CSSA

2023
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAMINATION ROOM

Centre Number

Student Number

Mathematics Advanced

Morning Session
Monday, 7 August 2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for use with this paper
- Use the Multiple-Choice Answer Sheet provided for Section I
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I - 10 marks (pages 2–6)
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–32)
- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10.

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1 Which of the following sets of ordered pairs represent a one-to-many relationship?

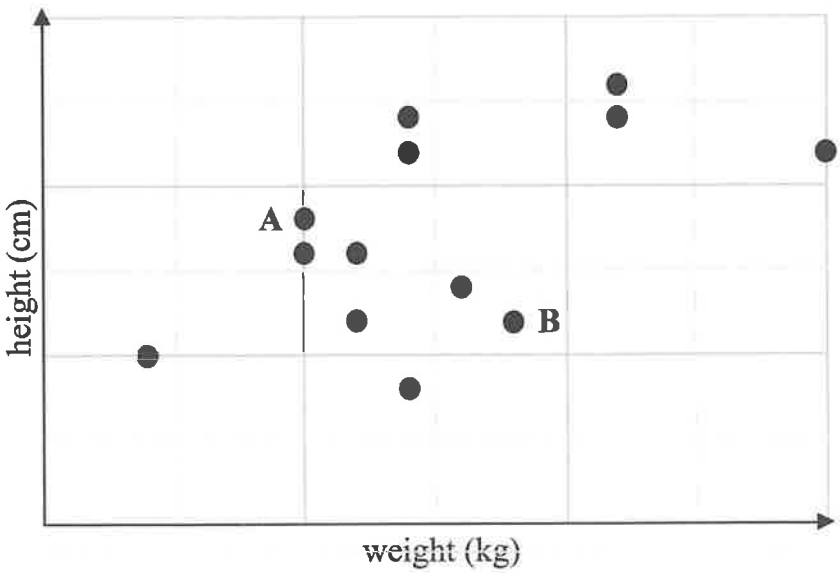
A. $(1,1),(1,2),(2,3),(3,4)$

B. $(1,1),(2,2),(3,2),(4,3)$

C. $(1,1),(2,2),(3,3),(4,4)$

D. $(1,1),(1,2),(2,3),(3,3)$

2 The height and weight of a group of students is shown on the scatterplot below.



Which of the following statements is true?

A. Student **A** is taller and weighs more than Student **B**.

B. Student **A** is taller and weighs less than Student **B**.

C. Student **A** is shorter and weighs more than Student **B**.

D. Student **A** is shorter and weighs less than Student **B**.

3 In a mathematics examination, 25% of students scored less than 40 marks, 75% of students scored less than 86 marks and the highest mark was 96.

Given that the distribution is symmetrical with no outliers, what is the lowest mark and the median of this data set?

	Lowest mark	Median
A.	30	63
B.	30	68
C.	36	63
D.	36	68

4 In an arithmetic series, the sum of the first three terms is 21 and the sum of the first four terms is 36.

What is the sum of the first five terms of this series?

A. 15

B. 51

C. 55

D. 57

5 Let $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{x-1}$.

What is the domain and range for both $f\left(g(x)\right)$ and $g\left(f(x)\right)$?

	Domain	Range
A.	$[-1,1]$	$(-\infty,\infty)$
B.	$(0,\infty)$	$(-\infty,0)\cup(0,\infty)$
C.	$[0,\infty)$	$(0,\infty)$
D.	$(1,\infty)$	$(0,\infty)$

6 The limiting sum of a geometric series is $\frac{10}{9}$ and the sum of its first three terms is $\frac{26}{25}$.

What is the common ratio of this series?

A. 1.39

B. 1.25

C. 0.40

D. 0.25

7 In a bag of n coloured discs, exactly 7 are red.

Two discs are selected at random, without replacement.

The probability that both discs are red is $\frac{21}{55}$.

Which quadratic equation could be used to find the value of n?

A. $n^2 + n + 110 = 0$

B. $n^2 - n - 110 = 0$

C. $n^2 + n - 110 = 0$

D. $n^2 - n + 110 = 0$

8 A particle is moving in a straight line such that its displacement x metres at time t seconds from the origin is given by the equation $x = k\cos(at + b)$, where a, b and k are constants.

The particle is stationary when $t = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \ldots$.

The initial acceleration of the particle is $6\sqrt{3}$ ms^{-2}.

What could be the values of a, b and k?

