Midterm Exam

GRA 41564 Accounting, Valuation and Financial Economics

November 10, 2024

Question 1: Reformulation

1. Reformulate the income statement to be suitable for financial statement analysis and valuation.

Reformulated PNDL	2023	2022	2021
Revenues (O)	1 802 184,00	1 430 208,00	749 381,00
Other operating income (O)	24 080,00	8 040,00	4 060,00
Total revenues and other operating income	*1 826 264,00*	*1 438 248,00*	*753 441,00*
Voyage expenses and commissions (O)	618 595,00	605 544,00	392 697,00
Ship operating expenses (O)	176 533,00	175 164,00	164 246,00
Administrative expenses (O)	53 528,00	47 374,00	26 424,00
Contingent rental income (O)		- 623,00	- 3 606,00
-Total operating expenses	848 656,00	827 459,00	579 761,00
EBITDA	**977 608,00**	**610 789,00**	**173 680,00**
Depreciation (O)	230 942,00	165 170,00	165 205,00
EBIT	**746 666,00**	**445 619,00**	**8 475,00**
- Income tax expense	- 205,00	- 412,00	- 4 633,00
-/+ Tax shield from NFE	- 19 818,92	6 722,32	- 4 130,50
Operating tax expense	- 20 023,92	6 310,32	- 8 763,50
NOPAT	**726 642,08**	**451 929,32**	**- 288,50**
Finance income (F)	18 065,00	1 479,00	121,00
Finance expense (F)	- 171 336,00	- 45 330,00	- 44 244,00
Gain on marketable securities (F)	22 989,00	58 359,00	7 677,00
Share of results of associated companies (F)	3 383,00	14 243,00	- 724,00
Foreign currency translation gain (loss) (F)	- 39,00	226,00	28,00
Dividends received (F)	36 852,00	1 579,00	18 367,00
Net Financial Expense (NFE)	**- 90 086,00**	**30 556,00**	**- 18 775,00**
+/- Tax Shield from NFE	19 818,92	- 6 722,32	4 130,50
Net income	**656 375,00**	**475 763,00**	**- 14 933,00**

2. Reformulate the balance sheet to NOA format step by step, i.e., doing the TA format, then the CE format, and finally, the NOA format.

Here are some classifications of assets and liabilities into Operating (O) and Financing (F) that we thought were unclear and could be assumed:

- **Derivative Instruments Receivable** we assume that it's Operating and that it's insuring operational risks (not financial).
- **Investment in Associated Companies** is also assumed to be Operating, since in this task we are given that "associates and joint ventures operate in the same industry as Frontline", and therefore these investments are likely to support Frontline's core business activities.
- We assume that **Right-of-Use Assets** are also operating. In fact, it can be classified as both operating and financing, depending on the nature of each lease (p.63, 13.2 Annual Report

2023).

- **Other Non-Current Assets** is operating since "Advances paid in respect of vessel upgrades in relation to exhaust gas cleaning systems ("EGCS") and ballast water treatment systems ("BWTS") are included within "other non-current assets", until such time as the equipment is installed on a vessel, at which point it is transferred to "Vessels and equipment".(p.58, 8.3 Annual Report 2023).

- **Prepaid Consideration** was classified as Operating since it has direct association with vessel acquisitions and are basically payments made in advance for the acquisition of vessels.

- **Current portion of obligations under leases** is assumed operating and is the same as Right-of-Use Assets, can be both operating or financing, but we keep it operating.

Assets	2023	2022
Current Assets		
Cash and cash equivalents (F)	308 322,00	254 525,00
Marketable securities (F)	7 432,00	236 281,00
Trade and other receivables (O)	124 647,00	139 467,00
Related party receivables (O)	19 292,00	13 485,00
Inventories (O)	135 161,00	107 114,00
Voyages in progress (O)	110 061,00	110 638,00
Prepaid expenses and accrued income (O)	15 753,00	14 255,00
Other current assets (O)	7 258,00	5 285,00
Total current assets	**727 926,00**	**881 050,00**
Non-current Assets		
Newbuildings (O)		47 991,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00
Right-of-use assets (O)	2 236,00	3 108,00
Goodwill (O)	112 452,00	112 452,00
Derivative instruments receivable (O)	39 117,00	53 993,00
Investment in associated companies (O)	12 386,00	16 302,00
Loan notes receivable (O)		1 388,00
Prepaid consideration (O)	349 151,00	
Other non-current assets (O)	6 329,00	1 507,00
Total non-current assets	**5 154 840,00**	**3 887 393,00**
Total Assets	**5 882 766,00**	**4 768 443,00**

