Section I – Multiple Choice 1 mark per question **5 Marks**

Use the multiple choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

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1. Which of the following is the derivative of $\log(2x^2 + 1)$?

$\frac{4x}{2x^2+1}$

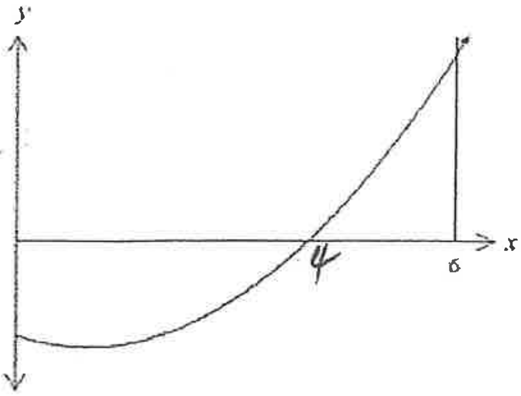
(A) $\frac{2x^2}{2x^2+1}$

(B) $\frac{4x}{2x^2+1}$

(C) $\frac{2x}{2x^2+1}$

(D) $\frac{4x^2}{2x^2+1}$

2. The diagram be low shows the graph of $y = x^2 - 2x - 8$.



$0 = (x-4)(x+2)$
$x = 4$

What is the correct expression for the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \leq x \leq 6$?

(A) $\int_0^5 (x^2 - 2x - 8)\ dx + \left|\int_5^6 (x^2 - 2x - 8)\ dx\right|$

(B) $\left|\int_0^4 (x^2 - 2x - 8)\ dx\right| + \int_4^6 (x^2 - 2x - 8)\ dx$

(C) $\int_0^4 (x^2 - 2x - 8)\ dx + \left|\int_4^6 (x^2 - 2x - 8)\ dx\right|$

(D) $\left|\int_0^5 (x^2 - 2x - 8)\ dx\right| + \int_5^6 (x^2 - 2x - 8)\ dx$

3. Which of the following will give the correct equation of the tangent to the curve $y = f(x)$ at the point $x = a$?

(A) $y - f(a) = \left(\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}\right)(x - a)$

(B) $y - f(a) = \left(\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}\right)(x - a)$

(C) $y - f(x) = \left(\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}\right)(x - a)$

(D) $y - f(x) = \left(\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}\right)(x - a)$

4. The indefinite integral of $\int 2x(x^2-3)^3$ is:

$y = (x^2-3)^4$
$y' = 4(x^2-3)^3 \times 2x$
$= 8x(x^2-3)^3$

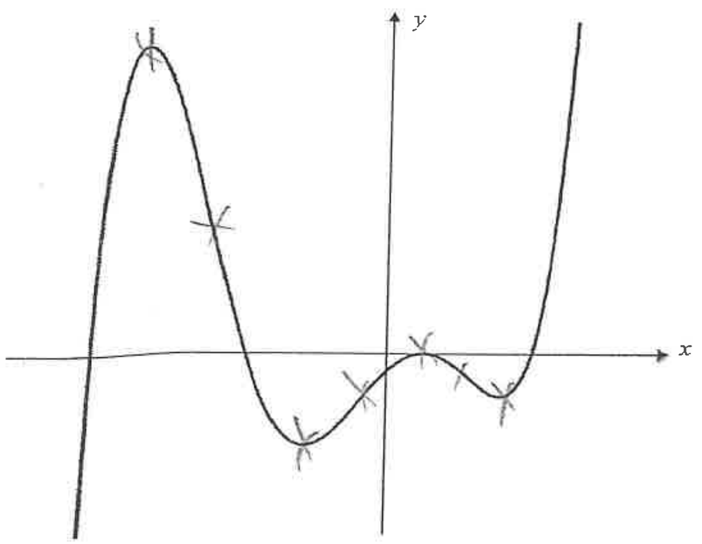
(A) $\frac{1}{4}(x^2-3)^4 + C$

(B) $\frac{x}{4}(x^2-3)^2 + C$

(C) $\frac{1}{8}(x^2-3)^4 + C$

(D) $2x(x^2-3)^3 + C$

5. The graph of $y = f(x)$ is shown below.



What is the number of stationary points and points of inflection in the graph of $y = f(x)$?

