EE 102
Probability and Statistics in Electrical Engineering
MIDTERM 1

NAME: SOLUTION MANUAL

Problem 1	35	
Problem 2	35	
Problem 3	30	
Total	100	

Notes:

- Show your work for full/partial credit
- In the exam, $P[A]$ denotes the probability of event A happening.

Problem 1. For a random variable X, the probability mass function (PMF) is given as

$$P_X(x) = \begin{cases} \beta(x^2 + x + 1) & x = -2, -1, 0, 1, 2 \\ 0 & otherwise \end{cases}$$

7 a) Find β such that $p_X(x)$ is a valid PMF.

7 b) Write the sample space of X.

7 c) Is X a discrete random variable? Explain your reasoning.

7 d) Is X a <u>discrete uniform</u> random variable? Explain your reasoning.

7 e) Calculate the conditional probability $P[X = -1|X < 0]$.

a) $P_X(x) = \begin{cases} 3\beta & x=-2 \\ \beta & x=-1 \\ \beta & x=0 \\ 3\beta & x=1 \\ 7\beta & x=2 \end{cases}$ $\qquad \sum_x P_X(x) = 1$

$\Rightarrow 15\beta = 1 \Rightarrow \beta = \frac{1}{15}$

b) $S_X = \{-2, -1, 0, 1, 2\}$

c) YES, sample space is countable

d) NO, $P_X(x)$ is not uniform

e) $P[X=-1 | X<0] = \dfrac{P[\{X=-1\} \cap \{X<0\}]}{P[X<0]}$

$= \dfrac{P[X=-1]}{P[X<0]} = \dfrac{\beta}{4\beta} = \dfrac{1}{4}$

Problem 2. Consider three boxes given in Figure 1. Box #1 contains 2 black balls and 3 white balls. Box #2 contains 1 black ball and two white balls. Box #3 contains 1 black ball and one white ball.

A FAIR die is thrown, and a box is selected according to the number showing on the die: Box 1 is selected if the die shows a number less than 3. Box 2 is selected if the die shows 3. Box 3 is selected if the die shows a number larger than 3.

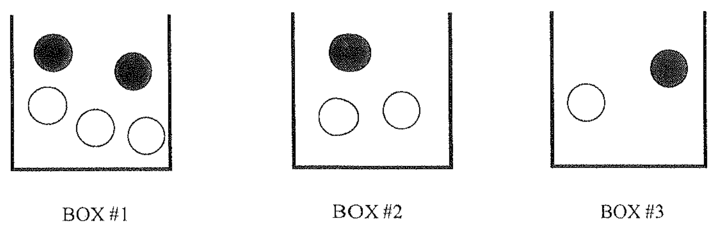

Figure 1: Problem 2

Two balls are drawn from the selected box WITHOUT REPLACEMENT.

12 a) Find the probability of picking two white balls.

12 b) If two white balls are picked, what is the probability that Box#1 was selected?

11 c) How does your answer change if the sampling is done WITH REPLACEMENT in parts a and b?

a) $P[WW] = P[WW \mid Box\#1]\, P[Box\#1] + P[WW \mid Box\#2]\, P[Box\#2] + P[WW \mid Box\#3]\, P[Box\#3]$

$P[Box\#1] = P[Die < 3] = \frac{2}{6}$

$P[Box\#2] = P[Die = 3] = \frac{1}{6}$

$P[Box\#3] = P[Die > 3] = \frac{3}{6}$

$P[WW] = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{6} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot 0 \cdot \frac{3}{6} = \frac{7}{45} = 0.155$

b) $P[Box\#1 \mid WW] = \dfrac{P[WW \mid Box\#1]\, P[Box\#1]}{P[WW]}$

$= \dfrac{\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{6}}{7/45} = \dfrac{\frac{1}{10}}{\frac{7}{45}} = \frac{1}{10} \cdot \frac{45}{7} = \frac{9}{14} = 0.642$

$$\frac{3}{25}+\frac{2}{27}+\frac{1}{8}$$

c) $$P[WW] = \frac{3}{5}\cdot\frac{\cancel{3}}{5}\cdot\frac{\cancel{2}}{\cancel{6}} + \frac{2}{3}\cdot\frac{\cancel{2}}{3}\cdot\frac{1}{\underset{3}{\cancel{6}}} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{\cancel{3}}{\underset{2}{\cancel{6}}} = \cancel{0.4672}\ 0.3191$$

$$P[\text{Box }\#1 \mid WW] = \frac{\frac{3}{5}\ \frac{\cancel{3}}{5}\ \frac{\cancel{2}}{\cancel{6}}}{\cancel{0.4672}\ 0.3191} = \cancel{0.2568}\ 0.3761$$

Problem 3. You are supposed to come to school 5 days a week for a full semester (15 weeks), and with probability $p = 0.05$ you get sick on a given day, independently of other days. Let X be the total number of days you are sick in a given semester.

10 a) What type of random variable is X? Specify its parameters clearly.

5 b) What is the probability that you get sick at most 7 days in a given semester?

15 c) On the sick days you spent money for regular medicine which costs 10\$. On some of the sick days, the regular medicine does not heal you and you buy a stronger version which costs 40\$. This happens on 10% of the sick days.

Let M denote the amount of money you spent for medicine on any day. Find the sample space and the pmf for M.

a) $n = 15 \times 5 = 75$ days

$X \sim \text{Binomial}(75, 0.05)$

b) $P[X \leq 7] = \sum_{k=0}^{7} \binom{75}{k} p^k (1-p)^{75-k}$

c) $S_M = \{0, 10, 50\}$ 5

10

$P[M=0] = P[\text{not sick}] = (1-p) = 1-0.05 = 0.95$

$P[M=10] = P[\text{sick \& regular med works}]$

$= 0.05 \times \frac{9}{10} = 0.045$

$P[M=50] = 0.05 \times \frac{1}{10} = 0.005$