

Your Paper

You

November 7, 2024

Question 1: Reformulation

1. Reformulate the income statement to be suitable for financial statement analysis and valuation.

Reformulated PNDL	2023	2022	2021
Revenues (O)	1 802 184,00	1 430 208,00	749 381,00
Other operating income (O)	24 080,00	8 040,00	4 060,00
<i>Total revenues and other operating income</i>	<i>1 826 264,00</i>	<i>1 438 248,00</i>	<i>753 441,00</i>
Voyage expenses and commissions (O)	618 595,00	605 544,00	392 697,00
Ship operating expenses (O)	176 533,00	175 164,00	164 246,00
Administrative expenses (O)	53 528,00	47 374,00	26 424,00
Contingent rental income (O)		- 623,00	- 3 606,00
-Total operating expenses	848 656,00	827 459,00	579 761,00
EBITDA	977 608,00	610 789,00	173 680,00
Depreciation (O)	230 942,00	165 170,00	165 205,00
EBIT	746 666,00	445 619,00	8 475,00
- Income tax expense	- 205,00	- 412,00	- 4 633,00
-/+ Tax shield from NFE	- 19 818,92	6 722,32	- 4 130,50
Operating tax expense	- 20 023,92	6 310,32	- 8 763,50
NOPAT	726 642,08	451 929,32	- 288,50
Finance income (F)	18 065,00	1 479,00	121,00
Finance expense (F)	- 171 336,00	- 45 330,00	- 44 244,00
Gain on marketable securities (F)	22 989,00	58 359,00	7 677,00
Share of results of associated companies (F)	3 383,00	14 243,00	- 724,00
Foreign currency translation gain (loss) (F)	- 39,00	226,00	28,00
Dividends received (F)	36 852,00	1 579,00	18 367,00
Net Financial Expense (NFE)	- 90 086,00	30 556,00	- 18 775,00
+/- Tax Shield from NFE	19 818,92	- 6 722,32	4 130,50
Net income	656 375,00	475 763,00	- 14 933,00

2. Reformulate the balance sheet to NOA format step by step, i.e., doing the TA format, then the CE format, and finally, the NOA format.

Here are some classifications of assets and liabilities into Operating (O) and Financing (F) that we thought were unclear and could be assumed:

- **Derivative Instruments Receivable** we assume that it's Operating and that it's insuring operational risks (not financial).
- **Investment in Associated Companies** is also assumed to be Operating, since in this task we are given that "associates and joint ventures operate in the same industry as Frontline", and

therefore these investments are likely to support Frontline's core business activities.

- We assume that **Right-of-Use Assets** are also operating. In fact, it can be classified as both operating and financing, depending on the nature of each lease (p.63, 13.2 Annual Report 2023).
- **Other Non-Current Assets** is operating since "Advances paid in respect of vessel upgrades in relation to exhaust gas cleaning systems ("EGCS") and ballast water treatment systems ("BWTS") are included within "other non-current assets", until such time as the equipment is installed on a vessel, at which point it is transferred to "Vessels and equipment".(p.58, 8.3 Annual Report 2023).
- **Prepaid Consideration** was classified as Operating since it has direct association with vessel acquisitions and are basically payments made in advance for the acquisition of vessels.
- **Current portion of obligations under leases** is assumed operating and is the same as Right-of-Use Assets, can be both operating or financing, but we keep it operating.

