Optimisation Modern Power Systems 1

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Task 1.a)	70%	30%
Task 1.b)	30%	70%
Task 1.c)	50%	50%
Task 2.b)	50%	50%

Table 1: Contribution of each member to project tasks. Repository: GitHub link

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Contents

1 Consumer Flexiblity

1.1 Problem Setup

We have a scenario where there is a prosumer, who can buy and sell electricity to the grid and has his own solar panels installed in his home. the consumer is fully flexible and can ramp up and down to his max capacity in one hour. Similarly the solar can also ramp/down to it´s max capacity in one hour. both the solar and prosumer have a max load and capacity of 3 kW per hour respectively. The solar has known power production but can be fully curtailed. We know also the spot prices for the electricity in each hour along with grid fees for importing/exporting electricity. There is also a penalty for importing/exporting more than a certain amount per hour.

To solve this problem, we first defined the variables needed to setup the optimization problem.

Table 2: Decision Variables

Variable	**Description**	**Bounds**
i_t	Imported power from the grid at time t [kW]	$i_t \geq 0$
e_t	Exported power to the grid at time t [kW]	$e_t \geq 0$
l_t	Flexible load consumption at time t (fraction of max load)	$0 \leq l_t \leq 1$
s_t	Solar power utilized at time t [kW]	$0 \leq s_t \leq c_t^f \cdot S^{\max}$
z_t^{imp}	Excess imports at time t [kW]	$z_t^{imp} \geq 0$
z_t^{exp}	Excess exports at time t [kW]	$z_t^{exp} \geq 0$

With:

- $P^{\text{imp}}, P^{\text{exp}}$: Import/export fee [DKK/kWh]
- Π_t: electricty spot price for a time t [DKK/kWh]
- Λ^{imp}: Penalty fee for exceeding maximum import capacity [DKK/kWh]
- Λ^{exp}: Penalty fee for exceeding maximum export capacity [DKK/kWh]
- $L^{\max}, L^{\min}$: Max and min flexible load [kW]
- $S^{\max}$: Installed PV capacity [kW]
- CF_t: PV capacity factor at time t
- $I^{\max}, E^{\max}$: Import/export limits [kW]
- $R^{\uparrow}_{\text{FFL}}, R^{\downarrow}_{\text{FFL}}$: Ramp rates of flexible load
- $R^{\uparrow}_{\text{PV}}, R^{\downarrow}_{\text{PV}}$: Ramp rates of PV
- L_t^{ref}: Hourly profile ratio for a time t (fraction of max load)

Prosumer Optimization Model

Objective: Minimize total procurement cost and penalties

$$\min_{i_t, e_t, z_t^{imp}, z_t^{exp}, l_t} \quad \sum_{t \in \mathcal{T}} \Big[(\Pi_t + P^{\text{imp}}) i_t - (\Pi_t - P^{\text{exp}}) e_t + \Lambda^{imp} z_t^{imp} + \Lambda^{exp} z_t^{exp} \Big] \tag{1}$$

We first pay the electricity price when importing in addition to the grid tariff, whereas when we are exporting the electricity we will receive the electricity price, but still have to by the export tariff(P^{exp}), which is why the electricity price sign is negative in this case.

Subject to:
Hourly constraints, $\forall t \in \mathcal{T}$:

$$\begin{aligned}
L^{\min} \;\le\; l_t \cdot L^{\max} \;\le\; L^{\max} & \qquad \text{(FFL bounds)} \qquad (2)\\
0 \;\le\; CF_t \cdot S_t \;\le\; S^{\max} & \qquad \text{(Solar availability)} \qquad (3)\\
i_t - e_t + S_t \;=\; l_t \cdot L^{\max} & \qquad \text{(Energy balance)} \qquad (4)\\
z_t^{imp} \;\ge\; i_t - I^{\max} & \qquad \text{(Excess import)} \qquad (5)\\
z_t^{exp} \;\ge\; e_t - E^{\max} & \qquad \text{(Excess export)} \qquad (6)
\end{aligned}$$

The solar power produced (S_t) is limited by the capacity factor(CF_t) for of the solar panels. The excess imports/exports(z_t) will be positive always, since we defined both variables with a lower bound of zero.

Ramp-rate constraints, $\forall t > 0$:

$$\begin{aligned}
(l_t - l_{t-1}) \cdot L^{\max} \;\le\; R^{\uparrow}_{\text{FFL}} \cdot L^{\max} & \qquad \text{(FFL ramp-up)} \qquad (7)\\
(l_{t-1} - l_t) \cdot L^{\max} \;\le\; R^{\downarrow}_{\text{FFL}} \cdot L^{\max} & \qquad \text{(FFL ramp-down)} \qquad (8)\\
S_t - S_{t-1} \;\le\; R^{\uparrow}_{\text{PV}} S^{\max} & \qquad \text{(PV ramp-up)} \qquad (9)\\
S_{t-1} - S_t \;\le\; R^{\downarrow}_{\text{PV}} S^{\max} & \qquad \text{(PV ramp-down)} \qquad (10)
\end{aligned}$$

Daily energy requirement:

$$\sum_{t \in \mathcal{T}} l_t L^{\max} \;\ge\; L^{\min}_{\text{daily}} \tag{11}$$

The $L^{\min}_{\text{daily}}$ is the minimum demand needed to be satisfied for the prosumer and is calculated multiplying the minimum number of hours required at full load, which in our case was 8 h by the max load of our prosumer which was 3 kW. Hence the minimum energy requirement for the whole day has to be at least greater than 24 kWh. By defining it this way we are not constraining our prosumer to just run 8h at full load rather the loads are more spread out.

Dual Problem of the Prosumer Optimization Model

Dual variables:

- $\alpha_t^{\min} \geq 0$: Dual for $l_t L_{\max} \geq_{\min}$ (FFL lower bound)
- $\alpha_t^{\max} \geq 0$: Dual for $l_t L_{\max} \leq L_{\max}$ (FFL upper bound)
- $\beta_t \geq 0$: Dual for $S_t \leq CF_t S_{\max}$ (PV availability)
- γ_t (free): Dual for energy balance $i_t - e_t + S_t \;=\; l_t \cdot L^{\max}$
- $\delta_t^I \geq 0$: Dual for excess import $z_t^{imp} \geq i_t - I_{\max}$
- $\delta_t^E \geq 0$: Dual for excess export $z_t^{exp} \geq e_t - E_{\max}$
- $\rho_t^{\uparrow}, \rho_t^{\downarrow} \geq 0$: Duals for FFL ramp up/down
- $\sigma_t^{\uparrow}, \sigma_t^{\downarrow} \geq 0$: Duals for PV ramp up/down
- $\mu \geq 0$: Dual for daily energy requirement $\sum_t l_t L_{\max} \geq L_{\min}^{\text{daily}}$

Dual objective:

$$\max \quad \sum_{t \in \mathcal{T}} L_{\max} \, (\alpha_t^{\min} - \alpha_t^{\max}) + \sum_{t \in \mathcal{T}} CF_t S_{\max} \, \beta_t + \sum_{t \in \mathcal{T}} I_{\max} \, \delta_t^I + \sum_{t \in \mathcal{T}} E_{\max} \, \delta_t^E + L_{\min}^{\text{daily}} \, \mu \tag{12}$$

Dual constraints: For all $t \in \mathcal{T}$

$$(p^{\text{imp}} + \pi_t) - \gamma_t + \delta_t^I \geq 0 \qquad \text{(imports } I_t) \tag{13}$$

$$(p^{\text{exp}} - \pi_t) + \gamma_t + \delta_t^E \geq 0 \qquad \text{(exports } E_t) \tag{14}$$

$$-\gamma_t + L_{\max}(\alpha_t^{\min} - \alpha_t^{\max}) + \rho_t^{\uparrow} - \rho_t^{\downarrow} + \mu \leq 0 \qquad \text{(FFL } L_t) \tag{15}$$

$$\gamma_t + \beta_t + \sigma_t^{\uparrow} - \sigma_t^{\downarrow} \leq 0 \qquad \text{(PV } S_t) \tag{16}$$

$$\lambda^I - \delta_t^I \geq 0 \qquad \text{(excess import } Z_t^I) \tag{17}$$

$$\lambda^E - \delta_t^E \geq 0 \qquad \text{(excess export } Z_t^E) \tag{18}$$

Dual variable bounds: All duals ≥ 0 except γ_t which is free.

2 Interpretation and Properties of the Dual Problem

The dual problem provides important economic insights into the prosumer optimization model. Its variables and constraints have clear interpretations:

2.1 Dual Variables and Economic Meaning

- $\alpha_t^{\min} \geq 0$: Marginal value of relaxing the minimum flexible load at hour t. If positive, increasing $L_t^{\min}$ would increase total cost.
- $\alpha_t^{\max} \geq 0$: Marginal benefit of relaxing the maximum flexible load at hour t. If positive, the upper bound is binding and limits cost reduction.
- $\beta_t \geq 0$: Marginal value of increasing PV availability at hour t. Positive values indicate that more PV could reduce procurement cost.

- γ_t (free): Marginal cost of energy at hour t, representing the value of the energy balance constraint.
- $\delta_t^I \geq 0$, $\delta_t^E \geq 0$: Marginal cost associated with violating import/export limits. Positive values indicate binding constraints.
- $\rho_t^{\uparrow}, \rho_t^{\downarrow} \geq 0$: Marginal cost of FFL ramp-up/ramp-down constraints. Positive values indicate binding ramp limits.
- $\sigma_t^{\uparrow}, \sigma_t^{\downarrow} \geq 0$: Marginal cost of PV ramp-up/ramp-down constraints. Positive values indicate ramping limitations.
- $\mu \geq 0$: Marginal cost of meeting the total daily energy requirement. Positive values show that relaxing the daily minimum reduces total cost.

2.2 Dual Objective Interpretation

The dual objective:

$$\max \quad \sum_{t\in\mathcal{T}} L_{\max}(\alpha_t^{\min} - \alpha_t^{\max}) + \sum_{t\in\mathcal{T}} CF_t S_{\max}\beta_t + \sum_{t\in\mathcal{T}} I_{\max}\delta_t^I + \sum_{t\in\mathcal{T}} E_{\max}\delta_t^E + L_{\min}^{\text{daily}}\mu \tag{19}$$

represents the maximum total value of relaxing all constraints. Each term shows how much reducing or relaxing a constraint would impact total procurement cost.

