Implementation and comparison of register allocation to translate to $x86\,$

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1 Introduction

Compilation refers to the process of translating from one language to another, most often from a high-level programming language intended for humans to work with, to machine- or bytecode intended to be executed on a target architecture. This process can be divided into several distinct phases, which are grouped into one of two stages colloquially referred to as the *frontend* and *backend*, the former translating a high-level programming language to an *intermediate representation* (IR) and the latter translating IR to executable machine code of a target architecture or bytecode of a target *virtual machine* (VM).

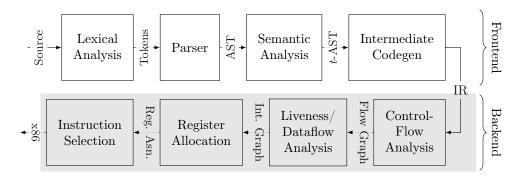


Figure 1: Compiler phases, backend highlighted

Most operations of a general-purpose programming language are translated to a set of control, logic, and arithmetic instructions to be executed sequentially on a computer processor: a single circuit/chip, referred to as the *central processing unit* (CPU), the design of which has varied and evolved over time.

Most CPUs are register machines, in that they use a limited set of general-purpose registers (GPRs) to store working values in combination with random access memory (RAM) for mid-term, and other I/O peripherals for long-term storage. This can largely be attributed to performance, as register machines routinely outperform stack machines (Shi et al., 2008) that are often used in VMs. Because of the limited amount of GPRs available simultaneously, a crucial part of the backend stage for an optimizing compiler is assigning each variable of the source program to a GPR in such a way that maximizes performance without sacrificing correctness.

The process of assigning each variable to a GPR is referred to as register allocation, and can be approached in several different ways. This paper will seek to implement graph coloring and linear scan and evaluate them in terms of runtime performance after compilation. The primary sources will be Modern Compiler Implementation in ML (Appel, 1997) and Compilers: Principles, techniques, and tools (Aho et al., 2014), in addition to publications concerning the linear scan approach.

2 Control Flow Analysis

The backend of a compiler takes some form of IR as input, usually a linear sequence of instructions for each separate function. This representation is close to the level of an actual processor by design, but it isn't immediately useful for the further analysis steps needed to generate optimized code for the target architecture. The control flow of a given program refers to the order in which instructions are executed. While the flow of most instructions is linear, in the sense that the next instruction executed is located immediately after, some transfer the flow of execution elsewhere or even terminate it.

A continuous flow of instructions is referred to as a *basic block*, defined as a sequence of instructions with no branches in or out except for the first instruction (referred to as a *leader*, immediately following either the function entry or label) and the last (referred to as a *terminator*, as it either terminates or transfers the flow of execution).

Basic blocks represent a single node in a *control flow graph* (CFG), which is a directed graph whose edges denote transfer of control flow. The unconditional branch terminator always transfers control flow to the block labelled, meaning only one successor will follow, whereas conditional branching could transfer to either

of the two, but because control flow analysis is not concerned with data, it simply adds both as successors. While each block has 0-2 immediate successors, the amount of predecessors is unbounded as the amount of branches targeting a specific leader is unlimited.

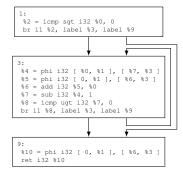


Figure 2: CFG of @square of square.11

2.1 Building a graph

2.2 Parameterized over individual instructions

2.3 Parameterized over basic blocks

With an input stream of instructions

Each function defined in an LLVM program is constructed with the help of

3 Liveness Analysis

Translating IR with an unbounded number of variables to a CPU with a bounded number of registers involves the process of assigning each variable a register such that no value that can be used in the future is overwritten. Variables that are in use at a given program point are considered 'live', and although variables can be assigned the same register, variables that are live at the same time (i.e. at intersecting program points) cannot, in which case they are also said to be in interference with one another. Variables that are not in interference can be assigned the same register, and finding the precise points at which any variable is live is trivial for linear sequences of instructions. However, when conditional branching is introduced, deriving the path of execution becomes undecidable because of the halting problem.

