



Fundamentals of Traffic Operations and Control

Lab 1: Traffic Data from Urban Inductive Loop Detectors and the Macroscopic
Fundamental Diagram

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Group C October 2023

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Introduction

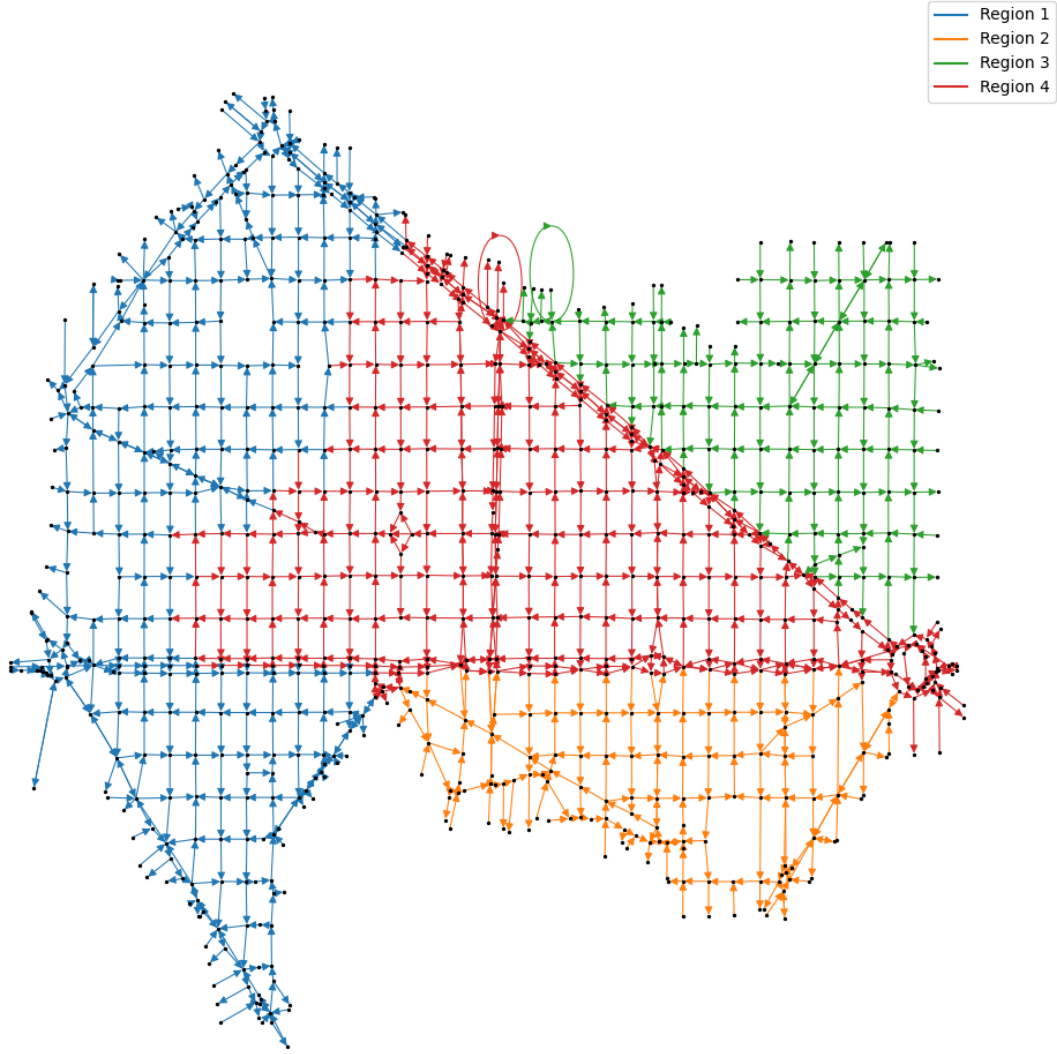


Figure 1: Network of Barcelona

The Figure 1 represents the Network of Barcelona streets, it's composed of Nodes and Links. The network is separated into 4 regions as you can see on the figure. We will in the future section analyse in details the traffic in this Network and try to identify the difference between regions. For reasons of comprehension, we will during all our analysis use the same colour for each region as shown on Figure 1.

The data we have available for our analysis is the flow of vehicle every 90 seconds of each links. We also have available the occupancy in [%] for each link.

1 Links congestion (Step 1)

In this section we will analyse the distribution of the level of congestion on each link at three different times:

- At 60 min refer on Figure 2
- At 90 min refer on Figure 3
- At 120 min refer on Figure 4

On the figures there is a grey scale that represents the level of congestion from 0 to 100 [%]. The darker the arrow, the more congested the link. We can see from the figures that the more time passes, the more congested the number of links becomes. This indicates that demand is increasing, and that network capacity is not sufficient to meet demand. Demand will reach a maximum and then reduce, but we do not observe any reduction in congestion over time in these three snapshots. The peak in demand has certainly not been reached, or it has been reached but the network has not returned to its initial state. Region 2 seems to be spared from congestion. Region 2 seems to be spared from congestion, but this is not at all the case for region 1, which had very few congested links at 60 minutes but becomes very dark over time.