2.3 Dual Constraints Interpretation

Each dual constraint ensures that the marginal contribution of a variable does not exceed its cost:

- **Imports** i_t: Cost of buying electricity plus penalty must cover the marginal value from energy balance.
- **Exports** e_t: Net export fee minus revenue must cover the marginal value from energy balance.
- **Flexible load** l_t: Marginal value of FFL bounds, ramp limits, and daily minimum cannot exceed the cost of consuming energy.
- **PV** S_t: Value of available PV limited by capacity factor and ramping must be less than or equal to marginal benefit.
- **Excess imports/exports** z_t^{imp}, z_t^{exp}: Penalties must cover the dual contributions.

2.4 Key Properties

- **Non-negativity**: Most duals are ≥ 0, meaning relaxing a constraint cannot increase cost.
- **Free variable**: γ_t can be positive or negative, reflecting true marginal energy cost at hour t.
- **Economic interpretation**: Dual variables act as shadow prices, showing the cost impact of small changes in constraints.
- **Complementary slackness**:
  - If a primal constraint is not binding, its dual variable $= 0$.

– If a dual variable > 0, the corresponding primal constraint is active at the optimum.

2.5 Scenarios

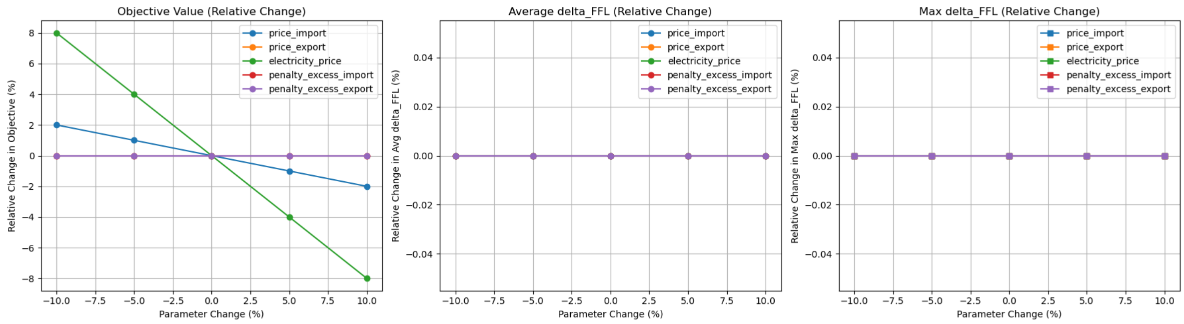


Figure 1: Sensitivity of Profitability and Flexibility

It seems that the only financial metrics that have an impact on the profitability are the importing fee and electricity price. The exporting fee and penalties have no impact since, the model will try to not buy excess electricity as it is too expensive, which is why the penalties have no impact on the model. As we can see, the prosumer only imports power at the very start of the day,

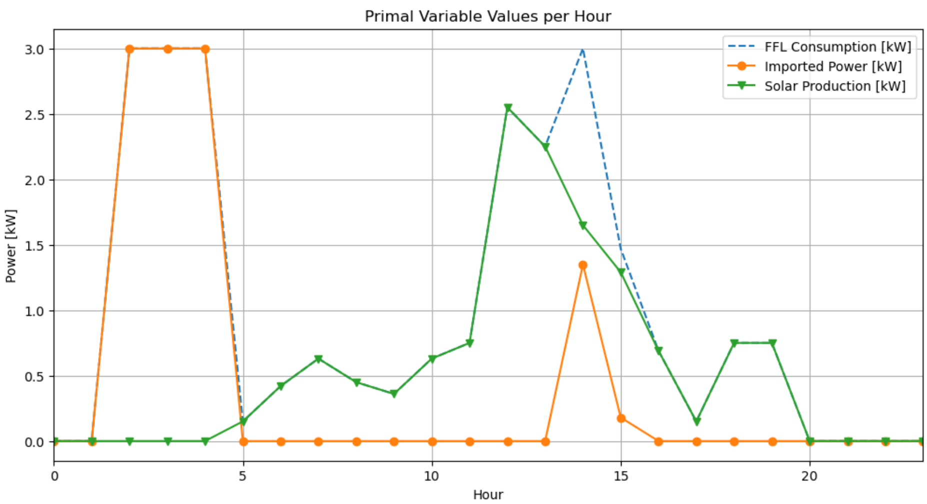


Figure 2: Power Flows

due to there being no solar power available for the first five consecutive hours, but once the solar production begins, the imports drop to 0. This is also why the largest difference in loads occurs between hour 4 and 5, along with the hour where solar production reaches peak, at hour 13, where the solar capacity jumps from 0.25 to 0.85, the largest change in solar production. The flexibility of the system itself is independent of the costs and is dependent on the capacity of our buses and ramping rates of our generators.

3 Task 1b: Flexibility Preferences and Optimization

3.1 Adaptation of the Optimization Problem

The consumer now minimizes both energy procurement cost and discomfort from shifting flexible load away from a reference profile. To do so the system will have an auxiliary term that will set weight values to each part of the optimization, this way a more thoroughly research of the flexibility will be done. The updated objective is:

$$\min_{i_t, e_t, z_t^{imp}, z_t^{exp}, l_t, d_t} \sum_{t \in \mathcal{T}} [\beta \cdot \text{Cost}_t + (1 - \beta) \cdot d_t] \tag{20}$$

Where:

$$\text{Cost}_t = \left[\left(\Pi_t + P^{\text{imp}} \right) i_t - \left(\Pi_t - P^{\text{exp}} \right) e_t + \Lambda^{imp} z_t^{imp} + \Lambda^{exp} z_t^{exp} \right] \tag{21}$$

New Constraints:

$$d_t \geq (x_t^{FFl} - X_t^{ref}) \cdot L^{max} \tag{22}$$

$$d_t \leq -(x_t^{FFl} - X_t^{ref}) \cdot L^{max} \tag{23}$$

This constraints will limit the deviation only to positive numbers, in other terms, it is getting only absolute values so that the objective function does not get misleading results.

New Notations:

- β: Weighting parameter for cost vs discomfort ($0 \leq \beta \leq 1$)
- d_t: Discomfort variable
- x_t^{FFl}: Flexible load at time t
- X_t^{ref}: Reference load at time t

This approach allows the consumer to trade off between cost savings and comfort, reflecting realistic flexibility preferences.

3.2 Dual Variables and Their Impact on the Objective Value

The Lagrangian is:

$$\mathcal{L} = \sum_{t=1}^{T} [\beta \cdot \text{Cost}_t + (1 - \beta) \cdot d_t] + \sum_j \lambda_j g_j(x) \tag{24}$$

where $g_j(x)$ are the constraints and λ_j are dual variables. In the optimization model, each constraint $g_j(x)$ has an associated dual variable λ_j. The value of λ_j represents the sensitivity of the objective value to changes in the corresponding constraint.

Main constraints and their duals:

Compared to the previous optimization problem, the dual for the daily load will be removed since it is no longer necessary, and two new duals will appear.

- $\mu \geq 0$: Dual for daily energy requirement $\sum_t l_t L_{\max} \geq L_{\min}^{\text{daily}}$ (removed)
- $\phi_t^+ \geq 0$: Dual for positive discomfort $d_t \geq (x_t^{FFL} - X_t^{ref}) \cdot L_{\max}$ (added)
- $\phi_t^- \geq 0$: Dual for negative discomfort $d_t \leq -(x_t^{FFL} - X_t^{ref}) \cdot L_{\max}$ (added)

With all these things considered, the Lagrange function would be like this:

$$
\begin{aligned}
\mathcal{L} = \quad & \sum_t [\beta \cdot \text{Cost}_t + (1-\beta) \cdot d_t] && (25)\\
& + \sum_t \alpha_t^{\min}\big(l_t L_{\max} - L_{\min}\big) + \sum_t \alpha_t^{\max}\big(L_{\max} - l_t L_{\max}\big) && (26)\\
& + \sum_t \beta_t\big(CF_t S_{\max} - S_t\big) + \sum_t \gamma_t\big(i_t - e_t + S_t - l_t L_{\max}\big) && (27)\\
& + \sum_t \delta_t^I\big(z_t^{imp} - i_t + I_{\max}\big) + \sum_t \delta_t^E\big(z_t^{exp} - e_t + E_{\max}\big) && (28)\\
& + \sum_t \rho_t^{\uparrow}\big(R_t^{FFL,\uparrow} - (l_t - l_{t-1})\big) + \sum_t \rho_t^{\downarrow}\big(R_t^{FFL,\downarrow} - (l_{t-1} - l_t)\big) && (29)\\
& + \sum_t \sigma_t^{\uparrow}\big(R_t^{PV,\uparrow} - (S_t - S_{t-1})\big) + \sum_t \sigma_t^{\downarrow}\big(R_t^{PV,\downarrow} - (S_{t-1} - S_t)\big) && (30)\\
& + \sum_t \phi_t^+\big(d_t - (x_t^{FFL} - X_t^{ref})L_{\max}\big) + \sum_t \phi_t^-\big(-d_t - (x_t^{FFL} - X_t^{ref})L_{\max}\big). && (31)
\end{aligned}
$$

Dual Objective:

$$
\begin{aligned}
\max \sum_t \alpha_t^{\min} L_{\min} + \sum_t \alpha_t^{\max} L_{\max} + \sum_t \beta_t\, CF_t S_{\max} + \sum_t \delta_t^I I_{\max} + \sum_t \delta_t^E E_{\max} \\
+ \sum_t \rho_t^{\uparrow} R_t^{FFL,\uparrow} + \sum_t \rho_t^{\downarrow} R_t^{FFL,\downarrow} + \sum_t \sigma_t^{\uparrow} R_t^{PV,\uparrow} + \sum_t \sigma_t^{\downarrow} R_t^{PV,\downarrow}
\end{aligned} \quad (32)
$$

Note: The γ_t and $\phi_t^{\pm}$, constraints have RHS = 0, so they do not contribute directly to the dual objective, but appear in the dual constraints.

