

1. Assume the mathematical function is $F(n)$.

Assume the numbers of operations of `compareTo()`, `queue.offer()`, `queue.poll()`, `queue.size()`, and `queue.peek()` are 1.

The method of `sortQueue` is shown below:

```
public static <T extends Comparable<T>> void sortQueue(Queue<T> queue) {
    int size = queue.size();          // 2 operations
    int counter = 0;                  // 1 operation

    //Total operations for the whole loop:
    //(6n - 2) + 7 * (n + (n - 1) + ... + 1) + 4 * (2 + 3 + 4 + (n - 1))
    for (int k = 1; k < size; k++) {    // Assume the size is n, so we have 1 + n operations
        T max = queue.poll();          // 2 * (n - 1) operations

        // Total operations for the first nested loop are :
        // 7 * (n + (n - 1) + (n - 2) + ... + 1) + 1
        for (int i = 0; i < size - k; i++) {
            if (max.compareTo(queue.peek()) >= 0) {
                queue.offer(queue.poll());
            } else {
                T temp = max;
                max = queue.poll();
                queue.offer(temp);
            }
        }
        counter++;                    // (n - 1) operations
        queue.add(max);                // (n - 1) operations

        // Total operations for the second nested loop are:
        // 4 * (2 + 3 + 4 + ... + (n - 1)) + 1 operations
        for (int j = counter; j >= 2; j--) {
            queue.offer(queue.poll());
        }
    }
}
```

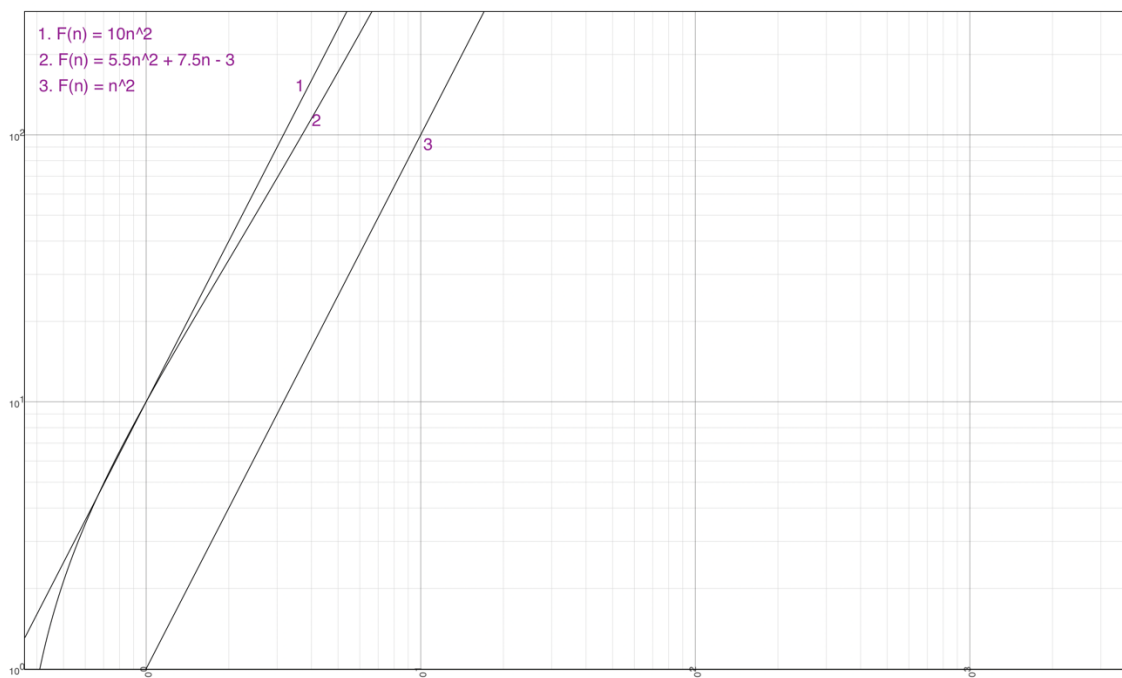
From the diagram above, when we add all operations, we can get(in worst case):

$F(n) = (6n - 2) + 7 * (n + (n - 1) + \dots + 1) + 4 * (2 + 3 + 4 + (n - 1)) + 3$

Therefore, $F(n) = 5.5n^2 + 7.5n - 3$, and we pick $n_0 = 1$ and $c = 10$.

When $n \geq 1$, we know $F(n) = 5n^2 + 7n - 3 \leq 10n^2$, so $F(n)$ is $O(10n^2)$.

The diagram is shown below:



Hence, the runtime complexity of my `sortQueue` algorithm is $O(n^2)$.

2. Assume the time complexity of the implementation is $F(n)$
 Assume the number of operation of `numbers.length` is 1.
 The implementation is shown below:

```
public static int findMissingNumber(int[] numbers) {
    int commonDifference = (numbers[numbers.length - 1] - numbers[0]) / numbers.length; //6 operations
    return helpFindMissingNumber(numbers, 0, numbers.length - 1, commonDifference); // 1 operations
}

private static int helpFindMissingNumber(int[] numbers, int start, int end, int commonDifference) {
    if (end - start + 1 == 2) { // 3 operations
        return numbers[start] + commonDifference; // 2 operations
    }

    int middle = (start + end) / 2; // 3 operations
    if ((numbers[middle] - numbers[start]) / (middle - start) != commonDifference) {
        return helpFindMissingNumber(numbers, start, middle, commonDifference);
    } else {
        return helpFindMissingNumber(numbers, middle, end, commonDifference);
    }
}
```

From the diagram above, we can easily know that the asymptotic bound in Big O for `findMissingNumber` is $O(1)$.

From the method of `helpFindMissingNumber`, we know that this method is recursive method. Assume the time complexity of the `helpFindMissingNumber` method is $T(n)$, so we have the recursive expression below:

$$T(n) = \begin{cases} O(1) & (n = 2) \\ T\left(\frac{n}{2}\right) + O(1) & (n \geq 3) \end{cases}$$

Therefore, $T(n) = O(1) + T\left(\frac{n}{2}\right) = O(1) + O(1) + T\left(\frac{n}{4}\right) = \dots = \log_2 n * O(1)$
 Hence, the asymptotic bound in Big O for `helpFindMissingNumber` is $O(\log_2 n)$.
 Hence, the time complexity of whole implementation $F(n)$ is $O(\log_2 n) + O(1)$, which is $O(\log_2 n)$.