Cooperative Collision Avoidance Scheme Design and Analysis in V2X-based Driving Systems

Ruoqi Deng, Boya Di, Lingyang Song
School of Electrical Engineering and Computer Science, Peking University, Beijing, China,
Email: {ruoqi.deng, boya.di, lingyang.song}@pku.edu.cn.

***Abstract*—In this paper, we consider a cooperative autonomous driving system where a vehicle overtakes the one in front based on collective perception. To avoid collisions with vehicles on the other lane, we propose a V2X-based cooperative collision avoidance scheme. The overtaking vehicle estimates its distance with the neighbors via V2V communications and decides whether to overtake or not. Two cases where the distance information is obtained independently and cooperatively are taken into account. We derive the probability of collision avoidance, and analyze the influence of different factors such as speed and density of vehicles on the system performance. Simulation results verify our analysis and show the improvement brought by the cooperative case compared to the independent case.**

***Index Terms*—Automated driving, V2X, cooperative collision avoidance.**

I. Introduction

With the rapid development of automated driving, the importance of traffic reliability and safety is attracting more and more attention. Nowadays automated driving heavily relies on a variety of onboard sensors to learn about surrounding driving context and to make optimal driving decisions [1]. However, this approach is not sufficient enough to deliver efficient and large-scale road safety and traffic management services due to the restricted awareness capability. To solve this issue, vehicle-to-everything (V2X) communications [2] are developed with the promise of much safer, efficient, and scalable traffic flows. Vehicles can share information with its neighborhood and then extend their awareness range beyond on-board capabilities, which will reduce the collision probability.

A typical safety-critical application is the V2X-assisted lane changing where a vehicle intends to overtake the one traveling ahead [2] [3]. For the overtaking vehicle, it may collide with vehicles on the other lane when the vehicle ahead of it blocks its view. One intuitive solution is that the vehicle ahead can perceive neighboring vehicles on the other lane via cameras and forward such information to the overtaking vehicle [2]. However, it can only obtain rough information of the surroundings and there still exists high risk of collision. To enhance the collision avoidance, one considers utilizing the V2X capability of autonomous vehicles, since it can well support reliable services. Through V2V links assigned by the base station (BS) between the overtaking vehicle and others, driving information of vehicles on the other lane can be obtained, based on which potential collision can be further avoided [3].

Challenges have arisen in such a V2X enabled collision avoidance scenario. For one thing, detection of the vehicle with the largest collision probability is difficult, especially in a two-lane scenario. For the other, theoretical analysis of the system is complex due to the interaction between multiple vehicles. To solve the above issues, we develop a V2X-based collision avoidance (V2X-CA) scheme in the overtaking scenario. In the proposed scheme, the overtaking vehicle utilizes the received signal strength (RSS) to estimate the distance between itself and the vehicle with the largest collision probability. The overtaking vehicle can then decide whether to overtake the vehicle in front of it. Since the vehicle ahead can also obtain information of vehicles on the other lane, we also consider the cooperative case where the overtaking decision can be jointly made by two vehicles to improve the collision avoidance probability.

Several works in the literature have considered collision avoidance based on the RSS measurement in V2X networks. In [4], RSS indication in V2V networks has been used to predict vehicle collisions, which is regardless of the distance estimation. In [5], an RSS-based algorithm has been proposed for collision warning, where the estimated position information has been exchanged between any two vehicles. Other collision avoidance methods have explored a path planning perspective [6] [7], bringing a huge computational burden in practice.

The main contributions of this paper can be summarized as follows. (1) A cooperative avoidance scheme based on distance estimation strategy is proposed. Both the independent case and cooperative case are taken into account. (2) We derive the probability that the overtaking vehicle successfully detects the vehicle with the largest collision probability in these two cases. The probability of collision avoidance in these two cases is also derived. (3) Simulation results verify our analysis and show that the collision in lane changing can be avoided effectively by our proposed scheme.

The rest of this paper is organized as follows. In Section II, the system model is presented, and the V2X-based collision avoidance schemes are proposed. In Section III, we analyze the distance estimation strategy, and derive the probability that the overtaking vehicle can successfully detect the vehicle with the largest collision probability. In Section IV, collision avoidance probability is analyzed for both independent and cooperative cases. The simulation results are presented in Section V. Finally, the conclusion is drawn in Section V.

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II. System Model

In this section, we first describe a cellular automated vehicular network where the collision avoidance of connected vehicles is required. The resource allocation scheme is then introduced, followed by our proposed V2X-based collision avoidance scheme.

A. Scenario Description

As shown in Fig. 1, we consider a two-lane vehicular network consisting of a group of automated vehicles. Vehicle B is immediately followed by vehicle C travelling from east to west. The group of vehicles travelling on the opposite lane is denoted by $\mathcal{A}$=$\{A_{-i}, \ldots, A_{-1}, A_0, A_1, \ldots, A_i, \ldots\}$, in which vehicle A_0 denotes the vehicle right ahead of vehicle B[1]. We assume that the vehicles in $\mathcal{A}$ are Poissonly distributed with density ρ. $A_i(i \geqslant 1)$ represents vehicles travelling behind vehicle A_0 and A_{-i} represents vehicles travelling ahead of vehicle A_0. Vehicle C intends to overtake vehicle B via the opposite lane if vehicle C perceives no vehicle in front of it. However, with vehicle B blocking the view of vehicle C, vehicle C is not able to perceive vehicle A_0 through the camera, leading to potential collision between vehicles C and A_0. To avoid such collision, the cooperation between vehicle C and its neighbors needs considering.