Assets	2023	2022	Liabilities and Equity	2023	2022
Current Assets			Current liabilities		
Cash and cash equivalents (F)	308 322,00	254 525,00	Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Marketable securities (F)	7 432,00	236 281,00	Current portion of obligations under leases (O) Assumption	1 104,00	1 024,00
Trade and other receivables (O)	124 647,00	139 467,00	Related party payables (O)	47 719,00	31 248,00
Related party receivables (O)	19 292,00	13 485,00	Trade and other payables (O)	98 232,00	81 533,00
Inventories (O)	135 161,00	107 114,00	Total current liabilities	409 054,00	391 659,00
Voyages in progress (O)	110 061,00	110 638,00	Non-current liabilities		
Prepaid expenses and accrued income (O)	15 753,00	14 255,00	Long-term debt (F)	3 194 464,00	2 112 460,00
Other current assets (O)	7 258,00	5 285,00	Obligations under leases (F)	1 430,00	2 372,00
Total current assets	727 926,00	881 050,00	Other non-current payables (O)	472,00	2 053,00
Non-current Assets			Total non-current liabilities	3 196 366,00	2 116 885,00
Newbuildings (O)		47 991,00	Total Liabilities	3 605 420,00	2 508 544,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00	Equity		
Right-of-use assets (O)	2 236,00	3 108,00	Share capital	222 623,00	222 623,00
Goodwill (O)	112 452,00	112 452,00	Additional paid in capital	604 687,00	604 687,00
Derivative instruments receivable (O)	39 117,00	53 993,00	Contributed surplus	1 004 094,00	1 004 094,00
Investment in associated companies (O)	12 386,00	16 302,00	Accumulated other reserves	415,00	454,00
Loan notes receivable (O)		1 388,00	Retained earnings	445 999,00	428 513,00
Prepaid consideration (O)	349 151,00	-	Total equity attributable to the shareholders of the Company	2 277 818,00	2 260 371,00
Other non-current assets (O)	6 329,00	1 507,00	Non-controlling interest	472,00	472,00
Total non-current assets	5 154 840,00	3 887 393,00	Total Equity	2 277 346,00	2 259 899,00
Total Assets	5 882 766,00	4 768 443,00	Total liabilities and equity	5 882 766,00	4 768 443,00

From our balance sheet we group together the items into 7 groups. For assets we assign each item to either Operating Non-Current Assets(ONCA), Financial assets(FA) or Operating Current Assets(OCA). Our liabilities and equity items are assigned to either Equity(E), Operating Non-Current Liabilities(ONCL), Interest Bearing Debt(IBD), or Operating Current Liabilities(OCL).

TA Format					
Assets	2023	2022	Liabilities and Equity	2023	2022
Newbuildings (O)	-	47 991,00	Equity		
Vessels and equipment (O)	4 633 169,00	3 650 652,00	Share capital	222 623,00	222 623,00
Right-of-use assets (O)	2 236,00	3 108,00	Additional paid in capital	604 687,00	604 687,00
Goodwill (O)	112 452,00	112 452,00	Contributed surplus	1 004 094,00	1 004 094,00
Loan notes receivable (O)		1 388,00	Accumulated other reserves	415,00	454,00
Derivative instruments receivable (O)	39 117,00	53 993,00	Retained earnings	445 999,00	428 513,00
Investment in associated companies (O)	12 386,00	16 302,00	Total equity attributable to the shareholders of the Company	2 277 818,00	2 260 371,00
Prepaid consideration (O)	349 151,00	-	Non-controlling interest	472,00	472,00
Other non-current assets (O)	6 329,00	1 507,00	E	2 277 346,00	2 259 899,00
ONCA	5 154 840,00	3 887 393,00	Other non-current payables (O)	472,00	2 053,00
Cash and cash equivalents (F)	308 322,00	254 525,00	ONCL	472,00	2 053,00
Marketable securities (F)	7 432,00	236 281,00	Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
FA	315 754,00	490 806,00	Long-term debt (F)	3 194 464,00	2 112 460,00
Trade and other receivables (O)	124 647,00	139 467,00	Obligations under leases (F)	1 430,00	2 372,00
Related party receivables (O)	19 292,00	13 485,00	IBD	3 457 893,00	2 392 686,00
Inventories (O)	135 161,00	107 114,00	Related party payables (O)	47 719,00	31 248,00
Voyages in progress (O)	110 061,00	110 638,00	Trade and other payables (O)	98 232,00	81 533,00
Prepaid expenses and accrued income (O)	15 753,00	14 255,00	Current portion of obligations under leases (O) Assumption	1 104,00	1 024,00
Other current assets (O)	7 258,00	5 285,00	OCL	147 055,00	113 805,00
OCA	412 172,00	390 244,00			
TA (ONCA+FA+OCA)	5 882 766,00	4 768 443,00	Total E+ONCL+IBD+OCL	5 882 766,00	4 768 443,00

To go from TA format to CE format we subtract ONCL from ONCA to get NONCA(Net Operating Non-current Assets). We also subtract OCL from OCA to get NOWC(Net Operating Working Capital). Finally we get CE (capital employed) by adding up FA, NOWC and NONCA.