Liabilities and Equity	2023	2022
Current liabilities		
Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Current portion of obligations under leases (O) Assumption	1 104,00	1 024,00
Related party payables (O)	47 719,00	31 248,00
Trade and other payables (O)	98 232,00	81 533,00
Total current liabilities	**409 054,00**	**391 659,00**
Non-current liabilities		
Long-term debt (F)	3 194 464,00	2 112 460,00
Obligations under leases (F)	1 430,00	2 372,00
Other non-current payables (O)	472,00	2 053,00
Total non-current liabilities	**3 196 366,00**	**2 116 885,00**
Total Liabilities	**3 605 420,00**	**2 508 544,00**
Equity		
Share capital	222 623,00	222 623,00
Additional paid in capital	604 687,00	604 687,00
Contributed surplus	1 004 094,00	1 004 094,00
Accumulated other reserves	415,00	454,00
Retained earnings	445 999,00	428 513,00
Total equity attributable to the shareholders of the Company	**2 277 818,00**	**2 260 371,00**
Non-controlling interest	- 472,00	- 472,00
Total Equity	**2 277 346,00**	**2 259 899,00**
Total liabilities and equity	**5 882 766,00**	**4 768 443,00**

From our balance sheet we group together the items into 7 groups. For assets we assign each item to either Operating Non-Current Assets(ONCA), Financial assets(FA) or Operating Current Assets(OCA). Our liabilities and equity items are assigned to either Equity(E), Operating Non-Current Liabilities(ONCL), Interest Bearing Debt(IBD), or Operating Current Liabilities(OCL).

TA Format

Assets	2023	2022
Newbuildings (O)	-	47 991,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00
Right-of-use assets (O)	2 236,00	3 108,00
Goodwill (O)	112 452,00	112 452,00
Loan notes receivable (O)		1 388,00
Derivative instruments receivable (O)	39 117,00	53 993,00
Investment in associated companies (O)	12 386,00	16 302,00
Prepaid consideration (O)	349 151,00	-
Other non-current assets (O)	6 329,00	1 507,00
ONCA	**5 154 840,00**	**3 887 393,00**
Cash and cash equivalents (F)	308 322,00	254 525,00
Marketable securities (F)	7 432,00	236 281,00
FA	**315 754,00**	**490 806,00**
Trade and other receivables (O)	124 647,00	139 467,00
Related party receivables (O)	19 292,00	13 485,00
Inventories (O)	135 161,00	107 114,00
Voyages in progress (O)	110 061,00	110 638,00
Prepaid expenses and accrued income (O)	15 753,00	14 255,00
Other current assets (O)	7 258,00	5 285,00
OCA	**412 172,00**	**390 244,00**
TA (ONCA+FA+OCA)	**5 882 766,00**	**4 768 443,00**

Liabilities and Equity	2023	2022
Equity		
Share capital	222 623,00	222 623,00
Additional paid in capital	604 687,00	604 687,00
Contributed surplus	1 004 094,00	1 004 094,00
Accumulated other reserves	415,00	454,00
Retained earnings	445 999,00	428 513,00
Total equity attributable to the shareholders of the Company	**2 277 818,00**	**2 260 371,00**
Non-controlling interest	- 472,00	- 472,00
E	**2 277 346,00**	**2 259 899,00**
Other non-current payables (O)	472,00	2 053,00
ONCL	**472,00**	**2 053,00**
Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Long-term debt (F)	3 194 464,00	2 112 460,00
Obligations under leases (F)	1 430,00	2 372,00
IBD	**3 457 893,00**	**2 392 686,00**
Related party payables (O)	47 719,00	31 248,00
Trade and other payables (O)	98 232,00	81 533,00
Current portion of obligations under leases (O) Assumption	1 104,00	1 024,00
OCL	**147 055,00**	**113 805,00**
Total E+ONCL+IBD+OCL	**5 882 766,00**	**4 768 443,00**

To go from TA format to CE format we subtract ONCL from ONCA to get NONCA(Net Operating Non-current Assets). We also subtract OCL from OCA to get NOWC(Net Operating Working Capital). Finally we get CE (capital employed) by adding up FA, NOWC and NONCA.

