

# Concepts in Parallel Programming

Subtitle

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Arrows are a general interface for computation and an alternative to Monads for API design. In contrast to Monad-based parallelism, we explore the use of Arrows for specifying generalised parallelism. Specifically, we define an Arrow-based language and implement it using multiple parallel Haskells. As each parallel computation is an Arrow, such parallel Arrows (PArrows) can be readily composed and transformed as such. To allow for more sophisticated communication schemes between computation nodes in distributed systems, we utilise the concept of Futures to wrap direct communication. To show that PArrows have similar expressive power as existing parallel languages, we implement several algorithmic skeletons and four benchmarks. Benchmarks show that our framework does not induce any notable performance overhead. We conclude that Arrows have considerable potential for composing parallel programs and for producing programs that can execute on multiple parallel language implementations.



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# Introduction

Functional languages have a long history of being used for experimenting with novel parallel programming paradigms. Haskell, which we focus on in this paper, has several mature implementations. We regard here in-depth Glasgow parallel Haskell or short GpH (its Multicore SMP implementation, in particular), the *Par* Monad, and Eden, a distributed memory parallel Haskell. These languages represent orthogonal approaches. Some use a Monad, even if only for the internal representation. Some introduce additional language constructs. Section 3.2 gives a short overview over these languages.

A key novelty in this paper is to use Arrows to represent parallel computations. They seem a natural fit as they can be thought of as a more general function arrow ( $\rightarrow$ ) and serve as general interface to computations while not being as restrictive as Monads (Hughes, 2000). Section 3.1.4 gives a short introduction to Arrows.

We provide an Arrows-based type class and implementations for the three above mentioned parallel Haskell. Instead of introducing a new low-level parallel backend to implement our Arrows-based interface, we define a shallow-embedded DSL for Arrows. This DSL is defined as a common interface with varying implementations in the existing parallel Haskell. Thus, we not only define a parallel programming interface in a novel manner – we tame the zoo of parallel Haskell. We provide a common, very low-penalty programming interface that allows to switch the parallel implementations at will. The induced penalty is in the single-digit percent range, with means typically under 2% overhead in measurements over the varying cores configuration (Section 7). Further implementations, based on HdpH or a Frege implementation (on the Java Virtual Machine), are viable, too.

## 1.1 Contributions

We propose an Arrow-based encoding for parallelism based on a new Arrow combinator  $parEvalN :: [arr\ a\ b] \rightarrow arr\ [a]\ [b]$ . A parallel Arrow is still an Arrow, hence the resulting parallel Arrow can still be used in the same way as a potential sequential version. In this paper we evaluate the expressive power of such a formalism in the context of parallel programming.

- We introduce a parallel evaluation formalism using Arrows. One big advantage of this specific approach is that we do not have to introduce any new types, facilitating composability (Section 4).
- We show that PArrow programs can readily exploit multiple parallel language implementations. We demonstrate the use of GpH, a *Par* Monad, and Eden. We do not re-implement all the parallel internals, as this functionality is hosted in the *ArrowParallel* type class, which abstracts all parallel implementation logic. The implementations can easily be swapped, so we are not bound to any specific one.

This has many practical advantages. For example, during development we can run the program in a simple GHC-compiled variant using GpH and afterwards deploy it on a cluster by converting it into an Eden program, by just replacing the *ArrowParallel* instance and compiling with Eden’s GHC variant (Section 4).

- We extend the PArrows formalism with *Futures* to enable direct communication of data between nodes in a distributed memory setting similar to Eden’s Remote Data (Dieterle et al., 2010b). Direct communication is useful in a distributed memory setting because it allows for inter-node communication without blocking the master-node. (Section 5.1)
- We demonstrate the expressiveness of PArrows by using them to define common algorithmic skeletons (Section 4.4), and by using these skeletons to implement four benchmarks (Section 7).
- We practically demonstrate that Arrow parallelism has a low performance overhead compared with existing approaches, e.g. the mean over all cores of relative mean overhead was less than 3.5% and less than 0.8% for all benchmarks with GpH and Eden, respectively. As for *|Par|* Monad, the mean of mean overheads was in favour of PArrows in all benchmarks (Section 7).

PArrows are open source and are available from <https://github.com/s4ke/Parrows>.



## Related Work

### 2.1 Parallel Haskell

The non-strict semantics of Haskell, and the fact that reduction encapsulates computations as closures, makes it relatively easy to define alternate parallelisations. A range of approaches have been explored, including data parallelism (Chakravarty et al., 2007, Keller et al. (2010)), GPU-based approaches (Mainland and Morrisett, 2010, obsidian-phd), software transactional memory (Harris et al., 2005, Perfumo et al. (2008)). The Haskell–GPU bridge Accelerate (Chakravarty et al., 2011, Clifton-Everest et al. (2014), McDonell et al. (2015)) is completely orthogonal to our approach. A good survey of parallel Haskell can be found in Marlow (2013).

Our PArrow implementation uses three task parallel languages as backends: the GpH (Trinder et al., 1996, Trinder et al. (1998)) parallel Haskell dialect and its multicore version (Marlow et al., 2009), the *Par* Monad (Marlow et al., 2011, Foltzer et al. (2012)), and Eden (Loogen et al., 2005, Loogen (2012)). These languages are under active development, for example a combined shared and distributed memory implementation of GpH is available (Aljabri et al., 2014, Aljabri et al. (2015)). Research on Eden includes low-level implementation (Berthold, 2008, Berthold (2016)), skeleton composition (Dieterle et al., 2016), communication (Dieterle et al., 2010b), and generation of process networks (Horstmeyer and Loogen, 2013). The definitions of new Eden skeletons is a specific focus (Hammond et al., 2003, Berthold and Loogen (2006), Berthold et al. (2009b), Berthold et al. (2009c), Dieterle et al. (2010a), Encina et al. (2011), Dieterle et al. (2013), Janjic et al. (2013))

Other task parallel Haskell related to Eden, GpH, and the *Par* Monad include the following. Hd pH (Maier et al., 2014, Stewart (2016)) is an extension of *Par* Monad to heterogeneous clusters. LVish (Kuper et al., 2014) is a communication-centred extension of the *Par* Monad.

## 2.2 Algorithmic skeletons

Algorithmic skeletons were introduced by Cole (1989). Early publications on this topic include (Danelutto et al., 1992, Darlington et al. (1993), Botorog and Kuchen (1996), Lengauer et al. (1997), Gorlatch (1998)). Rabhi and Gorlatch (2003) consolidated early reports on high-level programming approaches. Types of algorithmic skeletons include *map*-, *fold*-, and *scan*-based parallel programming patterns, special applications such as divide-and-conquer or topological skeletons.

The *farm* skeleton (Hey, 1990, Peña and Rubio (2001), Poldner and Kuchen (2005)) is a statically task-balanced parallel *map*. When tasks' durations cannot be foreseen, a dynamic load balancing (*workpool*) brings a lot of improvement (Rudolph et al., 1991, Hammond et al. (2003), Hippold and Rünger (2006), Berthold et al. (2008), Marlow 2009). For special tasks *workpool* skeletons can be extended with dynamic task creation (Priebe, 2006, Dinan et al. (2009), Brown and Hammond (2010)). Efficient load-balancing schemes for *workpools* are subject of research (Blumofe and Leiserson, 1999, Acar et al. (2000), Nieuwpoort et al. (2001), Chase and Lev (2005), Olivier and Prins (2008), Michael et al. (2009)).

The *fold* (or *reduce*) skeleton was implemented in various skeleton libraries (Kuchen, 2002, Karasawa and Iwasaki (2009), Buono et al. (2010), Dastgeer et al. (2011)), as also its inverse, *scan* (Bischof and Gorlatch, 2002, Harris et al. (2007)). Google *map-reduce* (Dean and Ghemawat, 2008, Dean and Ghemawat (2010)) is more special than just a composition of the two skeletons (Lämmel, 2008, Berthold et al. (2009b)).

The effort is ongoing, including topological skeletons (Berthold and Loogen, 2006), special-purpose skeletons for computer algebra (Berthold et al., 2009c, Lobachev (2011), Lobachev (2012), Janjic et al. (2013)), iteration skeletons (Dieterle et al., 2013). The idea of Linton et al. (2010) is to use a parallel Haskell to orchestrate further software systems to run in parallel. Dieterle et al. (2016) compare the composition of skeletons to stable process networks.

## 2.3 Arrows

Arrows were introduced by Hughes (2000) as a less restrictive alternative to Monads, in essence they are a generalised function arrow  $\rightarrow$ . Hughes (2005) presents a tutorial on Arrows. Jacobs (2009), Lindley et al. (2011), Atkey (2011) develop theoretical background of Arrows. (Paterson, 2001) introduced a new notation for Arrows. Arrows have applications in information flow research (Li and Zdancewic, 2006, Li

and Zdancewic (2010), Russo et al. (2008)), invertible programming (Alimarine et al., 2005), and quantum computer simulation (Vizzotto, 2006). But probably most prominent application of Arrows is Arrow-based functional reactive programming, AFRP (Nilsson et al., 2002, Hudak et al. (2003), Czaplicki and Chong (2013)). (Liu et al., 2009) formally define a more special kind of Arrows that capsule the computation more than regular Arrows do and thus enable optimisations. Their approach would allow parallel composition, as their special Arrows would not interfere with each other in concurrent execution. In contrast, we capture a whole parallel computation as a single entity: our main instantiation function *parEvalN* makes a single (parallel) Arrow out of list of Arrows. Huang et al. (2007) utilise Arrows for parallelism, but strikingly different from our approach. They use Arrows to orchestrate several tasks in robotics. We, however, propose a general interface for parallel programming, while remaining completely in Haskell.

### 2.3.1 Arrows in other languages

Although this work is centred on Haskell implementation of Arrows, it is applicable to any functional programming language where parallel evaluation and Arrows can be defined. Basic definitions of PArrows are possible in the Frege language<sup>1</sup> (which is basically Haskell on the JVM). However, they are beyond the scope of this work, as are similar experiments with the Eta language<sup>2</sup>, a new approach to Haskell on the JVM.

(Achten et al., 2004, Achten et al. (2007)) use an Arrow implementation in Clean for better handling of typical GUI tasks. (Dagand et al., 2009) used Arrows in OCaml in the implementation of a distributed system.

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<sup>1</sup>GitHub project page at <https://github.com/Frege/frege>

<sup>2</sup>Eta project page at <http://eta-lang.org>



## Background

This section gives a short overview of Arrows (Section 3.1.4) and of GpH, the *Par* Monad, and Eden, the three parallel Haskell's which we base our DSL on (Section 3.2).

### 3.1 Functional Programming

#### 3.1.1 Why Functional Programming?

Hughes (1990) describes the fundamental idea of functional programming like this:

Functional programming is so called because its fundamental operation is the application of functions to arguments. A main program itself is written as a function that receives the program's input as its argument and delivers the program's output as its result. Typically the main function is defined in terms of other functions, which in turn are defined in terms of still more functions, until at the bottom level the functions are language primitives.

Functional programming is also often - wrongly - only defined by what it does not allow programmers to do. Hughes (1990) furthermore describes this aspect elegantly while naming the usual advantages of functional programs:

The special characteristics and advantages of functional programming are often summed up more or less as follows. Functional programs contain no assignment statements, so variables, once given a value, never change. More generally, functional programs contain no side-effects at all. A function call can have no effect other than to compute its result. This eliminates a major source of bugs, and also makes the order of execution irrelevant — since no side-effect can change an expression's value, it can be evaluated at any time. This relieves the programmer of the burden of prescribing the flow of control. Since expressions can be

evaluated at any time, one can freely replace variables by their values and vice versa — that is, programs are “referentially transparent”. This freedom helps make functional programs more tractable mathematically than their conventional counterparts.

[...]

Even a functional programmer should be dissatisfied with these so-called advantages, because they give no help in exploiting the power of functional languages. One cannot write a program that is particularly lacking in assignment statements, or particularly referentially transparent. There is no yardstick of program quality here, and therefore no ideal to aim at.

To argue that there is merit in functional programming besides having fewer error-prone features Hughes (1990) goes also into detail about one of the actual aspects why functional programming matters - composability. He does this by showing how higher order functions help in expressing programs in a modular way.

### 3.1.2 A Short introduction to Haskell

In the following section, we will give a short introduction to functional programming with Haskell. While this will give a good idea of how programming in Haskell works, this is not aimed to be a complete tutorial on Haskell, but merely a quick overview over the most relevant features of the language used in this thesis. The following is loosely based on the book „Learn you a haskell for great good!“ (Michaelson, 2013).

#### Functional Programming vs Imperative Programming

In order to ease the introduction to functional programming, we will give a short introduction to functional programming in Haskell in this section by comparing the general style of imperative C code to functional Haskell using the example of the Fibonacci sequence.

For starters, take a look at the iterative implementation of the Fibonacci sequence in Fig. 3.1. It contains assignments and a loop, but since in pure<sup>1</sup> functional programming we do not use assignment statements, we can not express the Fibonacci

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<sup>1</sup>Pure code is code without side-effects. Assignments are side-effects

sequence in an iterative way with loops<sup>2</sup>. In fact, loops can only compute anything meaningful if assignment statements are part of the programming language.

```
int fib( int n ) {
    int pre = 0;
    int cur = 1;
    int res = 0;
    for ( int i = 0; i < n; ++i ) {
        res = pre + cur;
        pre = cur;
        cur = res;
    }
    return cur;
}
```

**Figure 3.1:** Iterative Fibonacci in C

If we translate this Fibonacci example into a recursive definition (Fig. 3.2), however, we get pure functional C code without any assignment statements, that resembles the Haskell variant in Fig. 3.3. Also note how the flow of the programming without requiring any modifiable state.

```
int fib( int n ) {
    if ( n <= 0 )
        return 0;
    else if ( n == 1 )
        return 1;
    else
        return fib( n - 2 ) + fib( n - 1 );
}
```

**Figure 3.2:** Recursive Fibonacci in C

In functional languages like Haskell we only express computations in this matter by composition of functions (recursion is also in essence a composition of a function with itself). Because of this and since we can not change the state of any associated variables, we generally also do not have to worry about the order of execution in functional programs. In general, we can say that in functional programming we primarily focus on what information is required and by which transformations to compute it instead of how we perform them and how we track the changes in state.<sup>3</sup>

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<sup>2</sup>It is however possible to introduce monadic DSLs in Haskell that mimic C style behaviour, see <https://hackage.haskell.org/package/ImperativeHaskell-2.0.0.1>.

<sup>3</sup>from <https://docs.microsoft.com/en-us/dotnet/visual-basic/programming-guide/concepts/linq/functional-programming-vs-imperative-programming>

```

fib :: Int → Int
fib n
  | n ≤ 0 = 0
  | n ≡ 1 = 0
  | otherwise =
    (fib (n - 2))
    + (fib (n - 1))

```

**Figure 3.3:** Standard Fibonacci in Haskell.

Haskell being a functional language does not mean, that we do not have the usual problem of a too small call-stack size encountered when programming with recursion. While Haskell programs can naturally handle much bigger call-stacks without overflowing, at some point the limit will be reached and the program will crash. But since the class of tail-recursive programs is equivalent to the class of all recursive programs (which is in turn equivalent to all imperative programs), this is no big problem: We can just translate our *fib* definition into a tail-recursive variant (Fig. 3.4) which Haskell’s compiler is capable of automatically translating into looping machine code.

```

fib :: Int → Int
fib n
  | n ≤ 0 = 0
  | otherwise = fib' n 0 1
where
  fib' :: Int → Int → Int → Int
  fib' n prev res
    | n ≡ 0 = res
    | otherwise = fib' (n - 1) res (res + prev)

```

**Figure 3.4:** Tail Recursive Fibonacci in Haskell.

## Functions

As already mentioned above, the basic building blocks of a Haskell program are functions. They are defined as follows:

```

f :: Int → Int → Int
f x y = multiply x y

```



Here, we declared a function  $f$  which takes two arguments of type  $Int$  and returns yet another  $Int$ . In the definition we say that  $f$  is basically the function *multiply*, which we define as:

$$\begin{aligned} multiply &:: Int \rightarrow Int \rightarrow Int \\ multiply\ x\ y &= x * y \end{aligned}$$

Since  $f$  and *multiply* seem to be the same, we can even write this relationship directly:

$$\begin{aligned} f &:: Int \rightarrow Int \rightarrow Int \\ f &= multiply \end{aligned}$$

We can do so because in Haskell functions can be treated just like any other type. For example, if we wanted to have another function  $g$  which applied  $f$  on two lists of integers, we can write

$$\begin{aligned} g &:: [Int] \rightarrow [Int] \rightarrow [Int] \\ g &= zipWith\ f \end{aligned}$$

where *zipWith* would be of type  $(Int \rightarrow Int \rightarrow Int) \rightarrow [Int] \rightarrow [Int] \rightarrow [Int]$ .

## Type inference

Taking the example function  $g$  from the previous section, it does not make sense to be so restrictive in terms of which type to allow in such a function since all it does is apply some function to zip two lists. Thankfully, in Haskell we can define functions in a completely generic way such that we can write the actual type of *zipWith* as  $(a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$  as in it can zip a list containing some  $as$  with a list containing a list of  $bs$  with a function  $a \rightarrow b \rightarrow c$  to get a list of  $cs$ . If we use this function in the context of our function  $g$  it is then specialized into the  $Int$  form.