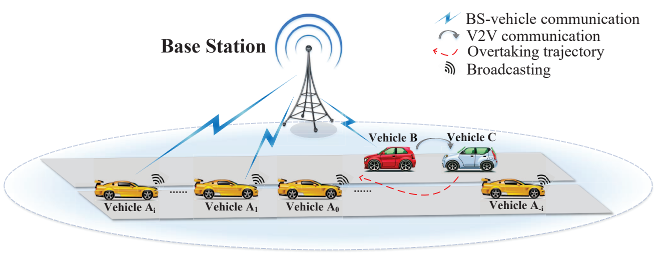

Fig. 1. System model of the cooperative collision avoidance for connected automated vehicles

To solve the above problem, we consider utilizing the V2X capacity of vehicles in $\mathcal{A}$ to assist vehicle C in deciding whether to overtake vehicle B. Note that each vehicle is required to broadcast its safety-critical information such as velocity and direction to its neighborhood periodically, occupying the time-frequency resource scheduled by the BS [2]. Therefore, vehicles B and C can receive the real-time information of vehicles in $\mathcal{A}$. Taking advantage of A_0's safety-critical information, vehicle C can estimate the distance between vehicle A_0 and itself, based on which it determines whether to overtake vehicle B or not.

In this scenario, the signal received by vehicle j from vehicle i can be given by

$$y_j = (PG \cdot d_{i,j}^{-\alpha})^{\frac{1}{2}} g_{i,j} x_i + I_{i,j} + n_{i,j}, \tag{1}$$

where x_i is the source signal transmitted by vehicle i, P is the fixed transmit power of each vehicle, G is the constant power gain factor introduced by amplifier and antenna, $d_{i,j}$ is the distance between vehicles i and j, $g_{i,j}$ is an independent Rayleigh fading coefficient, and α represents the path loss coefficient. $I_{i,j}$ is the interference in vehicle i-j link, and $n_{i,j}$ represents the Gaussian noise with zero mean and N_0 as variance.

[1]The scenario in this paper is applicable to two cases where vehicles in $\mathcal{A}$ travel in either the same or opposite direction to vehicle C. Without loss of generality, we take the opposite-direction case as an example.

B. V2X-based Collision Avoidance Scheme

In this subsection we first illustrate the communication process between vehicles, then we consider the user scheduling in cellular networks. Finally we propose the V2X-based collision avoidance scheme.

When vehicle C intends to overtake vehicle B, it first sends the transmission request to the BS. After receiving the request, the BS schedules frequency resources for vehicle C to broadcast the information of its overtaking intension. The BS then allocates orthogonal time-frequency resources neighboring vehicles, i.e., for vehicles in $\mathcal{A}$, to broadcast their safety-critical information to vehicles B and C. Note that the interference to vehicles B and C only comes from other mobile devices sharing the same time-frequency resources. To ensure the fairness of information transmission for each vehicle, the BS uses a round robin based user scheduling scheme to allocate spectrum resources. Therefore, adjacent vehicles occupy different time-frequency resources.

We now propose the V2X-CA scheme in which the strategy for vehicle C to accurately obtain the information of vehicle A_0 is designed. In other words, vehicle C is required to select vehicle A_0 from $\mathcal{A}$. Since both vehicle C and vehicle B have the V2X capability, the following two cases will be considered.

1) Vehicle C detects vehicle A_0 independently : We assume that vehicle C can receive the safety-critical information of all vehicles in its communication range. Note that vehicle C can not directly obtain the accurate coordinates of vehicles on the opposite lane and that the vehicles behind it have no impact on its overtaking decision. Therefore vehicle C will first filter out the safety-critical information from the vehicles behind it according to the orienting direction of the signal, i.e., the direction of arrival, by using the signal-direction discriminator [8] or digital receiver array [9]. After that it estimates the distances between itself and vehicles in $\mathcal{A}$ ahead of it based on the received signal strength. The larger $SINR$ is, the shorter the distance is. Since vehicles B and C are very close to each other, vehicle A_0 can be regarded as the vehicle right in front of both vehicles B and C, i.e., we have $A_0 = argmax_{A_i \in \mathcal{A}} SINR_{A_i,C}$ [2]. Once vehicle C has accurately detected vehicle A_0 from $\mathcal{A}$, it can obtain vehicle A_0's detailed information such as velocity and relative location.

2) Vehicles B and C cooperate to detect vehicle A_0: In this case, both vehicle C and vehicle B can estimate the distances between itself and vehicles in $\mathcal{A}$ based on the

[2]Vehicles in $\mathcal{A}$ broadcast messages every $100ms$ as mentioned above. In general, the distance between vehicles is at least 1 meter. Vehicles move runs at 108 kilometers per hour. Therefore the deviation of the distance estimation brought by delay of the message can be ignored.

received signal strength, after which vehicle B will share its distance estimation information with vehicle C. Therefore, vehicle C can detect the position of A_0 based on the estimation of itself and vehicle B.

III. Distance Estimation Strategy Analysis

In this section, we first present the interference and outage analysis. We then analyze the probability that vehicle C can accurately select vehicle A_0 from the set $\mathcal{A}$ based on the distance estimation strategy.

