

Science Behind Sudoku

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 9 | | 1 | | | 3 | | |
| | 1 | | | 6 | | | 2 | 4 |
| 7 | | | 3 | 8 | | | | |
| | | | | | | 4 | | 6 |
| | 8 | 3 | | | | 1 | 9 | |
| 2 | | 7 | | | | | | |
| | | | | 9 | 3 | | | 5 |
| 6 | 7 | | | 2 | | | 8 | |
| | | 9 | | | 4 | | 6 | |

Palash Badjatya
and
Vipul Singh

Single Position Technique

- A cell must be assigned a digit if in its row/ column/ block, that is the only position available to the digit.

3 in C6; 4 in R2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 4 | | 5 | | 9 | 8 | | |
| | | | | 3 | | | 7 | |
| 3 | | 8 | | 1 | | | | 4 |
| | | | | | 7 | | | |
| 4 | | | 9 | 6 | 5 | | | 7 |
| | | | 3 | | | | | |
| 8 | | | | 9 | | 4 | | 3 |
| | 2 | | | 5 | | | | |
| | | 3 | 6 | | 4 | | 9 | 2 |

Single Candidate Technique

- If there is only one possible candidate for a cell, it must be assigned.

R1C5; then R6C5

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 4 | | | | | | 8 | |
| 2 | 1 | | | | | | | |
| 3 | 8 | | | 1 | | | 6 | 7 |
| 7 | 5 | 8 | 6 | 2 | 1 | 3 | 4 | 9 |
| 9 | 2 | 3 | 4 | 5 | 7 | 8 | 1 | 6 |
| 4 | 6 | 1 | | | | 7 | 5 | 2 |
| 1 | | 4 | 2 | 7 | 8 | 6 | | 5 |
| 5 | 7 | 6 | 1 | 3 | | | 2 | 8 |
| 8 | | 2 | | | | | 7 | |

Candidate Lines Technique

- If cells accepting a digit in a block are localized to a row (column), remove that digit from rest of the row (column)
- If cells accepting a digit in a row (column) are localized to a block, remove digit from rest of the block.

6 in B7 (localized to line)

| | | | | | | | | |
|-----------------------------|-----------------------------|-----------------------------|--|---|-----------------------------|---------------------------|--|-----------------------------|
| 4 | ⁵ ₈ | 9 | ¹ ₆ | 7 | ^{1 2} ₆ | 3 | ^{5 6} ₈ | ² _{8 6} |
| 3 | 1 | _{7 8} | 5 | 9 | ² ₆ | 4 | _{7 8} ⁶ | ² _{8 6} |
| 6 | ₇ ⁵ | 2 | 4 | 8 | 3 | 1 | ₇ ⁵ | 9 |
| 5 | 3 | 4 | 7 | 6 | 8 | 9 | 2 | 1 |
| 2 | 9 | ₈ ⁶ | 3 | 1 | 4 | 7 | ₈ ⁶ | 5 |
| 7 | ₈ ⁶ | 1 | 2 | 5 | 9 | ₈ ⁶ | 4 | 3 |
| ¹ _{8 9} | 4 | 5 | ¹ _{8 9} ⁶ | 3 | 7 | 2 | ¹ _{8 9} ⁶ | ₈ ⁶ |
| ¹ _{8 9} | _{7 8} ⁶ | _{7 8} ⁶ | ¹ _{8 9} ⁶ | 2 | 5 | ₈ ⁶ | 3 | 4 |
| ¹ _{8 9} | 2 | 3 | ¹ _{8 9} ⁶ | 4 | ¹ ₆ | 5 | ¹ _{8 9} ⁶ | 7 |

3 in C5 (localized to block)

| | | | | | | | | |
|-----------------------------|---|---|---------------------------|-----------------------------|---------------------------------|---------------------------------|---------------------------|-----------------------------|
| ^{8 9} | 6 | 3 | 5 | ² ₉ | 1 | ² _{8 9} | 4 | 7 |
| ⁴ ₉ | 7 | 5 | 8 | 6 | ^{2 3} ₄ | ^{2 3} ₉ | ^{1 3} | ^{1 2} ₉ |
| ⁴ _{8 9} | 1 | 2 | ⁴ ₉ | 7 | ³ ₄ | ³ _{6 8 9} | 5 | ⁶ ₉ |
| 5 | 4 | 8 | 2 | 1 | 9 | 7 | 6 | 3 |
| 1 | 2 | 6 | 3 | 4 | 7 | 5 | 9 | 8 |
| 3 | 9 | 7 | 6 | 8 | 5 | 1 | 2 | 4 |
| 2 | 3 | 9 | 7 | 5 | ⁶ ₈ | 4 | ¹ ₈ | ¹ ₆ |
| 6 | 8 | 4 | 1 | ^{2 3} ₉ | ^{2 3} | ^{2 3} ₉ | 7 | 5 |
| 7 | 5 | 1 | ⁴ ₉ | ^{2 3} ₉ | ^{2 3} _{4 6 8} | ^{2 3} _{6 8 9} | ³ ₈ | ² _{6 9} |

Multiple Lines Technique

- If two blocks at the same level have a digit confined to two lines, the third block must have the digit in the remaining line.

