Laboratory Session : April 27, 2023

Exercises due on : May 14, 2023

# Exercise 1

- the number of claims received by an insurance company during a week follows a Poisson distribution with unknown mean ($\mu$)
- the number of claims, per week, observed ovr a ten week period are:

  `5, 8, 4, 6, 11, 6, 6, 5, 6, 4`

(a) suppose to use a prior uniform distribution for $\mu$

- find the posterior distribution for $\mu$ and compute the posterior mean, median and variance
- plot the posterior distribution and the 95% credibility interval

(b) suppose to use a Jeffreys' prior for $\mu$ ($g(\mu) \propto 1/sqrt\mu$)

- find the posterior distribution for $\mu$ and compute the posterior mean, median and variance
- plot the posterior distribution and the 95% credibility interval

(c) evaluate a 95% credibility interval for the results obtained with both priors. Compare the result with that obtained using a normal approximation for the posterior distribution, with the same mean and standard deviation

# Exercise 2

- a well established and diffused method for detecting a disease in blood fails to detect the presence of disease in 15% of the patients that actually have the disease.
- A young UniPD startUp has developed an innovative method of screening. During the qualification phase, a random sample of $n = 75$ patients known to have the disease is screened using the new method.

(a) what is the probability distribution of $y$, the number of times the new method fails to detect the disease ?

(b) on the $n = 75$ patients sample, the new method fails to detect the disease in $y = 6$ cases. What is the frequentist estimator of the failure probability of the new method ?

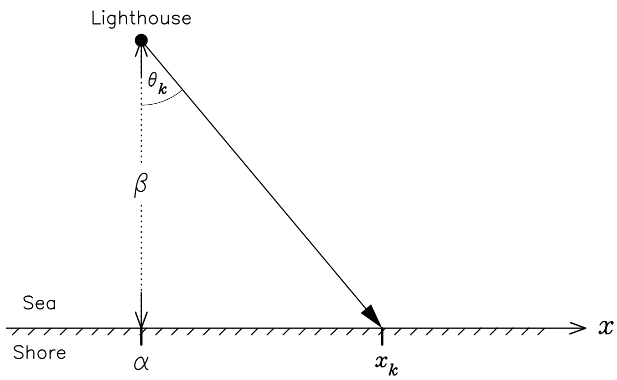
(c) setup a bayesian computation of the posterior probability, assuming a beta distribution with mean value 0.15 and standard deviation 0.14. Plot the posterior distribution for $y$, and mark on the plot the mean value and variance

(d) Perform a test of hypothesis assuming that if the probability of failing to the detect the desease in ill patients is greater or equal than 15%, the new test is no better that the traditional method. Test the sample at a 5% level of significance in the Bayesian way.

(e) Perform the same hypothesis test in the classical frequentist way.

## Exercise 2

- given the problem of the lightouse discussed last week, study the case in which both the position along the shore ($\alpha$) and the distance out at sea ($\beta$) are unknown




## Exercise 3

- given the Signal over Background example discussed last week, analyze and discuss the following cases:

(a) vary the sampling resolution of used to generate the data, keeping the same sampling range

```
xdat <- seq(from=-7*w, to=7*w, by=0.5*w)
```

- change the resolution $w = \{0.1, 0.25, 1, 2, 3\}$
- Check the effect on the results

(b) change the ratio $A/B$ used to simulate the data (keeping both positive in accordance with the prior)

- Check the effect on the results