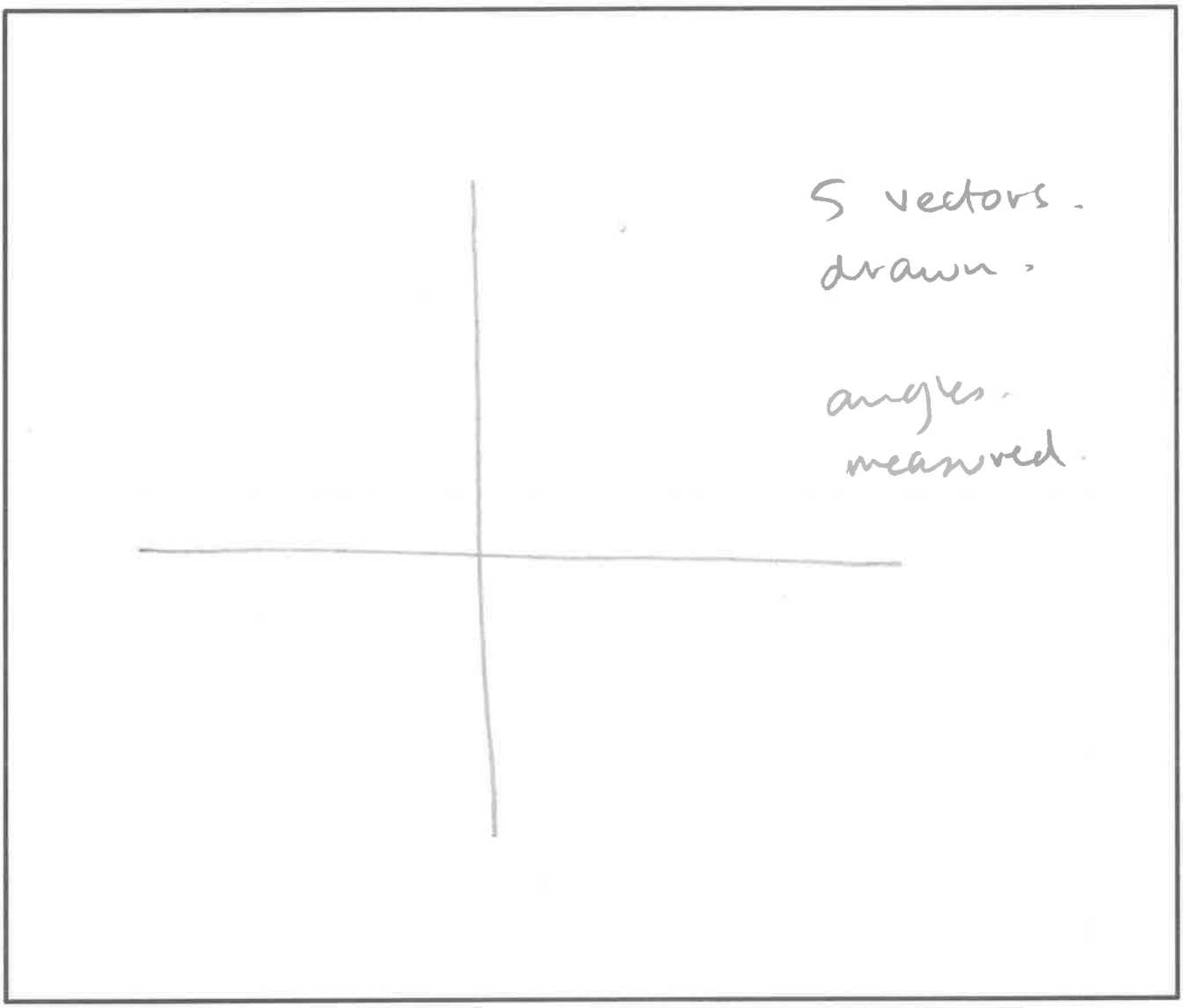
$f(x) = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$

$f(x) + f(x) = 2bx^6 + 2dx^4 + 2fx^2 + 2h = 0$

$\therefore b, d, f, h = 0$

ii) Make up 5 position vectors of your own.