CE Format

Capital Employed	2023	2022
Newbuildings (O)	-	47 991,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00
Right-of-use assets (O)	2 236,00	3 108,00
Goodwill (O)	112 452,00	112 452,00
Loan notes receivable (O)	-	1 388,00
Derivative instruments receivable (O)	39 117,00	53 993,00
Investment in associated companies (O)	12 386,00	16 302,00
Prepaid consideration (O)	349 151,00	-
Other non-current assets (O)	6 329,00	1 507,00
Other non-current payables (O)	- 472,00	- 2 053,00
NONCA	**5 154 368,00**	**3 885 340,00**
Trade and other receivables (O)	124 647,00	139 467,00
Related party receivables (O)	19 292,00	13 485,00
Inventories (O)	135 161,00	107 114,00
Voyages in progress (O)	110 061,00	110 638,00
Prepaid expenses and accrued income (O)	15 753,00	14 255,00
Other current assets (O)	7 258,00	5 285,00
Related party payables (O)	- 47 719,00	- 31 248,00
Trade and other payables (O)	- 98 232,00	- 81 533,00
Current portion of obligations under leases (O) Assumption	- 1 104,00	- 1 024,00
NOWC	**265 117,00**	**276 439,00**
NOA (NONCA + NOWC)	**5 419 485,00**	**4 161 779,00**
Cash and cash equivalents (F)	308 322,00	254 525,00
Marketable securities (F)	7 432,00	236 281,00
FA	**315 754,00**	**490 806,00**
Total CE (FA + NONCA + NOWC)	**5 735 239,00**	**4 652 585,00**

Equity and IBD	2023	2022
Share capital	222 623,00	222 623,00
Additional paid in capital	604 687,00	604 687,00
Contributed surplus	1 004 094,00	1 004 094,00
Accumulated other reserves	415,00	454,00
Retained earnings	445 999,00	428 513,00
Total equity attributable to the shareholders of the Company	**2 277 818,00**	**2 260 371,00**
Non-controlling interest	- 472,00	- 472,00
E	**2 277 346,00**	**2 259 899,00**
Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Long-term debt (F)	3 194 464,00	2 112 460,00
Obligations under leases (F)	1 430,00	2 372,00
IBD	**3 457 893,00**	**2 392 686,00**
Total E+IBD	**5 735 239,00**	**4 652 585,00**

To go from CE format to NOA(Net Operating Assets) format we move FA to the right, where we subtract it from IBD to get NIBD (Net Interest Bearing Debt). We also calculate NOA on the left by adding up NONCA and NOWC. In the end we see that the reformulated balance sheet is balanced due to the fact that NOA=E+NIBD.

NOA format

Invested Capital	2023	2022
Newbuildings (O)	-	47 991,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00
Right-of-use assets (O)	2 236,00	3 108,00
Goodwill (O)	112 452,00	112 452,00
Loan notes receivable (O)	-	1 388,00
Derivative instruments receivable (O)	39 117,00	53 993,00
Investment in associated companies (O)	12 386,00	16 302,00
Prepaid consideration (O)	349 151,00	-
Other non-current assets (O)	6 329,00	1 507,00
Other non-current payables (O)	- 472,00	- 2 053,00
NONCA	**5 154 368,00**	**3 885 340,00**
Trade and other receivables (O)	124 647,00	139 467,00
Related party receivables (O)	19 292,00	13 485,00
Inventories (O)	135 161,00	107 114,00
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Prepaid expenses and accrued income (O)	15 753,00	14 255,00
Other current assets (O)	7 258,00	5 285,00
Related party payables (O)	- 47 719,00	- 31 248,00
Trade and other payables (O)	- 98 232,00	- 81 533,00
Current portion of obligations under leases (O) Assumption	- 1 104,00	- 1 024,00
NOWC	**265 117,00**	**276 439,00**
NOA (NONCA + NOWC)	**5 419 485,00**	**4 161 779,00**