A. $a = 4, b = -\frac{5\pi}{6}, k = 3$

B. $a = 2, b = -\frac{5\pi}{6}, k = -3$

C. $a = 2, b = \frac{\pi}{6}, k = 3$

D. $a = 2, b = \frac{\pi}{6}, k = -3$

9 Consider the equation $\left(\sin\theta - \frac{\sqrt{2}}{2}\right)\left(\tan 2\theta - \frac{\sqrt{3}}{3}\right) = 0.$

How many solutions are there in the interval $0 \leq \theta \leq \pi$?

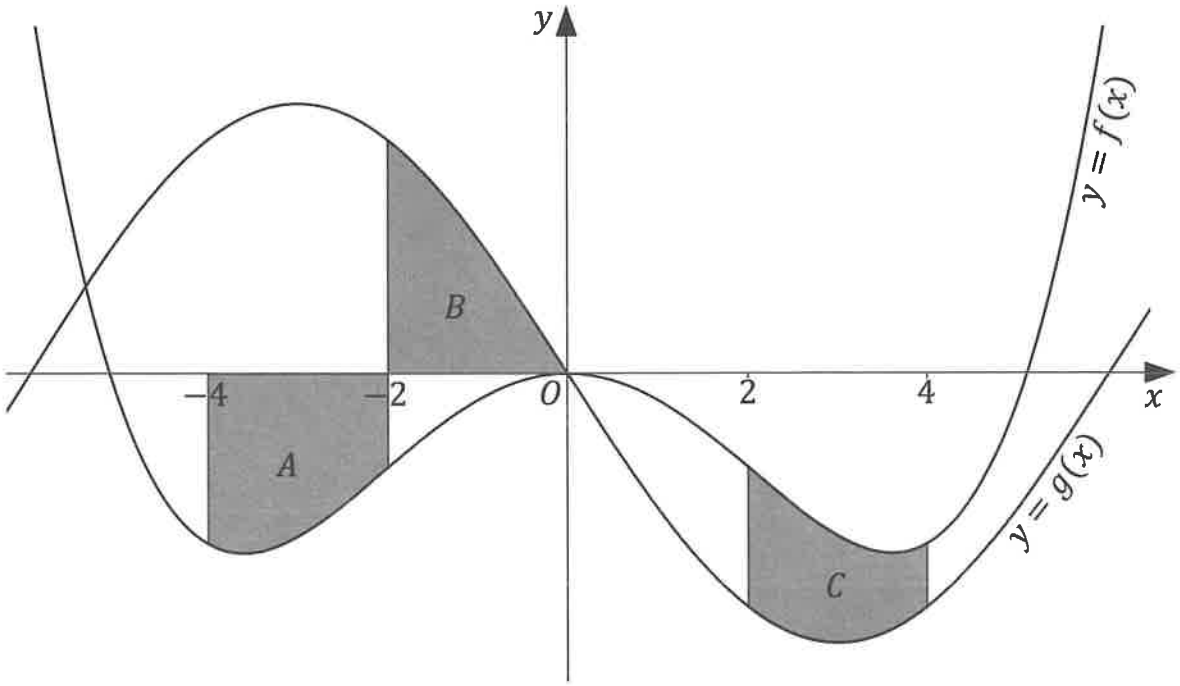
A. 4

B. 3

C. 2

D. 1

10 The graph of an even function, $f(x)$, and an odd function, $g(x)$ are shown below.



Let $A = \int_{-4}^{-2} f(x)dx$, $B = \int_{-2}^{0} g(x)dx$ and $C = \int_{2}^{4} f(x) - g(x)dx$.

Which of the following expressions equals $\int_{0}^{4} g(x)dx$?