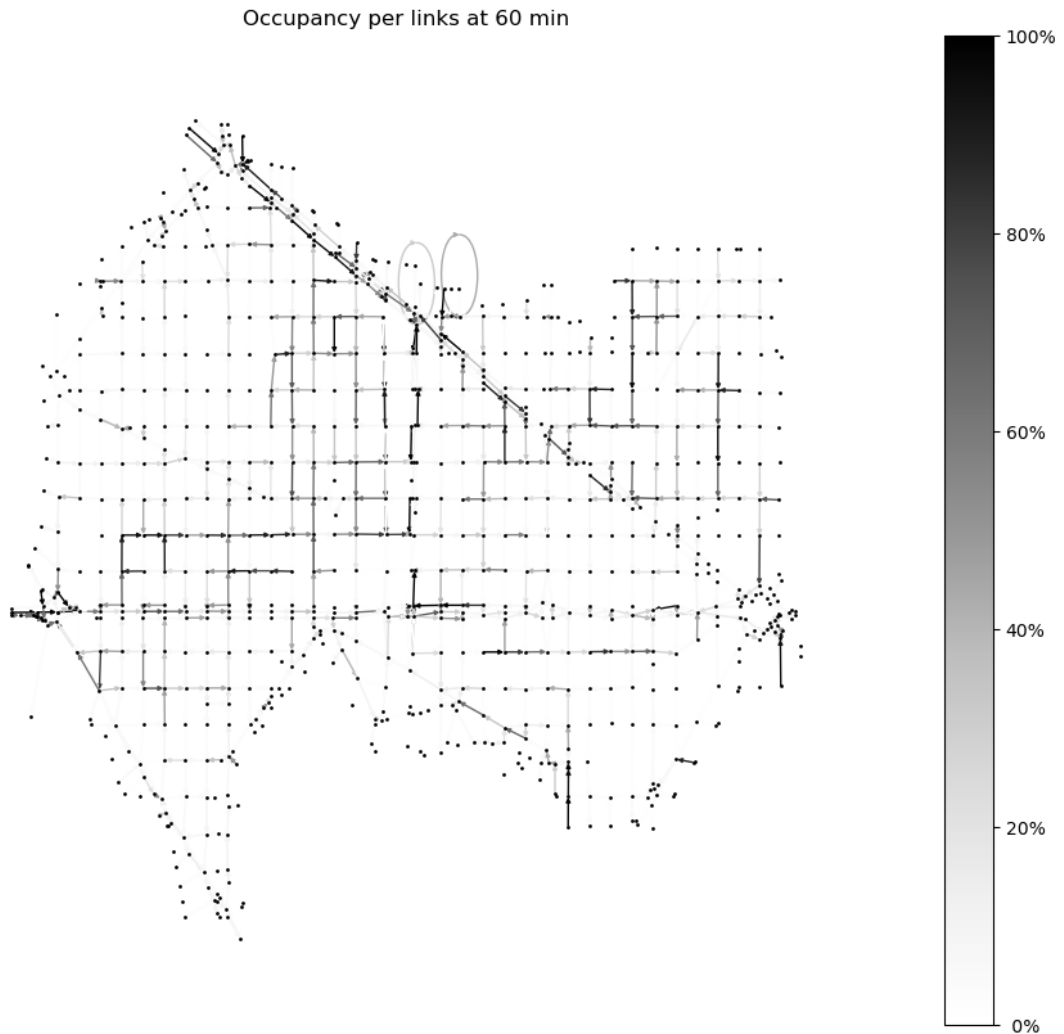


Figure 2: Occupancy per links at 60 min

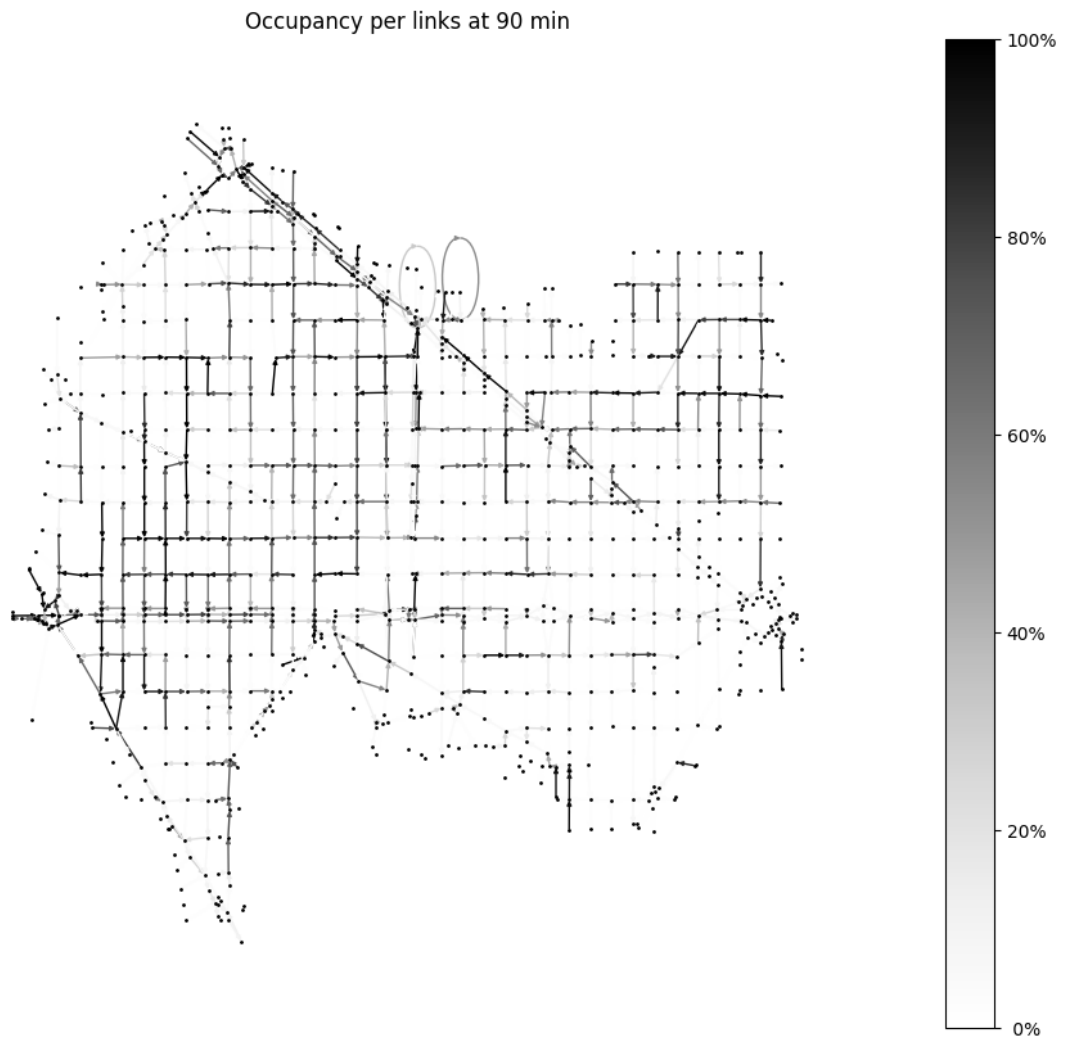


Figure 3: Occupancy per links at 90 min

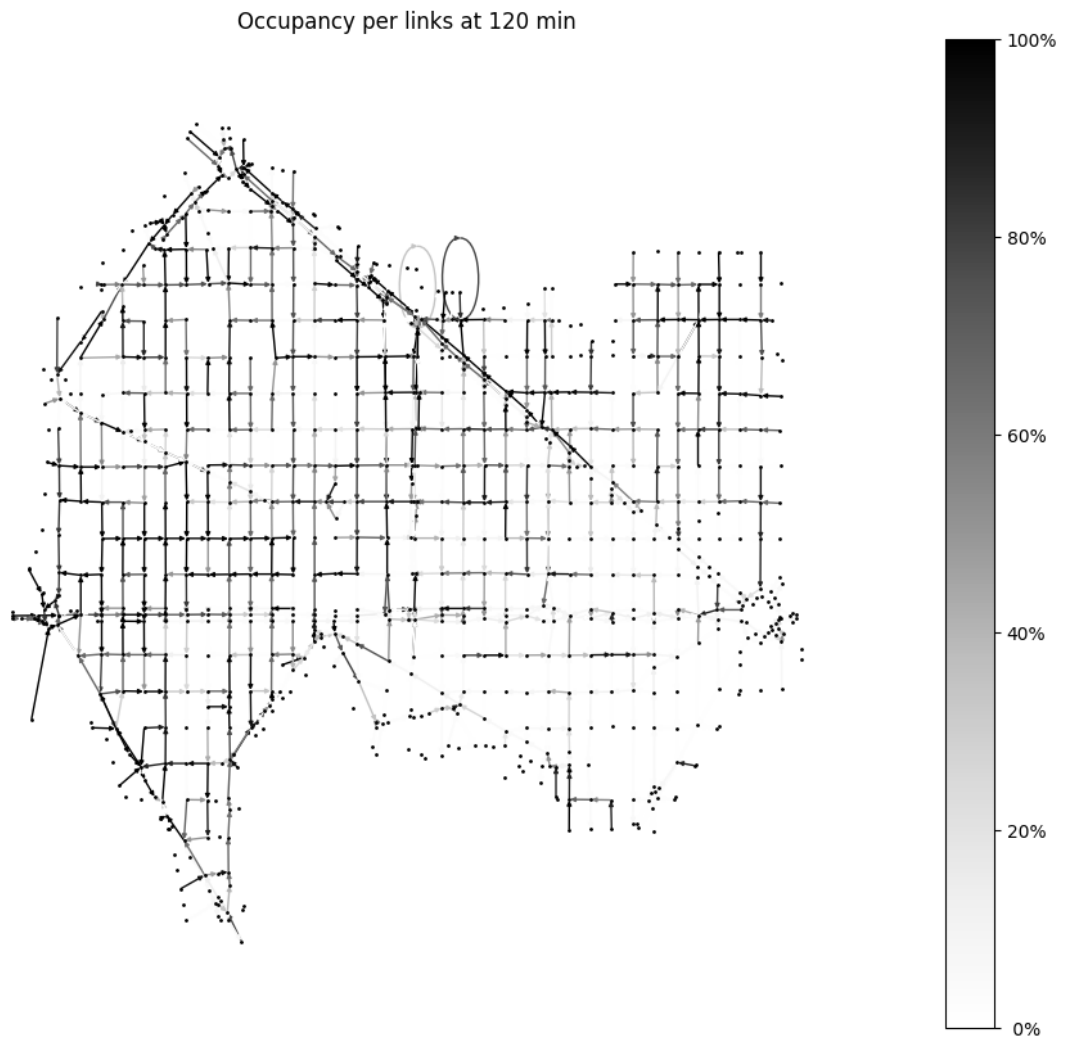


Figure 4: Occupancy per links at 120 min

2 Macroscopic Fundamental Diagram (Step 2)

We can plot a graph of the "Flow vs occupancy" with the given data. As a reminder, the unit of flow is the number of vehicles per hour that are able to travel a road, but this unit depends directly on the occupancy. The occupancy doesn't take into account the variable μ which is the amount of lanes of each link. It would be better to compare it with the density, this value takes into account μ and the size of a vehicle. We will come back to this difference in the next steps. For now we will try to analyse the "flow vs occupancy" in different scale:

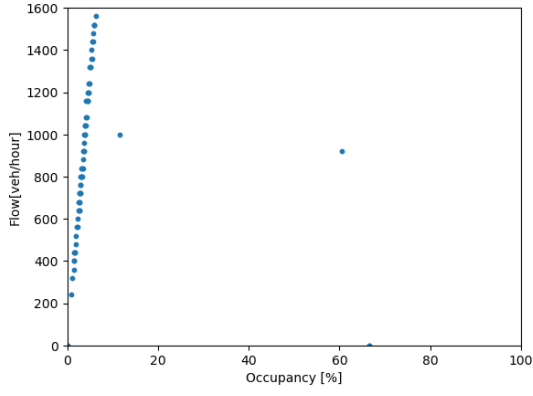
- For one link refer on Figure 5
- For two link refer on Figure 6
- For the whole network refer on Figure 7
- For the region scale refer on Figure 13

On this "Flow vs occupancy" figures we will try to recognise a Fundamental diagram. Is there a relationship between, the flow and the occupancy ? The volume in the data were given every 90 seconds, to convert it we used the following equation:

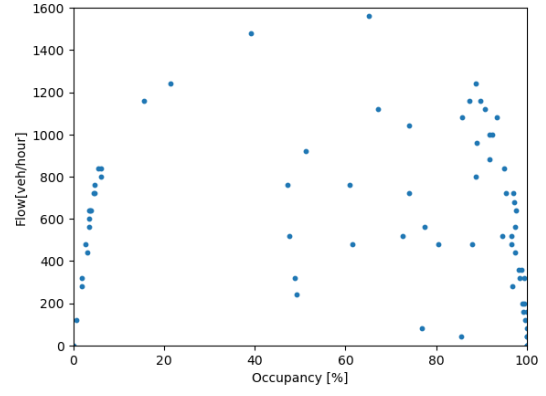
$$Flow[\frac{veh}{h}] = \frac{Flow[veh/90s]}{90[s]} \cdot 3600[\frac{s}{h}]$$

2.1 MFD for one random link

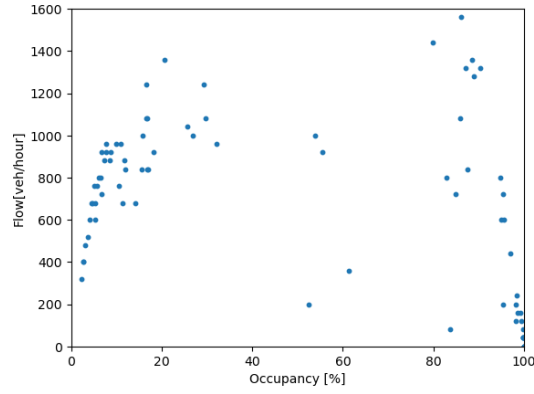
On Figure 5 we plot the "flow vs occupancy diagram" for some links that we chosed randomly. We see that their is no relationship between the flow and the congestion level. All link does not experience congestion.



(a) Flow vs occupancy of link 512



(b) Flow vs occupancy of link 514

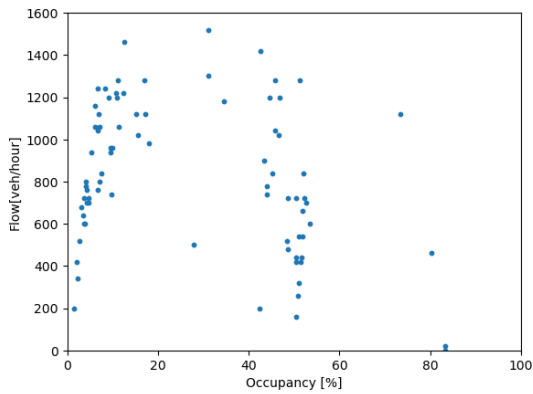


(c) Flow vs occupancy of link 516

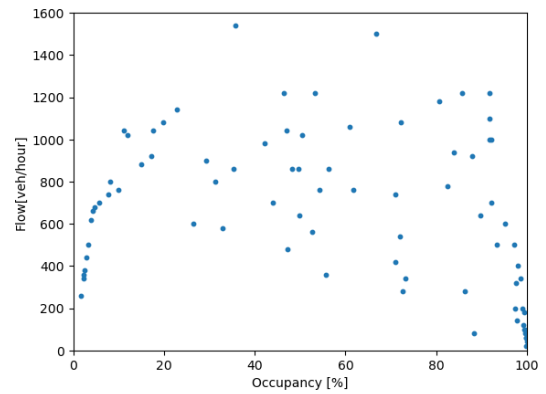
Figure 5: Flow vs occupancy of one link

2.2 MFD of two links

There is still no relationship between the flow and the occupancy. We plotted on Figure 6 the average of the link we plotted on Figure 5, to show that there is no clear common behaviour between those two links.



(a) Average Flow vs occupancy of link 512 and 516



(b) Average Flow vs occupancy of link 514 and 516

Figure 6: Average Flow vs occupancy over too link

2.3 MFD for the whole network

If we compute the average Flow and Occupancy for enough links, we see that there is a clear relationship. On the Figure 7, we easily see that there is an optimum regime for the flow and the occupancy for the whole system. The aggregation of links therefore gives us a relations of the average flow and the average occupancy.

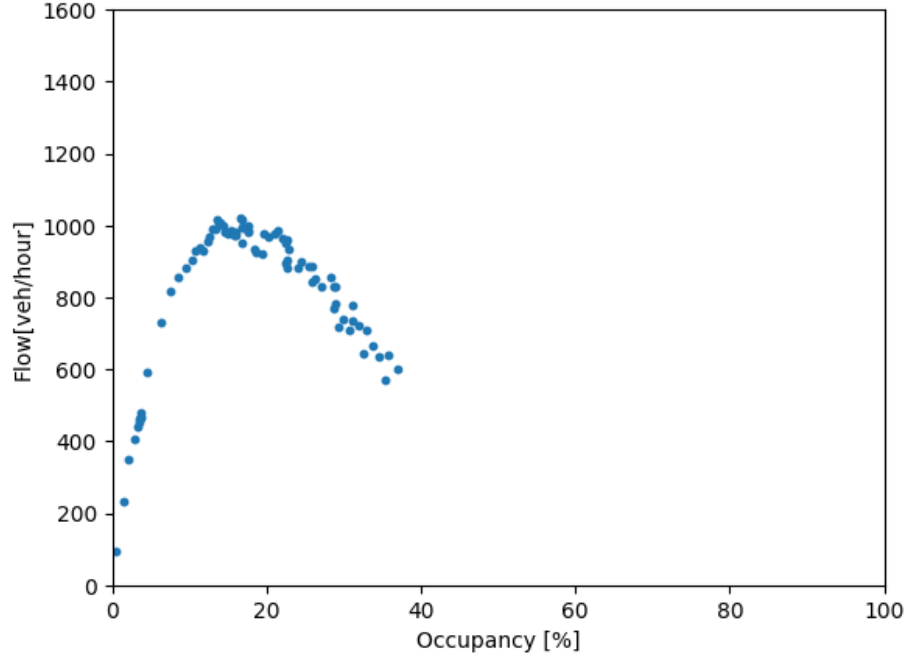


Figure 7: Flow vs occupancy of the 4 regions together

2.4 MFD on the 4 different Regions

Now that we showed that there is a relationship between the flow and the occupancy when we aggregate enough links, we can apply this to average the "flow vs occupancy" for all 4 regions. You can observe the results on Figure 13 and 14.

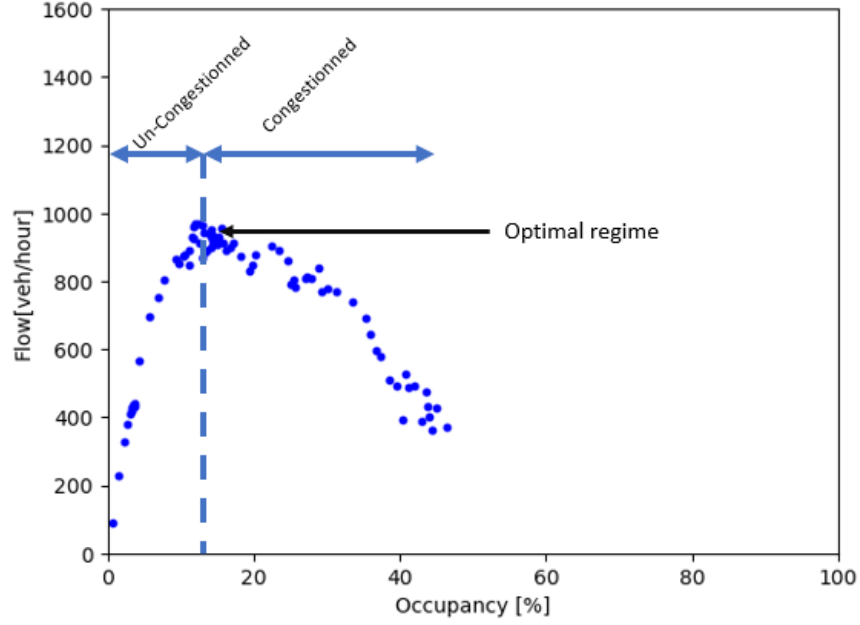


Figure 8: Regime description on region 1

We can see that every region experiences a un-congested regime and a congested regime as you can see on Figure 8. These two regimes are separated by an optimal regime. This optimal regime is the point where the highest volume of vehicles can be served. In the un-congested regime the flow is proportional to the occupancy, this is also called the free flow. All vehicles arriving can be served without experiencing any congestion. After passing the optimal regime, any new vehicle arriving will exceed the free flow and the regime will begin experiencing congestion. This observation we made to region 1 on Figure 8 can be done in all region on the figure 14. We can observe that the optimum occupancy is for all 4 regions between [15%-25%]. The Region 2 is the only region that doesn't experience congestion.

