Proof of (a). Consider

$$f\left(t_i|\alpha_i,\gamma\right)=\frac{\gamma}{\alpha_i}\left(\frac{t_i}{\alpha_i}\right)^{\gamma-1}\exp(-(t_i/\alpha_i)^{\gamma})$$

By taking log, we have the log-likelihood for the ith subject:

$$\begin{aligned}\ell\left(t_i|\alpha_i,\gamma\right) &= \log\left(\frac{\gamma}{\alpha_i}\right)+(\gamma-1)\log\left(\frac{t_i}{\alpha_i}\right)-\left(\frac{t_i}{\alpha_i}\right)^{\gamma} \\ &= \log\gamma-\log\alpha_i+(\gamma-1)\log\left(\frac{t_i}{\alpha_i}\right)-\left(\frac{t_i}{\alpha_i}\right)^{\gamma}\end{aligned}$$

Let $z_i=\frac{\log t_i-\log\alpha_i}{\delta}=\frac{\log\left(\frac{t_i}{\alpha_i}\right)}{\delta}$, $\alpha_i=\beta_0+\beta_1x_i+\beta_2u_i$ and $\delta=\frac{1}{\gamma}$. Then

$$\frac{t_i}{\alpha_i}=\exp\left(\delta z_i\right), \quad \log\left(\frac{t_i}{\alpha_i}\right)=\delta z_i, \quad \gamma\delta=1$$

Thus we can express $\ell\left(t_i|\alpha_i,\gamma\right)$ as:

$$\begin{aligned}\ell\left(t_i|\alpha_i,\gamma\right) &= \log\gamma-\log\alpha_i+(\gamma-1)\delta z_i-\left[\exp\left(\delta z_i\right)\right]^{\gamma} \\ &= \log\left(\frac{1}{\delta}\right)-\log\alpha_i+(\gamma-1)\delta z_i-\exp\left(\gamma\delta z_i\right) \\ &= -\log\delta-\log\alpha_i+(\gamma-1)\delta z_i-\exp\left(z_i\right) \\ &= -\log\delta-\log\alpha_i+z_i-\delta z_i-\exp\left(z_i\right) \\ &= -\log\delta-\log\alpha_i-\delta z_i+z_i-\exp\left(z_i\right)\end{aligned}$$

Since

$$\log\left(t_i\right)-\log\left(\alpha_i\right)=\delta z_i$$

we have:

$$\log\left(t_i\right)=\log\left(\alpha_i\right)+\delta z_i$$

Thus $\log\alpha_i+\delta z_i=\log\left(t_i\right)$ does not involve parameters and can be dropped and the resulting log likelihood is:

$$\ell\left(t_i|\alpha_i,\gamma\right)=-\log\delta+z_i-\exp\left(z_i\right)$$

For the whole sample, the log-likelihood is given by:

$$\ell\left(\alpha_i,\gamma\right)=\sum\ell\left(t_i|\alpha_i,\gamma\right)=-n\log\delta+\sum\left(z_i-\exp\left(z_i\right)\right)$$