Furthermore we can even define  $g$  without writing down the type definition and let the compiler determine the actual type of  $g$ .

$$g = zipWith\ f$$

While this is possible, it is generally encouraged to always specify the type of top-level functions for better readability, but sometimes this is useful for some nested helper functions.

## Function composition, higher-order functions, and function application

As we have seen, in Haskell, functions can be handled similar to data types. This way, we can for example define a function that computes a number to the power of four as

```
toThePowerOfFour :: Int → Int
toThePowerOfFour = toThePowerOfTwo ∘ toThePowerOfTwo
```

with  $\circ$  being the functional composition operator with type  $(\circ) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$ <sup>4</sup> and where *toThePowerOfTwo* is defined simply as *toThePowerOfTwo*  $x = \text{multiply } x \ x$ .

Another aspect of functions being similar to data types is that, in functional programming, we frequently use higher order functions to express calculations. We have seen this earlier with the use of *zipWith*. Other often used higher-order functions include mapping (*map* ::  $(a \rightarrow b) \rightarrow [a] \rightarrow [b]$ , i.e. convert a list of *as* into a list of *bs* with the given function  $a \rightarrow b$ ) and folding (e.g. *foldLeft* ::  $(b \rightarrow a \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$ , i.e. reduce the list with the given function  $b \rightarrow a \rightarrow b$  into a singular value given a starting value of type *b*). These are often used in some sort of composition like

```
euclidDistance :: [Int] → [Int] → Int
euclidDistance = sqrt ∘ foldLeft (+) 0 ∘ map (toThePowerOfTwo) ∘ zipWith (-)
```

Note that while this could have easily been written shorter as something along the lines of *sqrt (foldLeft (+) 0 (zipWith (\a b → toThePowerOfTwo (a + b))))* it is easy to see that the above declaration is easier to understand because of the simple steps the computation takes. We first zip the list of inputs with element-wise subtraction and then square this difference, sum these results up and finally take the square root. This is something we see a lot in Haskell code, complex computations can be expressed with the help of higher-order functions instead of having to write it manually which would obviously include. This is not only much shorter, but also easier to understand for other programmers which have to read-up on the implementation for some reason.

Something which is also quite useful in Haskell is the function application operator (*\$*) ::  $(a \rightarrow b) \rightarrow a \rightarrow b$  which allows for the application of a function  $a \rightarrow b$  to a given argument *a*. It is simply defined as:

---

<sup>4</sup>note the order of the arguments,  $g \circ f$  means to first apply *f* and then *g* and not the other way around

$$(\$) :: (a \rightarrow b) \rightarrow a \rightarrow b$$

$$f \$ x = f x$$

While the usecase for such an operator might not be immediately clear, it will, if we take a look at the following function  $listApp :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$  where we take a list of functions  $[a \rightarrow b]$  and apply them one-by-one with their respective input values from the input list  $[a]$  to generate a list of results  $b$ :

$$listApp :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$$

$$listApp = zipWith (\$)$$

Here, if we had not used the  $(\$)$  operator, we would have to write  $zipWith (\lambda f a \rightarrow f a)$  which obviously seems a bit redundant.

Something the  $(\$)$  operator is also used quite often is to write shorter code. For example, code snippets like

$$someFunc = f1 (f2 param1 (f3 param2 (f4 param3)))$$

can also be written without the braces as

$$someFunc = f1 \$ f2 param1 \$ f3 param2 \$ f4 param3$$

which is sometimes preferred to the brace-style, but is semantically identical.

## Conditional Computation

Haskell has different styles of dealing with conditional evaluation. We will now show the three the most common variants to express conditional statements.

The most obvious one in terms of functionality is the **if ... then ... else** construct:

$$myFunc :: Int \rightarrow Int$$

$$myFunc x = \text{if } x < 10 \text{ then } x * 2 \text{ else } x * 4$$

While having the same well-known semantics of any **if ... then ... else** like they could be found in imperative languages like e.g. C, in Haskell, being a functional language, the **else** is non-optional as expressions are required to be total.<sup>5</sup>

<sup>5</sup>total in terms of computation, unsuccessful calculations can still be expressed with constructs like *Maybe a*.

An alternative to this are guards, which make expressions easier to read if many alternatives are involved in a function:

```
myFunc :: Int → Int
myFunc x
  | x < 10 = x * 2
  | x < 12 = x * 3
  | x < 14 = x
  | x > 18 ∧ x < 20 = 42
  | otherwise = x * 4
```

Yet another technique for conditional computation is using pattern matching. For conditional statements we can use it by writing definitions of the function for specific values, like

```
myFunc :: Int → Int
myFunc 5 = 10
myFunc x @ {10} = x * 10
myFunc x = x * 2
```

, where the first matching definition is chosen during computation. Alternatively, we can do pattern matching with the help of case expressions:

```
myFunc :: Int → Int
myFunc x = case x of
  5 → 10
  x @ {10} = x * 10
  x = x * 2
```

These can be used just like ordinary expressions.

We can not, however, express boolean statements in this way. This is because pattern matching is done on the structure of the value that is being pattern matched. Later in this section we will see what other powerful things we can do with this technique.

**where, let**

While Haskell does not have variables, it still allows the programmer to name sub-expressions so that either the code becomes more clear or that it can be reused more easily. Here, two different variants are available: **where** and **let**.

With the help of **where** we can write code like the following:

```
whereReuse :: Double → Double → String
whereReuse a b
  | divided > 10 = "a is more than 10 times b"
  | divided ≡ 10 = "a is 10 times b"
  | otherwise = "a is less than 10 times b"
where divided = a / b
```

**where** is just syntactic sugar, though, and can not be used in expressions like  $f (a * 2 \text{ where } a = 3)$ . This is possible with **let** where we can write  $f (\text{let } a = 3 \text{ in } a * 2)$  or  $\text{let } a = 3 \text{ in } f (a * 2)$ . **let** can in contrast, however, not be used in conjunction with guards.

## Type safety

Haskell is a statically typed functional language. This means that during compilation all types are checked for compatibility and type declarations are not just treated as optional „hints“ to the type-checker. Pairing this with the pure aspect of the language means that Haskell programs seem to be correct more often if the program compiles than in imperative languages. The compiler essentially helps the programmer to write *semantically correct* instead of just syntactically correct code. It should be noted that this does not mean that testing can be omitted. It is still extremely important, but becomes less cumbersome because state is mostly a non-issue.

## Type classes

The example function *multiply* from above seems a bit restrictive as it only allows for the usage of *Ints*. *Ints* are obviously not the only type which can be multiplied. Haskell has a way to express this fact: type classes. We can express the type class *Multiplicable a* that encapsulates the contract of *multiply* on some type *a* as

```
class Multiplicable a where
  multiply :: a → a → a
```

With this class in place, we can then introduce instances - implementations of the contract - for specific types. For example the instance for *Int*, *Multiplicable Int* can be defined as

```
instance Multiplicable Int where
    multiply x y = x * y
```

Now if we want to use this new contract on a generic function  $f$ , we require a *Multiplicable* instance for every type that we want to use *multiply* on inside the function:

```
f :: Multiplicable a => a -> a -> a
f x y = multiply (multiply x y) x
```

Such a function  $f$  does work with the contract of *Multiplicable* instead of requiring some specific type. This way we can reuse many definitions in Haskell even though it is a statically typed language.

In Haskell we can also write type classes with more than one type parameter. This allows for encapsulation of contracts of arbitrary complexity. Furthermore type classes can itself have constraints placed on what types are allowed. Both can be seen here:

```
class (SomeClass a, SomeOtherClass b) => MyClass a b c where
    ...
```

## Lazy Evaluation

One thing that is not obvious when looking at the definitions from this chapter is that Haskell is a lazy language. This means that values are only evaluated when required. This has one major benefit: We get a Producer/Consumer pattern behaviour for free. For example if we have the lazy function  $producer :: Int \rightarrow [Int]$  producing some list of integers and some consumer consuming  $consumer :: [Int] \rightarrow Int$  this list. Then, in a program  $consumer \circ producer$ , *producer* generates the elements of the result-list as they are consumed. This also means that, if *consumer* only requires the first few elements of the list to compute the result, *consumer* does not produce unneeded results.

Laziness even allows us to express infinite streams, which can be helpful in some cases. As an example, an infinite list of ones is defined as

```
ones :: [Int]
ones = 1 : ones
```

Another good example where Laziness simplifies things is when branching is involved:

```
calculateStuff :: [Int] → Int
calculateStuff = if < someCondition >
  then doStuff list1 list2
  else doSomeOtherStuff list1
  where
    list1 = ...
    list2 = ...
```

Here, *list2* is not required in both branches of the if statement. Thanks to laziness it is therefore only evaluated upon a successful if-check. While such a behaviour is obviously possible in non-lazy languages the elegance of the above definition is apparent. We can define as many variables in the same clear way without having unnecessary computations or code dealing with conditional computation like nested **wheres**.

Usually laziness is beneficial to programs, but sometimes we require more control about when something is evaluated. This can be done for example with

## Custom types

As in any good programming language, in Haskell programmers obviously do not have to represent everything with only some base-set of types. Types are usually defined in three different ways. For starters, we can give types aliases with the **type** keyword like

```
-- Tuple of a and b
type Tuple a b = (a, b)

-- Tuple of ints
type IntTuple = (Int, Int)
```

, which are treated just like original  $(a, b)$  or  $Int, Int$  would. This means, we can use such types loosely and pass e.b. a `Tuple Int Int` into a function  $f :: (Int, Int) \rightarrow \dots$ . The same also holds for typeclasses.

The second way to declare types, **data** however declares new-types as in actual new types in the type system like

```

data Direction =
    North
  | NorthEast
  | East
  | SouthEast
  | South
  | SouthWest
  | West
  | NorthWest

```

, where *North* - *NorthWest* are called constructors.

**data** types are not limited to enum-style types though, they can also hold values, like the *Maybe a* type from Haskell. This type - which *may* hold a value *a* internally - is usually used as a return type for functions which not always return an actual result. We can define it as follows:

```

-- unnamed field
data Maybe a = Just a | Nothing

```

where values are created by calling the constructors with the appropriate parameters (if any), i.e. when passed into a function:  $f$  (*Just* 1). Furthermore, **data** constructors can have named fields defined like

```

-- named field
data Maybe a =
    Just { theThing :: a }
  | Nothing

```

where values are created by calling the constructor and passing the appropriate parameters to the properties, i.e.  $f$  (*Maybe* { *theThing* = 1 }) The final way to define custom types is via **newtype**:

```

-- unnamed field
newtype MyNewType a = Constructor a
-- named field
newtype MyOtherNewType a = Constructor { myOnlyThing :: a }

```

Types declared this way are similar to **data** types, but can only contain a single constructor with just a single field. Also, unlike **data**, constructors declared with



**newtype** are strict, meaning the compiler can optimize away the surrounding declaration. Everything else is handled exactly like with **data** types. **newtype** types are also a useful tool if we were to write a wrapper for a type while not wanting to inherit all instances of typeclasses, but are also often used when declaring more complicated types.

## Pattern Matching

While we have seen pattern matching as an alternative to **if...then...else** and guard statements, it can do more things. For example, if we have a datatype *MyType a* type defined as

```
data MyType a = SomeConstructor a | SomeOtherConstructor a
```

and we want to write a function *unwrap :: MyType a → a* to unwrap the *a* value, we use pattern matching like this:

```
unwrap :: MyType a → a
unwrap (SomeConstructor x) = x
unwrap (SomeOtherConstructor x) = x
```

This type of unwrapping can also be done with the help of **case** statements so that we do not require a new function definition:

```
someFunc :: MyType a → a
someFunc t = case t of
  (SomeConstructor a) → ...
  (SomeOtherConstructor a) → ...
```

In Haskell programs we can also write unwrapping code with the help of the **let** notation for single constructor types like **let**  $(x, y) = \text{vec2d}$  **in**  $\text{sqrt } (x * x + y * y)$ . Predictably, we can do this with **where** as well with constructs like **where**  $(x, y) = \text{vec2d}$ .

Sometimes we only care about some part of the value. For example, in a definition of *maybeHead :: [a] → Maybe a*, which should return the first element of the list or *Nothing* if it is an empty list, we can write this with the help of wildcards (**\_**) as:

```
maybeHead :: Maybe a → a
maybeHead (x : _) = x
maybeHead [] = Nothing
```

We could even write

```
maybeHead :: Maybe a → a
maybeHead (x : _) = x
maybeHead _ = Nothing
```

as the second equation will only ever match when the list is empty.<sup>6</sup> Furthermore, in Haskell, functions *isJust* :: *Maybe a* → *Bool* when they only care about the structure of the type, can be written with only with wildcards:

```
isJust :: Maybe a → Bool
isJust (Just _) = True
isJust _ = False
```

Additionally, if we want to preserve laziness and we are 100% sure that a match will work (e.g. if we have called *isJust*), we can use irrefutable patterns like *~(Just a) = someMaybe*.

## Lambdas and Partial application

As Functions are just another type that can be passed into higher-order functions it makes sense to have a short-hand to write anonymous functions - lambdas. In Haskell they look like this:

```
λ(a, b) → a + b
```

This can easily be passed into functions, like `zipWith`<sup>^</sup> [While  $(\lambda(a, b) \rightarrow a + b)$  is obviously the same as  $(+)$ , we just write it as a lambda here for demonstration purposes]:

```
someFunc :: [Int] → [Int] → [Int]
someFunc xs ys = zipWith (λ(a, b) → a + b) xs ys
```

Here, we notice the reason for yet another feature in Haskell that is commonly used: Partial application. While the definition of *someFunc* is definitely not wrong, we could have written it more elegantly as

```
someFunc :: [Int] → [Int] → [Int]
someFunc = zipWith (λ(a, b) → a + b)
```

---

<sup>6</sup>A single element list has two forms in Haskell,  $a : []$  and  $[a]$  of which the latter is just syntactic sugar of the former.

where we this means that *someFunc* is defined as *zipWith* ::  $(a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$  partially applied with the passed lambda to get a function with type  $[a] \rightarrow [b] \rightarrow [c]$  which the compiler then automatically binds to the type of *someFunc* ::  $[Int] \rightarrow [Int] \rightarrow [Int]$ .

### 3.1.3 Monads

Functional programmers try to avoid mutable state at all cost, but programs that do not only just compute some function usually involve some sort of it. So, doesn't this make Haskell useless being a pure functional language without *any* mutable state? No. Functional Programs generally just avoid *unnecessary* mutable state at all cost. The fact of the matter is that in functional programming, we can represent mutable state as well, but we do so in a meaningful and controlled manner.

While in most computations, we could represent state, by passing it into every function that can possibly change it and returning it alongside of the actual returned value like

```
comp :: MyState → Int → (Int, MyState)
comp curState x = (x + 3, nextState)
  where nextState = changeState curState
```

this can become unnecessarily complicated to handle by hand. A better alternative is the use of monads, which are the main concept generally used in computations involving some sort of mutable state. The typeclass for the *Monad* typeclass can be defined as

```
class Monad m where
  (≫=) :: m a → (a → m b) → m b
  (≫)  :: m a → m b → m b
  m ≫ k = m ≻= \_ → k
  return :: a → m a
```

Thinking of Monads as computations, we can come up with the following explanation: *return* is used to create a computation *m a* just returning some given value *a*. Next,  $(\gg=)$  is used to compose some monadic computation *m a* returning some *a* with a monadic function  $a \rightarrow m b$  to return some computation *m b* returning some *b*. Finally,  $(\gg)$  is used to define the order of two monadic computations *m a* and *m b* so that *m a* is computed before *m b* while discarding the result of the first one as can also be seen in its default implementation above.

Given this definition of a Monad, we can now take a look at the well known example for a Monad - the *State* Monad. It is defined as (Michaelson, 2013)

```
newtype State s a = State { runState :: s → (a, s) }
```

where a *State s a* encapsulates a stateful computation on some state type *s* yielding some value of type *a*. For easier understanding it is often useful to think of *State s a* just as a usability wrapper around a function  $s \rightarrow (a, s)$  that returns some *a* if we pass it some state *s*. The *State* monad therefore only contains the „blueprint“ of the computation that can only be run if we start it by passing a state. The instance for the Monad typeclass can then be defined as

```
instance Monad (State s) where
  (State h) >>= f = State $ \s → let (a, newState) = h s
    (State g) = f a
    in g newState
  return x = State { runState = \s → (x, s) }
```

where we declare the Monad deliberately on top of *State s* meaning that *State* itself is not a monad, but it is a monad together with some state representation *s*.<sup>7</sup> Note how the operations are defined here: *return* encapsulates the given value  $x :: a$  inside the internal function and therefore is equal to the identity  $id :: a \rightarrow a$  function on tuples with one parameter already applied.<sup>8</sup> In the composition operator  $\gg=$ , the monadic computation  $State\ h :: State\ s\ a$  is composed with the function  $f :: a \rightarrow State\ s\ b$  into a new monadic computation of type  $State\ s\ b$ . The internal function of the state is essentially taken out of the first argument and composed with the second argument inside the returned Monad.

