EE 102
Probability and Statistics in Electrical Engineering
MIDTERM 1
SPRING 2014

NAME: SOLUTION

Problem 1	35	
Problem 2	35	
Problem 3	30	
Total	100	

+5 (extra credit)

Notes:

- Show your work for full/partial credit
- In the exam, $P[A]$ denotes the probability of event A happening.
- Show your work explicitly.

Problem 1. For a discrete random variable X, the cumulative distribution function (CDF) is given as in Fig. 1.

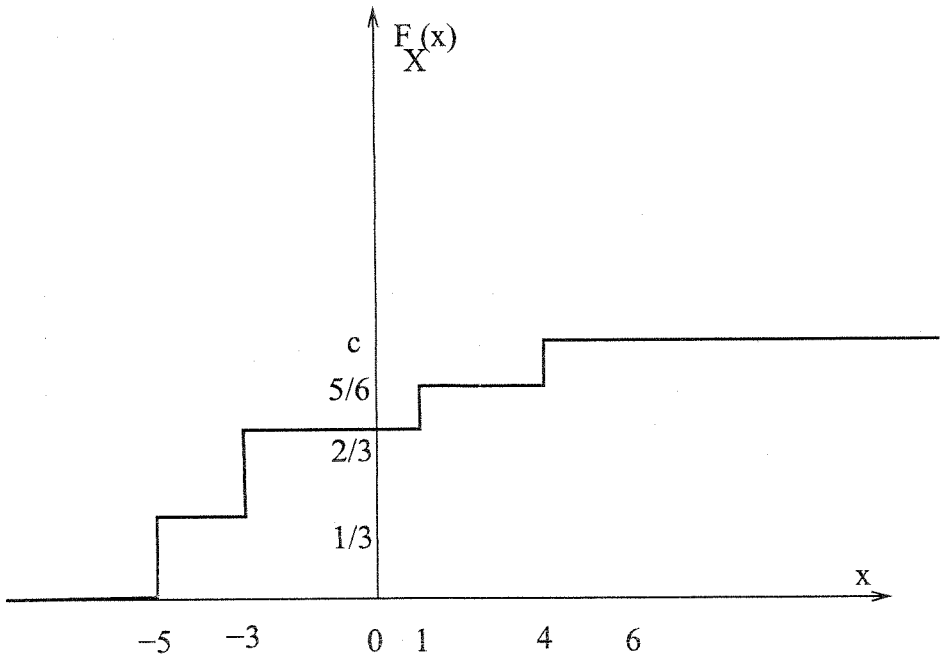


Figure 1: CDF for Random Variable X.

10 a) Write the sample space for X.

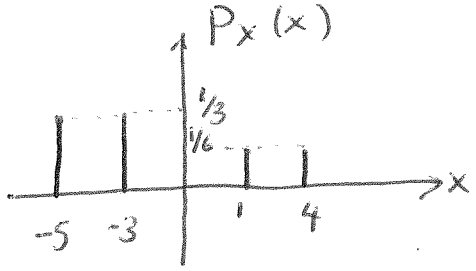
10 b) Find and sketch the probability mass function (PMF) for X.

5 c) What kind of random variable is X? Explain your reasoning.

10 d) Calculate the conditional probability $P[X > 2|X < 6]$.

a) $S_X = \{-5, -3, 1, 4\}$

b) $P_X(x) = \begin{cases} 1/3 & x = -5 \text{ or } x = -3 \\ 1/6 & x = 1 \text{ or } x = 4 \\ 0 & \text{o.w.} \end{cases}$



c) X is a discrete random variable since the sample space is countable.

d) $P[X > 2 | X < 6] = \dfrac{P[2 < X < 6]}{P[X < 6]} = \dfrac{1/6}{1} = \dfrac{1}{6}$

Problem 2. A player has three decks of cards: RED, BLUE, and GREEN. The RED deck has the standard 52 cards, but the BLUE deck is missing all the spades, and the GREEN deck is missing Ace of Hearts.

Hint: Remember that a standard deck of playing cards is 52-cards. There are 4 suits in a deck: clubs, hearts, diamonds and spades. Each suit has 13 cards. Each deck contains 4 aces-one from each suit.

a) A FAIR die is thrown, and a deck is selected according to the number showing on the die: RED deck is selected if the die shows a number less than 3. BLUE deck is selected if the die shows a number greater than 3. GREEN deck is selected if the die shows exactly 3.

From the selected deck, we pick FOUR cards.

10 a1) Find the probability of picking exactly two aces.

10 a2) If two aces are picked, what is the probability that the BLUE deck was selected?

b) The player mixes all the decks and then picks FOURS cards (the player is not biased by the color of the cards).