a) Graph them and measure their angle of inclination to the positive direction of the x axis **1**



b) Use trigonometry to confirm the direction of each of your vectors **1**

shift tan y coord / x coord

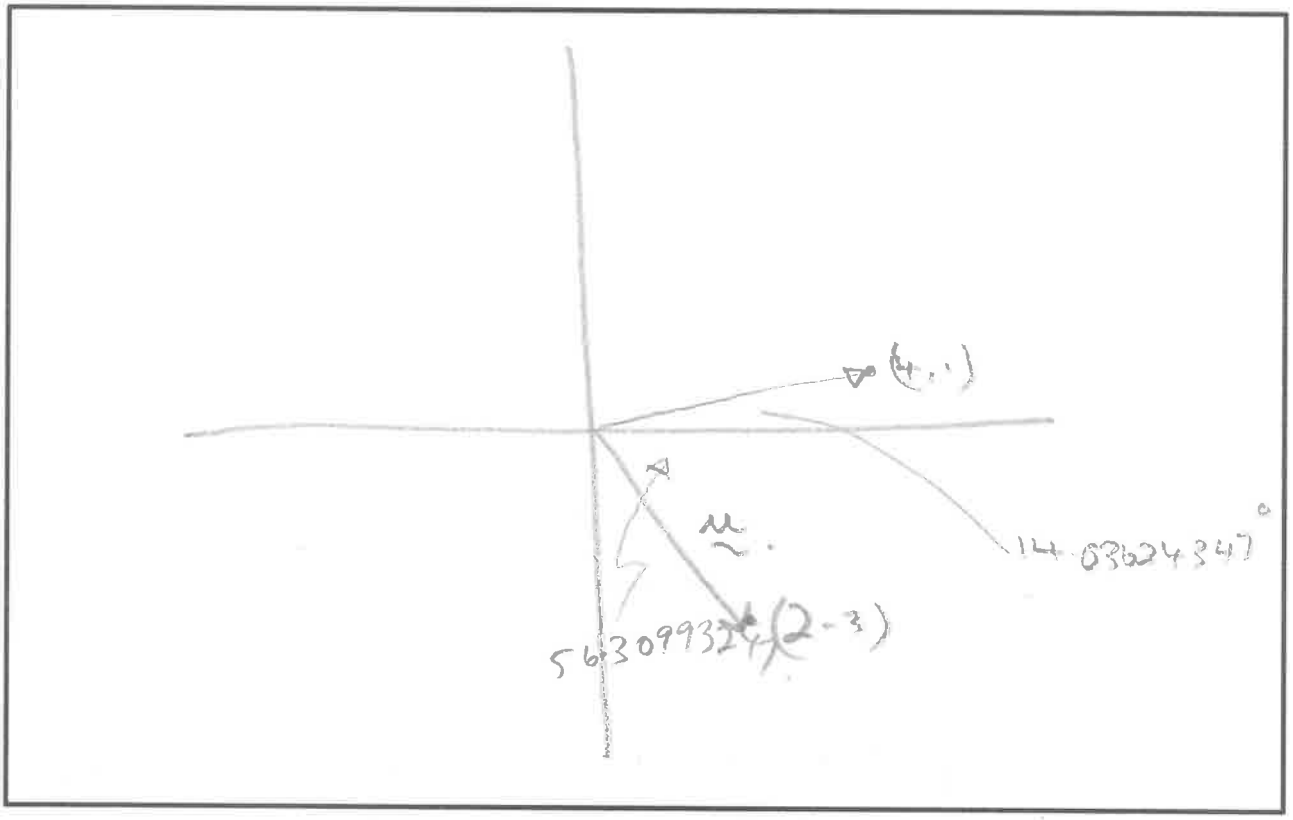
Question 4 **10 marks**

i) Consider the vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} + \underset{\sim}{j}$

a) Write each as column vectors and find the dot product **1**

$\begin{bmatrix} 2 \\ -3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $8 - 3 = 5$

b) Graph $\underset{\sim}{u}$ and $\underset{\sim}{v}$ as position vectors and measure the angle between them. **1**



c) Use the alternate form of the dot product formula and your measured angle to verify your calculation in part a) **1**