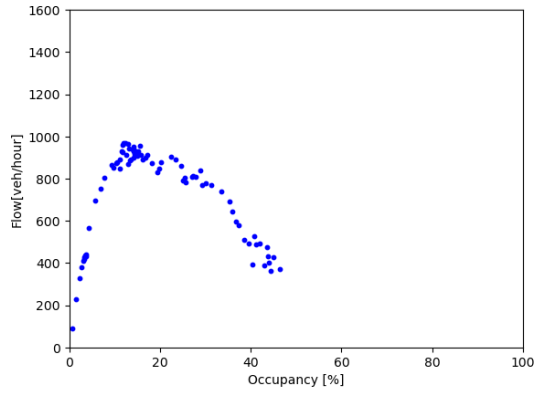


Figure 9: Flow vs occupancy of the region 1

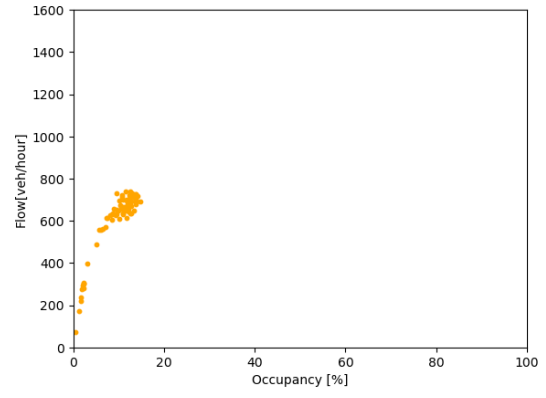


Figure 10: Flow vs occupancy of the region 2

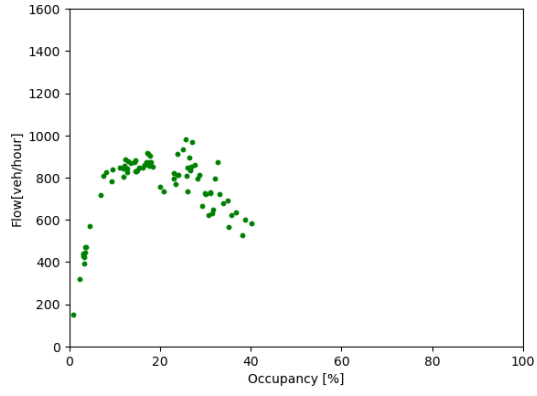


Figure 11: Flow vs occupancy of the region 3

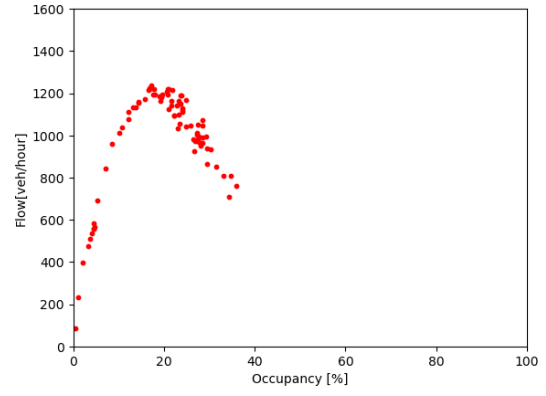


Figure 12: Flow vs occupancy of the region 4

Figure 13: Average flow vs average occupancy of the 4 different regions

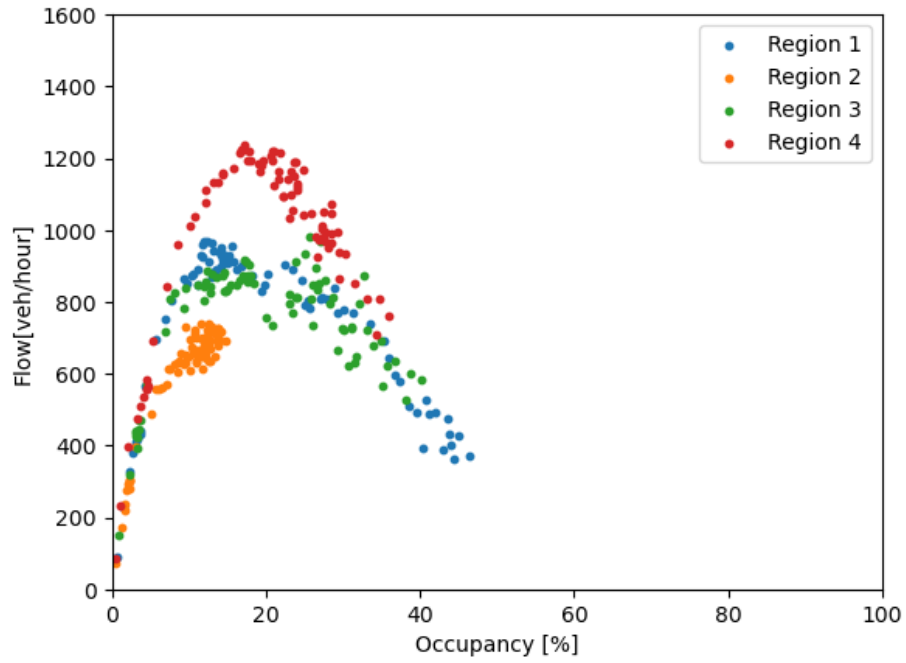


Figure 14: Flow vs occupancy of the 4 regions together

3 Assessment of the space mean speed (Step 3)

In this part, the variation of the mean speed (or space mean speed) u , is assessed. The graphics of the mean speed with respect to density and time are plotted above for each region separately and the entire network. Those quantities can be computed using the following relationships.

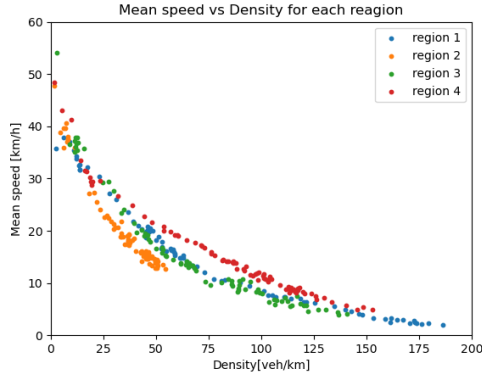
$$Density\ k = \frac{\frac{Occupancy}{100} \cdot \mu}{L + L_D} [veh/km] \quad (1)$$

$$Linkspeed\ [km/h] = \frac{Volume\ V[veh/h]}{Density\ k[veh/km]} \quad (2)$$

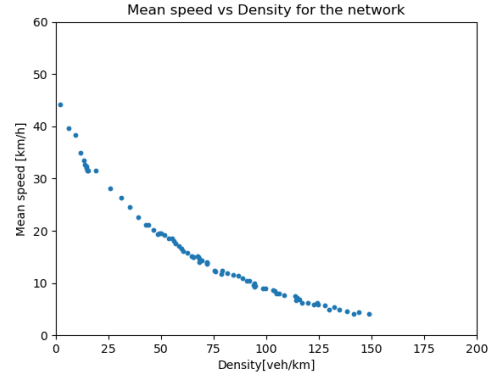
$$Mean\ Speed\ u\ [km/h] = \frac{\sum V \cdot length_{links}}{\sum k \cdot length_{links}} [km/h] \quad (3)$$

where μ is the number of lanes for a given link and V is the volume of traffic, $L = 5$ [m] the average vehicle length, and $L_D = 2$ [m] is the detector length.

3.1 Mean speed with respect to density



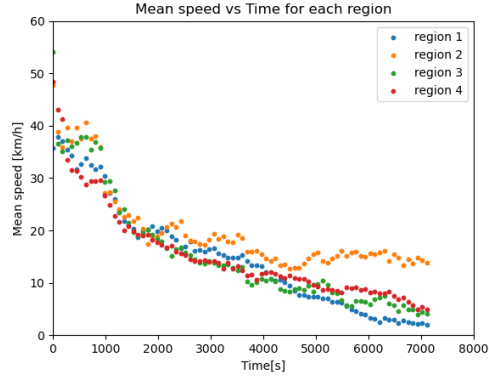
(a) Mean speed vs Density for each region



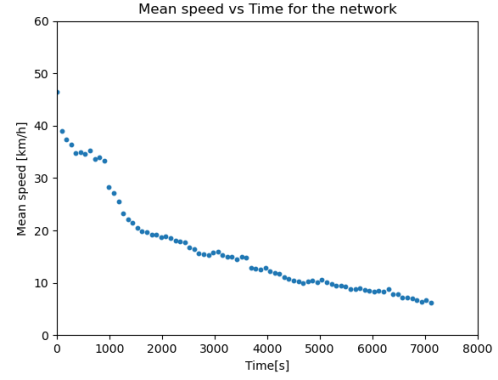
(b) Mean speed vs Density for the network

The behaviour of each region is described by a decreasing hyperbolic curve (typical of an over saturated regime) with relatively low scarcity region wise. The mean speed varies from 0 to 55 [km/h] (which seems reasonable for a city) and the density varies from 0 to 180 [veh/km]. An interesting point is that for region 2 the speed never reaches 0, which means that this part of the network is never completely jammed at any moment in the analysed data and the mean density of this part of the network is a little lower than the rest.