Suppose a function that calls another:

```
define i32 @countcall(i32 %x0) {
    %x1 = add i32 %x0, 1
    call ptr @subproc()
    ret i32 %x1
5 }
```

Because of the halting problem, static analysis cannot determine if the call to @subproc will return for every possible implementation. So when assigning %x1 a register it is undecidable whether the variable must be live in the last return instruction, i.e. needs to live across the call to another function which can overwrite several registers depending on calling conventions. Because of this, any sound approach to liveness analysis will be an approximation.

A sound albeit very naive approach is to consider every variable live at every program point, such that every variable is in interference with one another, producing a fully connected interference graph. Such a heuristic is greedy in the sense that it picks an assignment known to be safe for the least amount of preprocessing work possible. However, this can be a very inefficient assignment at runtime: a number of n

variables greater than k working registers causes n-k variables to be spilled to memory, which can greatly reduce performance, but allows for translation in constant time.

3.1 Dataflow Analysis

Another approach, which is a much more precise approximation, is a specific variant of the dataflow analysis as described in *Modern Compiler Implementation in ML* (Appel, 1997) and *Compilers: Principles, Techniques, and Tools* (Aho et al., 2014). In general, dataflow analysis is the process of finding the possible paths in which data may propagate through different branches of execution. While several applications of this exist (like constant propagation, reaching definitions, available expressions etc.), one that is immediately beneficial in the case of liveness analysis is one that traverses a CFG in the reverse order of execution (i.e. *backwards* flow), and extracts any variable that *may* be used in execution (also referred to as *backwards may* analysis).

This algorithm calculates which program points each variable may be accessed from with some conservative constraints known to maintain correctness. Specifically, these are the transfer and control-flow constraints.

The transfer constraint is based on a transfer function that describes how liveness is affected across instructions. For each instruction, there is a transfer function that describes how liveness changes from one point to the one immediately after. For example, as an arithmetic operation needs to be assigned a new temporary variable, the liveness of a new variable is propagated to all instructions executed subsequently. This is done by applying the transfer function to the current live-out set to stop further propagation of variables defined by the currently visited instruction.

$$in [n] = use [n] \cup (out [n] - def [n])$$

$$(1)$$

Control-flow constraints on the other hand propagate the use of variables to previously executed instructions, expecting these to be defined somewhere further up the CFG. This is done by propagating the union of the *live-in* set associated with all immediate successor nodes. This is also referred to as the *meet operator*, whose operator depends on the type of dataflow alanysis as well, but for liveness analysis a union is performed on the previos *live-in* variables.

$$out [n] = \bigcup_{s \in succ[n]} in [s]$$
 (2)

Initially, two sets are associated with each instruction: the *live-in* and *live-out* sets, which are the sets of variables that are live respectively before and after execution. Then the following two equations are applied iteratively until a fixed point is reached, i.e. a point in which neither in[n] or out[n] is changed for all n instructions.

The simplest equations to implement are the def and use functions, as all of the values of interest are located immediately within the instruction itself and not hidden behind some layer of indirection:

```
let def (s : S.SS.t) (insn : Cfg.insn) =
     match insn with Insn (Some dop, _) -> S.SS.add dop s | _ -> s
   let use (s : S.SS.t) (insn : Cfg.insn) =
4
     let op o s = match o with Ll.Id i -> S.SS.add i s | _ -> s in
5
     let po s o = op o s in
6
     match insn with
       Insn (_, AllocaN (_, (_, o)))
       Insn (_, Bitcast (_, o, _))
        Insn (_, Load (_, 0))
10
            (_, Ptrtoint (_, o, _))
       Insn
       Insn
            (_, Sext (_, o, _))
12
            (_, Trunc (_, o, _))
13
        Insn (_,
                 Zext (_, o, _)) ->
14
15
       Insn (_, Binop (_, _, 1, r))
16
       Insn (_, Icmp (_, _, l, r))
17
```

```
| Insn (_, Store (_, l, r)) ->
| op l s |> op r
| Insn (_, Call (_, _, args)) -> List.map snd args |> List.fold_left po s
| Insn (_, Gep (_, bop, ops)) -> List.fold_left po (op bop s) ops
| Insn (_, Select (c, (_, l), (_, r))) -> op c s |> op l |> op r
| Insn (_, PhiNode (_, ops)) -> List.map fst ops |> List.fold_left po s
| Term (Ret (_, Some o) | Cbr (o, _, _)) -> op o s
| _ -> s
```