(A) 3 stationary points and 3 points of inflection

(B) 3 stationary points and 4 points of inflection

(C) 4 stationary points and 3 points of inflection

(D) `4 stationary points and 4 points of inflection

End of Section I

Section II

30 Marks

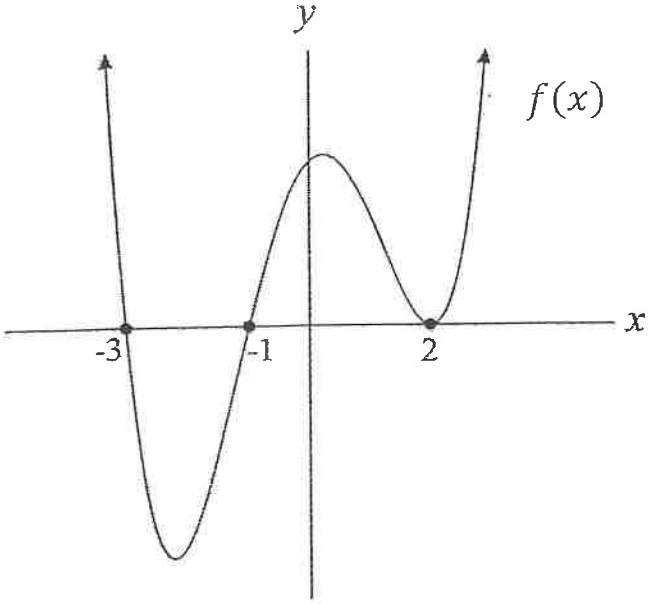
Attempt Questions 6 to 8.
Allow about 40 minutes for this section.

In Questions 6 to 8 your responses should include relevant mathematical reasoning and/or calculations.

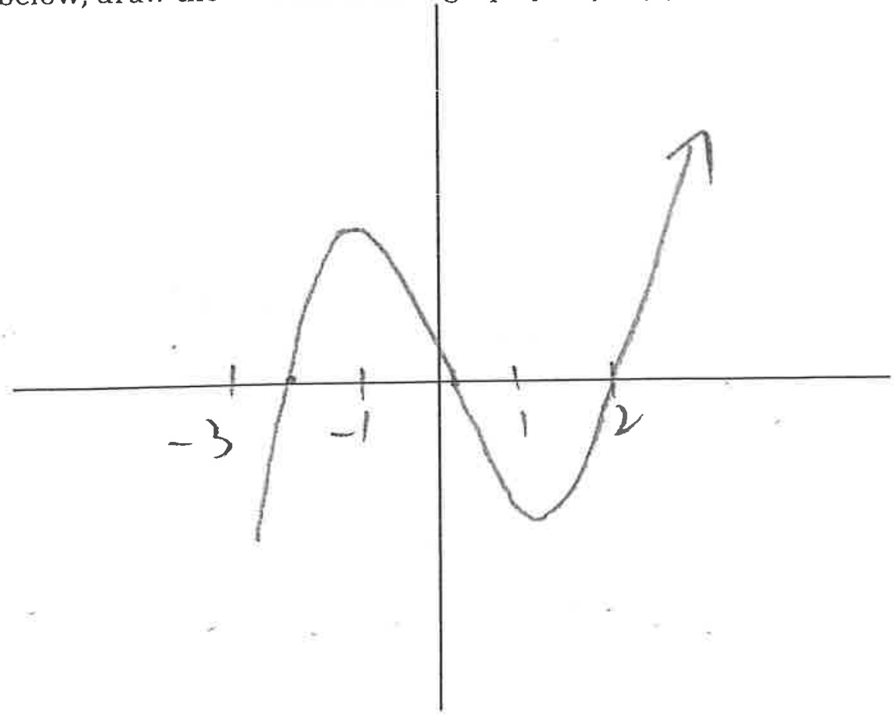
Question 6 (12 marks)

Write your solutions in the spaces provided **Marks**

(a) The diagram below shows the graph of a quartic function $y = f(x)$. 2



In the space below, draw the first derivative graph $y = f'(x)$.