Additionally, we have helper operations to use this construct with. The first is  $put :: s \rightarrow State\ s\ ()$  which overwrites the current state returning a unit  $()$  as result:

```
put :: s → State s ()
put newState = State { runState = \s → ((), newState) }
```

The second one is  $get :: State\ s\ s$  which returns the current state, but does not change it:

```
get :: State s s
get = State { runState = \s → (s, s) }
```

<sup>7</sup>We can't have declared *State* a monad anyways since the Monad is a typeclass with just one type parameter

<sup>8</sup>With the help of  $curry :: ((a, b) \rightarrow c) \rightarrow a \rightarrow b \rightarrow c$ , we could have therefore also written  $return\ x = State\ \{ runState = (curry\ id)\ x \}$

With these operations, we can easily write stateful programs like this one:<sup>9</sup>

```
type Stack = [Int]
empty :: Stack
empty = []

pop :: State Stack Int
pop = get >>= (\(x : xs) → put xs >> return x)

push :: Int → State Stack ()
push a = State $ \xs → ((), a : xs)

peek :: State Stack Int
peek = get >>= \((x : xs) → return x

computeStateful :: State Stack Int
computeStateful = push 10 >>
  push 20 >>
  pop >>= \a →
    (pop >>= \b → push (a + b)) >>
  peek

-- main program inside the IO monad
main :: IO ()
main = print (evalState computeStateful empty)
```

Here, *computeStateful* first pushes some values on top of a stack represented by a list *[Int]* (the actual state inside of the *State* monad) and then *pops* these values and *pushes* their sum back on the stack to finally *peek* the actual value which then is the result of the computation. To make writing such code easier, Haskell has syntactic sugar called *do* notation. With it we can write the above method *computeStateful* in a way that resembles imperative-style code (but with side-effects clearly encapsulated) as:

```
computeStateful :: State Stack Int
computeStateful = do
  push 10
  push 20
  a ← pop
  b ← pop
  push (a + b)
  peek
```

---

<sup>9</sup>inspired and adapted from <https://gist.github.com/sdiehl/8d991a718f7a9c80f54b>

In this example, we can also see the direct relationship between ( $\gg$ ) and simple new lines and the ( $\gg=$ ) operator and the special  $\leftarrow$  operator in *do* notation which facilitates the binding to a variable.<sup>10</sup>

Other often used Monads in the Haskell eco-system include the *Writer* monad, which is useful for e.g. logging or the *IO* monad, which is used to encapsulate I/O computations as well as compiler internal operations. Furthermore, as one of many other applications, Monads are used in some parallel Haskell as we will see later in this thesis.

### 3.1.4 Arrows

Arrows were introduced by Hughes (2000) as a general interface for computation and a less restrictive generalisation of Monads. Hughes (2000) motivates the broader interface of Arrows with the example of a parser with added static meta-information that can not satisfy the monadic bind operator ( $\gg=$ )  $:: m\ a \rightarrow (a \rightarrow m\ b) \rightarrow m\ b$  (with  $m$  being a Monad).<sup>11</sup>

An Arrow  $arr\ a\ b$  represents a computation that converts an input  $a$  to an output  $b$ . This is defined in the *Arrow* type class shown in Fig. 3.5. To lift an ordinary function to an Arrow,  $arr$  is used, analogous to the monadic *return*. Similarly, the composition operator  $\gg$  is analogous to the monadic composition  $\gg=$  and combines two Arrows  $arr\ a\ b$  and  $arr\ b\ c$  by „wiring“ the outputs of the first to the inputs to the second to get a new Arrow  $arr\ a\ c$ . Lastly, the *first* operator takes the input Arrow  $arr\ a\ b$  and converts it into an Arrow on pairs  $arr\ (a, c)\ (b, c)$  that leaves the second argument untouched. It allows us to save input across Arrows. Fig. 3.6 shows a graphical representation of these basic Arrow combinators. The most prominent instances of this interface are regular functions ( $\rightarrow$ ) and the Kleisli type (Fig. 3.5), which wraps monadic functions, e.g.  $a \rightarrow m\ b$ .

Hughes also defined some syntactic sugar (Fig.3.7): *second*, *\*\*\** and *&&&*. *second* is the mirrored version of *first* (Appendix 10.1). The *\*\*\** function combines *first* and *second* to handle two inputs in one arrow, and is defined as follows:

$$\begin{aligned} (***)\ &::\ Arrow\ arr \Rightarrow arr\ a\ b \rightarrow arr\ c\ d \rightarrow arr\ (a, c)\ (b, d) \\ f\ ***\ g\ &= first\ f\ \gg\ second\ g \end{aligned}$$

<sup>10</sup>( $\gg=$ ) is also often called *bind* in languages which do not support operator overloading.

<sup>11</sup>In the example a parser of the type *Parser s a* with static meta information  $s$  and result  $a$  is shown to not be able to use the static  $s$  without applying the monadic function  $a \rightarrow m\ b$ . With Arrows this is possible.

```

class Arrow arr where
  arr :: (a → b) → arr a b
  (>>>) :: arr a b → arr b c → arr a c
  first :: arr a b → arr (a, c) (b, c)

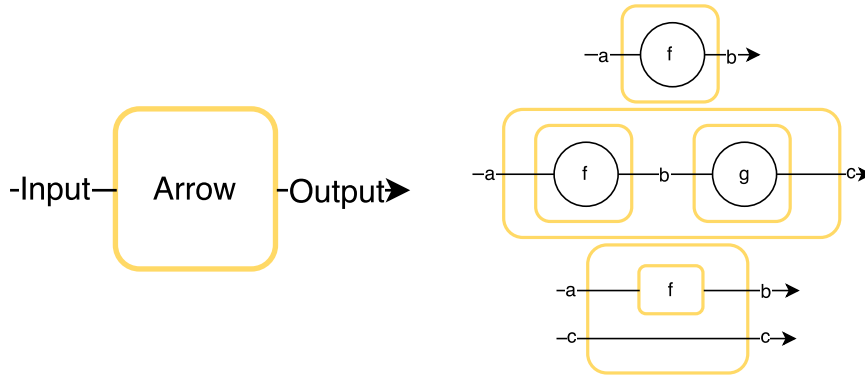
instance Arrow (→) where
  arr f = f
  f >>> g = g ∘ f
  first f = λ(a, c) → (f a, c)

data Kleisli m a b = Kleisli { run :: a → m b }

instance Monad m ⇒ Arrow (Kleisli m) where
  arr f = Kleisli (return ∘ f)
  f >>> g = Kleisli (λa → f a >>= g)
  first f = Kleisli (λ(a, c) → f a >>= λb → return (b, c))

```

**Figure 3.5:** The *Arrow* type class and its two most typical instances.



**Figure 3.6:** Schematic depiction of an *Arrow* (left) and its basic combinators *arr*, *>>>* and *first* (right).

The *&&&* combinator, which constructs an *Arrow* that outputs two different values like *\*\*\**, but takes only one input, is:

```

(&&&) :: Arrow arr ⇒ arr a b → arr a c → arr a (b, c)
f &&& g = arr (λa → (a, a)) >>> f *** g

```

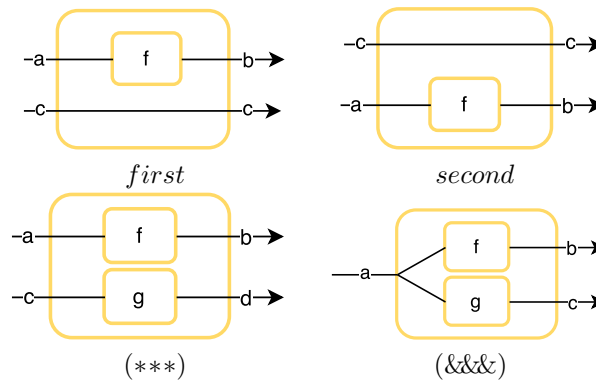
A first short example given by Hughes on how to use *Arrows* is addition with *Arrows*:

```

add :: Arrow arr ⇒ arr a Int → arr a Int → arr a Int
add f g = f &&& g >>> arr (λ(u, v) → u + v)

```

As we can rewrite the monadic bind operation (*>>=*) with only the *Kleisli* type into  $m\ a \rightarrow Kleisli\ m\ a\ b \rightarrow m\ b$ , but not with a general *Arrow*  $arr\ a\ b$ , we can intuitively



**Figure 3.7:** Visual depiction of syntactic sugar for Arrows.

get an idea of why Arrows must be a generalisation of Monads. While this also means that a general Arrow can not express everything a Monad can, Hughes (2000) shows in his parser example that this trade-off is worth it in some cases.

## 3.2 Short introduction to parallel Haskell

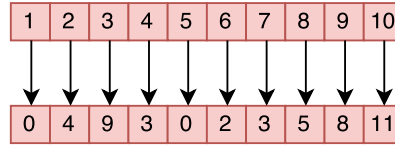
In 3.1, we cited Hughes (1990) saying that in functional programming, the order of evaluation is irrelevant. In parallel programs this is not the case, as at least some kind of structure of evaluation is required to have actual speedup in programs. In the following we will take a look at how parallelism can be achieved in Haskell programs in general. Now, one might think that we would want side effects (parallel evaluation is a side-effect) and require the need to think about order of evaluation in a pure functional program seems a bit odd. The fact of the matter is that functional programs only aim to avoid *unnecessary* side-effects and in the case of parallelism it is obvious that some amount of side-effects are required. Also, parallel Haskell generally aim to encapsulate all the necessary and complicated code in a way such that the room for code-breaking errors is almost impossible. If some parallel evaluation code is written in a sub-optimal way, only the performance is affected, but not the result, which will always be tractable no matter the order of evaluation.<sup>12</sup>

In its purest form, parallel computation (on functions) can be looked at as the execution of some functions  $a \rightarrow b$  in parallel or  $parEvalN :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$ , as also Fig. 3.8 symbolically shows.

In this section, we will implement this non-Arrow version which will later be adapted for usage in our Arrow-based parallel Haskell.

<sup>12</sup>Some exceptions using unsafe and non-deterministic operations exist, though. These situations can however only be achieved if the programmer actively chooses to use these kinds of operations.





**Figure 3.8:** Schematic illustration of *parEvalN*. A list of inputs is transformed by different functions in parallel.

There exist several parallel Haskell already. Among the most important are probably GpH (based on *par* and *pseq* „hints“, Trinder et al. (1996), Trinder et al. (1998)), the *Par* Monad (a Monad for deterministic parallelism, Marlow et al. (2011), Foltzer et al. (2012)), Eden (a parallel Haskell for distributed memory, Loogen et al. (2005), Loogen (2012)), HdpH (a Template Haskell-based parallel Haskell for distributed memory, Maier et al. (2014), Stewart (2016)) and LVish (a *Par* extension with focus on communication, Kuper et al. (2014)).

As the goal of this paper is not to re-implement yet another parallel runtime, but to represent parallelism with Arrows, we base our efforts on existing work which we wrap as backends behind a common interface. For this paper we chose GpH for its simplicity, the *Par* Monad to represent a monadic DSL, and Eden as a distributed parallel Haskell.

LVish and HdpH were not chosen as the former does not differ from the original *Par* Monad with regard to how we would have used it in this paper, while the latter (at least in its current form) does not comply with our representation of parallelism due to its heavy reliance on Template Haskell.

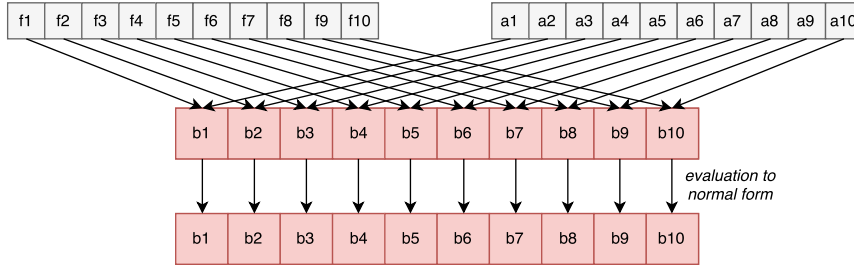
We will now go into some detail on GpH, the *Par* Monad and Eden, and also give their respective implementations of the non-Arrow version of *parEvalN*.

### 3.2.1 Glasgow parallel Haskell – GpH

GpH (Marlow et al., 2009, Trinder et al. (1998)) is one of the simplest ways to do parallel processing found in standard GHC.<sup>13</sup> Besides some basic primitives (*par* and *pseq*), it ships with parallel evaluation strategies for several types which can be applied with *using* :: *a* → *Strategy a* → *a*, which is exactly what is required for an implementation of *parEvalN*.

$$\begin{aligned}
 \text{parEvalN} &:: (\text{NFData } b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b] \\
 \text{parEvalN } fs \ as &= \text{let } bs = \text{zipWith } (\$) \ fs \ as \\
 &\quad \text{in } bs \text{ 'using' } \text{parList } \text{rdeepseq}
 \end{aligned}$$

<sup>13</sup>The Multicore implementation of GpH is available on Hackage under <https://hackage.haskell.org/package/parallel-3.2.1.0>, compiler support is integrated in the stock GHC.



**Figure 3.9:** *parEvalN* (GpH).

In the above definition of *parEvalN* we just apply the list of functions  $[a \rightarrow b]$  to the list of inputs  $[a]$  by zipping them with the application operator  $\$$ . We then evaluate this lazy list  $[b]$  according to a *Strategy*  $[b]$  with the *using*  $:: a \rightarrow \text{Strategy } a \rightarrow a$  operator. We construct this strategy with *parList*  $:: \text{Strategy } a \rightarrow \text{Strategy } [a]$  and *rdeepseq*  $:: \text{NFData } a \Rightarrow \text{Strategy } a$  where the latter is a strategy which evaluates to normal form. Other strategies like e.g. evaluation to weak head normal form are available as well. It also allows for custom *Strategy* implementations to be used. Fig. 3.9 shows a visual representation of this code.

### 3.2.2 *Par* Monad

The *Par* Monad<sup>14</sup> introduced by (Marlow et al., 2011), is a Monad designed for composition of parallel programs. Let:

$$\begin{aligned} \text{parEvalN} &:: (\text{NFData } b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b] \\ \text{parEvalN } fs \ as &= \text{runPar } \$ \\ &\quad (\text{sequenceA } (\text{map } (\text{return} \circ \text{spawn}) (\text{zipWith } (\$) \ fs \ as))) \gg= \text{mapM } \text{get} \end{aligned}$$

The *Par* Monad version of our parallel evaluation function *parEvalN* is defined by zipping the list of  $[a \rightarrow b]$  with the list of inputs  $[a]$  with the application operator  $\$$  just like with GpH. Then, we map over this not yet evaluated lazy list of results  $[b]$  with *spawn*  $:: \text{NFData } a \Rightarrow \text{Par } a \rightarrow \text{Par } (\text{IVar } a)$  to transform them to a list of not yet evaluated forked away computations  $[\text{Par } (\text{IVar } b)]$ , which we convert to  $\text{Par } [\text{IVar } b]$  with *sequenceA*. We wait for the computations to finish by mapping over the *IVar*  $b$  values inside the *Par* Monad with *get*. This results in  $\text{Par } [b]$ . We execute this process with *runPar* to finally get  $[b]$ . While we used *spawn* in the definition above, a head-strict variant can easily be defined by replacing *spawn* with *spawn\_*  $:: \text{Par } a \rightarrow \text{Par } (\text{IVar } a)$ . Fig. 3.10 shows a graphical representation.

<sup>14</sup>The *Par* Monad can be found in the *monad-par* package on Hackage under <https://hackage.haskell.org/package/monad-par-0.3.4.8/>.

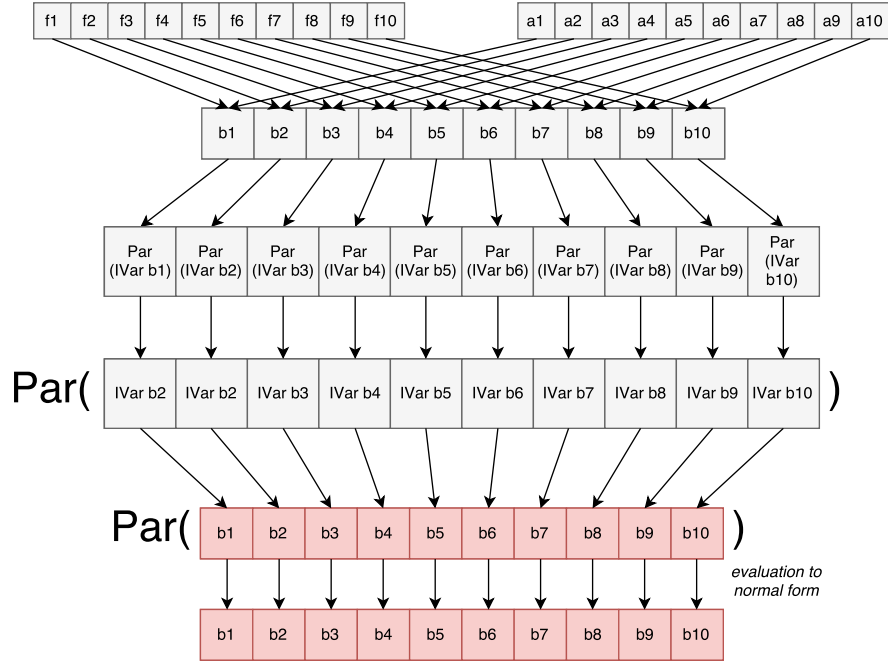


Figure 3.10: *parEvalN* (Par Monad).

