$$\begin{aligned}
K_3 * K_4 &= m_0 * K_4 + m_1 * K_4 + m_2 * K_4 + m_3 * K_4 + m_8 * K_4 + m_9 * K_4 \\
&\qquad + m_A * K_4 + m_B * K_4 \\
&= 0 + m_1 * m_1 + m_2 * m_2 + 0 + 0 + m_9 * m_9 \\
&\qquad + m_A * m_A + 0 \\
&= 0 + m_1 + m_2 + 0 + 0 + m_9 \\
&\qquad + m_A + 0 \\
&= m_1 + m_2 + m_9 + m_A
\end{aligned}$$

3.13 Map-entered Variables

The K map becomes unwieldy even with five variables; with more than five variables the K map rapidly becomes unmanageable. The map-entered variable reduces the required map size, thereby extending the K map's practical usefulness. The number of function variables equals the number of K map dimensions plus the number of map-entered variables. How are variables entered into a map?

Coefficients of terms are entered into squares.

Up to this point the only K map entries have been 1, 0, and – (for don't care). This set of entries implies all variables in a function's equation are assigned a dimension of a K map.

Literals may be interpreted as coefficients of terms.

K map entries are coefficients of product terms in the equation. Consider the simplest case. When one literal in a product term of n literals is treated as a coefficient, does the literal replace the 1 normally entered into the K map? The answer is yes. This idea is the basis for map-entered variables. In effect this idea is the same as the table-entered variable idea (Section 3.11.4).

3.13.1 Writing the Map

The equation $g = xy + yz'$ has three variables: x, y, z. The number of K map variables reduces to two if x is treated as a coefficient. When x or x' is not in a term, the coefficient is 1.

$$g = xy + yz' = x(y) + 1(yz')$$

We know how to map the yz' term because the coefficient is 1. It maps into square 2. The coefficient of the y term is x. If the coefficient is one, the y term would map into squares 3 and 2. So, instead of 1 enter the coefficient x into squares 3 and 2 (Figure 3.22). The final step is to logically OR multiple entries in a square. In this case $1 + x$ becomes 1 (Figure 3.23). Next, we show why the method is a general solution.

FIGURE 3.22
K Map for $g = xy + yz'$ with map-entered variable x

$g = xy + yz'$

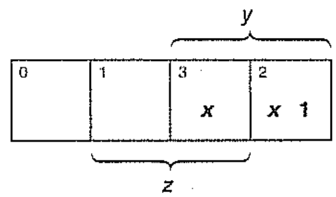


FIGURE 3.23
K Map for $g = xy + yz'$ after logical OR of entries

$g = xy + yz'$

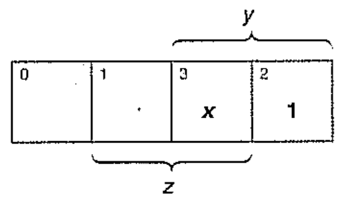


FIGURE 3.24
K Map for $g = xy + yz'$ with all minterms mapped

$g(x,y,z)$

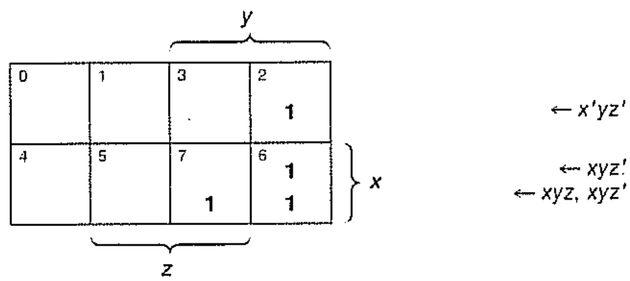


FIGURE 3.25
K Map for $g = xy + yz'$ with map entered variable x

$g = xy + yz' = xyz + xyz' + xyz' + x'yz'$

$g(x,y,z)$

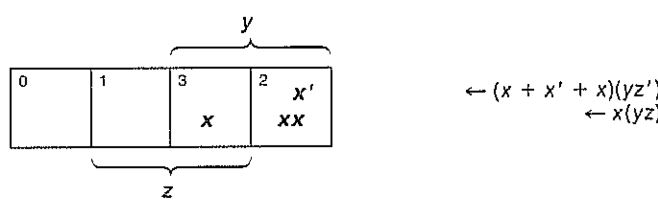
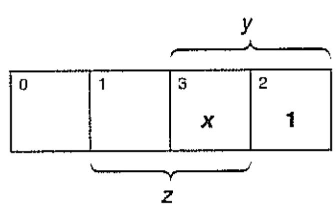


FIGURE 3.26
K Map for $g = xy + yz'$ with map entered variable x

$g = xy + yz'$

$g(x,y,z)$



The equation $g = xy + yz'$ has three variables: x, y, z. Designate x as a map-entered variable and then reduce the K map to the two dimensions y, z.

$$g = xy + yz'$$

(form all minterms)

$$g = xy(z + z') + (x + x')yz'$$

$$g = xyz + xyz' + xyz' + x'yz'$$

(make x, x' coefficients)

$$g = x(yz) + x(yz') + x(yz') + x'(yz')$$

Map the four minterms into a 3-variable K map (Figure 3.24).

Collapse the x dimension. The four 1s from Figure 3.24 convert to three x and one x' because x or x' is a coefficient of each two dimensional minterm (Figure 3.25).