Equity and net interest-bearing debt	2023	2022
Share capital	222 623,00	222 623,00
Additional paid in capital	604 687,00	604 687,00
Contributed surplus	1 004 094,00	1 004 094,00
Accumulated other reserves	415,00	454,00
Retained earnings	445 999,00	428 513,00
Total equity attributable to the shareholders of the Company	**2 277 818,00**	**2 260 371,00**
Non-controlling interest	- 472,00	- 472,00
E	**2 277 346,00**	**2 259 899,00**
Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Long-term debt (F)	3 194 464,00	2 112 460,00
Obligations under leases (F)	1 430,00	2 372,00
Cash and cash equivalents (F)	- 308 322,00	- 254 525,00
Marketable securities (F)	- 7 432,00	- 236 281,00
NIBD	**3 142 139,00**	**1 901 880,00**
Total E + NIBD	**5 419 485,00**	**4 161 779,00**

3. Find the FCFF, the FCFE, and the cash surplus for the year 2023.

CASH FLOW	2023
NOPAT	726 642,08
+Depreciation	230 942,00
-Change in NOWC	11 322,00
-Change in NONCA	- 1 499 970,00
FCFF	**- 531 063,92**
+Change in NIBD excluding cash	1 294 056,00
Net Financial Expense	- 90 086,00
+/-Tax-shield from NFE	19 818,92
FCFE	**692 725,00**
-S. dividends	- 638 928,00
Cash surplus	**53 797,00**

In this cash flow analysis, we start with NOPAT, add back depreciation (a non-cash expense), and adjust for changes in NOWC and NONCA to calculate Free Cash Flow to the Firm (FCFF). We then account for changes in net interest-bearing debt (NIBD) and adjust for net

financial expenses and their tax shield to arrive at Free Cash Flow to Equity (FCFE). Finally, we subtract dividends to determine the cash surplus.

4. Show that the calculated cash surplus for 2023 is correct.

We know that the cash at the end of the year must be equal to the cash at the beginning of the year plus the cash surplus. Therefore, we first calculate what the cash at the end of the year should be according to the cash surplus we calculated and the cash at the beginning of the year.

cash beginning period	254 525,00
+Cash surplus	53 797,00
Cash end of the period	308 322,00

Following, we compare our result with the actual end of year cash and cash equivalents which we find in the Frontline Annual Report 2023.

Net cash used in investing activities		**(1,235,456)**	(257,320)	(374,419)
Financing activities				
Net proceeds from issuance of shares	**20**	**—**	—	52,447
Proceeds from issuance of debt	**17**	**1,609,449**	651,248	403,868

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Repayment of debt	**17**	**(536,587)**	(597,834)	(219,521)
Repayment of obligations under leases	**18**	**(862)**	(2,123)	(9,284)
Lease termination payments	**22**	**—**	(4,456)	—
Cash dividends paid	**8**	**(638,928)**	(33,393)	—
Net cash provided by financing activities		**433,072**	13,442	227,510
Net change in cash and cash equivalents		**53,797**	141,452	(61,648)
Cash and cash equivalents at beginning of year		**254,525**	113,073	174,721
Cash and cash equivalents at end of year		**308,322**	254,525	113,073
Supplemental disclosure of cash flow information:				
Income taxes paid	**7**	**122**	199	4,986

See accompanying Notes that are an integral part of these Consolidated Financial Statements.

As we can see from the Annual Report, our calculated end of year cash and cash equivalents balance is the same as the one listed in the Annual Report, confirming that the calculated cash surplus for 2023 is correct.

Question 2: A three-period consumption-savings model

The utility function associated with this exercise is:

$$U^{(c_1,c_2,c_3)} = \frac{c_1^{1-\gamma}}{1-\gamma} + \beta\frac{c_2^{1-\gamma}}{1-\gamma} + \beta^2\frac{c_3^{1-\gamma}}{1-\gamma}$$

The budget constraints are:

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

Where $y_3 = 0$ and we assume inherited wealth so $a_0 = \underline{a}$.

