CSSA

2023
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAMINATION ROOM

Centre Number

Student Number

Mathematics Extension 1

Afternoon Session
Friday, 11 August 2023

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- A normal cumulative distribution function table is provided
- Use the Multiple-Choice Answer Sheet provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 70

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–10)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10

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1 A bag contains a large number of red, green and blue marbles. Arlo will take out some marbles, without looking, and needs to be certain he will have at least four marbles of the same colour. What is the smallest number of marbles that he must take out to ensure this?

A. 4

B. 5

C. 10

D. 13

2 Which of the following is equivalent to $\frac{d}{dx}\left(2\sin^{-1}\frac{x}{2}\right)$?

A. $\dfrac{1}{\sqrt{1-x^2}}$

B. $\dfrac{2}{\sqrt{1-x^2}}$

C. $\dfrac{2}{\sqrt{4-x^2}}$

D. $\dfrac{1}{2\sqrt{4-x^2}}$

3 What is the coefficient of x^5 in the expansion of $(3+2x)^7$?

A. 21

B. 128

C. 1344

D. 6048

4 What is the maximum value of $15\sin\theta - 8\cos\theta$?

A. 7

B. 15

C. 17

D. 23

5 A given Bernoulli experiment has a probability of success p. It is repeated n times, giving an expected value of 0.5 and a variance of 0.45. What is the value of n?

A. 5

B. 10

C. 25

D. 50

6 Which of the following gives the complete domain of $y = 5\cos^{-1}\left(\frac{2-x}{3}\right)$?

A. $[1,5]$

B. $[-1,5]$

C. $[-5,1]$

D. $[-5,-1]$

7 Consider the function $f(x) = x^2 + 2$, with domain $x \geq 0$.
What is the gradient of the tangent to $y = f^{-1}(x)$ at $x = 3$?