A. Interference and Outage Analysis

As mentioned in Section II, the interference from mobile devices will affect the decisions of vehicle B and vehicle C. We denote the set of other mobile devices in this area as ψ_m, deployed following the independent homogeneous PPP with density ρ_m. Since the subchannel is allocated randomly, the possibility that a mobile device occupies the same subchannel of vehicles in $\mathcal{A}$ is $\frac{1}{K}$. Therefore, when vehicle C receives safety-critical information from vehicle i, the interference it suffers from other mobile devices can be given by

$$I_{i,C} = \sum\nolimits_{k \in \psi'_m} P_m d_{k,C}^{-4} h_m, \tag{2}$$

where P_m denotes the average transmit power of mobile devices, h_m is the small-scale fading coefficient following an exponential distribution, i.e., $h_m \sim \exp(1)$, and ψ'_m is a thinning homogeneous PPP of ψ_m with the density of $\rho'_m = \frac{\rho_m}{K}$. Following the method in [10] to avoid the singularity of the path loss law $d_{k,C}^{-4}$, we replace it by $\min\{1, d_{k,C}^{-4}\}$. The average interference from other mobile devices to the vehicle i – vehicle C link can then be given by [10]

$$\mathbb{E}I_{i,C} = 2P_m\rho'_m\pi. \tag{3}$$

The outage event in the channel from vehicle i to vehicle j happens when vehicle j can not successfully decode the message from vehicle i. This outage probability can be expressed by:

$$P_{i,j}^{out} = P(SINR_{i,j} < \Upsilon_0), \tag{4}$$

where Υ_0 denotes the threshold of $SINR_{i,j}$. For Rayleigh flat fading, $|g_{i,j}|^2$ is exponentially distributed with parameter $\lambda_{i,j}$, and thus, we have

$$P_{i,j}^{out} = P(\frac{PG|\hat{g}_{i,C}|^2 d_{i,C}^{-4}}{I_{i,C} + N_0} < \Upsilon_0) \sim \frac{\lambda_{i,j}(N_0 + \mathbb{E}I_{i,C})\Upsilon_0}{PGd_{i,j}^{-4}}, \tag{5}$$

where $\mathbb{E}I_{i,C}$ can be calculated according to (3).

B. Probability of Obtaining Vehicle A_0's Information

In this subsection, we consider the following two cases.

1) Vehicle C detects vehicle A_0 independently: Define the probability that vehicle C can obtain vehicle A_0's detailed information independently as P_{COA_0I}. Only when the outage event in A_0–C link does not happen and vehicle C successfully detects vehicle A_0, can vehicle C obtain vehicle A_0's detailed information. P_{COA_0I} is then given by

$$P_{COA_0I} = (1 - P_{A_0,C}^{out}) \cdot P_{CDA_0I}, \tag{6}$$

where $P_{A_0,C}^{out}$ can be calculated by eq.(5), and P_{CDA_0I} denotes the probability that vehicle C successfully detects vehicle A_0 independently. P_{CDA_0I} can be derived as below.

We consider a practical system where the accurate channel state information (CSI) is hard to be obtained. The impact of imperfect CSI can be depicted by

$$\hat{g}_{i,j} = \kappa g_{i,j} + \sqrt{1-\kappa^2}\varepsilon, \tag{7}$$

where $g_{i,j}$ denotes the ideal channel coefficients of the i–j link, and $\hat{g}_{i,j}$ is the practical channel coefficients of the i–j link, which is uncorrelated with $g_{i,j}$. ε is a Gaussian random variable with zero mean and variance $1/\lambda_{i,j}$, and $\kappa(0 \le \kappa \le 1)$ is the correlation coefficient. It can be derived that $|\hat{g}_{i,j}|^2$ is also exponentially distributed with mean $\lambda_{i,j}$. We set $h_{i,j} = |\hat{g}_{i,j}|^2$ for simplicity.

Due to the imperfect CSI, vehicle C may mistake vehicle A_i as A_0 if $SINR_{i,C} > SINR_{A_0,C}$ holds. In other words, only when $SINR_{i,C} < SINR_{A_0,C}$ holds for each vehicle $A_i (i > 0)$, can vehicle C detect vehicle A_0 successfully. For each vehicle i, the probability of $SINR_{i,C} < SINR_{A_0,C}$ can then be expressed as[3]

$$\begin{aligned} P_{i,C} &= P(SINR_{i,C} < SINR_{A_0,C}) \\ &= P(\frac{PG|\hat{g}_{i,C}|^2 d_{i,C}^{-4}}{I_{i,C} + N_0} < \frac{PG|\hat{g}_{A_0,C}|^2 d_{A_0,C}^{-4}}{I_{A_0,C} + N_0}). \end{aligned} \tag{8}$$

Proposition 1: The probability in eq.(8) can be given by

$$\begin{aligned} P_{i,C} &= \int_{\mathcal{D}_{i,C}} f_{|\hat{g}_{i,C}|^2}(h_{i,C}) f_{|\hat{g}_{A_0,C}|^2}(h_{A_0,C}) f_{d_{A_0,C},d_{i,C}}(d_0, d_i) \\ &\qquad dh_{i,C} dh_{A_0,C} dd_0 dd_i \\ &= \int_0^1 \frac{i(1-x)^{i-1}}{1+x^4} dx \\ &= F[\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\}, \{\frac{i+1}{4}, \frac{i+2}{4}, \frac{i+3}{4}, \frac{i+4}{4}\}, -1], \end{aligned} \tag{9}$$

where $x = d_{A_0,C}/d_{i,C}$, $F(\cdot)$ is the hypergeometric function and $\mathcal{D}_{i,C}$ is the integral domain defined by the detection metric of the independent case.

Proof. See Appendix A.