A. $A + B + C$

B. $A - B + C$

C. $-A + B - C$

D. $-A - B - C$

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Instructions

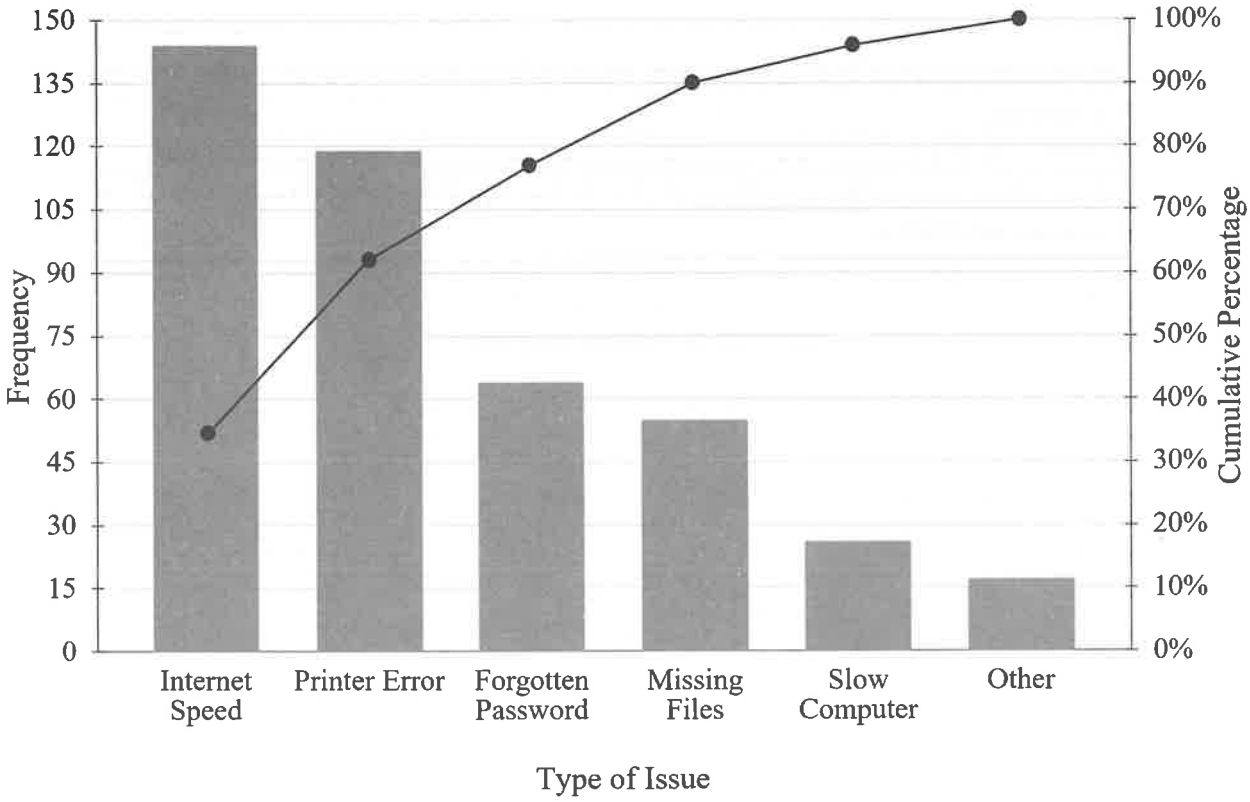
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 33–35. If you use this space, clearly indicate which question you are answering.

Section II begins on page 8

Question 11 (2 marks)

A company records the type of issues for IT support over a period of time. **2**

The results are shown in the Pareto chart below.



What percentage of issues were due to "Printer Error" or "Forgotten Password"?

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Question 12 (2 marks)

Find the equation of the normal to the curve $y = x^3 - 2x$ at the point $(-1, 1)$. **2**

Question 13 (2 marks)

The time it takes to complete a task varies inversely with the number of people assigned. **2**

It takes 5 people to complete a task in 4 hours.

Find the amount of time it would take 8 people to complete the same task.

Question 14 (5 marks)

The table below gives the present value interest factors for an annuity of \$1 per period, for various interest rates, r and numbers of periods, n.

Table of present value interest factors						
	Interest rate per period, r (as a decimal)					
n	0.0025	0.0030	0.0035	0.0040	0.0045	0.0050
46	43.40237	42.90711	42.41960	41.93970	41.46727	41.00219
47	44.29164	43.77578	43.26816	42.76863	42.27703	41.79322
48	45.17869	44.64186	44.11376	43.59425	43.08315	42.58032
49	46.06354	45.50534	44.95642	44.41658	43.88567	43.36350
50	46.94617	46.36624	45.79613	45.23564	44.68459	44.14279

(a) Anne plans to invest \$500 each month for 4 years at 4.2% per annum. Use the table to calculate the present value of the annuity. **2**

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(b) Calculate the future value of Anne's investment. **1**

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(c) Trevor decides to take out a personal loan of \$49 000 to buy a car. Use the table to determine the total interest paid if the loan is paid in full after 46 months at an interest rate of 0.5% per month. **2**

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Question 15 (3 marks)

(a) Find $\frac{dy}{dx}$, given $y = (\ln x)^2$. **1**

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(b) Hence, or otherwise, find $\int_1^e \frac{\ln(x^2)}{2x}\,dx$. **2**

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Question 16 (3 marks)

The graph of the primitive function of $f(x) = \tan^2 x$ crosses the x-axis at $x = \frac{\pi}{3}$. **3**

Find the primitive function, leaving all values in exact form.