5 in B1 and B3

| | | | | | | | | |
|----------------------------|---------------------|---------------------|-----------------------|---------------------------------------|---|------------------------|---------------------|----------------------------|
| ² 5 6 | 1 | 4 | ^{5 6} 7 8 | ³ 5 6 ^{7 8} | ³ 5 6 ^{7 8} | 9 | ³ 5 8 | ^{2 3} 5 6 8 |
| ² 5 6 | 3 | ² 5 6 | 4 | 9 | 1 | ⁶ 8 | 7 | ² 5 6 8 |
| 7 | 8 | 9 | 2 | ³ 5 6 | ³ 5 6 | ³ 6 | 4 | 1 |
| ³ 5 9 | 4 | 1 | ^{5 6} 7 8 | ³ 5 6 ^{7 8} | ³ 5 6 ^{7 8 9} | 2 | ³ 5 8 | ³ 5 8 |
| 8 | 6 | ^{2 3} 5 | ¹ 5 | ^{2 3} 5 | ³ 5 | 4 | 9 | 7 |
| ^{2 3} 5 9 | ² 5 9 | 7 | ⁵ 8 | ^{2 3} 4 5 8 | ³ 4 5 8 9 | 1 | 6 | ³ 5 8 |
| 1 | 7 | ³ 5 6 | 9 | ^{5 6} 8 | 2 | ³ 6 8 | ³ 8 | 4 |
| ^{4 5 6} 9 | ⁵ 9 | ^{5 6} | 3 | 1 | ^{4 5 6} 8 | 7 | 2 | ⁶ 8 9 |
| ^{2 3} 4 6 9 | ² 9 | 8 | ⁶ 7 | ^{4 6} 7 | ^{4 6} 7 | 5 | 1 | ³ 6 9 |

Naked Pair/Tuples Technique

- If for a group of k inter-dependent cells, the set of all candidates is of size k , remove those k digits from everywhere else in the line or block.
- Usually look for $k=2,3$.

{1,4} in C5 as well as B5

| | | | | | | | | |
|---------------------------|---------------------------|-----|-------------------------------|-----------------------------|---|-------------------------------|-------------------------------|-------------------------------|
| 9 | 7 | 3 | ⁴ ₈ | 5 | 1 | 2 | ⁴ ₈ | 6 |
| 2 | 5 | 4 | ⁶ ₈ | ³ ₆ | 9 | ^{1 3} ₈ | 7 | ^{1 3} |
| 1 | 8 | 6 | ² ₄ | ^{2 3} ₄ | 7 | 5 | ³ ₄ | 9 |
| ³ ₄ | ¹ ₉ | 7 8 | ^{4 5} _{7 9} | ¹ ₄ | 6 | ^{1 3} _{4 8} | ³ _{4 5 8} | 2 |
| 6 | 2 | 5 | ⁴ ₇ | 8 | 3 | ¹ ₄ | 9 | ¹ ₇ |
| ³ ₄ | ¹ ₉ | 7 8 | ^{4 5} _{7 9} | ¹ ₄ | 2 | ^{1 3} _{4 8} | 6 | ^{1 3} _{7 5} |
| 5 | 6 | 1 | 3 | 7 | 4 | 9 | 2 | 8 |
| 7 | 4 | 2 | 1 | 9 | 8 | 6 | ³ ₅ | ³ ₅ |
| 8 | 3 | 9 | ² ₆ | ² ₆ | 5 | 7 | 1 | 4 |