3.3 KKT Conditions for the Flexibility Optimization Problem

The Karush–Kuhn–Tucker (KKT) conditions represent the necessary optimality criteria for this nonlinear optimization problem. They combine the primal feasibility (original constraints), dual feasibility (sign and freedom of dual variables), stationarity (first-order conditions of the Lagrangian), and complementary slackness (linking primal and dual activeness).

1. Primal Feasibility

$$L_{\min} \le l_t L_{\max} \le L_{\max}, \tag{33}$$
$$S_t \le CF_t S_{\max}, \tag{34}$$
$$i_t - e_t + S_t - l_t L_{\max} = 0, \tag{35}$$
$$Z_t^I \ge I_t - I_{\max}, \quad z_t^{exp} \ge e_t - E_{\max}, \tag{36}$$
$$l_t - l_{t-1} \le R_t^{FFL,\uparrow}, \quad l_{t-1} - l_t \le R_t^{FFL,\downarrow}, \tag{37}$$
$$S_t - S_{t-1} \le R_t^{PV,\uparrow}, \quad S_{t-1} - S_t \le R_t^{PV,\downarrow}, \tag{38}$$
$$d_t - (x_t^{FFL} - X_t^{ref})L_{\max} \ge 0, \quad -d_t - (x_t^{FFL} - X_t^{ref})L_{\max} \ge 0 \tag{39}$$

These are the original constraints of the optimization problem. They describe what is physically or operationally possible for the system (loads, storage, rates, etc.). Mathematically, these are the constraints the primal (consumer's) optimization must satisfy.

Interpretation of Each Group:

- $L_t^{\min} \le l_t L_{\max} \le L_{\max}$**:** The flexible load L_t must remain between its minimum and maximum normalized levels. This ensures the consumer cannot go below a comfort threshold or exceed device capacity.
- $S_t \le CF_t S_{\max}$**:** PV generation (or storage output) cannot exceed the available capacity given by the capacity factor CF_t.
- $i_t - e_t + S_t - l_t L_{\max} = 0$**:** Energy balance constraint — imports, exports, storage, and load must be balanced at every time t. This ensures total inflow equals total outflow.
- $z_t^{imp} \ge i_t - I_{\max}, \quad z_t^{exp} \ge e_t - E_{\max}$**:** Import/export limit constraints — imports and exports cannot exceed their rated capacities. The slack variables Z_t^I and Z_t^E represent possible over-limit deviations (penalized in the objective).
- $l_t - l_{t-1} \le R_t^{FFL,\uparrow}$, $l_{t-1} - l_t \le R_t^{FFL,\downarrow}$**:** Ramp-rate limits for flexible loads — the change in load between two consecutive periods cannot exceed the allowed ramp-up or ramp-down rates.
- $S_t - S_{t-1} \le R_t^{PV,\uparrow}$, $S_{t-1} - S_t \le R_t^{PV,\downarrow}$**:** Similar ramp-rate limits, but for PV generation or storage charge/discharge transitions.
- $d_t - (x_t^{FFL} - X_t^{ref})L_{\max} \ge 0$, $-d_t - (x_t^{FFL} - X_t^{ref})L_{\max} \ge 0$**:** Discomfort bounds — these inequalities ensure that d_t measures the absolute deviation of the flexible load x_t^{FFL} from the reference load profile X_t^{ref}. The discomfort variable thus captures both upward and downward deviations from the preferred consumption pattern.

2. Dual Feasibility

$$\alpha_t^{\min}, \alpha_t^{\max}, \beta_t, \delta_t^I, \delta_t^E, \rho_t^{\uparrow}, \rho_t^{\downarrow}, \sigma_t^{\uparrow}, \sigma_t^{\downarrow}, \phi_t^{+}, \phi_t^{-} \ge 0, \quad \gamma_t \text{ free.}$$

This specifies which dual variables (Lagrange multipliers) must be nonnegative. The variable γ_t is free because it corresponds to an equality constraint (the power balance).

3. Stationarity

This is the **first-order optimality condition**: At optimality, the gradient of the Lagrangian with respect to each primal variable must be zero. In other words, no small change in a primal variable can improve the objective while respecting the constraints.

$$\frac{\partial \mathcal{L}}{\partial L_t} : \alpha_t^{\min} L_{\max} - \alpha_t^{\max} L_{\max} - \gamma_t L_{\max} - \rho_t^{\uparrow} + \rho_t^{\downarrow} + \rho_{t+1}^{\uparrow} - \rho_{t+1}^{\downarrow} = 0, \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial S_t} : -\beta_t + \gamma_t - \sigma_t^{\uparrow} + \sigma_t^{\downarrow} + \sigma_{t+1}^{\uparrow} - \sigma_{t+1}^{\downarrow} = 0, \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : \gamma_t - \delta_t^I = 0, \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial E_t} : -\gamma_t - \delta_t^E = 0, \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial d_t} : (1-\beta) + \phi_t^+ - \phi_t^- = 0, \tag{44}$$

$$\frac{\partial \mathcal{L}}{\partial X_t^{FFL}} : -L_{\max}(\phi_t^+ + \phi_t^-) = 0. \tag{45}$$

Each derivative equation expresses an equilibrium between the direct impact of a variable on the objective function and the indirect penalties imposed by constraints.

Interpretations:

- $\frac{\partial \mathcal{L}}{\partial L_t}$: The marginal effect of changing load L_t is balanced by the constraint multipliers on load limits and ramping. It means that all opposing forces (cost, power balance, and flexibility limits) are in equilibrium.
- $\frac{\partial \mathcal{L}}{\partial d_t}$: Balances discomfort cost $(1-\beta)$ against the dual pressures from both positive and negative deviation limits (ϕ_t^+, ϕ_t^-).
- $\frac{\partial \mathcal{L}}{\partial X_t^{FFL}}$: Shows that deviations of flexible load from the reference profile only matter if discomfort constraints are active (i.e., ϕ_t^+ or $\phi_t^- > 0$).

4. Complementary Slackness

$$\alpha_t^{\min}(l_t L_{\max} - L_{\min}) = 0, \qquad \alpha_t^{\max}(L_{\max} - l_t L_{\max}) = 0, \tag{46}$$

$$\beta_t(CF_t S_{\max} - S_t) = 0, \qquad \delta_t^I(z_t^{imp} - i_t + I_{\max}) = 0, \tag{47}$$

$$\delta_t^E(z_t^{exp} - e_t + E_{\max}) = 0, \qquad \rho_t^{\uparrow}(R_t^{FFL,\uparrow} - (l_t - l_{t-1})) = 0, \tag{48}$$

$$\rho_t^{\downarrow}(R_t^{FFL,\downarrow} - (l_{t-1} - l_t)) = 0, \qquad \sigma_t^{\uparrow}(R_t^{PV,\uparrow} - (S_t - S_{t-1})) = 0, \tag{49}$$

$$\sigma_t^{\downarrow}(R_t^{PV,\downarrow} - (S_{t-1} - S_t)) = 0, \qquad \phi_t^+(d_t - (x_t^{FFL} - X_t^{ref})L_{\max}) = 0, \tag{50}$$

$$\phi_t^-(-d_t - (x_t^{FFL} - X_t^{ref})L_{\max}) = 0. \tag{51}$$

Complementary slackness links each dual variable with its corresponding constraint: if a constraint is not binding (has slack), its dual variable is zero. If the constraint is tight, the dual variable is *positive, indicating how much the objective would improve if the constraint were relaxed.*

3.4 Qualitative Discussion of Structural and Optimality Impacts

The introduction of the weighting parameter β transforms the optimization problem from a single-objective cost minimization into a bi-objective formulation that balances energy procurement cost and consumer discomfort. Specifically, the updated objective function (20) creates a continuum between two behavioral extremes: for $\beta = 1$, the consumer seeks pure cost minimization, while for $\beta = 0$, the optimization focuses entirely on maintaining comfort by adhering to the reference load profile. For intermediate values of β, the optimal solution represents a trade-off between these two competing objectives.

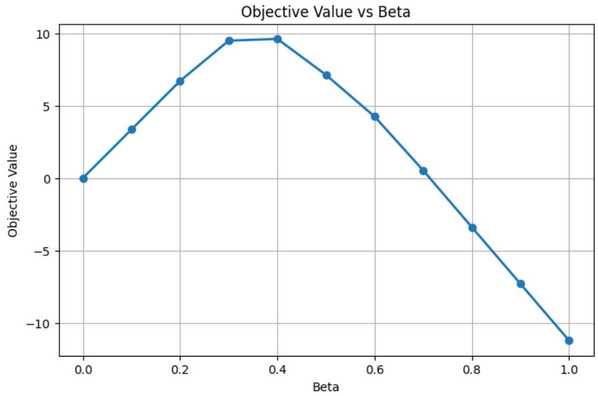
Structurally, the inclusion of the discomfort variable d_t and its corresponding constraints introduces additional coupling between the flexible load x_t^{FFL} and its reference profile X_t^{ref}. These constraints define tolerance bands around the preferred consumption pattern and determine how much deviation is permitted depending on the consumer's flexibility preference. Consequently, the Lagrangian now includes new dual variables φ_t^+ and φ_t^- associated with these discomfort bounds, modifying the system of KKT conditions and changing which constraints are active at optimality.

When β is low, the consumer strongly prioritizes comfort, leading to solutions where the flexible load closely follows the reference profile and the discomfort constraints become binding. In this regime, the optimizer accepts higher energy procurement costs to maintain comfort. Conversely, when β is high, the optimization prioritizes cost reduction, allowing larger deviations from the reference load. Here, the discomfort constraints are typically non-binding, and economic or physical limits such as import/export capacities, ramp-rate limits, or energy balance dominate the solution. For intermediate values of β, multiple constraints interact, and the resulting solution lies on the Pareto frontier between cost efficiency and comfort preservation.

