A Study Of Quantum Entanglement And Nonlocality

Neil Smith

Supervised by Dr Pieter Kok and Dr Mark Howard

The Department Of Physics And Astronomy

What Are Quantum Entanglement And Nonlocality?

In quantum mechanics, particles can behave in ways that are forbidden according to classical physics. Particles can interact or influence each other no matter how far apart they are separated, we say the particles behave nonlocally. In order to observe this nonlocal behaviour, the particles need to be connected or quantum entangled. This entanglement is very fragile and sensitive and so not all particles are entangled, but we can make entangled pairs of particles in the lab. We can even use this nonlocality as a resource in new quantum technologies to perform tasks quicker and more efficiently than current technologies based on classical physics.

What we observe when we make a measurement on a particle is completely random, but it obeys some probability distribution. So you would think that when you make a measurement on two different particles the outcomes would be totally independent or uncorrelated. What's bizzare about nonlocality is that when we make measurements on entangled pairs of particles the outcomes are correlated! This means that what we do to one particle somehow instantaneously affects the other, no matter how far apart they are!

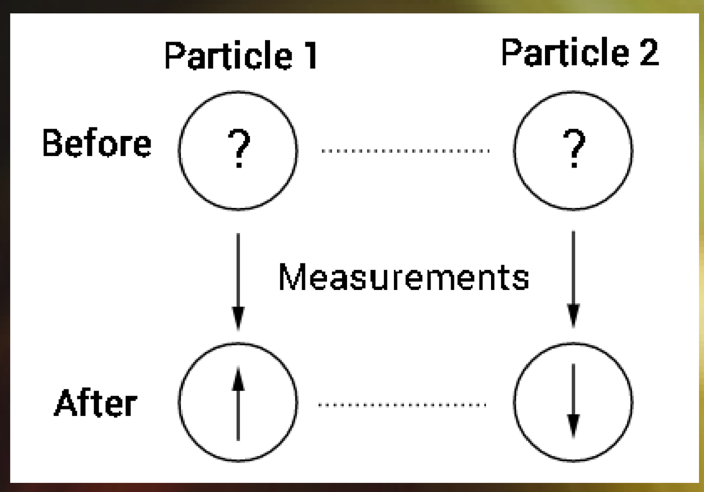

Fig. 1: Two entangled particles being measured. We always find the second particle to be in the opposite state to the first, no matter how far apart they are separated.

What Are Some Applications Of Nonlocality?

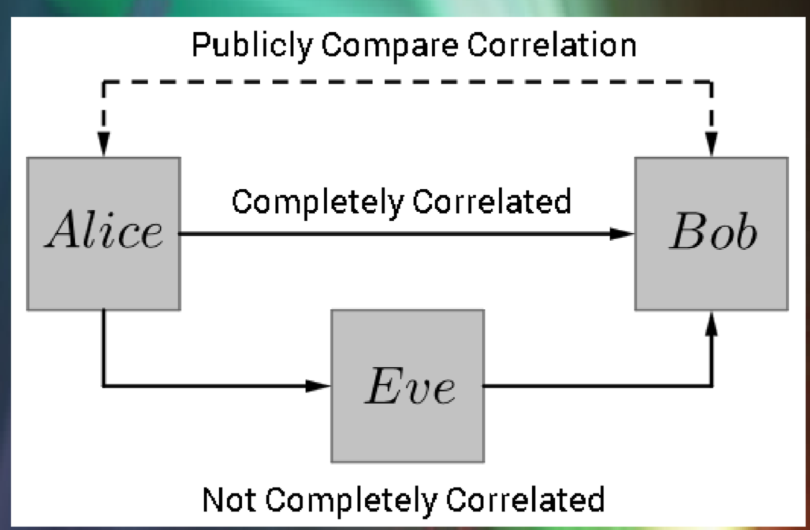

Fig. 2: An eavesdropper trying to listen in will change the state of the entangled particles and they will always be detected.

We can use the nonlocality of quantum systems to do things that are currently not possible with current technology based on classical physics. We can use nonlocality to create a secure communication system. Suppose you send an encrypted message to another person using entangled particles. By checking that the entangled particles are still correlated you can determine if someone has gotten hold of the entangled particles and therefore the message during the transmission. When an eavesdropper listens in they disrupt the correlation of the entangled particles. If they are not correlated then you know someone was eavesdropping.