{2,7,9} in R2

| | | | | | | | | |
|---------------------------|-------------------------------|-----------------------------|-----------------------------|------------------|-----------------------------|-------------------------------|---------------------------|-----------------------------|
| 1 | ² ₅ | 6 | ² ₅ | 8 | 9 | 7 | 3 | 4 |
| 8 | 4 | ² _{7 9} | 3 | ^{5 6} | ² ₇ | ^{1 2} _{5 9} | ² ₉ | ¹ _{6 9} |
| ⁵ ₉ | 3 | ² _{7 9} | 1 | ^{4 5 6} | ² _{4 7} | ² _{5 9} | 8 | ⁶ ₉ |
| 3 | 8 | 5 | 4 | 9 | 1 | 6 | 7 | 2 |
| 7 | 6 | 4 | 8 | 2 | 3 | ¹ ₉ | 5 | ¹ ₉ |
| 2 | ¹ ₉ | ¹ ₉ | 6 | 7 | 5 | 3 | 4 | 8 |
| 6 | 7 | 8 | ² _{5 9} | ^{4 5} | ² ₄ | ² ₉ | 1 | 3 |
| ⁵ ₉ | ^{1 2} _{5 9} | ^{1 2} ₉ | ² _{5 9} | 3 | 8 | 4 | 6 | 7 |
| 4 | ² ₉ | 3 | 7 | 1 | 6 | 8 | ² ₉ | 5 |

Find a naked tuple in B9

| | | | | | | | | |
|---|-----------------------------|-----------------------------|-----------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|-------------------------------|
| 8 | 9 | 1 | 5 | 7 | 6 | 2 | 3 | 4 |
| 6 | ³ ₅ | ³ _{4 5} | 1 | ^{2 3} | ² ₄ | 9 | 7 | 8 |
| 2 | ³ ₇ | ³ _{4 7} | 9 | ³ ₈ | ⁴ ₈ | 5 | 1 | 6 |
| 7 | 8 | 6 | ^{2 3} ₄ | ^{2 3} ₅ | ² _{4 5} | ³ ₄ | 9 | 1 |
| 5 | 1 | ² ₉ | ^{2 3} ₄ | 6 | ² _{4 8 9} | ³ _{4 7 8} | ^{2 3} _{4 8 7} | |
| 3 | 4 | ² ₉ | 7 | ² _{5 8 9} | 1 | 6 | ⁵ ₈ | ² ₅ |
| 9 | ^{2 3} ₅ | 8 | ² ₆ | ^{1 2} ₅ | 7 | ^{1 3} ₄ | ^{4 5 6} | ³ ₅ |
| 4 | ² _{7 5} | ² _{7 5} | ² ₆ | ^{1 2} _{5 9} | 3 | ¹ _{7 8} | ^{5 6} ₈ | ⁵ _{7 9} |
| 1 | 6 | ³ _{7 5} | 8 | 4 | ⁵ ₉ | ³ ₇ | 2 | ³ _{7 5 9} |

Hidden Subsets Technique

- If for a group of k digits, the set of all locations in a block or line is of size k , remove all other candidates in those k locations.
- Duality with naked tuple technique.

C7 – {7,9} form hidden subset while
 {4,6,8} form naked tuple

| | | | | | | | | |
|---|---|---------------------------|---|-----------------------------|---------------------------|-----------------------------|---|---------------------------|
| 8 | ³ ₆ | ³ ₆ | 9 | 4 | 5 | 1 | 7 | 2 |
| 9 | 4 | 2 | 1 | 7 | 6 | 3 | ⁵ ₈ | ⁵ ₈ |
| 1 | 5 | 7 | 2 | 8 | 3 | ⁴ ₆ | ⁴ ₆ | 9 |
| 5 | 9 | 4 | 7 | ^{1 2} ₆ | ^{1 2} | ⁶ ₈ | ³ ₆ ₈ | ³ ₈ |
| ⁶ ₇ | 1 | 8 | 3 | ⁶ ₉ | 4 | 2 | ^{5 6} ₉ | ⁵ ₇ |
| ³ ₇ ⁶ | 2 | ³ ₆ | 5 | ⁶ ₉ | 8 | ⁶ _{7 9} | 1 | 4 |
| ³ _{4 7} ⁶ | ³ ₇ ⁶ | 1 | 8 | ^{2 3} ₉ | ² ₉ | 5 | ³ _{4 9} | ³ ₇ |
| ³ ₄ | ³ ₈ | 9 | 6 | 5 | 7 | ⁴ ₈ | 2 | 1 |
| 2 | ³ _{7 8} | 5 | 4 | ^{1 3} ₉ | ¹ ₉ | ^{7 8 9} | ³ _{8 9} | 6 |