1. Derive the intertemporal budget constraint.

To start off, we write out the functions for savings in each period.

$$a_1 = a_0$$

$$a_2 = (1+r)a_1 + y_1 - c_1$$

$$a_3 = (1+r)a_2 + y_2 - c_2$$
$$a_4 = (1+r)a_3 - c_3 = 0$$

As we can see, we can input the function for a_1 into the function of a_2 which yields

$$a_2 = (1+r)a_0 + y_1 - c_1$$

The new equation for a_2 can be can be substituted into the equation for a_3 to get:

$$a_3 = (1+r)^2 a_0 + (1+r)y_1 - (1+r)c_1 + y_2 - c_2$$

Continuing, we substitute our new equation for a_3 into the equation for a_4 and equate it to zero, as savings in a_4 is zero:

$$0 = (1+r)^3 a_0 + (1+r)^2 y_1 - (1+r)^2 c_1 + (1+r)y_2 - (1+r)c_2 - c_3$$

By Moving all consumption terms to the LHS and leaving the income terms(including inherited savings a_0) to the RHS, we are left with the equation:

$$(1+r)^2 c_1 + (1+r)c_2 + c_3 = (1+r)^3 a_0 + (1+r)^2 y_1 + (1+r)y_2$$

What remains is to divide both sides by $(1+r)^2$ to get the answer to this question. This gives us the intertemporal budget contraint:

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = y_1 + \frac{y_2}{(1+r)} + (1+r)a_0$$

2. Derive the Euler equations.

By the use of the utility function defined above, we set up the Lagrangian for this exercise:

$$\mathcal{L}(c;\lambda) = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) - \lambda\left(c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} - (1+r)a_0 - y_1 - \frac{y_2}{(1+r)}\right)$$

Where the u functions are partitions of the utility function U, defined as:

$$u(c_1) = \frac{c_1^{1-\gamma}}{1-\gamma} \qquad u(c_2) = \frac{c_2^{1-\gamma}}{1-\gamma} \qquad u(c_3) = \frac{c_3^{1-\gamma}}{1-\gamma}$$

Continuing, we solve for the First-Order conditions:

$$\text{I:} \quad c_1^{-\gamma} - \lambda = 0$$
$$\text{II:} \quad \beta c_2^{-\gamma} - \frac{\lambda}{1+r} = 0$$
$$\text{III:} \quad \beta^2 c_3^{-\gamma} - \frac{\lambda}{(1+r)^2} = 0$$

The roman numerals are identifiers making it easier to distinguish which equations we refer to. Now that we have the FOCs, we start by solving for lambda in equation I:

$$\text{I:} \quad \lambda = c_1^{-\gamma}$$

Using this definition of λ in equation II, we get that:

$$\text{II:} \quad \beta c_2^{-\gamma} = \frac{\lambda}{1+r} \quad \rightarrow \quad c_1^{-\gamma} = \beta(1+r)c_2^{-\gamma}$$

Using the definition of λ to solve equation III we get:

$$\text{III:} \quad \beta^2 c_3^{-\gamma} - \frac{\lambda}{(1+r)^2} = 0 \quad \rightarrow \quad c_1^{-\gamma} = \beta^2(1+r)^2 c_3^{-\gamma}$$

Lastly, we can use the definition of $c_1^{1-\gamma}$ derived from equation II to solve equation III for $c_2^{1-\gamma}$ instead. This results in the equation:

$$c_2^{-\gamma} = \frac{\beta^2(1+r)^2 c_3^{-\gamma}}{\beta(1+r)} \quad \rightarrow \quad c_2^{-\gamma} = \beta(1+r)c_3^{-\gamma}$$

Hence, we conclude that the Euler equations for this exercise is:

$$c_i^{-\gamma} = \beta(1+r)c_{i+1}^{-\gamma}, \qquad \text{for } i = 1, 2$$

3. Compute optimal consumption and savings for the first period.

We start off by rewriting the Euler equations:

$$c_1^{-\gamma} = \beta(1+r)c_2^{-\gamma} \quad \rightarrow \quad c_2 = \beta^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}c_1$$

$$c_1^{-\gamma} = \beta^2(1+r)^2c_3^{-\gamma} \quad \rightarrow \quad c_3 = \beta^{\frac{2}{\gamma}}(1+r)^{\frac{2}{\gamma}}c_1$$

Now that we have defined c_2 and c_3 as functions of c_1, we can now use the intertemporal budget constraint and solve for c_1:

$$c_1\left(1+\beta^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}-1}+\beta^{\frac{2}{\gamma}}(1+r)^{\frac{2}{\gamma}-2}\right) = y_1 + \frac{y_2}{(1+r)} + (1+r)a$$

From this equation, we can conclude that the optimal consumption for the first period is:

$$c_1 = \frac{y_1 + \frac{y_2}{(1+r)} + (1+r)a}{\left(1+\beta^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}-1}+\beta^{\frac{2}{\gamma}}(1+r)^{2(\frac{1}{\gamma}-1)}\right)}$$

And optimal savings for the first period is:

$$a_2 = (1+r)a_0 + y_1 - c_1$$

4. Discuss the impact of the discount factor, and the elasticity of intertemporal substitution (EIS = $1/\gamma$) on savings.

