while x<y do x := x+x

$\rho(x) = \alpha_1$ $\quad \sigma(\alpha_1) = 1$

$\rho(y) = \alpha_2$ $\quad \sigma(\alpha_2) = 2$

Lemma 1:

$$\dfrac{\dfrac{\langle x, \rho_0, \sigma_0\rangle \to 1 \quad \langle x, \rho_0, \sigma_0\rangle \to 1}{\langle x + x, \rho_0, \sigma_0\rangle \to 2}}{\langle x := x+x, \rho_0, \sigma_0\rangle \to \underbrace{\sigma_0[\alpha_1/2]}_{\sigma_1}}$$

Lemma 2:

$$\dfrac{\dfrac{\langle x, \rho_0, \sigma_1\rangle \to 2 \quad \langle y, \rho_0, \sigma_1\rangle \to 2}{\langle x < y, \rho_0, \sigma_1\rangle \to \text{false}}}{\langle \text{while } x<y \text{ do } x := x+x, \rho_0, \sigma_1\rangle \to \sigma_1}$$

Hauptrechnung:

$$\dfrac{\dfrac{\langle x, \rho_0, \sigma_0\rangle \to 1 \quad \langle y, \rho_0, \sigma_0\rangle \to 2}{\langle x < y, \rho_0, \sigma_0\rangle \to \text{true}} \quad \text{Lemma 1} \quad \text{Lemma 2}}{\langle \text{while } x<y \text{ do } x := x+x, \rho_0, \sigma_0\rangle \to \sigma_1}$$