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Question 17 (3 marks)

Consider the function $f(x) = -\sin\left(\frac{1}{2}(x-\pi)\right)$.

(a) Show that $f(x)$ is even. **1**

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(b) Hence, or otherwise, find the area bounded by the curve $y = f(x)$ and the x-axis in the interval $[-\pi, \pi]$. **2**

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Question 18 (5 marks)

Jason buys a ticket in a lottery, where the first prize is \$10 000.

Tickets cost \$20 each with the probability of winning a prize shown in the discrete distribution table below.

Prize (\$)	0	20	100	500	5000	10 000
$P(X = x)$	0.75	m	0.03	0.002	0.001	0.0001

(a) Find the value of m. **1**

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(b) Show that Jason is expected to lose \$5.66 every time he plays this lottery. **2**

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(c) Find the variance of this discrete distribution. **2**

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Question 19 (7 marks)

Ben is cooking a pizza. The temperature of the pizza oven is modelled by the equation

$$T = 175 - 150 \times (0.9)^{0.2t},$$

where T is the temperature of the oven (°C) at time t minutes since the oven was switched on.

(a) What was the initial temperature of the oven? **1**

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(b) Calculate the average rate at which the temperature changes per minute from $t = 25$ to $t = 75$, correct to two decimal places. **3**

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(c) Ben can place the pizza in the oven when the instantaneous rate of change of temperature is less than or equal to 1°C per minute. **3**

Given that Ben turned the oven on at 10:30 am, find the earliest time that Ben can place the pizza in the oven, correct to the nearest minute.

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Question 20 (3 marks)

A group of students were surveyed on the type of pets they own. **3**

The results are summarised in the following incomplete two-way table.

	Cat	No Cat
Dog	A	32
No Dog	45	35

Given that a student owns a dog, the probability that they also own a cat is 60%.

Find the probability that a student chosen at random owns both a cat and a dog.

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Question 21 (2 marks)

The relative level of a noise compared to a reference level is calculated by the formula **2**

$$R = 20 \log_{10} \left(\frac{N}{N_0} \right),$$

where R is the relative noise level measured in decibels (dB), N is the level of a noise and N_0 is the maximum noise level permitted.

The relative noise level at a construction site when operating machinery was found to be 8 dB higher than the maximum noise level permitted.

Show that the level of noise produced by the machinery was approximately 2.5 times greater than the maximum noise level permitted.

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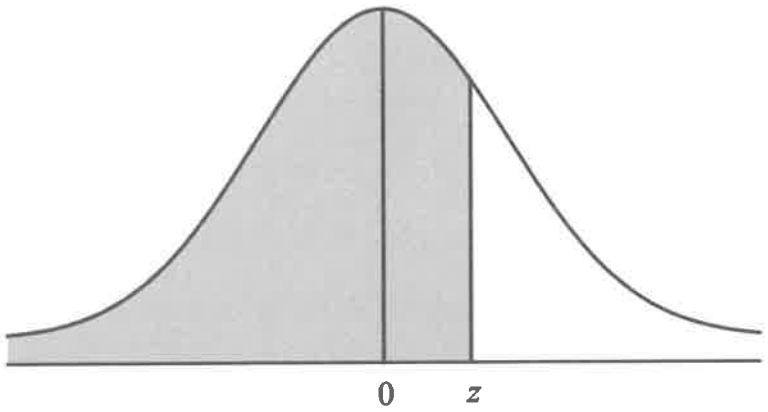
Question 22 (3 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table below gives the probability that this random variable lies below z for different values of z.

z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.500	0.540	0.579	0.618	0.655	0.691	0.726	0.758	0.788	0.816
1	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



(a) Calculate the probability that the random variable lies above -1.2. **1**

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(b) A factory fills bottles with water. The actual amount of water in each bottle is normally distributed with a mean of 602 mL and a standard deviation of 2.5 mL. **2**

Using the values in the table, determine the percentage of bottles that contain between 599 mL and 604 mL of water.