Proposition 2: For vehicle C, when the number of vehicles in vehicle C's communication range behind vehicle A_0 is k, the probability of detecting vehicle A_0 independently can then be given by

$$\begin{aligned} P_{CDA_0I} = &\int_{h_0>0, 0<d_0<d_1<\cdots<d_k} \prod_{i=1}^{k} (1 - e^{-h_0 (\frac{d_i}{d_0})^4}) \\ &\cdot e^{-h_0} e^{-d_k} dh_0 dd_0 \cdots dd_k. \end{aligned} \tag{10}$$

[3]In this paper, we set $\alpha = 4$ for simplicity [11].

where $h_0 = |\hat{g}_{A_0,C}|^2$.
Proof. See Appendix B.

2) Vehicles B and C cooperate to detect vehicle A_0: Define the probability that vehicles B and C cooperate to obtain vehicle A_0's detailed information as P_{COA_0B}. Only when vehicle C and vehicle B successfully detect vehicle A_0 cooperatively and the outage event in the A_0–B–C link or A_0–C link does not happen, can vehicle C obtain vehicle A_0's information. Therefore, P_{COA_0B} can be given as

$$P_{COA_0B} = \{1 - P^{out}_{A_0,C}[1-(1-P^{out}_{A_0,B})(1-P^{out}_{B,C})]\} \cdot P_{CDA_0B}, \quad (11)$$

where $P^{out}_{A_0,C}$, $P^{out}_{A_0,B}$ and $P^{out}_{B,C}$ can be calculated by eq.(5), and P_{CDA_0B} denotes the probability that vehicles B and C cooperate to detect vehicle A_0. P_{CDA_0B} can be derived as below.

In this case, vehicles B and C cooperate to judge whether A_i is the target vehicle based on the signal strength $SINR_{i,B}$ and $SINR_{i,C}$. The metric for detecting vehicle A_i can be described as below. Only when $\beta_B SINR_{i,B} + \beta_C SINR_{i,C} < \beta_B SINR_{A_0,B} + \beta_C SINR_{A_0,C}$ holds for each vehicle $A_i (i > 0)$, can vehicle C detect the target vehicle A_0, where β_B and β_C are weight factors for the A_i – B link and A_i – C link, respectively. Mathematically, the probability that A_i will not be detected as A_0 is given in eq.(12).

Proposition 3: $P_{i,CB}$ in eq.(12) is given in eq.(13), where $\mathcal{D}_{i,CB}$ is the defined by the detecting metric of the cooperative case, $x = (\beta_B h_{A_0,B} + \beta_C h_{A_0,C})/(\beta_B h_{i,B} + \beta_C h_{i,C})$, and we have set $d_{i,B} = d_{i,C}, d_{A_0,B} = d_{A_0,C}$ for simplicity.
Proof. See Appendix A.

Proposition 4: When the number of vehicles in vehicle C's communication range behind vehicle A_0 is k, the probability that vehicles B and C cooperate to detect vehicle A_0 successfully can then be given by eq.(14), where $t_i = d_i/d_0$, $\hat{h}_0 = \beta_B |\hat{g}_{A_0,B}|^2 + \beta_C |\hat{g}_{A_0,C}|^2$. The numeric results of P_{CDA_0B} can be obtained by numerical integration.[4]
Proof. See Appendix B.

For generalization, we define the overall probability for vehicle C to successfully obtain the information from vehicle A_0 as P_{COA_0}, which equals to P_{COA_0I} for the independent case and P_{COA_0B} for the cooperative case.

Based on the theoretical results in Fig.2, we have $P_{CDA_0B} > P_{CDA_0I}$, implying that the cooperation between vehicles B and C further improves the collision avoidance. Therefore, it can be derived from (6) and (11) that $P_{COA_0B} > P_{COA_0I}$, i.e., the cooperative case performs better than the independent one theoretically in terms of P_{COA_0}.

IV. Collision Avoidance Probability Analysis

In this section, we analyze the probability of collision avoidance.

[4]For large k, we use the product of sequences $P_{i,C}$ and $P_{i,CB}$ to approximate the integral in (10) and (14). Consequently, we provide a theoretical lower bound of P_{CDA_0I} and P_{CDA_0B} for the value in real scenario.

The probability of collision avoidance can be given by

$$P_{\text{final}} = P_{COA_0} \cdot P_{CAA_0} + \sum_i P_{CMA_i} \cdot P_{CAA_i}, i > 0, \quad (15)$$

where P_{CAA_0} denotes the probability that vehicle C detects vehicle A_0 and avoids the collision, P_{CMA_i} denotes the probability that vehicle C mistakes vehicle A_i for vehicle A_0, and P_{CAA_i} denotes the probability that vehicle C avoids the collision when it mistakes vehicle A_i for vehicle A_0. Note that vehicle C may give up overtaking vehicle B even though overtaking does not lead to collision due to distance estimation error. Considering traffic efficiency, we exclude this condition in (15). The derivation of P_{CAA_0}, P_{CMA_i} and P_{CAA_i} will be given as follows.