### 3.2.3 Eden

Eden (Loogen et al., 2005, Loogen (2012)) is a parallel Haskell for distributed memory and comes with MPI and PVM as distributed backends.<sup>15</sup> It is targeted towards clusters, but also functions well in a shared-memory setting with a further simple backend. However, in contrast to many other parallel Haskell, in Eden each process has its own heap. This seems to be a waste of memory, but with distributed programming paradigm and individual GC per process, Eden yields good performance results on multicores, as well (Berthold et al., 2009a, Aswad et al. (2009)).

While Eden comes with a Monad *PA* for parallel evaluation, it also ships with a completely functional interface that includes a *spawnF* :: (*Trans a*, *Trans b*) ⇒ [*a* → *b*] → [*a*] → [*b*] function that allows us to define *parEvalN* directly:

$$\begin{aligned} \text{parEvalN} &:: (\text{Trans } a, \text{Trans } b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b] \\ \text{parEvalN} &= \text{spawnF} \end{aligned}$$

#### Eden TraceViewer

To comprehend the efficiency and the lack thereof in a parallel program, an inspection of its execution is extremely helpful. While some large-scale solutions

<sup>15</sup>The projects homepage can be found at <http://www.mathematik.uni-marburg.de/~eden/>. The Hackage page is at <https://hackage.haskell.org/package/edenmodules-1.2.0.0/>.

exist (Geimer et al., 2010), the parallel Haskell community mainly utilises the tools Threadscope (Wheeler and Thain, 2009) and Eden TraceViewer<sup>16</sup> (Berthold and Loogen, 2007). In the next sections we will present some *trace visualisations*, the post-mortem process diagrams of Eden processes and their activity.

The trace visualisations are colour-coded. In such a visualisation (Fig. 5.2), the  $x$  axis shows the time, the  $y$  axis enumerates the machines and processes. The visualisation shows a running process in green, a blocked process is red. If the process is „runnable“, i.e. it may run, but does not, it is yellow. The typical reason for this is GC. An inactive machine, where no processes are started yet, or all are already terminated, shows as a blue bar. A communication from one process to another is represented with a black arrow. A stream of communications, e.g. a transmitted list is shown as a dark shading between sender and receiver processes.

---

<sup>16</sup>See <http://hackage.haskell.org/package/edentv> on Hackage for the last available version of Eden TraceViewer.

## Parallel Arrows

While Arrows are a general interface to computation, we introduce here specialised Arrows as a general interface to *parallel computations*. We present the *ArrowParallel* type class and explain the reasoning behind it before discussing some parallel Haskell implementations and basic extensions.

### 4.1 The *ArrowParallel* type class

A parallel computation (on functions) can be seen as execution of some functions  $a \rightarrow b$  in parallel, as our *parEvalN* prototype shows (Section 3.2). Translating this into Arrow terms gives us a new operator *parEvalN* that lifts a list of Arrows  $[arr\ a\ b]$  to a parallel Arrow  $arr\ [a]\ [b]$ . This combinator is similar to *evalN* from Appendix 10.1, but does parallel instead of serial evaluation.

$$parEvalN :: (Arrow\ arr) \Rightarrow [arr\ a\ b] \rightarrow arr\ [a]\ [b]$$

With this definition of *parEvalN*, parallel execution is yet another Arrow combinator. But as the implementation may differ depending on the actual type of the Arrow *arr* - or even the input *a* and output *b* - and we want this to be an interface for different backends, we introduce a new type class *ArrowParallel arr a b*:

```
class Arrow arr => ArrowParallel arr a b where
  parEvalN :: [arr a b] -> arr [a] [b]
```

Sometimes parallel Haskells require or allow for additional configuration parameters, e.g. an information about the execution environment or the level of evaluation (weak head normal form vs. normal form). For this reason we introduce an additional *conf* parameter as we do not want *conf* to be a fixed type, as the configuration parameters can differ for different instances of *ArrowParallel*.

```
class Arrow arr => ArrowParallel arr a b conf where
  parEvalN :: conf -> [arr a b] -> arr [a] [b]
```

By restricting the implementations of our backends to a specific *conf* type, we also get interoperability between backends for free. We can parallelize one part of a program using one backend, and parallelize the next with another one.

## 4.2 *ArrowParallel* instances

With the type class defined, we will now give implementations of it with GpH, the *Par* Monad and Eden.

### 4.2.1 Glasgow parallel Haskell

The GpH implementation of *ArrowParallel* is implemented in a straightforward manner in Fig. 4.1, but a bit different compared to the variant from Section 3.2.1. We use  $evalN :: [arr\ a\ b] \rightarrow arr\ [a]\ [b]$  (definition in Appendix 10.1, think *zipWith* (\$) on Arrows) combined with  $withStrategy :: Strategy\ a \rightarrow a \rightarrow a$  from GpH, where *withStrategy* is the same as *using*  $:: a \rightarrow Strategy\ a \rightarrow a$ , but with flipped parameters. Our *Conf a* datatype simply wraps a *Strategy a*, but could be extended in future versions of our DSL.

```

data Conf a = Conf (Strategy a)
instance (ArrowChoice arr) =>
  ArrowParallel arr a b (Conf b) where
    parEvalN (Conf strat) fs =
      evalN fs >>>
      arr (withStrategy (parList strat))

```

**Figure 4.1:** GpH *ArrowParallel* instance.

#### 4.2.2 *Par* Monad

As for GpH we can easily lift the definition of *parEvalN* for the *Par* Monad to Arrows in Fig. 4.2. To start off, we define the *Strategy a* and *Conf a* type so we can have a configurable instance of *ArrowParallel*:

```

type Strategy a = a → Par (IVar a)
data Conf a = Conf (Strategy a)

```

Now we can once again define our *ArrowParallel* instance as follows: First, we convert our Arrows  $[arr\ a\ b]$  with *evalN* (*map* ( $\ggg arr\ strat$ ) *fs*) into an Arrow  $arr\ [a]\ [(Par\ (IVar\ b))]$  that yields composable computations in the *Par* monad. By combining the result of this Arrow with *arr sequenceA*, we get an Arrow  $arr\ [a]\ (Par\ [IVar\ b])$ . Then, in order to fetch the results of the different threads, we map over the *IVars* inside the *Par* Monad with *arr* ( $\gg\ mapM\ get$ ) – our intermediary Arrow is of type  $arr\ [a]\ (Par\ [b])$ . Finally, we execute the computation *Par*  $[b]$  by composing with *arr runPar* and get the final Arrow  $arr\ [a]\ [b]$ .

```

instance (ArrowChoice arr)  $\Rightarrow$  ArrowParallel arr a b (Conf b) where
  parEvalN (Conf strat) fs =
    evalN (map ( $\ggg$  arr strat) fs)  $\ggg$ 
    arr sequenceA  $\ggg$ 
    arr ( $\gg=$  mapM Control.Monad.Par.get)  $\ggg$ 
    arr runPar

```

**Figure 4.2:** *Par* Monad *ArrowParallel* instance.

### 4.2.3 Eden

For both the GpH Haskell and *Par* Monad implementations we could use general instances of *ArrowParallel* that just require the *ArrowChoice* type class. With Eden this is not the case as we can only spawn a list of functions, which we cannot extract from general Arrows. While we could still manage to have only one instance in the module by introducing a type class

```

class (Arrow arr)  $\Rightarrow$  ArrowUnwrap arr where
  unwrap :: arr a b  $\rightarrow$  (a  $\rightarrow$  b)

```

we avoid doing so for aesthetic reasons. For now, we just implement *ArrowParallel* for normal functions and the Kleisli type in Fig. 4.3, where *Conf* is simply defined as **data** *Conf* = *Nil* since Eden does not have a configurable *spawnF* variant.



```

instance (Trans a, Trans b)  $\Rightarrow$  ArrowParallel ( $\rightarrow$ ) a b Conf where
    parEvalN _ = spawnF
instance (ArrowParallel ( $\rightarrow$ ) a (m b) Conf,
    Monad m, Trans a, Trans b, Trans (m b))  $\Rightarrow$ 
    ArrowParallel (Kleisli m) a b conf where
        parEvalN conf fs =
            arr (parEvalN conf (map ( $\lambda$ (Kleisli f)  $\rightarrow$  f) fs))  $\ggg$ 
            Kleisli sequence

```

**Figure 4.3:** Eden *ArrowParallel* instance.

## 4.2.4 Default configuration instances

While the configurability in the instances of the *ArrowParallel* instances above is nice, users probably would like to have proper default configurations for many parallel programs as well. These can also easily be defined as we can see by the example of the default implementation of *ArrowParallel* for the GpH:

```

instance (NFData b, ArrowChoice arr, ArrowParallel arr a b (Conf b))  $\Rightarrow$ 
    ArrowParallel arr a b () where
        parEvalN _ fs = parEvalN (defaultConf fs) fs
        defaultConf :: (NFData b)  $\Rightarrow$  [arr a b]  $\rightarrow$  Conf b
        defaultConf fs = stratToConf fs rdeepseq
        stratToConf :: [arr a b]  $\rightarrow$  Strategy b  $\rightarrow$  Conf b
        stratToConf _ strat = Conf strat

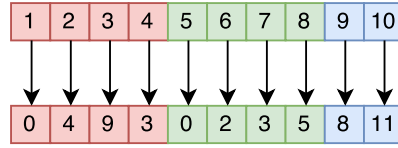
```

The other backends have similarly structured implementations which we do not discuss here for the sake of brevity. We can, however, only have one instance of *ArrowParallel* *arr* *a* *b* () present at a time, which should not be a problem, though.

Up until now we discussed Arrow operations more in detail, but in the following sections we focus more on the data-flow between the Arrows, now that we have seen that Arrows are capable of expressing parallelism. We do explain new concepts with more details if required for better understanding, though.

## 4.3 Extending the interface

With the *ArrowParallel* type class in place and implemented, we can now define other parallel interface functions. These are basic algorithmic skeletons that are used to define more sophisticated skeletons.



**Figure 4.4:** *parEvalNLazy* depiction.

```

parEvalNLazy :: (ArrowParallel arr a b conf, ArrowChoice arr, ArrowApply arr) =>
  conf -> ChunkSize -> [arr a b] -> (arr [a] [b])
parEvalNLazy conf chunkSize fs =
  arr (chunksOf chunkSize) >>>
  evalN fchunks >>>
  arr concat
where
  fchunks = map (parEvalN conf) (chunksOf chunkSize fs)

```

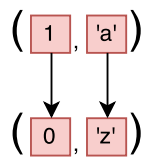
**Figure 4.5:** Definition of *parEvalNLazy*.

### 4.3.1 Lazy *parEvalN*

The function *parEvalN* fully traverses the list of passed Arrows as well as their inputs. Sometimes this might not be feasible, as it will not work on infinite lists of functions like e.g. *map (arr ∘ (+)) [1..]* or just because we need the Arrows evaluated in chunks. *parEvalNLazy* (Figs. 4.4, 4.5) fixes this. It works by first chunking the input from *[a]* to *[[a]]* with the given *chunkSize* in *arr (chunksOf chunkSize)*. These chunks are then fed into a list *[arr [a] [b]]* of chunk-wise parallel Arrows with the help of our lazy and sequential *evalN*. The resulting *[[b]]* is lastly converted into *[b]* with *arr concat*.

### 4.3.2 Heterogeneous tasks

We have only talked about the parallelization of Arrows of the same set of input and output types until now. But sometimes we want to parallelize heterogeneous types as well. We can implement such a *parEval2* combinator (Figs. 4.6, 10.16) which combines two Arrows *arr a b* and *arr c d* into a new parallel Arrow *arr (a, c) (b, d)* quite easily with the help of the *ArrowChoice* type class. Here, the general idea is



**Figure 4.6:** *parEval2* depiction.

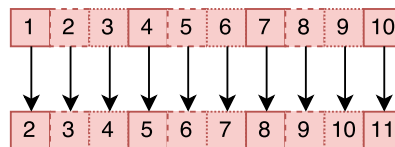
to use the `+++` combinator which combines two Arrows `arr a b` and `arr c d` and transforms them into `arr (Either a c) (Either b d)` to get a common Arrow type that we can then feed into `parEvalN`.

## 4.4 Basic *map*-based skeletons

Now we have developed Parallel Arrows far enough to define some useful algorithmic skeletons that abstract typical parallel computations. We start here with some basic *map*-based skeletons. The essential differences between these skeletons presented here are in terms of order of evaluation and work distribution but still provide the same semantics as a sequential *map*.

### 4.4.1 Parallel *map* and laziness

The `parMap` skeleton (Figs. 10.7, 10.8) is probably the most common skeleton for parallel programs. We can implement it with `ArrowParallel` by repeating an Arrow `arr a b` and then passing it into `parEvalN` to obtain an Arrow `arr [a] [b]`. Just like `parEvalN`, `parMap` traverses all input Arrows as well as the inputs. Because of this, it has the same restrictions as `parEvalN` as compared to `parEvalNLazy`. So it makes sense to also have a `parMapStream` (Figs. 10.9, 10.10) which behaves like `parMap`, but uses `parEvalNLazy` instead of `parEvalN`. Implementing these skeletons is straightforward as in Appendix 10.3 in Figs. 10.8 and 10.10.



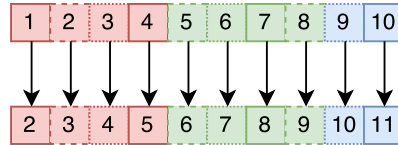
**Figure 4.7:** *farm* depiction.

```

farm :: (ArrowParallel arr a b conf,
        ArrowParallel arr [a] [b] conf, ArrowChoice arr) =>
        conf -> NumCores -> arr a b -> arr [a] [b]
farm conf numCores f =
    unshuffle numCores >>>
    parEvalN conf (repeat (mapArr f)) >>>
    shuffle

```

**Figure 4.8:** *farm* definition.



**Figure 4.9:** *farmChunk* depiction.

#### 4.4.2 Statically load-balancing parallel *map*

Our *parMap* spawns every single computation in a new thread (at least for the instances of *ArrowParallel* we presented in this paper). This can be quite wasteful and a statically load-balancing *farm* (Figs. 4.7, 4.8) that equally distributes the workload over *numCores* workers seems useful. The definitions of the helper functions *unshuffle*, *takeEach*, *shuffle* (Fig. 10.11) originate from an Eden skeleton<sup>1</sup>.

Since a *farm* is basically just *parMap* with a different work distribution, it has the same restrictions as *parEvalN* and *parMap*. We can, however, define *farmChunk* (Figs. 4.9, 10.14) which uses *parEvalNLazy* instead of *parEvalN*. It is basically the same definition as for *farm*, but with *parEvalNLazy* instead of *parEvalN*.

<sup>1</sup>Available on Hackage under <https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/src/Control-Parallel-Eden-Map.html>.

## Further development of Parallel Arrows

### 5.1 Futures

Consider the outline parallel Arrow combinator in Fig. 5.1. In a distributed environment this first evaluates all `[arr a b]` in parallel, sends the results back to the master node, rotates the input once (in the example we require *ArrowChoice* for this) and then evaluates the `[arr b c]` in parallel to then gather the input once again on the master node. Such situations arise, e.g. in scientific computations when data distributed across the nodes needs to be transposed. A concrete example is 2D FFT computation (Gorlatch and Bischof, 1998, Berthold et al. (2009c)).

While the example could be rewritten into a single *parEvalN* call by directly wiring the Arrows together before spawning, it illustrates an important problem. When using a *ArrowParallel* backend that resides on multiple computers, all communication between the nodes is done via the master node, as shown in the Eden trace in Figure 5.2. This can become a serious bottleneck for a larger amount of data and number of processes as e.g. Berthold et al. (2009c) showcases.

This is only a problem in distributed memory (in the scope of this paper) and we should allow nodes to communicate directly with each other. Eden already provides „remote data“ that enable this (Alt and Gorlatch, 2003, Dieterle et al. (2010b)). But as we want code using our DSL to be implementation agnostic, we have to wrap this concept. We do this with the *Future* type class (Fig. 5.3). We require a *conf* parameter here as well, but only so that Haskell's type system allows us to have multiple Future implementations imported at once without breaking any dependencies similar to what we did with the *ArrowParallel* type class earlier.

Since *RD* is only a type synonym for a communication type that Eden uses internally, we have to use some wrapper classes to fit that definition, though, as Fig. 10.5 shows. Technical details are in Appendix, in Section 10.3.

