2023

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

CSSA

MARKING GUIDELINES

Mathematics Extension I

Section I
10 marks

Multiple Choice Answer Key

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	C	ME11-5	E1
2	C	ME11-3	E1-E2
3	D	ME11-5, ME11-6	E1-E2
4	C	ME12-3	E2
5	A	ME12-5	E2
6	B	ME11-2	E2-E3
7	B	ME11-1, ME11-2	E3-E4
8	A	ME12-2	E3
9	A	ME11-5	E4
10	D	ME12-4	E4

Question 1 (1 mark)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E1*

Solution	Mark
By the pigeonhole principle, $3 \times 3 + 1 = 10$ marbles must be chosen to guarantee that there are at least 4 marbles (pigeons) of the same colour (pigeonholes). Hence C	1

Question 2 (1 mark)
Outcomes Assessed: *ME11-3*
Targeted Performance Bands: *E1-E2*

Solution	Mark
$\frac{d}{dx}\left(2\sin^{-1}\frac{x}{2}\right) = 2\times\frac{\frac{1}{2}}{\sqrt{1-(\frac{x}{2})^2}}$ $= 2\times\frac{\frac{1}{2}}{\frac{1}{2}\sqrt{4-x^2}} = \frac{2}{\sqrt{4-x^2}}$ Hence C	1

Question 3 (1 mark)
Outcomes Assessed: *ME11-5, ME11-6*
Targeted Performance Bands: *E1-E2*

Solution	Mark
The x^5 term will be ${}^7C_5(3)^2(2x)^5 = 6048x^5$. Hence the coefficient is 6048. Hence D	1

Question 4 (1 mark)
Outcomes Assessed: *ME12-3*
Targeted Performance Bands: *E2*

Solution	Mark
$15\sin\theta - 8\cos\theta = R\sin(\theta-\alpha)$, where $\alpha = \tan^{-1}\left(\frac{8}{15}\right)$ and $R = \sqrt{15^2+8^2} = 17$. Hence C	1

Question 5 (1 mark)
Outcomes Assessed: *ME12-5*
Targeted Performance Bands: *E2*

Solution	Mark
$np = E(X) = 0.5$ and $\mathrm{Var}(X) = np(1-p) = 0.45$ Hence $0.45 = 0.5\times(1-p)$, and so $p = 0.1$. $0.45 = n\times 0.1\times 0.9$ $n = \frac{0.45}{0.09} = 5$ Hence A	1

Question 6 (1 mark)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E2-E3*

Solution	Mark
The domain of an inverse cosine function will give: $-1 \le \dfrac{2-x}{3} \le 1$ $-3 \le 2-x \le 3$ $-5 \le -x \le 1$ $5 \ge x \ge -1$ Which is the interval $[-1,5]$. Hence B	1

Question 7 (1 mark)
Outcomes Assessed: *ME11-1, ME11-2*
Targeted Performance Bands: *E3-E4*

Solution	Mark
For $f^{-1}(3)$ we need $3 = x^2 + 2$ and since $x \ge 0$, $x = 1$. So the relevant point on $y = f(x)$ is $(1,3)$, which has gradient $f'(1) = 2 \times 1 = 2$. So the point on $y = f^{-1}(x)$ is $(3,1)$ with a gradient of $\frac{1}{2}$. Hence B	1

Question 8 (1 mark)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E3*

Solution	Mark
Vertical slopes along the line $x = 0$ rules out C and D. Testing the point $(4,2)$ in A gives $\dfrac{dy}{dx} = \frac{1}{2} - 2 < 0$, which matches the diagram. Testing the point $(4,2)$ in B gives $\dfrac{dy}{dx} = \frac{1}{2} + 2$, which does not match the diagram. Hence A	1

