# Regulated Grammars and Automata

#### Alexander Meduna and Petr Zemek

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Based on

Alexander Meduna and Petr Zemek Regulated Grammars and Automata Springer, New York, pp. 680, 2014

Supported by IT4I Centre of Excellence CZ.1.05/1.1.00/02.0070.

# Outline



#### Part I: An Introduction to the Book

Basic Idea General Info Contents

#### Part II: A Sample: One-Sided Random Context Grammars

Basic Idea

Definitions and Examples

Generative Power

Normal Forms

Reduction

Other Topics of Investigation



 $\bullet$  a grammar or an automaton based upon a finite set of rules  $\ensuremath{\textit{R}}$ 

#### Example

A context-free grammar with the set of rules *R*:

$$R: \qquad \begin{array}{c} S \rightarrow ABC \\ A \rightarrow aA \\ B \rightarrow bB \\ C \rightarrow cC \\ A \rightarrow a \\ B \rightarrow b \end{array}$$

 $C \rightarrow c$ 



- a grammar or an automaton based upon a finite set of rules R
- a regulation over R

#### Example

A context-free grammar with the set of rules R:

 $R: 1: S \rightarrow ABC$ 

 $2: A \rightarrow aA$ 

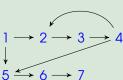
 $3: B \rightarrow bB$ 

4:  $C \rightarrow cC$ 

 $5: A \rightarrow a$ 

 $6: B \rightarrow b$ 

7:  $C \rightarrow c$ 





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#### Example

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$$S \Rightarrow ABC$$
 [1]



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$$\begin{array}{ccc} S & \Rightarrow & ABC & [1] \\ & \Rightarrow & aABC & [2] \end{array}$$



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#### Example

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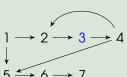
$$2: A \rightarrow aA$$

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$$S \Rightarrow ABC$$
 [1]

$$\Rightarrow aABC$$
 [2  $\Rightarrow aAbBC$  [3



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#### Example

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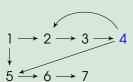
$$2: A \rightarrow aA$$

$$3: B \rightarrow bB$$

$$\mathbf{4}\colon\thinspace C\to\,cC$$

6: 
$$B \rightarrow b$$

7: 
$$C \rightarrow c$$



$$S \Rightarrow ABC \qquad [1]$$

$$\Rightarrow aABC \qquad [2]$$

$$\Rightarrow$$
 aAbBC [3]  $\Rightarrow$  aAbBcC [4]



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#### Example

$$R: 1: S \rightarrow ABC$$

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$$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4$$

$$\begin{array}{cccc} S & \Rightarrow & ABC & [1] \\ & \Rightarrow & aABC & [2] \\ & \Rightarrow & aAbBC & [3] \\ & \Rightarrow & aAbBcC & [4] \end{array}$$

$$\Rightarrow$$
 aabBcC [5]



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#### Example

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$$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4$$

$$S \Rightarrow ABC$$
 [1]

$$\Rightarrow$$
 aABC [2]  $\Rightarrow$  aAbBC [3]

$$\Rightarrow aAbBC [3]$$
$$\Rightarrow aAbBcC [4]$$

$$\Rightarrow$$
 aabBcC [5]

$$\Rightarrow$$
 aabBcC [5]

$$\Rightarrow$$
 aabbcC [6]



- a grammar or an automaton based upon a finite set of rules R
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#### Example

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#### Example

R: 1: 
$$S \rightarrow ABC$$
  
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$$C \rightarrow cC$$

$$4: C \rightarrow CC$$

$$5: A \rightarrow a$$

$$6: B \rightarrow b$$

$$7\colon C\to c$$

$$\begin{array}{c}
1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \\
\downarrow \\
5 \longrightarrow 6 \longrightarrow 7
\end{array}$$

$$L(G) = \{a^n b^n c^n : n \ge 1\}$$

## General Info





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### Motivation and Subject

- an important trend in formal language theory
- since 1990, no book has been published on the subject although many papers have discussed it

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## Motivation and Subject

- an important trend in formal language theory
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## Purpose

- theoretical: to summarize key results on the subject
- practical: to demonstrate applications of regulated grammars and automata



