Student Number

2023 **HSC ASSESSMENT TASK 3**

Mathematics Advanced

Year 12

General Instructions

- Working time – 45 minutes
- Weighting 25%
- Write using black or blue pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the end of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:

35

Section I – 5 marks

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – 30 marks

- Attempt Questions 6 – 7
- Allow about 37 minutes for this section
- Write your solutions in the space provided

Section	Marks
Section I	**/5**
Section II	**/30**
Total marks	**/35**

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Section I

5 marks

Attempt Questions 1 to 5.

Allow about 8 minutes to complete this section.

Use the multiple-choice answer sheet for Questions 1-5.

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1 A particle that moves in a straight line with a displacement of *x* metres at time *t* seconds. For which of the following conditions could the particle be speeding up?

A. $\frac{dx}{dt} < 0$ and $\frac{d^2x}{dt^2} > 0$

B. $\frac{dx}{dt} > 0$ and $\frac{d^2x}{dt^2} < 0$

C. $\frac{dx}{dt} > 0$ and $\frac{d^2x}{dt^2} > 0$

D. $\frac{dx}{dt} < 0$ and $\frac{d^2x}{dt^2} = 0$

2 At time t a particle has displacement and acceleration functions $x(t)$ and $a(t)$. For which of the following functions does $x(t) = a(t)$?

A. $x(t) = 3\,sin(t) - e^t$

B. $x(t) = 3\,cos(t) - e^{-t}$

C. $x(t) = e^t - e^{-t}$

D. $x(t) = 3\,sin(t) - 3\,cos(t)$

3 The correct expression for the integral $\int \sin\frac{x}{5}dx$ is:

A. $-\cos\frac{x}{5} + C$

B. $5\cos\frac{x}{5} + C$

C. $-5\cos\frac{x}{5} + C$

D. $\frac{1}{5}\cos\frac{x}{5} + C$

4 If $\log_e 3x = \log_e y - 2\log_e z$ where $x, y, z > 0$, which of the following is true?

A. $x = \frac{y - z^2}{3}$

B. $x = \frac{y}{3z^2}$

C. $\log_e 3x = \frac{y}{z^2}$

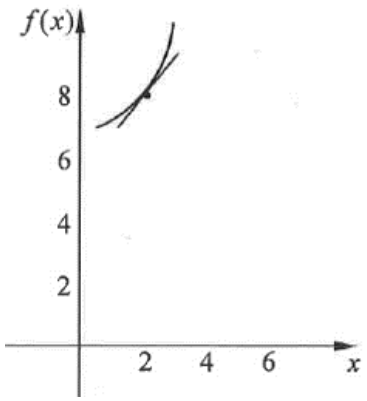
D. $\log_e 3x = \frac{\log_e y}{\log_e z^2}$

5 A function has the following properties:

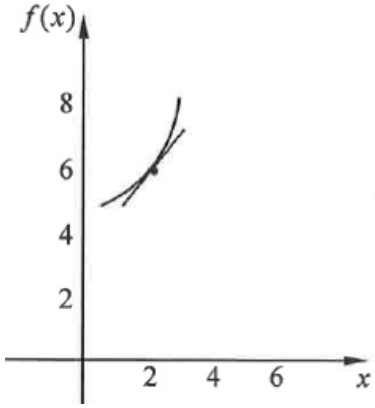
$f(2) = 8$ $\quad f'(2) = 6$ $\quad f''(2) = 2$

Which sketch best matches the graph of the function near $x = 2$?

A.



B.



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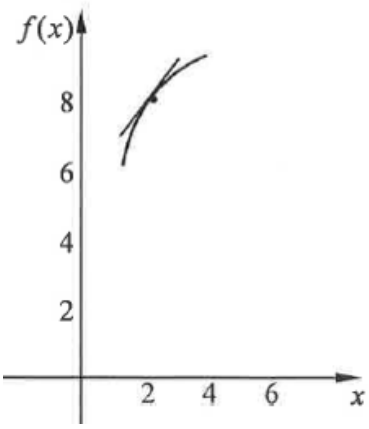
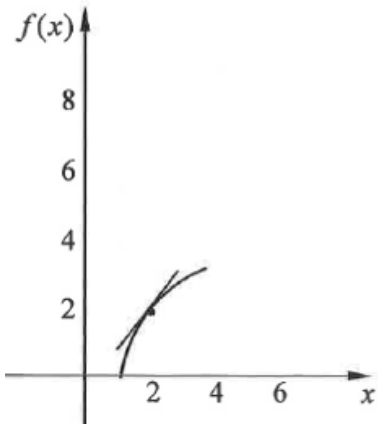


D.