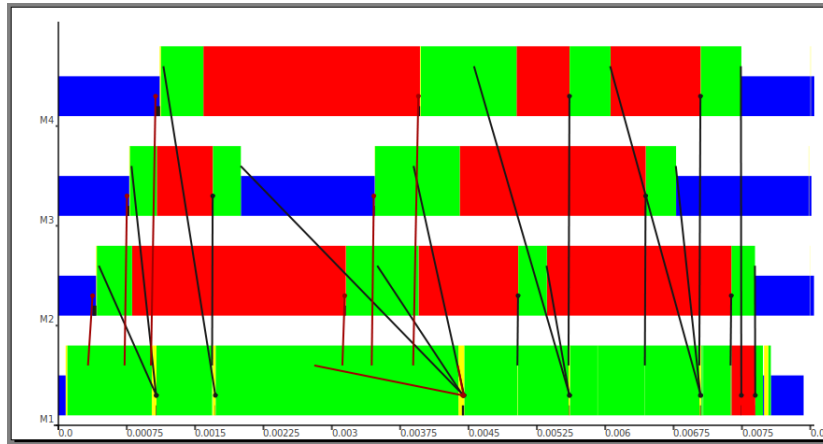
For GpH and *Par* Monad, we can simply use *BasicFutures* (Fig. 10.6), which are just simple wrappers around the actual data with boiler-plate logic so that the type

```

someCombinator :: (ArrowChoice arr,
  ArrowParallel arr a b (),
  ArrowParallel arr b c ()) =>
  [arr a b] -> [arr b c] -> arr [a] [c]
someCombinator fs1 fs2 =
  parEvalN () fs1 >>>
  rightRotate >>>
  parEvalN () fs2

```

**Figure 5.1:** The outline combinator.



**Figure 5.2:** Communication between 4 Eden processes without Futures. All communication goes through the master node. Each bar represents one process. Black lines represent communication. Colours: blue  $\hat{=}$  idle, green  $\hat{=}$  running, red  $\hat{=}$  blocked, yellow  $\hat{=}$  suspended.

class is satisfied. This is because the concept of a *Future* does not change anything for shared-memory execution as there are no communication problems to fix. Nevertheless, we require a common interface so the parallel Arrows are portable across backends. The implementation can also be found in Section 10.3. In our communication example we can use this *Future* concept for direct communication between nodes as shown in Fig. 5.4.

In a distributed environment, this gives us a communication scheme with messages going through the master node only if it is needed – similar to what is shown in the trace visualisation in Fig. 5.5. One especially elegant aspect of the definition in Fig. 5.3 is that we can specify the type of *Future* to be used per backend with full interoperability between code using different backends, without even requiring to know about the actual type used for communication. We only specify that there has to be a compatible Future and do not care about any specifics as can be seen in Fig. 5.4. With the PArrows DSL we can also define default instances *Future fut a ()* for each backend similar to how *ArrowParallel arr a b ()* was defined in Section 4. Details can be found in Section 10.3.

```

class Future fut a conf | a conf → fut where
  put :: (Arrow arr) ⇒ conf → arr a (fut a)
  get :: (Arrow arr) ⇒ conf → arr (fut a) a

```

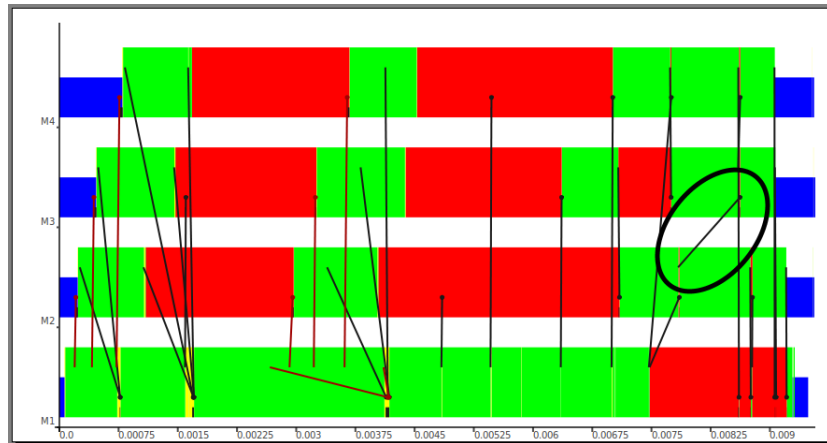
**Figure 5.3:** The *Future* type class.

```

someCombinator :: (ArrowChoice arr,
  ArrowParallel arr a (fut b) (),
  ArrowParallel arr (fut b) c (),
  Future fut b ()) ⇒
  [arr a b] → [arr b c] → arr [a] [c]
someCombinator fs1 fs2 =
  parEvalN () (map (>>> put ()) fs1) >>>
  rightRotate >>>
  parEvalN () (map (get ()) >>>) fs2)

```

**Figure 5.4:** The outline combinator in parallel.



**Figure 5.5:** Communication between 4 Edent processes with Futures. Other than in Fig. 5.2, processes communicate directly (one example message is highlighted) instead of always going through the master node (bottom bar).

## 5.2 Advanced topological skeletons

Even though many algorithms can be expressed by *parMaps*, some problems require more sophisticated skeletons. The Eden library leverages this problem and already comes with more predefined skeletons<sup>1</sup>, among them a *pipe*, a *ring*, and a *torus* implementation (Loogen et al., 2003). These seem like reasonable candidates to be ported to our Arrow-based parallel Haskell. We aim to showcase that we can express more sophisticated skeletons with parallel Arrows as well.

If we were to use the original definition of *parEvalN*, however, these skeletons would produce an infinite loop with the GpH and *Par* Monad which during runtime would result in the program crash. This materialises with the usage of *loop* of the *ArrowLoop* type class and we think that this is due to difference of how evaluation is done in these backends when compared to Eden. An investigation of why this difference exists is beyond the scope of this work, we only provide a workaround for these types of skeletons as such they probably are not of much importance outside of a distributed memory environment. However our workaround enables users of the DSL to test their code within a shared memory setting. The idea of the fix is to provide a *ArrowLoopParallel* type class that has two functions – *loopParEvalN* and *postLoopParEvalN*, where the first is to be used inside an *loop* construct while the latter will be used right outside of the *loop*. This way we can delegate to the actual *parEvalN* in the spot where the backend supports it.

```
class ArrowParallel arr a b conf =>
  ArrowLoopParallel arr a b conf where
  loopParEvalN :: conf -> [arr a b] -> arr [a] [b]
  postLoopParEvalN :: conf -> [arr a b] -> arr [a] [b]
```

As Eden has no problems with the looping skeletons, we use this instance:

```
instance (ArrowChoice arr, ArrowParallel arr a b Conf) =>
  ArrowLoopParallel arr a b Conf where
  loopParEvalN = parEvalN
  postLoopParEvalN _ = evalN
```

As the *Par* Monad and GpH have problems with *parEvalN* inside of *loop* their respective instances for *ArrowLoopParallel* look like this:

```
instance (ArrowChoice arr, ArrowParallel arr a b (Conf b)) =>
  ArrowLoopParallel arr a b (Conf b) where
```

---

<sup>1</sup>Available on Hackage: <https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/Control-Parallel-Eden-Topology.html>.



$$\begin{aligned} \text{loopParEvalN } \_ &= \text{evalN} \\ \text{postLoopParEvalN} &= \text{parEvalN} \end{aligned}$$

### 5.2.1 Parallel pipe

The parallel *pipe* skeleton is semantically equivalent to folding over a list  $[arr\ a\ a]$  of Arrows with  $\ggg$ , but does this in parallel, meaning that the Arrows do not have to reside on the same thread/machine. We implement this skeleton using the *ArrowLoop* type class which provides us with the  $\text{loop} :: arr\ (a, b)\ (c, b) \rightarrow arr\ a\ c$  combinator allowing us to express recursive fix-point computations in which output values are fed back as input. For example

$$\text{loop}\ (arr\ (\lambda(a, b) \rightarrow (b, a : b)))$$

which is the same as

$$\text{loop}\ (arr\ \text{snd} \ \&\&\&\ arr\ (\text{uncurry}\ (:)))$$

defines an Arrow that takes its input  $a$  and converts it into an infinite stream  $[a]$  of it. Using *loop* to our advantage gives us a first draft of a pipe implementation (Fig. 5.6) by plugging in the parallel evaluation call  $\text{loopParEvalN}\ \text{conf}\ fs$  inside the second argument of  $\&\&\&$  and then only picking the first element of the resulting list with  $\text{arr}\ \text{last}$  outside of the *loop*.

However, using this definition directly will make the master node a potential bottleneck in distributed environments as described in Section 5.1. Therefore, we introduce a more sophisticated version that internally uses Futures and obtain the final definition of *pipe* in Fig. 5.7.

Sometimes, this *pipe* definition can be a bit inconvenient, especially if we want to pipe Arrows of mixed types together, i.e.  $arr\ a\ b$  and  $arr\ b\ c$ . By wrapping these two Arrows inside a bigger Arrow  $arr\ (([a], [b]), [c])\ (([a], [b]), [c])$  suitable for *pipe*, we can define *pipe2* as in Fig. 5.8.

Extensive use of *pipe2* over *pipe* with a hand-written combination data type will probably result in worse performance because of more communication overhead from the many calls to *parEvalN* inside of *evalN*. Nonetheless, we can define a parallel piping operator  $| \ggg |$ , which is semantically equivalent to  $\ggg$  similarly to other parallel syntactic sugar from Appendix 10.4.

```

pipeSimple :: (ArrowLoop arr, ArrowLoopParallel arr a a conf) =>
  conf -> [arr a a] -> arr a a
pipeSimple conf fs =
  loop (arr snd &&&
    (arr (uncurry (:)) >>> lazy) >>> loopParEvalN conf fs)) >>>
  arr last

```

**Figure 5.6:** Simple *pipe* skeleton. The use of *lazy* (Fig. 10.12) is essential as without it programs using this definition would never halt. We need to ensure that the evaluation of the input  $[a]$  is not forced fully before passing it into *loopParEvalN*.

```

pipe :: (ArrowLoop arr,
  ArrowLoopParallel arr (fut a) (fut a) conf,
  Future fut a conf) =>
  conf -> [arr a a] -> arr a a
pipe conf fs = unliftFut conf (pipeSimple conf (map (liftFut conf) fs))
liftFut :: (Arrow arr, Future fut a conf, Future fut b conf) =>
  conf -> arr a b -> arr (fut a) (fut b)
liftFut conf f = get conf >>> f >>> put conf
unliftFut :: (Arrow arr, Future fut a conf, Future fut b conf) =>
  conf -> arr (fut a) (fut b) -> arr a b
unliftFut conf f = put conf >>> f >>> get conf

```

**Figure 5.7:** *pipe* skeleton definition with Futures.

```

pipe2 :: (ArrowLoop arr, ArrowChoice arr,
  ArrowLoopParallel arr (fut (([a],[b]),[c])) (fut (([a],[b]),[c])) conf,
  Future fut (([a],[b]),[c]) conf) =>
  conf -> arr a b -> arr b c -> arr a c
pipe2 conf f g =
  (arr return &&& arr (const [])) &&& arr (const []) >>>
  pipe conf (replicate 2 (unify f g)) >>>
  arr snd >>>
  arr head
where
  unify :: (ArrowChoice arr) =>
    arr a b -> arr b c -> arr (([a],[b]),[c]) (([a],[b]),[c])
  unify f' g' =
    (mapArr f' *** mapArr g') *** arr (const []) >>>
    arr (\((b,c),a) -> ((a,b),c))
(|>>>|) :: (ArrowLoop arr, ArrowChoice arr,
  ArrowLoopParallel arr (fut (([a],[b]),[c])) (fut (([a],[b]),[c])) (),
  Future fut (([a],[b]),[c]) ()) =>
  arr a b -> arr b c -> arr a c
(|>>>|) = pipe2 ()

```

**Figure 5.8:** Definition of *pipe2* and  $(|>>>|)$ , a parallel  $>>>$ .

## 5.2.2 Ring skeleton

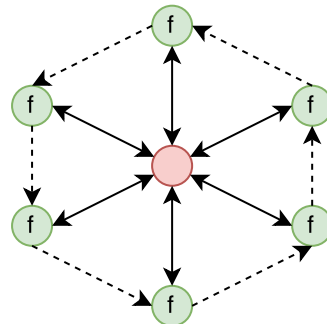
Eden comes with a ring skeleton<sup>2</sup> (Fig. 5.9) implementation that allows the computation of a function  $[i] \rightarrow [o]$  with a ring of nodes that communicate with each other. Its input is a node function  $i \rightarrow r \rightarrow (o, r)$  in which  $r$  serves as the intermediary output that gets sent to the neighbour of each node. This data is sent over direct communication channels, the so called „remote data“. We depict it in Appendix, Fig. 10.15.

We can rewrite this functionality easily with the use of *loop* as the definition of the node function,  $\text{arr } (i, r) \rightarrow (o, r)$ , after being transformed into an Arrow, already fits quite neatly into *loop*'s signature:  $\text{arr } (a, b) \rightarrow (c, b) \rightarrow \text{arr } a \rightarrow c$ . In each iteration we start by rotating the intermediary input from the nodes  $[fut \ r]$  with *second* (*rightRotate*  $\ggg$  *lazy*) (Fig. 10.12). Similarly to the *pipe* from Section 5.2.1 (Fig. 5.6), we have to feed the intermediary input into our *lazy* (Fig. 10.12) Arrow here, or the evaluation would fail to terminate. The reasoning is explained by Loogen (2012) as a demand problem.

Next, we zip the resulting  $([i], [fut \ r])$  to  $((i, fut \ r))$  with  $\text{arr } (\text{uncurry } \text{zip})$ . We then feed this into our parallel Arrow  $\text{arr } [(i, fut \ r)] \rightarrow [(o, fut \ r)]$  obtained by transforming our input Arrow  $f :: \text{arr } (i, r) \rightarrow (o, r)$  into  $\text{arr } (i, fut \ r) \rightarrow (o, fut \ r)$  before *repeating* and lifting it with *loopParEvalN*. Finally we unzip the output list  $[(o, fut \ r)]$  list into  $([o], [fut \ r])$ .

Plugging this Arrow  $\text{arr } ([i], [fut \ r]) \rightarrow [(o, fut \ r)]$  into the definition of *loop* from earlier gives us  $\text{arr } [i] \rightarrow [o]$ , our ring Arrow (Fig. 5.10). To make sure this algorithm has speedup on shared-memory machines as well, we pass the result of this Arrow to *postLoopParEvalN* *conf* (*repeat* ( $\text{arr } \text{id}$ )). This combinator can, for example, be used to calculate the shortest paths in a graph using Warshall's algorithm.

<sup>2</sup>Available on Hackage: <https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/Control-Parallel-Eden-Topology.html>.



**Figure 5.9:** *ring* skeleton depiction.

```

ring :: (Future fut r conf,
        ArrowLoop arr,
        ArrowLoopParallel arr (i, fut r) (o, fut r) conf,
        ArrowLoopParallel arr o o conf) =>
  conf -> arr (i, r) (o, r) -> arr [i] [o]
ring conf f =
  loop (second (rightRotate >>> lazy) >>>
        arr (uncurry zip) >>>
        loopParEvalN conf (repeat (second (get conf) >>> f >>> second (put conf))) >>>
        arr unzip) >>>
  postLoopParEvalN conf (repeat (arr id))

```

**Figure 5.10:** *ring* skeleton definition.

### 5.2.3 Torus skeleton

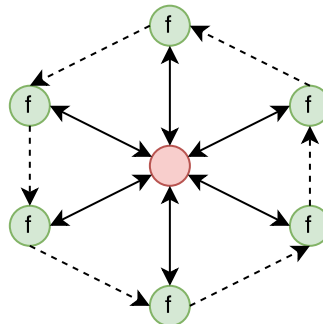
If we take the concept of a *ring* from Section 5.2.2 one dimension further, we obtain a *torus* skeleton (Fig. 5.11, 5.12). Every node sends and receives data from horizontal and vertical neighbours in each communication round. With our Parallel Arrows we re-implement the *torus* combinator<sup>3</sup> from Eden – yet again with the help of the *ArrowLoop* type class.

Similar to the *ring*, we start by rotating the input (Fig. 10.12), but this time not only in one direction, but in two. This means that the intermediary input from the neighbour nodes has to be stored in a tuple  $([[fut\ a]], [[fut\ b]])$  in the second argument (*loop* only allows for two arguments) of our looped Arrow of type

$$arr\ ([[c]], ([[fut\ a]], [[fut\ b]]))\ ([[d]], ([[fut\ a]], [[fut\ b]]))$$

and our rotation Arrow becomes

<sup>3</sup>Available on Hackage: <https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/Control-Parallel-Eden-Topology.html>.



**Figure 5.11:** *torus* skeleton depiction.

```
second ((mapArr rightRotate >>> lazy) *** (arr rightRotate >>> lazy))
```

instead of the singular rotation in the ring as we rotate `[[fut a]]` horizontally and `[[fut b]]` vertically. Then, we zip the inputs for the input Arrow with

```
arr (uncurry3 zipWith3 lazyzip3)
```

from `(([[c]], ([[fut a]], [[fut b]])))` to `[[c, fut a, fut b]]`, which we then evaluate in parallel.

This, however, is more complicated than in the *ring* case as we have one more dimension of inputs that needs to be transformed. We first have to *shuffle* all the inputs to then pass them into `loopParEvalN conf (repeat (ptorus conf f))` to get an output of `[(d, fut a, fut b)]`. We then unshuffle this list back to its original ordering by feeding it into `arr (uncurry unshuffle)` which takes the input length we saved one step earlier as additional input to get a result matrix `[[[(d, fut a, fut b)]]]`. Finally, we unpack this matrix with `arr (map unzip3) >>> arr unzip3 >>> threetotwo` to get `(([[d]], ([[fut a]], [[fut b]])))`.

This internal looping computation is once again fed into *loop* and we also compose a final `postLoopParEvalN conf (repeat (arr id))` for the same reasons as explained for the *ring* skeleton.

As an example of using this skeleton, Loogen et al. (2003) showed the matrix multiplication using the Gentleman algorithm (Gentleman, 1978). An adapted version can be found in Fig. 5.13.