$\underset{\sim}{u} \cdot \underset{\sim}{v} = |u| \times |b| \times \cos\theta$

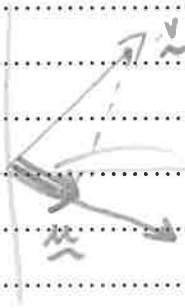
$\sqrt{13} \times \sqrt{17} \times$ $= 4.999999$

pretty close

iv) Consider the vector $\underset{\sim}{u} = \sqrt{3}\underset{\sim}{i} - \underset{\sim}{j}$ and the vector $\underset{\sim}{v} = \sqrt{2}\underset{\sim}{i} + \sqrt{2}\underset{\sim}{j}$

a) Explain what is meant by the expression $proj_{\underset{\sim}{u}}\underset{\sim}{v}$ and calculate this vector showing all working. **1**

the projection of $\underset{\sim}{v}$ onto $\underset{\sim}{u}$



$(\underset{\sim}{v} \cdot \hat{\underset{\sim}{u}})\hat{\underset{\sim}{u}}$

$\begin{bmatrix}\sqrt{2}\\ \sqrt{2}\end{bmatrix}\begin{bmatrix}\frac{\sqrt{3}}{2}\\ -\frac{1}{2}\end{bmatrix}\left(\frac{\sqrt{3}}{2}i - \frac{1}{2}j\right)$

$\frac{\sqrt{3}}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}}$

b) Include a graph of the vector you have found and highlight both its magnitude and direction. In your graph include $\underset{\sim}{u}$ and $\underset{\sim}{v}$ **1**

$\frac{\sqrt{3}-1}{\sqrt{2}}$

$\frac{3-\sqrt{3}}{2\sqrt{2}}i : -\frac{\sqrt{3}+1}{2\sqrt{2}}j$

iii) Consider the vector $\underset{\sim}{a} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$

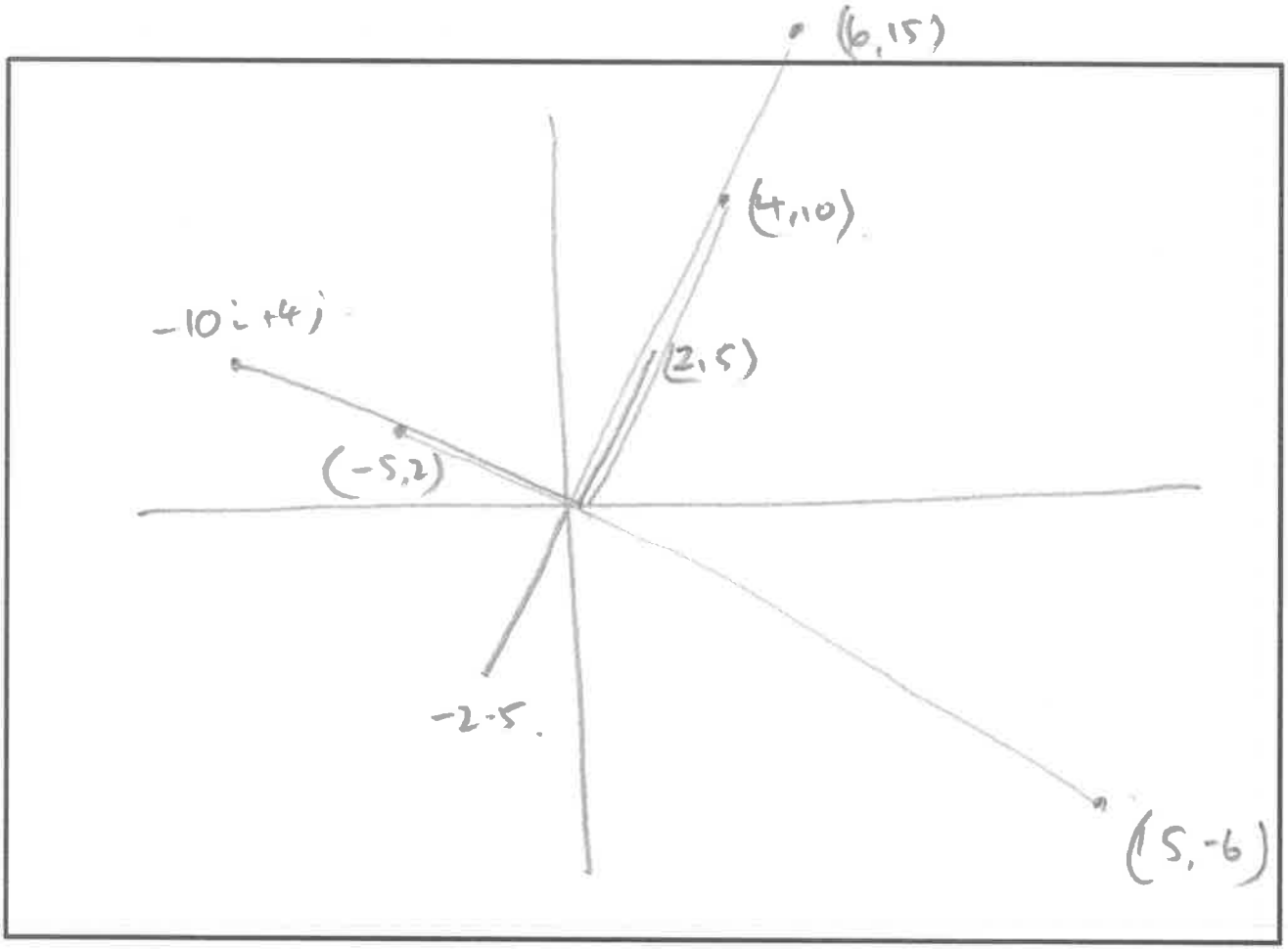
a) give in components form 3 parallel vectors (one with the opposite direction). **1**