#### **Focus**

- power
- transformation
- reduction



#### **Focus**

- power
- transformation
- reduction

## Organization

- 9 parts
- 22 chapters



## Approach and Features

- theoretically oriented treatment of regulated grammars and automata
- emphasis on algorithms
- intuitive explanation
- many examples
- application perspectives



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- theoretically oriented treatment of regulated grammars and automata
- emphasis on algorithms
- intuitive explanation
- many examples
- application perspectives

#### **Book Audience**

- computer scientists: professionals, professors, Ph.D. students
- mathematicians
- linguists

# Contents (1/4)



## Part I Introduction and Terminology

- 1 Introduction
- 2 Mathematical Background
- 3 Rudiments of Formal Language Theory

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## Part I Introduction and Terminology

- 1 Introduction
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- 7 On Erasing Rules and Their Elimination
- 8 Extension of Languages Resulting from Regulated Grammars
- 9 Sequential Rewriting over Word Monoids

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## Part VIII Applications

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# Part II: A Sample: One-Sided Random Context Grammars



- a variant of random context grammars
- $(A \rightarrow x, U, W) \in P$



- a variant of random context grammars
- $(A \rightarrow X, U, W) \in P$
- $P = P_L \cup P_R$



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- $(A \rightarrow X, U, W) \in P_R$
- $P = P_I \cup P_R$

$$\dots \qquad \boxed{A} \xrightarrow{\dots}$$

#### Illustration

$$(A \rightarrow X, \{B, C\}, \{D\}) \in P_L$$

**bBcECbAcD** 



- a variant of random context grammars
- $(A \rightarrow X, U, W) \in P_R$
- $P = P_l \cup P_R$

#### Illustration

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- $(A \rightarrow X, U, W) \in P_R$
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#### Illustration

$$(A \rightarrow X, \{B, C\}, \{D\}) \in P_L$$

$$\overleftarrow{bBcECb} A cD \Rightarrow bBcECb x cD$$

## Definitions



#### Definition

A one-sided random context grammar is a quintuple

$$G = (N, T, P_L, P_R, S)$$

#### where

- N is an alphabet of nonterminals;
- T is an alphabet of terminals  $(N \cap T = \emptyset)$ ;
- $P_L$  and  $P_R$  are two finite sets of *rules* of the form

$$(A \rightarrow X, U, W)$$

where  $A \in N$ ,  $x \in (N \cup T)^*$ , and  $U, W \subseteq N$ ;

•  $S \in N$  is the starting nonterminal.

## Definitions



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•  $S \in N$  is the starting nonterminal.

#### Definition

If  $(A \rightarrow x, U, W) \in P_L \cup P_R$  implies that  $|x| \ge 1$ , then G is propagating.

## Definitions (Continued)



#### Definition

The *direct derivation* ⇒ is defined as

$$uAv \Rightarrow uxv$$

if and only if

$$(A \rightarrow X, U, W) \in P_L, U \subseteq alph(u), \text{ and } W \cap alph(u) = \emptyset$$

or

$$(A \rightarrow x, U, W) \in P_R, U \subseteq alph(v), \text{ and } W \cap alph(v) = \emptyset$$

Note: alph(y) denotes the set of all symbols appearing in string y

## Definitions (Continued)



#### Definition

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or

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#### Definition

The language of G is defined as

$$L(G) = \{ w \in T^* : S \Rightarrow^* w \}$$

where  $\Rightarrow^*$  is the reflexive-transitive closure of  $\Rightarrow$ .

# Example



### Example

Consider the one-sided random context grammar

$$G = (\{S, A, B, \bar{A}, \bar{B}\}, \{a, b, c\}, P_L, P_R, S)$$

where  $P_L$  contains

$$\begin{array}{ll} (\mathcal{S} \rightarrow \mathcal{A}\mathcal{B}, \emptyset, \emptyset) & (\bar{\mathcal{B}} \rightarrow \mathcal{B}, \{\mathcal{A}\}, \emptyset) \\ (\mathcal{B} \rightarrow \mathcal{b}\bar{\mathcal{B}}\mathcal{C}, \{\bar{\mathcal{A}}\}, \emptyset) & (\mathcal{B} \rightarrow \varepsilon, \emptyset, \{\mathcal{A}, \bar{\mathcal{A}}\}) \end{array}$$

and  $P_R$  contains

$$\begin{array}{ll} (A \to \alpha \bar{A}, \{B\}, \emptyset) & (A \to \varepsilon, \{B\}, \emptyset) \\ (\bar{A} \to A, \{\bar{B}\}, \emptyset) & \end{array}$$



