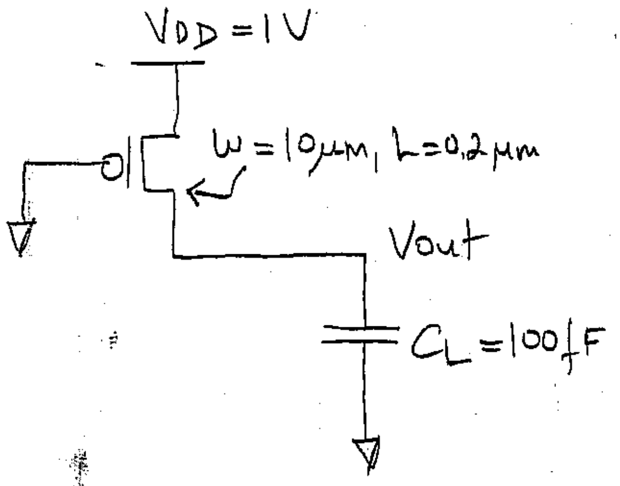
CmpE 110 MIDTERM 2 / S 06 Dr. Ahmet BINDAL

(1)

40 pts



$\mu_p = 100\ cm^2/Vsec$

$C_{ox} = 6.9\times10^{-7}\ F/cm^2$

$V_{TP} = 0.2V$

At $t=0$ $V_{out} = 0V$ ⇒ plot $V_{out}(t)$ according to the transistor parameters above. How long does it take for V_{out} to reach 1 V ? Show your calculations clearly.

$$I_D = \frac{\mu_p C_{ox} W}{L}\left[(V_{SG}-V_{TP})V_{SD} - \frac{V_{SD}^2}{2}\right]$$

(2) $Y = A(B+CD)$

60 pts (a) Implement the circuit without using an output inverter.

(b) Assume $T_R = \frac{T_F}{2}$ ⇒ size the transistors according to a minimum geometry of 1μ.

NOTE: Implement Y so that you get minimal rise and fall times.

(1)

$$I_{DSAT} = \frac{\mu_p C_{ox} W}{2L} (V_{SG} - V_{Tp})^2$$

$\mu_p = 100 \ cm^2/Vsec$

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{50 \times 10^{-8}} = 6.9 \times 10^{-7} \ F/cm^2$

$V_{SG} = 1V \quad V_{Tp} = 0.2V$

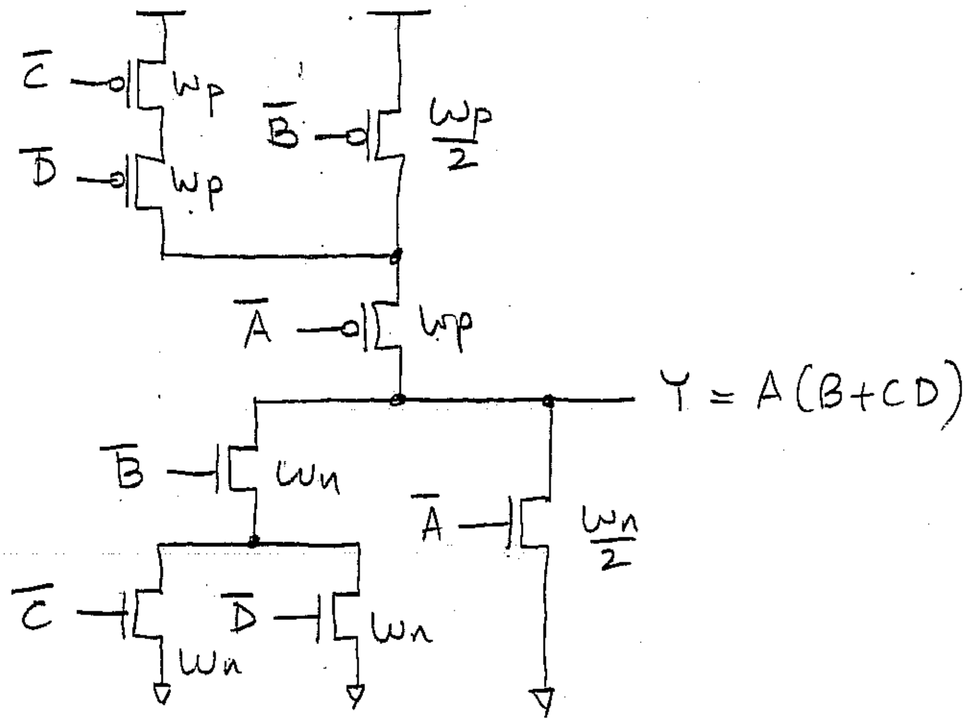
$W = 10 \mu m \quad L = 0.2 \mu m$

$$I_{DSAT} = \frac{100 \times 6.9 \times 10^{-7} \times 10}{2 \times 0.2} (\underbrace{1 - 0.2}_{0.8})^2 = 1.1 \ mA$$

$$C_L = 100 fF \Rightarrow \quad T = \frac{100 \times 10^{-15} \times 1}{1.1 \times 10^{-3}} = 90.91 \ psec.$$

(2) (a) $Y = A(B + CD) \Rightarrow \overline{Y} = \overline{A} + \overline{(B + CD)}$

$$= \overline{A} + \overline{B} \cdot (\overline{C} + \overline{D})$$



$$T_F = 2.2\, R_{Neq}\, C_L = 2.2\, C_L\, 2 R_N = 2.2\, C_L\, \cancel{2}\, \frac{P}{\cancel{2} W_n}$$

$$T_R = 2.2\, R_{PEQ}\, C_L = 2.2\, C_L\, 3 R_P = 2.2\, C_L\, 3\, \frac{P}{W_P}$$

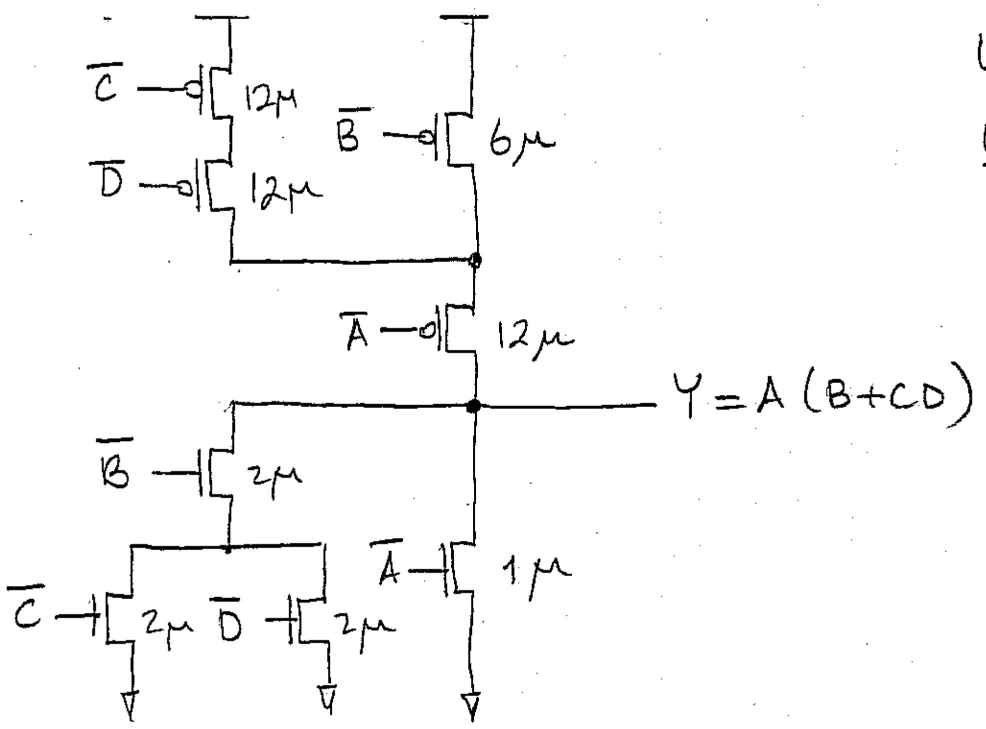
$$T_R = \frac{T_F}{2} \Rightarrow \frac{3}{W_p} = \frac{1}{2} \cdot \frac{1}{W_n} \Rightarrow \boxed{W_p = 6 W_n}$$

Minimum geometry is $1\,\mu m$ $\Rightarrow$ $\frac{W_n}{2} = 1\mu$

$W_n = 2\mu$

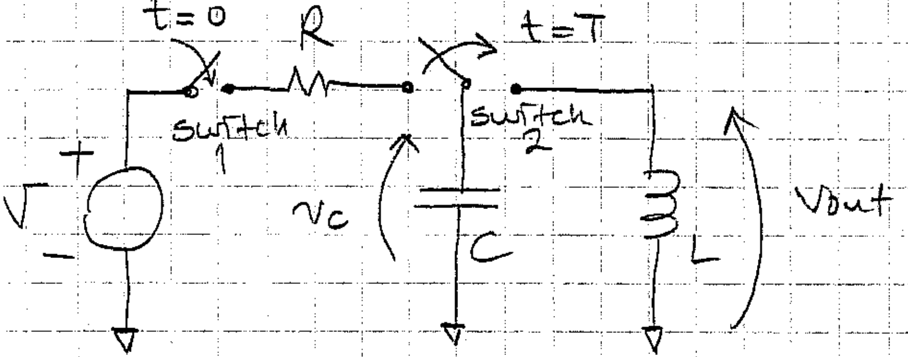
$W_p = 12\mu$

$\frac{W_p}{2} = 6\mu$.



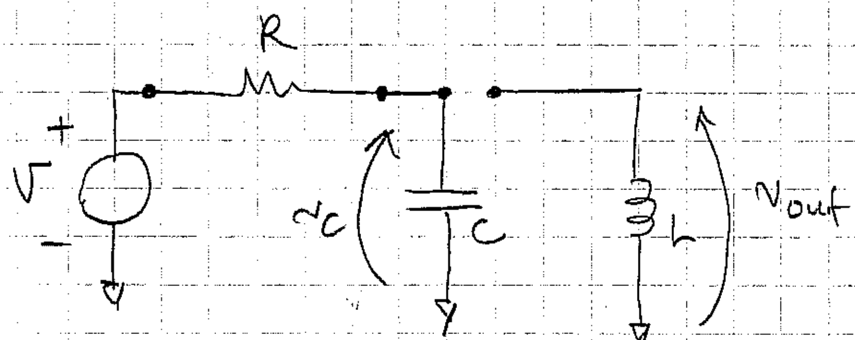
CmpE 110 Midterm II Dr. Ahmet BINDAL

The circuit below is given:

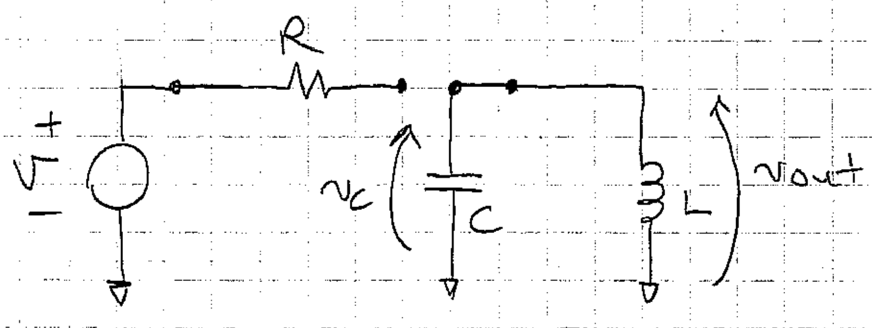


At $t=0$, switch 1 is closed; switch 2 is flipped to R side shown as below. $v_c(0) = 0$ V & $i_L(0) = 0$ A.

$t=0$

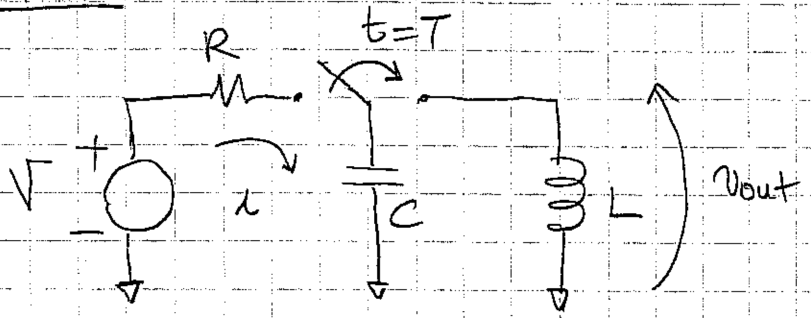


At $t=T$, while switch 1 is in the same position, switch 2 flips to L side shown as below.



Find the expression for Vout(t) from $t=0$ to ∞ & plot it.