(b) For the function $f(x) = x^3(4-x)$:

i) Find the coordinates of any stationary points. 2

$y = 4x^3 - x^4$ $\qquad 0 = x^3(4-x)$

$y' = 12x^2 - 4x^3$ $\qquad x = 0, 4$

$0 = 4x^2(3-x)$

$x = 0, \quad x = 3$

S.P $(0,0) \quad (3,27)$

ii) Determine the nature of the stationary points. 3

$y'' = 24x - 12x^2$

$x = 0, y'' = 0 \quad \therefore$ possible horizontal S.P

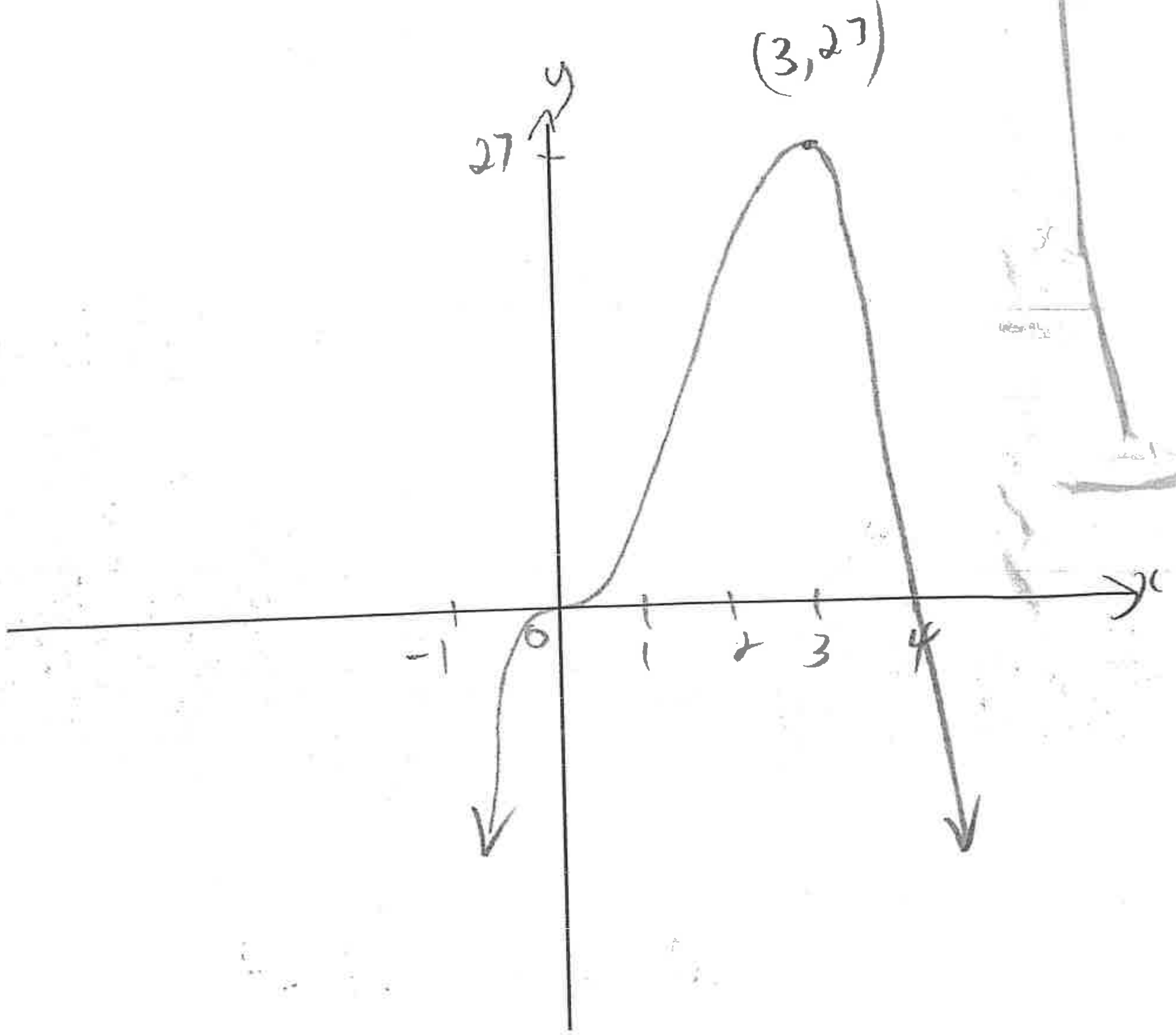
Test

x	-1	0	1
y''	-36	$-$	12
	$\cap$		$\cup$

change of concavity $\therefore (0,0)$ horizontal pt of inflexion

$x = 3 \quad y'' = 24(3) - 12(3)^2$
$= -36 < 0 \quad \therefore$ max T.P at $(3,27)$

iii) Draw the graph showing all relevant points. **2**



c) Find the definite integral of $\int 5x^4 + 3x^2 - 2\ dx$ **1**

$= \frac{5x^5}{5} + \frac{3x^3}{3} - 2x + C$

$= x^5 + x^3 - 2x + C$

Question 7 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) By firstly differentiating $y = \sqrt{2x^2 - 4}$, find $\int \frac{x}{\sqrt{2x^2-4}}\, dx$. **2**