End of Section I

Section II

30 marks

Attempt Questions 6 – 7
Allow about 37 minutes to complete this section

Question 6 (15 marks)

a. Differentiate $e^{x\cos x}$. **2**

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b. Find the exact value of $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} sec^2(2x)\,dx$. **2**

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c. Find $\int \frac{x-1}{3x^2-6x}\, dx$. **2**

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d. (i) Differentiate $\log_e(\sin x)$. **1**

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(ii) Hence find $\int \cot x\, dx$. **1**

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e. Find the equation of the normal to the curve $y = 2e^{2x} - e^{x}$ at the point where $x = 0$, in general form. **3**

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f. Consider the function $y = 2\sin x + \sin^2 x$.

(i) Show that $\frac{dy}{dx} = 2\cos x\,(1 + \sin x)$. **1**

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(ii) Find the co-ordinates of the minimum turning point in the domain $[0, 2\pi]$. **3**

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Question 7 (15 marks)

a. A radioactive substance decomposes from 100 grams to 90 grams in 2 hours according to the law $\frac{dM}{dt} = -kM$.

(i) Show $M = M_o e^{-kt}$ is a solution and find show $k = -\frac{1}{2}\log_e\left(\frac{9}{10}\right)$. **2**

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(ii) Calculate the half-life of this radioactive substance, correct to the nearest minute. **2**

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(iii) What percentage of the original amount has decayed after 6 hours? **1**

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b. A particle moving in a straight line has velocity $v = 3e^{t} - 12e^{-2t}$.

The particle is initially at the origin, with *t* is measured in seconds and v in metres per second.

(i) Find the initial velocity of the particle. **1**

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(ii) Determine by calculation if the particle is ever at rest. **2**

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(iii) Find the displacement of the particle when *t* = 4 seconds, correct to 1 decimal place. **2**

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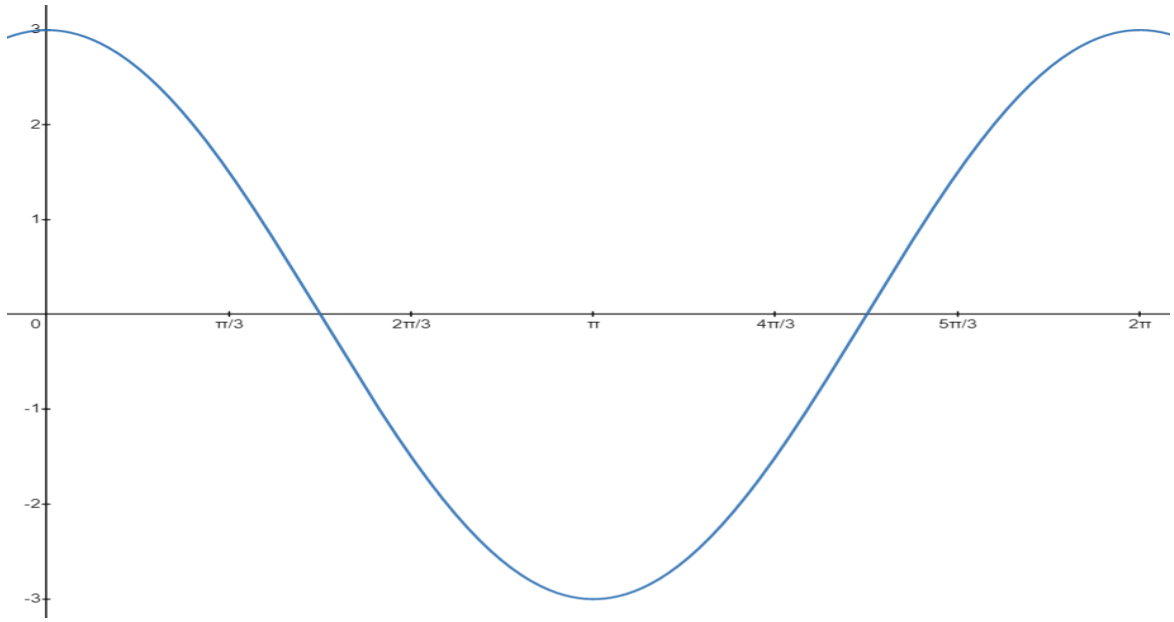
c. The graph below shows the curve $y = 3\cos x$.



(i) On the same graph draw $y = \sin 2x$ with domain $[0, 2\pi]$. **2**

(ii) Hence find the exact area bounded by the curves. **3**

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End of Task

EXTRA WRITING SPACE

EXTRA WRITING SPACE

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2023 Year 12 – Mathematics Advanced Assessment 3

Multiple Choice Answer Sheet

Student Number

Instructions for use:

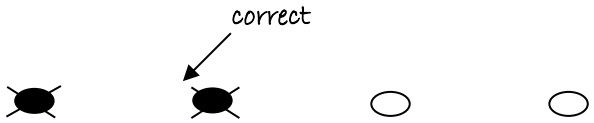
• Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ● C ⬭ D ⬭

• If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

• If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



• Attempt all multiple-choice questions.

Question	**1**	A ○	B ○	C ○	D ○
	2	A ○	B ○	C ○	D ○
	3	A ○	B ○	C ○	D ○
	4	A ○	B ○	C ○	D ○
	5	A ○	B ○	C ○	D ○