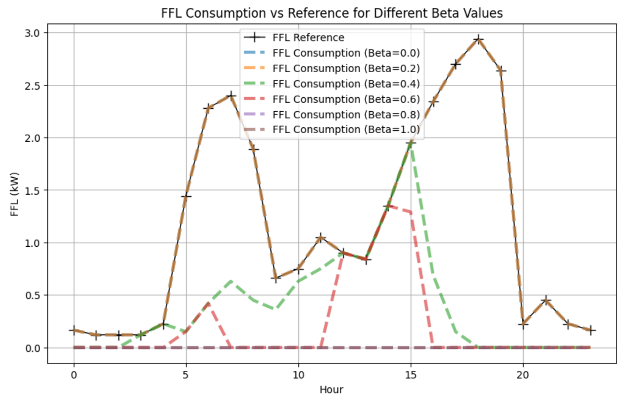
Comparing these outcomes to the findings from Question 1(a).iii, it can be concluded that the earlier results still hold in the limit where $\beta \to 1$, as the optimization again behaves as a pure cost minimization problem. However, as β decreases, the inclusion of comfort considerations fundamentally alters the optimal structure, causing a shift in the set of binding constraints and the sensitivity of the objective to parameter changes. Ultimately, the model captures a realistic trade-off: higher flexibility and profit potential for cost-oriented consumers, and higher comfort but greater cost for comfort-oriented consumers.

3.5 Scenario Analysis and Insights

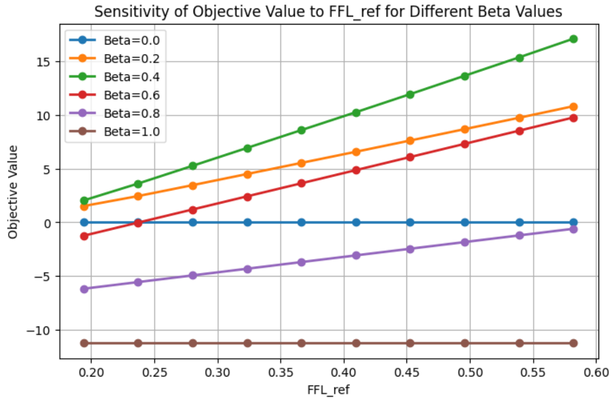
To perform the scenario analysis, all model parameters were systematically perturbed within a range of $\pm 50\%$ to identify the most influential (binding) factors. A series of simulations were conducted for multiple values of β, specifically $\beta \in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$. The corresponding results are illustrated in the following figures.



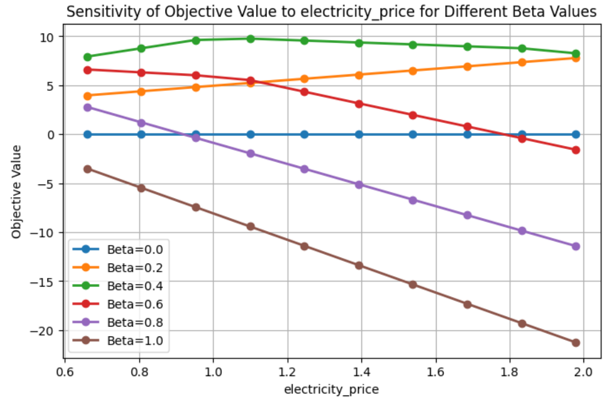
(a) Objective vs β.



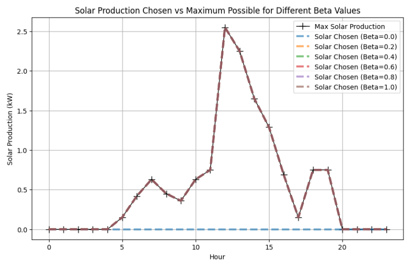
(b) Load tracking



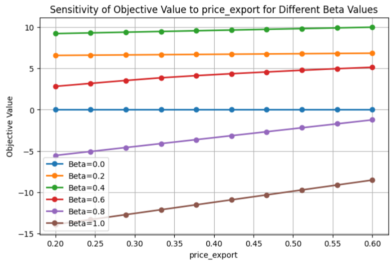
(c) Load reference.



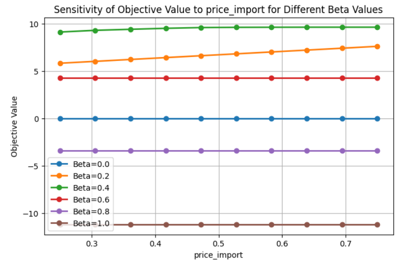
(d) Electricity price



(e) Solar production



(f) Export prices



(g) Import prices

Figure 3: Sensitivity of Dual Parameters to Objective Function Weighting.

When $\beta \to 0$ the system just tries to stick to the load reference, taking energy wherever it can (grid or solar panels). Since no costs are taken into account in this case, wether it takes one or another is irrelevant for the system.

First, as the objective function tries to minimize costs while keeping the load reference, the system will use all the possible solar energy to not incur into losses and at the same time buy energy from the grid occasionally. As you can see in the above charts, the electricity price acts like a doublededged sword, as the β increases and you are willing to get energy from the grid (paying for it), it has a negative impact on our costs and of course an increase of this price will harm the system´s revenue, but something really different will happen with greater betas.

On the other hand, as you reach higher levels of β ($\beta \to 1$), the system no longer cares about the load and its only purpose is to reduce cost and even further get profit. As proven in the charts, for β above 0.4 the increase in the electricity price has positive impact in our system which means that now instead of using the solar power for the load requirements it´s been sold in the grid, that way the system gets better profit as the electricity prices increases.

Another key insight is how different import and export prices vary the objective, since imports only have a real effect when trying to stick to the load $\beta \in [0.1 - 0.3]$ and export prices have a greater impact when selling energy to the grid $\beta \in [0.5 - 1]$.

Finally, the Load reference always will have harm our objective as the value increases, obviously cause the more load the system has cover with the same amount of generation, will incur in buying energy from the grid. That will happen with $\beta \in [0.1 - 0.9]$, but for $\beta \in [0, 1]$ the load reference will not matter at all, because they either do not care about the costs or how much the reference is covered.

This further proves all the qualitative analysis brought up before, showing how import and export prices have a greater impact in the system as well as the electricity price, when cost savings are prior ($\beta \to 1$) and how the load reference is when ($\beta \to 0.4$).

4 Task 1c: Battery implementation

4.1 Adaptation of the Optimization Problem

The prosumer has now decided to add a battery to further increase his flexibility. He is still trying to fulfill a reference hourly load profile. Hence, the objective function is still the same, as question 1b). Except now we have 3 new variables introduced for the battery,

With:

- $soc_t \leq B_{cap}$: the State of Charge of battery at hour t
- $e_t^{ch} \leq CH^{max,ratio} \cdot B_{cap}$: Energy delivered to battery at the start of hour t
- $e_t^{disch} \leq DISCH^{max,ratio} \cdot B_{cap}$: Energy delivered from battery at the start of hour t

The B_{cap} is the capacity of our battery in kWh. The $max_{charge,ratio}$ is a constant which is used to calculate the the power of charging. Hence why when we calculate it per hour, it is equivalent to the energy transferred.

Furthermore, we add a new constraint for the SOC as,

$$soc_t = soc_{t-1} + \eta_c \cdot e_{t-1}^{ch} - \frac{e_{t-1}^{disch}}{\eta_d} \qquad \forall t > 0 \tag{52}$$

Where η_c/η_d are the charging and discharging efficiencies. Along with this, We are told the initial (SOC_{init}) and final state of charge of the battery should be 50 % of the capacity. Hence we added the constraints,

$$soc_0 = soc_{init} \cdot B_{cap} \tag{53}$$
$$soc_{24} = soc_0 \tag{54}$$

We could have also made it so that, both the final and initial state of charge could have been greater than equal to the 50 % boundary. If the inequality is set as =, it will always be binding whereas for the other case (≤), it will be binding if the dual variable is non-zero and non-binding if the dual variable is zero.

Along with this, the new energy balance equation is,

$$i_t - e_t + S_t + e_t^{disch} - e_t^{ch} = l_t \cdot L_{max}, \qquad \forall t \in \mathcal{T} \tag{55}$$

The Objective Function will be the same as in 1b), but slightly different. β was used to make a deep sensitivity analysis in the previous part of the report, but from now on, the deviation will be substituted by a fixed value that will account for the economic consequences of the deviation. This value will have the following measurement (DKK/kWh) and will be 1.7 times the electricity price at each time slot (It is assumed that the TSO gives a lot of importance to following the reference load).

$$Price_{penalty,t} = \Pi_t \cdot 1.7 \tag{56}$$

Previous Objective:

$$\min_{i_t, e_t, z_t^{imp}, z_t^{exp}, l_t, d_t} \sum_{t \in \mathcal{T}} [\beta \cdot \text{Cost}_t + (1 - \beta) \cdot d_t] \tag{57}$$

New Objective:

$$\min_{i_t, e_t, z_t^{imp}, z_t^{exp}, l_t, d_t, e_t^{ch}, e_t^{disch}, soc_t} \sum_{t \in \mathcal{T}} [\text{Cost}_t + Price_{penalty,t} \cdot d_t] \tag{58}$$

4.2 Dual Variables and Their Impact on the Objective Value

The Lagrangian is:

$$\mathcal{L} = \sum_{t \in \mathcal{T}} [\text{Cost}_t + Price_{penalty,t} \cdot d_t] + \sum_j \lambda_j g_j(x) \tag{59}$$

Main constraints and their duals:

- $\gamma_t \geq 0$: Dual for daily energy requirement $\sum_t i_t - e_t + S_t = \ell_t L_{\text{max}}$ (removed)
- $\gamma_t \geq 0$: Dual for daily energy requirement $\sum_t i_t - e_t + S_t + e_t^{\text{disch}} - e_t^{\text{ch}} = \ell_t L_{\text{max}}$ (added)

- $\psi_t \geq 0$: $soc_t = soc_{t-1} + \eta_c\, e^{\text{ch}}_{t-1} - \frac{e^{\text{disch}}_{t-1}}{\eta_d}$(added)
- $\psi^0 \geq 0$:Dual for positive discomfort $soc_0 = soc_{\text{init}} \cdot B_{\text{cap}}$ (added)
- $\psi^{24} \geq 0$:Dual for positive discomfort $soc_{24} = soc_0$ (added)