A. Derivation of P_{CAA_0}

Even if vehicle C accurately receives the information from vehicle A_0, the collision between vehicle C and vehicle A_0 may also happen due to the distance estimation error. In fact, the collision will not happen only if the velocity of vehicles A_0 and C and their distance satisfy the following inequality:

$$L \triangleq \begin{cases} (2t_s + \frac{l_B + l_C + d_{BC}}{v_C - v_B}) v_{\mathcal{A}} + \frac{l_B + l_C + d_{BC}}{v_C - v_B} v_C \le d_{A_0C}, & (a) \\ -(2t_s + \frac{l_B + l_C + d_{BC}}{v_C - v_B}) v_{\mathcal{A}} + \frac{l_B + l_C + d_{BC}}{v_C - v_B} v_C \le d_{A_0C}, & (b) \end{cases} \quad (16)$$

where (a) and (b) depict the cases where vehicles in $\mathcal{A}$ travel in the same direction and opposite direction to vehicle C, respectively[5], t_s denotes the time for vehicle C to react and make a turn, l_B and l_C denote the length of vehicles B and C, $v_{\mathcal{A}}$ denotes the velocity of vehicles in $\mathcal{A}$, v_B and v_C denote the velocity of vehicles B and C. Therefore, The probability that vehicle C successfully detects vehicle A_0 and avoids the collision can be calculated by

$$P_{CAA_0} = 1 - P(\tilde{d}_{A_0,C} > L > d_{A_0,C}) - P(\tilde{d}_{A_0,C} < L < d_{A_0,C}), \quad (17)$$

where $\tilde{d}_{A_0,C}$ denotes vehicle C's estimation of the distance between itself and vehicle A_0 according to $SINR_{A_0,C}$.

Proposition 5: The probability that vehicle C successfully detects vehicle A_0 and avoids the collision can be given by

$$P_{CAA_0} = \int_0^L \rho e^{-\rho d_0} \frac{L^4}{L^4 + d_0^4} dd_0 + \int_L^\infty \rho e^{-\rho d_0} \frac{d_0^4}{L^4 + d_0^4} dd_0. \quad (18)$$

Proof. See Appendix C.

B. Derivation of P_{CMA_i}

Note that the collision may happen if a wrong vehicle, i.e., $A_i (i > 0)$ is mistaken as vehicle A_0. In this case, the $SINR$ of the link involving vehicle A_i is the largest. The mistaking probability P_{CMA_i} can be derived as below.

When vehicle C detects vehicle A_0 independently, the probability of mistaking vehicle A_i for vehicle A_0, i.e., P_{CMA_iI}, is expressed in eq.(19), where $n_1 = d_0/d_i$, $n_2 = d_2/d_1$ for

[5]In this paper we assume that vehicle C overtakes with its maximum velocity. Therefore vehicle C will not collide with vehicles behind it.

$$P_{i,CB} = P(\beta_B \frac{PG|\hat{g}_{i,B}|^2 d_{i,B}^{-4}}{I_{i,B}} + \beta_C \frac{PG|\hat{g}_{i,C}|^2 d_{i,C}^{-4}}{I_{i,C}} < \beta_B \frac{PG|\hat{g}_{A_0,B}|^2 d_{A_0,B}^{-4}}{I_{A_0,B}} + \beta_C \frac{PG|\hat{g}_{A_0,C}|^2 d_{A_0,C}^{-4}}{I_{A_0,C}}) \tag{12}$$

$$= \int_{D_{i,CB}} f_{|\hat{g}_{i,B}|^2}(h_{i,B}) f_{|\hat{g}_{i,C}|^2}(h_{i,C}) f_{|\hat{g}_{A_0,B}|^2}(h_{A_0,B}) f_{|\hat{g}_{A_0,C}|^2}(h_{A_0,C}) f_{d_{A_0,C},d_{i,C}}(d_0,d_i) dh_{i,B} dh_{i,C} dh_{A_0,B} dh_{A_0,C} dd_0 dd_i$$

$$= 1 - \int_0^1 \frac{2\beta_B\beta_C(1-\beta_B\beta_C)x^3 + (1 - 2\beta_B\beta_C + 4\beta_B^2\beta_C^2)x^2 + 2\beta_B\beta_C(1-\beta_B\beta_C)x}{(x+1)^2(\beta_B x + \beta_C)^2(\beta_C x + \beta_B)^2}(1 - x^{1/4})^i dx, \tag{13}$$

$$P_{CDA_0B} = \int_{h_0>0, 1<t_1<t_2<\cdots<t_k} \prod_{i=1}^{k} (1 - \frac{\beta_B e^{-\frac{\hat{h}_0}{\beta_B} t_i^4} - \beta_C e^{-\frac{\hat{h}_0}{\beta_C} t_i^4}}{\beta_B - \beta_C}) \cdot \frac{(e^{-\frac{\hat{h}_0}{\beta_B}} - e^{-\frac{\hat{h}_0}{\beta_C}})k!}{(\beta_B - \beta_C) t_k^{k+1}} d\hat{h}_0 dt_1 \cdots dt_k, \tag{14}$$

$$P_{CMA_iI} \approx \begin{cases} P(SINR_1 > SINR_0, SINR_1 > SINR_2) & i = 1 \\ P(SINR_i > SINR_0, SINR_i > SINR_1) & i > 1 \end{cases}$$
$$= \begin{cases} \int_{0<n_1<1<n_2} (1 - \frac{1}{1+n_1^4} - \frac{1}{1+n_2^4} + \frac{1}{1+n_1^4+n_2^4}) \frac{2}{n_2^3} dn_1 dn_2 \\ \int_{0<n_1<n_2<1} (1 - \frac{1}{1+n_1^4} - \frac{1}{1+n_2^4} + \frac{1}{1+n_1^4+n_2^4}) i(i-1)(1-n_2)^{i-2} dn_1 dn_2, \end{cases} \tag{19}$$

$i = 1$ and d_1/d_i for $i > 1$, and i represents the ID of vehicles in $\mathcal{A}$. The proof can be found in Appendix D.