Where s.ss is a Set.s module built over the symbol type found in lib/symbol.ml:

```
type symbol = string * int
(* ... *)
module SS = Set.Make (struct
type t = symbol
let compare (_, n1) (_, n2) = compare n1 n2
end)
type set = SS.t
```

With the actual fixed-point iteration performed as follows:

```
let dataflow (insns : Cfg.insn list) (ids : Cfg.G.V.t array) (g : Cfg.G.t) =
     let insns = List.mapi (fun i v -> (i, v)) insns |> List.rev in
     let in_ = Array.init (List.length insns) (fun _ -> S.SS.empty) in
3
     let out = Array.init (List.length insns) (fun _ -> S.SS.empty) in
4
     let rec dataflow () =
5
       let flowout = (* ... *)
6
       let flowin = (* ... *)
       let flow changed insn = changed || flowout insn || flowin insn in
       if List.fold_left flow false insns then dataflow () else (in_, out)
9
10
     dataflow ()
11
```

The flowin function correponds to the *live-in* equation (1) and is implemented as follows:

```
let flowin (i, insn) =
let newin = S.SS.union (use insn) (S.SS.diff out.(i) (def insn)) in
let changed = not (S.SS.equal newin in_.(i)) in
if changed then in_.(i) <- newin;
changed</pre>
```

And the flowout function which correponds to the live-out equation (2) implemented as follows:

```
let flowout (i, _) =
let newout =
let succ = Cfg.G.succ g ids.(i) in
List.fold_left
(fun s v -> S.SS.union s in_.(Cfg.G.V.label v))
S.SS.empty succ
in
let changed = not (S.SS.equal newout out.(i)) in
if changed then out.(i) <- newout;
changed</pre>
```

3.2 Interference Graph

The purpose for conducting dataflow analysis as above is finding variables that may be assigned the same register. This is done by building an interference graph, which is an undirected graph, whose nodes represent variables and edges that signify interference between them, i.e. variables a and b live at overlapping program points is represented with an edge (a, b).

Constructing an interference graph only depends on the *live-out* set and type of instruction. If the instruction defines a variable, said variable is in interference with all variables in the *live-out* set. There is one exception however: according to the Appel text, move instructions (i.e. phi nodes in the case of SSA form) are given special consideration. The purpose of phi nodes is to copy/move a certain value from a certain predecessor, so they are not necessarily in conflict for being live at the same time. Rather it would often benefit if they were assigned the same register to spare unnecessary moves.

Because of this, for any phi node of the form

$$a = \Phi(b_1, ..., b_n)$$

add edges to all live-out variables not in B:

$$\forall b_j \in (out[i] \setminus B), add_edge(a, b_j) \text{ where } B = \{b_1, ..., b_n\}$$

For any other instruction that defines a variable a

$$\forall b_i \in B, add \ edge(a, b_i).$$

Although the Appel text (Appel, 1997) describes interference of variables with concrete registers as well as overlapping variables, this isn't considered in this implementation.

4 Graph Coloring

Once an interference graph is constructed, the actual assignments can be found using graph coloring. Although this has long been known to be NP-complete, the heuristic as introduced in both the Appel and Aho et al. texts is a linear time approximation to this problem. It is based on an iterative approach wherein nodes known to be colorable are removed until either an empty graph remains, in which case the original graph G is k-colorable or nodes with more than k neighbours remain. In this case a node is chosen to be spilled to the stack and removed. This is then repeated.

Let k be the number of working registers available on the target architecture. After building the interference graph G, a node n with fewer than k neighbors is chosen. As n has at most k-1 neighbors, it can be removed safely, effectively simplifying G. It is pushed to a stack to preserve the order in which they are removed so that G can be rebuilt and the corresponding registers can be assigned correctly once a k-colorable assignment is found.

4.1 Coloring by simplification

4.2 Coalescing

Coalescing is the process of eliminating moves/copies of data from one GPR to another by combining their interference graph nodes. This is similar to but not the same as the lack of interference edges between variables that are subject to move operations. This is because variables a and b may still be assigned different registers or even spilled if, for instance, either of them are of significant degree. Coalescing joins nodes a and b to node ab preserving the edges of both to maintain soundness.