3.2 Mean speed with respect to time



(a) Mean speed vs Time for each region



(b) Mean speed vs Time for the network

The behaviour that is described illustrates the evolution of the congestion in the urban area, the mean speed decreases through time, as more vehicles enter the network. Which means the level of congestion increase with the time. As expected from the analysis of the previous graphics, region 2 trends to have a higher mean speed thanks to a lower density, compared to the rest of the network.

4 Production and accumulation MFD (Step 4)

In order to have an other indicator of the performance of each region, the production P and the accumulation A are computed for the cases of a single link, two different links, and finally for each regions.

The production and accumulation are computed for a single link as following:

$$\text{production } P = \text{volume} \cdot \text{lenght} [\text{veh} \cdot \text{km}/\text{h}] \quad (4)$$

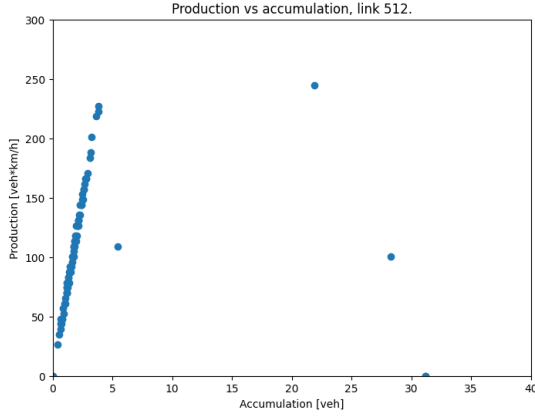
$$\text{accumulation } A = \text{density} \cdot \text{lenght} [\text{veh}] \quad (5)$$

With the length being the length of the link.

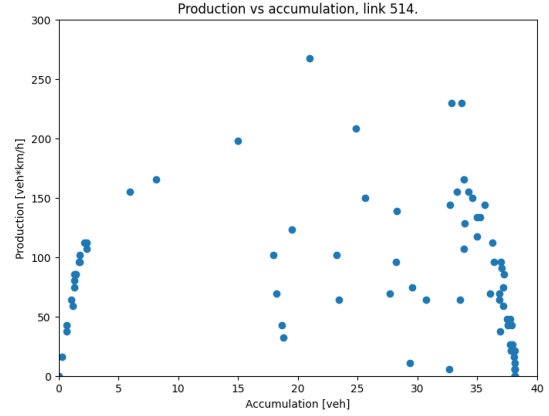
The density used for the calculation of the accumulation (total number of vehicles in the studied links) is the same as computed in step 3. The accumulation and production are dependant of respectively the volume and the occupancy, similarly to the MFD in step 2, with the added length and amount of lanes parameters being taken into account in the MFD.

When considering an ensemble of multiples links, the production and accumulation of each links are summed.

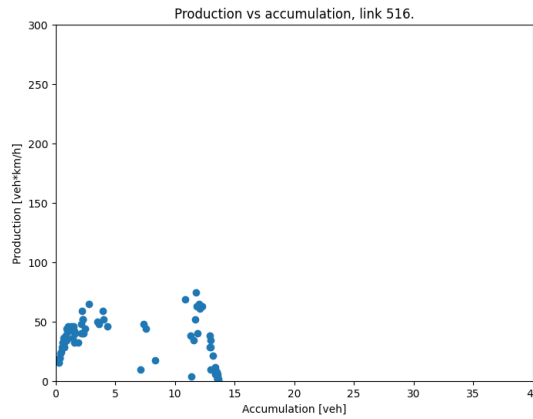
4.1 Production and accumulation for a single link



(a) Production vs accumulation of link 512



(b) Production vs accumulation of link 514



(c) Production vs accumulation of link 516

Figure 17: Production vs accumulation, single links

The trend of the production vs accumulation in single links is similar to the flow vs occupation trend, but the scale and the distribution differ from the flow vs occupancy MFD. This is expected, as the length and amount of lanes are added as parameters, which are constant for a single link. As for the flow vs occupancy MFD, it is hard to observe any optimal regime, even if free-flow slopes seems to appear, and a congested regime is observed for some links (514 and 516 here).

However, we can expect changes in the MFDs of larger links sets, such as regions, and we can also expect a more precise MFD with un-congested, optimal and congested regimes. As in step 2, link 512 does not have a congestionned behavior. Link 516 has not changed its appearance, but it has become much less widely distributed than the other links. The production of link 516 is lower than that of links 512 and 514. This observation could not be made in step 2.

4.2 Production and accumulation for two different links

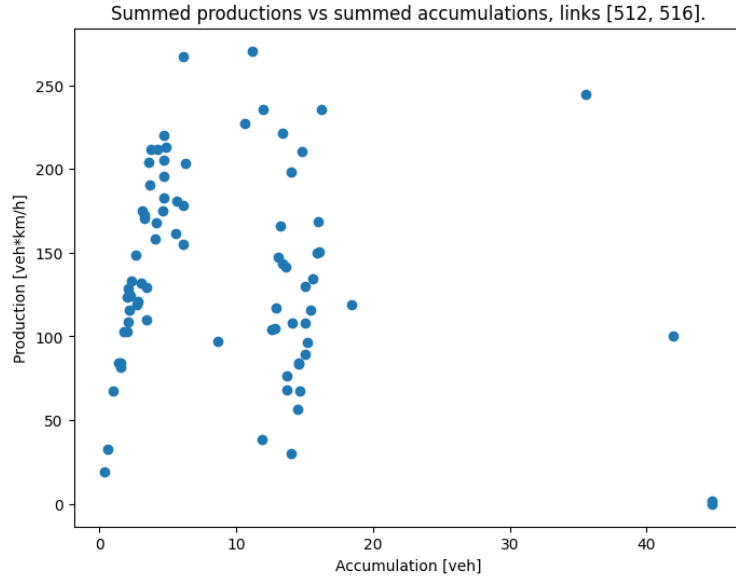


Figure 18: Summed production vs summed accumulation, links 512 and 516

When adding the accumulation and production of two different links, the aspect of the graph changes, even if the trend is similar; the influence of the new parameters is observable, and we can conclude that the two links have different lengths and/or different amount of lanes.

Indeed, the two linear shapes observed, resulting of the combination of the two graphs, also observed in the average flow vs occupancy in part 2.2, are modified in their position in the MFD. Furthermore, a number of vehicles is directly given, instead of a percentage of occupancy, giving a macro-scale point of view of the simulation. Also, the scales of the links, influenced by their lenght and number of lanes are now derivable from the MFD, having a direct influence on production.

Therefore, this MFD seems to give a broader range of information on the links considered.

4.3 Production and accumulation per region

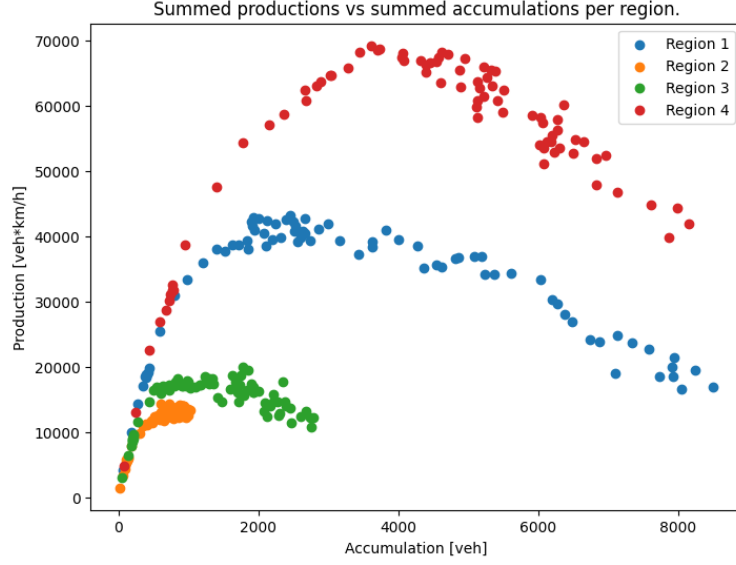


Figure 19: Summed production vs summed accumulation, per region

We can observe a similar trend to the MFDs computed at step 2, with an un-congestionned part where the production increase with the number of vehicles in the element. Similarly, the production is majored, at the optimal regime, and seems to decrease as the element become congestionned.

However, some differences appears when comparing the MFDs, especially for the entire regions. Firstly, the differences of production between the regions are much bigger than the differences in flow. This can be explained again by the length and amount of lanes being taken into account. Indeed, longer (or more) links in regions 1 and 4 increase the production of these regions, compared to regions 2 and 3, with smaller links lengths or less links.

Similarly, a more important amount of lanes in a region compared to the others will stretch its accumulation, as the density increase. Therefore, we can assume that region 1 and 4 have more lanes than the other regions.