If we compare the trace from a call using our Arrow definition of the *torus* (Fig. 5.14) with the Eden version (Fig. 5.15) we can see that the behaviour of the Arrow version and execution times are comparable. We discuss further benchmarks on larger clusters in more detail in the next section.

```

torus :: (Future fut a conf, Future fut b conf,
        ArrowLoop arr, ArrowChoice arr,
        ArrowLoopParallel arr (c, fut a, fut b) (d, fut a, fut b) conf,
        ArrowLoopParallel arr [d] [d] conf) =>
  conf -> arr (c, a, b) (d, a, b) -> arr [[c]] [[d]]
torus conf f =
  loop (second ((mapArr rightRotate >>> lazy) *** (arr rightRotate >>> lazy)) >>>
        arr (uncurry3 (zipWith3 lazyzip3)) >>>
        arr length &&& (shuffle >>> loopParEvalN conf (repeat (ptorus conf f))) >>>
        arr (uncurry unshuffle) >>>
        arr (map unzip3) >>> arr unzip3 >>> threetotwo) >>>
  postLoopParEvalN conf (repeat (arr id))
ptorus :: (Arrow arr, Future fut a conf, Future fut b conf) =>
  conf ->
  arr (c, a, b) (d, a, b) ->
  arr (c, fut a, fut b) (d, fut a, fut b)
ptorus conf f =
  arr (λ~(c, a, b) -> (c, get conf a, get conf b)) >>>
  f >>>
  arr (λ~(d, a, b) -> (d, put conf a, put conf b))

```

**Figure 5.12:** *torus* skeleton definition. *lazyzip3*, *uncurry3* and *threetotwo* definitions are in Fig. 10.13.

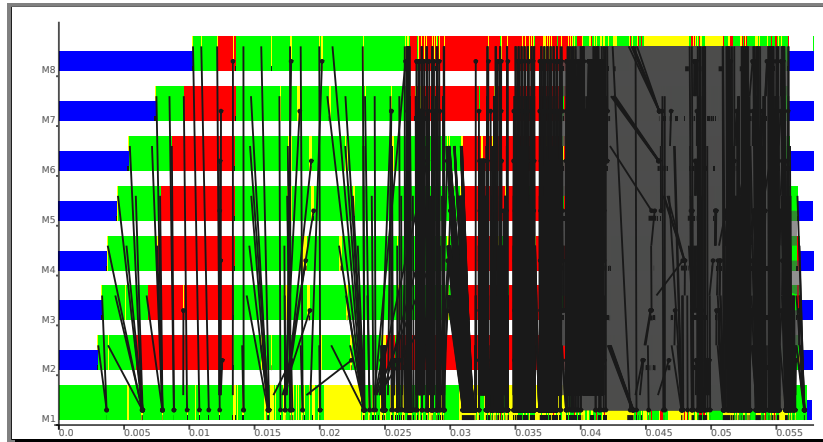
```

type Matrix = [[Int]]
prMM_torus :: Int -> Int -> Matrix -> Matrix -> Matrix
prMM_torus numCores problemSizeVal m1 m2 =
  combine $ torus () (mult torusSize) $ zipWith zip (split1 m1) (split2 m2)
  where torusSize = (floor ∘ sqrt) $ fromIntegral $ numCoreCalc numCores
        combine x = concat (map ((map (concat)) ∘ transpose) x)
        split1 x = staggerHorizontally (splitMatrix (problemSizeVal 'div' torusSize) x)
        split2 x = staggerVertically (splitMatrix (problemSizeVal 'div' torusSize) x)

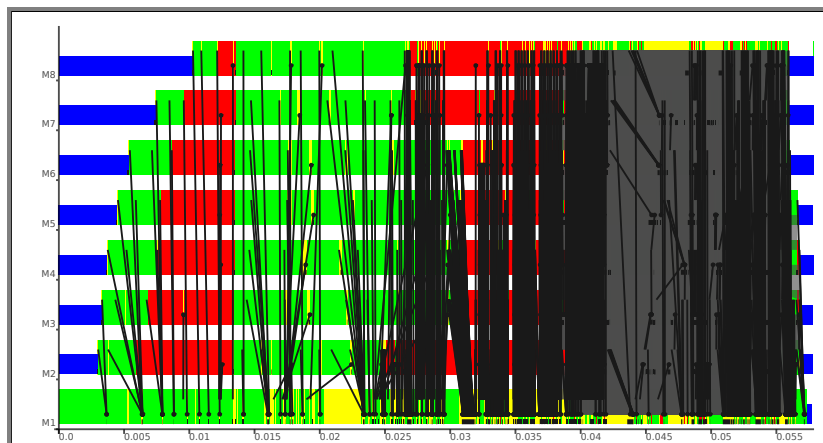
-- Function performed by each worker
mult :: Int -> ((Matrix, Matrix), [Matrix], [Matrix]) -> (Matrix, [Matrix], [Matrix])
mult size ((sm1, sm2), sm1s, sm2s) = (result, toRight, toBottom)
  where toRight = take (size - 1) (sm1 : sm1s)
        toBottom = take (size - 1) (sm2 : sm2s)
        sms = zipWith prMM (sm1 : sm1s) (sm2 : sm2s)
        result = foldl1' matAdd sms

```

**Figure 5.13:** Adapted matrix multiplication in Eden using a the *torus* skeleton. *prMM\_torus* is the parallel matrix multiplication. *mult* is the function performed by each worker. *prMM* is the sequential matrix multiplication in the chunks. *splitMatrix* splits the Matrix into chunks. *staggerHorizontally* and *staggerVertically* pre-rotate the matrices. *matAdd* calculates  $A + B$ . Omitted definitions can be found in 10.17.



**Figure 5.14:** Matrix multiplication with *torus* (PArrows).



**Figure 5.15:** Matrix multiplication with *torus* (Eden).



## Experiment: Cloud Haskell Backend



## Performance results and discussion

The preceding section has shown that PArrows are expressive. This section evaluates the performance overhead of this compositional abstraction in comparison to GpH and the *Par* Monad on shared memory architectures and Eden on a distributed memory cluster. We describe our measurement platform, the benchmark results – the shared-memory variants (GpH, *Par* Monad and Eden CP) followed by Eden in a distributed-memory setting, and conclude that PArrows hold up in terms of performance when compared to the original parallel Haskell.

### 7.1 Measurement platform

We start by explaining the hardware and software stack and outline the benchmark programs and motivation for choosing them. We also shortly address hyper-threading and why we do not use it in our benchmarks.

#### 7.1.1 Hardware and software

The benchmarks are executed both in a shared and in a distributed memory setting using the Glasgow GPG Beowulf cluster, consisting of 16 machines with 2 Intel® Xeon® E5-2640 v2 and 64 GB of DDR3 RAM each. Each processor has 8 cores and 16 (hyper-threaded) threads with a base frequency of 2 GHz and a turbo frequency of 2.50 GHz. This results in a total of 256 cores and 512 threads for the whole cluster. The operating system was Ubuntu 14.04 LTS with Kernel 3.19.0-33. Non-surprisingly, we found that hyper-threaded 32 cores do not behave in the same manner as real 16 cores (numbers here for a single machine). We disregarded the hyper-threading ability in most of the cases.

Apart from Eden, all benchmarks and libraries were compiled with Stack's<sup>1</sup> lts-7.1 GHC compiler which is equivalent to a standard GHC 8.0.1 with the base package in version 4.9.0.0. Stack itself was used in version 1.3.2. For GpH in its Multicore

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<sup>1</sup>see <https://www.haskellstack.org/>

variant we used the parallel package in version 3.2.1.0<sup>2</sup>, while for the *Par* Monad we used monad-par in version 0.3.4.8<sup>3</sup>. For all Eden tests, we used its GHC-Eden compiler in version 7.8.2<sup>4</sup> together with OpenMPI 1.6.5<sup>5</sup>.

Furthermore, all benchmarks were done with help of the bench<sup>6</sup> tool in version 1.0.2 which uses criterion ( $\geq 1.1.1.0$  &&  $< 1.2$ )<sup>7</sup> internally. All runtime data (mean runtime, max stddev, etc.) was collected with this tool.

We used a single node with 16 real cores as a shared memory test-bed and the whole grid with 256 real cores as a device to test our distributed memory software.

## 7.1.2 Benchmarks

We measure four benchmarks from different sources. Most of them are parallel mathematical computations, initially implemented in Eden. Table 7.1 summarises.

**Table 7.1:** The benchmarks we use in this paper.

| Name              | Area        | Type                     | Origin           | Source                            |
|-------------------|-------------|--------------------------|------------------|-----------------------------------|
| Rabin–Miller test | Mathematics | <i>parMap + reduce</i>   | Eden             | Lobachev (2012)                   |
| Jacobi sum test   | Mathematics | <i>workpool + reduce</i> | Eden             | Lobachev (2012)                   |
| Gentleman         | Mathematics | <i>torus</i>             | Eden             | Loogen et al. (2003)              |
| Sudoku            | Puzzle      | <i>parMap</i>            | <i>Par</i> Monad | Marlow et al. (2011) <sup>8</sup> |

Rabin–Miller test is a probabilistic primality test that iterates multiple (here: 32–256) „subtests“. Should a subtest fail, the input is definitely not a prime. If all  $n$  subtest pass, the input is composite with the probability of  $1/4^n$ .

Jacobi sum test or APRCL is also a primality test, that however, guarantees the correctness of the result. It is probabilistic in the sense that its run time is not certain. Unlike Rabin–Miller test, the subtests of Jacobi sum test have very different durations. Lobachev (2011) discusses some optimisations of parallel APRCL. Generic parallel implementations of Rabin–Miller test and APRCL were presented in Lobachev (2012).

<sup>2</sup>see <https://hackage.haskell.org/package/parallel-3.2.1.0>

<sup>3</sup>see <https://hackage.haskell.org/package/monad-par-0.3.4.8>

<sup>4</sup>see [http://www.mathematik.uni-marburg.de/~eden/?content=build\\_eden\\_7\\_&navi=build](http://www.mathematik.uni-marburg.de/~eden/?content=build_eden_7_&navi=build)

<sup>5</sup>see <https://www.open-mpi.org/software/ompi/v1.6/>

<sup>6</sup>see <https://hackage.haskell.org/package/bench>

<sup>7</sup>see <https://hackage.haskell.org/package/criterion-1.1.1.0>

<sup>8</sup>actual code from: <http://community.haskell.org/~simonmar/par-tutorial.pdf> and <https://github.com/simonmar/parconc-examples>

„Gentleman“ is a standard Eden test program, developed for their *torus* skeleton. It implements a Gentleman’s algorithm for parallel matrix multiplication (Gentleman, 1978). We ported an Eden-based version (Loogen et al., 2003) to PArrows.

A parallel Sudoku solver was used by Marlow et al. (2011) to compare *Par* Monad to GpH, we ported it to PArrows.

### 7.1.3 What parallel Haskells run where

The *Par* monad and GpH – in its multicore version (Marlow et al., 2009) – can be executed on shared memory machines only. Although GpH is available on distributed memory clusters, and newer distributed memory Haskells such as HdpH exist, current support of distributed memory in PArrows is limited to Eden. We used the MPI backend of Eden in a distributed memory setting. However, for shared memory Eden features a „CP“ backend that merely copies the memory blocks between disjoint heaps. In this mode, Eden still operates in the „nothing shared“ setting, but is adapted better to multicore machines. We call this version of Eden „Eden CP“.

## 7.2 Benchmark results

We compare the PArrow performance with direct implementations of the benchmarks in Eden, GpH and the *Par* Monad. We start with the definition of mean overhead to compare both PArrows-enabled and standard benchmark implementations. We continue comparing speedups and overheads for the shared memory implementations and then study OpenMPI variants of the Eden-enabled PArrows as a representative of a distributed memory backend. We plot all speedup curves and all overhead values in the supplementary materials.

### 7.2.1 Defining overhead

We compare the mean overhead, i.e. the mean of relative wall-clock run time differences between the PArrow and direct benchmark implementations executed multiple times with the same settings. The error margins of the time measurements, supplied by criterion package<sup>9</sup>, yield the error margin of the mean overhead.

Quite often the zero value lies in the error margin of the mean overhead. This means that even though we have measured some difference (against or even in favour of PArrows), it could be merely the error margin of the measurement and the difference

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<sup>9</sup><https://hackage.haskell.org/package/criterion-1.1.1.0>

might not be existent. We are mostly interested in the cases where above issue does not persist, we call them *significant*. We often denote the error margin with  $\pm$  after the mean overhead value.

## 7.2.2 Shared memory

### Speedup

The Rabin–Miller benchmark showed almost linear speedup for both 32 and 64 tasks, the performance is slightly better in the latter case: 13.7 at 16 cores for input  $2^{11213} - 1$  and 64 tasks in the best case scenario with Eden CP. The performance of the Sudoku benchmark merely reaches a speedup of 9.19 (GpH), 8.78 (*Par* Monad), 8.14 (Eden CP) for 16 cores and 1000 Sudokus. In contrast to Rabin–Miller, here the *GpH* seems to be the best of all, while Rabin–Miller profited most from Eden CP (i.e. Eden with direct memory copy) implementation of PArrows. Gentleman on shared memory has a plummeting speedup curve with GpH and *Par* Monad and logarithmically increasing speedup for the Eden-based version. The latter reached a speedup of 6.56 at 16 cores.

### Overhead

For the shared memory Rabin–Miller benchmark, implemented with PArrows using Eden CP, GpH, and *Par* Monad, the overhead values are within single percents range, but also negative overhead (i.e. PArrows are better) and larger error margins happen. To give a few examples, the overhead for Eden CP with input value  $2^{11213} - 1$ , 32 tasks, and 16 cores is 1.5%, but the error margin is around 5.2%! Same implementation in the same setting with 64 tasks reaches  $-0.2\%$  overhead, PArrows apparently fare better than Eden – but the error margin of 1.9% disallows this interpretation. We focus now on significant overhead values. To name a few:  $0.41\% \pm 7 \cdot 10^{-2}\%$  for Eden~CP and 64 tasks at 4 cores;  $4.7\% \pm 0.72\%$  for GpH, 32 tasks, 8 cores;  $0.34\% \pm 0.31\%$  for *Par* Monad at 4 cores with 64 tasks. The worst significant overhead was in case of GpH with  $8\% \pm 6.9\%$  at 16 cores with 32 tasks and input value  $2^{11213} - 1$ . In other words, we notice no major slow-down through PArrows here.

For Sudoku the situation is slightly different. There is a minimal significant ( $-1.4\% \pm 1.2\%$  at 8 cores) speed *improvement* with PArrows Eden CP version when compared with the base Eden CP benchmark. However, with increasing number of cores the error margin reaches zero again:  $-1.6\% \pm 5.0\%$  at 16 cores. The *Par* Monad shows

a similar development, e.g. with  $-1.95\% \pm 0.64\%$  at 8 cores. The GpH version shows both a significant speed improvement of  $-4.2\% \pm 0.26\%$  (for 16 cores) with PArrows and a minor overhead of  $0.87\% \pm 0.70\%$  (4 cores).

The Gentleman multiplication with Eden CP shows a minor significant overhead of  $2.6\% \pm 1.0\%$  at 8 cores and an insignificant improvement at 16 cores. Summarising, we observe a low (if significant at all) overhead, induced by PArrows in the shared memory setting.

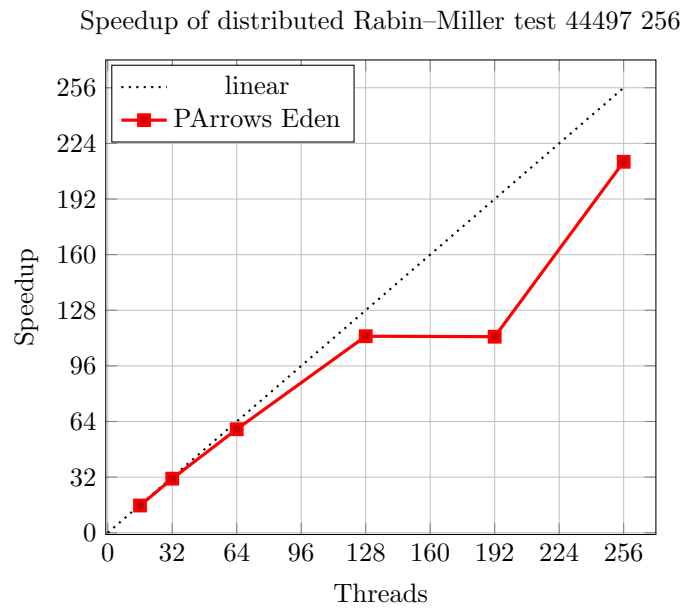
## 7.2.3 Distributed memory

### Speedup

The speedup of distributed memory Rabin–Miller benchmark with PArrows and Eden showed an almost linear speedup excepting around 192 cores where an unfortunate task distribution reduces performance. As seen in Fig. 7.1, we reached a speedup of 213.4 with PArrows at 256 cores (vs. 207.7 with pure Eden). Because of memory limitations, the speedup of Jacobi sum test for large inputs (such as  $2^{4253} - 1$ ) could be measured only in a massively distributed setting. PArrows improved there from 9193s (at 128 cores) to 1649s (at 256 cores). A scaled-down version with input  $2^{3217} - 1$  stagnates the speedup at about 11 for both PArrows and Eden for more than 64 cores. There is apparently not enough work for that many cores. The Gentleman test with input 4096 had an almost linear speedup first, then plummeted between 128 and 224 cores, and recovered at 256 cores with speedup of 129.