Since all edges are preserved, the resulting node ab may be of a much higher degree. Because of this, only strategies that produce a k-colorable graph are worth considering, as additional spills negate the purpose entirely.

5 Linear Scan

6 Instruction Selection

6.1 LLVM-- instruction set

The intermediate representation emitted by the frontend of a compiler serves as a stepping stone independent of the target architecture. The LLVM infrastructure is the industry standard in terms of bridging this gap and was consequently the library used to translate the semantically annotated abstract syntax tree to executable machine code in the 2022 compilers course. As this project is an expansion on this, it follows naturally to build on this.

The instruction set used in this paper will be a union of the sets used in the 2022 and 2023 compilers courses in order to work as a drop-in replacement of LLVM for either of the two respective source languages: Tiger and Dolphin. This instruction set is a subset of the one used in practice, as, for instance, neither of the languages implemented support exception handling, floating point operations and so on, and instead only strive to cover the basics of compilers.

The instructions included are trunc has only been added in order to help cover more generated LLVM. The branching for most of these is trivial: after successful execution the flow of all but br, ret and unreachable will unconditionally attempt to execute the next instruction in memory (i.e. increment the instruction pointer by instruction length).

Because of this, these instructions are referred to as *terminators*, as their purpose is to disrupt what had otherwise been a linear flow from the beginning of this continus sequence of instructions, henceforth referred to as a *basic block*.

6.2 Translating to x86

Translating each IR instruction to x86 correctly is a matter of eliminating unintended side-effects. Each LLVM- instruction is defined to have only one purpose, as concepts such as calling conventions, stack frames, or a FLAGS register are completely abstracted over in order to remain platform independent.

In contrast, the x86 instruction set architecture (ISA) is targeting a complex instruction set computer (CISC) family of processors, the instructions of which perform a much broader set of operations Appel, 1997, p. 190. This is in part due to pipelining, i.e. an abstraction over the concrete implementation of the actual processor, which in turn is made for the sake of performance.

An example of this would be the division/remainder operation: since integer division is a non-trivial iterative process wherein both the quotient and remainder is needed throughout, the result of both of these is stored in the <code>%rax</code> and <code>%rdx</code> registers respectively. This means two operations are performed simultaneously regardless of which value is used, hence they need to be restored before executing the next instruction, as any variables assigned to <code>%rax</code> or <code>%rdx</code> will be overwritten.

add	addx	{}
mul	imul	{ %rax }
sdiv	idivx	$\{\%$ rax, $\%$ rdx $\}$
srem	idivx	{%rax, %rdx
call	callx	$\{\%$ rax, caller saved registers $\}$

6.3 Asserting correctness

Validating the correctness of each instruction translation is a complicated task given how low-level it is. One approach is attaching a debugger and examining the state before and after the translated sequence of x86 instructions is executed. The result of applying a binary operation, for instance, should only affect the assigned register.

7 Further Optimization

8 Evaluation

Relevant metrics by which to compare these variations are naturally the performance of the code generated at runtime, but also the time efficiency at compile-time. While the code written isn't expected to outperform LLVM, due to the time complexity of the linear scan and simplification algorithms implemented their respective runtime performances are expected to outperform the builtin graph coloring algorithm.

Another factor worth considering would be memory usage, caches misses, garbage collection (although not relevant to this as it doesn't use GC), vectorization (loop optimizations by SIMD) etc.

8.1 Benchmarking

There are different approaches to assessing how well a compiler backend translates IR to machine code targeting a specific architecture and platform. One of the primary means is to simply measure the time it takes to execute the code generated by it.

Since the target architecture is x86-64, all benchmarks are executed natively on the fastest processor available to be used consistently at the time of writing. This is to maximize the sample size and in turn reduce uncertainty, but the clock speed at which the benchmarks are performed is irrelevant to how fast the generated code is. Because of this, the metric recorded is the amount of CPU cycles spent executing from start to finish instead of actual time in seconds, as the time measure only works as a more imprecise estimate of the amount of underlying work/execution steps performed on the processor.

Needless to say, CPU cycles isn't a perfect measurement either, but it works to eliminate the clock speed as well as negate much of the time the scheduler allocates for other processes. Scheduling still has a negative impact on execution because of the cache misses caused by context switching, but outside of executing each program in immediate mode which isn't possible on Linux or macOS this is a sensible estimate. To further reduce context switching, benchmarks are made on a fresh restart with minimal background processes running.