To conclude, the Production vs Accumulation plots are more useful to analyse a region capacity than the one we plotted in Step 2. The optimal regime is now expressed for a certain amount of accumulation which is a number of vehicles. It is more informative than percentage of occupancy which does not includes key parameters as the number of lanes and their length.

5 MFD estimation (Step 5)

Now that we have plotted production and accumulation over each region, we will fit a cubic function into it. We will then try 3 different strategies to produce MFDs without considering all links but only half of them.

5.1 Fitting a cubic function over the plots from Step 4

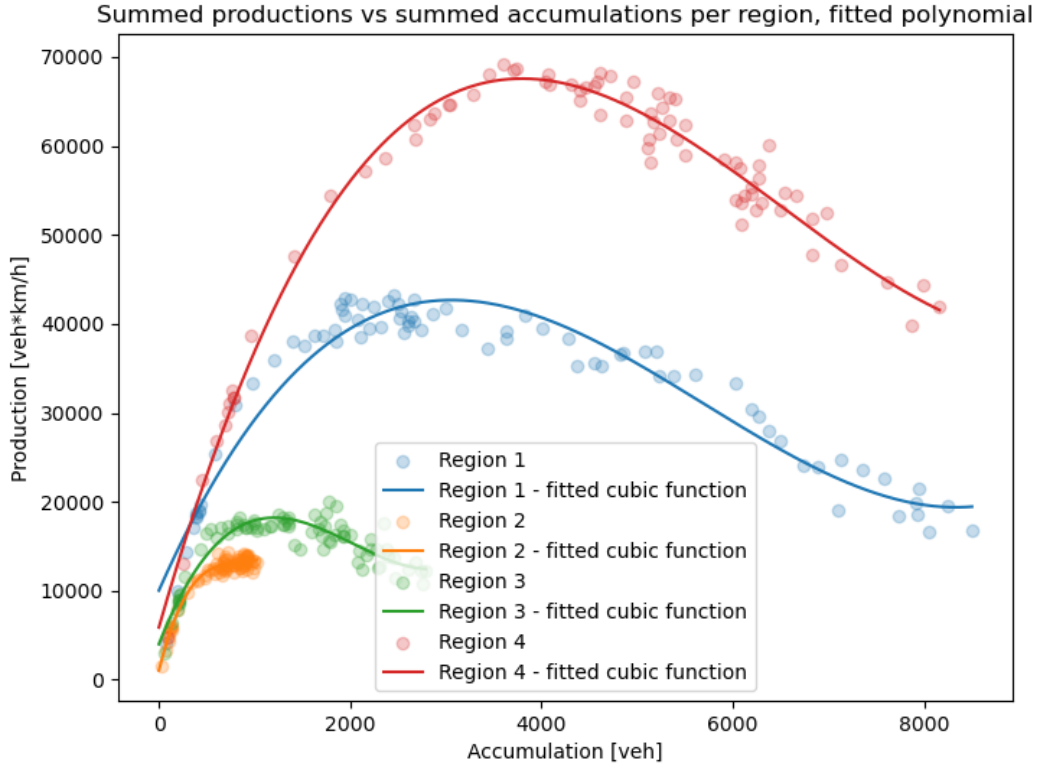


Figure 20: Fitted cubic polynomials over production vs accumulation plots

Here, we aimed to have a continuous function representing the Macroscopic Fundamental Diagram (MFD) of each region. The Figure 20 represents the cubic polynomials we obtain by minimising the square error. Those polynomial can be expressed as the following :

$$\text{Region 1 : } 10037 + 24x - 0.0054x^2 + (3.18 \times 10^{-7}) * x^3$$

$$\text{Region 2 : } 1051 + 42x - 0.051x^2 + (2.05 \times 10^{-5}) * x^3$$

$$\text{Region 3 : } 4007 + 27x - 0.016x^2 + (2.67 \times 10^{-6}) * x^3$$

$$\text{Region 4 : } 5891 + 38x - 0.0070x^2 + (3.57 \times 10^{-7}) * x^3$$

As we can see, each polynomial follows a similar curve but over a different scale. The flat end of some curves is due to the use of cubic polynomials which creates odd functions

5.2 Sampling the network to approximate MFDs

Now, we aim to find a strategy to approximate the MFDs from each region using only a fraction of the links. Three different strategies will be used to select half of the links :

1. Maximising the number of lanes
2. Maximising the longest link length
3. Maximising the average flow

Then, for each region, the production and accumulation will be computed over the selected links and scaled by the ratio between the total length of the selected links and the total length of the links in the region. Cubic polynomials will then be fitted on the resulting production and accumulation.

5.2.1 Strategy 1 : Maximising the number of lanes

For this strategy, we have selected the links based on the number of lanes, keeping in each region only the links with the higher number of lanes. This is a discrete metric, and therefore there are many links with the median value. Those have been separated only based on the linkID in order to strictly keep 50% of the links.

The results are shown in Figure 21. As we can see, both the production and the accumulation seem overestimated, up to a factor of 50%. This is not a surprising result, as the roads with multiples lanes will host more vehicles than roads with only one lanes. It can however be observed that for regions 1 and 3 the increase in production is even bigger than the increase in accumulation, while it is smaller in region 2.

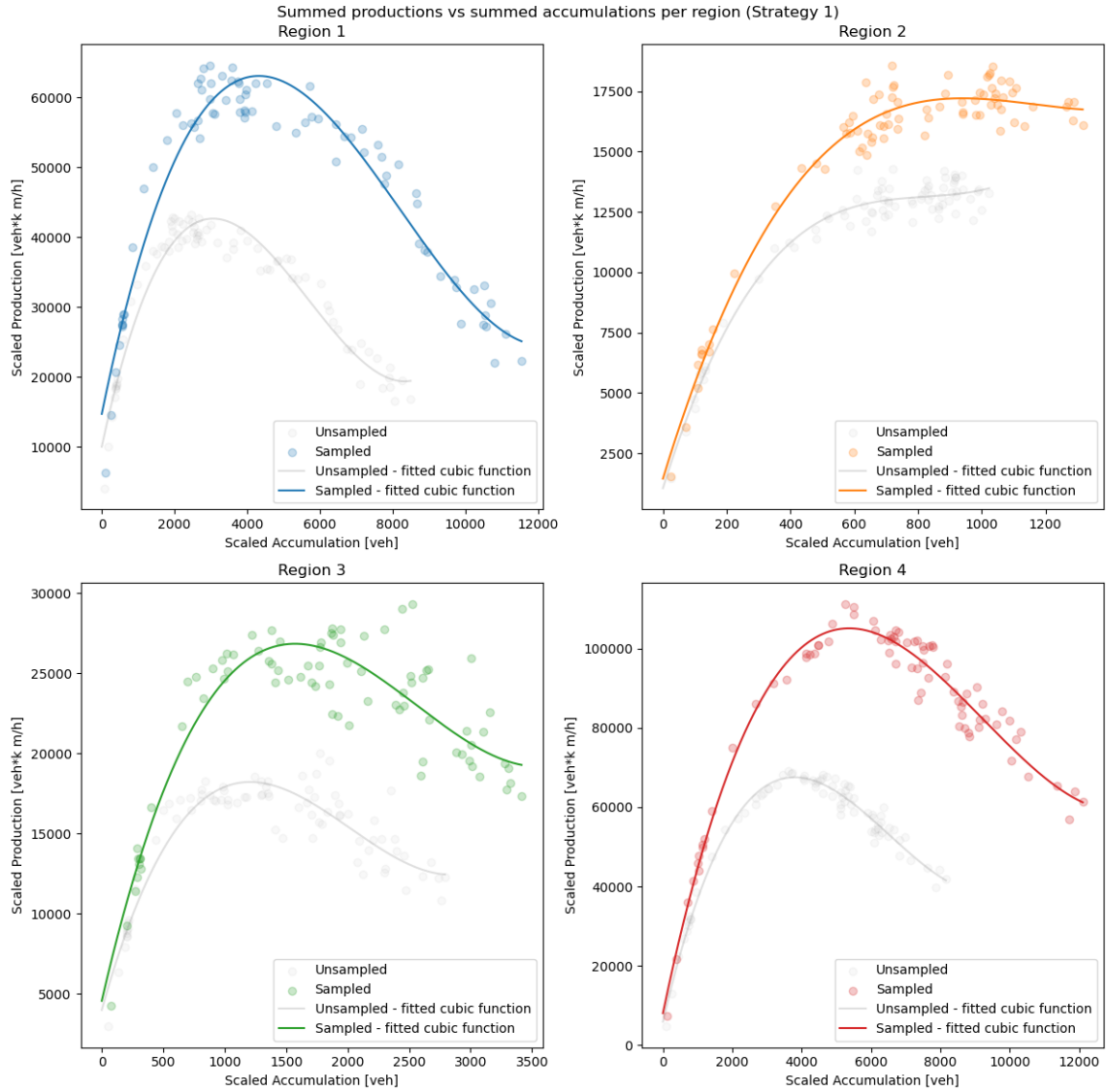


Figure 21: MFDs using Strategy 1

5.2.2 Strategy 2 : Maximising the link length

In this case, for each region, the links were sorted by their length and only the upper half kept. The results are shown in figure 22 : Here, the MFDs from the sampled network are very similar to