b) Also give in component form 3 vectors which are at right angles to $\underset{\sim}{a}$ **1**

$-2i - 5j$ $4i + 10j$ $6i + 15j$

$-5i + 2j$ ~~$-4i + 10j$~~ $-10i + 4j$ $1.5i - 6j$

c) Graph all 7 vectors in the space below **1**



Section II Description **10**
marks

You must make a video, no longer than 4 minutes, explaining a particular aspect of vectors from chapter 8. You should view your work as a teaching resource. The video must clearly be your own work. There must be audio of you speaking or vision of you demonstrating.

Your video can include work on a white-board, video of the computer, spoken or acting talent or any other work which teaches your message.

Every video will need to be submitted in an agreed format so there is no problem viewing them.

You should also produce a single page of written work which will be a part of your presentation. It should contain a summary of the point you are making, a couple of examples to enhance your message, and 3 marks worth of original exam type questions type question worth 3 marks. Its solution should be included on the back of this sheet.

The idea is for the presentation to convey mathematically correct material and not be overly complicated. Your work will be marked according to the following rubric

Mark Range	Description of response... VIDEO and written support
9-10	A coherent and fluent video, which contains sophisticated and engaging mathematics thus delivering an effective concept. The video addresses the page of supporting evidence and provides structured insight for the examples and hints for the exam question. The support sheet clearly conveys a summary of your main message, with thoughtful examples and 3 marks of exam type questions which stem from your message
7-8	A competent video which contains mostly correct mathematics, and which teaches a concept to a competent level. The video fits soundly with the support sheet and provides some guidance for the examples and some clues for the exam question. The support sheet conveys a summary of your main message, with sound examples and 3 marks of exam type questions which stem from your message.
5-6	A satisfactory video and support page with some errors or omissions. Teaching of the concept is satisfactory. Connections between the video and the support sheet are weaker or poorly sequenced. Your support sheet is mostly complete.
3-4	An attempt is made to construct a video and support page which provides a lesson on vectors. The connections between the video and the support sheet might be incorrect or incomplete.
0-2	Little or no attempt is made to construct a meaningful video and support page. Unclear or unconvincing message. Limited or no connection to the support sheet.

The video must be submitted in the agreed format on a thumb drive and your supporting page in both PDF and hard copy by 10am on Friday 30 April.

One teacher will view and grade all the videos, together with supporting written work. Videos will be stopped at the 4 minute point.

The marks will be published at the conclusion of the marking.

The intention at the end of the process is to make all the videos available to all of the participants.

The video is to be all your own work but collaboration is expected in the development stages in week 10. Iron out problems before the holidays if possible. No questions will be considered during the holidays or after Wednesday 28 April.