EIS is a measure of a consumers willingness to substitute future consumption for current consumption. How much is saved depends on the consumption. As EIS increase, current consumption decreases which in turn increases savings. Thus, it follows that a decrease in EIS decrease savings. The discount rate is the consumers preference to consume now or later. In this model, a higher discount factor gives a lower consumption now which leads to more savings.

Question 3: Portfolio Diversification

Exercise 1

1. Find the average return for the stock and the bond funds.

$$\mu_S = 9,40\% \qquad \mu_B = 3,94\%$$

2. Find the standard deviation of the returns for the stock and the bond funds.

$$\sigma_S = 19,55\% \qquad \sigma_B = 5,71\%$$

3. Find the covariance of the returns of the stock and the bond funds.

$$Cov(S,B) = 0,00165$$

4. Find the correlation of the returns of the stock and the bond funds.

$$Corr(S,B) = 0,1478$$

5. Find the minimum variance portfolio and its weights.

5. MINIMUM VARIANCE PORTFOLIO	
Wstock	0,042
Wbond	0,958
Return	4,17%
Variance	0,0031
SD	0,0559

We use the following formulas to calculate Minimum Variance Portfolio:

$$W_1 = \frac{\sigma_2^2 - \rho_{1,2}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2}$$

$$W_2 = 1 - W_1$$

6. Find and plot the investment opportunity set.

We created an opportunity set for stocks and bonds with various weight combinations, ranging from -50% to 150% for each asset. For each combination, we calculated the resulting Return, Standard Deviation (SD), and Sharpe Ratio, where the Sharpe Ratio measures return relative to risk, indicating the efficiency of each portfolio mix. The data suggests that the Sharpe Ratio is maximized around a 40% weight in stocks and a 60% weight in bonds.

6. OPPORTUNITY SET				
Wa	Wb	Return	SD	Sharpe Ratio
1,5	-0,5	12,1%	29,04%	31,43%
1,4	-0,4	11,6%	27,13%	31,64%
1,3	-0,3	11,0%	25,22%	31,86%
1,2	-0,2	10,5%	23,32%	32,12%
1,1	-0,1	9,9%	21,43%	32,41%
1	0	9,4%	19,55%	32,73%
0,9	0,1	8,9%	17,69%	33,08%
0,8	0,2	8,3%	15,85%	33,48%
0,7	0,3	7,8%	14,04%	33,90%
0,6	0,4	7,2%	12,28%	34,32%
0,5	0,5	6,7%	10,58%	34,67%
0,4	0,6	6,1%	8,99%	34,73%
0,3	0,7	5,6%	7,57%	34,03%
0,2	0,8	5,0%	6,44%	31,54%
0,1	0,9	4,5%	5,76%	25,76%
0	1	3,9%	5,71%	16,43%
-0,1	1,1	3,4%	6,29%	6,23%
-0,2	1,2	2,8%	7,37%	-2,09%
-0,3	1,3	2,3%	8,75%	-8,00%
-0,4	1,4	1,8%	10,32%	-12,07%
-0,5	1,5	1,2%	12,01%	-14,93%

We present two graphs illustrating opportunity sets for portfolio construction under different conditions: (1) without short-selling (where negative weights are not allowed) and (2) with short-selling (where negative weights are permitted). In each graph, the blue line represents the opportunity set, showing the range of risk-return combinations achievable with varying weights. The red line corresponds to Exercise 8 and represents the capital market line.

Allowing short-selling expands the range of risk-return profiles, as shown by the extended curve in the second graph, indicating more aggressive portfolio combinations.