Question 9 (1 mark)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E4*

Solution	Mark
If you fix the principal, you can arrange the other participants 12! ways without restriction. If you sit all the year group pairs together, you can the order of year groups in 6! ways and each of the 6 pairs can also be swapped. So the number of ways you can arrange all groups in pairs is $6! \times 2^6$. Therefore the number of ways you can seat the 13 participants if at least one pair of students from a year group sits apart is the complement of this, being $12! - 6! \times 2^6$. Hence A	1

Question 10 (1 mark)
Outcomes Assessed: *ME12-4*
Targeted Performance Bands: *E4*

Solution	Mark
Let $F(x)$ be the indefinite integral function of $f(x)$, as in $F'(x) = f(x)$. For all options, LHS $= F(a) - F(0)$. For option A: $\text{RHS} = k\int_0^{ak} f(kx)\,dx = k\left[\frac{1}{k}F(kx)\right]_0^{ak} = F(ak^2) - F(0) \neq \text{LHS}.$ For option B: $\text{RHS} = \frac{1}{k}\int_0^{ak} f(kx)\,dx = \frac{1}{k}\left[\frac{1}{k}F(kx)\right]_0^{ak} = \frac{1}{k^2}\left(F(ak^2) - F(0)\right) \neq \text{LHS}.$ For option C: $\text{RHS} = k\int_0^{ak} f\left(\frac{x}{k}\right)dx = k\left[kF\left(\frac{x}{k}\right)\right]_0^{ak} = k^2\left(F(a) - F(0)\right) \neq \text{LHS}.$ For option D: $\text{RHS} = \frac{1}{k}\int_0^{ak} f\left(\frac{x}{k}\right)dx = \frac{1}{k}\left[kF\left(\frac{x}{k}\right)\right]_0^{ak} = F(a) - F(0) = \text{LHS}.$ Hence D	1

Section II

60 marks

Question 11 (14 marks)

Question 11(a) (2 marks)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E1*

Criteria	Marks
• correct solution	2
• progress towards correct solution	1

Sample Answer:

$$|5x-1|<5$$
$$-5<5x-1<5$$
$$-4<5x<6$$
$$-\frac{4}{5}<x<\frac{6}{5}$$

Question 11(b) (2 marks)
Outcomes Assessed: *ME12-4*
Targeted Performance Bands: *E1-E2*

Criteria	Marks
• correct solution	2
• some progress towards correct solution	1

Sample Answer:

Using the reference sheet, $\displaystyle\int \frac{1}{9+25x^2}\,dx = \frac{1}{5}\int \frac{5}{3^2+(5x)^2}\,dx = \frac{1}{15}\tan^{-1}\left(\frac{5x}{3}\right)+c$

Question 11(c) (i) (1 mark)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E2*

Criteria	Mark
• correct solution	1

Sample Answer:

Sum of roots given by $\alpha+\beta+\gamma=-\frac{b}{a}=-3$.

Question 11(c) (ii) (2 marks)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E2*

Criteria	Marks
• correct solution	2
• some progress towards correct expression for $\alpha^2+\beta^2+\gamma^2$	1

Sample Answer:

$$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)=(-3)^2-2(-4)=17$$

Question 11(d) (i) (2 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E1-E2*

Criteria	Marks
• Correct solution	2
• Attempts to apply the dot product	1

Sample Answer:

$$\underset{\sim}{u}\cdot\underset{\sim}{v}=\begin{pmatrix}2\\3\end{pmatrix}\cdot\begin{pmatrix}5\\-12\end{pmatrix}=10-36=-26$$

Question 11(d) (ii) (1 mark)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E2*

Criteria	Mark
• Correct solution	1

Sample Answer:

$$|\underset{\sim}{v}|=\sqrt{(-5)^2+12^2}=13$$

Question 11(d) (iii) (2 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E2*

Criteria	Marks
• Correct solution	2
• Some progress to apply a formula for projection	1