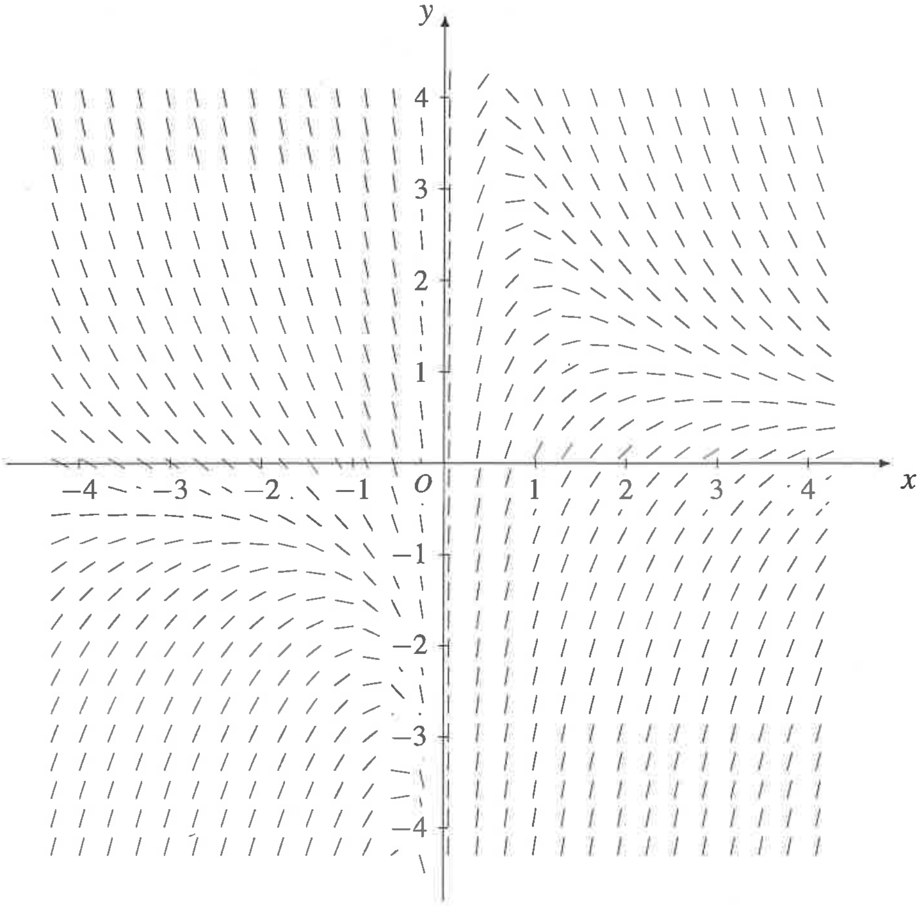
A. $\frac{1}{6}$

B. $\frac{1}{2}$

C. 2

D. 6

8 The slope field for a differential equation is sketched below.



Which of the following is the correct differential equation for the slope field?

A. $\dfrac{dy}{dx}=\dfrac{2}{x}-y$

B. $\dfrac{dy}{dx}=\dfrac{2}{x}+y$

C. $\dfrac{dy}{dx}=\dfrac{2}{y}-x$

D. $\dfrac{dy}{dx}=\dfrac{2}{y}+x$

9 A school principal calls a meeting with two student representatives from each year group from Year 7 to Year 12. The 12 students and the principal sit at a round table. In how many ways can the 13 participants sit around the table if at least one pair of students from a year group sits apart?

A. $12! - 6! \times 2^6$

B. $13! - 6! \times 2^6$

C. $12! - 5! \times 2^5$

D. $13! - 5! \times 2^5$

10 Let $f(x)$ be a continuous function with $a > 0$ and $k > 0$. Which of the following is true?

A. $\int_0^a f(x)\,dx = k\int_0^{ak} f(kx)\,dx$

B. $\int_0^a f(x)\,dx = \frac{1}{k}\int_0^{ak} f(kx)\,dx$

C. $\int_0^a f(x)\,dx = k\int_0^{ak} f\left(\frac{x}{k}\right)\,dx$

D. $\int_0^a f(x)\,dx = \frac{1}{k}\int_0^{ak} f\left(\frac{x}{k}\right)\,dx$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Your responses for Questions 11–14 should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks)

(a) Solve $|5x - 1| < 5$. **2**

(b) Find $\int \frac{1}{9 + 25x^2}\,dx$. **2**

(c) The polynomial $P(x) = x^3 + 3x^2 - 4x - 5$ has roots α, β and γ.

(i) Find $\alpha + \beta + \gamma$. **1**

(ii) Find $\alpha^2 + \beta^2 + \gamma^2$. **2**

(d) For the vectors $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$, and $\underset{\sim}{v} = 5\underset{\sim}{i} - 12\underset{\sim}{j}$, find each of the following:

(i) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **2**

(ii) $|\underset{\sim}{v}|$ **1**

(iii) the projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$ **2**

(e) In how many ways can the letters of the word POSSIBILITY be arranged so that the two Ss are together? **2**

Question 12 (15 marks)

(a) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2}{x^3 e^y}$, where $y(1) = 0$. **3**

Express your solution in the form $y = f(x)$.

(b) Prove the identity $\tan\left(\dfrac{\pi}{4} + x\right) - \tan\left(\dfrac{\pi}{4} - x\right) = 2\tan(2x)$. **3**

(c) Given $|\underset{\sim}{a}| = 2$, $|\underset{\sim}{b}| = 3$ and $\underset{\sim}{a} \cdot \underset{\sim}{b} = 5$, calculate the length of $3\underset{\sim}{a} - 2\underset{\sim}{b}$. **3**

(d) Find the exact value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x - \cos x)^2 \, dx$. **3**

(e) Use mathematical induction to prove that $23^n - 1$ is divisible by 11, for all positive integers n. **3**

Question 13 (16 marks)

(a) (i) Show that $\dfrac{d}{dx}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$. **1**