### Overhead

We use our mean overhead quality measure and the notion of significance also for distributed memory benchmarks. The mean overhead of Rabin–Miller test in the distributed memory setting ranges from 0.29% to  $-2.8\%$  (last value in favour of PArrows), but these values are not significant with error margins  $\pm 0.8\%$  and  $\pm 2.9\%$  correspondingly. A sole significant (by a very low margin) overhead is  $0.35\% \pm 0.33\%$  at 64 cores. We measured the mean overhead for Jacobi benchmark for an input of  $2^{3217} - 1$  for up to 256 cores. We reach the flattering value  $-3.8\% \pm 0.93\%$  at 16 cores in favour of PArrows, it was the sole significant overhead value. The value for 256 cores was  $0.31\% \pm 0.39\%$ . Mean overhead for distributed Gentleman multiplication was also low. Significant values include  $1.23\% \pm 1.20\%$  at 64 cores and  $2.4\% \pm 0.97\%$  at 256 cores. It took PArrows 64.2 seconds at 256 cores to complete the benchmark.



**Figure 7.1:** Speedup of the distributed Rabin–Miller benchmark using PArrows with Eden.

Similar to the shared memory setting, PArrows only imply a very low penalty with distributed memory that lies in lower single-percent digits at most.



## 7.3 Discussion

**Table 7.2:** Overhead in the shared memory benchmarks. Bold marks values in favour of PArrows.

| Benchmark             | Base      | Mean of mean overheads | Maximum normalised stdDev | Runtime for 16 cores (s) |
|-----------------------|-----------|------------------------|---------------------------|--------------------------|
| Sudoku 1000           | Eden CP   | <b>-2.1%</b>           | 5.1%                      | 1.17                     |
|                       | GpH       | <b>-0.82%</b>          | 0.7%                      | 1.11                     |
|                       | Par Monad | <b>-1.3%</b>           | 2.1%                      | 1.14                     |
| Gentleman 512         | Eden CP   | <i>0.81%</i>           | 6.8%                      | 1.66                     |
| Rabin–Miller 11213 32 | Eden CP   | <i>0.79%</i>           | 5.2%                      | 5.16                     |
|                       | GpH       | <i>3.5%</i>            | 6.9%                      | 5.28                     |
|                       | Par Monad | <b>-2.5%</b>           | 19.0%                     | 5.84                     |
| Rabin–Miller 11213 64 | Eden CP   | <i>0.21%</i>           | 1.9%                      | 10.3                     |
|                       | GpH       | <i>1.6%</i>            | 1.3%                      | 10.6                     |
|                       | Par Monad | <b>-4.0%</b>           | 17.0%                     | 11.4                     |

**Table 7.3:** Overhead in the distributed memory benchmarks. Bold marks values in favour of PArrows.

| Benchmark | Base | Mean of mean overheads | Maximum normalised stdDev | Runtime for 256 cores (s) |
|-----------|------|------------------------|---------------------------|---------------------------|
| 4096      | Eden | <i>0.67%</i>           | 1.5%                      | 110.0                     |
| 44497 256 | Eden | <b>-0.5%</b>           | 2.9%                      | 165.0                     |
| 3217      | Eden | <b>-0.74%</b>          | 1.6%                      | 635.0                     |

PArrows performed in our benchmarks with little to no overhead. Tables 7.2 and 7.3 clarify this once more: The PArrows-enabled versions trade blows with their vanilla counterparts when comparing the means over all cores of the mean overheads. If we combine these findings with the usability of our DSL, the minor overhead induced by PArrows is outweighed by their convenience and usefulness to the user.

PArrows are an extendable formalism, they can be easily ported to further parallel Haskells while still maintaining interchangeability. It is straightforward to provide further implementations of algorithmic skeletons in PArrows.

## Discussion

In this thesis we have seen that...



# Conclusion

Arrows are a generic concept that allows for powerful composition combinators. To our knowledge we are first to represent *parallel* computation with Arrows, and hence to show their usefulness for composing parallel programs. We have shown that for a generic and extensible parallel Haskell, we do not have to restrict ourselves to a monadic interface. We argue that Arrows are better suited to parallelise pure functions than Monads, as the functions are already Arrows and can be used directly in our DSL. Arrows are a better fit to parallelise pure code than a monadic solution as regular functions are already Arrows and can be used with our DSL in a more natural way. We use a non-monadic interface (similar to Eden or GpH) and retain composability. The DSL allows for a direct parallelisation of monadic code via the Kleisli type and additionally allows to parallelise any Arrow type that has an instance for *ArrowChoice*. (Some skeletons require an additional *ArrowLoop* instance.)

We have demonstrated the generality of the approach by exhibiting PArrow implementations for Eden, GpH, and the *Par* Monad. Hence, parallel programs can be ported between task parallel Haskell implementations with little or no effort. We are confident that it will be straightforward to add other task-parallel Haskell. Our measurements of four benchmarks on both shared and distributed memory platforms shows that the generality and portability of PArrows has very low performance overheads, i.e. never more than  $8\% \pm 6.9\%$  and typically under 2%.

## 9.1 Future work

Our PArrows DSL can be expanded to other task parallel Haskell, and a specific target is HdpH (Maier et al., 2014). Further Future-aware versions of Arrow combinators can be defined. Existing combinators could also be improved, for example more specialised versions of `>>>` and `***` combinators are viable.

In ongoing work we are expanding both our skeleton library and the number of skeleton-based parallel programs that use our DSL. It would also be interesting to see a hybrid of PArrows and Accelerate (McDonnell et al., 2015). Ports of our approach to other languages such as Frege, Eta, or Java directly are at an early development stage.



## Appendix

### 10.1 Utility Arrows

Following are definitions of some utility Arrows used in this paper that have been left out for brevity. We start with the *second* combinator from Hughes (2000), which is a mirrored version of *first*, which is for example used in the definition of *\*\*\**:

```
second :: Arrow arr => arr a b -> arr (c, a) (c, b)
second f = arr swap >>> first f >>> arr swap
  where swap (x, y) = (y, x)
```

Next, we give the definition of *evalN* which also helps us to define *map*, and *zipWith* on Arrows. The *evalN* combinator in Fig. 10.1 converts a list of Arrows  $[arr\ a\ b]$  into an Arrow  $arr\ [a]\ [b]$ .

```
evalN :: (ArrowChoice arr) => [arr a b] -> arr [a] [b]
evalN (f : fs) = arr listcase >>>
  arr (const []) ||| (f *** evalN fs >>> arr (uncurry (:)))
  where listcase [] = Left ()
        listcase (x : xs) = Right (x, xs)
evalN [] = arr (const [])
```

**Figure 10.1:** The definition of *evalN*.

The *mapArr* combinator (Fig. 10.2) lifts any Arrow  $arr\ a\ b$  to an Arrow  $arr\ [a]\ [b]$ . The original inspiration was from Hughes (2005), but the definition was then unified with *evalN*.

```
mapArr :: ArrowChoice arr => arr a b -> arr [a] [b]
mapArr = evalN o repeat
```

**Figure 10.2:** The definition of *map* over Arrows.

Finally, with the help of *mapArr* (Fig. 10.2), we can define *zipWithArr* (Fig. 10.3) that lifts any Arrow  $arr\ (a, b)\ c$  to an Arrow  $arr\ ([a], [b])\ [c]$ .

```

zipWithArr :: ArrowChoice arr ⇒ arr (a, b) c → arr ([a], [b]) [c]
zipWithArr f = (arr (λ(as, bs) → zipWith (,) as bs)) >>> mapArr f

```

**Figure 10.3:** *zipWith* over Arrows.

These combinators make use of the *ArrowChoice* type class which provides the `|||` combinator. It takes two Arrows *arr a c* and *arr b c* and combines them into a new Arrow *arr (Either a b) c* which pipes all *Left a*'s to the first Arrow and all *Right b*'s to the second Arrow:

```

(|||) :: ArrowChoice arr a c → arr b c → arr (Either a b) c

```

## 10.2 Profunctor Arrows

In Fig. 10.4 we show how specific Profunctors can be turned into Arrows. This works because Arrows are strong Monads in the bicategory *Prof* of Profunctors as shown by Asada (2010). In Standard GHC (`>>>`) has the type `(>>>) :: Category cat ⇒ cat a b → cat b c → cat a c` and is therefore not part of the *Arrow* type class like presented in this paper.<sup>1</sup>

```

instance (Category p, Strong p) ⇒ Arrow p where
    arr f = dimap id f id
    first = first'

instance (Category p, Strong p, Costrong p) ⇒ ArrowLoop p where
    loop = loop'

instance (Category p, Strong p, Choice p) ⇒ ArrowChoice p where
    left = left'

```

**Figure 10.4:** Profunctors as Arrows.

## 10.3 Additional function definitions

We have omitted some function definitions in the main text for brevity, and redeem this here. We begin with warping Eden's build-in *RemoteData* to *Future* in Figure 10.5.

<sup>1</sup>For additional information on the type classes used, see: <https://hackage.haskell.org/package/profunctors-5.2.1/docs/Data-Profunctor.html> and <https://hackage.haskell.org/package/base-4.9.1.0/docs/Control-Category.html>.



```

data RemoteData a = RD { rd :: RD a }
put' :: (Arrow arr) => arr a (BasicFuture a)
put' = arr BF
get' :: (Arrow arr) => arr (BasicFuture a) a
get' = arr ( $\lambda(\sim(BF\ a)) \rightarrow a$ )
instance NFData (RemoteData a) where
  rnf = rnf o rd
instance Trans (RemoteData a)
instance (Trans a) => Future RemoteData a Conf where
  put _ = put'
  get _ = get'
instance (Trans a) => Future RemoteData a () where
  put _ = put'
  get _ = get'

```

**Figure 10.5:** *RD-based RemoteData version of Future for the Eden backend.*

Next, we have the definition of *BasicFuture* in Fig. 10.6 and the corresponding *Future* instances.

```

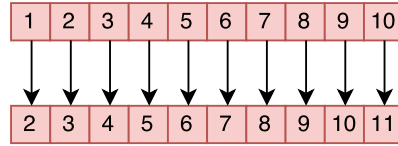
data BasicFuture a = BF a
put' :: (Arrow arr) => arr a (BasicFuture a)
put' = arr BF
get' :: (Arrow arr) => arr (BasicFuture a) a
get' = arr ( $\lambda(\sim(BF\ a)) \rightarrow a$ )
instance NFData a => NFData (BasicFuture a) where
  rnf (BF a) = rnf a
instance Future BasicFuture a (Conf a) where
  put _ = put'
  get _ = get'
instance Future BasicFuture a () where
  put _ = put'
  get _ = get'

```

**Figure 10.6:** *BasicFuture* type and its *Future* instance for the *Par* Monad and GpH.

Figures 10.7–10.10 show the definitions and a visualisations of two parallel *map* variants, defined using *parEvalN* and its lazy counterpart.

Arrow versions of Eden’s *shuffle*, *unshuffle* and the definition of *takeEach* are in Fig. 10.11. Similarly, Fig. 10.12 contains the definition of Arrow versions of Eden’s *lazy* and *rightRotate* utility functions. Fig. 10.13 contains Eden’s definition of *lazyzip3* together with the utility functions *uncurry3* and *threetotwo*. The full definition of



**Figure 10.7:** *parMap* depiction.

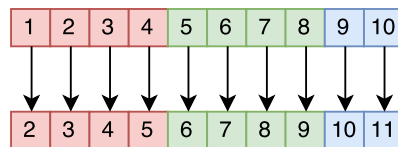
$$\begin{aligned} \text{parMap} &:: (\text{ArrowParallel } \text{arr } a \ b \ \text{conf}) \Rightarrow \text{conf} \rightarrow (\text{arr } a \ b) \rightarrow (\text{arr } [a] \ [b]) \\ \text{parMap } \text{conf } f &= \text{parEvalN } \text{conf } (\text{repeat } f) \end{aligned}$$

**Figure 10.8:** *parMap* definition.

*farmChunk* is in Fig. 10.14. Eden definition of *ring* skeleton is in Fig. 10.15. It follows Loogen (2012).

The *parEval2* skeleton is defined in Fig. 10.16. We start by transforming the  $(a, c)$  input into a two-element list  $[Either \ a \ c]$  by first tagging the two inputs with *Left* and *Right* and wrapping the right element in a singleton list with *return* so that we can combine them with *arr* (*uncurry*  $(:)$ ). Next, we feed this list into a parallel Arrow running on two instances of  $f \text{ +++ } g$  as described in the paper. After the calculation is finished, we convert the resulting  $[Either \ b \ d]$  into  $([b], [d])$  with *arr partitionEithers*. The two lists in this tuple contain only one element each by construction, so we can finally just convert the tuple to  $(b, d)$  in the last step.

Furthermore, Fig. 10.17 contains the omitted definitions of *prMM* (sequential matrix multiplication), *splitMatrix* (which splits the a matrix into chunks), *staggerHorizontally* and *staggerVertically* (to pre-rotate the matrices), and lastly *matAdd*, that calculates  $A + B$  for two matrices  $A$  and  $B$ .



**Figure 10.9:** *parMapStream* depiction.

```

parMapStream :: (ArrowParallel arr a b conf, ArrowChoice arr, ArrowApply arr) =>
  conf -> ChunkSize -> arr a b -> arr [a] [b]
parMapStream conf chunkSize f = parEvalNLazy conf chunkSize (repeat f)

```

**Figure 10.10:** *parMapStream* definition.

```

shuffle :: (Arrow arr) => arr [[a]] [a]
shuffle = arr (concat o transpose)
unshuffle :: (Arrow arr) => Int -> arr [a] [[a]]
unshuffle n = arr (\xs -> [takeEach n (drop i xs) | i <- [0..n-1]])
takeEach :: Int -> [a] -> [a]
takeEach n [] = []
takeEach n (x : xs) = x : takeEach n (drop (n-1) xs)

```

**Figure 10.11:** *shuffle*, *unshuffle*, *takeEach* definition.

```

lazy :: (Arrow arr) => arr [a] [a]
lazy = arr (\x -> (x : xs) -> x : lazy xs)
rightRotate :: (Arrow arr) => arr [a] [a]
rightRotate = arr $ \list -> case list of
  [] -> []
  xs -> last xs : init xs

```

**Figure 10.12:** *lazy* and *rightRotate* definitions.

```

lazyzip3 :: [a] -> [b] -> [c] -> [(a, b, c)]
lazyzip3 as bs cs = zip3 as (lazy bs) (lazy cs)
uncurry3 :: (a -> b -> c -> d) -> (a, (b, c)) -> d
uncurry3 f (a, (b, c)) = f a b c
threetotwo :: (Arrow arr) => arr (a, b, c) (a, (b, c))
threetotwo = arr $ \x -> (a, b, c) -> (a, (b, c))

```

**Figure 10.13:** *lazyzip3*, *uncurry3* and *threetotwo* definitions.

```

farmChunk :: (ArrowParallel arr a b conf, ArrowParallel arr [a] [b] conf,
  ArrowChoice arr, ArrowApply arr) =>
  conf -> ChunkSize -> NumCores -> arr a b -> arr [a] [b]
farmChunk conf chunkSize numCores f =
  unshuffle numCores >>>
  parEvalNLazy conf chunkSize (repeat (mapArr f)) >>>
  shuffle

```

**Figure 10.14:** *farmChunk* definition.

```

ringSimple :: (Trans i, Trans o, Trans r) => (i -> r -> (o, r)) -> [i] -> [o]
ringSimple f is = os
  where (os, ringOuts) = unzip (parMap (toRD $ uncurry f) (zip is $ lazy ringIns))
        ringIns = rightRotate ringOuts
toRD :: (Trans i, Trans o, Trans r) => ((i, r) -> (o, r)) -> ((i, RD r) -> (o, RD r))
toRD f (i, ringIn) = (o, release ringOut)
  where (o, ringOut) = f (i, fetch ringIn)
rightRotate :: [a] -> [a]
rightRotate [] = []
rightRotate xs = last xs : init xs
lazy :: [a] -> [a]
lazy~(x : xs) = x : lazy xs

```

**Figure 10.15:** Eden’s definition of the *ring* skeleton.

```

parEval2 :: (ArrowChoice arr,
  ArrowParallel arr (Either a c) (Either b d) conf) =>
  conf -> arr a b -> arr c d -> arr (a, c) (b, d)
parEval2 conf f g =
  arr Left *** (arr Right >>> arr return) >>>
  arr (uncurry (:)) >>>
  parEvalN conf (replicate 2 (f +++ g)) >>>
  arr partitionEithers >>>
  arr head *** arr head

```

**Figure 10.16:** *parEval2* definition.

```

prMM :: Matrix -> Matrix -> Matrix
prMM m1 m2 = prMMTr m1 (transpose m2)
  where
    prMMTr m1' m2' = [[sum (zipWith (*) row col) | col <- m2'] | row <- m1']
splitMatrix :: Int -> Matrix -> [[Matrix]]
splitMatrix size matrix = map (transpose o map (chunksOf size)) $ chunksOf size $ matrix
staggerHorizontally :: [[a]] -> [[a]]
staggerHorizontally matrix = zipWith leftRotate [0..] matrix
staggerVertically :: [[a]] -> [[a]]
staggerVertically matrix = transpose $ zipWith leftRotate [0..] (transpose matrix)
leftRotate :: Int -> [a] -> [a]
leftRotate i xs = xs2 ++ xs1 where
  (xs1, xs2) = splitAt i xs
matAdd = chunksOf (dimX x) $ zipWith (+) (concat x) (concat y)

```

**Figure 10.17:** *prMMTr*, *splitMatrix*, *staggerHorizontally*, *staggerVertically* and *matAdd* definition.

## 10.4 Syntactic sugar

Next, we also give the definitions for some syntactic sugar for PArrows, namely `|***|` and `|&&&|`. For basic Arrows, we have the `***` combinator (Fig. 3.7) which allows us to combine two Arrows `arr a b` and `arr c d` into an Arrow `arr (a, c) (b, d)` which does both computations at once. This can easily be translated into a parallel version `|***|` with the use of `parEval2`, but for this we require a backend which has an implementation that does not require any configuration (hence the `()` as the `conf` parameter:

$$\begin{aligned} (|***|) &:: (\text{ArrowChoice } arr, \text{ArrowParallel } arr \text{ (Either } a \text{ c) (Either } b \text{ d) } ())) \Rightarrow \\ &\quad arr \ a \ b \rightarrow arr \ c \ d \rightarrow arr \ (a, c) \ (b, d) \\ (|***|) &= \text{parEval2 } () \end{aligned}$$

We define the parallel `|&&&|` in a similar manner to its sequential pendant `&&&` (Fig. 3.7):

$$\begin{aligned} (|&&&|) &:: (\text{ArrowChoice } arr, \text{ArrowParallel } arr \text{ (Either } a \ a) \text{ (Either } b \ c) \ ())) \Rightarrow \\ &\quad arr \ a \ b \rightarrow arr \ a \ c \rightarrow arr \ a \ (b, c) \\ (|&&&|) \ f \ g &= (arr \ \$ \ \lambda a \rightarrow (a, a)) \ggg f \ |***| \ g \end{aligned}$$

## 10.5 Experimental Cloud Haskell Backend code

Finally, we include the Template Haskell based code generator to make the experimental Cloud Haskell backend easier to use and a version of the main Sudoku benchmark program as an example.