When vehicles B and C cooperate to detect vehicle A_0, the probability of mistaking vehicle A_i for vehicle A_0, i.e., P_{CMA_iB}, can be expressed in eq.(20), where $n_1 = d_0/d_i$, $n_2 = d_2/d_1$ for $i = 1$ and d_1/d_i for $i > 1$,$\hat{h}_1 = \beta_B|\hat{g}_{1,B}|^2 + \beta_C|\hat{g}_{1,C}|^2$, i represents ID of vehicles in $\mathcal{A}$, and $\mathcal{D}_{A_iB} = \{(\hat{h}, n_1, n_2) : \hat{h}_1 > 0, 0 < n_1 < n_2 < 1\}$. The case that $\beta_B = \beta_C$ can be obtained by letting $\beta_B - \beta_C \to 0$ in eq.(20). The proof can be found in Appendix D.

For generalization, we define the overall probability that vehicle C mistakes vehicle A_i for vehicle A_0 as P_{CMA_i}, which equals to P_{CMA_iI} for the independent case and P_{CMA_iB} for the cooperative case.

C. Derivation of P_{CAA_i}

If vehicle C mistakes vehicle A_i for vehicle A_0, similar to the analysis of P_{CAA_0}, the probability of collision avoidance in this case can be calculated by

$$P_{CAA_i} = 1 - P(\tilde{d}_{A_i,C} > L > d_{A_i,C}) - P(\tilde{d}_{A_i,C} < L < d_{A_i,C}), \tag{21}$$

where $\tilde{d}_{A_i,C}$ denotes vehicle C's estimation of the distance between itself and vehicle A_i according to $SINR_{A_i,C}$.

Proposition 6: The probability that vehicle C avoids the collision when it mistakes vehicle A_i for vehicle A_0 can be given by

$$P_{CAA_i} = \int_0^\infty \frac{\rho^{i+1}}{i!} e^{-\rho d_i} \frac{d_i^i L^4}{L^4 + d_i^4} dd_i + \int_L^\infty \frac{\rho^{i+1}}{i!} e^{-\rho d_i} \frac{(d_i - L)^i (d_i^4 - L^4)}{L^4 + d_0^4} dd_0. \tag{22}$$

Proof. See Appendix C.

V. Simulation Results

In this section, we provide the simulation results for our proposed V2X-CA scheme to validate the theoretical analysis. Simulation parameters are set up based on the existing works and 3GPP specifications [2] as given in Table I.

TABLE I
PARAMETERS FOR SIMULATION

Parameters	Values
Vehicle C's communication range r_{max} (m)	2000
Distance for vehicle C to decode information correctly (m)	300
Transmit power of mobile devices P_m (dBm)	38
Vehicle's transmit power P (dBm)	23
Antenna amplify gain G (dBi)	15
System bandwidth (kHZ)	180
Time for vehicle C to react and to make a turn t_s (s)	1.8
Length of vehicle B l_B (m)	4
Length of vehicle C l_C (m)	4
Velocity of vehicles in $\mathcal{A}$ v_{A_0} (m/s)	15-30
Velocity of vehicle B v_B (m/s)	15
Velocity of vehicle C v_C (m/s)	30

Fig.2 depicts the probability of obtaining vehicle A_0's information successfully in both independent and cooperative cases versus the density of vehicles in $\mathcal{A}$. The lower bound of the probability is depicted by eq.(10) and (14). We observe that as the density (ρ) of vehicles in $\mathcal{A}$ grows, the probability slightly drops. Moreover, compared with the independent case, the probability is larger in the cooperative case, implying that cooperative connections between vehicles can help obtain vehicle A_0's information more accurately.

Fig.3 shows the probability of collision avoidance versus the density of vehicles in $\mathcal{A}$. The lower bound of the probability

$$P_{CMA_iB} \approx \begin{cases} P(SINR_1 > SINR_0, SINR_1 > SINR_2) & i=1 \\ P(SINR_i > SINR_0, SINR_i > SINR_1) & i>1 \end{cases}$$

$$= \begin{cases} \int_{\mathcal{D}_{A_iB}} (1 - \frac{\beta_B e^{-\frac{\hat{h}_1}{\beta_B} n_1^4} - \beta_C e^{-\frac{\hat{h}_1}{\beta_C} n_1^4}}{\beta_B - \beta_C})(1 - \frac{\beta_B e^{-\frac{\hat{h}_1}{\beta_B} n_2^4} - \beta_C e^{-\frac{\hat{h}_1}{\beta_C} n_2^4}}{\beta_B - \beta_C}) \cdot \frac{2(e^{-\frac{\hat{h}_1}{\beta_B}} - e^{-\frac{\hat{h}_1}{\beta_C}})}{(\beta_B - \beta_C) n_2^3} d\hat{h}_1 dn_1 dn_2 \\ \int_{\mathcal{D}_{A_iB}} (1 - \frac{\beta_B e^{-\frac{\hat{h}_1}{\beta_B} n_1^4} - \beta_C e^{-\frac{\hat{h}_1}{\beta_C} n_1^4}}{\beta_B - \beta_C})(1 - \frac{\beta_B e^{-\frac{\hat{h}_1}{\beta_B} n_2^4} - \beta_C e^{-\frac{\hat{h}_1}{\beta_C} n_2^4}}{\beta_B - \beta_C}) \frac{(e^{-\frac{\hat{h}_1}{\beta_B}} - e^{-\frac{\hat{h}_1}{\beta_C}})}{(\beta_B - \beta_C)} i(i-1)(1-n_2)^{i-2} d\hat{h}_1 dn_1 dn_2, \end{cases} \tag{20}$$

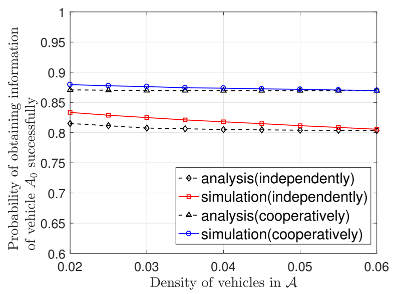

Fig. 2. The probability of obtaining vehicle A_0's information successfully in two cases vs. density of vehicles in $\mathcal{A}$