Solution:



$$V = iR + v_c \rightarrow 0 = R\frac{di}{dt} + \frac{dv_c}{dt} = R\frac{di}{dt} + \frac{i}{C}$$

Thus:

$$\frac{di}{dt} + \frac{i}{RC} = 0 \Rightarrow i = Ae^{-\frac{t}{RC}}$$

$$v_c = V - Ri = V - RAe^{-\frac{t}{RC}} \quad \text{but } v_c(0) = 0$$

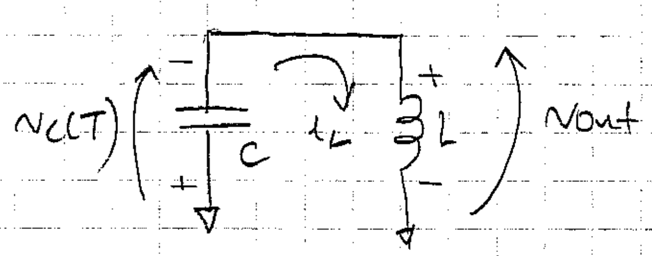
$$0 = V - RA \Rightarrow A = \frac{V}{R} \Rightarrow i = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$v_c = V\left(1 - e^{-\frac{t}{RC}}\right)$$

Thus:

$v_c(T) = V\left(1 - e^{-\frac{T}{RC}}\right)$ as initial condition when the switch is flipped.

$t \geq T$



$$v_c + v_L = 0 \Rightarrow \frac{dv_c}{dt} + \frac{dv_L}{dt} = 0 \qquad \frac{i_L}{C} + \frac{d}{dt}\left(L\frac{di_L}{dt}\right) = 0$$

But,

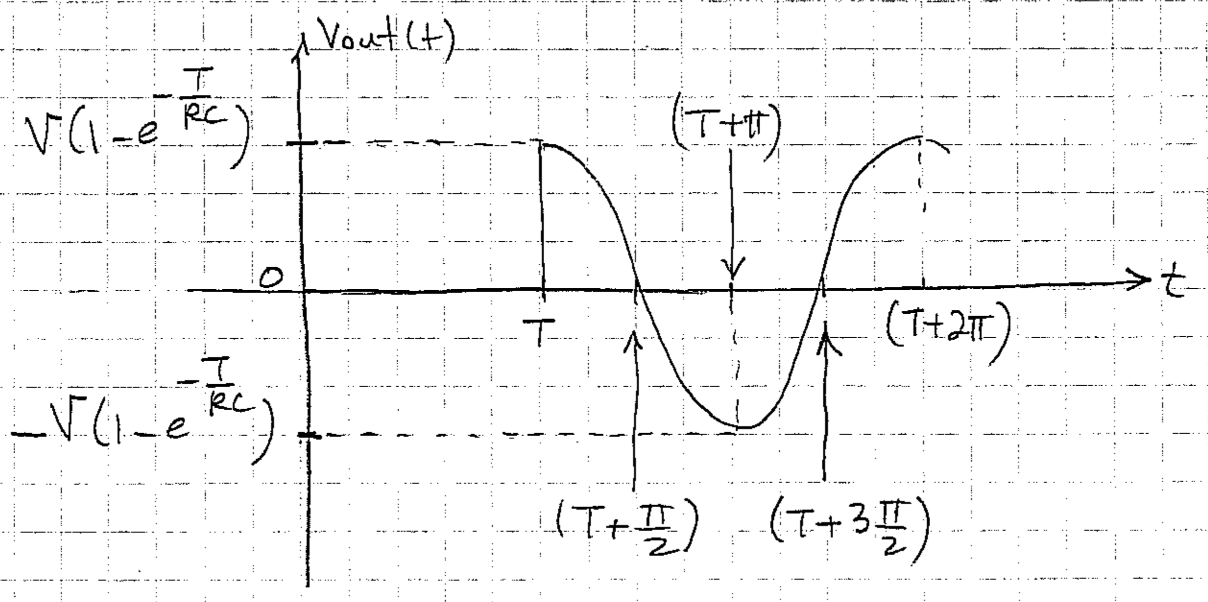
$$V_{out} = L B_1 s_1 e^{s_1(t-T)} - L B_1 s_2 e^{s_2(t-T)}$$

$$= L\left(-\frac{V}{2}\right) j \sqrt{\frac{C}{L}} \left(1 - e^{-\frac{T}{RC}}\right) \left[\frac{j}{\sqrt{LC}} e^{\frac{j}{\sqrt{LC}}(t-T)} + \frac{j}{\sqrt{LC}} e^{-\frac{j}{\sqrt{LC}}(t-T)} \right]$$

$$= L\left(-\frac{V}{2}\right) j \sqrt{\frac{C}{L}} \left(1 - e^{-\frac{T}{RC}}\right) \frac{j}{\sqrt{LC}} \left[\cos\frac{(t-T)}{\sqrt{LC}} + \cancel{j \sin\frac{(t-T)}{\sqrt{LC}}} + \cos\frac{(t-T)}{\sqrt{LC}} - \cancel{j \sin\frac{(t-T)}{\sqrt{LC}}} \right]$$

$$= \cancel{L} \frac{V}{\cancel{2}} \sqrt{\frac{\cancel{C}}{\cancel{L}}} \left(1 - e^{-\frac{T}{RC}}\right) \frac{1}{\cancel{\sqrt{LC}}} \cancel{2} \cos\frac{(t-T)}{\sqrt{LC}}$$

$$V_{out} = V\left(1 - e^{-\frac{T}{RC}}\right) \cos\frac{(t-T)}{\sqrt{LC}}$$



$$\frac{d^2 i_L}{dt^2} + \frac{i_L}{LC} = 0 \qquad s^2 + \frac{1}{LC} = 0 \qquad s_1 = \frac{j}{\sqrt{LC}}, \quad s_2 = -\frac{j}{\sqrt{LC}}$$

$$i_L = B_1 e^{s_1(t-T)} + B_2 e^{s_2(t-T)}$$

$$v_{out} = L\frac{di_L}{dt} = L B_1 s_1 e^{s_1(t-T)} + L B_2 s_2 e^{s_2(t-T)}$$

$$v_{out}(T) = v_C(T) = V\left(1 - e^{-\frac{T}{RC}}\right) \Rightarrow$$

$$\frac{V}{L}\left(1 - e^{-\frac{T}{RC}}\right) = B_1 s_1 + B_2 s_2 \qquad \text{— Eqn. 1}$$

We also know that $i_L(T) = 0\,A$

$$0 = B_1 + B_2 \qquad \text{— Eqn. 2}$$

Substitute $B_2 = -B_1$ in Eqn. 1

$$\frac{V}{L}\left(1 - e^{-\frac{T}{RC}}\right) = B_1(s_1 - s_2) \Rightarrow B_1 = \frac{V}{L}\,\frac{\left(1 - e^{-\frac{T}{RC}}\right)}{(s_1 - s_2)}$$

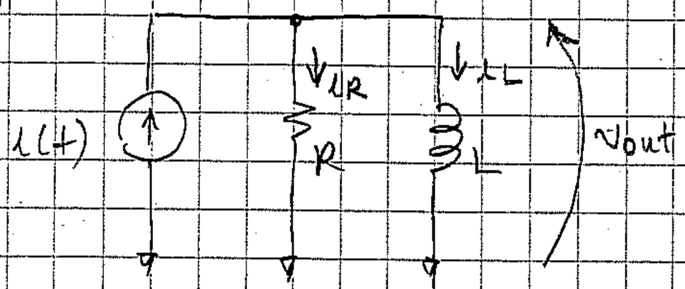
But, $s_1 - s_2 = \frac{j}{\sqrt{LC}} + \frac{j}{\sqrt{LC}} = \frac{2j}{\sqrt{LC}} \Rightarrow$

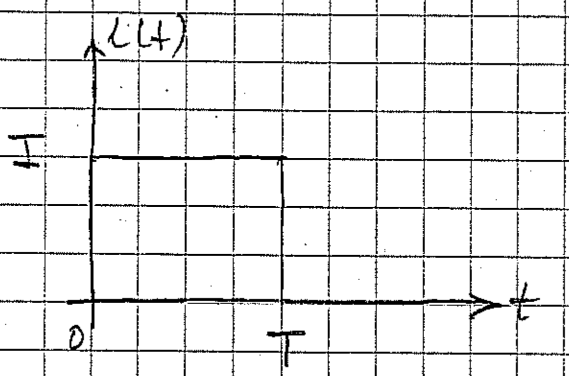
$$B_1 = \frac{V}{L}\,\frac{\left(1 - e^{-\frac{T}{RC}}\right)}{2j}\cdot\sqrt{LC} = -\frac{V}{2}j\sqrt{\frac{C}{L}}\left(1 - e^{-\frac{T}{RC}}\right)$$

$$B_2 = \frac{V}{2}j\sqrt{\frac{C}{L}}\left(1 - e^{-\frac{T}{RC}}\right)$$

CmpE110 MIDTERM #1 Dr. Ahmet Bindal

(1)

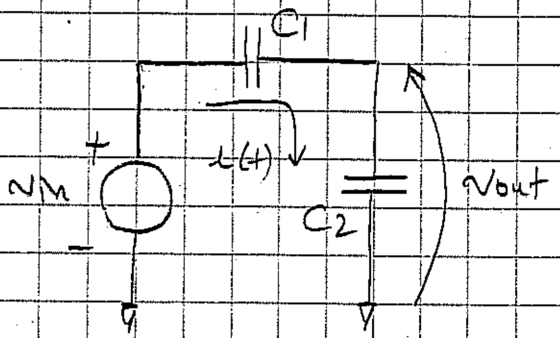


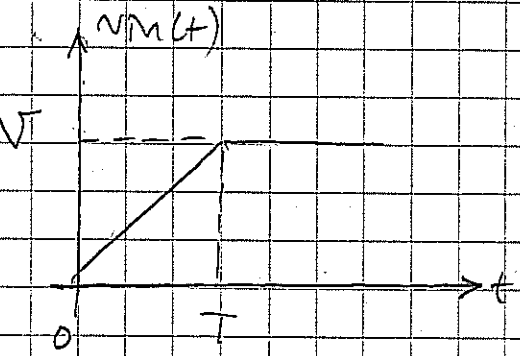


30pts (a) Solve $i_L(t)$ and $v_{out}(t)$ using time-domain analysis

20pts (b) Plot $i_L(t)$ and $v_{out}(t)$

(2)



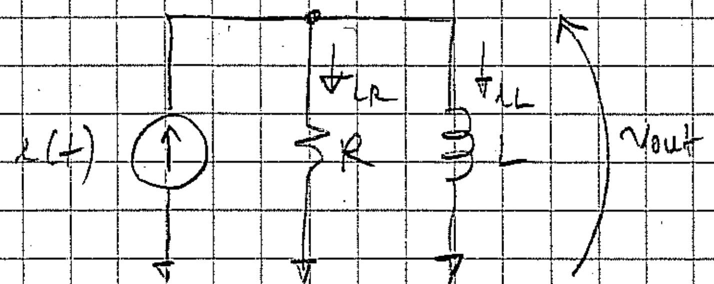


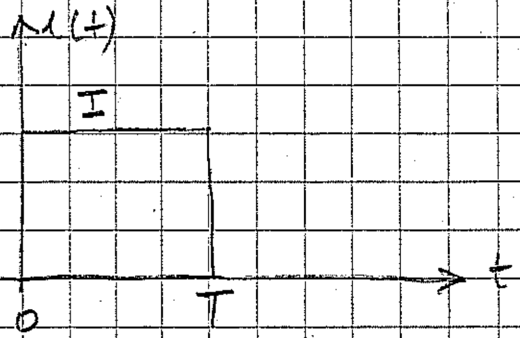
30pts (a) Solve $i(t)$ and $v_{out}(t)$ using time-domain analysis

20pts (b) Plot $i(t)$ and $v_{out}(t)$

(1)

(a) For $t \leq T$:





$$\left.\begin{aligned} i(t) &= i_R + i_L \\ i_R &= \frac{v_{out}}{R} \\ v_{out} &= L\frac{di_L}{dt} \end{aligned}\right\} \qquad i_R R = L\frac{di_L}{dt} \qquad i_R = \frac{L}{R}\frac{di_L}{dt}$$

$$i(t) = \frac{L}{R}\frac{di_L}{dt} + i_L \quad \text{or} \quad \frac{di_L}{dt} + \frac{R}{L}i_L = \frac{R}{L}i(t)$$

$$i_L(t) = Ae^{-\frac{R}{L}t} + i_{L_{part}} \quad \text{where} \quad i_{L_{part}} = I$$

$$i_L(t) = Ae^{-\frac{R}{L}t} + I, \quad \text{but} \quad i_L(0) = 0 = A + I$$

$$i_L(t) = I\left(1 - e^{-\frac{R}{L}t}\right) \;\&\; v_{out} = IL\left[0 - e^{-\frac{R}{L}t}\cdot\left(-\frac{R}{L}\right)\right] = IRe^{-\frac{R}{L}t}$$