CE Format						
Capital Employed	2023	2022		Equity and IBD	2023	2022
Newbuildings (O)	-	47 991,00		Share capital	222 623,00	222 623,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00		Additional paid in capital	604 687,00	604 687,00
Right-of-use assets (O)	2 236,00	3 108,00		Contributed surplus	1 004 094,00	1 004 094,00
Goodwill (O)	112 452,00	112 452,00		Accumulated other reserves	415,00	454,00
Loan notes receivable (O)	-	1 388,00		Retained earnings	445 999,00	428 513,00
Derivative instruments receivable (O)	39 117,00	53 993,00		Total equity attributable to the shareholders of the Company	2 277 818,00	2 260 371,00
Investment in associated companies (O)	12 386,00	16 302,00		Non-controlling interest	- 472,00	- 472,00
Prepaid consideration (O)	349 151,00	-		E	2 277 346,00	2 259 899,00
Other non-current assets (O)	6 329,00	1 507,00		Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Other non-current payables (O)	- 472,00	- 2 053,00		Long-term debt (F)	3 194 464,00	2 112 460,00
NONCA	5 154 368,00	3 885 340,00		Obligations under leases (F)	1 430,00	2 372,00
Trade and other receivables (O)	124 647,00	139 467,00		IBD	3 457 893,00	2 392 686,00
Related party receivables (O)	19 292,00	13 485,00				
Inventories (O)	135 161,00	107 114,00				
Voyages in progress (O)	110 061,00	110 638,00				
Prepaid expenses and accrued income (O)	15 753,00	14 255,00				
Other current assets (O)	7 258,00	5 285,00				
Related party payables (O)	- 47 719,00	- 31 248,00				
Trade and other payables (O)	- 98 232,00	- 81 533,00				
Current portion of obligations under leases (O) Assumption	- 1 104,00	- 1 024,00				
NOWC	265 117,00	276 439,00				
NOA (NONCA + NOWC)	5 419 485,00	4 161 779,00				
Cash and cash equivalents (F)	308 322,00	254 525,00				
Marketable securities (F)	7 432,00	236 281,00				
FA	315 754,00	490 806,00				
Total CE (FA + NONCA + NOWC)	5 735 239,00	4 652 585,00		Total E+IBD	5 735 239,00	4 652 585,00

To go from CE format to NOA(Net Operating Assets) format we move FA to the right, where we subtract it from IBD to get NIBD (Net Interest Bearing Debt). We also calculate NOA on the left by adding up NONCA and NOWC. In the end we see that the reformulated balance sheet is balanced due to the fact that $NOA = E + NIBD$.

NOA format						
Invested Capital	2023	2022		Equity and net interest-bearing debt	2023	2022
Newbuildings (O)	-	47 991,00		Share capital	222 623,00	222 623,00
Vessels and equipment (O)	4 633 169,00	3 650 652,00		Additional paid in capital	604 687,00	604 687,00
Right-of-use assets (O)	2 236,00	3 108,00		Contributed surplus	1 004 094,00	1 004 094,00
Goodwill (O)	112 452,00	112 452,00		Accumulated other reserves	415,00	454,00
Loan notes receivable (O)	-	1 388,00		Retained earnings	445 999,00	428 513,00
Derivative instruments receivable (O)	39 117,00	53 993,00		Total equity attributable to the shareholders of the Company	2 277 818,00	2 260 371,00
Investment in associated companies (O)	12 386,00	16 302,00		Non-controlling interest	- 472,00	- 472,00
Prepaid consideration (O)	349 151,00	-		E	2 277 346,00	2 259 899,00
Other non-current assets (O)	6 329,00	1 507,00		Short-term debt and current portion of long-term debt (F)	261 999,00	277 854,00
Other non-current payables (O)	- 472,00	- 2 053,00		Long-term debt (F)	3 194 464,00	2 112 460,00
NONCA	5 154 368,00	3 885 340,00		Obligations under leases (F)	1 430,00	2 372,00
Trade and other receivables (O)	124 647,00	139 467,00		Cash and cash equivalents (F)	- 308 322,00	- 254 525,00
Related party receivables (O)	19 292,00	13 485,00		Marketable securities (F)	- 7 432,00	- 236 281,00
Inventories (O)	135 161,00	107 114,00		NIBD	3 142 139,00	1 901 880,00
Voyages in progress (O)	110 061,00	110 638,00				
Prepaid expenses and accrued income (O)	15 753,00	14 255,00				
Other current assets (O)	7 258,00	5 285,00				
Related party payables (O)	- 47 719,00	- 31 248,00				
Trade and other payables (O)	- 98 232,00	- 81 533,00				
Current portion of obligations under leases (O) Assumption	- 1 104,00	- 1 024,00				
NOWC	265 117,00	276 439,00				
NOA (NONCA + NOWC)	5 419 485,00	4 161 779,00		Total E + NIBD	5 419 485,00	4 161 779,00