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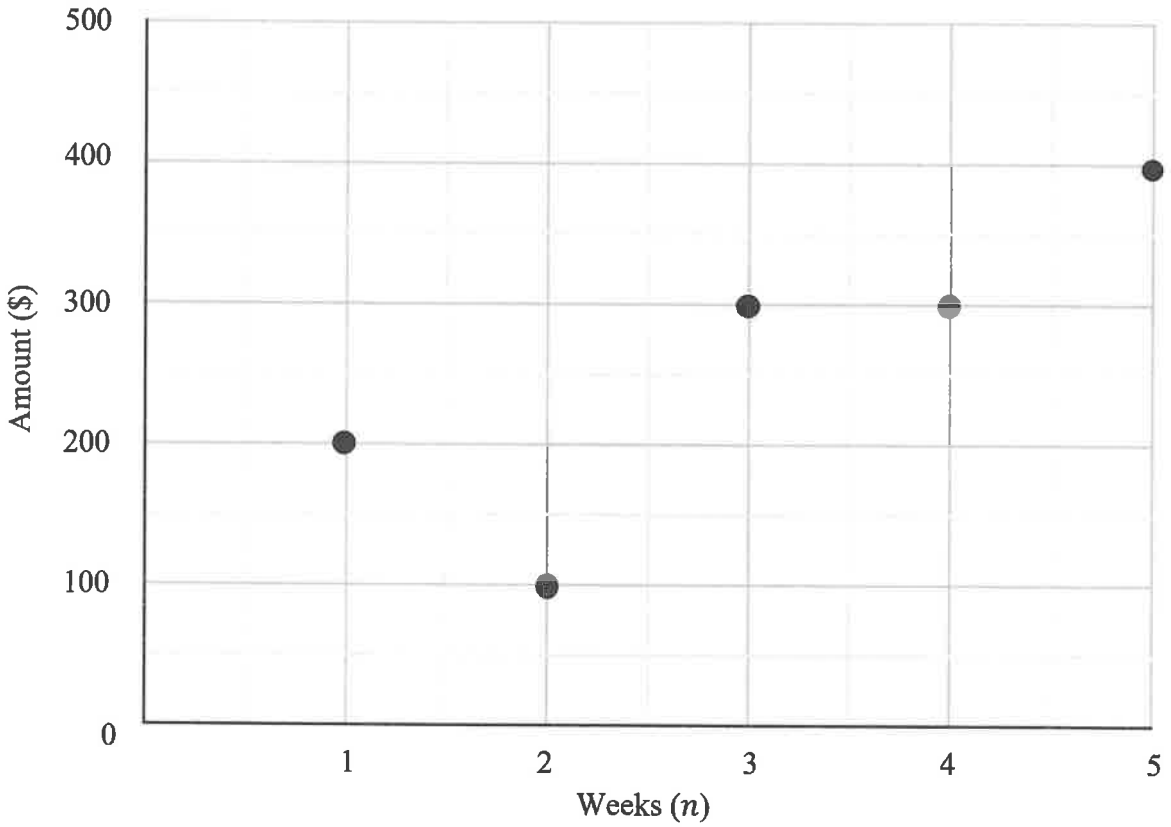
Question 23 (5 marks)

Ivy has plotted the income that she has received each week for the past five weeks on the graph below. **5**



Over the same five weeks, Ivy's expenses which were originally $200 per week, increase by $20 each week.

Write the equation of the least-squares regression line for Ivy's income, determine the equation of Ivy's expenses and draw both lines neatly on the graph above.

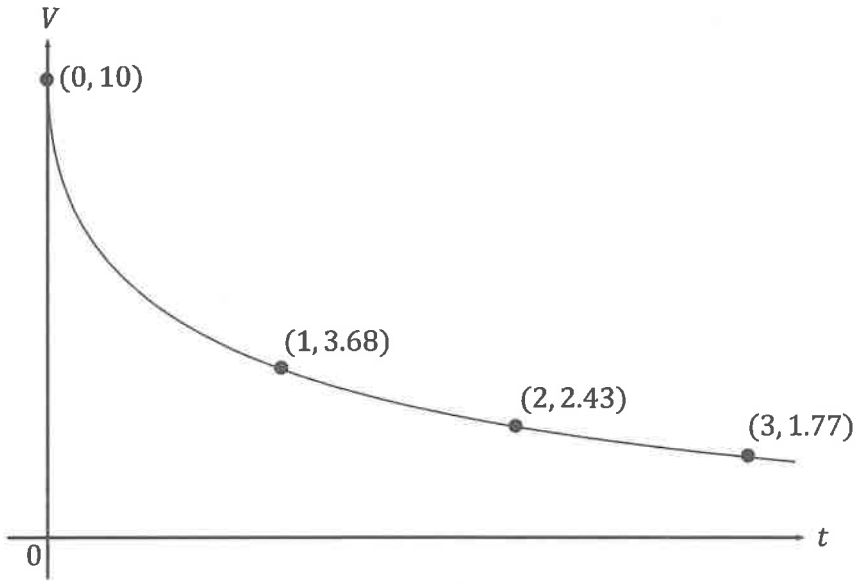
Income = ...