For $t \geq T$: $\frac{di_L}{dt} + \frac{R}{L}i_L = 0$ since $i(t \geq T) = 0$ A

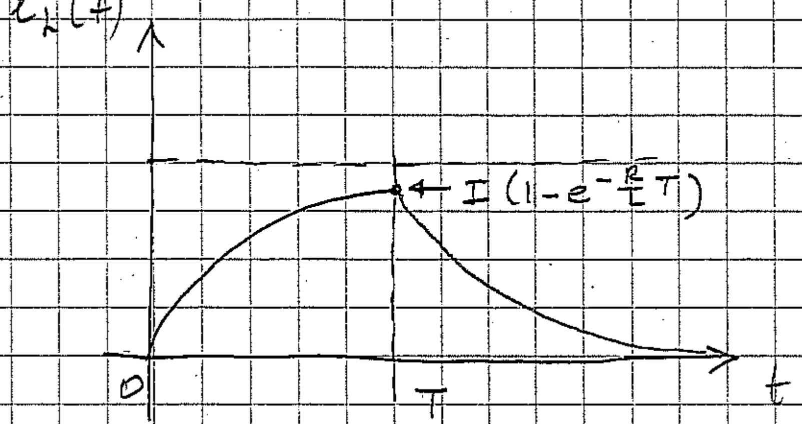
$$i_L = Be^{-\frac{R}{L}(t-T)} \qquad \text{But, } i_L(T) = I\left(1 - e^{-\frac{R}{L}T}\right) = B \text{ from } t \leq T$$

$$i_L(t) = I\left(1 - e^{-\frac{R}{L}T}\right)e^{-\frac{R}{L}(t-T)}$$

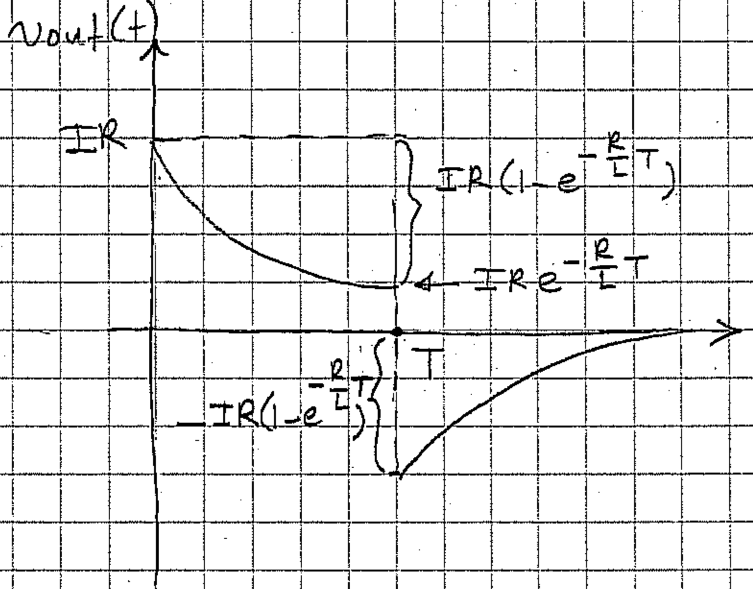
$$v_{out} = \cancel{L}I\left(1 - e^{-\frac{R}{L}T}\right)e^{-\frac{R}{L}(t-T)}\left(-\frac{R}{\cancel{L}}\right) = -IR\left(1 - e^{-\frac{R}{L}T}\right)e^{-\frac{R}{L}(t-T)}$$

Continue 1

(b)



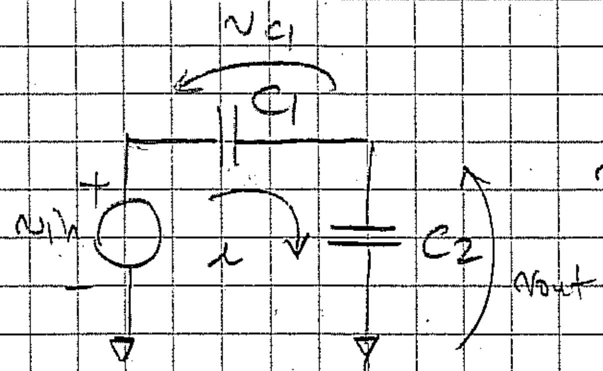
$i_L(t)$
$I(1-e^{-\frac{R}{L}T})$
0
T
t



$v_{out}(t)$
IR
$IR(1-e^{-\frac{R}{L}T})$
$IRe^{-\frac{R}{L}T}$
T
$-IR(1-e^{-\frac{R}{L}T})$

(2)

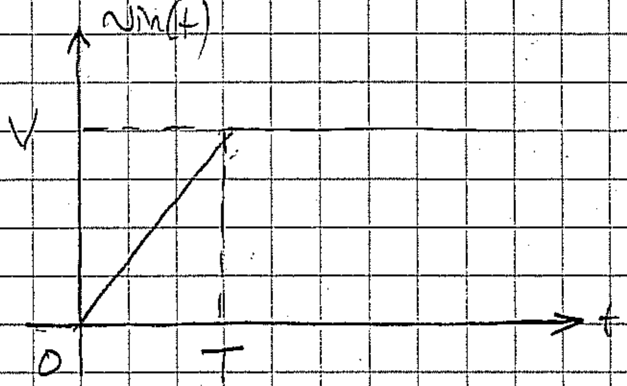
(a)



$v_{in} = v_{C1} + v_{out} \Rightarrow \frac{dv_{in}}{dt} = \frac{dv_{C1}}{dt} + \frac{dv_{out}}{dt}$

Thus; $\frac{dv_{in}}{dt} = \frac{i}{C_1} + \frac{i}{C_2}$

$$i = \frac{C_1 C_2}{(C_1+C_2)} \frac{dv_{in}}{dt}$$



From the graph:

$$v_{in}(t) = \frac{V}{T}t \qquad 0 \le t \le T$$
$$= V \qquad t \ge T$$

Thus: $\frac{dv_{in}}{dt} = \frac{V}{T} \Rightarrow \;\rightarrow i = \frac{C_1 C_2}{(C_1+C_2)} \frac{V}{T}$ for $0 \le t \le T$

Since $i = C_2 \frac{dv_{out}}{dt} \Rightarrow V_{out} = \frac{1}{C_2}\int_0^T i(t)\,dt$

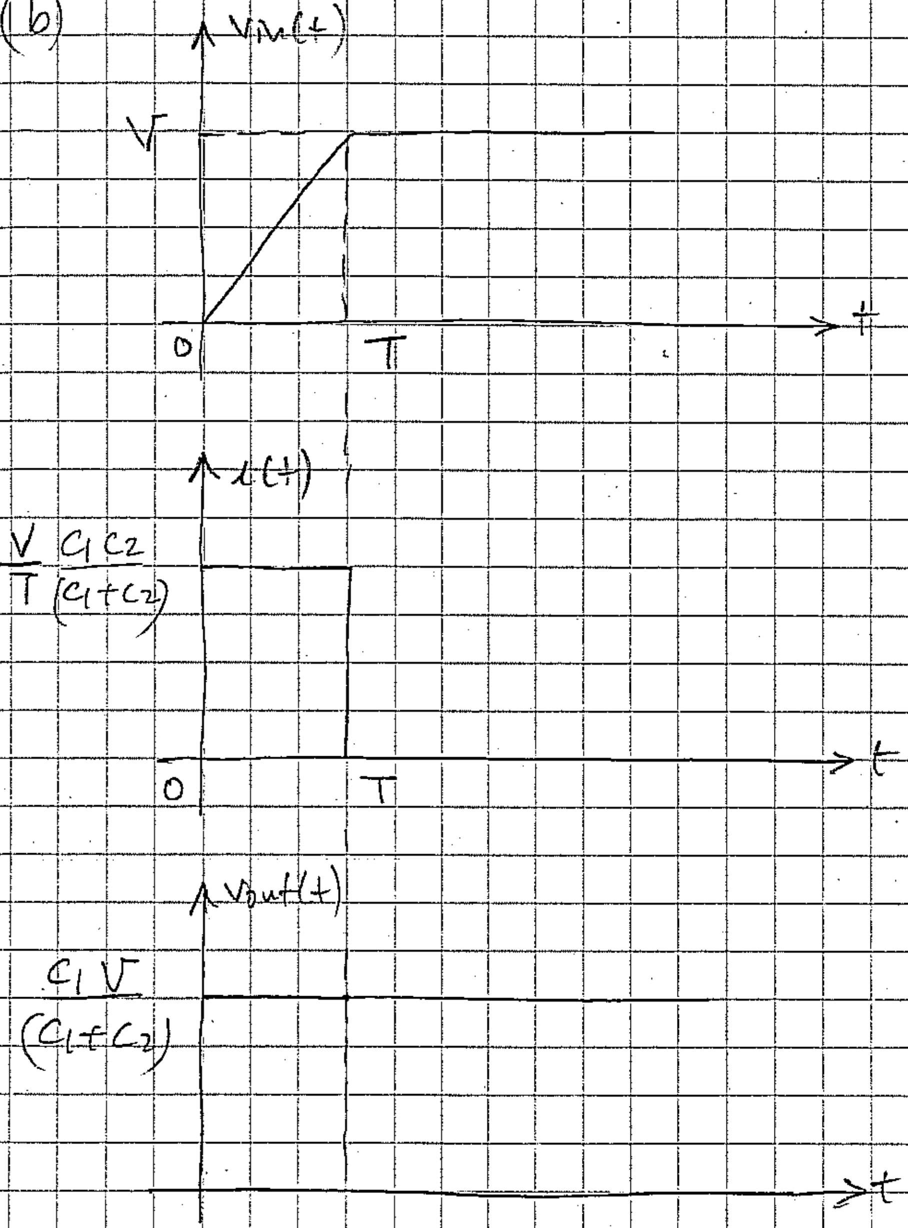
$$= \frac{1}{\cancel{C_2}} \cdot \frac{C_1 \cancel{C_2}}{(C_1+C_2)} \frac{V}{\cancel{T}} \cancel{T}$$

$\rightarrow v_{out} = \frac{C_1}{(C_1+C_2)} V$ for $0 \le t \le T$

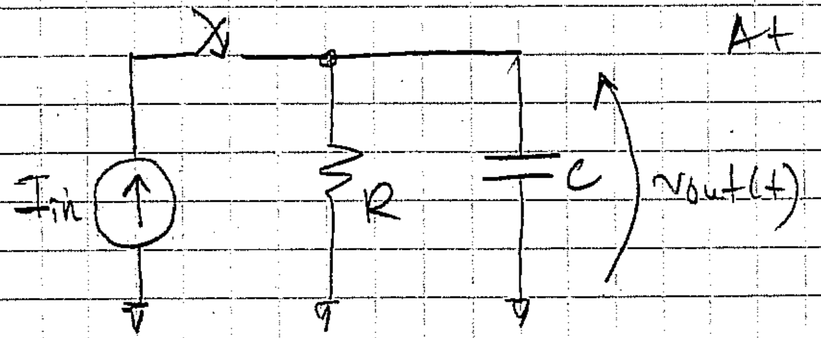
For $t \ge T$ $\quad v_{in}(t) = V \quad \frac{dv_{in}}{dt} = 0 \Rightarrow i = 0$ A

$$v_{out} = \frac{C_1}{C_1+C_2} V$$

(b)



① Consider the circuit below:



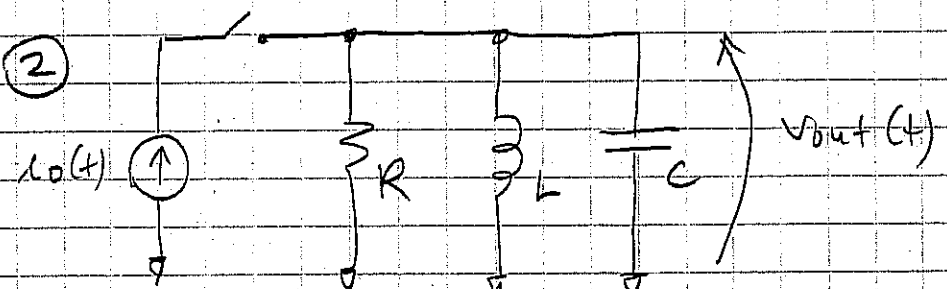
At $t=0^+$ switch closes.

$v_{out}(0) = 0\,V$

I_{in} is a constant current

Using time-domain analysis, derive $v_{out}(t)$ & plot it.

