

To keep everything consistent, the EM algorithm was run for a fixed 30 iterations on the polarlights image using 6 different initial conditions for “ $\pi$ ” and “ $\mu$ ”. For each of the 6 runs, the initial values for  $\pi$  were randomized and then unitized so they summed to 1. The same process was used for each topic’s  $\mu$  vector. It can be easier to detect origins of variation by looking at the progression of iterations. These images by iteration are available individually in the submission directory under “fig/images\_by\_iteration/20\_segments/polarlights\_randStart/#” or as animated .gif files under “fig/images\_by\_iteration/animated\_gifs”. Although each run converges on very similar segmentations, there are slight differences. Depending on the random initial values for  $\pi$  and  $\mu$ , the first assignments of pixels to segments is different for each of the 6 trials. Certain colors appear in the early iterations and then morph more towards the original image by later iterations. Obviously the center shifts to shades of green, the right blues, and the left purples. These generalities hold across runs despite the difference in initial conditions, as they are the “ground truth” assignments looking at the original image. The variance in the final images reflects the fact that the EM algorithm converges to find parameters that will obtain local maxima and likelihood of the data given the model’s parameters. (LOCAL maxima not GLOBAL maxima). Intuitively, the process will find different maximizations given different initial conditions. If one cluster is assigned a high  $\pi$  value originally, it will be relatively easier for this cluster to achieve assignments... while for a cluster that was originally assigned a low  $\pi$  value, it will struggle to achieve assignments (relatively).