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4 x\, dx$. **3**

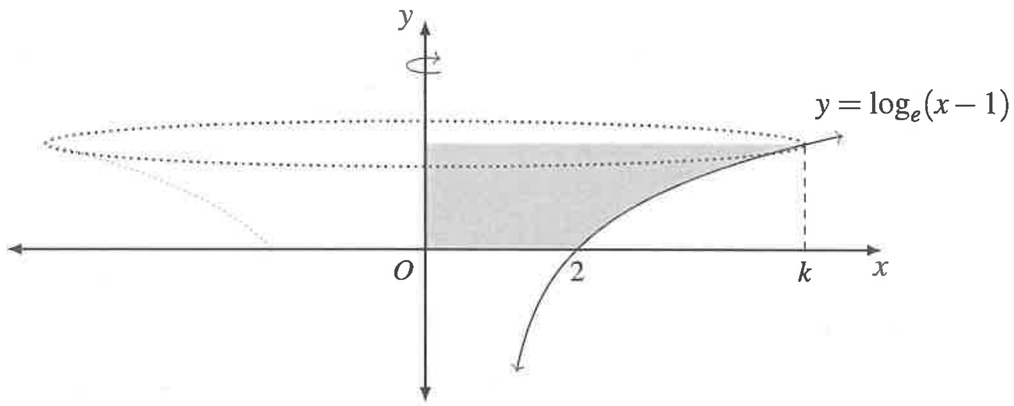
(b) A cumulative probability distribution function for a continuous random variable is given by $F(x) = \dfrac{3}{\pi}\tan^{-1}(\sqrt{x}),\ 0 \le x \le a$. Find the value of a. **3**

(c) (i) Show that $\cos x + \cos 5x = 2\cos 3x \cos 2x$. **1**

(ii) Hence, or otherwise, solve $\cos x + \cos 5x = \cos 2x$ for $x \in [0, \pi]$. **2**

(d) Use the substitution $u = \tan^{-1} x$ to find $\displaystyle\int_0^{\sqrt{3}} \frac{1}{(1+x^2)^{3/2}}\, dx$. **3**

(e) The shaded region shown below is bounded by the positive x- and y-axes, the curve $y = \log_e(x-1)$, and a horizontal line which intersects the curve where the x-coordinate is the rational number k. **3**



The volume V formed when this region is rotated around the y-axis is given by

$$V = \frac{\pi(27 + \log_e 16)}{2}.$$

Find the value of k.

Question 14 (15 marks)

(a) *You may use the Normal Cumulative Distribution Function Table on page 11 to answer Question 14(a).*

LithiumCorp claims that the batteries it manufactures for mobile phones will last for at least 10 hours of continuous video streaming. The video streaming time for their batteries is normally distributed with a mean of 10 hours 15 minutes and a standard deviation of 30 minutes. **3**

A potential customer, BananaPhones, will test 25 randomly selected batteries and if more than 50 percent of the batteries allow at least 10 hours of video streaming it will agree to purchase batteries from LithiumCorp.

Calculate the probability that LithiumCorp will be successful in supplying its batteries to BananaPhones. Give your answer correct to the nearest percent.

(b) A water tank is a prism with a base of area A square metres. Torricelli's Law states that the rate of change in the volume V cubic metres of a leaking tank with water level h metres at time t hours is given by the differential equation **5**