②

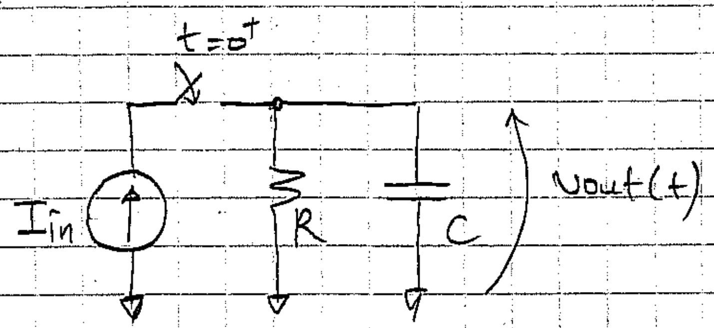


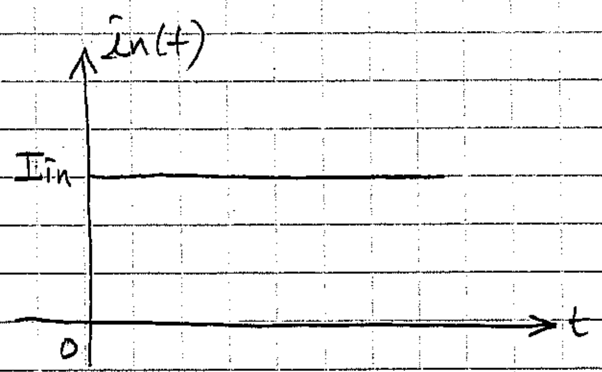
(a) Find $H(s) = \dfrac{V_{out}(s)}{I_o(s)}$ & find the poles

(b) Find the value of R in terms of L & C such that the circuit is stable but v_{out} undershoots (overdamped), the circuit is stable but v_{out} is critical (critically damped) & the circuit is stable but V_{out} overshoots (underdamped). Plot the location of poles & approximate V_{out} for each case

(c) What is the value of R to make the circuit oscillatory? Find the frequency of oscillation. Plot the location of pole

(1)

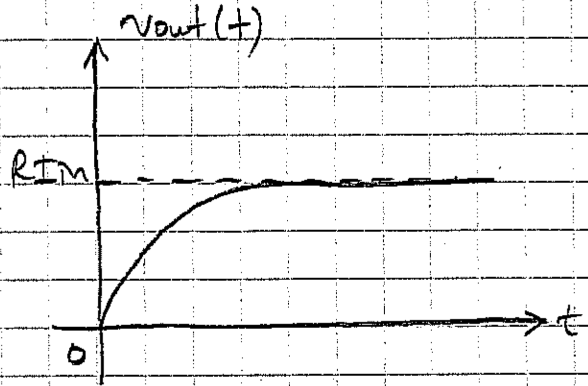




$v_{out}(0) = 0\,V$

$$\left.\begin{aligned} I_{in} &= i_R + i_C \\ v_{out} &= i_R R \\ i_C &= C\frac{dv_{out}}{dt} \end{aligned}\right\} \quad I_{in} = \frac{v_{out}}{R} + C\frac{dv_{out}}{dt}$$

$$\frac{dv_{out}}{dt} + \frac{1}{RC} v_{out}(t) = \frac{I_{in}}{C}$$



$v_{out} = v_{out\,gen} + v_{out\,part}$

$v_{out\,gen} = Ae^{st}$ $\quad v_{out\,part} = K$

Char. eqn: $s + \frac{1}{RC} = 0 \Rightarrow s = -\frac{1}{RC}$

$v_{out\,gen} = Ae^{-\frac{t}{RC}}$

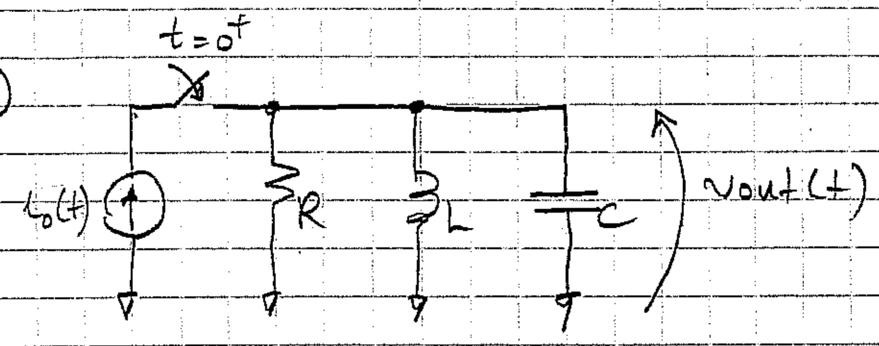
For $v_{out\,part}$: $\frac{1}{R\cancel{C}} \cdot K = \frac{I_{in}}{\cancel{C}} \Rightarrow K = RI_{in}$

$v_{out} = Ae^{-\frac{t}{RC}} + RI_{in}$

$v_{out}(0) = A + RI_{in} = 0 \Rightarrow A = -RI_{in}$

$v_{out}(t) = RI_{in}\left(1 - e^{-\frac{t}{RC}}\right)$

(2)



(a) $I_0(s) = I_R(s) + I_L(s) + I_C(s)$

$$= \frac{V_{out}(s)}{R} + \frac{V_{out}(s)}{sL} + V_{out}(s)\cdot sC = V_{out}(s)\left(\frac{1}{R} + \frac{1}{sL} + sC\right)$$

$$I_0(s) = V_{out}(s)\cdot\frac{s^2LCR + R + sL}{sRL} = V_{out}(s)\cdot\frac{\cancel{L}C\cancel{R}\,(s^2 + s/RC + 1/LC)}{s\cancel{R}\cancel{L}}$$

$$H(s) = \frac{V_{out}(s)}{I_0(s)} = \frac{s}{C\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right)}$$

Poles $= s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$ $\qquad s_{1,2} = \dfrac{-\frac{1}{RC} \mp \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$

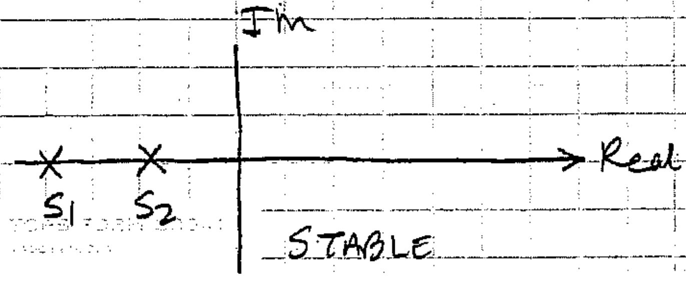
$$= -\frac{1}{2RC} \mp \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Let $\alpha \triangleq \frac{1}{2RC}$, $\omega_0 \triangleq \frac{1}{\sqrt{LC}}$ $\Rightarrow$

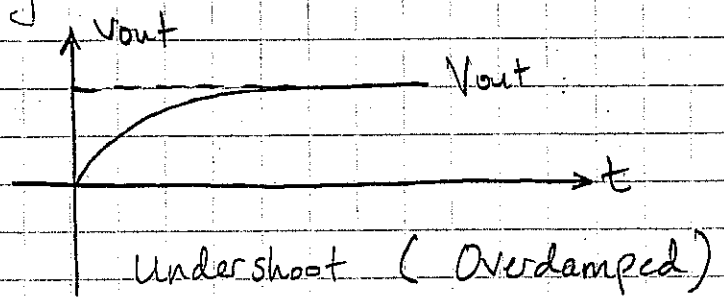
$$s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - \omega_0^2}$$

(b) (1) If $\alpha > \omega_0 \Rightarrow \frac{1}{2RC} > \frac{1}{\sqrt{LC}} \Rightarrow R < \frac{\sqrt{LC}}{2C} \Rightarrow R < \frac{1}{2}\sqrt{\frac{L}{C}}$

Then, both roots lay on the LHS real axis



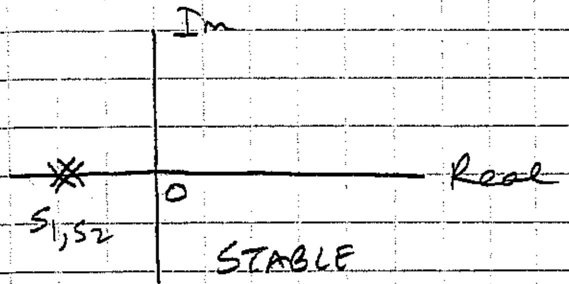
STABLE



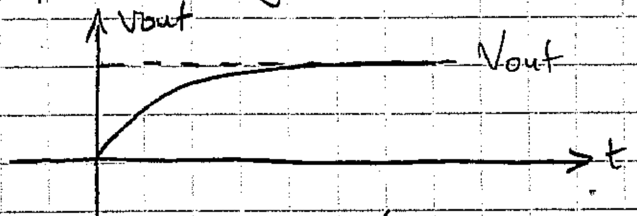
Undershoot (Overdamped)

(2) $\alpha = \omega_0 \quad s_{1,2} = -\alpha \Rightarrow R = \frac{1}{2}\sqrt{\frac{L}{C}}$

Then both roots are equal & lay on the LHS real axis.



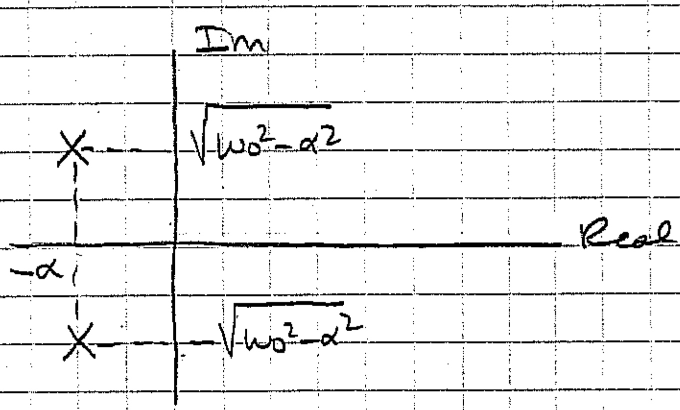
STABLE



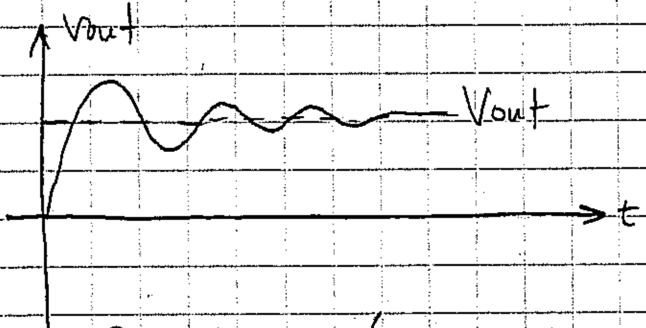
Critical (Critically damped)

(3) $\alpha < \omega_0 \quad s_{1,2} = -\alpha \mp j\sqrt{\omega_0^2 - \alpha^2} \Rightarrow R > \frac{1}{2}\sqrt{\frac{L}{C}}$

Then the roots lay on the LHS plane



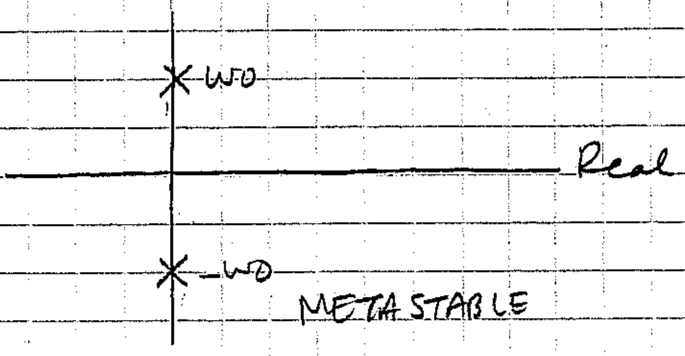
STABLE



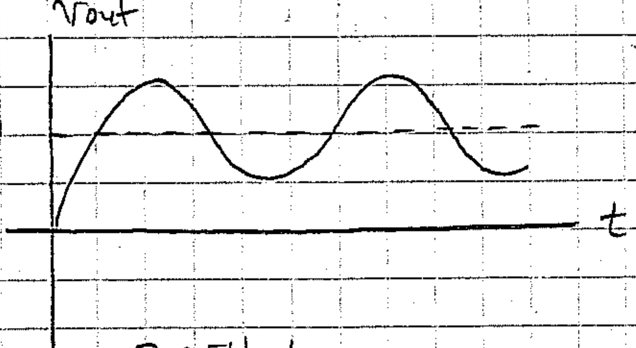
Overshoot (Underdamped)

(c) If $R = \infty\ \Omega$ (open circuit) $\Rightarrow \alpha = 0$

$s_{1,2} = \mp j\omega_0$



METASTABLE



Oscillatory

$f_0 = \frac{1}{2\pi\sqrt{LC}}$ = oscillation frequency.

Design the following logic function and size the transistors such that $T_R = 2\ T_F$.

$$f = \overline{A}.B + C$$

NOTE: Do NOT consider Elmore delay components.
Do NOT use any inverters at the output.