Sample Answer:

$$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\underset{\sim}{v}\cdot\underset{\sim}{v}}\underset{\sim}{v} = \frac{\begin{pmatrix}2\\3\end{pmatrix}\cdot\begin{pmatrix}5\\-12\end{pmatrix}}{\begin{pmatrix}5\\-12\end{pmatrix}\cdot\begin{pmatrix}5\\-12\end{pmatrix}}\begin{pmatrix}5\\-12\end{pmatrix} = \frac{-26}{13^2}\begin{pmatrix}5\\-12\end{pmatrix} = \begin{pmatrix}-\frac{10}{13}\\\frac{24}{13}\end{pmatrix}$$

Question 11(e) (2 marks)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E1-E2*

Criteria	Marks
• Correct solution in factorial or numerical form	2
• Finds the number of arrangements without dealing with the restriction	1

Sample Answer:
11 letters, with $2\times$ S and $3\times$ I gives $\frac{11!}{3!2!}$, which is wrong given the restriction. Instead treat the letters S as one unit, meaning you need to permute 10 items where 3 of them are indistinguishable, so $\frac{10!}{3!} = 604\,800$.

Question 12 (15 marks)

Question 12(a) (3 marks)
Outcomes Assessed: *ME11-2, ME12-4*
Targeted Performance Bands: *E2-E3*

Criteria	Marks
• Correct solution	3
• Significant progress towards solving the equation	2
• Correctly separates the equation into x and y functions	1

Sample Answer:

$$\frac{dy}{dx} = \frac{2}{x^3e^y}$$
$$\int e^y\,dy = \int \frac{2}{x^3}\,dx$$
$$e^y = \frac{2x^{-2}}{-2} + c$$

sub in $y(1) = 0$ gives $1 = -1 + c \Rightarrow c = 2$

$$e^y = 2 - \frac{1}{x^2}$$
$$y = \log_e\left|2 - \frac{1}{x^2}\right|$$

Question 12(b) (3 marks)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E3*

Criteria	Marks
• Correct solution	3
• Correctly simplifies and finds common denominator	2
• Expands double angle for tan correctly	1

Sample Answer:
RTP: $\tan\left(\frac{\pi}{4}+x\right)-\tan\left(\frac{\pi}{4}-x\right)=2\tan(2x)$.

$$\begin{aligned}\text{LHS} &= \frac{\tan\frac{\pi}{4}+\tan x}{1-\tan x\tan\frac{\pi}{4}}-\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan x\tan\frac{\pi}{4}}\\ &= \frac{1+\tan x}{1-\tan x}-\frac{1-\tan x}{1+\tan x}\\ &= \frac{(1+\tan x)^2-(1-\tan x)^2}{(1-\tan x)(1+\tan x)}\\ &= \frac{1+2\tan x+\tan^2 x-1+2\tan x-\tan^2 x}{1-\tan^2 x}\\ &= \frac{4\tan x}{1-\tan^2 x}=2\tan 2x=\text{RHS}\end{aligned}$$

Question 12(c) (3 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E2-E3*

Criteria	Marks
• Correct solution	3
• Significant progress towards finding the length of $3\underset{\sim}{a}-2\underset{\sim}{b}$	2
• Some progress towards expanding the length of $3\underset{\sim}{a}-2\underset{\sim}{b}$ or work of similar merit	1

Sample Answer:

Let $d=|3\underset{\sim}{a}-2\underset{\sim}{b}|$

$$\begin{aligned}d^2 &= |3\underset{\sim}{a}-2\underset{\sim}{b}|^2\\ &= \left(3\underset{\sim}{a}-2\underset{\sim}{b}\right)\cdot\left(3\underset{\sim}{a}-2\underset{\sim}{b}\right)\\ &= 3\underset{\sim}{a}\cdot 3\underset{\sim}{a}-3\underset{\sim}{a}\cdot 2\underset{\sim}{b}-2\underset{\sim}{b}\cdot 3\underset{\sim}{a}+2\underset{\sim}{b}\cdot 2\underset{\sim}{b}\\ &= 9|\underset{\sim}{a}|^2-12\underset{\sim}{a}\cdot\underset{\sim}{b}+4|\underset{\sim}{b}|^2\\ &= 9\times 4-12\times 5+4\times 9=12\end{aligned}$$