7. Find the tangency portfolio.

We have calculated tangency portfolio for stocks and bonds with the following formulas:

$$w_B = \frac{\left[E(r_B) - r_f\right]\sigma_S^2 - \left[E(r_S) - r_f\right]\sigma_B\sigma_S\rho_{BS}}{\left[E(r_B) - r_f\right]\sigma_S^2 + \left[E(r_S) - r_f\right]\sigma_B^2 - \left[E(r_B) - r_f + E(r_S) - r_f\right]\sigma_B\sigma_S\rho_{BS}}$$

$$w_S = 1 - w_B$$

As shown, the tangency portfolio has weights (43.3% in stocks and 56.7% in bonds) that are close to the weights where the Sharpe Ratio was maximized in Exercise 6. Calculating the Sharpe Ratio for this portfolio would yield an even higher value, confirming it as the tangency portfolio. This supports its position on the efficient frontier as the portfolio with the highest risk-adjusted return, making it the optimal choice for maximizing returns per unit of risk.

TANGENCY PORTFOLIO	
Wstock	0,433
Wbond	0,567
Return	0,063
SD	0,095

8. Draw the capital market line.

To plot the Capital Market Line (CML), we calculated it for various risk levels, ranging from 0% to 15%.The red line in Exercise 6 represents the CML, starting at the risk-free rate of 3% and tangent to the Opportunity set. This tangency point indicates the optimal portfolio on the Opportunity set, where the highest risk-adjusted return (Sharpe Ratio) can be achieved.

Risk	CML
0%	3,00%
1%	3,35%
2%	3,69%
3%	4,04%
4%	4,39%
5%	4,74%
6%	5,08%
7%	5,43%
8%	5,78%
9%	6,13%
15%	8,21%

Exercise 2

1. Find the efficient frontier not allowing for short selling.

To begin our analysis, we calculated the Covariance Matrix, which measures the covariances between different asset pairs (A, B, C, D, E, and F).

Covariance Matrix						
	A	B	C	D	E	F
A	0,0235	0,0179	0,0270	0,0264	0,0123	0,0074
B	0,0179	0,0213	0,0296	0,0306	0,0096	0,0123
C	0,0270	0,0296	0,0498	0,0484	0,0172	0,0174
D	0,0264	0,0306	0,0484	0,0556	0,0234	0,0205
E	0,0123	0,0096	0,0172	0,0234	0,0247	0,0066
F	0,0074	0,0123	0,0174	0,0205	0,0066	0,0902

In the table below, we identify various portfolio combinations with targeted returns ranging from 0.055 to 0.085. Using a solver, we minimize the portfolio risk (standard deviation, SD) by adjusting the asset weights, with constraints set to achieve each specified return level. Negative weights are not allowed in this case, meaning short-selling is excluded. The solver optimizes the weight distribution across assets (A to F) to obtain the lowest possible risk for each target return, thereby constructing efficient portfolios for different risk-return preferences.

Not allowing negative weights							
SD	0,1459	0,124961	0,126432	0,131187	0,142875	0,17048	0,2358
Return	0,055	0,06	0,065	0,07	0,075	0,08	0,085
Variance	0,021287	0,015615	0,015985	0,01721	0,020413	0,029063	0,055602
A	0	0,1127	0,3331	0,5534	0,7605	0,3903	0
B	1	0,4182	0,2150	0,0117	0	0	0
C	0	0	0	0	0	0	0
D	0	0	0	0	0,0044	0,3742	1
E	0	0,3923	0,3413	0,2902	0,0372	0	0
F	0	0,0768	0,1107	0,1446	0,1979	0,2355	0
Return	0,055	0,06	0,065	0,07	0,075	0,08	0,085
Sum	1	1	1	1	1	1	1

The following table shows the optimized portfolio where there is no constraint on return; the objective is solely to minimize risk (standard deviation, SD). The solver adjusts the weights of the assets (A to F) to achieve the lowest possible portfolio risk. In this configuration, the resulting standard deviation is 0.1250, with a corresponding return of 0.0603.

Optimal Portfolio(No constraint on return)	
SD	0,1250
Variance	0,0156
A	0,1276
B	0,4045
C	0
D	0
E	0,3888
F	0,0791
Return	0,0603

The plotted Efficient Frontier of both sub-problems of exercise 2 can be found at the end of the next sub-problem.