Cmpt 110 Quiz #2 Dr. Ahmet Birdal

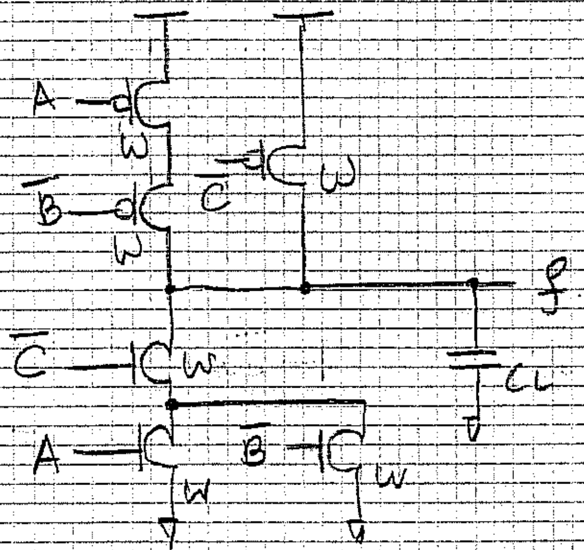
Solution:

Since we are not allowed to use any inverter at the output we need to find out $\overline{f}$.

$$f = \overline{A}\cdot B + C$$

$$\overline{f} = \overline{\overline{A}\cdot B + C}$$

$$= (A+\overline{B})\cdot\overline{C}$$



$$T_F = 2.2\, C_L R_{neq} = 2.2\, C_L (R_n + R_n)$$
$$= 2.2\, C_L (2R_n)$$

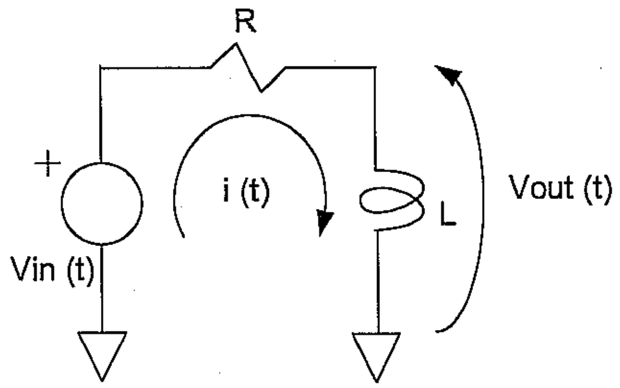
$$T_R = 2.2\, C_L R_{peq} = 2.2\, C_L (R_p + R_p)$$
$$= 2.2\, C_L (2R_p)$$

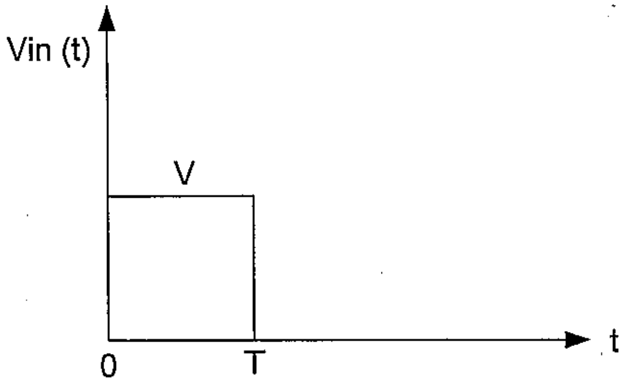
$$2T_F = T_R \Rightarrow 2R_n = R_p$$

$$R_n = \frac{K}{2W_n} \qquad R_p = \frac{K}{W_p}$$

$$2\,\frac{K}{2W_n} = \frac{K}{W_p} \Rightarrow \boxed{W_p = W_n = W}$$

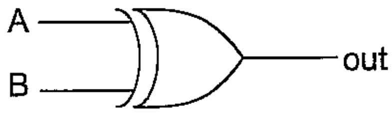
1.





(a) If Vin(t) is given above, find i(t) using time-domain analysis. Note that i(0) = 0 in the inductor.
(b) Compute Vout(t) using time-domain analysis.

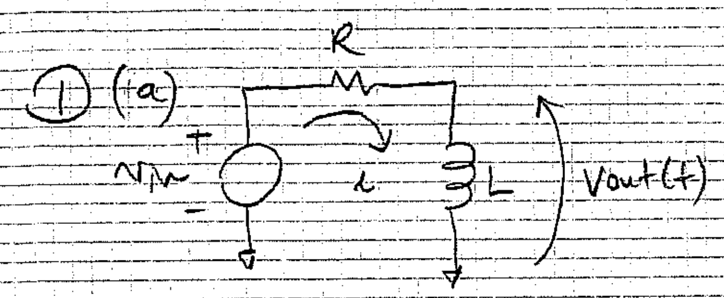
2.



(a) Write output function of this gate in terms of its inputs, A and B (Form a min-term function).
(b) Implement the above gate with CMOS circuitry.

CmpE 110 MIDTERM solutions:

① (a)



$\underline{0 \le t \le T}$:

$$v_{in} = Ri + L\frac{di}{dt} = V$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

general soln: $\frac{di_g}{dt} + \frac{R}{L} i_g = 0$

$$s + \frac{R}{L} = 0 \qquad s = -\frac{R}{L}$$

$$i_g(t) = A e^{-\frac{R}{L}t}$$

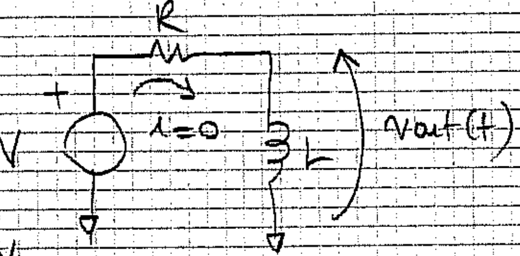
particular soln:

$$i_p(t) = K$$

$$0 + \frac{R}{L}K = \frac{V}{L} \Rightarrow K = \frac{V}{R}$$

Thus: $i = i_g + i_p = A e^{-\frac{R}{L}t} + \frac{V}{R}$

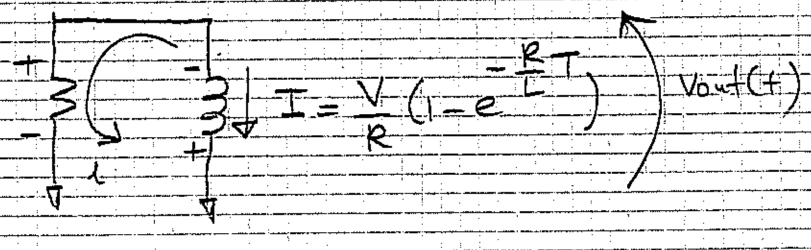
At $t=0$ the circuit looks like:



$$i(0) = A + \frac{V}{R} = 0 \Rightarrow A = -\frac{V}{R}$$

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad 0 \le t \le T$$

$\underline{T \le t < \infty}$:



$$v_R + v_L = 0 \Rightarrow iR + L\frac{di}{dt} = 0$$

$$i(t) = B e^{-\frac{R}{L}(t-T)} \qquad \text{but} \qquad i(T) = I = B$$

Thus:

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}T}\right) e^{-\frac{R}{L}(t-T)}$$

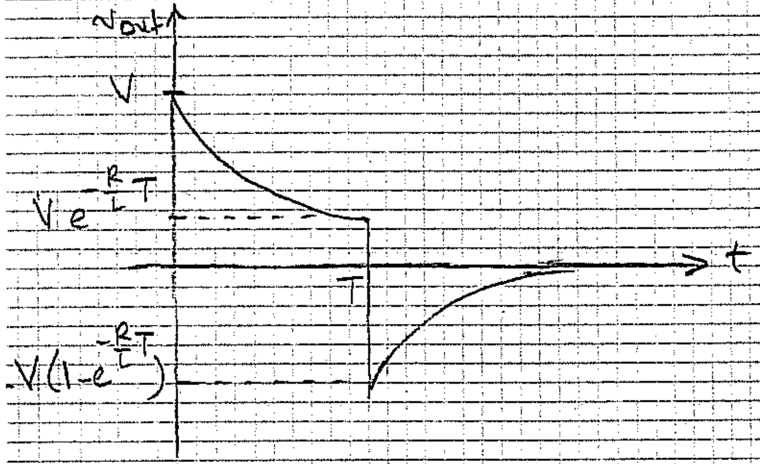
(b) $v_{out} = L\frac{di}{dt} = L\frac{V}{R}\left[0 - e^{-\frac{R}{L}t}\cdot\left(-\frac{R}{L}\right)\right] = \cancel{L}\frac{V}{\cancel{R}}\cdot\frac{\cancel{R}}{\cancel{L}}\, e^{-\frac{R}{L}t}$

$$v_{out} = V e^{-\frac{R}{L}t} \qquad 0 \le t \le T$$

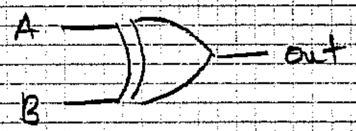
However for $T \le t < \infty$:

$$v_{out} = L\frac{di}{dt} = \frac{V}{\cancel{R}}\left(1-e^{-\frac{R}{L}T}\right)\left(-\frac{\cancel{R}}{\cancel{L}}\right)e^{-\frac{R}{L}(t-T)}\cdot\cancel{L}$$

$$= -V\left(1-e^{-\frac{R}{L}T}\right)e^{-\frac{R}{L}(t-T)}$$

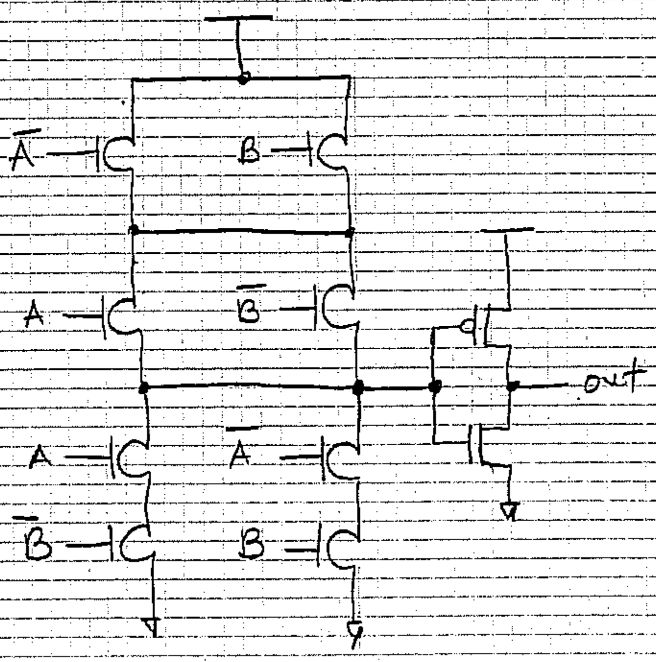


(2)



(a) $out = A \oplus B = A\bar{B} + \bar{A}B$

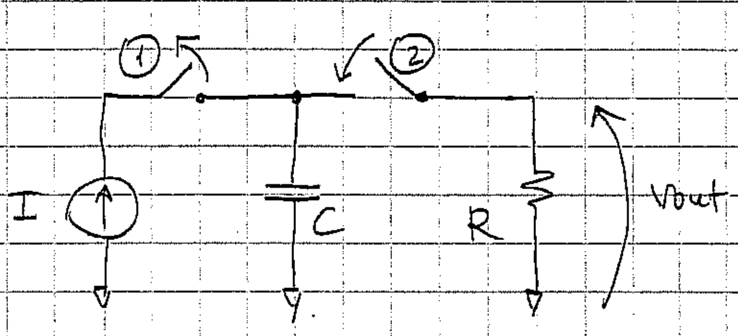
(b)



CmpE110 QUIZ #1 / F03 Dr. Ahmet BINDAL

<MAKE UP>

The following circuit is given:

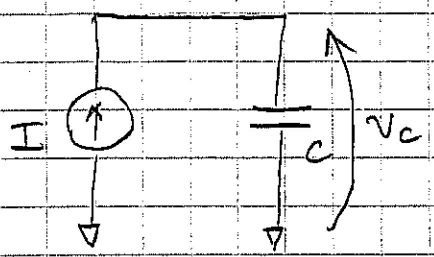


At $t=0$ switch 1 is closed switch 2 is opened.

$t=T$ switch 1 is opened switch 2 closed.

Plot the waveform at vout from $t=0$ to $t=\infty$.

