Your Paper

You

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Question 1: Reformulation

Not finished this section

Question 2: A three-period consumption-savings model

The utility function ascociated with this exercise is:

$$U^{(c_1,c_2,c_3)} = \frac{c_1^{1-\gamma}}{1-\gamma} + \beta\frac{c_2^{1-\gamma}}{1-\gamma} + \beta^2\frac{c_3^{1-\gamma}}{1-\gamma}$$

The budget constraints are:

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

Where $y_3 = 0$ and we assume inherited wealth so $a_0 = \underline{a}$.

1. Derive the intertemporal budget constraint.

To start off, we write out the functions for savings in each period.

$$a_1 = a + y_1 - c_1$$

$$a_2 = (1+r)a_1 + y_2 - c_2$$

As we can see, we can input the function for a_1 into the function of a_2 which yields

$$a_2 = (1+r)(a + y_1 - c_1) + y_2 - c_2$$

Now since there is no income in period three, the consumption in period three is limited by the savings and prices. Defined as a function we have:

$$c_3 = (1+r)a_2$$

We continue to expand this function by substituting in our expanded function of a_2 to get:

$$c_3 = (1+r)^2(a + y_1 - c_1) + (1+r)y_2 - (1+r)c_2$$

By Moving all consumption terms to the LHS and the income terms(including inherited savings a) to the RHS, we are left with the equation:

$$(1+r)^2c_1 + (1+r)c_2 + c_3 = (1+r)^2a + (1+r)^2y_1 + (1+r)y_2$$

What remains is to divide both sides by $(1+r)^2$ to get the answer to this question. This gives us the intertemporal budget contraint:

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = a + y_1 + \frac{y_2}{(1+r)}$$

2. Derive the Euler equations.

By the use of the utility function defined above, we set up the Lagrangian for this exercise:

$$\mathcal{L}(c;\lambda) = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) - \lambda\left(c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} - a - y_1 - \frac{y_2}{(1+r)}\right)$$

Where the u functions are partitions of the utility function U, defined as:

$$u(c_1) = \frac{c_1^{1-\gamma}}{1-\gamma} \qquad u(c_2) = \frac{c_2^{1-\gamma}}{1-\gamma} \qquad u(c_3) = \frac{c_3^{1-\gamma}}{1-\gamma}$$

Continuing, we solve for the First-Order conditions:

$$\text{I:} \quad c_1^{-\gamma} - \lambda = 0$$

$$\text{II:} \quad \beta c_2^{-\gamma} - \frac{\lambda}{1+r} = 0$$

$$\text{III:} \quad \beta^2 c_3^{-\gamma} - \frac{\lambda}{(1+r)^2} = 0$$

The roman numerals are identifiers making it easier to distinguish which equations we refer to. Now that we have the FOCs, we start by solving for lambda in equation I:

$$\text{I:} \quad \lambda = c_1^{-\gamma}$$

Using this definition of λ in equation II, we get that:

$$\text{II:} \quad \beta c_2^{-\gamma} = \frac{\lambda}{1+r} \quad \rightarrow \quad c_1^{-\gamma} = \beta(1+r)c_2^{-\gamma}$$

Using the definition of λ to solve equation III we get:

$$\text{III:} \quad \beta^2 c_3^{-\gamma} - \frac{\lambda}{(1+r)^2} = 0 \quad \rightarrow \quad c_1^{-\gamma} = \beta^2(1+r)^2 c_3^{-\gamma}$$

Lastly, we can use the definition of $c_1^{1-\gamma}$ derived from equation II to solve equation III for $c_2^{1-\gamma}$ instead. This results in the equation:

$$c_2^{-\gamma} = \frac{\beta^2(1+r)^2 c_3^{-\gamma}}{\beta(1+r)} \quad \rightarrow \quad c_2^{-\gamma} = \beta(1+r)c_3^{-\gamma}$$

Hence, we conclude that the Euler equations for this exercise is:

$$c_i^{-\gamma} = \beta(1+r)c_{i+1}^{-\gamma}, \qquad \text{for } i = 1, 2$$

3. Compute optimal consumption and savings for the first period.

Have solved by hand on notability but not yet written in LaTeX.

Question 3: Portfolio Diversification

Linh will solve this

References