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CmpE 110 **Midterm/Fall 2013** **Dr. Ahmet Bindal**

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Question 1 (50 points):
The following circuit is given:

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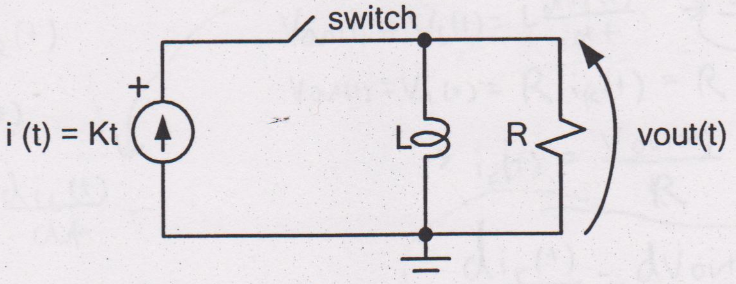

In this circuit, the switch is closed at t =0sec. The input current is linearly varying with time, i(t) = Kt, with a slope of K (a constant value) as shown.
Assuming that the current stored in the inductor, iL(0) = 0A. Find the expression for vout (t) and plot it using **time-domain analysis** (diff equation method).

Question 2 (50 points):
The following circuit is given:

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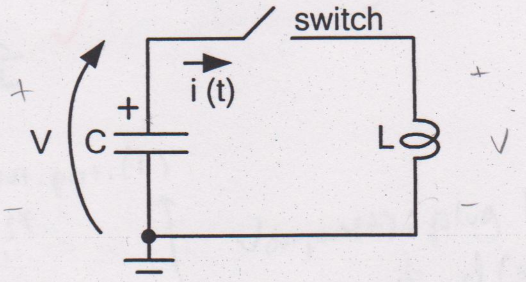

In this circuit, the switch is closed at t =0sec. Assuming the voltage across the capacitor at t = 0sec, is 0V at the direction shown in the figure, determine the expression for current, i(t), the voltage across the inductor, $v_L(t)$, the voltage across the capacitor, vc (t) using **time-domain analysis**. Plot each waveform.

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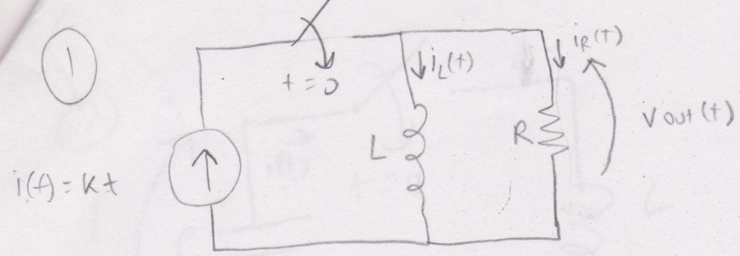


$i_L(0) = 0A$

Find $V_{out}(t)$

$i(t) = i_L(t) + i_R(t)$

$i_R(t) = i(t) - i_L(t)$

$$\frac{di_r(t)}{dt} = \frac{di(t)}{dt} - \frac{di_L(t)}{dt}$$

$V_{out}(t) = V_L(t) = L\frac{di_L(t)}{dt} \rightarrow \frac{di_L(t)}{dt} = \frac{V_{out}(t)}{L}$

$V_{out}(t) = V_L(t) = R \cdot i_R(t) = R\left(i(t) - i_L(t)\right) \rightarrow$

$\rightarrow i_r(t) = \frac{V_{out}(t)}{R}$

$$\frac{di_r(t)}{dt} = \frac{dV_{out}(t)}{R}$$

$$\frac{1}{R}\frac{dV_{out}(t)}{dt} = K - \frac{V_{out}(t)}{L}$$

$$\mathcal{L}\left(\frac{1}{R}\frac{dv_{out}(t)}{dt} + \frac{1}{L}V_{out}(t) = K\right)$$

$$\frac{s}{R} + \frac{1}{L} = 0 \rightarrow s = -\frac{R}{L}$$

$V_{out}(t) = V_{out.gen.}(t) + V_{out.part.}(t)$

$V_{out\,gen}(t) = Ae^{st}$

$V_{out\,part}(t)$: plug K into dif.eq:

$\cancel{\frac{1}{R}\cdot\frac{d(K)}{dt} + \frac{1}{L}(K) = 0}$

$\cancel{-\frac{1}{R} = \frac{1}{L}K \rightarrow K = -\frac{L}{R}}$

NO!