Switch (1) is closed & (2) is opened

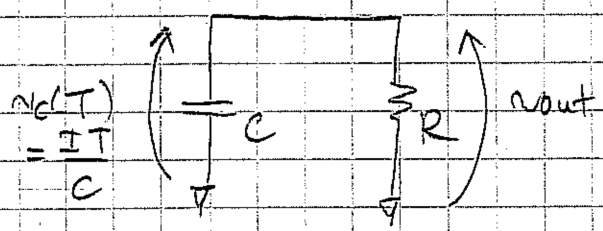


$$I = C\frac{dv_c}{dt}$$

$$v_c = \int_0^t \frac{I}{C}\,dt = \frac{I}{C}t$$

At $t = T$ $\quad v_c(T) = \frac{IT}{C}$

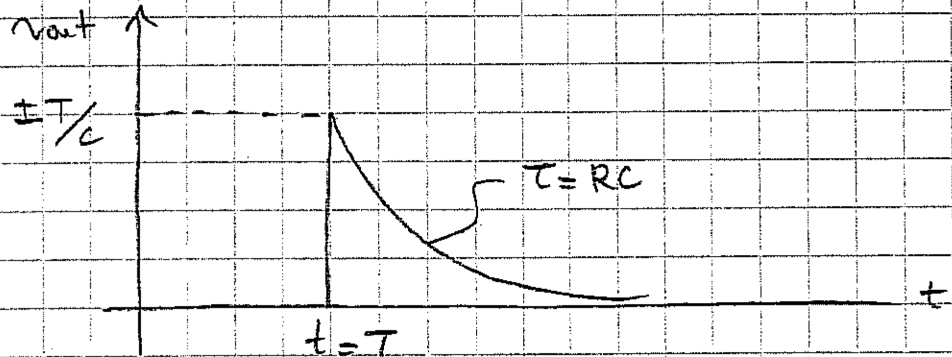
Now, at $t = T$ switch (1) is opened & (2) is closed.



Thus; $\quad v_{out}(T) = v_c(T) = \frac{IT}{C}$

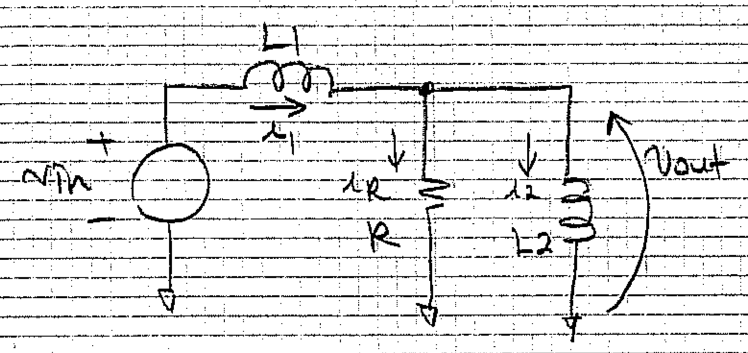
$$v_{out}(t) = \frac{IT}{C}e^{-\frac{t}{RC}} \quad \text{for } t \geq T$$

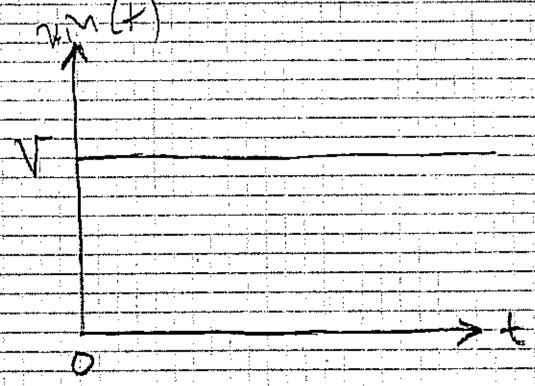
The resultant waveform:



CmpE 110 MIDTERM 3 Dr. Ahmet Bindal

(1)





50 pts

$i_1(0) = 0A$ & $i_2(0) = 0A$

find the expression for vout only.

(2) $out = AB\overline{C} + D$

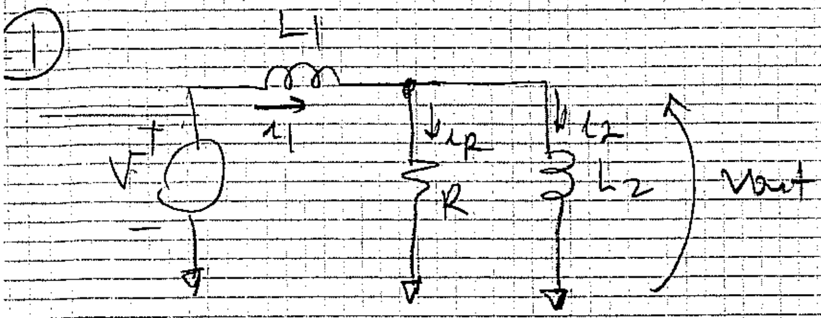
50 pts

(a) Implement the output function above with CMOS

(b) Find the transistor sizes for $T_R = T_F$ if the minimum width is W.

NOTE: Do NOT use the output inverter in (a).

1



$$i_1 = i_R + i_2 \quad (1)$$

$$V_i = v_{L1} + v_{out} = L_1 \frac{di_1}{dt} + R i_R \quad (2)$$

$$i_R R = L_2 \frac{di_2}{dt} \quad (3)$$

From (1) $i_2 = i_1 - i_R$ substitute it to (3)

$$i_R R = L_2 \frac{d}{dt}(i_1 - i_R) = L_2 \frac{di_1}{dt} - L_2 \frac{di_R}{dt}$$

$$\frac{di_1}{dt} = \frac{1}{L_2}\left(i_R R + L_2 \frac{di_R}{dt}\right) \quad \text{substitute into (2)}$$

$$V = R i_R + \frac{L_1}{L_2}\left(i_R R + L_2 \frac{di_R}{dt}\right)$$

$$= R i_R + \frac{L_1}{L_2} R i_R + L_1 \frac{di_R}{dt}$$

$$\frac{di_R}{dt} + i_R \frac{R}{L_1}\left(1 + \frac{L_1}{L_2}\right) = \frac{V}{L_1}$$

Let $K \triangleq \frac{R}{L_1}\left(1 + \frac{L_1}{L_2}\right) \Rightarrow \frac{di_R}{dt} + i_R K = \frac{V}{L_1}$

General solution =

$$\frac{di_R}{dt} + K i_R = 0 \qquad i_R = Ae^{st} = Ae^{-Kt}$$

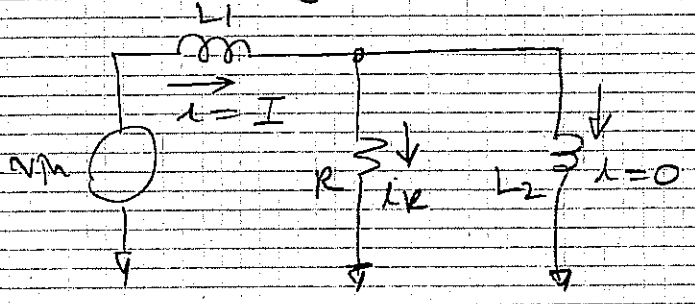
particular solution = $i_R = M$

Thus, substituting particular solution into the DE:

$$0 + MK = \frac{V}{L_1} \Rightarrow M = \frac{V}{KL_1}$$

$$i_R = Ae^{-Kt} + \frac{V}{KL_1}$$

Initially:



$$i_R(0) = 0 = A + \frac{V}{KL_1}$$

$$A = -\frac{V}{KL_1}$$

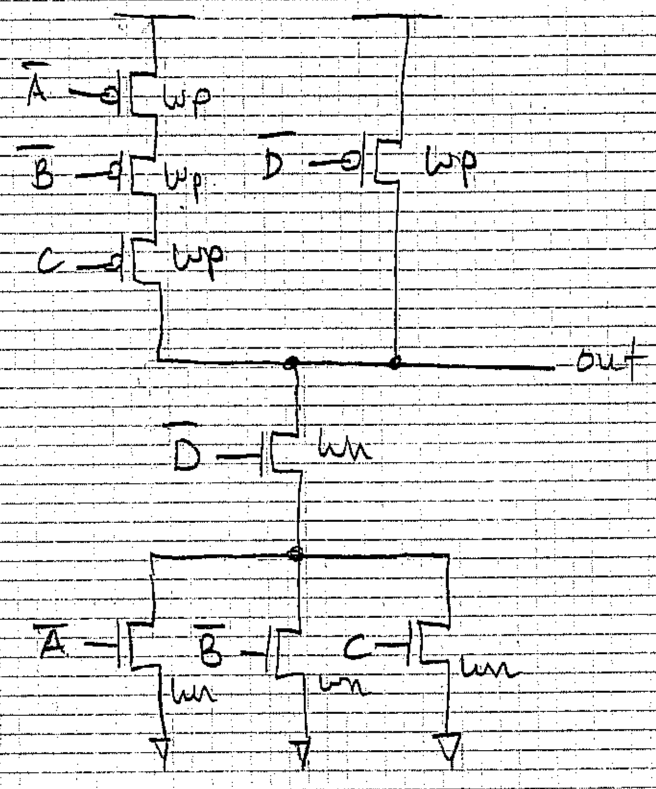
$$i_R = \frac{V}{KL_1}\left(1 - e^{-Kt}\right) = \frac{V}{\cancel{L_1}\frac{R}{\cancel{L_1}}\left(1+\frac{L_1}{L_2}\right)} \cdot \left[1 - e^{-\frac{R}{L_1}\left(1+\frac{L_1}{L_2}\right)t}\right]$$

$$v_{out} = R\,i_R = \frac{V}{\left(1+\frac{L_1}{L_2}\right)}\left[1 - e^{-\frac{R}{L_1}\left(1+\frac{L_1}{L_2}\right)t}\right]$$

$$\Rightarrow v_{out} = \frac{V L_2}{(L_1+L_2)}\left[1 - e^{-\frac{R}{L_1}\left(1+\frac{L_1}{L_2}\right)t}\right]$$

(2) $out = \overline{AB\overline{C} + D} = (\overline{A} + \overline{B} + C)\,\overline{D}$

(a)



(b)

$$T_R = 2.2\, R_{peq}\, C_L = 2.2 \times 3 \frac{K}{W_p} C_L$$

$$T_F = 2.2\, R_{neq}\, C_L = 2.2 \times 2 \frac{K}{2W_n} C_L$$

$$T_R = T_F \Rightarrow \frac{3}{W_p} = \frac{1}{W_n} \Rightarrow W_p = 3W_n$$

Let $W_n = W \Rightarrow W_p = 3W$

The following function is given:

$$f = \overline{A} + \overline{B}.\left[\overline{C} + \overline{D}.(\overline{E} + \overline{F})\right]$$

This circuit needs to drive a large fan-out composed of a large capacitor, therefore, an inverter needs to be added to its output.

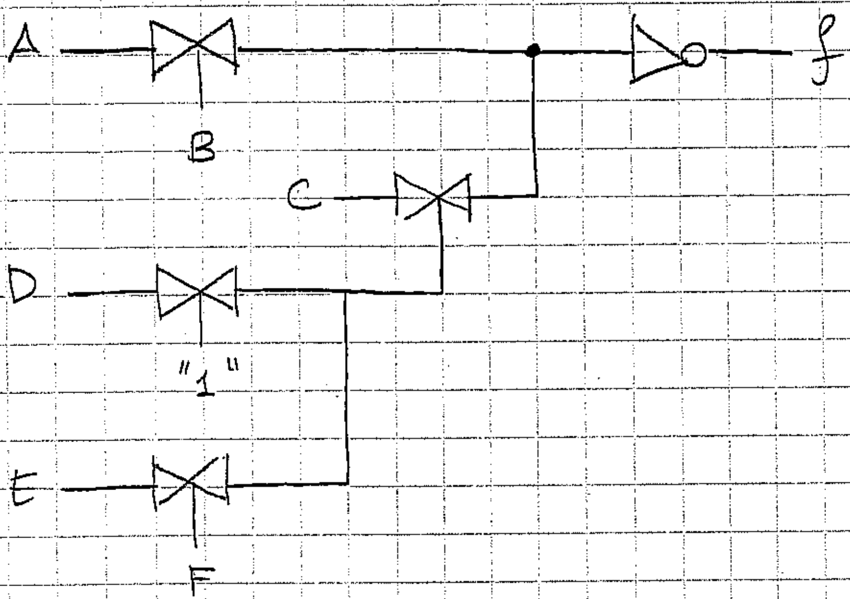
Implement the function, f, with minimum number of elements using transmission gate logic.

CmpE 110 QUIZ #3 Dr. Ahmet BINDAL

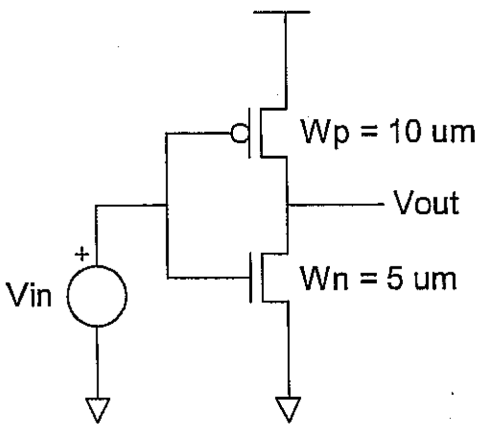
Since an inverter needs to be used at the output of f:

$$f = \overline{\overline{(\bar{A}+\bar{B})\left[\bar{C}+\bar{D}(\bar{E}+\bar{F})\right]}} = \overline{\overline{(\bar{A}+\bar{B})} + \overline{\left[\bar{C}+\bar{D}(\bar{E}+\bar{F})\right]}}$$

$$= \overline{AB + C(D+EF)}$$



An inverter below is given.
$V_{DD} = 3$ V
$V_{TN} = V_{TP} = 0.6$ V
Mobility for electrons (u_n) = 1200 cm^2/Vsec
Mobility for holes (u_p) = 600 cm^2/Vsec
$C_{OX} = 6.9 \times 10^{-7}$ F/cm^2
L = 0.25 um



The current through a MOSFET is defined as:

$$I_D = \frac{\mu C_{OX} W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

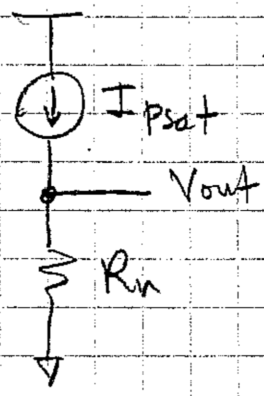
(a) Compute the static DC current when Vin = 2 V ($C_{OUT} = 0$ F).
(b) Compute the output voltage when Vin = 2 V.