Hence $d=\sqrt{12}=2\sqrt{3}$

Question 12(d) (3 marks)
Outcomes Assessed: *ME12-3*
Targeted Performance Bands: *E2-E3*

Criteria	Marks
• correct solution	3
• some progress towards correct integration or correct substitution into integration	2
• some progress towards the correct expansion of the identity	1

Sample Answer:

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x - \cos x)^2\,dx = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin^2 x + \cos^2 x - 2\sin x\cos x)\,dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1 - \sin 2x)\,dx = \left[x + \frac{\cos 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \left(\frac{3\pi}{4} - \frac{\cos\frac{3\pi}{2}}{2}\right) - \left(\frac{\pi}{4} - \frac{\cos\frac{\pi}{2}}{2}\right) = \frac{\pi}{2}$$

Question 12(e) (3 marks)
Outcomes Assessed: *ME12-1, ME12-7*
Targeted Performance Bands: *E3*

Criteria	Marks
• correct solution	3
• correct applies the $n = k$ step	2
• proves the statement is true for $n = 1$	1

Sample Answer:
Step 1 Prove the result true for $n = 1$.
$23^1 - 1 = 22 = 2 \times 11$
Therefore, the result is true for $n = 1$.

Step 2 Assume the result is true for some integer $n = k$
That is, $23^k - 1 = 11p,\ p \in \mathbb{Z}$

Step 3 Prove the result true for $n = k + 1$
i.e. Prove that $23^{k+1} - 1 = 11q,\ q \in \mathbb{Z}$

$$\begin{aligned} \text{LHS} &= 23^{k+1} - 1 \\ &= 23 \times 23^k - 1 \\ &= 23 \times (11p + 1) - 1 \qquad \text{By Step 2} \\ &= 23 \times 11p + 23 - 1 \\ &= 11(23p + 2) \end{aligned}$$

And since $23p + 2 \in \mathbb{Z}$, the result is true by the principle of mathematical induction.

Question 13 (16 marks)

Question 13(a) (i) (1 mark)

Outcomes Assessed: *ME12-1*

Targeted Performance Bands: *E2-E3*

Criteria	Mark
• correct solution	1

Sample Answer:

$$\begin{aligned}\frac{d}{dx}\left(\tan^3 x\right) &= 3\tan^2 x \times \sec^2 x \\ &= 3\left(\sec^2 x - 1\right)\sec^2 x \\ &= 3\sec^4 x - 3\sec^2 x\end{aligned}$$

Question 13(a) (ii) (3 marks)

Outcomes Assessed: *ME12-1, ME12-4*

Targeted Performance Bands: *E2-E3*

Criteria	Marks
• correct solution	3
• significant progress towards connecting part (i) to evaluating the integral	2
• some progress towards a solution	1

Sample Answer:

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \sec^4 x\,dx &= \int_0^{\frac{\pi}{4}} \left(\sec^4 x - \sec^2 x + \sec^2 x\right)\,dx \\ &= \frac{1}{3}\int_0^{\frac{\pi}{4}} \left(3\sec^4 x - 3\sec^2 x\right)\,dx + \int_0^{\frac{\pi}{4}} \sec^2 x\,dx \\ &= \frac{1}{3}\left[\tan^3 x\right]_0^{\frac{\pi}{4}} + \left[\tan x\right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{3}\left(1^3 - 0^3\right) + (1 - 0) \\ &= \frac{4}{3}\end{aligned}$$

Question 13(b) (3 marks)
Outcomes Assessed: *ME12-5*
Targeted Performance Bands: *E2-E3*

Criteria	Marks
• correct solution	3
• correctly substitutes for $F(a) = 1$	2
• tests for $F(0)$	1