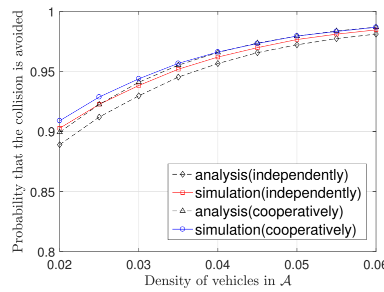

Fig. 3. The probability of collision avoidance vs. density of vehicles in $\mathcal{A}$

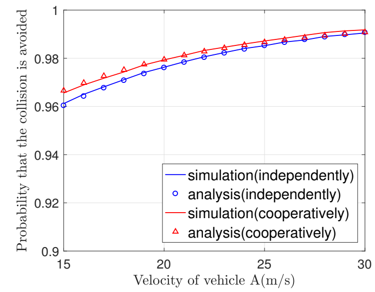

Fig. 4. The probability of collision avoidance vs. speed of vehicles in $\mathcal{A}$

is depicted by eq.(15). We observe that the collision avoidance probability increases with the density ρ. The main reason is that as the density of vehicles in $\mathcal{A}$ grows, the distance between vehicles decreases, thereby increasing the probability that vehicle C decides not to overtake vehicle B.

Fig.4 illustrates the probability of collision avoidance versus speed of vehicles in $\mathcal{A}$. We observe that the probability increases with the speed of vehicles in $\mathcal{A}$. It can be seen that the expression in (15) perfectly matches with the simulation results. Both Fig.3 and Fig.4 show that cooperative connection can improve probability of obtaining vehicle A_0's information successfully, thereby enhancing the collision avoidance performance.

VI. Conclusion

In this paper, a new V2X-based collision avoidance scheme in the overtaking scenario was proposed. In the scheme, vehicles estimated the distance between each other via V2X communications, and both the independent and cooperative detection cases were taken into account. The theoretical values of the probability of collision avoidance based on the proposed scheme were derived correspondingly, which matched the simulation results. It was shown that, by analysis and simulation results, the proposed scheme can avoid the collision efficiently. Moreover, the accuracy of detecting the vehicle with the largest collision probability and the probability of collision avoidance can be improved when the cooperation between vehicles are considered.

VII. Acknowledgement

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Appendix A

Proof. Since $|\hat{g}_{i,C}|^2$ has exponential distribution and $d_{i,C}$ has Gamma distribution, which are scale family. Therefore, when considering the quotient of random variable from the same scale family, we can set $\lambda_{i,j} = 1$ and $\rho = 1$. Then we have $f_{|\hat{g}_{i,C}|^2}(x) = e^{-x}$. Further we have

$$f_{\frac{|\hat{g}_{A_0,C}|^2}{|\hat{g}_{i,C}|^2}}(z) = \int_0^\infty y f_{|\hat{g}_{A_0,C}|^2}(zy) f_{|\hat{g}_{i,C}|^2}(y) dy = \frac{1}{(z+1)^2}. \tag{23}$$

Set $\hat{h}_i = \beta_B |\hat{g}_{i,B}|^2 + \beta_C |\hat{g}_{i,C}|^2$, then the PDF of $\hat{h}_i$ can be written as

$$f(\hat{h}_i) = \frac{e^{-\hat{h}_i/\beta_C} - e^{\hat{h}_i/\beta_B}}{\beta_C - \beta_B}. \tag{24}$$

The joint PDF of $d_{A_0,C}$ and $d_{i,C}$ is given by [12]

$$f_{d_{A_0,C}, d_{i,C}}(d_0, d_i) = e^{-d_i} \frac{(d_i - d_0)^{i-1}}{(i-1)!}, 0 < d_0 < d_i. \tag{25}$$

By substitution $x = d_{A_0,C}/d_{i,C}, y = d_{i,C}$, we obtain

$$f_{d_{A_0,C}/d_{i,C}}(x) = i(1-x)^{i-1}, \quad 0 < x < 1. \tag{26}$$

Therefore, we have

$$P_{i,C} = \int_0^\infty i(1-x)^{i-1} \int_{z^4}^\infty \frac{1}{(z+1)^2} = \int_0^1 \frac{i(1-x)^{i-1}}{1+x^4} dx. \tag{27}$$

For $P_{i,CB}$, substitute $I_{i,C}$ by $\mathbb{E}I_{i,C}$ and set $d_{i,B} = d_{i,C}, d_{A_0,B} = d_{A_0,C}$, we have

$$P_{i,CB} = P(\hat{h}_0/\hat{h}_i > (d_{A_0,C}/d_{i,C})^4). \tag{28}$$

Similar to (23), we can derive the PDF of $\hat{h}_0/\hat{h}_i$ and then obtain $P_{i,CB}$ as shown in eq.(13).