HINT: Consider the equivalent circuits of nfet and pfet at this bias condition.

For $V_{in} = 2\,V \Rightarrow$ nfet is in linear region

pfet is in saturation region.

Eq. circuit =



$$I_{psat} = \frac{\mu_p C_{ox} W_p}{2L}(V_{SGp} - V_T)^2$$

$$= \frac{600 \times 6.9 \times 10^{-7} \times 10}{2 \times 0.25}(1 - 0.6)^2$$

$$= 1.32\ mA$$

For nfet $\quad I_D \simeq \frac{\mu_n C_{ox} W}{L}(V_{GSn} - V_T) V_{DSn} = I_{psat}$

$$= \frac{1200 \times 6.9 \times 10^{-7} \times 5}{0.25}(2 - 0.6) V_{out} = 1.32 \times 10^{-3}$$

$$V_{out} = \frac{1.32 \times 10^{-3}}{0.023} = 57\ mV$$

(1) $sum = A \oplus B \oplus Cin$
$Cout = AB + Cin(A + B)$

(a) Draw the schematic in CMOS implementing sum and Cout. Assume to use an output inverter due to high load capacitance, CL.

(b) Draw the schematic in transmission gates implementing sum and Cout. Also use an output inverter due to high load capacitance, CL.

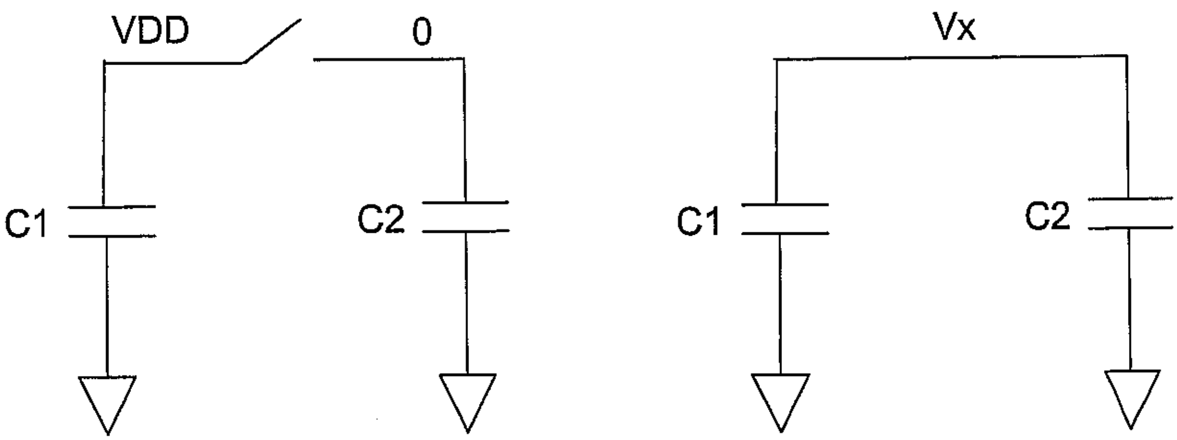
(2) (a) Obtain only the pfet tree for the following function,
$out = \overline{E} + \overline{D}(\overline{A} + \overline{BC})$

(b) Now construct the nfet tree separately from the pfet tree obtained in (a), yielding the same function, *out.*

(c) Now join the nfet and pfet trees and size them according to Rinv = 4 (inverter ratio) in units of W.
The minimum width must be W/2 in the schematic.

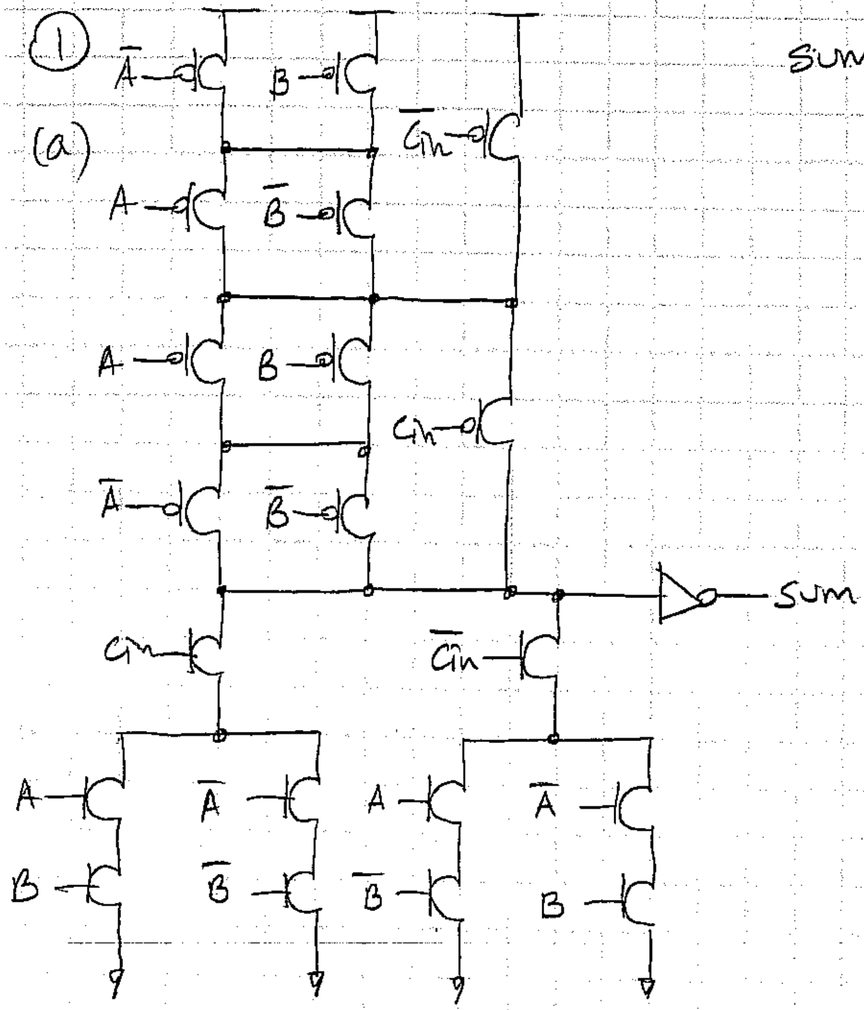
(d) Find TR/TF (the ratio of rise time to fall time).

(3) Find Vx after the switch closes.



1

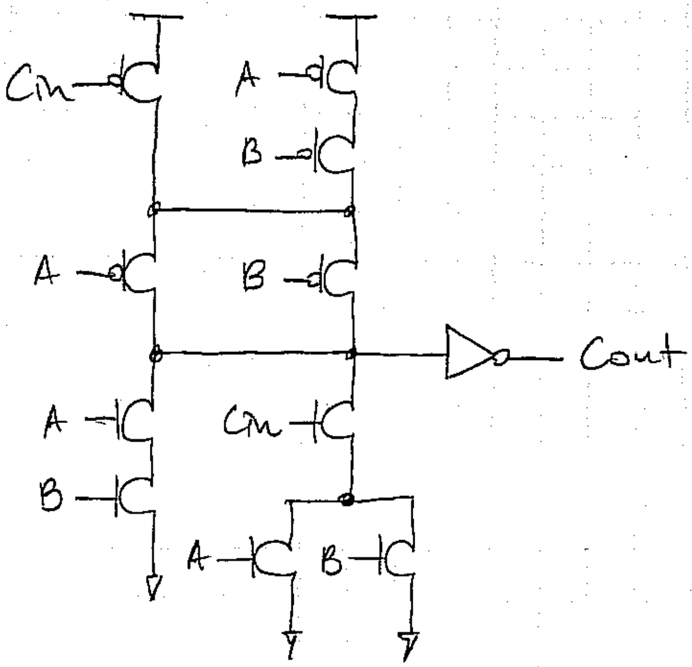
(a)



$$Sum = A \oplus B \oplus C = (A\bar{B} + \bar{A}B) \oplus C_{in}$$

$$= \overline{(A\bar{B} + \bar{A}B)}\, C_{in} + (A\bar{B} + \bar{A}B)\, \overline{C_{in}}$$

$$= (\bar{A}\bar{B} + AB)\, C_{in} + (A\bar{B} + \bar{A}B)\, \overline{C_{in}}$$



$$C_{out} = AB + C_{in}(A + B)$$

(b) $\text{Sum} = \overline{\overline{A \oplus B \oplus C}} = \overline{\overline{(\bar{A}B + A\bar{B}) \oplus C_{in}}}$

$$= \overline{\overline{(\bar{A}B + A\bar{B})\,\overline{C_{in}} + (AB + \bar{A}\bar{B})\,C_{in}}}$$

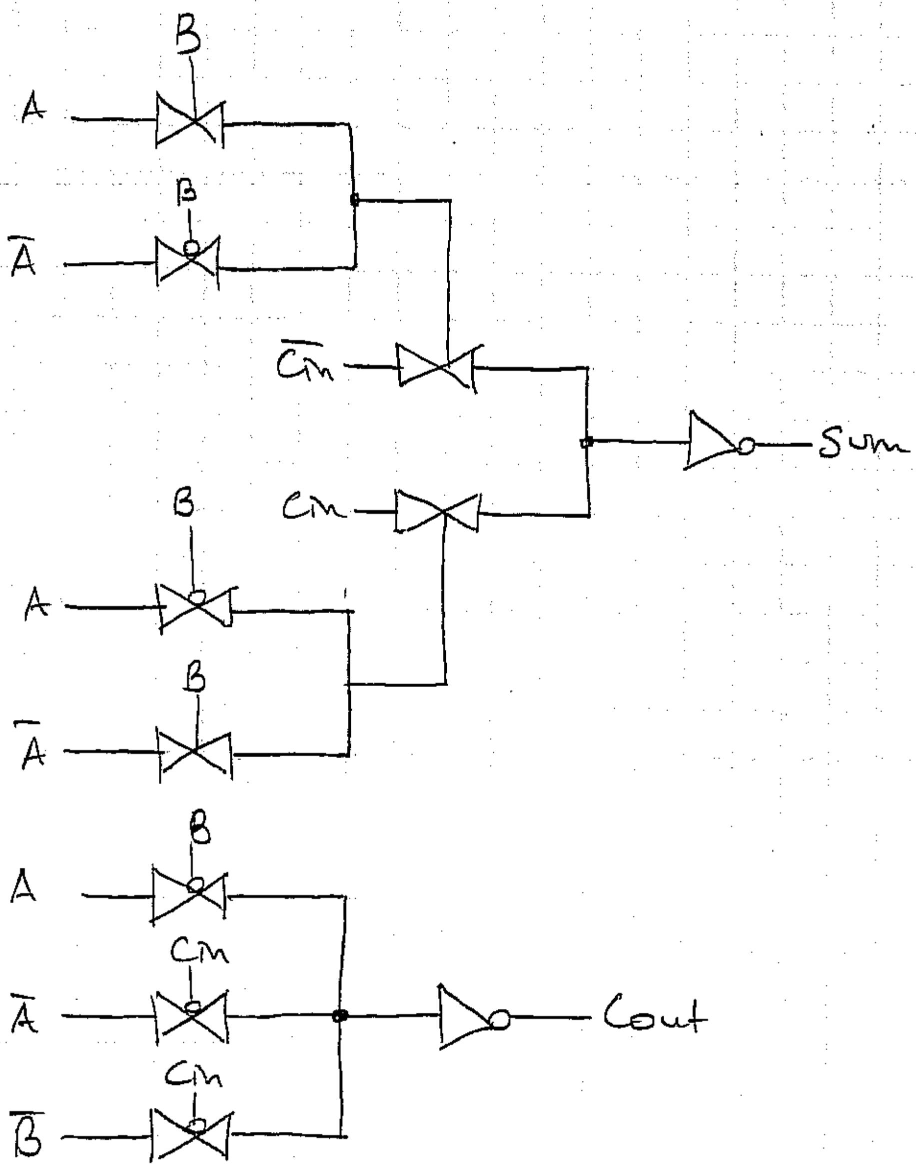
$$= \overline{\left[(AB + \bar{A}\bar{B}) + C_{in}\right]\left[(A\bar{B} + \bar{A}B) + \overline{C_{in}}\right]}$$

$$= \overline{(AB + \bar{A}\bar{B})(A\bar{B} + \bar{A}B) + \overline{C_{in}}(AB + \bar{A}\bar{B}) + C_{in}(A\bar{B} + \bar{A}B)}$$

