Homework 2 Report

Notations:

- P_t is power of base station (in dBm)
- G_t is gain of transmitter antenna (in dB)
- G_r is gain of receiver antenna (in dB)
- h_t, h_r are height of transmitter and receiver (in m)
- d is the distance (in m)
- T is temperature (in Kelvin)
- B is bandwidth (in Hz)
- k is Boltzmann constant = 1.38×10^{-23}

For convenience, we give the cells on grid coordinates (i, j) by the following axis-i and axis-j direction, and the central BS is $(0, 0)$:

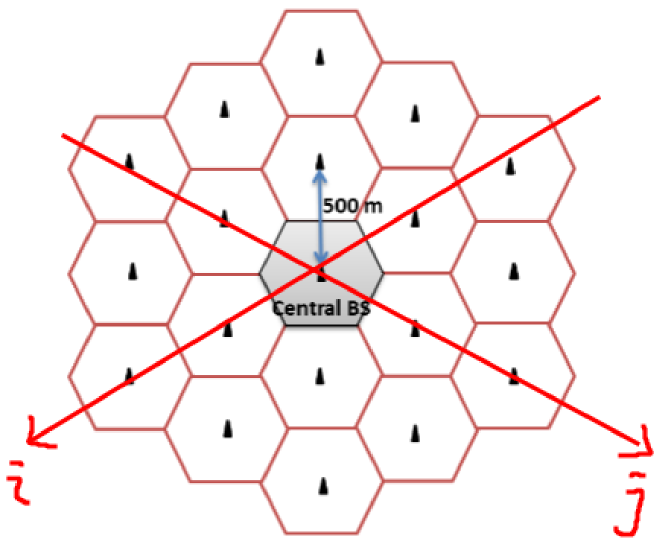
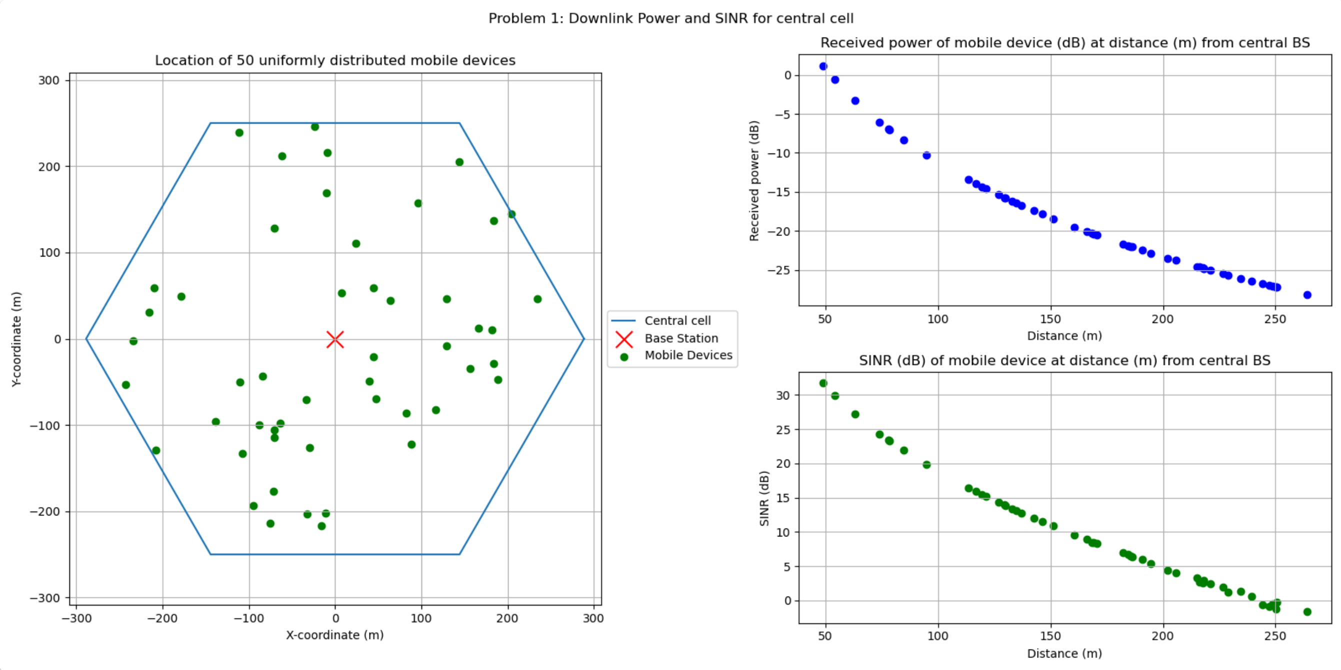


