Linear Algebra – Revision

Question 1. Vectors

(a) Multiply the following vector by the scalar. What is the geometric interpretation of this operation?

- $2 \cdot \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}$

(b) What is a column vector and row vector? How to transform one into the other?

(c) Calculate the length of the following vectors and normalize them.

- $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$
- $\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$
- $\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$

(d) Calculate the distance between A and B: (i) Euclidean distance; (ii) Manhattan distance:

- $A(1, 2, 3), B(3, 4, 5)$
- $A(2, 2, 3, 3, 6), B(8, 9, 1, 2, 8)$

(e) For the following pairs of vectors (i) add them; (ii) calculate their scalar product. What is the geometric interpretation of scalar multiplication? (Hint: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$). What is the scalar product of perpendicular vectors?

- $\begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$
- $\begin{pmatrix} 3 \\ -1 \\ 1 \\ -5 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -5 \\ -2 \end{pmatrix}$
- $\begin{pmatrix} -1 \\ 5 \\ 2 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \\ -2 \end{pmatrix}$

Question 2. Matrices

(a) Transpose the following matrices:

- $\begin{pmatrix} 1 & -1 & 6 \\ 5 & 0 & 2 \end{pmatrix}^{\mathrm{T}}$
- $\begin{pmatrix} -2 & 7 & 6 & 3 \\ 1 & 5 & 2 & -3 \\ 4 & -1 & 0 & 2 \end{pmatrix}^{\mathrm{T}}$

(b) Multiply the following pairs of matrices. What are the necessary conditions for matrix multiplication?

- $\begin{pmatrix} 3 & 2 \\ 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$
- $\begin{pmatrix} 6 & 4 & 1 \\ 0 & 3 & 5 \end{pmatrix} \begin{pmatrix} 6 & -4 \\ 0 & 2 \\ 3 & -3 \end{pmatrix}$
- $\begin{pmatrix} -2 & 2 \\ 1 & 0 \\ 0 & 4 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 4 & 0 \\ 4 & 1 & 4 \end{pmatrix}$
- $\begin{pmatrix} 1 & -1 & 6 \\ 5 & 0 & 2 \end{pmatrix} \begin{pmatrix} -2 & 7 & 6 & 3 \\ 1 & 5 & 2 & -3 \\ 4 & -1 & 0 & 2 \end{pmatrix}$

Question 3. Planes and hyperplanes

(a) What are the equations of planes or hyperplanes with the following normal vectors?

- $\begin{pmatrix} -9 \\ 1 \\ -5 \end{pmatrix}$
- $\begin{pmatrix} -2 \\ 5 \\ -8 \\ 4 \\ 9 \end{pmatrix}$
- $\begin{pmatrix} -4 \\ 9 \\ 0 \\ 1 \end{pmatrix}$

(b) What are the normal vectors of planes or hyperplanes with the following equations?

- $8x_1 + 5x_2 - x_3 - 3 = 0$
- $x_1 + 9x_2 - 7x_3 + 3x_4 + 7x_5 - 4x_6 + 8 = 0$
- $-7x_1 - x_2 + 7x_3 - 9x_4 + 8x_6 - 3 = 0$