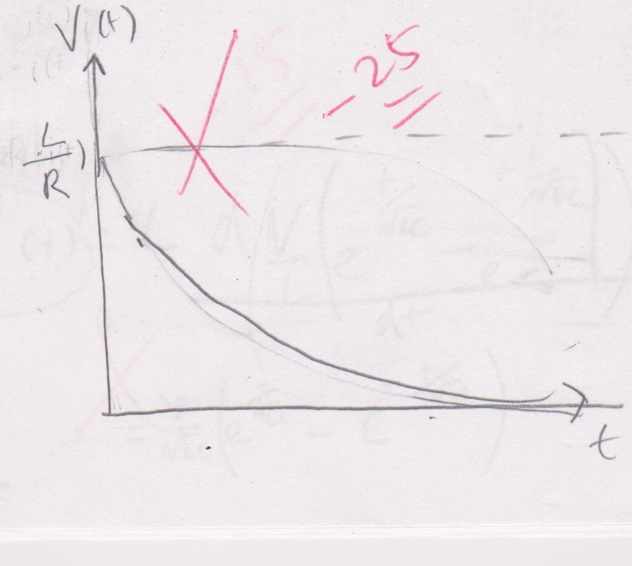
$V_{out}(t) = Ae^{st} - \frac{L}{R}$

@ $t=0$:

$V_{out}(0) = A - \frac{L}{R} = 0$

$A = \frac{L}{R}$

$V_{out}(t) = \frac{L}{R}\left(e^{-\frac{Rt}{L}}\right)$



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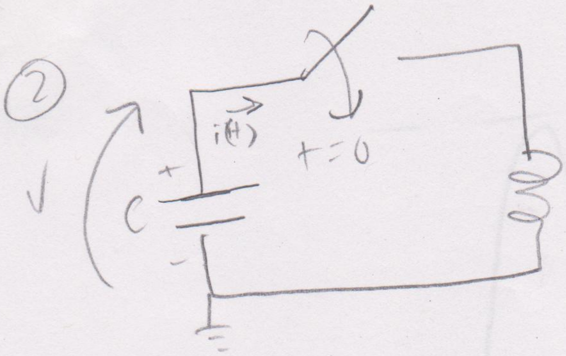
② 

$V_c(0) = 0$

Find $i(t)$, $V_c(t)$, $V_L(t)$

$$i_c = C\frac{dV_c(t)}{dt} = C\frac{dV_L(t)}{dt}$$

$$V_L = L\frac{di_L(t)}{dt}$$

$$i_c(t) = LC\frac{d^2 i_L(t)}{dt^2} =$$

$$V_c'(t) = V_L(t)$$

$$-i_c(t) = i(t) = i_L(t)$$

$$\frac{1}{LC}\left(\frac{d^2 i_L(t)}{dt} + i_c(t) = 0\right)$$

$$\frac{d^2 i_L(t)}{dt} + \frac{i_L(t)}{LC} = 0$$

$$s^2 = \frac{1}{LC} \qquad \alpha = 0 \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$s = \pm j\sqrt{\frac{1}{LC}}$$

$$i(t) = Ae^{s_1 t} + Be^{s_2 t} + 0$$

$$i(0) = 0 = A + B \rightarrow B = -A$$

$$\frac{di(0)}{dt} = s_1 A e^{s_1 t} + s_2 B e^{s_2 t}$$

$$= s_1 A + s_2 B = \frac{V_L}{L}$$

$$= s_1 A - s_2 A = \frac{V_L}{L}$$

$$= A(s_1 - s_2) = \frac{V_L}{L}$$

$$A = \frac{V_L}{L}$$

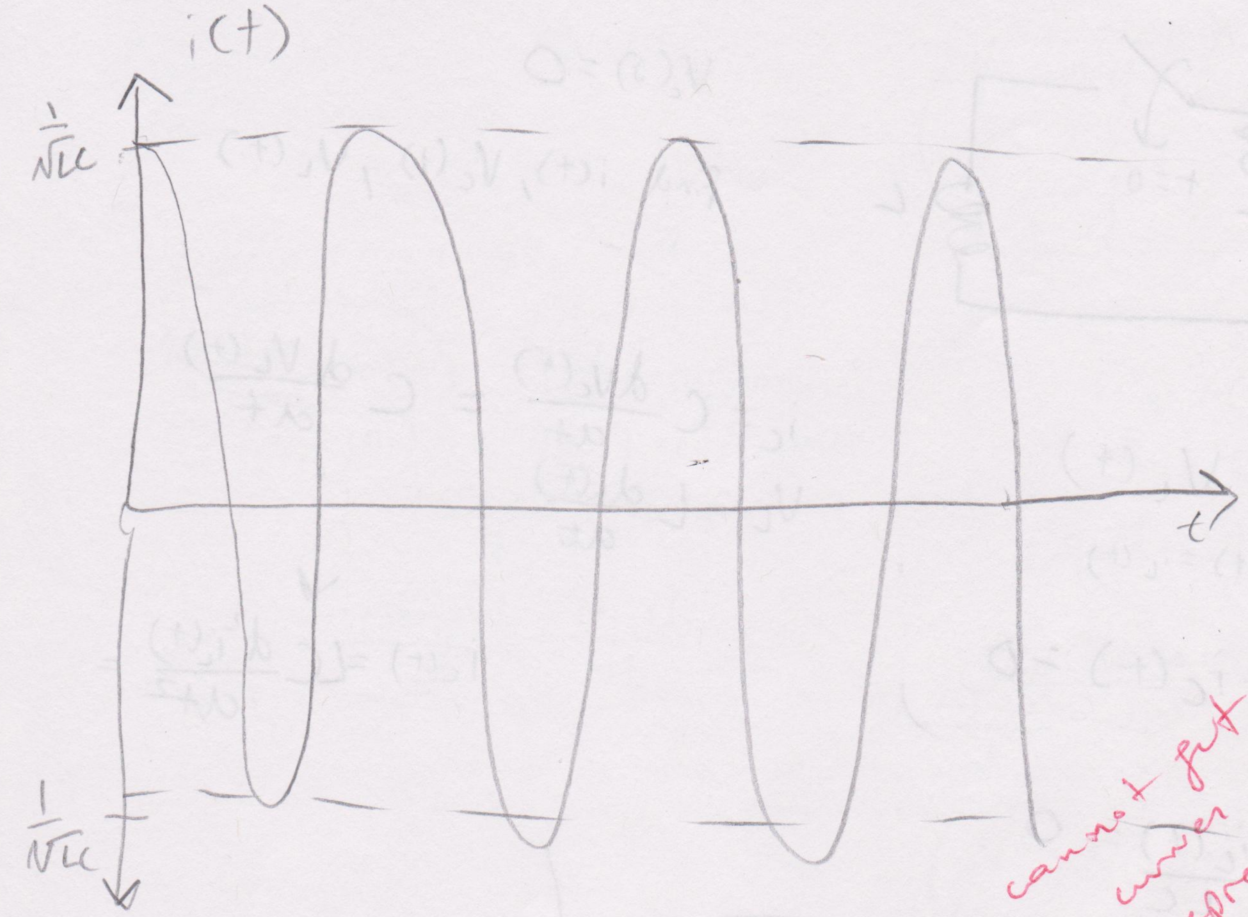
$$i(t) = \frac{V}{L}\left(e^{\frac{t}{\sqrt{LC}}} - e^{-\frac{t}{\sqrt{LC}}}\right)$$