$$P_{L}: (S \to AB, \emptyset, \emptyset) \qquad P_{R}: (A \to \alpha \bar{A}, \{B\}, \emptyset)$$

$$(B \to b\bar{B}c, \{\bar{A}\}, \emptyset) \qquad (\bar{A} \to A, \{\bar{B}\}, \emptyset)$$

$$(\bar{B} \to B, \{A\}, \emptyset) \qquad (A \to \varepsilon, \{B\}, \emptyset)$$

$$(B \to \varepsilon, \emptyset, \{A, \bar{A}\}) \qquad (S \to AB, \emptyset, \emptyset)]$$



$$P_{L}: (S \rightarrow AB, \emptyset, \emptyset) \qquad P_{R}: (A \rightarrow \alpha \bar{A}, \{B\}, \emptyset)$$

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$$(B \rightarrow E, \emptyset, \{A, \bar{A}\}) \qquad (A \rightarrow E, \{B\}, \emptyset)$$

$$(B \rightarrow E, \emptyset, \{A, \bar{A}\}) \qquad (S \rightarrow AB, \emptyset, \emptyset)$$

$$(B \rightarrow E, \emptyset, \{A, \bar{A}\}) \qquad [(S \rightarrow AB, \emptyset, \emptyset)]$$

$$(A \rightarrow E, \{B\}, \emptyset) \qquad (A \rightarrow E, \{B\}, \emptyset)$$

$$(B \rightarrow E, \emptyset, \{A, \bar{A}\}) \qquad [(A \rightarrow AB, \emptyset, \emptyset)] \qquad (A \rightarrow AB, \emptyset, \emptyset)$$

$$(B \rightarrow E, \emptyset, \{A, \bar{A}\}) \qquad (A \rightarrow E, \{B\}, \emptyset)$$



$$\begin{array}{lll} P_{L} \colon & (S \to AB, \emptyset, \emptyset) & P_{R} \colon & (A \to \alpha \bar{A}, \{B\}, \emptyset) \\ & (B \to b \bar{B} c, \{\bar{A}\}, \emptyset) & (\bar{A} \to A, \{\bar{B}\}, \emptyset) \\ & (\bar{B} \to B, \{A\}, \emptyset) & (A \to \varepsilon, \{B\}, \emptyset) \\ & (B \to \varepsilon, \emptyset, \{A, \bar{A}\}) & \\ & S & \Rightarrow & AB \\ & \Rightarrow & \alpha \bar{A} B \\ & \Rightarrow & \alpha \bar{A} b \bar{B} c & [(S \to AB, \emptyset, \emptyset)] \\ & \Rightarrow & \alpha \bar{A} b \bar{B} c & [(B \to b \bar{B} c, \{\bar{A}\}, \emptyset)] \end{array}$$



$$\begin{array}{lll} P_L\colon & (S\to AB,\emptyset,\emptyset) & P_R\colon & (A\to \alpha\bar{A},\{B\},\emptyset) \\ & (B\to b\bar{B}C,\{\bar{A}\},\emptyset) & (\bar{A}\to A,\{\bar{B}\},\emptyset) \\ & (\bar{B}\to B,\{A\},\emptyset) & (A\to \varepsilon,\{B\},\emptyset) \\ & (B\to \varepsilon,\emptyset,\{A,\bar{A}\}) & & & & & & & & \\ S & \Rightarrow & AB & & & & & & & & \\ & \Rightarrow & \alpha\bar{A}B & & & & & & & & \\ & \Rightarrow & \alpha\bar{A}b\bar{B}c & & & & & & & & \\ & \Rightarrow & \alpha Ab\bar{B}c & & & & & & & \\ & \Rightarrow & \alpha Ab\bar{B}c & & & & & & & \\ \end{array}$$