Sample Answer:

$$F(0) = 0, \text{ so we need } F(a) = 1,$$
$$\text{so: } \frac{3}{\pi}\tan^{-1}(\sqrt{a}) = 1$$
$$\tan^{-1}(\sqrt{a}) = \frac{\pi}{3}$$
$$\sqrt{a} = \sqrt{3}$$
$$\therefore \quad a = 3$$

Question 13(c) (i) (1 mark)
Outcomes Assessed: *ME12-3*
Targeted Performance Bands: *E1-E2*

Criteria	Mark
• correct solutions	1

Sample Answer:
From the reference sheet, $\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$

$$\text{RHS} = 2\cos 3x\cos 2x = \cos(3x-2x)+\cos(3x+2x) = \cos x + \cos 5x = \text{LHS}$$

Question 13(c) (ii) (2 marks)
Outcomes Assessed: *ME12-3*
Targeted Performance Bands: *E2-E3*

Criteria	Marks
• correct solution	2
• solves one part of the equation	1

Sample Answer:

$$\cos x + \cos 5x = \cos 2x, \quad x \in [0, \pi]$$
$$2\cos 3x\cos 2x = \cos 2x$$
$$\cos 2x(2\cos 3x - 1) = 0$$
$$\therefore \cos 2x = 0 \quad \text{or} \quad \cos 3x = \frac{1}{2} \quad x \in [0, \pi]$$
$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

Question 13(d) (3 marks)
Outcomes Assessed: *ME12-1, ME12-4*
Targeted Performance Bands: *E3*

Criteria	Marks
• correct solution	3
• finds correct limits and integrand or work of similar merit	2
• finds correct limits or work of similar merit	1

Sample Answer:

$$\int_0^{\sqrt{3}} \frac{1}{(1+x^2)^{3/2}}\,dx \qquad \text{let } u = \tan^{-1}x \quad \text{when } x=0,\, u=0$$

$$\text{so } du = \frac{1}{1+x^2}\,dx \quad \text{when } x=\sqrt{3},\, u=\frac{\pi}{3}$$

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{1+x^2}\,(1+x^2)}\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{\sqrt{1+\tan^2 u}}\,du \qquad \text{and for } 0\le u\le\frac{\pi}{3},\ \sqrt{1+\tan^2 u}=\sec u$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{\sec u}\,du = \int_0^{\frac{\pi}{3}} \cos u\,du$$

$$= \Big[\sin u\Big]_0^{\frac{\pi}{3}} = \frac{\sqrt{3}}{2}$$

Question 13(e) (3 marks)
Outcomes Assessed: *ME12-4*
Targeted Performance Bands: *E3-E4*

Criteria	Marks
• correct solution	3
• correct integration OR correct substitution of limits	2
• correct expression for the volume including limits	1

Sample Answer:

$$V = \pi\int_0^q x^2\,dy \qquad \text{where } q=\log_e(k-1),\text{ and } x=e^y+1$$

$$\frac{V}{\pi} = \int_0^q (e^y+1)^2\,dy$$

$$= \int_0^q \left(e^{2y}+2e^y+1\right)dy = \left[\frac{1}{2}e^{2y}+2e^y+y\right]_0^q$$

$$= \left(\frac{1}{2}e^{2q}+2e^q+q\right)-\left(\frac{1}{2}e^0+2e^0+0\right) \qquad \text{note } e^q=k-1$$

$$\frac{\pi(27+\log_e 16)}{2\pi} = \frac{1}{2}(k-1)^2+2(k-1)+\log_e(k-1)-\frac{5}{2}$$

$$27+2\log_e 4 = (k-1)^2+4(k-1)+2\log_e(k-1)-5$$

Now by equating irrational expressions, $\log_e(k-1)=\log_e(4)$, hence $k=5$.