Expenses = ...

Question 24 (3 marks)

A deflated weather balloon at a scientific research station is being inflated such that the rate of air flow can be modelled by the equation $V = 10e^{-\sqrt{t}}$, where V is the rate of air flow at time t.

The graph of $V = 10e^{-\sqrt{t}}$ is shown below with the flow rates at hourly intervals.



(a) Using three applications of the Trapezoidal rule, estimate the volume of air in the balloon at time $t = 3$. **2**

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(b) A safety shutoff valve is programmed to be activated before the balloon reaches its maximum volume, after which the balloon will burst. **1**

By referring to the graph, or otherwise, explain why the Trapezoidal rule is a valid method of estimation in this context.

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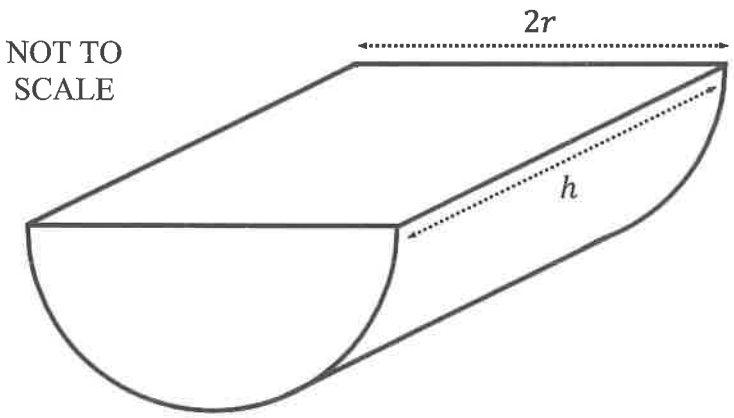
Question 25 (6 marks)

A closed water tank in the shape of a half-cylinder with diameter $2r$ metres and height h metres is to be made from sheet metal as shown below.



The surface area of the tank is A square metres and the volume of the tank is V cubic metres.

(a) Show $A = \pi r^2 + \frac{2V}{\pi}\left(\frac{2+\pi}{r}\right)$, given that $V = \frac{1}{2}\pi r^2 h$. **2**

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Question 25 continues on page 21

Question 25 (continued)

(b) Show that the amount of sheet metal used is minimised when $r = \sqrt[3]{\dfrac{V(2+\pi)}{\pi^2}}$. **3**

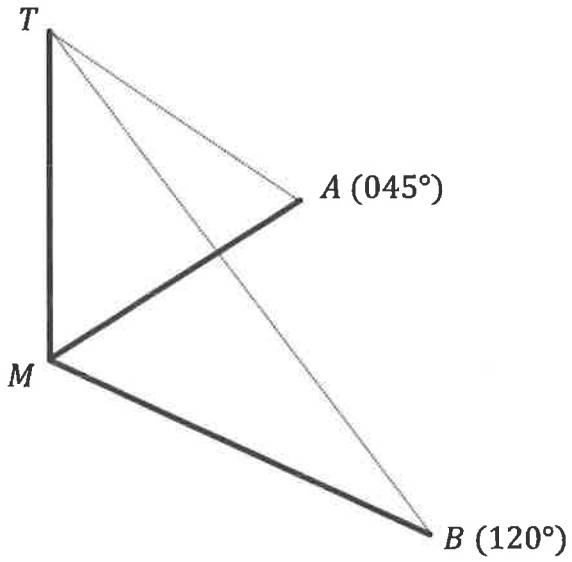
(c) Hence find the height of the tank, given that the volume of the tank is 10 cubic metres. **1**

End of Question 25

Question 26 (5 marks)

Michael installs a vertical radio mast on level ground at M. Two pegs are installed on the ground at A and B on bearings of 045° and 120° respectively from M.