APPENDIX B

Proof. The joint PDF of $d_0, \cdots, d_i$ is given by [12]

$$f_{d_{A_0,C},\cdots,d_{A_i,C}}(d_0,\cdots,d_i) = e^{-d_i},\ 0 < d_0 < \cdots < d_i. \tag{29}$$

Consider the independent case first. Since $\{d_{i,C}\}_{i=0}^{k}$ and $\{|\hat{g}_{i,C}|^2\}_{i=0}^{k}$ are independent, we have

$$\begin{aligned} &f_{d_{A_0,C},\cdots,d_{k,C},|\hat{g}_{A_0,C}|^2,\cdots,|\hat{g}_{k,C}|^2}(d_0,\cdots,d_k,h_0,\cdots,h_k) \\ &= e^{-(h_0+\cdots+h_k)} \cdot e^{-d_k}. \end{aligned} \tag{30}$$

where $h_i = |\hat{g}_{i,C}|^2$, and the integral domain defined by the detection metric of the independent case. Therefore, by integrating over $h_1, \cdots, h_k$ and applying

$$\int_0^{h_0(\frac{d_i}{d_0})^4} e^{-h_i} dh_i = 1 - e^{-h_0(\frac{d_i}{d_0})^4}, \tag{31}$$

P_{CDA_0I} is obtained.

For the cooperative case, h_i is replaced by $\hat{h}_i = \beta_B|\hat{g}_{i,B}|^2 + \beta_C|\hat{g}_{i,C}|^2$. Then P_{CDA_0B} in (14) can be obtained similarly.

APPENDIX C

Proof. According to $SINR_{A_0,C}$, vehicle C's estimation of the distance between itself and vehicle A_0 $\tilde{d}_{A_0,C}$ is

$$\tilde{d}_{A_0,C} = Q^{\frac{1}{4}} \cdot d_{A_0,C}, \tag{32}$$

where $Q = |\hat{g}_{A_0,C}|^2/|g_{A_0,C}|^2$, whose PDF is $f_Q(q) = 1/(1+q)^2$. The probability that vehicle C successfully detects vehicle A_0 and avoid the collision is

$$P_{CAA_0} = 1 - P(L^4 > d_{A_0,C}^4 > L^4 Q) - P(L^4 < d_{A_0,C}^4 < L^4 Q). \tag{33}$$

By double integral we obtain

$$P(L^4 > d_{A_0,C}^4 > L^4 \cdot Q) = \int_0^L \rho e^{-\rho d_0} \frac{d_0^4}{L^4 + d_0^4} dd_0, \tag{34}$$

$$P(L^4 < d_{A_0,C}^4 < L^4 \cdot Q) = \int_L^\infty \rho e^{-\rho d_0} \frac{L^4}{L^4 + d_0^4} dd_0. \tag{35}$$

Similarly, for vehicle A_i,

$$\begin{aligned} &P(Q' \cdot d_{i,C}^4 > L^4 > d_{A_0,C}^4) \\ &= \int_0^\infty \frac{\rho^{i+1} e^{-\rho d_i} d_i^{i+4}}{i!(L^4 + d_i^4)} dd_i - \int_L^\infty \frac{\rho^{i+1} e^{-\rho d_i}}{i!} \frac{(d_i - L)^i d_i^4}{L^4 + d_i^4} dd_i. \end{aligned} \tag{36}$$

$$P(Q' \cdot d_{i,C}^4 < L^4 < d_{A_0,C}^4) = \int_L^\infty \frac{\rho^{i+1}}{i!} e^{-\rho d_i} \frac{(d_i - L)^i L^4}{L^4 + d_i^4} dd_i. \tag{37}$$

where $Q' = |g_{i,C}|^2/|\hat{g}_{i,C}|^2$.

APPENDIX D

Proof. The PDF of $d_{A_0,C}$, $d_{1,C}$ and $d_{i,C}$ is given by [12]

$$f_{d_{0,C},d_{1,C},d_{i,C}}(d_0,d_1,d_i) = e^{-d_i} \frac{(d_i - d_1)^{i-2}}{(i-2)!}. \tag{38}$$

We consider the independent case first. Then we have

$$\begin{aligned} &f_{d_{A_0,C},d_{1,C},d_{i,C},h_0,h_1,h_i}(d_0,d_1,d_i,h_0,h_1,h_i) \\ &= e^{-(h_0+h_1+h_i)} e^{-d_i} \frac{(d_i - d_1)^{i-2}}{(i-2)!}, \end{aligned} \tag{39}$$

where $0 < d_0 < d_1 < d_i, h_0, h_1, h_i > 0$. For $i = 1$, $h_1 \cdot d_1^{-4} > h_0 \cdot d_0^{-4}, h_1 \cdot d_1^{-4} > h_2 \cdot d_2^{-4}$. Thus we have

$$\begin{aligned} P_{CMA_1I} = \int_{0<d_0<d_1<d_2} &[1 - \frac{1}{1 + (\frac{d_0}{d_1})^4} - \frac{1}{1 + (\frac{d_2}{d_1})^4} \\ &+ \frac{1}{1 + (\frac{d_0}{d_1})^4 (\frac{d_2}{d_1})^4}] e^{-d_2} dd_0 dd_1 dd_2 \end{aligned} \tag{40}$$

where in we integrate over h_0, h_2 then over h_1. Then P_{CMA_1I} is obtained by making the substitution $n_1 = d_0/d_1, n_2 = d_2/d_1, n_3 = d_1$ and integrating over n_3.

For $i \geq 2$, the integral domain is $h_i \cdot d_i^{-4} > h_j \cdot d_j^{-4}$, where $j = 0, 1$. Similar to P_{CMA_1I}, we firstly integrate over h_0, h_1 then over h_i. We then make the substitution $n_1 = d_0/d_i, n_2 = d_1/d_i, n_3 = d_i$, integrate over n_3 and P_{CMA_iI} is obtained.

In the cooperative case, replace h_i by $\hat{h}_i = \beta_B|\hat{g}_{i,B}|^2 + \beta_C|\hat{g}_{i,C}|^2$, and the rest of the proof follows that of independent case.

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