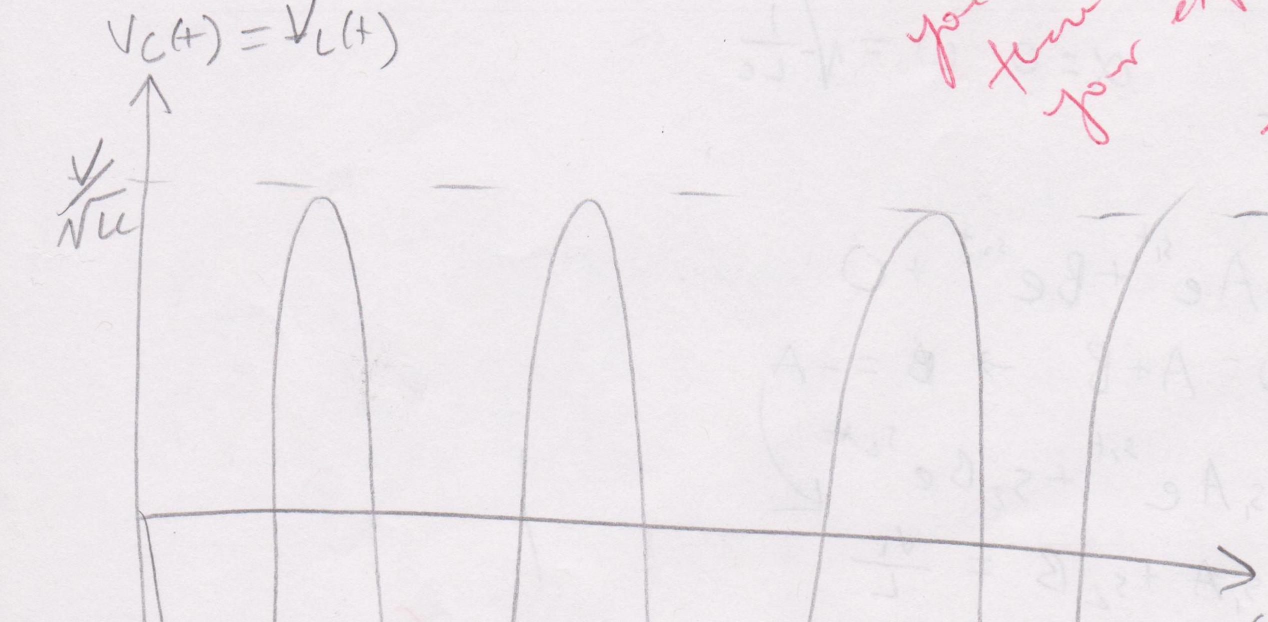
-15

$$V_L(t) = V_c(t) = L\frac{d\left(\frac{V}{L}\left(e^{\frac{t}{\sqrt{LC}}} - e^{-\frac{t}{\sqrt{LC}}}\right)\right)}{dt}$$

$$= \frac{V}{\sqrt{LC}}\left(e^{\frac{t}{\sqrt{LC}}} - e^{-\frac{t}{\sqrt{LC}}}\right)$$

Waveforms on next page





you cannot get these curves from your expressions!

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