Nonlocality also has important applications in communication complexity tasks. A Communication complexity task is where we have some people or parties who need to communicate information in order to solve a problem. If they use an entangled quantum system to communicate they can calculate the answer quicker and with less communication. An example is information flow in circuits. If a computer uses entangled systems, the circuit design can be improved.

What Is The Goal Of This Work?

These new quantum technologies use nonlocality as a resource and need quantum systems that exhibit non-locality in order to work. The more nonlocality in the quantum system the better the device works. The goal therefore is to find the quantum systems that exhibit the most nonlocality. The first step towards finding the most nonlocal quantum systems is to find the quantum systems that do not exhibit any nonlocality at all.

Mathematically, each quantum system can be represented as a point in a space of probabilities. All the quantum systems that are not entangled can be represented as a box in a space of probabilities, we call this the local box. Quantum systems that lie outside of this box are entangled and exhibit nonlocality, the further away the quantum system is from the box the more nonlocality it exhibits.

We can find this local box of quantum systems by finding the faces of the box. We can generate planes, or Bell Inequalities, in this probability space that touch the box. However, not all of these planes will touch a face of the box, but just an edge or corner. We need a way of finding which of these planes represent the faces of the box.

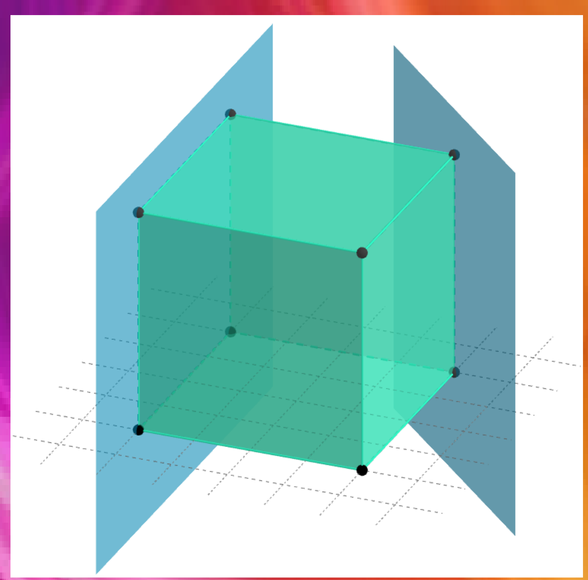

Fig. 3: The nonlocal quantum systems form a box in probability space. We can find the box by finding the planes that line up exactly with the face of the box.

What Did I Do?

I created an algorithm that calculates whether or not a given plane or Bell Inequality represents a face of the local box. We do know the corners of this local box and we can use this to help find the box. The algorithm calculates how many corners of the local box lie on the plane, this number is called the facet dimension of the Bell Inequality. The more corners that lie on the plane then the more it aligns with a face.

Looking at Figure 3 we can see that if one corner lies on the plane then we know the plane only intersects one corner. If two lie on the plane, the plane intersects an edge. If four lie on the plane then it is a face. The Bell Inequalities that are faces are called tight Bell Inequalities and have a facet dimension that is equal to its spatial dimension. We know the spatial dimension of these Bell Inequalities and so can know when we have found a face.

What Did I Find?

The algorithm was tested against a series of known tight Bell Inequalities for which we do know the spatial dimension. Table 1 shows the different Bell Inequalities that the algorithm was tested against and the results. The list of numbers $D_{m_k,k}$ describes the type of quantum system, how many particles are part of the system and the measurements made upon them. The algorithm also calculates the maximum classical correlation or bound of the particles, we can also use this to check the algorithm works. It produced the correct results for each Bell Inequality and we can be confident that it works.

I am now continuing with this work in my fourth year project where I am starting to apply the algorithm to new scenarios for which we do not know the tight Bell Inequalities, in the hopes of finding these local boxes.

Test	$D_{m_k,k}$	Expected Dimension	Expected Bound	Calculated Dimension	Calculated Bound	Result
1	((2;2),(2;2))	7	2	7	2	Pass
2	((2;2;2),(2;2;2))	14	0	14	0	Pass
3	((2;2),(2;2),(2;2))	25	6	25	6	Pass
4	((2;2;2),(2;2;2),(2;2;2))	62	8	62	8	Pass
5	((2;2),(2;2),(2;2),(2;2))	79	2	79	2	Pass
6	((2;2),(2;2),(2;2),(2;2),(2;2))	241	2	241	2	Pass
7	((2;2),(2;2;2))	10	0	10	0	Pass

Tab. 1: The Bell Inequalities the program was tested against and the results. It passes all of the tests.