R5 – find a hidden subset

| | | | | | | | | |
|--------------------|---------------|----------------------|---------------------------|----------------------------|----------------------|--------------------|----------------------|---------------------|
| <div>4 7</div> | 2 | 8 | <div>76 16</div> | <div>14 46 7</div> | 3 | 9 | 5 | |
| 3 | <div>45</div> | 6 | <div>2 9</div> | <div>5 9</div> | <div>42 45</div> | 1 | 7 | 8 |
| <div>79</div> | 1 | <div>5 9</div> | 8 | 3 | <div>75</div> | 6 | 4 | 2 |
| 1 | 3 | 7 | 5 | 2 | 9 | 8 | 6 | 4 |
| <div>4 9</div> | <div>45</div> | <div>25 59</div> | <div>36 76</div> | 8 | <div>36 76</div> | <div>2 9</div> | 1 | <div>3 79</div> |
| 6 | 8 | <div>2 9</div> | 1 | 4 | <div>3 7</div> | 5 | <div>23 79</div> | <div>3 9</div> |
| 5 | 6 | 1 | <div>23 9</div> | 7 | 8 | 4 | <div>23 9</div> | <div>3 9</div> |
| 2 | 9 | 3 | 4 | <div>15</div> | <div>15</div> | 7 | 8 | 6 |
| 8 | 7 | 4 | <div>23 6 9</div> | <div>6 9</div> | <div>23 6</div> | <div>2 9</div> | 5 | 1 |

XWing Technique

- 1 digit : 2 cells/column for 2 columns
- If these 4 cells form vertices of a rectangle, remove the digit from other cells in those rows.

7 in C5 and C9

| | | | | | | | | |
|-------------------|----------------|---------------------|--------------------------|----------------|---|--------------------------|------------------------|--------------------------|
| 4 | 3 | ¹ 7 8 | ¹ 7 | 6 | 9 | 5 | ¹ 8 | 2 |
| 9 | _{7 8} | 6 | ¹ 4 5 7 | ₇ 5 | 2 | 3 | ¹ 4 8 | ¹ 4 7 |
| ¹ 7 | 5 | 2 | ¹ 4 7 | 8 | 3 | ¹ 4 7 | 9 | 6 |
| 2 | 6 | ₇ 4 5 9 | 3 | ₇ 5 | 8 | ¹ 4 7 9 | ¹ 4 | ¹ 4 7 9 |
| 3 | 1 | ₇ 5 9 | ₇ 5 | 4 | 6 | ₇ 9 | 2 | 8 |
| 8 | ₇ 4 | ₇ 4 | 9 | 2 | 1 | 6 | 3 | 5 |
| ¹ 7 | 2 | ¹ 3 7 | 6 | 9 | 4 | 8 | 5 | ¹ 3 |
| 6 | 9 | ¹ 4 | 8 | 3 | 5 | 2 | 7 | ¹ 4 |
| 5 | ₈ 4 | ₈ 4 3 | 2 | 1 | 7 | ₉ 4 | 6 | ₉ 4 3 |

Figure 4.1. Rule 4.1 at work. In gray, candidates that have been eliminated by other techniques

Swordfish Technique

- Generalize XWing to k lines.
- Digit occurs in 2 cells/line for k lines.
- Edge-parallel path between the $2k$ cells.

Digit 1 in rows 4,5 and 9

| | | | | | | | | |
|----------------|----------------|-----------------------|-------------------|-----------------------------------|-----------------------------------|---------------------|-------------------|--------------------------|
| 6 | 4 | 5 | 7 | ^{1 2} | 9 | ^{1 2 3} | 8 | ^{1 2 3} |
| 8 | ^{1 3} | ^{1 2 3} 7 | 4 | ^{1 2} | 5 | 6 | ¹ 7 | 9 |
| ^{1 2} | 9 | ^{1 2} 7 | 3 | 6 | 8 | ^{1 2} 4 | 5 | ^{1 2} 4 7 |
| 3 | 7 | 9 | ¹ 5 | 8 | 2 | ¹ 4 | 6 | ^{1 2} 4 5 |
| 4 | 8 | ^{1 2} | ¹ 5 | 7 | 6 | 9 | 3 | ^{1 2} 5 |
| ^{1 2} | 5 | 6 | 9 | ¹ 4 ³ | ¹ 4 ³ | 8 | ¹ 7 | ^{1 2} 4 7 |
| 7 | 2 | 8 | 6 | ¹ 4 ³ | ¹ 4 ³ | 5 | 9 | ^{1 3} |
| 9 | ^{1 3} | 4 | 8 | 5 | ^{1 3} | 7 | 2 | 6 |
| 5 | 6 | ^{1 3} | 2 | 9 | 7 | ^{1 3} | 4 | 8 |

References

- Examples from “A to Z of Sudoku” by Narendra Jussien
- “Logical World of Puzzles”, blog by ex-National Champion Rohan Rao (~World Rank 15)