Additionally, all benchmarks are deterministic, so the value of each virtual variable will stay the same at every instruction step of execution when compared to another run with the same starting conditions. This means that the cycles measured over infinitely many runs will converge towards a 'perfect' run with no cache misses as caused by context switching. So across n runs, the run least affected by context switching will be the one with the least CPU cycles, so is the one most representative of the actual performance without noise

Measurements are made with the perf utility, which relies on the hardware counters (HC) registers of modern microprocessors, which are special purpose registers meant to record performance data live during execution. Because the analysis is integrated into the chip on which it is executing, very little overhead is incurred, and is therefore the go-to for estimating native performance.

8.1.1 Measurements

Each .11 file present in the benches directory is listed here in a table denoting the input paramater on the left column and type of allocator used.

8.1.1.1 fib.ll

The 'dumb' fib is a very naive approach to finding the nth number of the Fibonacci sequence by calling itself recursively twice (decrementing n before each), which causes an exponential growth in function calls.

fib	clang	greedy	simple	linear
40	1.000000	1.438693	1.441270	0.73
41	1.000000	1.441035	1.440626	0.73
42	1.000000	1.447140	1.446094	0

8.2 Comparison to other work and ideas for future work

9 Conclusion

References

- Aho, A. V., lan, M. S., Sethi, R., & Ullmann, J. D. (2014). Compilers: Principles, techniques, and tools (2nd ed., internat. ed.). Pearson Education Limited.
- Appel, A. W. (1997). Modern compiler implementation in ml. Cambridge University Press.
- Shi, Y., Casey, K., Ertl, M., & Gregg, D. (2008). Virtual machine showdown: Stack versus registers. *ACM transactions on architecture and code optimization*, 4(4), 1–36.

A LLVM- instruction set

```
    binary operations add, and, ashr, lshr, mul, or, sdiv, srem, shl, sub and xor
    integer comparison icmp (conditions being eq, ne, sge, sgt, sle and slt)
    memory/address operations alloca, gep, load and store
    mov operations gep, phi, ptrtoint, trunc, and zext
    control flow operations br, call and ret
    a nullary block terminator unreachable
```

B Benchmarks

B.1 benches/fib.11

```
1000 declare i32 @atoi(i8*)
1001 define i32 @fib(i32 %n0) {
      %cn = icmp sle i32 %n0, 2
1002
     br i1 %cn, label %base, label %rec
1003
1004 base:
1005
     ret i32 1
1006 rec:
1007
      %n1 = sub i32 %n0, 1
      %v0 = call i32 @fib(i32 %n1)
1008
      n2 = sub i32 n0, 2
1009
      v1 = call i32 @fib(i32 %n2)
      %v2 = add i32 %v1, %v2
      ret i32 %v2
1012
1013 }
1014 define i32 @main(i32 %argc, i8** %argv) {
      %arg1ptr = getelementptr i8*, i8** %argv, i64 1
1015
      %arg1 = load i8*, i8** %arg1ptr
1016
      %n = call i32 @atoi(i8* %arg1)
      call i32 @fib(i32 %n)
1018
      ret i32 0
1019
1020 }
```

B.1.1 make fib-clang

```
1000 $ make fib-clang
1001 clang -00 -target x86_64-unknown-darwin benches/fib.ll -o fib-clang
```

B.1.1.1 make bench fib-clang 42

```
0 average unshared stack size
1006
1007
                     765 page reclaims
                      0 page faults
1008
                       0 swaps
                      0 block input operations
                      0 block output operations
                      0 messages sent
1012
                      0 messages received
1014
                      0 signals received
                      0 voluntary context switches
1016
                     29 involuntary context switches
            11000239251 instructions retired
1017
             4962802652 cycles elapsed
1018
               1708928 peak memory footprint
```