Since $(x + x' + x) = 1$ the two x's and one x' in square 2 of Figure 2.25 are replaced by a 1 to obtain the final map (Figure 3.26) with map entered variable x.

3.13.2 Reading the Map

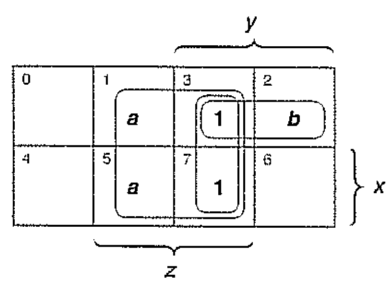
Here is one algorithm for reading K maps with map entered variables.

1's are treated as don't cares because a + a' = 1.

A. Set all map entered variables to 0. Read all 1s.
B. Restore one map entered variable and read the map while treating all 1s as don't cares. Set the map entered variable back to 0.
C. If there is another map-entered variable, repeat step two or else quit.

EXAMPLE 3.32

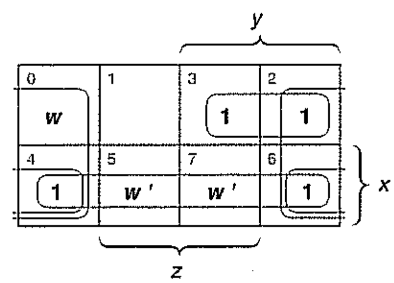
$g(x, y, z, a, b)$



1. Step A. Set $a = b = 0$. Read the column of 1s as yz.
2. Step B. Restore a. With $b = 0$ and the 1s as don't cares read the four square cluster above z as az because a is the coefficient. Set $a = 0$.
3. Step C. b is another map-entered variable. Go to Step B.
4. Step B. Restore b. With $a = 0$ and the 1s as don't cares read the two square cluster below y as $bx'y$ because b is the coefficient. Set $b = 0$.
5. Step C. There are no more map entered variables. Quit.

Therefore $$g = yz + az + bx'y$$

EXAMPLE 3.33



1. Step A. Set $w = w' = 0$ (strange but correct). Read the 1s as $x'y + xz'$.
2. Step B. Restore w. With $w' = 0$ and the 1s as don't cares, read the four-square cluster above z' as wz' because w is the coefficient. Set $w = 0$.
3. Step C. w' is another map-entered variable. Go to Step B.
4. Step B. Restore w'. With $w = 0$ and the 1s as don't cares, read the four-square cluster in row x as $w'x$ because w' is the coefficient. Set w' to 0.
5. Step C. There are no more map-entered variables. Quit.

Therefore $$\begin{aligned} g &= wz' + w'x + xz' + x'y \\ &= wz' + w'x + x'y \end{aligned}$$

Note: xz' is a consensus term.

Exercise 3.19 Read the K map.

0 a	1 1	3 1	2
4 1	5	7	6 1

One Answer: $g = ax'y' + (x \text{ xor } z)$ ■

Exercise 3.20 Read the K map.

0	1 1	3 a	2 1
4 1	5	7 1	6 b

One Answer: $g = ax'y + bxy + (x \text{ xor } y \text{ xor } z)$ ■

Example 3.34 Reading a K-map with Map-Entered Variables

$g(w,x,y,z,a,b,c)$

0	1 bc	3 $d+ef$	2
4 1	5 1	7	6
C a	D a	F	E
8	9	B 1	A

1. Step A. Set $a = bc = d + ef = 0$. Read the ones as $w'xy' + wx'yz$.

2. Step B. Restore a. With $bc = d + ef = 0$ and the 1s as don't cares, read the four-square cluster to the left of x as axy' because a is the coefficient. Set $a = 0$.
3. Step C. bc is another map-entered variable. Go to step B.
4. Step B. Restore bc. With $a = d + ef = 0$ and the 1s as don't cares read the two square cluster in the $y'z$ column as $bcw'y'z$ because bc is the coefficient. Set $bc = 0$.
5. Step C. $(d + ef)$ is another map-entered variable. Go to step B.
6. Step B. Restore $(d + ef)$. With $a = bc = 0$ and the 1s as don't cares, read the two square cluster in the yz column as $(d + ef)x'yz$ because $(d + ef)$ is the coefficient. Set $(d + ef) = 0$.
7. Step C. There are no more map-entered variables. Quit.

Therefore $g = w'xy' + wx'yz + axy' + bcw'y'z + (d + ef)x'yz$

SUMMARY

Axiomatic Basis of Switching Algebra

Huntington proved that six axioms provide a complete basis for *Boolean algebra.*

Principle of Duality

If a Boolean statement is true then the dual of the statement is true.

Constants

There are only two constants in switching algebra: *true T and false F* represented by the symbols 1 and 0 respectively. The symbols 1 and 0 are not numbers here. Different meanings may be given to 1 and 0.

Variables

Switching algebra is not an algebra of numbers. Switching algebra is an algebra of *states* represented by the constants 0 and 1. A variable is in state 0 or it is in state 1.

> Variables are assigned the constants 0 or 1. Again, we do not say numbers because 0 does not mean "zero" and 1 does not mean "one" in this algebra. 1 and 0 may mean a switch is on or off, a voltage is high or low, or some other binary pair (Table 3.2). Nevertheless there is no harm thinking in terms of numbers 0 and 1 so long as the context is not ignored.