Fig. 1

1. Downlink



Position

Referencing [this stackoverflow question](), I applied rejection sampling to sample 50 points uniformly within the hexagon, by bounding the hexagon of radius R with a rectangle of size $2R \times \sqrt{3}R$ and repeatedly generate point until it is within the hexagon:

```
def gen_hexagon(n, r, x, y):
        # Uniformly plot n points in hexagon with radius r centered at (x, y) using rejection sampling
        xlist = []
        ylist = []
        i = 0
        half_L = math.sqrt(3) * r / 2
        while i < n:
                x0 = np.random.uniform(-r, r)
                y0 = np.random.uniform(-half_L, half_L)
                if within_hexagon
                        xlist.append(x0 + x)
                        ylist.append(y0 + y)
                i += 1

return xlist, ylist
```

The `within_hexagon` condition can be checked with 4 inequalities (i.e. tilted sides of the hexagon) One can also use three basis vector to span the hexagon, by choosing two of the bases and forms three parallelogram.

Power

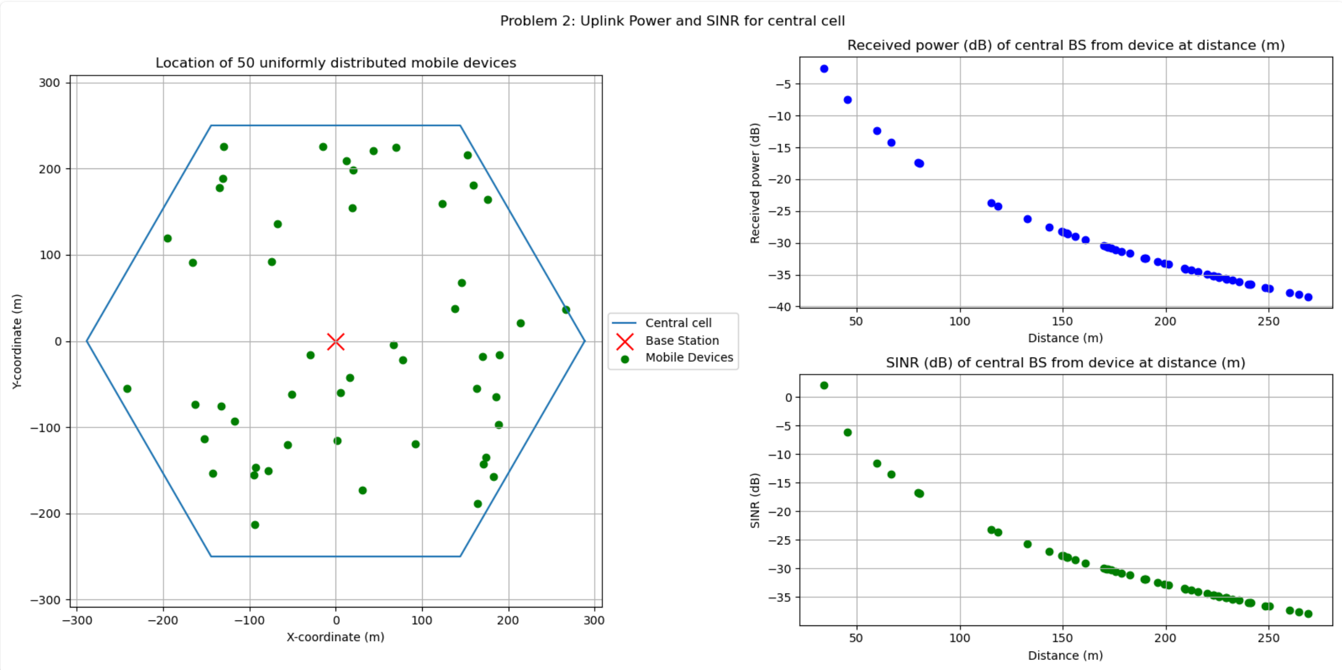
Similar to Homework 1, by two-way ground model, received power from device at distance d (m) is:
$S = P_t + G_t + G_r + 10 \times log(h_t^2 h_r^2 / d^4) - 30.$

SINR

We calculate SINR of device $k, k = 1 \ldots 50$ by $\mathrm{SINR} = S^k_{(0,0)}/(N+I)$ where $N = k \times T \times B, I = \sum_{p \in C} S^P_p - S^k_{(0,0)}$ where $P = \{(0,0),(1,0),\ldots\}$ is the set of all BS cell coordinates, and S^k_p denotes the received power of device k from BS at p. Simply put, I is the received power from all BS except the central BS.

One thing worth noticing is that not all point is on the curve proportional to $d^{-\beta}$, especially for those devices at more than 300m from the central BS. This makes sense because devices at the edges of cell tend to receive more interference than those close to the BS → less SINR.