2. Find the efficient frontier allowing for short selling.

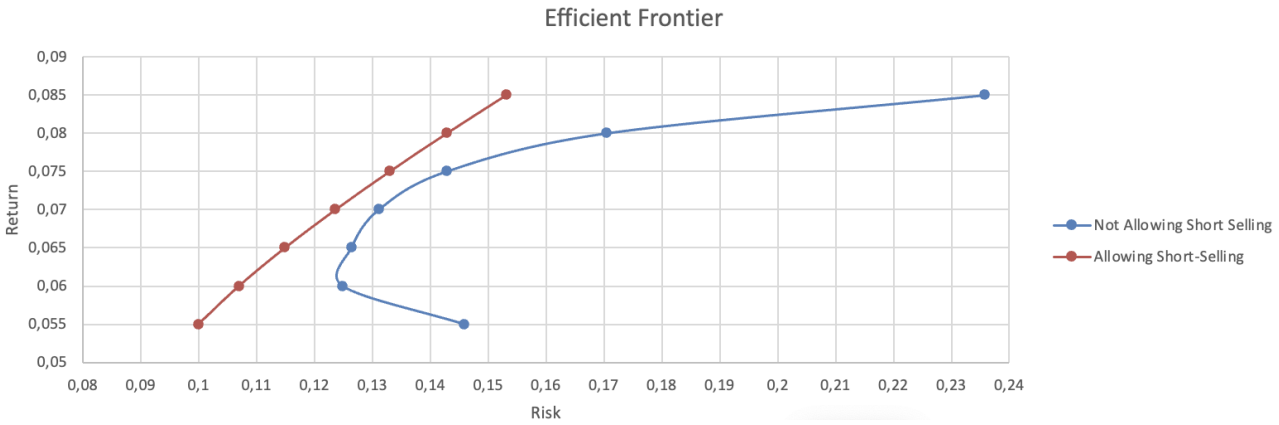
The following table presents a set of optimized portfolios where short-selling (negative weights) is allowed, with target returns ranging from 0.055 to 0.085. For each specified return, the solver minimizes the portfolio risk (standard deviation, SD) by adjusting the asset weights, allowing for both positive and negative allocations, constrained by the requirement that the weights sum up to 1. Allowing negative weights introduces the possibility of leveraging or hedging through short positions, resulting in lower risk (SD) for some return levels compared to portfolios without short-selling. The weights vary significantly across assets (A to F), with some assets having negative weights in specific portfolios. This flexibility enables more aggressive risk-return profiles, reflecting a broader range of efficient portfolios on the opportunity set.

Allowing negative weights							
SD	0,09997	0,106971	0,114937	0,123679	0,133043	0,142908	0,15318
Return	0,055	0,06	0,065	0,07	0,075	0,08	0,085
Variance	0,00999	0,011443	0,013211	0,015297	0,017701	0,020423	0,02346
A	0,44036	0,5563	0,6722	0,7881	0,9040	1,0200	1,13588
B	0,6397	0,4948	0,3500	0,2051	0	0	-0,22942
C	-0,31648	0	0	0	-1	-1	-0,62027
D	-0,22454	0	0	0	0,2230	0,3349	0,4468
E	0,34785	0,3009	0,2539	0,2069	0,1600	0	0,06601
F	0,1131	0,1277	0,1424	0,1570	0,1717	0,1863	0,20099
Return	0,055	0,06	0,065	0,07	0,075	0,08	0,085
Sum	1	1	1	1	1	1	1

Our final table displays the optimized portfolio with no constraint on return, focusing solely on minimizing risk (standard deviation, SD) while allowing negative weights (short-selling). The resulting standard deviation is 0.0859, with a return of 0.0347.

Optimal Portfolio (No constraint on return)	
SD	0,0859
Variance	0,0074
A	-0,0298
B	1,2272
C	-0,1111
D	-0,6784
E	0,5384
F	0,0537
Return	0,0347

This plot illustrates the efficient frontiers for portfolios with and without the option to allow negative weights (short-selling). The red curve represents the efficient frontier for portfolios without short-selling, showing the risk-return trade-off when all asset weights are constrained to be positive. The blue curve represents the efficient frontier with short-selling allowed, enabling the portfolio to take on both positive and negative weights.



All shown tables/graphs are derived from the Excel file in Appendix.