The distance from A to M is 100 m and the area of triangle AMB is 10 432 m^2.



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(a) Find the distance between the base of the mast, M, and the peg at B. **2**

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Question 26 continues on page 23

Question 26 (continued)

(b) Supporting wires are connected from each peg to the top of the mast at T. **3**

By considering triangle AMT and triangle BMT, find the height of the mast, given that the total length of the supporting wires is 380 m.

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End of Question 26

Question 27 (4 marks)

Consider the function $f(x) = (x + 1)^2$. **4**

The following transformations were applied to the graph of $y = f(x)$ in the order shown below.

- Horizontal translation to the right by 4 units
- Horizontal dilation by a factor of $\frac{4}{3}$
- Vertical dilation by a factor of $\frac{1}{3}$
- Vertical translation down by 3 units
- Reflection in the x-axis

Find the equation of the transformed function and sketch its graph, showing the location of the vertex and the axis intercepts.

Question 28 (7 marks)

A particle is moving along the x-axis so that its position, in metres, at time t seconds is given by

$$x = 2\ln(t^2 + 3) - t$$

(a) Determine the times when the particle is at rest. **2**

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(b) Find the distance travelled by the particle in the first second. **2**

Leave your answer in the form $\ln\left(\frac{a}{b}\right) - c$, where a, b and c are integers.

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(c) Find the maximum velocity of the particle. **3**

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Question 29 (7 marks)

Consider the function $f(x)=3x\left(1-\frac{x}{4}\right)^3+1$.

(a) By showing that $f'(x)=3\left(1-\frac{x}{4}\right)^2(1-x)$, find any stationary points and determine their nature. **3**

Question 29 continues on page 27

Question 29 (continued)

(b) Explain why the graph of $y = f(x)$ must cross the x-axis at least once between $x = 5$ and $x = 6$. **1**

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(c) Sketch the graph of $y = f(x)$ in the interval $[0, 6]$, showing the stationary points, the y-intercept and the approximate location of the x-intercept. **3**

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End of Question 29

Question 30 (5 marks)

Aida has \$500 000 in an investment account earning interest at a rate of 3% per annum compounded annually.

At the end of the first year, Aida withdraws \$25 000 for expenses.

Each subsequent year, the amount that Aida withdraws increases by 5% due to rising expenses.

Let A_n be the balance of Aida's investment account at the end of the nth year.

(a) Show that $A_3 = 500000(1.03)^3 - 25000(1.05^2 + (1.05)(1.03) + 1.03^2)$. **2**

Question 30 continues on page 29

Question 30 (continued)

(b) For how many complete years will Aida be able to withdraw money from this account in order to meet her expenses? **3**

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End of Question 30

Question 31 (5 marks)

The time taken, t minutes, for a machine to produce car parts can be modelled by a continuous random variable with probability density function given by

$$f(t)=\begin{cases} k(t-1)^2, & 1\le t\le 2 \\ k\left(2-\frac{t}{2}\right), & 2<t\le 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{3}{4}$. **2**

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Question 31 continues on page 31

Question 31 (continued)

(b) Let $F(t)$ be the cumulative distribution function of $f(t)$, where $F(2.16) = 0.3652$. **3**

Find the least amount of time taken to produce 88% of the parts.

End of Question 31

Question 32 (3 marks)

A heater converts electrical power, measured in watts, into heat in such a way that the heat produced is given by the function $h(x) = \left(f(x)\right)^2$. **3**

The heater is plugged into a faulty socket which outputs $x = g(t)$ watts at time t seconds.

The power output at time $t = 1$ is 3 watts and is increasing at a rate of 4 watts per second.

By considering $h'(x)$ and using the fact that $f'(x) = f'\left(g(t)\right) \times g'(t)$, find the rate of change of the heat produced at time $t = 1$, given that $f(3) = 5$ and $f'(3) = 2$.

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End of Examination

Question 10, 11, 22, 24-26: Images created by 2023 CSSA Trial HSC Examination Mathematics Advanced Convenor.

Section II extra writing space

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EXAMINERS

Michael Zaouk (Convenor)	The McDonald College, North Strathfield
Marie Donaghy	Casimir Catholic College, Marrickville
Dr Alan McCarthy	The University of Notre Dame Australia
Penny Pachos	MLC School, Burwood
Julie Sligar	St Gregory's College, Campbelltown