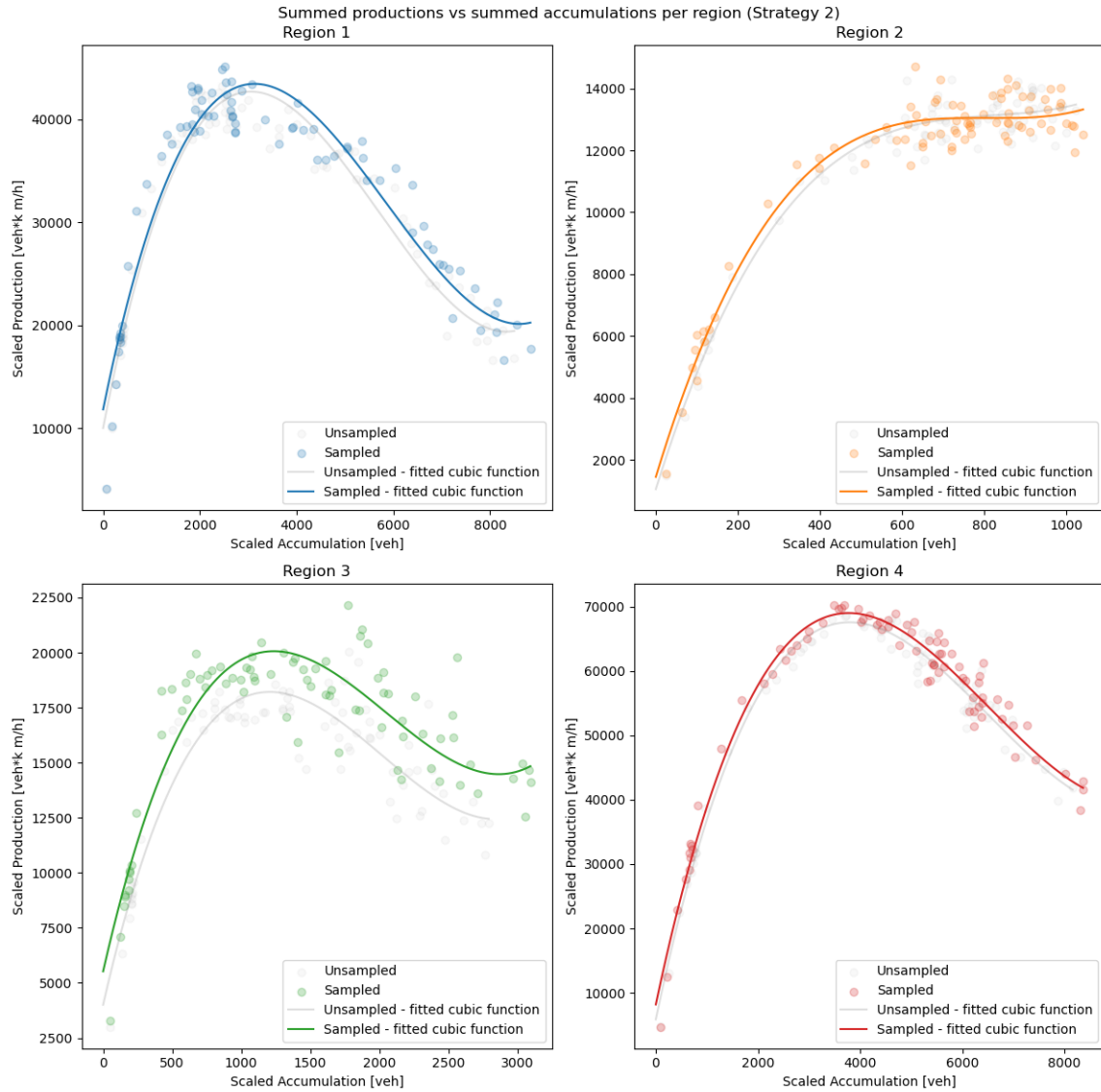


Figure 22: MFDs using Strategy 2

the MFDs from step 4, with only a small shift in the production of the third region that can be observed.

5.2.3 Strategy 3 : Maximising the average flow

For this last strategy, the links with the highest average flow over the period have been selected in each region. As Figure 23 shows, the resulting accumulations and productions are again up-scaled compared to the unsampled results. Compared to the first strategy, the effect is higher and more symmetrical between production and accumulation. A question that arises would be whether the scaling factor that was used (based on the link lengths) could be replaced by another one (for example, one based on the flow) and give better results.

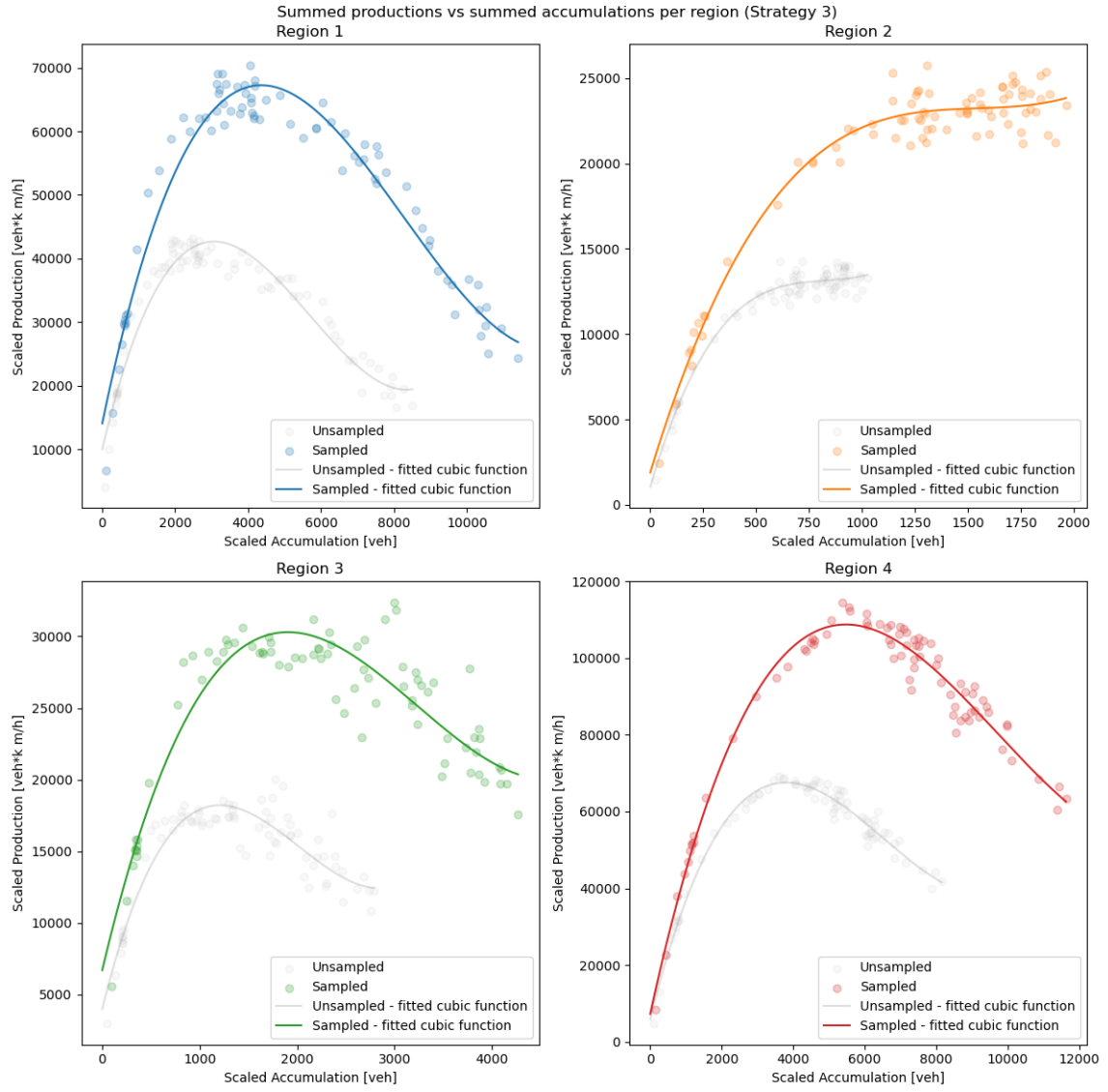


Figure 23: MFDs using Strategy 3

5.3 Comparison between the strategies

Figure 24 shows, for each region, the results from each strategy and from the full dataset. A clear difference can be seen between the different outcomes, with the second strategy giving the outcome nearest to the original curve while the third strategy is the further away.