With all these things considered, the Lagrange function would be like this:

$$\begin{aligned}
\mathcal{L} = & \sum_t \left[\text{Cost}_t + Price_{penalty,t} \cdot d_t\right] \\
& + \sum_t \alpha_t^{\min}(\ell_t L_{\max} - L_{\min}) + \sum_t \alpha_t^{\max}(L_{\max} - \ell_t L_{\max}) \\
& + \sum_t \beta_t(C_{F,t} S_{\max} - S_t) \\
& + \sum_t \gamma_t\big(i_t - e_t + S_t + e_t^{\text{disch}} - e_t^{\text{ch}} - \ell_t L_{\max}\big) \\
& + \sum_t \delta_t^I(z_t^{\text{imp}} - i_t + I_{\max}) + \sum_t \delta_t^E(z_t^{\text{exp}} - e_t + E_{\max}) \\
& + \sum_t \rho_t^{\uparrow}(R_t^{F,L,\uparrow} - (\ell_t - \ell_{t-1})) + \sum_t \rho_t^{\downarrow}(R_t^{F,L,\downarrow} - (\ell_{t-1} - \ell_t)) \\
& + \sum_t \sigma_t^{\uparrow}(R_t^{PV,\uparrow} - (S_t - S_{t-1})) + \sum_t \sigma_t^{\downarrow}(R_t^{PV,\downarrow} - (S_{t-1} - S_t)) \\
& + \sum_t \varphi_t^{+}(d_t - (x_t^{FFL} - X_t^{\text{ref}})L_{\max}) + \sum_t \varphi_t^{-}(-d_t - (x_t^{FFL} - X_t^{\text{ref}})L_{\max}) \\
& + \sum_{t>0} \psi_t \left(soc_t - soc_{t-1} - \eta_c e^{\text{ch}}_{t-1} + \frac{e^{\text{disch}}_{t-1}}{\eta_d}\right) \\
& + \psi^0(soc_0 - soc_{\text{init}} B_{\text{cap}}) + \psi^{24}(soc_{24} - soc_0).
\end{aligned} \tag{60}$$

1. Dual Objective:

$$\begin{aligned}
\max = \sum_t \Big[& - \alpha_t^{\min} L_{\min} + \alpha_t^{\max} L_{\max} + \beta_t (C_{F,t} S_{\max}) \\
& + \delta_t^I I_{\max} + \delta_t^E E_{\max} \\
& + \rho_t^{\uparrow} R_t^{F,L,\uparrow} + \rho_t^{\downarrow} R_t^{F,L,\downarrow} \\
& + \sigma_t^{\uparrow} R_t^{PV,\uparrow} + \sigma_t^{\downarrow} R_t^{PV,\downarrow} \\
& - (\varphi_t^{+} + \varphi_t^{-})(x_t^{FFL} - X_t^{\text{ref}}) L_{\max} \Big] \\
& - \psi^0 \big(soc_{\text{init}} B_{\text{cap}}\big)
\end{aligned} \tag{61}$$

4.3 KKT Conditions for the Flexibility Optimization Problem

1. Primal Feasibility Conditions

For optimality, all primal variables must satisfy the feasibility requirements defined by the model constraints. These conditions ensure that the physical, operational, and behavioral limits of the system are respected at every time step $t \in \mathcal{T}$:

$$\begin{aligned}
&\ell_t L_{\max} \geq L_{\min}, && \ell_t L_{\max} \leq L_{\max}, \\
&S_t \leq C_{F,t} S_{\max}, && i_t - e_t + S_t + e_t^{\text{disch}} - e_t^{\text{ch}} = \ell_t L_{\max}, \\
&z_t^{\text{imp}} - i_t + I_{\max} \geq 0, && z_t^{\text{exp}} - e_t + E_{\max} \geq 0, \\
&\ell_t - \ell_{t-1} \leq R_t^{F,L,\uparrow}, && \ell_{t-1} - \ell_t \leq R_t^{F,L,\downarrow}, \\
&S_t - S_{t-1} \leq R_t^{PV,\uparrow}, && S_{t-1} - S_t \leq R_t^{PV,\downarrow}, \\
&d_t \geq (x_t^{FFL} - X_t^{\text{ref}}) L_{\max}, && d_t \leq -(x_t^{FFL} - X_t^{\text{ref}}) L_{\max}, \\
&soc_t = soc_{t-1} + \eta_c e_{t-1}^{\text{ch}} - \frac{e_{t-1}^{\text{disch}}}{\eta_d}, && \forall t > 0, \\
&soc_0 = soc_{\text{init}} B_{\text{cap}}, && soc_{24} = soc_0.
\end{aligned} \tag{62}$$

Additionally, all decision variables are subject to non-negativity and physical limits:

$$\ell_t,\ S_t,\ i_t,\ e_t,\ e_t^{\text{ch}},\ e_t^{\text{disch}},\ z_t^{\text{imp}},\ z_t^{\text{exp}},\ d_t \geq 0, \quad \forall t \in \mathcal{T}. \tag{63}$$

2. Dual feasibility:

$$\alpha_t^{\min}, \alpha_t^{\max}, \beta_t, \delta_t^I, \delta_t^E, \rho_t^\uparrow, \rho_t^\downarrow, \sigma_t^\uparrow, \sigma_t^\downarrow, \phi_t^+, \phi_t^- \psi_t \geq 0, \quad \gamma_t \text{ free.}$$

3. Stationary:

$$\text{(a) Load } \ell_t: \quad L_{\max}(\alpha_t^{\min} - \alpha_t^{\max} - \gamma_t) + (-\rho_t^\uparrow + \rho_{t+1}^\uparrow) + (\rho_t^\downarrow - \rho_{t+1}^\downarrow) = 0, \quad \forall t. \tag{64}$$

$$\text{(b) Storage level } S_t: \quad \gamma_t + (\sigma_t^\uparrow - \sigma_{t+1}^\uparrow) + (-\sigma_t^\downarrow + \sigma_{t+1}^\downarrow) = 0, \quad \forall t. \tag{65}$$

$$\text{(c) Import } i_t: \quad \partial_{i_t} \text{Cost}_t + \gamma_t - \delta_t^I = 0, \quad \forall t. \tag{66}$$

$$\text{(d) Export } e_t: \quad \partial_{e_t} \text{Cost}_t - \gamma_t - \delta_t^E = 0, \quad \forall t. \tag{67}$$

$$\text{(e) Charge } e_t^{\text{ch}}: \quad \partial_{e_t^{\text{ch}}} \text{Cost}_t - \gamma_t - \eta_c \psi_{t+1} = 0, \quad \forall t, \tag{68}$$

$$\text{(f) Discharge } e_t^{\text{disch}}: \quad \partial_{e_t^{\text{disch}}} \text{Cost}_t + \gamma_t + \frac{\psi_{t+1}}{\eta_d} = 0, \quad \forall t. \tag{69}$$

$$\text{(g) SOC } soc_t\ (t > 0): \quad \psi_t - \psi_{t+1} - \beta_t + \gamma_t = 0, \quad \forall t > 0. \tag{70}$$

$$\text{(h) Slack variables } z_t^{\text{imp}}, z_t^{\text{exp}}: \quad \partial_{z_t^{\text{imp}}} \text{Cost}_t + \delta_t^I = 0, \quad \partial_{z_t^{\text{exp}}} \text{Cost}_t + \delta_t^E = 0, \quad \forall t. \tag{71}$$

$$\text{(i) Discomfort } d_t: \quad Price_{penalty} + \varphi_t^+ - \varphi_t^- = 0, \quad \forall t. \tag{72}$$

Interpretation. The dual maximizes the sum of the constant Lagrangian contributions (limits, capacity factors, ramp allowances, penalties related to reference deviation, SOC initial condition, etc.) under the linear stationarity constraints above. (68)–(69) couple the battery charge/discharge marginality with SOC multipliers ψ_{t+1};

4. Complementary Slackness Conditions

Remains the same as part 1.b) $\forall t \in \mathcal{T}$ this is beacuse Equality constraints such as the energy balance:

$$i_t - e_t + S_t + e_t^{\text{disch}} - e_t^{\text{ch}} = \ell_t L_{\max}$$

and the SOC dynamics:

$$soc_t = soc_{t-1} + \eta_c e_{t-1}^{\text{ch}} - \frac{e_{t-1}^{\text{disch}}}{\eta_d}$$

do not produce complementary slackness relations, since their dual variables $(\gamma_t, \psi_t, \psi^0, \psi^{24})$ are free (unrestricted in sign).

If additional bound constraints exist (e.g. $S_t \geq 0$, $e_t^{\text{ch}} \geq 0$, $e_t^{\text{disch}} \geq 0$, $z_t^{\text{imp}} \geq 0$, etc.), then each would introduce an additional complementary slackness condition of the form

$$\lambda \,(\text{bound residual}) = 0, \quad \lambda \geq 0.$$

4.4 Qualitative Discussion of Structural and Optimality Impacts

Introducing storage dynamics and comfort-related constraints significantly changes both the structure of the optimization model and how its results should be interpreted economically. By explicitly modeling battery charging and discharging together with the state of charge (SOC) evolution, the problem becomes a dynamic and time-dependent system rather than a static allocation task. Each decision at time t influences future feasibility through the SOC trajectory, creating a direct trade-off between present and future energy use. The new energy balance couples imports, exports, flexible loads, and storage, producing a tighter and more realistic representation of how energy resources interact over time.

From an economic point of view, the dual variables reveal the marginal value of system resources and constraints. The multiplier γ_t, linked to the energy balance, represents the instantaneous value of electricity essentially acting as a time-dependent marginal price that measures the cost of supplying one additional unit of net energy. The SOC-related duals ψ_t describe the inter-temporal value of stored energy: when ψ_t is positive, holding energy in the battery is economically beneficial because it enables future savings or revenue when prices rise. Similarly, the comfort duals φ_t^+ and φ_t^- measure how much the system would be willing to pay to maintain user comfort, effectively assigning an economic meaning to behavioral preferences.