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$$P_{R}: (S \rightarrow AB, \emptyset, \emptyset) \qquad P_{R}: (A \rightarrow \alpha \bar{A}, \{B\}, \emptyset) \\ (B \rightarrow b\bar{B}c, \{\bar{A}\}, \emptyset) \qquad (\bar{A} \rightarrow A, \{\bar{B}\}, \emptyset) \\ (\bar{B} \rightarrow B, \{A\}, \emptyset) \qquad (A \rightarrow \varepsilon, \{B\}, \emptyset) \\ (B \rightarrow \varepsilon, \emptyset, \{A, \bar{A}\}) \qquad (S \Rightarrow AB \qquad [(S \rightarrow AB, \emptyset, \emptyset)] \\ \Rightarrow \alpha \bar{A}B \qquad [(A \rightarrow \alpha \bar{A}, \{B\}, \emptyset)] \\ \Rightarrow \alpha \bar{A}b\bar{B}c \qquad [(B \rightarrow b\bar{B}c, \{\bar{A}\}, \emptyset)] \\ \Rightarrow \alpha Ab\bar{B}c \qquad [(\bar{A} \rightarrow A, \{\bar{B}\}, \emptyset)] \\ \Rightarrow \alpha Ab\bar{B}c \qquad [(\bar{B} \rightarrow B, \{\bar{A}\}, \emptyset)] \\ \Rightarrow \alpha Bb\bar{B}c \qquad [(\bar{B} \rightarrow B, \{\bar{A}\}, \emptyset)]$$



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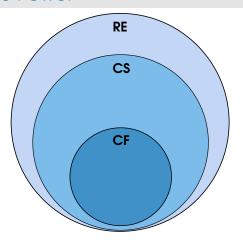
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$$L(G) = \{a^{n}b^{n}c^{n} : n \geq 0\}$$

## Generative Power





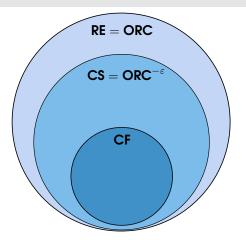
**RE** the family of recursively enumerable languages

**CS** the family of context-sensitive languages

**CF** the family of context-free languages

## Generative Power



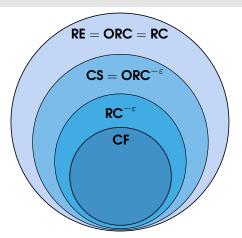


**ORC** the language family generated by one-sided random context grammars

 $\mathsf{ORC}^{-\varepsilon}$  the language family generated by propagating one-sided random context grammars

## Generative Power





- **RC** the language family generated by random context grammars
- $\mathbf{RC}^{-\varepsilon}$  the language family generated by propagating random context grammars



#### Results Structure

For any one-sided random context grammar, there is an equivalent one-sided random context grammar satisfying one of the following normal forms.



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#### Normal Form II

$$P_L \cap P_R = \emptyset$$



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#### Normal Form I

$$P_L = P_R$$

#### Normal Form II

$$P_L \cap P_R = \emptyset$$

#### Normal Form III

$$(A \rightarrow x, U, W) \in P_L \cup P_R$$
 implies that  $x \in NN \cup T \cup \{\varepsilon\}$ 



#### Results Structure

For any one-sided random context grammar, there is an equivalent one-sided random context grammar satisfying one of the following normal forms.

#### Normal Form I

$$P_L = P_R$$

#### Normal Form II

$$P_L \cap P_R = \emptyset$$

#### Normal Form III

$$(A \rightarrow X, U, W) \in P_L \cup P_R$$
 implies that  $X \in NN \cup T \cup \{\varepsilon\}$ 

#### Normal Form IV

$$(A \rightarrow X, U, W) \in P_L \cup P_R$$
 implies that  $U = \emptyset$  or  $W = \emptyset$ 

## Reduction



with respect to the total number of nonterminals

#### Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

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with respect to the total number of nonterminals

#### **Theorem**

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

 with respect to the number of right random context nonterminals

#### Definition

If  $(A \rightarrow x, U, W) \in P_R$ , then A is a right random context nonterminal.

## Reduction



with respect to the total number of nonterminals

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If  $(A \rightarrow x, U, W) \in P_R$ , then A is a right random context nonterminal.

#### **Theorem**

Any one-sided random context grammar can be converted to an equivalent one having no more than 2 right random context nonterminals.

# Reduction (Continued)



with respect to the number of right random context rules

#### Definition

If  $p \in P_R$ , then p is a right random context rule.

# Reduction (Continued)



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- special variants
  - one-sided permitting and forbidding grammars
  - left random context grammars and their variants



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- parsing-related versions
  - LL one-sided random context grammars
- one-sided versions of other grammars
  - left random context ETOL grammars