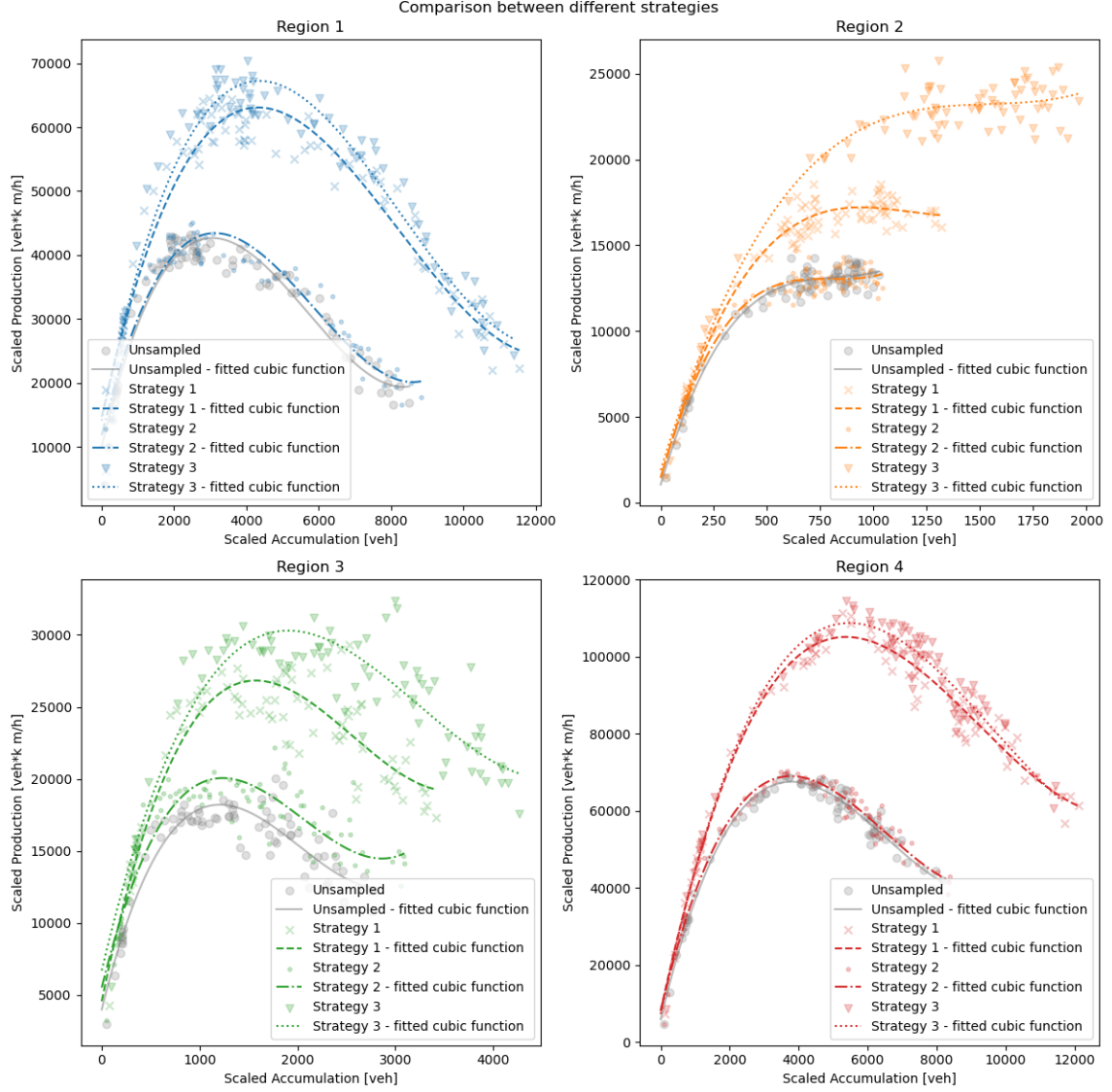


Figure 24: Comparison between the 3 strategies

To quantify this difference, we will look at different metrics :

Fitting errors We will compute, for each polynomial, the root mean squared error of the fitting process. This will provide an insight to how closely each polynomial fits the data it was processed on, but will not be very insightful on the correctness of each strategy.

Unsampled fitting error The root mean squared error for each polynomial will be computed again, but against the unsampled dataset. For the MFDs from Step 4, the result will be the same as the previous metric, but for each sampled MFDs it will evaluate whether the polynomials obtained could replace the polynomials computed on the whole dataset.

Point error For each point in the graph (which corresponds to the total production and accumulation in the region at a given time), the difference between the point from Step 4 and the points

from each strategy will be computed. The root mean square of these values will then be computed and compared. This will highlight the potential of each strategy to estimate the production and accumulation of the full network.

The results are given in the following table and represented in Figure 25:

		Fitting error	Unsampled fitting error	Point error
Region 1	Strategy 1	3.31e+04	1.74e+05	1.42e+05
	Strategy 2	2.46e+04	2.41e+04	7.44e+03
	Strategy 3	3.09e+04	2.01e+05	1.76e+05
	Unsampled	2.16e+04	-	-
Region 2	Strategy 1	6.33e+03	3.07e+04	3.15e+04
	Strategy 2	5.58e+03	5.25e+03	2.43e+03
	Strategy 3	9.36e+03	5.93e+04	8.45e+04
	Unsampled	4.80e+03	-	-
Region 3	Strategy 1	1.82e+04	7.37e+04	6.74e+04
	Strategy 2	1.44e+04	2.05e+04	1.46e+04
	Strategy 3	1.89e+04	1.02e+05	9.03e+04
	Unsampled	1.16e+04	-	-
Region 4	Strategy 1	3.50e+04	3.39e+05	2.74e+05
	Strategy 2	2.10e+04	2.29e+04	9.01e+03
	Strategy 3	3.37e+04	3.66e+05	3.09e+05
	Unsampled	1.86e+04	-	-

Table 1: Errors between the strategies

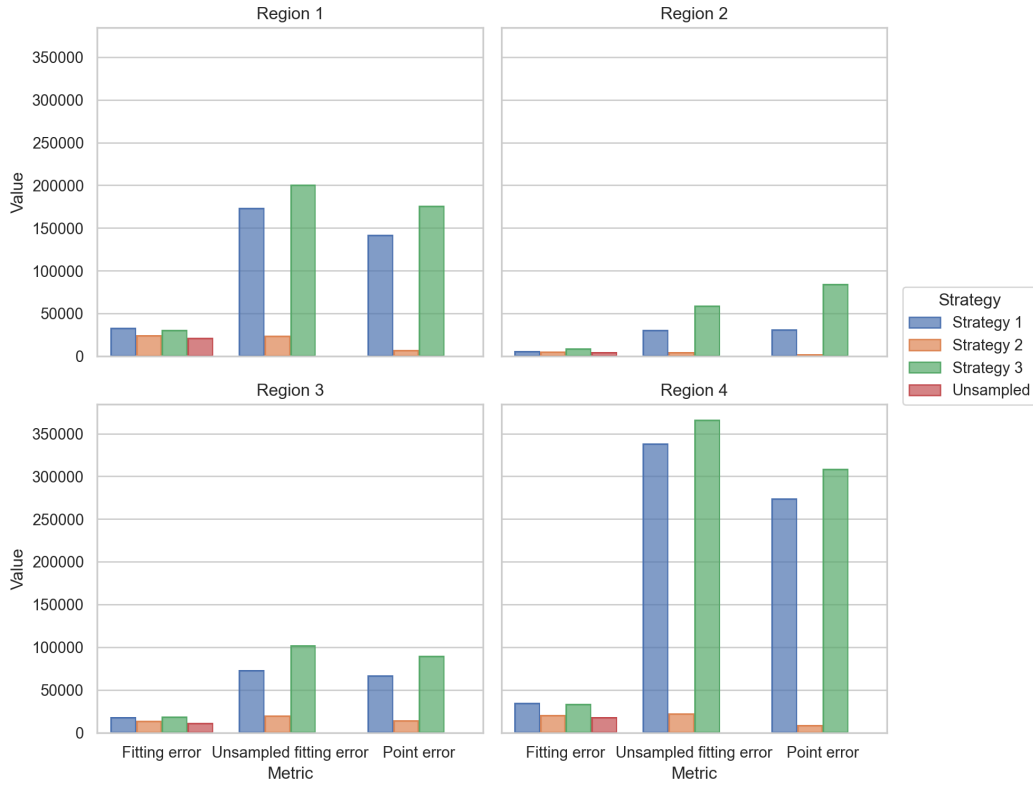


Figure 25: Errors between the strategies

This confirms the visual inspection from earlier : the MFD from the second strategy has the smallest difference with the unsampled MFD. The first and third strategy presents higher errors in all metrics, including in the fitting error : those strategies generate data points less consistent with a third degree polynomial.

In conclusion, sampling the edges of the network based on the link length is the strategy that gives the best results to approximate the whole network, even in different configurations. Samplings based