Removing the global daily energy constraint simplifies the overall structure and decentralizes decision-making over time. This results in locally interpretable shadow prices that make it easier to understand how each period contributes to the total cost. Complementary slackness conditions also indicate when operational limits become binding for example, when storage capacity, load flexibility, or comfort thresholds are fully used providing insight into where the system is constrained and where flexibility remains. Overall, the revised formulation strengthens both the operational and economic interpretation of the model: it captures the true temporal value of energy, clarifies the cost of comfort, and provides transparent signals for efficient and user-aware energy management.

4.5 Scenario Analysis and Insights

As it can be seen in the charts below, this is the load profile for a scenario where the penalty is 1.7 the electricity price. The load profile sticks perfectly to the reference. Moreover the first chart shows a pretty intuitive way of working of the battery installed. At first, when prices are low, the battery imports a lot of energy to be later used in hours 8 and 9 where the load reference is high (also the price) and the solar power is not enough to cover it. Something really similar happens later in the day. During the afternoon, the prices get really low and solar is on it´s peak, therefore all energy is stored and then used to cover the load for the rest of the day. The last thing the system does is import some energy to get back to the 50% state of charge set in the constraints.

Figure 5, just represents some scenarios where it is set up different penalties for the deviations, the most evident insights is how the system gives less importance to the load reference and gets more profit from doing arbitrage in the market.

Figure 6, shows how changing some parameters may affect the objective value. The most relevant points are the impact of changing the discomfort penalty and the storage capacity. The objective value gets to a limit when both are increased at a certain level, penalty is because at high prices the system never deviates from the reference, and for the storage capacity is because it reaches the import and export excess so it is no longer profitable to sell more energy in the market.

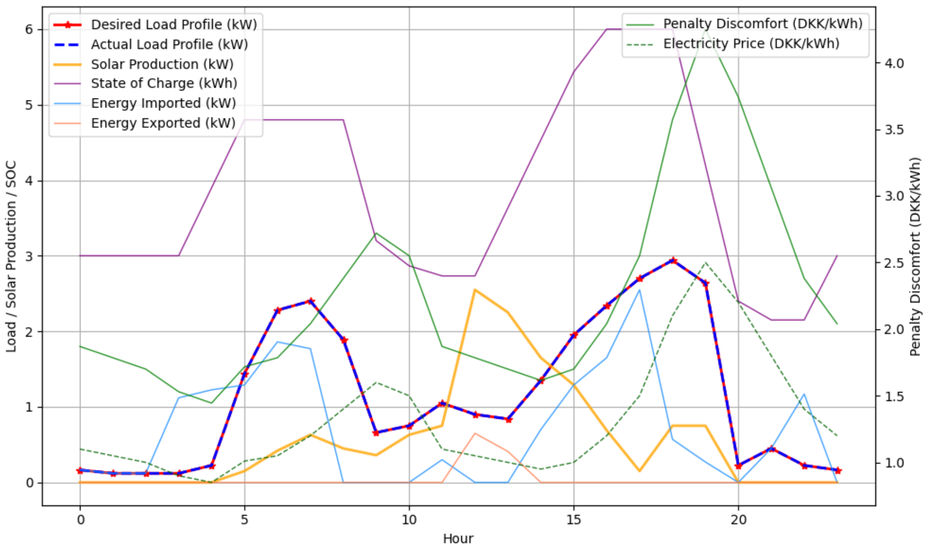


Figure 4: Load Profile with varying Discomfort

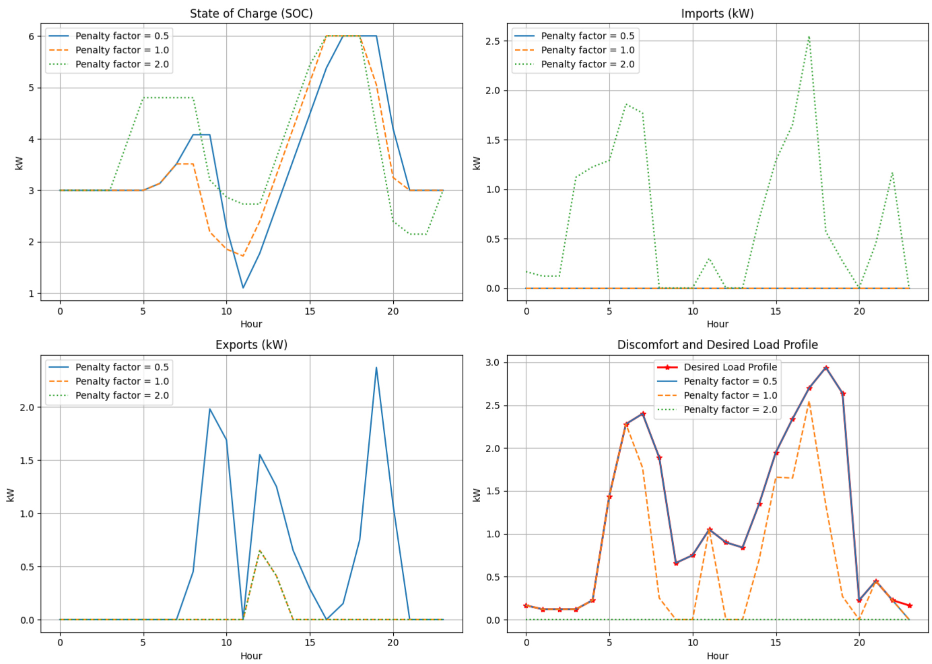


Figure 5: Load Profile with varying Discomfort

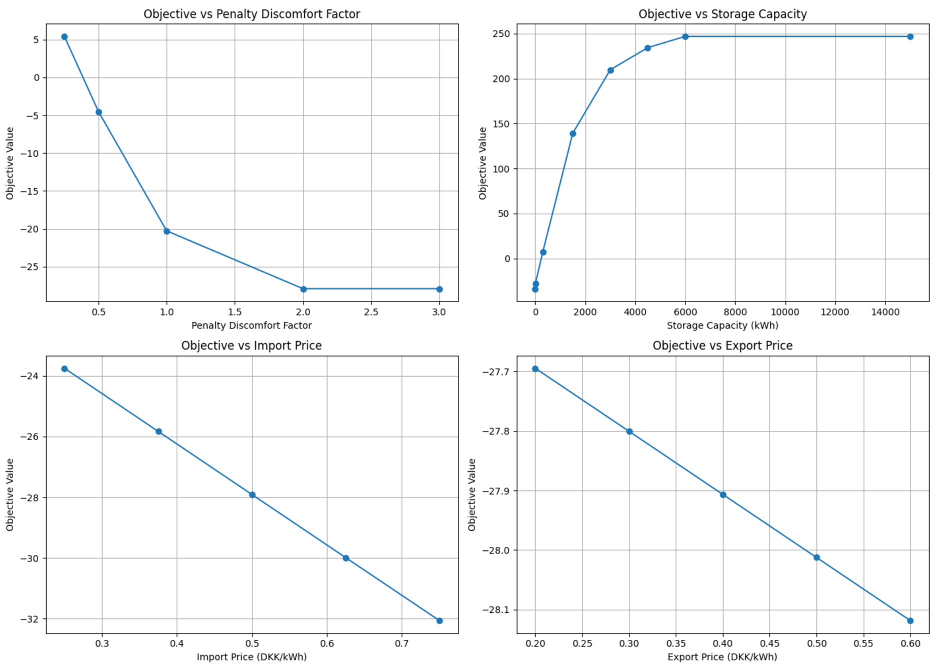


Figure 6: Load Profile with varying Discomfort

We also decided to plot the dual variables of the constraints every hour,

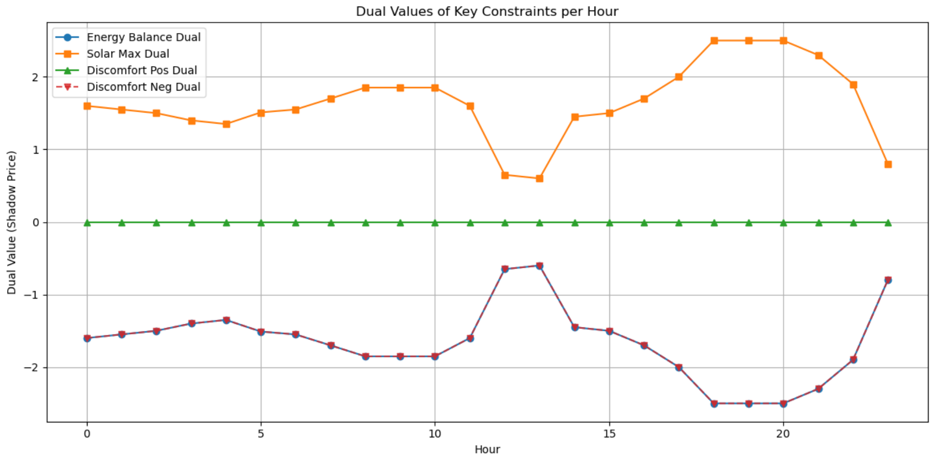


Figure 7: Dual Variables

Since the problem is setup as a maximization problem, positive dual means that relaxing the constraint increases the objective and a negative dual means relaxing the constraint decreases the objective. In hour case, at hour 18, the system is most sensitive to solar production constraint, negative discomfort constraint and energy balance constraint. For example, in this case relaxing the solar capacity by 1 kw, would increase our objective function by 2 units. Our positive discomfort is non-binding for all hours. This could also be interpreted as our actual load profile never exceeding the desired load profile which was true. This was done for when the weight of the discomfort is 1.

4 Value of Demand-Side Flexibility

(i) Formulation of the Optimization Problem

Given the following assumptions:

- Battery technical characteristics (charging/discharging power and energy capacity) scale linearly with a reference battery.
- Charging and discharging efficiencies are constant, independent of battery size.
- Capital cost scales linearly with the energy storage capacity.
- Battery life is 10 years, with no degradation.
- Input parameters such as load profiles, electricity prices, and PV production remain unchanged over time; future costs and revenues are not discounted.

