Homework 6
CS498df
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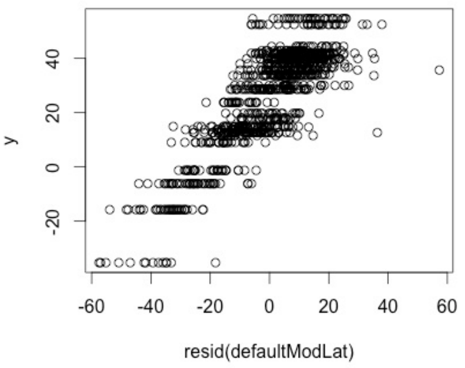
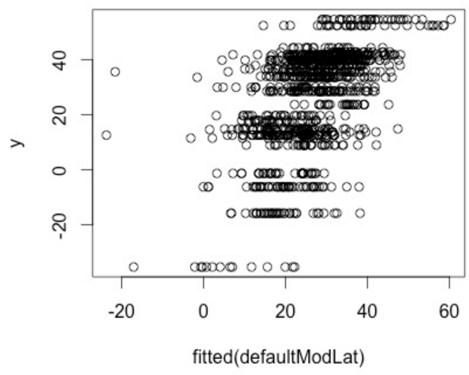
1.1 R^2 of latitude and longitude linear regressions against features:

latitude: R^2 = 0.2412
longitude: R^2 = 0.3182

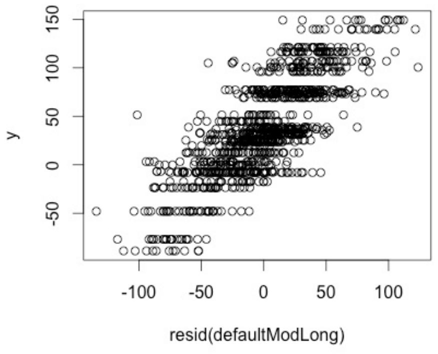
a. Residuals by Actual Latitude



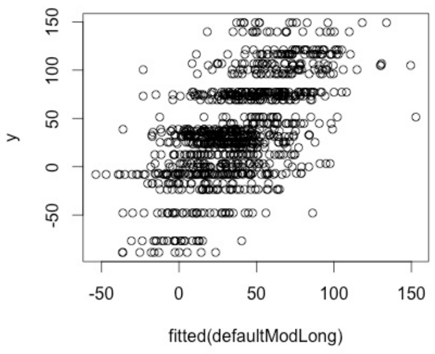
b. Predicted by Actual Latitude



c. Residuals by Actual Longitude



d. Predicted by Actual Longitude



1.2 We performed a boxcox transformation and found that applying it did not substantially improve performance of the model, either for longitude or latitude. The R^2 values for the models did not improve much; in fact for longitude it went down slightly:

boxcox transformed latitude: R^2 = 0.2546 (previously, 0.2412)
boxcox transformed longitude: R^2 = 0.3159 (previously, 0.3182)

1.3 For latitude and longitude, we tried 10 different values of alpha, ranging from 0 to 1 in increments of .1, where alpha = 0 indicates ridge regression and alpha=1 indicates lasso regression (and alpha values >0 but <1 indicate an 'elastic net' regression was applied). We also obtained cross validated MSE for the unregularized regression for comparison. All cross validation used 10 folds. All regularized regressions improved a lot on the unregularized model that contained all variables as predictors, approximately halving the MSE, and they were quite close to each other in terms of performance. lambda.min below indicates the lambda value (i.e. the regularization coefficient) that showed the best performance, ncoeffs indicates the number of coefficients that were kept in each model.

unregularized:
latitude – cross-validated MSE: 550
longitude – cross-validated MSE: 3934
regularized:
alpha: 0 (ridge regression)
latitude -- lambda.min: 7.6381632495138
longitude -- lambda.min: 3.73325959734027
latitude -- ncoeffs: 116
longitude -- ncoeffs: 116
latitude -- cross-validated MSE: 281.91133440168
longitude -- cross-validated MSE: 1890.58057894255
alpha: 0.1
latitude -- lambda.min: 3.80153184810457
longitude -- lambda.min: 2.69571545684487
latitude -- ncoeffs: 38
longitude -- ncoeffs: 85
latitude -- cross-validated MSE: 276.364087251967
longitude -- cross-validated MSE: 1863.1853181416
alpha: 0.2
latitude -- lambda.min: 2.28947928472423
longitude -- lambda.min: 1.01960494231266
latitude -- ncoeffs: 29
longitude -- ncoeffs: 81
latitude -- cross-validated MSE: 279.845357989028
longitude -- cross-validated MSE: 1873.82631311344
alpha: 0.3
latitude -- lambda.min: 1.52631952314949
longitude -- lambda.min: 0.898571818948289
latitude -- ncoeffs: 22
longitude -- ncoeffs: 78
latitude -- cross-validated MSE: 279.024481961714
longitude -- cross-validated MSE: 1894.33365633273
alpha: 0.4

```
latitude -- lambda.min: 1.14473964236212
longitude -- lambda.min: 0.55950758273938
latitude -- ncoeffs: 22
longitude -- ncoeffs: 70
latitude -- cross-validated MSE: 280.267024735496
longitude -- cross-validated MSE: 1868.31606921484
```

