Manhattan Distance

We will prove that Manhattan Distance is locally consistent, which implies global consistency, which implies admissibility.

Given two nodes n, m where m is a child of n, we'd like to prove that $h(n) \leq c(n,m) + h(m)$.
We know that when we move a white tile then $c(n,m) = 1$ and when we move a red tile then $c(n,m) = 30$. Lets check two cases:

Case 1: $h(n) \leq h(m)$
In this case, the heuristic considers m as "farther" to the solution than n. Since we move one tile each step, the heuristic value changes for at most 1. The inequality applies no matter the cost (which is non-negative, 1 or 30).

Case 2: $h(n) \geq h(m)$
In this case, the heuristic considers m as "closer" to the solution than n. Since we move one tile each step, the heuristic value changes for at most -1.
Therefore we can note $h(m) = h(n) - 1$. Since $c(n,m) \geq 1$,
we see that: $h(n) \leq c(n,m) + h(n) - 1 \implies h(n) \leq 1 + h(n) - 1 \implies h(n) \leq h(n)$
The inequality applies.

We have proved that MD is local consistent. Therefore:
MD is local consistent $\iff$ MD is global consistent $\implies$ MD is admissible.