Under these assumptions, the consumer's investment decision can be formulated as a Mixed-Integer Linear Program (MILP):

$$\max \quad \sum_{t\in T} \Big[-\left(P^{\text{imp}} + \Pi_t\right) i_t + \left(\Pi_t - P^{\text{exp}}\right) e_t \tag{73}$$

$$- \Lambda^{\text{imp}} z_t^{\text{imp}} - \Lambda^{\text{exp}} z_t^{\text{exp}} - \kappa_t^{\text{dis}} \delta_t \Big] \cdot 0.7 \tag{74}$$

$$- \frac{\phi_{\text{cap}}}{10 \times 365} \cdot B_{\text{factor}} \tag{75}$$

subject to:

- **Energy balance constraints:** ensure that imports, exports, PV generation, and battery charging/discharging satisfy the flexible load at every hour t.
- **Battery scaling:** technical limits for power and energy scale proportionally with the integer variable battery_factor.
- **State-of-charge (SOC) dynamics:**

$$SOC[t] = SOC[t-1] + \eta_c \, x_{\text{charge}}[t-1] - \frac{x_{\text{discharge}}[t-1]}{\eta_d},$$

with bounds $0.1 \cdot \text{battery\_factor} \leq SOC[t] \leq \text{battery\_factor}$.

- **Import/export limits:** grid capacity limits are respected; excess variables are penalized.
- **Integer constraint:** battery_factor $\in Z_+$ determines the number of installed battery units.
- **Profitability constraint:** ensures non-negative total profit over the assumed 10-year lifetime.

(ii) Impact of Assumptions and Possible Extensions

Impact of Simplifying Assumptions:

- *Linear scaling* allows a proportional increase in energy and power capacity, simplifying investment into an integer scaling problem.
- *Constant efficiency* maintains linearity in charging and discharging terms, preserving MILP tractability.
- *No degradation and no discounting* imply that future cash flows are treated equally, which may overstate profitability.
- *Static input parameters* neglect uncertainty in demand, PV output, or market prices, limiting realism.

Possible Model Extensions:

- Introducing nonlinear scaling or variable efficiency would create nonlinear (potentially nonconvex) constraints.
- Modeling *battery degradation* would require time-dependent capacity and efficiency parameters, leading to larger, more complex formulations.

- Incorporating *discount factors* or *stochastic forecasts* of prices and loads would yield multi-stage or robust optimization problems.
- These enhancements improve realism but at the cost of higher computational complexity and reduced tractability.

(iii) Implementation

A Python-based implementation using `Gurobi` was developed to solve the MILP. Key features include:

- Modular and transparent structure, allowing easy modification of cost parameters or system configurations.
- Explicit integer modeling of the investment decision variable battery_factor.
- Comprehensive constraint formulation, including SOC dynamics, power limits, and comfort penalties.
- Automated visualization of results: time-series of SOC, import/export flows, and load-solar interactions.

The implementation directly reproduces the mathematical formulation, ensuring consistency between theory and computation.

(iv) Numerical Experiment and Insights

Experiment Design:

To assess the value of demand-side flexibility through battery investment, several numerical experiments were performed, varying:

- Consumer flexibility (different load profiles).
- Battery capital cost (battery_cost_per_unit).
- Market conditions (import/export tariffs and electricity prices).

Each experiment was solved for discrete values of battery_factor, yielding metrics such as:

Total profit over lifetime, Optimal battery size, SOC pattern, Discomfort penalty, Import/Export balan

Scenarios:

- Baseline: no battery investment (battery_factor = 0).
- Reduced battery cost scenarios (e.g., 20%, 40%, 60% cost reductions).
- Increased consumer flexibility (wider comfort tolerance).

Results and Observations:

- *Profitability:* Battery investment becomes viable once marginal energy arbitrage benefits exceed the annualized cost per unit of capacity.

- *Flexibility effect:* Consumers with greater flexibility realize higher economic value from storage due to improved alignment with price signals.
- *Cost sensitivity:* A 20–30% decrease in capital cost can make investment profitable for typical consumers.
- *Operational behavior:* The SOC trajectory follows expected patterns—charging during PV surplus or low-price hours, discharging during price peaks.
- *Comfort trade-off:* High discomfort penalties constrain flexibility and reduce achievable profits.

Limitations:

- The model assumes perfect foresight and deterministic parameters.
- Battery degradation, maintenance, and replacement costs are omitted.
- Nonlinear effects such as round-trip efficiency losses or degradation-dependent capacity are not captured.

4.6 Model for Battery Sizing

Assuming a battery with a a C-rate of 1, has a capital cost of 415 USD/kWh as of 2025[1]. We can include this as an objective function to the sizing of our battery. So now B_{cap} becomes a free variable which will be optimized by the model.

Summary: The analysis demonstrates that *demand-side flexibility*, enabled through optimal battery sizing, can yield significant cost savings and improved system autonomy. However, the benefit is highly sensitive to both capital cost and consumer flexibility levels. Future work should integrate uncertainty, degradation, and dynamic pricing structures to reflect realistic investment decisions.

hence we write,

Parameters

- Prices and tariffs / economic parameters:
  - $p_t^{\text{mkt}} \in R$ — market energy price component at hour t (DKK/kWh),
  - $\tau^{\text{imp}}, \tau^{\text{exp}} \geq 0$ — grid import / export tariff components (DKK/kWh),
  - $\phi^{\text{cap}} \geq 0$ — capital cost of battery (DKK per kWh of installed capacity),
  - $\alpha_{\text{imp}}, \alpha_{\text{exp}} \geq 0$ — penalties for excess import/export (DKK/kWh),
  - $\kappa_{\text{dis}} \geq 0$ — weight/penalty for discomfort (DKK per unit of discomfort).
- Technical limits and profiles:
  - $P_t^{\text{pv}} \geq 0$ — available PV generation (capacity factor $\times$ panel size) at hour t (kW),
  - $L^{\max} \geq 0$ and $L^{\min} \geq 0$ — maximum and minimum flexible load (kW),
  - $G_{\text{imp}}^{\max}, G_{\text{exp}}^{\max} \geq 0$ — grid import/export capacity limits (kW),

- $P_{\text{pv}}^{\max} \geq 0$ — rated PV power (kW),
- η^{ch}, $\eta^{\text{dis}} \in (0, 1]$ — charging / discharging efficiencies,
- $r_{\max}^{\text{ch}}$, $r_{\max}^{\text{dis}} \geq 0$ — max charge/discharge power per unit capacity (kW per kWh) (or as ratio),
- $R_{\text{up}}^{\text{FFL}}$, $R_{\text{down}}^{\text{FFL}}$ — ramp-up / ramp-down rates for flexible load (per hour),
- $R_{\text{up}}^{\text{PV}}$, $R_{\text{down}}^{\text{PV}}$ — PV ramp limits (kW/h),
- $E_{\text{init}}^{\text{soc}}$, $E_{\text{final}}^{\text{soc}} \in [0, 1]$ — initial/final SOC fractions of installed capacity,
- $\mathcal{T}$ — set of hourly timesteps (index t).

Decision variables

- For each $t \in \mathcal{T}$:
  - $x_t^{\text{imp}} \in [0, G_{\text{imp}}^{\max}]$ — grid import (kW),
  - $x_t^{\text{exp}} \in [0, G_{\text{exp}}^{\max}]$ — grid export (kW),
  - $x_t^{\text{FFL}} \in [0, L^{\max}]$ — flexible load scheduled at hour t (kW),
  - $y_t^{\text{pv}} \in [0, P_{\text{pv}}^{\max}]$ — PV generation used at hour t (kW),
  - $c_t \geq 0$ — charging power (kW),
  - $d_t \geq 0$ — discharging power (kW),
  - $e_t \geq 0$ — state-of-charge (SOC) at the end of hour t (kWh),
  - $z_t^{\text{imp}} \geq 0$ — excess import above $G_{\text{imp}}^{\max}$ (kW),
  - $z_t^{\text{exp}} \geq 0$ — excess export above $G_{\text{exp}}^{\max}$ (kW),
  - $\delta_t \geq 0$ — discomfort slack (absolute deviation) at hour t (kW).
- Capacity decision:
  - $C^{\text{batt}} \geq 0$ — installed battery energy capacity (kWh).

Objective function

(Here written as *maximize profit*; you may convert sign to a minimization of net cost.)

$$\max_{(\cdot)} \sum_{t \in \mathcal{T}} \Bigg(\underbrace{\tau^{\text{exp}} x_t^{\text{exp}}}_{\text{export revenue}} - \underbrace{\left(\tau^{\text{imp}} + p_t^{\text{mkt}}\right) x_t^{\text{imp}}}_{\text{import cost}} - \underbrace{\alpha_{\text{imp}} z_t^{\text{imp}}}_{\text{import penalty}} - \underbrace{\alpha_{\text{exp}} z_t^{\text{exp}}}_{\text{export penalty}} - \underbrace{\kappa_{\text{dis}} \delta_t}_{\text{discomfort penalty}} \Bigg) - \underbrace{\phi^{\text{cap}} C^{\text{batt}}}_{\text{capital cost}} \tag{76}$$

Here we have made the capacity of the battery a free variable and the capital cost of the battery is fixed as the value stated before. In the sensitivity analysis we will vary this to see it impacts our sizing of the battery and if we even have a battery,