Uplink



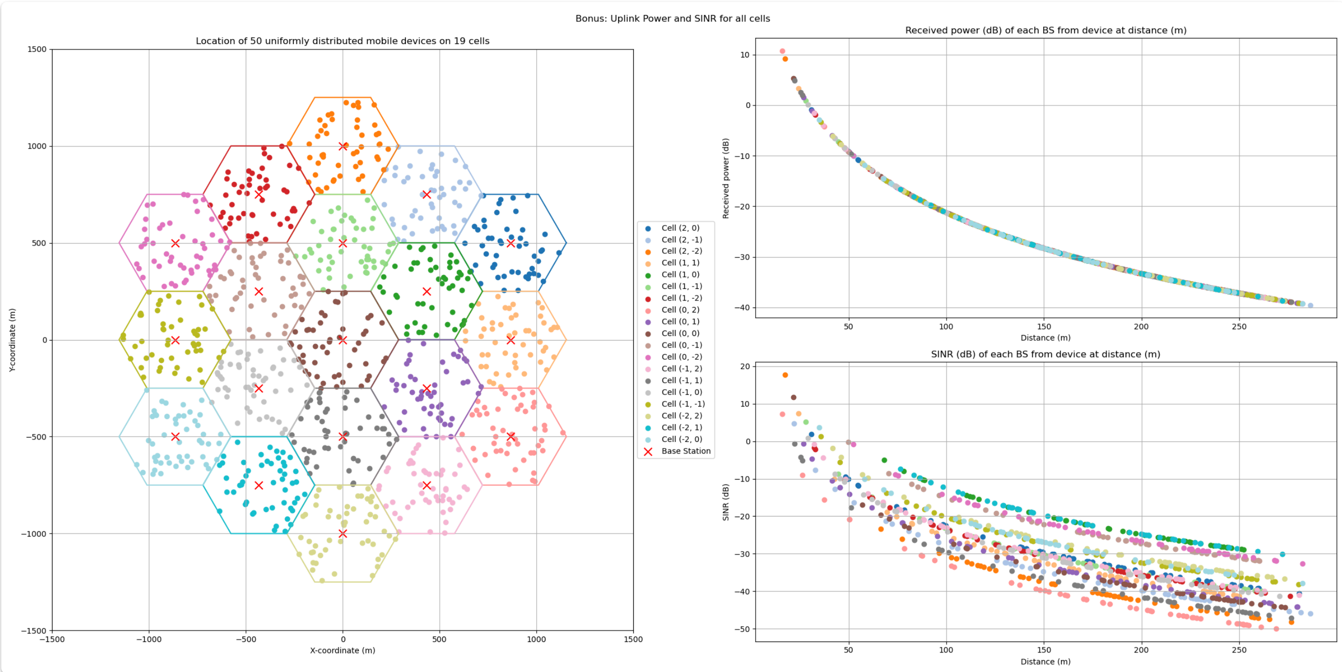
Position is plotted the same way as downlink, but with different positions.
Power is almost the same, just swapping h_t, h_r and updates P_t to 23dBm. (33dBm for BS, 22dBm for devices)

SINR

Reusing the notation from downlink section, we can write $\mathrm{SINR} = S^{(0,0)}_k/(N+I)$ where $N = k \times T \times B$ and $I = \sum_{i=1}^{50} S^{(0,0)}_i - S^{(0,0)}_k$

Note that the curve seems to be more "concave" in the middle compared to the power's plot, this can be seen by dividing $S^{(0,0)}_k$ from both numerator and denominator, and SINR is basically in the form of $1/(c \times d^\beta - 1)$ where c is some constant, which explain that it still roughly follows the shape of $d^{-\beta}$ but still slightly different.

Bonus



Position

Using our predefined coordinate, we can express the position of each BS with unit vectors, uniformly generate 50 points for each of 19 cells just like in previous two problems, and shift them to the respective BS position.

Power

Power follows calculation in the uplink section. Since each cell is independently generated, this means that when all BSs' received power are plotted together, they should fall on the same curve proportional to $d^{-\beta}$.

SINR

SINR follows the uplink section, but this time we sum over all 50×19 devices for interference, or $I = \sum_{i=1}^{50\times 19} S_i^{(0,0)} - S_k^{(0,0)}$.

We observe 19 different curves separated by cell, but devices within the same cell should fall on the same curve.

The reason to first observation is due to different position p of BS. Since the grid do not extend infinitely, cells on the edges e.g. cell $(0, -2)$ tends to get less interference due to some of the first-tier BS in CCI are missing (this is not definite, as we still can see cell like $(1, 0)$ having the worst SINR) In short, SINR on different BS is related to the layout of its position & all the devices.

The reason to second observation is similar to what we saw in uplink section, where the curve still roughly follows the shape of $d^{-\beta}$ within the BS.