alpha: 0.5

```
latitude -- lambda.min: 1.00508027544271
longitude -- lambda.min: 1.03402806744638
latitude -- ncoeffs: 22
longitude -- ncoeffs: 70
latitude -- cross-validated MSE: 277.16046491693
longitude -- cross-validated MSE: 1871.33352379455
```

alpha: 0.6

```
latitude -- lambda.min: 1.10721414468274
longitude -- lambda.min: 0.493090731001299
latitude -- ncoeffs: 19
longitude -- ncoeffs: 70
latitude -- cross-validated MSE: 288.050325580771
longitude -- cross-validated MSE: 1877.18456749995
```

alpha: 0.7

```
latitude -- lambda.min: 0.717914482459077
longitude -- lambda.min: 0.385102208120696
latitude -- ncoeffs: 21
longitude -- ncoeffs: 69
latitude -- cross-validated MSE: 279.277663109394
longitude -- cross-validated MSE: 1873.07213224524
```

alpha: 0.8

```
latitude -- lambda.min: 0.572369821181058
longitude -- lambda.min: 0.336964432105608
latitude -- ncoeffs: 21
longitude -- ncoeffs: 67
latitude -- cross-validated MSE: 275.150264102725
longitude -- cross-validated MSE: 1882.24272958121
```

alpha: 0.9

```
latitude -- lambda.min: 0.558377930801505
longitude -- lambda.min: 0.29952393964943
latitude -- ncoeffs: 21
longitude -- ncoeffs: 70
latitude -- cross-validated MSE: 279.397689255809
longitude -- cross-validated MSE: 1911.47712791714
```

alpha: 1 (lasso)

```
latitude -- lambda.min: 0.502540137721354
longitude -- lambda.min: 0.245623552536189
latitude -- ncoeffs: 21
longitude -- ncoeffs: 39
```

latitude -- cross-validated MSE: 280.020558327408
longitude -- cross-validated MSE: 1882.16665665947

2. We used different regularization schemes and found that an elastic net with alpha = .3 worked the best. Our optimal model achieved ~81% accuracy using 10-fold cross validation and on an 80/20 train-test split.

10-fold cross validated models used to select optimal alpha:

alpha: 0

lambda.min: 0.0147950762551908
ncoeffs: 30
cross-validated MSE: 0.193666666666667

alpha: 0.1

lambda.min: 0.000951038319441683
ncoeffs: 30
cross-validated MSE: 0.1891

alpha: 0.2

lambda.min: 0.000757160975927511
ncoeffs: 29
cross-validated MSE: 0.189233333333333

alpha: 0.3

lambda.min: 0.000968111203646219
ncoeffs: 27
cross-validated MSE: 0.188966666666667

alpha: 0.4

lambda.min: 0.000726083402734664
ncoeffs: 27
cross-validated MSE: 0.189133333333333

alpha: 0.5

lambda.min: 0.00101508109364243
ncoeffs: 26
cross-validated MSE: 0.189533333333333

alpha: 0.6

lambda.min: 0.000639893018195456
ncoeffs: 28
cross-validated MSE: 0.189266666666667

alpha: 0.7

lambda.min: 0.000601955826443263
ncoeffs: 27
cross-validated MSE: 0.189233333333333

alpha: 0.8

lambda.min: 0.000526711348137854
ncoeffs: 27
cross-validated MSE: 0.189233333333333

alpha: 0.9

lambda.min: 0.000513835603868887
ncoeffs: 27
cross-validated MSE: 0.189233333333333
alpha: 1
lambda.min: 0.000383935810917274
ncoeffs: 27
cross-validated MSE: 0.189333333333333

Accuracy on model retrained with 80/20 train-test split at alpha = .3: 0.809.

Here are the estimated betas for the optimal model, indicating which variables were excluded. mar1, mar2 and mar3 are indicator variables for the original MARRIAGE variable, edu1-edu6 are indicator variables for the original EDUCATION variable, and SEX has been converted to an indicator variable as well:

```
 (Intercept) -1.19e+00
LIMIT_BAL   -7.36e-07
SEX        -1.27e-01
AGE         2.71e-03
PAY_0       5.83e-01
PAY_2       7.53e-02
PAY_3       5.63e-02
PAY_4       2.77e-02
PAY_5       5.50e-02
PAY_6       .
BILL_AMT1   -2.03e-06
BILL_AMT2   .
BILL_AMT3    1.12e-08
BILL_AMT4   .
BILL_AMT5    8.67e-07
BILL_AMT6    6.79e-09
PAY_AMT1    -1.08e-05
PAY_AMT2    -7.78e-06
PAY_AMT3    -2.57e-06
PAY_AMT4    -3.94e-06
PAY_AMT5    -2.77e-06
PAY_AMT6    -3.42e-06
edu1        7.61e-02
edu2        .
edu3        .
edu4       -9.49e-01
edu5       -9.67e-01
edu6       -2.54e-01
mar1        2.27e-01
mar2        .
mar3        1.53e-01
```