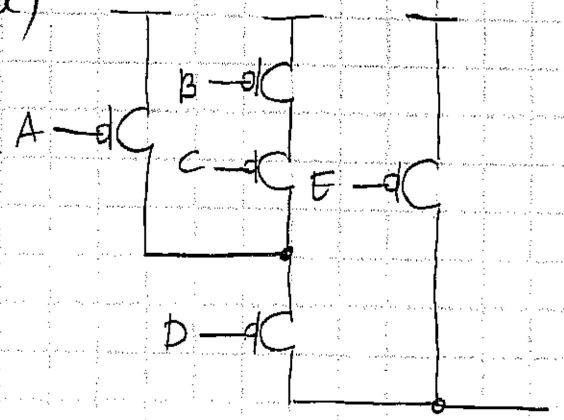
$$= \overline{\overline{C_{in}}\,(AB + \bar{A}\bar{B}) + C_{in}(A\bar{B} + \bar{A}B)}$$

$$C_{out} = \overline{\overline{AB + C_{in}(A + B)}} = \overline{(\bar{A} + \bar{B})(\overline{C_{in}} + \bar{A}\bar{B})} = \overline{\overline{C_{in}}\,(\bar{A} + \bar{B}) + \bar{A}\,\bar{B}}$$

$$= \overline{\overline{C_{in}}\,\bar{A} + \overline{C_{in}}\,\bar{B} + \bar{A}\,\bar{B}}$$



(2) (a)



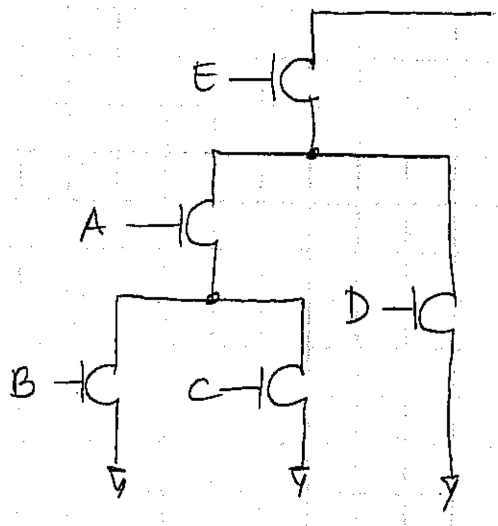
$out = \overline{E} + \overline{D}\,(\overline{A} + \overline{B}\,\overline{C})$

(b) Since $out = \overline{E} + \overline{D}\,(\overline{A} + \overline{B}\,\overline{C})$

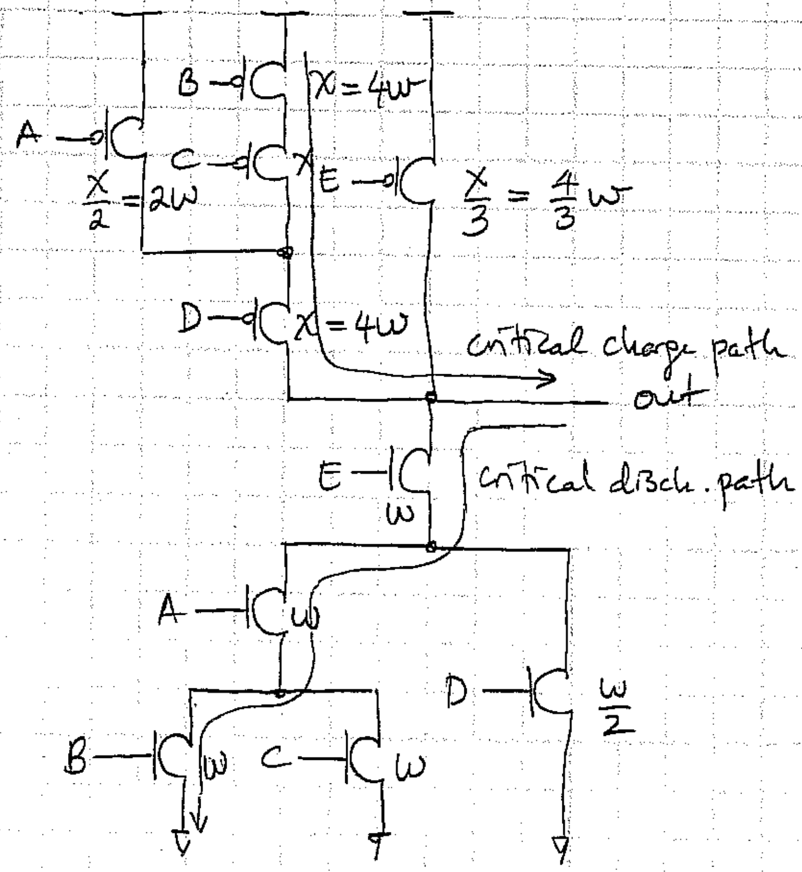
$\overline{out} = \overline{E\,\{D + [A\,(B+C)]\}} = E\,[D + A\,(B+C)]$

Thus, the nfet tree:

$\overline{out} = E\,[D + A(B+C)]$



(c)



$$R_{neq} = \frac{K}{2w} \times 3 = \frac{K}{2\left(\frac{w}{3}\right)} \Rightarrow w_{neq} = \frac{w}{3}$$

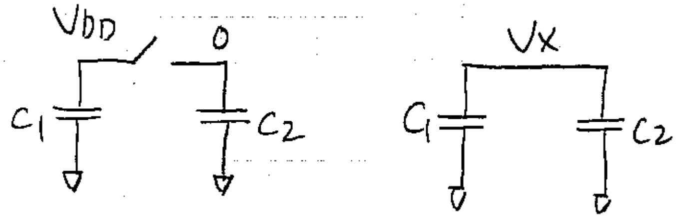
$$R_{peq} = \frac{K}{x} \times 3 = \frac{K}{\left(\frac{x}{3}\right)} \Rightarrow w_{peq} = \frac{x}{3}$$

Since $R_{NO} = 4 = \frac{w_{peq}}{w_{neq}} \Rightarrow 4 = \frac{x}{\cancel{3}} \cdot \frac{\cancel{3}}{w} \Rightarrow x = 4w$

(d)

$$\left.\begin{aligned} T_R &= 2.22\, C_L \frac{K}{4w} \times 3 \\ T_F &= 2.22\, C_L \frac{K}{2w} \times 3 \end{aligned}\right\} \quad T_R / T_F = \frac{1}{4w} \cdot 2w = 0.5$$

(3)



$Q_{total\,bef} = C_1 V_{DD}$ $\qquad$ $Q_{total\,after} = (C_1 + C_2) V_x$

$Q_{tot\,bef} = Q_{tot\,after}$

$C_1 V_{DD} = (C_1 + C_2) V_x$

$$V_x = \frac{C_1}{(C_1 + C_2)} V_{DD}$$

Implement the following function using CMOS.

$$out = \left(A + .\overline{B}C\right)\left(D + E\right)$$

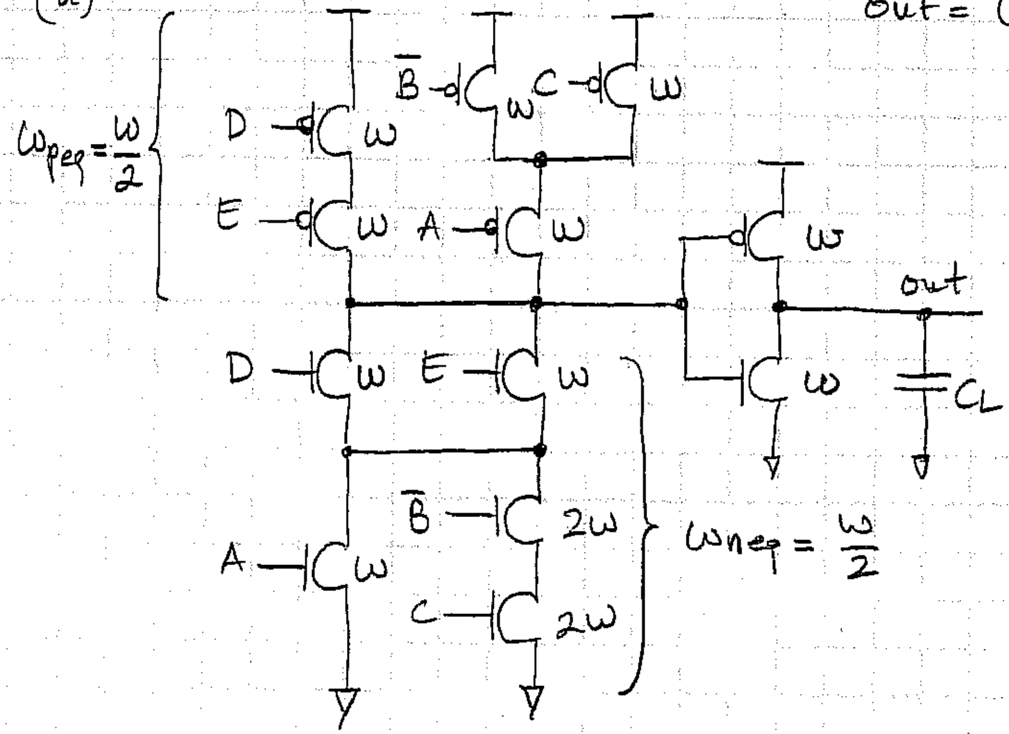
(a) Implement this function with an inverter at the output. Size the transistors such that the inverter ratio, Ri, is equal to 1. Use W as the minimum width.

(b) Implement the same function without an output inverter. Size the transistors such that TR = 2 TF. Use W as the minimum NMOS transistor width (PMOS width can be smaller).
TR = 2.22 CL Rpeq
TF = 2.22 CL Rneq
CL is the output load capacitor Rpeq and Rneq are th equivalent resistors in the charge and discharge paths, respectively.
Ignore the intrinsic capacitances and the resulting Elmore's effect.

CmpE 110 QUIZ #3 / SPRING 05 Dr. Ahmet BINDAL

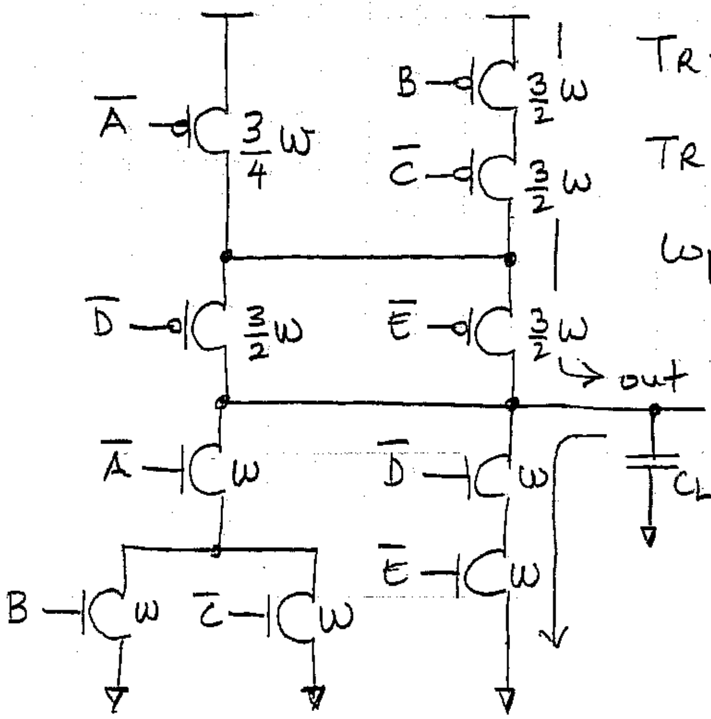
(a)

out = $(A+\bar{B}C)(D+E)$



$$R_T = \frac{W_{peq}}{W_{neq}} = \frac{W/2}{W/2} = 1$$

(b) $out = \overline{\overline{(A+\bar{B}C)(D+E)}} = \overline{\bar{A}(B+\bar{C})+\bar{D}\bar{E}}$



$$T_F = 2.22\, C_L\, 2R_n = 2.22\, C_L\, \cancel{2} \frac{K}{\cancel{2}W_n}$$

$$T_R = 2.22\, C_L\, 3R_p = 2.22\, C_L\, 3\frac{K}{W_p}$$

$$T_R = 2T_F \Rightarrow \frac{3}{W_p} = 2\frac{1}{W_n}$$

$W_p = \frac{3}{2} W_n$ Let $W_n = W$

Then; $W_p = \frac{3W}{2}$