

SOLUTIONS.

15-6-21

E1 T3.

Start here:

1. A ✓

2. C ✓

3. D ✓

4. B ✓

$$5a) \pi \int_{-2.75}^2 x^2 dy = \pi \int_{-2.75}^2 9 - y^2 dy \quad \checkmark$$

$$= \pi \left[ 9y - \frac{y^3}{3} \right]_{-2.75}^2 \quad \checkmark = \pi \left( 18 - \frac{8}{3} - (-24.75 + 6.9) \right)$$

$$= 33.15\pi$$

$$= 104.15 \quad \checkmark$$

$$b) y = -\frac{x^2}{115.2} \tan^2 \theta + x \tan \theta - \frac{x^2}{115.2}$$

$$10 = \frac{-1600}{115.2} \tan^2 \theta + 40 \tan \theta - \frac{1600}{115.2} \quad \checkmark$$

$$0 = -13.8889 \tan^2 \theta + 40 \tan \theta - 23.8889$$

$$-40 \pm \sqrt{40^2 - 4 \times -13.8889 \times -23.8889}$$

$$-2 \times 13.8889$$

$$\tan \theta = .8453584$$

$$\theta = 35^\circ 4'$$

$$2. \theta 34.639$$

$$63^\circ 50' \quad \checkmark$$

SOLS 15 JUNE 21 <sup>P2</sup> E1 T3.

c).  $\boxed{R = 13}$  ✓  
 if  $y = \frac{5}{13} \sin x - \frac{12}{13} \cos x$

AND  $\frac{y}{13} = \sin x \cos \alpha + \cos x \sin \alpha$

$\cos \alpha = \frac{5}{13} \quad \sin \alpha = \frac{-12}{13}$

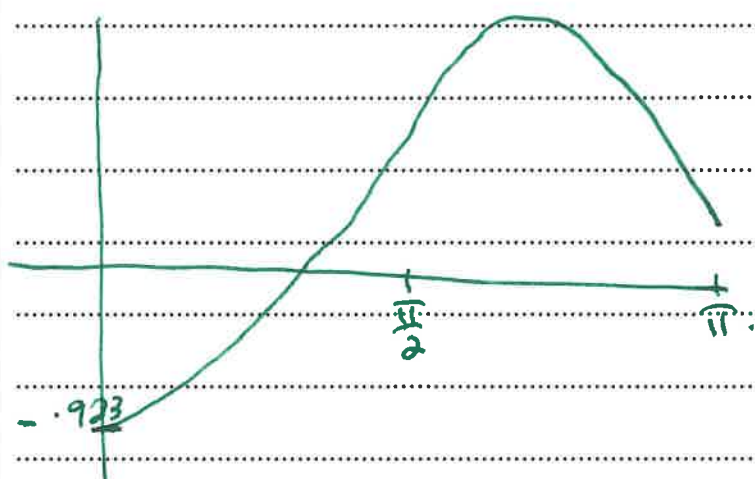
$\boxed{\alpha = -1.1760^\circ}$  ✓

ii)  $13 \cdot \sin(x - 1.1760)$

$x - 1.1760 = \frac{\pi}{2}$

MAX.

$x = 2.7468$  ✓



so MIN is when  $x = 0$  ✓

d).  $\int_0^{\pi} \cos^2 \frac{x}{4} dx = \int_0^{\pi} \frac{1}{2} + \frac{\cos \frac{x}{2}}{2} dx$  ✓

$= \left[ \frac{x}{2} + \sin \frac{x}{2} \right]_0^{\pi} = \frac{\pi}{2} + 1$  ✓

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15 JUNE 2021<sup>2</sup> SOLS P3.

Start here:

$$b) a) i) \sin\left(x + \frac{\pi}{2}\right) = \sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}$$

$$= 0 + \cos x \times 1$$

$$= \cos x \text{ as reqd}$$

ii) shift  $\sin x$   $\frac{\pi}{2}$  steps to the left.  
corresponding to the  $x + \frac{\pi}{2}$  transformation

$$b) i) \frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x} \quad \begin{matrix} u \\ v \end{matrix}$$

$$\frac{u'v - uv'}{v^2} = \frac{\cos x \times \cos x - \sin x (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \sec^2 x \text{ as reqd}$$

$$ii) \int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \int_0^{\frac{\pi}{4}} \frac{\tan^2 x + 1}{\sec^2 x} \sec^2 x - 1 \, dx.$$

$$= \left[ \tan x - x \right]_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$$

$$c) \text{ if } y = \sin^{-1} \frac{x}{\sqrt{2}} \quad \frac{dy}{dx} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{1 - \frac{x^2}{2}}}$$

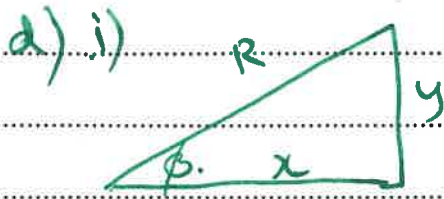
$$= \frac{1}{\sqrt{2(1 - \frac{x^2}{2})}} = \frac{1}{\sqrt{2 - x^2}}$$

P4.

$$I = \int_{\frac{1}{\sqrt{2}}}^1 \frac{1}{\sqrt{2-x}} dx = \left[ \sin^{-1} \frac{x}{\sqrt{2}} \right]_{\frac{1}{\sqrt{2}}}^1$$

$$= \frac{\pi}{4} - \frac{\pi}{6}$$

$$= \frac{\pi}{12}.$$



$$\frac{x}{R} = \cos \phi.$$

$$R = x \sec \phi \text{ as req'd}$$

ii) we want.  $\tan \phi x = x \tan \theta - g \frac{x^2 \sec^2 \theta}{2v^2}.$

$$0 = \left[ (\tan \theta - \tan \phi) - x \frac{g \sec^2 \theta}{2v^2} \right]$$

$$x = 0 \text{ or } x = (\tan \theta - \tan \phi) \frac{2v^2}{g \sec^2 \theta}.$$

$$x \sec \phi = \frac{2v^2}{g} (\tan \theta - \tan \phi) \frac{1}{\cos^2 \theta \sec \phi}.$$

iii)  $\frac{dR}{d\theta} = \frac{2v^2}{g} \sec \phi \left( \frac{d}{d\theta} (\tan \theta - \tan \phi) \cos^2 \theta \right).$

$$u \quad v = \cos^2 \theta.$$

$$u' = \sec^2 \theta \quad v' = -2 \sin \theta \cos \theta.$$

$$= \frac{2v^2}{g} \sec \phi \left( \cos^2 \theta \sec^2 \theta - 2 \sin \theta \cos \theta \left( \frac{\sin \theta}{\cos \theta} - \frac{\sin \phi}{\cos \phi} \right) \right)$$

$$= \frac{2v^2}{g} \sec \phi \left( 1 - 2 \sin^2 \theta + \frac{2 \sin \theta \cos \theta \sin \phi}{\cos \phi} \right).$$

$$= \frac{2v^2}{g} \sec^2 \phi \left( \cos 2\theta \cos \phi + \sin 2\theta \sin \phi \right).$$

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$$= \frac{2v^2}{g} \sec^2 \phi \cos(2\theta - \phi)$$

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b d) iv). we want  $\frac{dR}{d\theta} = 0$ .

ie.  $\cos(2\theta - \phi) = \cos \frac{\pi}{2}$

$$\begin{aligned} 2\theta - \phi &= \frac{\pi}{2} \\ \theta &= \frac{\pi}{4} + \frac{\phi}{2} \end{aligned} \quad \checkmark$$

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You may ask for an extra Writing Booklet if you need more space