$y = (2x^2 - 4)^{1/2}$

$y' = \frac{1}{2}(2x^2 - 4)^{-1/2} \times 4x$

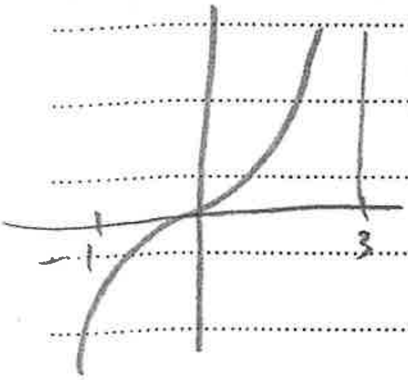
$= 2x(2x^2-4)^{-1/2}$

$\int \frac{x}{\sqrt{2x^2-4}}\, dx$

$y = \frac{1}{2}(2x^2-4)^{1/2} + C$

$y = \frac{1}{2}\sqrt{2x^2-4} + C$

(b) Find the area of the region bounded by the graph $y = x^3$, between $x = -1$ and $x = 3$. **2**



$A = \left| \int_{-1}^{0} x^3\, dx \right| + \int_{0}^{3} x^3\, dx$

$= \left| \frac{x^4}{4} \right|_{-1}^{0} + \left[\frac{x^4}{4} \right]_0^3$

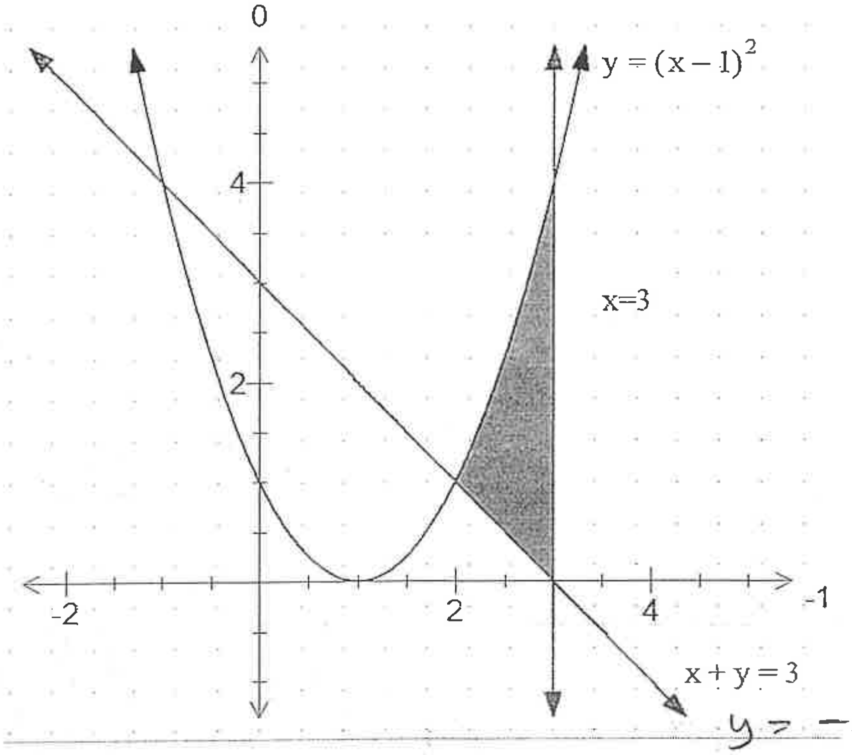
$= |0 - \frac{1}{4}| + \left(\frac{3^4}{4} - 0\right)$

$= \frac{1}{4} + \frac{81}{4}$

$= \frac{82}{4}$

$= 20\frac{1}{2}\, u^2$

(c) The diagram shows the shaded region bounded by the curve $y = (x-1)^2$ and the lines $x + y = 3$ and $x = 3$.