Question 14 (15 marks)

Question 14(a) (3 marks)

Outcomes Assessed: *ME12-5*

Targeted Performance Bands: *E3-E4*

Criteria	Marks
• correct solution (marker judgement about reasonable approximation)	3
• correctly evaluates z-score	2
• correctly evaluates b OR some progress towards a solution	1

Sample Answer:

The life of a battery $b \sim N(10.25, 0.25)$. The probability of a battery selected at random lasting longer than 10 hours is $P(b > 10) = P\left(Z > \dfrac{10 - 10.25}{0.5}\right) \approx 0.691$

Method 1 - using sample proportions

Let $\hat{p}$ be the proportion of batteries in the sample that last longer than 10 hours.

$$\hat{p} \sim N\left(0.691, \frac{0.691 \times 0.309}{25}\right)$$

$$\begin{aligned} \text{So} \quad P\left(Z > \frac{0.5 - 0.691}{0.09241}\right) &\approx P(Z > -2.07) \\ &\approx 0.9806 \ \text{(calculator)} \ \textbf{or} \approx 0.9808 \ \text{(table)} \\ &\approx 98\% \ \text{nearest percent} \end{aligned}$$

Method 2 - with continuity correction

Let X be the number of batteries in the sample of 25 that have a life greater than 10 hours

$$\begin{aligned} X \sim N(np, npq) &= X \sim N(17.275, 5.337975) \\ \text{So } P(X \geq 13) &= P(X > 12.5) \quad \text{(with continuity correction)} \\ &\approx P\left(Z > \frac{12.5 - 17.275}{\sqrt{5.337975}}\right) \\ &\approx P(Z > -2.067) \\ &\approx 0.9806 \ \text{(calculator)} \ \textbf{or} \approx 0.9808 \ \text{(table)} \\ &\approx 98\% \ \text{nearest percent} \end{aligned}$$

Method 3 - no continuity correction

$$\begin{aligned} X \sim N(np, npq) &= X \sim N(17.275, 5.337975) \\ \text{So } P(X \geq 13) &= P\left(Z > \frac{13 - 17.275}{\sqrt{5.337975}}\right) \\ &\approx P(Z > -1.85) \\ &\approx 0.96787 \ \text{(calculator)} \ \textbf{or} \approx 0.9678 \ \text{(table)} \\ &\approx 97\% \ \text{nearest percent} \end{aligned}$$

Question 14(b) (5 marks)
Outcomes Assessed: *ME12-4, ME12-7*
Targeted Performance Bands: *E3-E4*

Criteria	Marks
• correct solution	5
• substituting initial conditions	4
• integrating to get an expression in t	3
• correct application of chain rule AND finding $\frac{dV}{dh}$	2
• correct application of chain rule OR finding $\frac{dV}{dh}$	1

Sample Answer:
$\dfrac{dV}{dt} = -h\sqrt{h}$, while $V = Ah$, and since $A = 10$, $\quad \dfrac{dV}{dh} = 10$.

Now $\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt} = \frac{1}{10} \times -h\sqrt{h}$.

Reciprocating gives $\dfrac{dt}{dh} = -10h^{-3/2}$, then integrating:

$t = -10 \times h^{-1/2} \times -2 + c = \dfrac{20}{\sqrt{h}} + c$

When $t = 0$, $h = 4$, so $0 = \frac{20}{2} + c$, so $c = -10$.

Therefore, $t = 10\left(\dfrac{2}{\sqrt{h}} - 1\right)$

We want to find when $V = 1$ litre $= \frac{1}{1000}$ m^3. Therefore $\frac{1}{1000} = 10h$, so $h = \frac{1}{10000}$

$t = 10\left(\dfrac{2}{\sqrt{0.0001}} - 1\right) = 1990$ hours.

Question 14(c) (i) (1 mark)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E1-E2*

Criteria	Mark
• correct solution	1

Sample Answer:
When $10\sqrt{2}t = 10$, $t = \frac{1}{\sqrt{2}}$

$\therefore \underset{\sim}{r}\left(\frac{1}{\sqrt{2}}\right) = \begin{pmatrix} 10 \\ 10 - \frac{5}{2} + 1 \end{pmatrix}$. $\qquad$ Hence the height is 8.5 metres.