B.1.1.2 make bench fib-clang 43

```
1000 make bench fib-clang 43
1001 $ /usr/bin/time -al ./fib-clang 43
            2.71 real 2.49 user
                                                0.00 sys
1002
1003
                 2678784 maximum resident set size
                       0 average shared memory size
1004
                       0 average unshared data size
1005
1006
                       0 average unshared stack size
                      700 page reclaims
1007
1008
                       66 page faults
                       0 swaps
1009
1010
                       0 block input operations
                       0 block output operations
                       0 messages sent
1012
1013
                       0 messages received
                       0 signals received
1014
                      13 voluntary context switches
1015
                      32 involuntary context switches
             17797124508 instructions retired 8035601598 cycles elapsed
1017
1018
                1708928 peak memory footprint
```

B.1.1.3 make bench fib-clang 44

```
1000 $ make bench fib-clang 44
1001 /usr/bin/time -al ./fib-clang 44
           4.06 real
                         4.04 user
                2678784 maximum resident set size
1003
                      0 average shared memory size
1004
                       0 average unshared data size
1005
                      0 average unshared stack size
1006
                     700 page reclaims
1007
                     66 page faults
1008
                      0
1009
                         swaps
                      0 block input operations
                      0 block output operations
                      0 messages sent
                      0 messages received
                      0
                         signals received
1014
                      5 voluntary context switches
1015
                     29 involuntary context switches
1017
            28781328564 instructions retired
            12988681489 cycles elapsed
1018
```

B.1.1.4 make bench fib-clang 45

```
1000 $ make bench fib-clang 45
1001 /usr/bin/time -al ./fib-clang 45
                                                0.00 sys
1002
           6.54 real 6.51 user
                 2678784 maximum resident set size
1003
1004
                       0 average shared memory size
                       0 average unshared data size
1005
1006
                       0 average unshared stack size
                      700 page reclaims
1007
                      66 page faults
1008
                       0 swaps
                       0 block input operations
                       0 block output operations
1011
                       0 messages sent
                       0 messages received
1014
                       0 signals received
                      6 voluntary context switches
39 involuntary context switches
1016
             46555259189 instructions retired
             20968319385 cycles elapsed
1018
1019
                1708928 peak memory footprint
```

B.1.1.5 make bench fib-clang 46

```
1000 $ make bench fib-clang 46
1001 /usr/bin/time -al ./fib-clang 46
1002
           10.58 real
                            10.54 user
                                                  0.00 sys
                 2678784 maximum resident set size
1003
                       0 average shared memory size
1004
                        0 average unshared data size
1005
                        0 average unshared stack size
1006
                      700 page reclaims
1007
                       66 page faults
1008
                       0 swaps
                       0 block input operations
1010
                       0 block output operations
                       0 messages sent
                       0 messages received
1013
                       0 signals received
1014
1015
                       5 voluntary context switches
                      107 involuntary context switches
1016
             75314385847 instructions retired 33939361073 cycles elapsed
1017
1018
             1725376 peak memory footprint
```

B.1.1.6 make bench fib-clang 47

```
1000 $ make bench fib-clang 47
1001 /usr/bin/time -al ./fib-clang 47
       17.09 real 17.06 user
1002
                                           0.00 sys
              2678784 maximum resident set size
1003
               0 average shared memory size
1004
```

```
0 average unshared data size
1005
1006
                       0 average unshared stack size
                     766 page reclaims
1007
1008
                       0 page faults
                       0 swaps
1010
                       0 block input operations
                       0 block output operations
1011
                       0 messages sent
1013
                       0 messages received
                       0 signals received
1014
1015
                       0 voluntary context switches
                     100 involuntary context switches
1016
            121838577947 instructions retired
1017
1018
             54901966523 cycles elapsed
                 1708928 peak memory footprint
1019
```

B.1.1.7 make bench fib-clang 48

```
1000 $ make bench fib-clang 48
/usr/bin/time -al ./fib-clang 48
1002
           27.65 real
                             27.59 user
                                                  0.00 sys
                 2678784 maximum resident set size
1003
                        0 average shared memory size
1004
                        0 average unshared data size
1005
                        0 average unshared stack size
1006
1007
                      766 page reclaims
                       0 page faults
1008
1009
                        0 swaps
1010
                        0 block input operations
                        0 block output operations
1011
1012
                        0 messages sent
                        0 messages received
1013
                        0 signals received
1014
                        0 voluntary context switches
1015
            161 involuntary context switches
197130073055 instructions retired
1016
1017
             88835375488 cycles elapsed
1018
                1708928 peak memory footprint
1019
```