$y = -x + 3$

Find the area of the shaded region. 3

$$A = \int_2^3 (x-1)^2 - (-x+3)\ dx$$

$$= \int_2^3 x^2 - 2x + 1 + x - 3\ dx$$

$$= \int_2^3 x^2 - x - 2\ dx$$

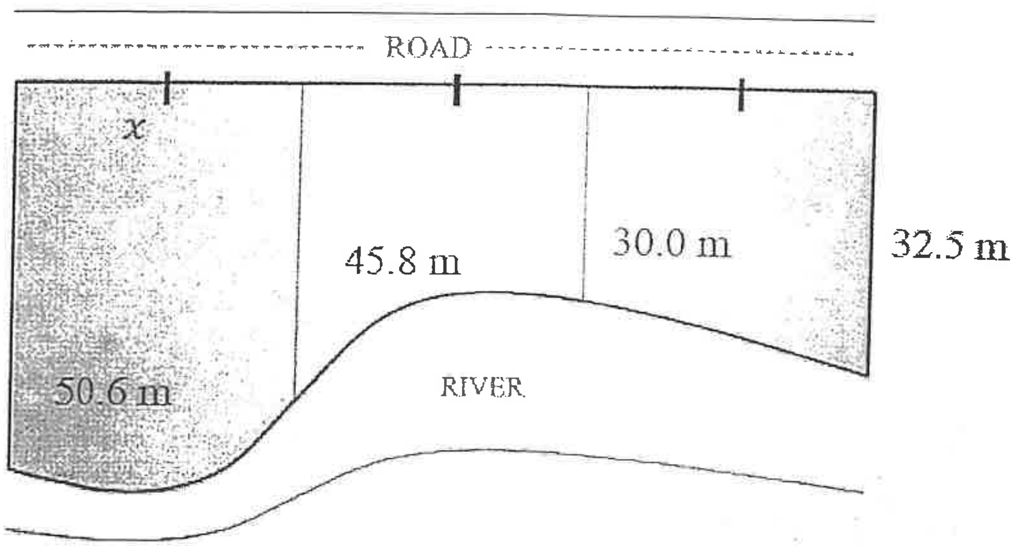
$$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_2^3$$

$$= \left(\frac{27}{3} - \frac{9}{2} - 6\right) - \left(\frac{8}{3} - \frac{4}{2} - 4\right)$$

$$= -1.5 - \left(-\frac{10}{3}\right)$$

$$= \frac{11}{6}\ u^2$$

(d) A field (shaded), is bordered on one side by a 120 metre of road and on the other side by a river. Measurements are taken from the road to the river, as shown.



(i) Find the value of x. **1**

$120 \div 3 = 40$ m.

(ii) Use the Trapezoidal Rule to find an approximation of the area of the field. Correct your answer to the nearest square metre. **2**

$A = \frac{\frac{b-a}{n}}{2}$ (first + last + 2 rest)

$= \frac{40}{2}(50.6 + 32.5 + 2(45.8 + 30.0))$

$= \frac{40}{2}(83.1 + 151.6)$

$= \frac{40}{2}(234.7)$

$= 4694 \text{ m}^2$

Question 8 (10 marks)

Write your solutions in the spaces provided **Marks**

(a) Differentiate:

i) $\frac{2x+1}{e^{2x}}$ **2**

$$y = \frac{2x+1}{e^{2x}}$$

$$y' = \frac{e^{2x}(2) - 2e^{2x}(2x+1)}{(e^{2x})^2} \quad 1$$

$$= \frac{\cancel{e^{2x}}(2 - 4x - 2)}{(e^{2x})^2} \quad = \frac{-4x}{e^{2x}}$$

ii) $\overset{u}{3x^4}\overset{v}{e^{2x}}$ **2**

$$y' = 3x^4(2e^{2x}) + 12x^3 e^{2x}$$

$$= 6x^4 e^{2x} + 12x^3 e^{2x}$$

$$= 6x^3 e^{2x}(x+2)$$

iii) $y = \log_e(3x^2 + 2x - 3)$ **1**

$$y' = \frac{6x+2}{3x^2+2x-3}$$

(b) If $\frac{dy}{dx} = e^x + 2$, find y when $x = 1$, given that $y = 4$ when $x = 0$. 3

$\int e^x + 2 \; dx$

$y = e^x + 2x + c$

$4 = e^0 + 2(0) + c$

$4 = 1 + c$

$c = 3$

$\therefore y = e^x + 2x + 3$

$x = 1 \quad y = e^1 + 2(1) + 3$

$y = e^1 + 5$

(c) If $y = \log_e x$, show that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$. 2

$y' = \frac{1}{x} \Rightarrow x^{-1}$

$y'' = -x^{-2}$

$= -\frac{1}{x^2}$

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{1}{x^2} + \left(\frac{1}{x}\right)^2$

$= -\frac{1}{x^2} + \frac{1}{x^2}$

$= 0$

End of Task