Question 14(c) (ii) (1 mark)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E2-E3*

Criteria	Mark
• correct solution	1

Sample Answer:

$$\underset{\sim}{r}(t) = \begin{pmatrix} 10\sqrt{2}t \\ 10\sqrt{2}t - 5t^2 + 1 \end{pmatrix} \qquad \text{differentiating w.r.t. time will give:}$$

$$\therefore \underset{\sim}{v}(t) = \begin{pmatrix} 10\sqrt{2} \\ 10\sqrt{2} - 10t \end{pmatrix}$$

Question 14(c) (iii) (2 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E3-E4*

Criteria	Marks
• correct solution (approximation not necessary)	2
• substitutes to find $\underset{\sim}{v}\left(\frac{1}{\sqrt{2}}\right)$	1

Sample Answer:
The ball hits the wall when $t = \frac{1}{\sqrt{2}}$

Now, $10\sqrt{2} - \dfrac{10}{\sqrt{2}} = 10\sqrt{2} - 5\sqrt{2}$

Hence, $\underset{\sim}{v}\left(\frac{1}{\sqrt{2}}\right) = \begin{pmatrix} 10\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}$

Now, $U = \left|\begin{pmatrix} 10\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}\right| = 5\sqrt{2}\sqrt{1^2+2^2} = 5\sqrt{10}.$

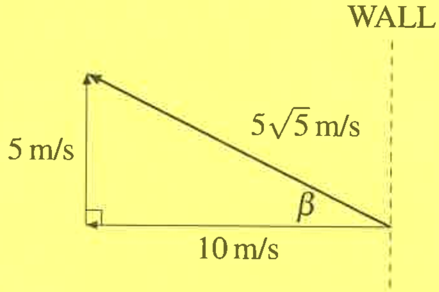
Furthermore, $\beta = \tan^{-1}\left(\frac{5\sqrt{2}}{10\sqrt{2}}\right) = \tan^{-1}\left(\frac{1}{2}\right) \approx 27°$

Question 14(c) (iv) (3 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E3-E4*

Criteria	Marks
• correct solution	3
• integrates from acceleration to get expression for $\underset{\sim}{v}(w)$	2
• finds conditions at the moment of rebound	1

Sample Answer:

Since $\beta = \tan^{-1}\left(\frac{1}{2}\right)$, and $\frac{1}{\sqrt{2}}U = 5\sqrt{5}$
The velocity of the ball as it leaves the wall is illustrated in the diagram on the right.



Let t seconds now be the time of flight from when the ball leaves the wall on its return journey.

$$\text{So} \quad \underset{\sim}{v}(0) = \begin{pmatrix} -10 \\ 5 \end{pmatrix}$$

$$\text{And also} \quad \underset{\sim}{r}(0) = \begin{pmatrix} 10 \\ 8.5 \end{pmatrix}$$

$$\text{Now} \quad \underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

$$\text{So} \quad \underset{\sim}{v}(t) = \begin{pmatrix} c_1 \\ -10t + c_2 \end{pmatrix} = \begin{pmatrix} -10 \\ 5 - 10t \end{pmatrix}$$

$$\text{and further} \quad \underset{\sim}{r}(t) = \begin{pmatrix} -10t + c_3 \\ 5t - 5t^2 + c_4 \end{pmatrix} = \begin{pmatrix} 10 - 10t \\ 5t - 5t^2 + 8.5 \end{pmatrix}$$

When the ball is at Q, then the horizontal component is zero.

$$10 - 10t = 0, \Rightarrow t = 1 \text{ second}$$

$$\therefore \quad \underset{\sim}{r}(1) = \begin{pmatrix} 0 \\ 5 - 5 + 8.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 8.5 \end{pmatrix} \text{ as required.}$$