

## Algoritmo de iteração de valor

### Algorithm 4.1: Value iteration algorithm

**Initialization:** The probability models p(r|s,a) and p(s'|s,a) for all (s,a) are known. Initial guess  $v_0$ .

**Goal:** Search for the optimal state value and an optimal policy for solving the Bellman optimality equation.

While  $v_k$  has not converged in the sense that  $||v_k - v_{k-1}||$  is greater than a predefined small threshold, for the kth iteration, do

For every state  $s \in \mathcal{S}$ , do

For every action  $a \in \mathcal{A}(s)$ , do

q-value:  $q_k(s, a) = \sum_r p(r|s, a)r + \gamma \sum_{s'} p(s'|s, a)v_k(s')$ 

Maximum action value:  $a_k^*(s) = \arg \max_a q_k(s, a)$ 

Policy update:  $\pi_{k+1}(a|s) = 1$  if  $a = a_k^*$ , and  $\pi_{k+1}(a|s) = 0$  otherwise

Value update:  $v_{k+1}(s) = \max_a q_k(s, a)$ 



# Algoritmo de iteração de política

## Algorithm 4.2: Policy iteration algorithm

**Initialization:** The system model, p(r|s,a) and p(s'|s,a) for all (s,a), is known. Initial guess  $\pi_0$ .

Goal: Search for the optimal state value and an optimal policy.

While  $v_{\pi_k}$  has not converged, for the kth iteration, do

Policy evaluation:

Initialization: an arbitrary initial guess  $v_{\pi_k}^{(0)}$ 

While  $v_{\pi_k}^{(j)}$  has not converged, for the jth iteration, do

For every state  $s \in \mathcal{S}$ , do

$$v_{\pi_k}^{(j+1)}(s) = \sum_a \pi_k(a|s) \left[ \sum_r p(r|s,a)r + \gamma \sum_{s'} p(s'|s,a)v_{\pi_k}^{(j)}(s') \right]$$

Policy improvement:

For every state  $s \in \mathcal{S}$ , do

For every action  $a \in \mathcal{A}$ , do

$$q_{\pi_k}(s,a) = \sum_r p(r|s,a)r + \gamma \sum_{s'} p(s'|s,a)v_{\pi_k}(s')$$

$$a_k^*(s) = \arg\max_a q_{\pi_k}(s,a)$$

$$\pi_k(s,a) = 1 \text{ if } s - s^* \text{ and } \pi_k(s,a) = 0 \text{ otherwise}$$

 $\pi_{k+1}(a|s) = 1$  if  $a = a_k^*$ , and  $\pi_{k+1}(a|s) = 0$  otherwise



# Algoritmo de iteração de política truncada

#### Algorithm 4.3: Truncated policy iteration algorithm

**Initialization:** The probability models p(r|s, a) and p(s'|s, a) for all (s, a) are known. Initial guess  $\pi_0$ .

**Goal:** Search for the optimal state value and an optimal policy.

While  $v_k$  has not converged, for the kth iteration, do

Policy evaluation:

Initialization: select the initial guess as  $v_k^{(0)} = v_{k-1}$ . The maximum number of iterations is set as  $j_{\text{truncate}}$ .

While  $j < j_{\text{truncate}}$ , do

For every state  $s \in \mathcal{S}$ , do

$$v_k^{(j+1)}(s) = \sum_a \pi_k(a|s) \left[ \sum_r p(r|s,a)r + \gamma \sum_{s'} p(s'|s,a)v_k^{(j)}(s') \right]$$
 Set  $v_k = v_k^{(j_{\text{truncate}})}$  Policy improvement: 
$$\sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a) \left[ r + \gamma v_k(s') \right]$$
 For every state  $s \in \mathcal{S}$ , do

For every action  $a \in \mathcal{A}(s)$ , do

$$q_{k}(s,a) = \sum_{r} p(r|s,a)r + \gamma \sum_{s'} p(s'|s,a)v_{k}(s')$$

$$a_{k}^{*}(s) = \arg\max_{a} q_{k}(s,a)$$

$$\pi_{k+1}(a|s) = 1 \text{ if } a = a_{k}^{*}, \text{ and } \pi_{k+1}(a|s) = 0 \text{ otherwise}$$



## Algoritmo: MC Básico

## Algorithm 5.1: MC Basic (a model-free variant of policy iteration)

**Initialization:** Initial guess  $\pi_0$ .

**Goal:** Search for an optimal policy.

For the kth iteration (k = 0, 1, 2, ...), do

For every state  $s \in \mathcal{S}$ , do

For every action  $a \in \mathcal{A}(s)$ , do

Collect sufficiently many episodes starting from (s,a) by following  $\pi_k$ 

Policy evaluation:

 $q_{\pi_k}(s,a) \approx q_k(s,a)$  = the average return of all the episodes starting from (s,a)

Policy improvement:

$$a_k^*(s) = \arg\max_a q_k(s, a)$$

 $\pi_{k+1}(a|s) = 1$  if  $a = a_k^*$ , and  $\pi_{k+1}(a|s) = 0$  otherwise



Algoritmo:
 MC com inícios
 exploratórios

#### Algorithm 5.2: MC Exploring Starts (an efficient variant of MC Basic)

**Initialization:** Initial policy  $\pi_0(a|s)$  and initial value q(s,a) for all (s,a). Returns(s,a) = 0 and  $\operatorname{Num}(s,a) = 0$  for all (s,a).

**Goal:** Search for an optimal policy.

For each episode, do

Episode generation: Select a starting state-action pair  $(s_0, a_0)$  and ensure that all pairs can be possibly selected (this is the exploring-starts condition). Following the current policy, generate an episode of length T:  $s_0, a_0, r_1, \ldots, s_{T-1}, a_{T-1}, r_T$ .

Initialization for each episode:  $g \leftarrow 0$ 

For each step of the episode,  $t = T - 1, T - 2, \dots, 0$ , do

$$g \leftarrow \gamma g + r_{t+1}$$

 $\mathsf{Returns}(s_t, a_t) \leftarrow \mathsf{Returns}(s_t, a_t) + g$ 

 $\operatorname{\mathsf{Num}}(s_t, a_t) \leftarrow \operatorname{\mathsf{Num}}(s_t, a_t) + 1$ 

Policy evaluation:

 $q(s_t, a_t) \leftarrow \mathsf{Returns}(s_t, a_t) / \mathsf{Num}(s_t, a_t)$ 

Policy improvement:

 $\pi(a|s_t) = 1$  if  $a = \arg\max_a q(s_t, a)$  and  $\pi(a|s_t) = 0$  otherwise