Assignment 2: What is the "current" situation?

2018 Fall EECS205002 Linear Algebra

Due: 2018/11/14

A simple electric circuit is a closed connection of batteries, resistors, wires. An electric circuit consists of voltage loops and current nodes. We usually know the voltage and the resistors in the network, and want to know the currents in each part of the network. Fortunately, with three simple laws below, we can find the currents.

- Ohm's Law: $V = IR$, voltage (V) equals to the current (I) times the resistor (R).

- Kirchhoff's Current Law (KCL): For any point in a network, the flow in equals the flow out.

- Kirchhoff's Voltage Law (KVL): Around any circuit the total drop equals the total rise.
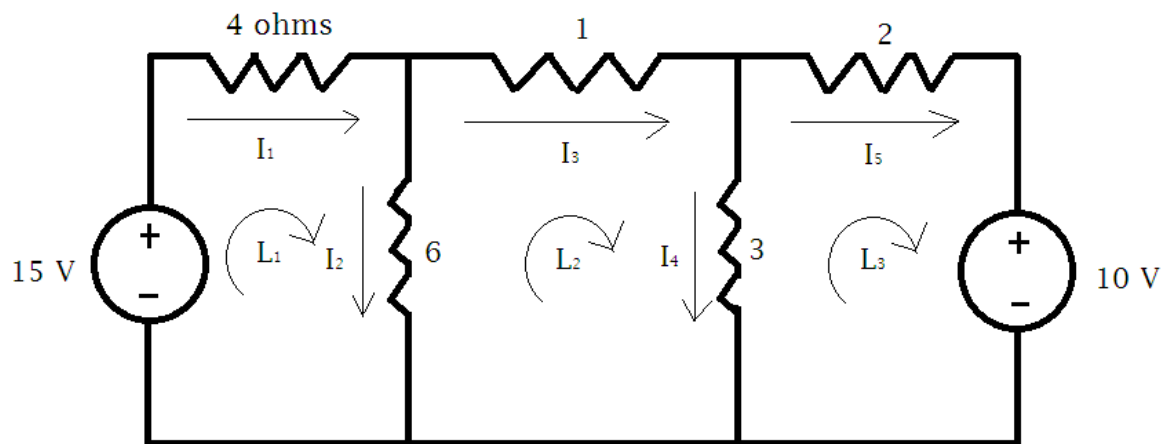

Figure 1: Example of a circuit

Figure shows an example. [1] Using KCL, we have

$$I_1 = I_2 + I_3$$
$$I_3 = I4 + I_5$$

and with KVL, we have

$$4I_1 + 6I_2 = 15$$
$$6I_2 - I_3 - 3I_4 = 0$$
$$3I_4 - 2I_5 = 10$$

Let $x^T = [I_1, I_2, I_3, I_4, I_5]$ be the unknown. We can use the above equations to solve $Ax = b$, where

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \\ 4 & 6 & 0 & 0 & 0 \\ 0 & 6 & -1 & -3 & 0 \\ 0 & 0 & 0 & 3 & -2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 0 \\ 15 \\ 0 \\ 10 \end{bmatrix}$$

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[1] https://physics.stackexchange.com/questions/96230

The solution is

$$x = \begin{bmatrix} 1.89 \\ 1.24 \\ 0.65 \\ 2.26 \\ -1.61 \end{bmatrix}.$$

1 Assignment in Python

1. Implement the function `mydet` using recursive definition, and compare its running time with `np.linalg.det`.

2. Design a simple circuit with 10 to 15 unknown currents, and draw the circuit.

3. Using the KCL and KVL to derive the equations, and write them in the $Ax = b$ form.

4. Solve your $Ax = b$ using

   (a) `numpy.linalg.solve`,

   (b) the adjoint of A (computing A^{-1} first and solve x by $A^{-1}b$) and

   (c) Cramer's Rule.

   Use `mydet` to compute the determinants. For each method, compute their residuals and running time.

2 Discussion

1. According to different matrix size n, what is the relation between running time of these experiments with matrix size n ?

3 Submission

1. Write a report in PDF file that includes (a) your circuit, (b) your matrix A and b, (c) A^{-1}, and (d) execution results for the third problem.

2. Python codes of the third problem.

3. Zip them and submit to iLMS system

4 Grading Policy

1. 詳見 assignment-2-評分標準A.pdf