Constraints

$$\text{(Bounds on PV and flexible load)} \quad 0 \le y_t^{\mathrm{pv}} \le P_t^{\mathrm{pv}}, \quad 0 \le x_t^{\mathrm{FFL}} \le L^{\max}, \quad \forall t \in \mathcal{T}, \tag{77a}$$

$$\text{(Hourly power balance)} \quad x_t^{\mathrm{imp}} - x_t^{\mathrm{exp}} + y_t^{\mathrm{pv}} + d_t - c_t = x_t^{\mathrm{FFL}}, \quad \forall t \in \mathcal{T}, \tag{77b}$$

$$\text{(Grid limits with excess variables)} \quad z_t^{\mathrm{imp}} \ge x_t^{\mathrm{imp}} - G_{\mathrm{imp}}^{\max}, \quad z_t^{\mathrm{exp}} \ge x_t^{\mathrm{exp}} - G_{\mathrm{exp}}^{\max}, \quad \forall t \in \mathcal{T}, \tag{77c}$$

$$\text{(PV availability / ramping)} \quad y_t^{\mathrm{pv}} \le P_t^{\mathrm{pv}}, \quad y_t^{\mathrm{pv}} - y_{t-1}^{\mathrm{pv}} \le R_{\mathrm{up}}^{\mathrm{PV}}, \quad y_{t-1}^{\mathrm{pv}} - y_t^{\mathrm{pv}} \le R_{\mathrm{down}}^{\mathrm{PV}}, \tag{77d}$$

$$\text{(Flexible load ramping)} \quad x_t^{\mathrm{FFL}} - x_{t-1}^{\mathrm{FFL}} \le R_{\mathrm{up}}^{\mathrm{FFL}}, \quad x_{t-1}^{\mathrm{FFL}} - x_t^{\mathrm{FFL}} \le R_{\mathrm{down}}^{\mathrm{FFL}}, \tag{77e}$$

$$\text{(Battery charging/discharging limits)} \quad 0 \le c_t \le r_{\max}^{\mathrm{ch}} C^{\mathrm{batt}}, \quad 0 \le d_t \le r_{\max}^{\mathrm{dis}} C^{\mathrm{batt}}, \tag{77f}$$

$$\text{(SOC dynamics, hourly energy accounting)} \quad e_t = e_{t-1} + \eta^{\mathrm{ch}} c_{t-1} - \frac{d_{t-1}}{\eta^{\mathrm{dis}}}, \quad \forall t > t_0, \tag{77g}$$

$$\text{(Initial / final SOC)} \quad e_{t_0} = E_{\mathrm{init}}^{\mathrm{soc}} C^{\mathrm{batt}}, \quad e_{t_{\mathrm{end}}} = E_{\mathrm{final}}^{\mathrm{soc}} C^{\mathrm{batt}}, \tag{77h}$$

$$\text{(SOC bounds)} \quad 0 \le e_t \le C^{\mathrm{batt}}, \quad \forall t \in \mathcal{T}, \tag{77i}$$

$$\text{(Discomfort: absolute deviation)} \quad \delta_t \ge x_t^{\mathrm{FFL}} - \ell_t^{\mathrm{ref}}, \quad \delta_t \ge \ell_t^{\mathrm{ref}} - x_t^{\mathrm{FFL}}, \quad \forall t \in \mathcal{T}, \tag{77j}$$

$$\text{(Nonnegativity)} \quad z_t^{\mathrm{imp}}, z_t^{\mathrm{exp}}, \delta_t \ge 0, \quad \forall t \in \mathcal{T}. \tag{77k}$$

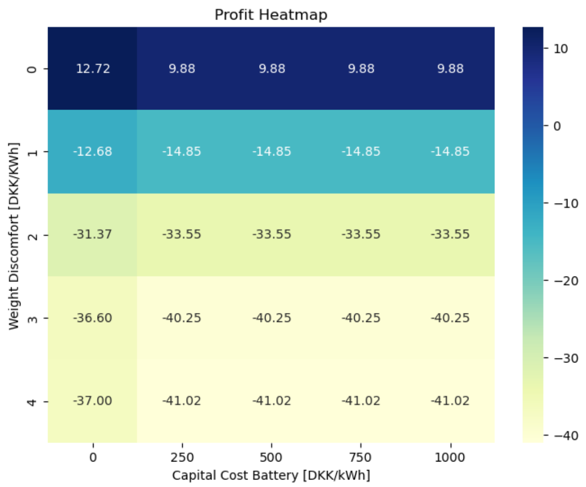
Equations 77f - 77i are referring to our batteries dynamics and based on our models decision of maximizing our profits, if the cost of the battery is too high it will use a smaller battery or none and along with that the discharging and charging powers will scale linearly accordingly.

Relaxing charging/discharging power limits or increasing ramp rates typically reduces the optimal battery energy capacity because the system can use power/faster controllability to cover short-term deviations rather than store energy. The effect on net imports/exports depends on price profiles and penalties — imports can fall if faster charging captures more local/cheap energy, but they can increase if the model opportunistically charges from the grid. Observed export spikes at the end of the horizon may be caused by the end-of-horizon SOC constraint interacting with price and PV timing; the exact effect depends on when prices and generation occur.

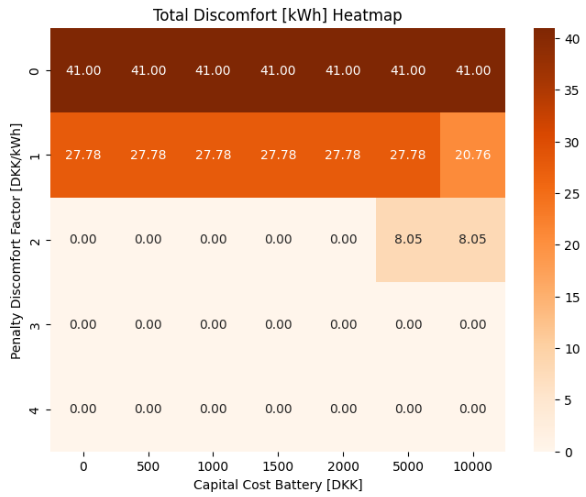
Another thing is that the solution may lead to the battery size being extremely large, if the capital cost of the battery is too cheap or the penalties become less than the actual spot price in the market, since it will start generating revenue by storing as much energy it can until the hour where the spot prices are relatively high.

Based on previous sensitivity analysis we know that the weight of the discomfort is the most sensitive parameter to change. Hence we decided to vary this weight along with the capital cost of the battery

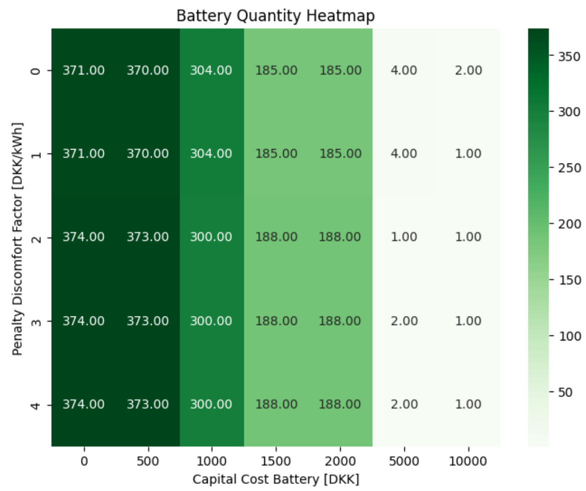
4.7 Scenarios



(a) Profit distribution across scenarios



(b) Discomfort level across scenarios



(c) Battery capacity utilization across scenarios

Figure 8: Comparison of key metrics (Profit, Discomfort, Battery Capacity) across different scenarios.

We observe that a battery is only installed or utilized when its costs are near zero or negligible. The consumer only makes profit if he is very flexible, i.e low discomfort weight. The weight of discomfort has more impact on the profits and discomfort than the actual cost of the battery.

Relaxing the state-of-charge constraints leads to slight improvements in profits, but the effect is limited. It also encourages the use of the battery even when its cost are moderately higher.

From a modeling perspective, we could introduce a cost associated with deviations from the initial and final state-of-charge and include it in the objective function. This would allow us to explicitly study how prioritizing battery conditions impacts the system's overall profitability and operation. Overall, the results suggest that profitability and consumer flexibility are inherently in tension in our system.

In conclusion if the consumer is extremely flexible and does not care about discomfort they will make money as long as the battery is not too expensive. Overall the cost of the battery has a less impact on our discomfort and profitability relative to the actual importance we give to that discomfort.

4.8 Interpretation of Dual Variables

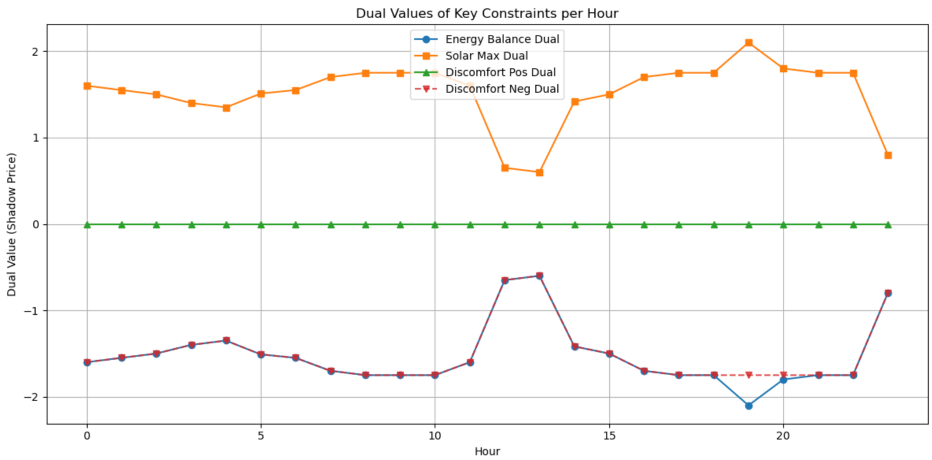


Figure 9: Dual Values of Constraints per hour

The dual value analysis in fig 9 provides insights into which constraints most strongly influence the profit-maximizing operation of the prosumer system. The energy balance constraint shows consistently negative dual values, indicating it is binding and directly limits profit, particularly during periods of high demand or low solar generation. The solar generation constraint exhibits positive dual values, suggesting that solar output frequently reaches its capacity limit; thus, expanding solar capacity would likely enhance profitability. In contrast, the positive flexibility (discomfort upper bound) constraint remains inactive, with dual values at zero, implying that additional upward flexibility offers no further economic benefit. The negative flexibility (discomfort lower bound) constraint, however, shows small negative values at certain hours, indicating that the ability to reduce flexible demand is sometimes restricted and affects the achievable profit. Overall, the results suggest that the system's profitability is primarily constrained by solar capacity and energy balance requirements, while the available flexibility is not fully utilized.

We could also improve the ramping rates for the battery to further improve our profitability.

A GitHub Repository

https://github.com/pulio12/Optimisation_PowerSystems_Assignment1

References

[1] I. E. A. (IEA), "Capital cost of utility-scale battery storage systems in the new policies scenario, 2017–2040." IEA Data Statistics, Chart, 2019. Last updated 7 February 2019; Licence: CC BY 4.0.