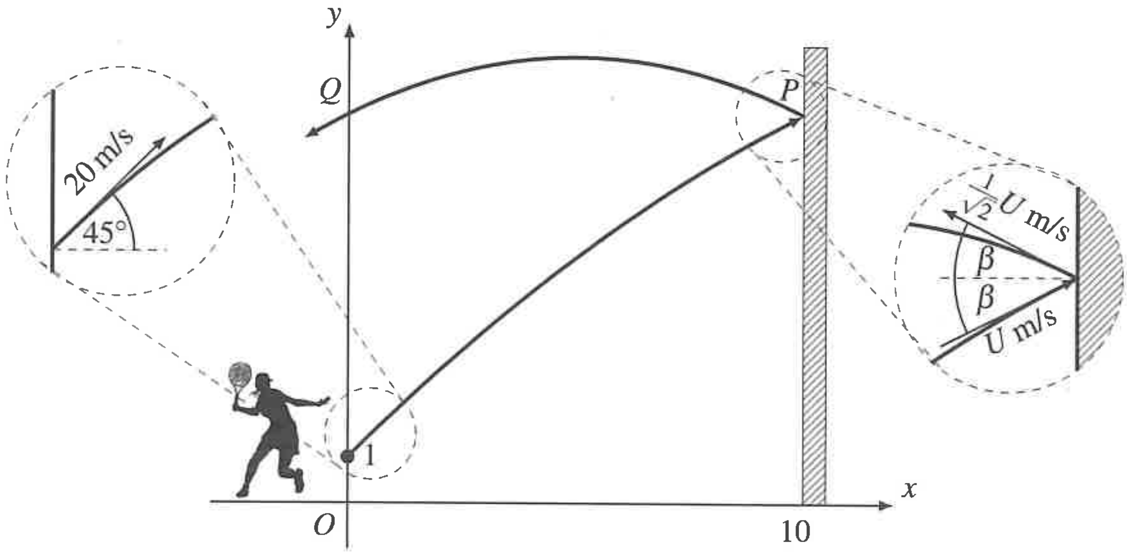
$$\frac{dV}{dt} = -h\sqrt{h}.$$

If a tank has a base with area 10 square metres and the initial height of the water is 4 metres, find how long it takes until the tank contains less than 1 litre of water.

Question 14 continues on page 10

Question 14 (continued)

(c) Charlie hits a tennis ball at an angle of 45° against a vertical wall 10 m away. The ball rebounds off the wall and passes directly over where it was first struck at point Q. The tennis ball is 1 m above the ground when Charlie hits it and has an initial speed of 20 m/s.



The position $\underset{\sim}{r}$ in metres of the ball t seconds after it is hit but before it hits the wall at P is given by the vector equation

$$\underset{\sim}{r}(t) = \begin{pmatrix} 10\sqrt{2}t \\ 10\sqrt{2}t - 5t^2 + 1 \end{pmatrix}. \qquad \text{Do NOT prove this.}$$

(i) Calculate the height at which the ball hits the wall. **1**

(ii) Find the velocity vector $\underset{\sim}{v}(t)$ of the ball during its flight from Charlie to the wall. **1**

(iii) Calculate the speed U m/s and the angle β at which the ball hits the wall. **2**

(iv) The ball rebounds off the wall at the same angle that it hits it, β, but its speed is reduced so that it leaves the wall with an initial speed of $\frac{1}{\sqrt{2}}U$ m/s. **3**

Show that the y-coordinates of the points P and Q in the diagram above are equal. Assume the acceleration due to gravity is 10 m/s^2.

End of Examination

EXAMINERS

David Houghton (Convenor)	Oxley College, Burradoo
Geoff Carroll	Sydney Grammar School, Darlinghurst
Stephen Ewington	Ascham School, Edgecliff
Rebekah Johnson	Loreto Kirribilli, Kirribilli
Svetlana Onisczenko	Meriden School, Strathfield

Questions 8, 13(c), and 14 (c) – Diagrams created by the 2023 CSSA Trial HSC Examinations Mathematics Extension 1 Committee

ST PIUS X COLLEGE
CHATSWOOD

fide et labore

St Pius X College 2023 Trial Examination Errata Sheet

MATHEMATICS EXTENSION 1

Delete:

Question 5	(1 mark)
Question 13 (b)	(3 marks)
Question 14 (a)	(3 marks)

Do not attempt deleted questions.

Replace deleted questions with the following:

Question 5

If $\dfrac{dN}{dt} = 0.1(N - 500)$ which of the following is the correct expression for N ?

(A) $-500 - 100e^{0.1t}$

(B) $-100 - 500e^{0.1t}$

(C) $500 + 100e^{0.1t}$

(D) $100 + 500e^{0.1t}$

Question 13 (b)

Using the t formulae solve the equation $\dfrac{7\sin x}{2} + 2\cos x = 4$ for $0 \leq x \leq 180°$. 3
Answer correct to the nearest minute.

Question 14 (a)

Consider $y = \sin^{-1}(x+1) + p\sin\left(\dfrac{\pi}{12}\right)$, where $p \in \mathbb{R}$ and $\sin\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. 3

(Do NOT prove this.)

Find the minimum value of p for which $y \geq 0$ for all x.