10 b1) Find the probability of picking exactly two aces.

5 b2) If two aces are picked, what is the probability that they are from BLUE deck?

(Extra Credit) → 5 c) Discuss and compare your results in Part-a and Part-b.

a) $P(\text{RED}) = P(\text{Die} < 3) = \frac{1}{3}$

$P(\text{GREEN}) = P(\text{Die} = 3) = \frac{1}{6}$

$P(\text{BLUE}) = P(\text{Die} > 3) = \frac{1}{2}$

$$P(\text{two aces} \mid \text{RED}) = \frac{\binom{4}{2}\binom{48}{2}}{\binom{52}{4}}$$

$$P(\text{two aces} \mid \text{GREEN}) = \frac{\binom{3}{2}\binom{48}{2}}{\binom{51}{4}}$$

$$P(\text{two aces} \mid \text{BLUE}) = \frac{\binom{3}{2}\binom{36}{2}}{\binom{39}{4}}$$

Using total prob. law,

$P(\text{two aces}) = P(\text{two aces} \mid \text{RED})\,P(\text{RED}) + P(\text{two aces} \mid \text{BLUE})P(\text{BLUE})$
$+ P(\text{two aces} \mid \text{GREEN})\,P(\text{GREEN})$

$= 0.0021$

Using Bayes thm,

$$P(\text{BLUE} \mid \text{two aces}) = \frac{P(\text{two aces} \mid \text{BLUE})\, P(\text{BLUE})}{P(\text{two aces})}$$

$$= \frac{\binom{3}{2}\binom{36}{2}\cdot\frac{1}{2} \Big/ \binom{39}{4}}{0.0221} = 0.5199$$

b)

Total = 142 cards

# of Aces = 10

$$P(\text{two aces}) = \frac{\binom{10}{2}\binom{132}{2}}{\binom{142}{4}} = 0.0240$$

$$P(\text{BLUE} \mid \text{two aces}) = \frac{\binom{3}{2}}{\binom{10}{2}} = 0.0667$$

Problem 3. Everyday, Mary asks her friend to pick a number (integer) between 0 and 4 uniformly and randomly, and then she buys that many number of chocolate bars and she eats them. She spends 4 dollars for each chocolate bar.

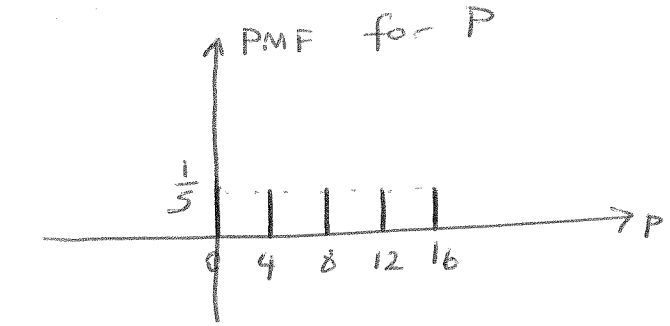
10 a) Let N denote the number of chocolate bars she eats each day. What type of random variable is N? Specify its parameters clearly.

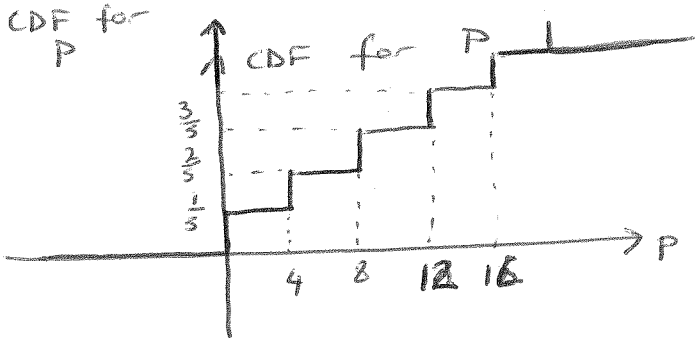
10 b) Let P denote the money she spends on chocolate each day. Sketch the CDF for P.

10 c) Each of the chocolate bars Mary buys has nuts in it with probability 0.4. Let M denote the number of chocolate bars with nuts in it. What is the probability that M is larger than 2?

a) $N \sim$ DiscreteUniform $(0,4)$

b) $P = 4N$





c) $P[M>2] = \sum_{n=3}^{4} P[M>2 \mid N=n]\, P[N=n]$

$= \left\{ \binom{3}{3} 0.4^3 0.6^0 + \left[\binom{4}{3} 0.4^3 0.6 + \binom{4}{4} 0.4^4 0.6^0 \right] \right\} \frac{1}{5}$

$= 0.0486$