3. Find the FCFF, the FCFE, and the cash surplus for the year 2023.

CASH FLOW	2023
NOPAT	726 642,08
+Depreciation	230 942,00
-Change in NOWC	11 322,00
-Change in NONCA	- 1 499 970,00
FCFF	- 531 063,92
+Change in NIBD excluding cash	1 294 056,00
Net Financial Expense	- 90 086,00
+/-Tax-shield from NFE	19 818,92
FCFE	692 725,00
-S. dividends	- 638 928,00
Cash surplus	53 797,00

In this cash flow analysis, we start with NOPAT, add back depreciation (a non-cash expense), and adjust for changes in NOWC and NONCA to calculate Free Cash Flow to the Firm (FCFF). We then account for changes in net interest-bearing debt (NIBD) and adjust for net financial expenses and their tax shield to arrive at Free Cash Flow to Equity (FCFE). Finally, we subtract dividends to

determine the cash surplus.

4. Show that the calculated cash surplus for 2023 is correct.

We know that the cash at the end of the year must be equal to the cash at the beginning of the year plus the cash surplus. Therefore, we first calculate what the cash at the end of the year should be according to the cash surplus we calculated and the cash at the beginning of the year.

cash beginning period	254 525,00
+Cash surplus	53 797,00
Cash end of the period	308 322,00

Following, we compare our result with the actual end of year cash and cash equivalents which we find in the Frontline Annual Report 2023.

Net cash used in investing activities		(1,235,456)	(257,320)	(374,419)
Financing activities				
Net proceeds from issuance of shares	20	—	—	52,447
Proceeds from issuance of debt	17	1,609,449	651,248	403,868

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Repayment of debt	17	(536,587)	(597,834)	(219,521)
Repayment of obligations under leases	18	(862)	(2,123)	(9,284)
Lease termination payments	22	—	(4,456)	—
Cash dividends paid	8	(638,928)	(33,393)	—
Net cash provided by financing activities		433,072	13,442	227,510
Net change in cash and cash equivalents		53,797	141,452	(61,648)
Cash and cash equivalents at beginning of year		254,525	113,073	174,721
Cash and cash equivalents at end of year		308,322	254,525	113,073

Supplemental disclosure of cash flow information:

Income taxes paid	7	122	199	4,986
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See accompanying Notes that are an integral part of these Consolidated Financial Statements.

As we can see from the Annual Report, our calculated end of year cash and cash equivalents balance is the same as the one listed in the Annual Report, confirming that the calculated cash surplus for 2023 is correct.

Question 2: A three-period consumption-savings model

The utility function associated with this exercise is:

$$U(c_1, c_2, c_3) = \frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma} + \beta^2 \frac{c_3^{1-\gamma}}{1-\gamma}$$

The budget constraints are:

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

Where $y_3 = 0$ and we assume inherited wealth so $a_0 = \underline{a}$.

1. Derive the intertemporal budget constraint.

To start off, we write out the functions for savings in each period.

$$a_1 = a_0$$

$$\begin{aligned}
a_2 &= (1+r)a_1 + y_1 - c_1 \\
a_3 &= (1+r)a_2 + y_2 - c_2 \\
a_4 &= (1+r)a_3 + y_3 - c_3 = 0
\end{aligned}$$

As we can see, we can input the function for a_1 into the function of a_2 which yields

$$a_2 = (1+r)a_0 + y_1 - c_1$$

The new equation for a_2 can be substituted into the equation for a_3 to get:

$$a_3 = (1+r)^2 a_0 + (1+r)y_1 - (1+r)c_1 + y_2 - c_2$$

Continuing, we substitute our new equation for a_3 into the equation for a_4 and equate it to zero, as savings in a_4 is zero:

$$0 = (1+r)^3 a_0 + (1+r)^2 y_1 - (1+r)^2 c_1 + (1+r)y_2 - (1+r)c_2 + y_3 - c_3$$

By Moving all consumption terms to the LHS and leaving the income terms(including inherited savings a_0) to the RHS, we are left with the equation:

$$(1+r)^2 c_1 + (1+r)c_2 + c_3 = (1+r)^3 a_0 + (1+r)^2 y_1 + (1+r)y_2$$

What remains is to divide both sides by $(1+r)^2$ to get the answer to this question. This gives us the intertemporal budget constraint:

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = y_1 + \frac{y_2}{(1+r)} + (1+r)a_0$$

2. Derive the Euler equations.

By the use of the utility function defined above, we set up the Lagrangian for this exercise:

$$\mathcal{L}(c; \lambda) = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) - \lambda \left(c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} - (1+r)a_0 - y_1 - \frac{y_2}{(1+r)} \right)$$

Where the u functions are partitions of the utility function U , defined as:

$$u(c_1) = \frac{c_1^{1-\gamma}}{1-\gamma} \quad u(c_2) = \frac{c_2^{1-\gamma}}{1-\gamma} \quad u(c_3) = \frac{c_3^{1-\gamma}}{1-\gamma}$$

Continuing, we solve for the First-Order conditions:

$$\text{I: } c_1^{-\gamma} - \lambda = 0$$

$$\text{II: } \beta c_2^{-\gamma} - \frac{\lambda}{1+r} = 0$$

$$\text{III: } \beta^2 c_3^{-\gamma} - \frac{\lambda}{(1+r)^2} = 0$$

The roman numerals are identifiers making it easier to distinguish which equations we refer to. Now that we have the FOCs, we start by solving for λ in equation I:

$$\text{I: } \lambda = c_1^{-\gamma}$$

Using this definition of λ in equation II, we get that:

$$\text{II: } \beta c_2^{-\gamma} = \frac{\lambda}{1+r} \rightarrow c_1^{-\gamma} = \beta(1+r)c_2^{-\gamma}$$

Using the definition of λ to solve equation III we get:

$$\text{III: } \beta^2 c_3^{-\gamma} - \frac{\lambda}{(1+r)^2} = 0 \rightarrow c_1^{-\gamma} = \beta^2 (1+r)^2 c_3^{-\gamma}$$

Lastly, we can use the definition of $c_1^{1-\gamma}$ derived from equation II to solve equation III for $c_2^{1-\gamma}$ instead. This results in the equation:

$$c_2^{-\gamma} = \frac{\beta^2 (1+r)^2 c_3^{-\gamma}}{\beta(1+r)} \rightarrow c_2^{-\gamma} = \beta(1+r) c_3^{-\gamma}$$

Hence, we conclude that the Euler equations for this exercise is:

$$c_i^{-\gamma} = \beta(1+r) c_{i+1}^{-\gamma}, \quad \text{for } i = 1, 2$$

3. Compute optimal consumption and savings for the first period.

We start off by rewriting the Euler equations:

$$\begin{aligned} c_1^{-\gamma} &= \beta(1+r) c_2^{-\gamma} \rightarrow c_2 = \beta^{\frac{1}{\gamma}} (1+r)^{\frac{1}{\gamma}} c_1 \\ c_1^{-\gamma} &= \beta^2 (1+r)^2 c_3^{-\gamma} \rightarrow c_3 = \beta^{\frac{2}{\gamma}} (1+r)^{\frac{2}{\gamma}} c_1 \end{aligned}$$

Now that we have defined c_2 and c_3 as functions of c_1 , we can now use the intertemporal budget constraint and solve for c_1 :

$$c_1 \left(1 + \beta^{\frac{1}{\gamma}} (1+r)^{\frac{1}{\gamma}-1} + \beta^{\frac{2}{\gamma}} (1+r)^{\frac{2}{\gamma}-2} \right) = y_1 + \frac{y_2}{(1+r)} + (1+r)a$$

From this equation, we can conclude that the optimal consumption for the first period is:

$$c_1 = \frac{y_1 + \frac{y_2}{(1+r)} + (1+r)a}{\left(1 + \beta^{\frac{1}{\gamma}} (1+r)^{\frac{1}{\gamma}-1} + \beta^{\frac{2}{\gamma}} (1+r)^{2(\frac{1}{\gamma}-1)} \right)}$$

And optimal savings for the first period is:

$$a_2 = (1+r)a_0 + y_1 - c_1$$

4. Discuss the impact of the discount factor, and the elasticity of intertemporal substitution ($\text{EIS} = 1/\gamma$) on savings.

EIS is a measure of a consumers willingness to substitute future consumption for current consumption. How much is saved depends on the consumption. As EIS increase, current consumption decreases which in turn increases savings. Thus, it follows that a decrease in EIS decrease savings. The discount rate is the consumers preference to consume now or later. In this model, a higher discount factor gives a lower consumption now which leads to more savings.

Question 3: Portfolio Diversification

Linh will solve this

References