The code generator can be found in Fig. 10.18. Here, if we enclose this in a Haskell module, the functions `mkEvalTasks` (to generate the `evalTasks` for the specific types), `mkRemotables` (to mark the evaluation tasks as remotable in Cloud Haskell) and `mkEvaluatables` (to create the `Evaluatable` instance) are the ones exposed to the user.

The Template Haskell version of the main Sudoku benchmark program can be found in Fig. 10.19<sup>2</sup>. We have to write type aliases for `Maybe Grid` (`MaybeGrid`) and `[Maybe Grid]` (`MaybeGridList`). We can then use these to generate the code required to evaluate these types in the Cloud Haskell backend with. In the `main` program we have two cases: a) the program is started in master mode and starts

<sup>2</sup>For the full code, see the GitHub repository at <https://github.com/s4ke/Parrows/blob/e1ab76018448d9d4ca3ed48ef1f0c5be26ae34ab/CloudHaskell/testing/Test.hs>

the computation, b) the program is started in slave mode and waits for computation requests. In order to launch this program and have speedup as well, we have to start slave nodes for each cpu core with commands like „<executable> slave 127.0.0.1 8000“ where the last parameter determines the port the slave will listen to and wait for requests on. Similarly a single master node can be started with „<executable> master 127.0.0.1 7999“ where, once again, the last parameter determines the communication port.

```

nested :: Type → Type → Type
nested a b = a 'AppT' (ParensT b)

tuple2 :: Type → Type → Type
tuple2 a b = (TupleT 2 'AppT' a) 'AppT' b

fn :: Type → Type → Type
fn a b = (ArrowT 'AppT' a) 'AppT' b

nameToFnName :: Name → Name
nameToFnName (Name (OccName str) _) = mkName $ ("_" ++ str ++ "_evalTaskImpl")

evalTaskFn :: Name → Name → Q [Dec]
evalTaskFn typeName fnName = do
  let sendPort = ConT $ mkName "SendPort"
      thunk = ConT $ mkName "Thunk"
      process = ConT $ mkName "Process"
      firstTup = (sendPort 'nested' (sendPort 'nested' (thunk 'nested' (ConT typeName))))
      secondTup = sendPort 'nested' (ConT typeName)
      procNil = process 'AppT' (TupleT 0)
  return [
    SigD fnName ((firstTup 'tuple2' secondTup) 'fn' procNil),
    FunD fnName [ Clause [] (NormalB (VarE $ mkName "evalTaskBase")) [] ]
  ]

evaluatableInstance :: Name → Name → Q [Dec]
evaluatableInstance typeName fnName = do
  let evaluatable = ConT $ mkName "Evaluatable"
      closure ← mkClosure fnName
  return [
    InstanceD (Nothing) [] (evaluatable 'nested' ConT typeName) [
      FunD (mkName "evalTask") [ Clause [] (NormalB closure) [] ]
    ]
  ]

mkEvalTasks :: [Name] → Q [Dec]
mkEvalTasks names = do
  let fnNames = map nameToFnName names
      (mapM (uncurry evalTaskFn) (zipWith (,) names fnNames)) >>= (return ∘ concat)

mkRemotables :: [Name] → Q [Dec]
mkRemotables names = do
  let fnNames = map nameToFnName names
      remotable fnNames

mkEvaluatables :: [Name] → Q [Dec]
mkEvaluatables names = do
  let fnNames = map nameToFnName names
      (mapM (uncurry evaluatableInstance) (zipWith (,) names fnNames)) >>= (return ∘ concat)

```

**Figure 10.18:** The Template Haskell code generator for the Cloud Haskell backend.

```

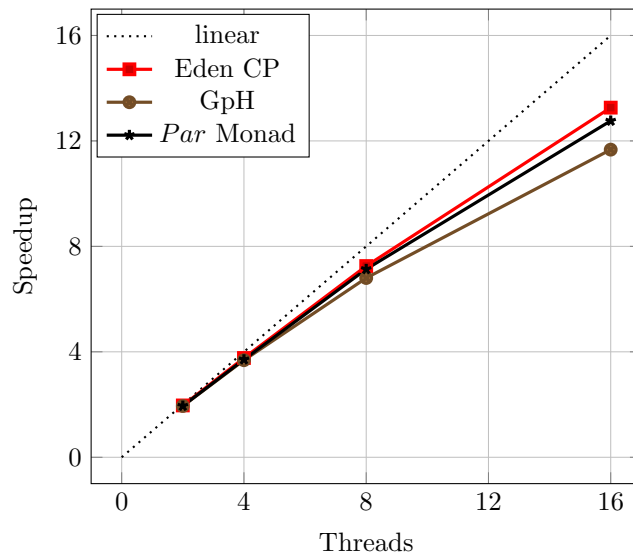
type MaybeGrid = Maybe Grid
type MaybeGridList = [Maybe Grid]
    -- remotable declaration for all eval tasks
$(mkEvalTasks [" MaybeGrid, " MaybeGridList])
$(mkRemotables [" MaybeGrid, " MaybeGridList])
$(mkEvaluatables [" MaybeGrid, " MaybeGridList])
myRemoteTable :: RemoteTable
myRemoteTable = Main.__remoteTable initRemoteTable
main :: IO ()
main = do
    args ← getArgs
    case args of
        ["master", host, port] → do
            conf ← startBackend myRemoteTable Master host port
            readMVar (workers conf) >>= print
            grids ← fmap lines $ readFile "sudoku.txt"
            print (length (filter isJust (farm conf 4 solve grids)))
        ["slave", host, port] → do
            startBackend myRemoteTable Slave host port
            print "slave shutdown."

```

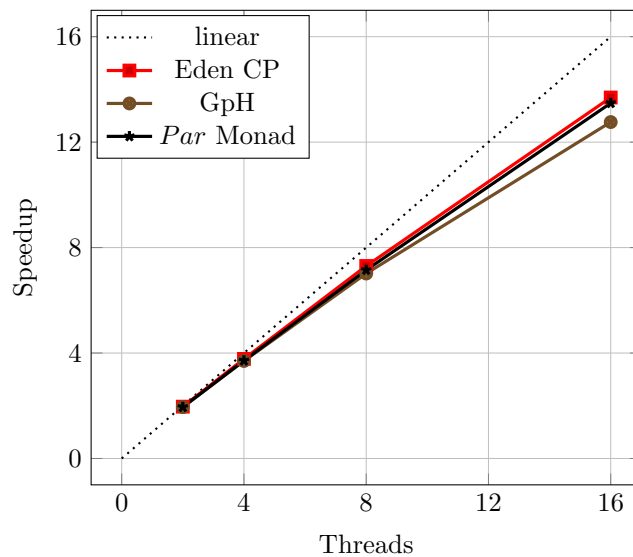
**Figure 10.19:** The Template Haskell version of the Sudoku benchmark program.



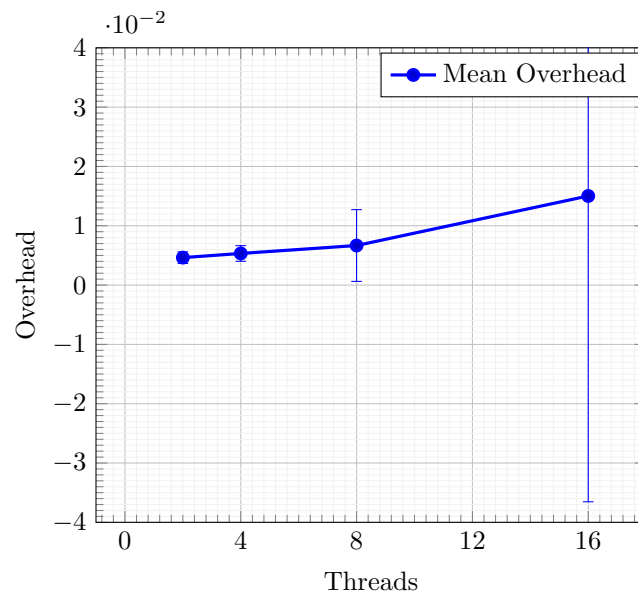
## 10.6 Shared Memory Benchmarks



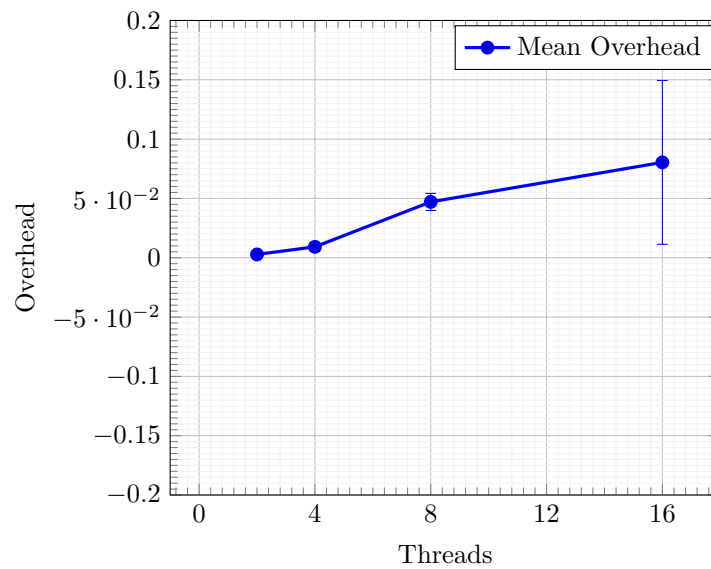
**Figure 10.20:** Parallel speedup of shared-memory Rabin-Miller test „11213 32“



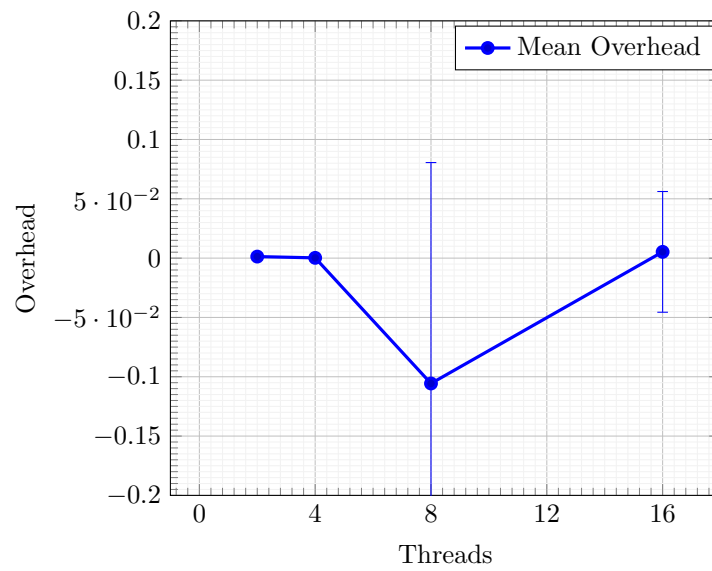
**Figure 10.21:** Parallel speedup of shared-memory Rabin-Miller test „11213 64“



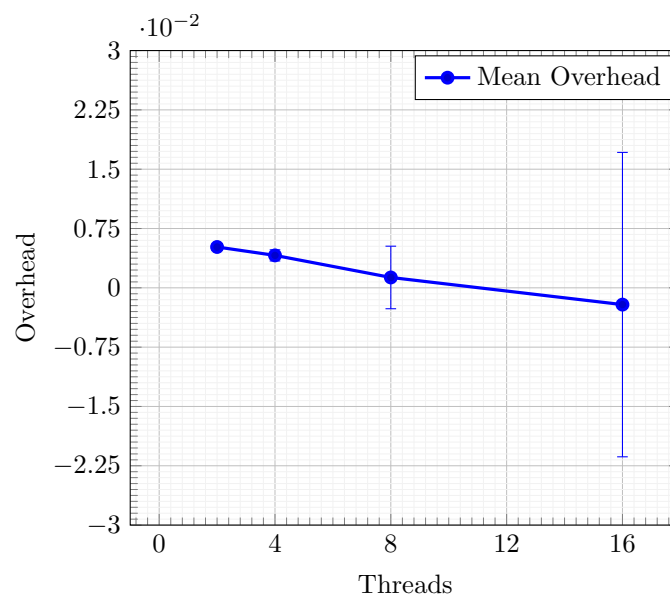
**Figure 10.22:** Mean overhead for shared-memory Rabin—Miller test „11213 32“ vs Eden CP



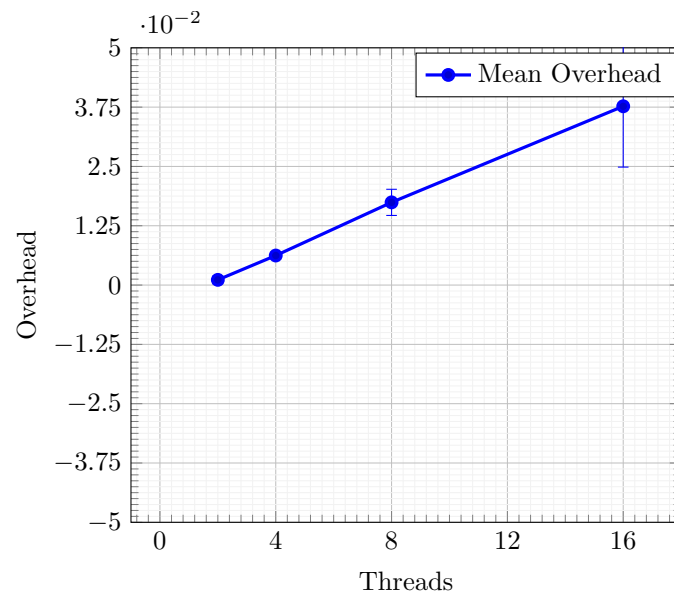
**Figure 10.23:** Mean overhead for shared-memory Rabin—Miller test „11213 32“ vs GpH



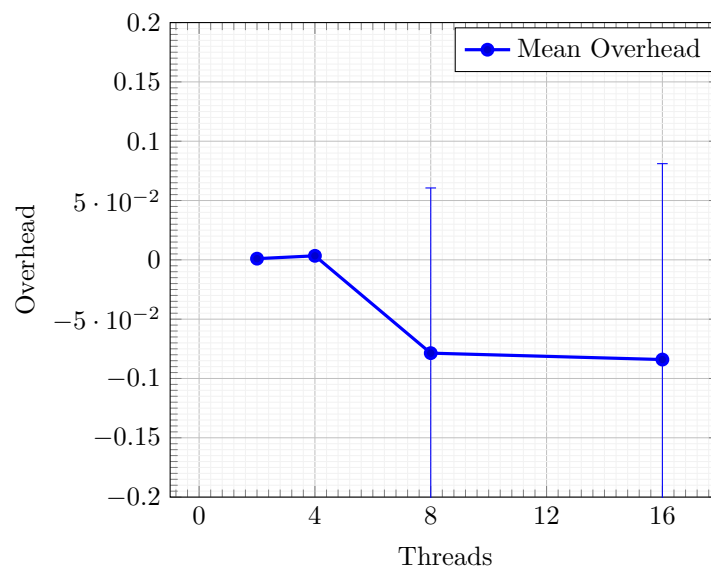
**Figure 10.24:** Mean overhead for shared-memory Rabin—Miller test „11213 32“ vs *Par monad*



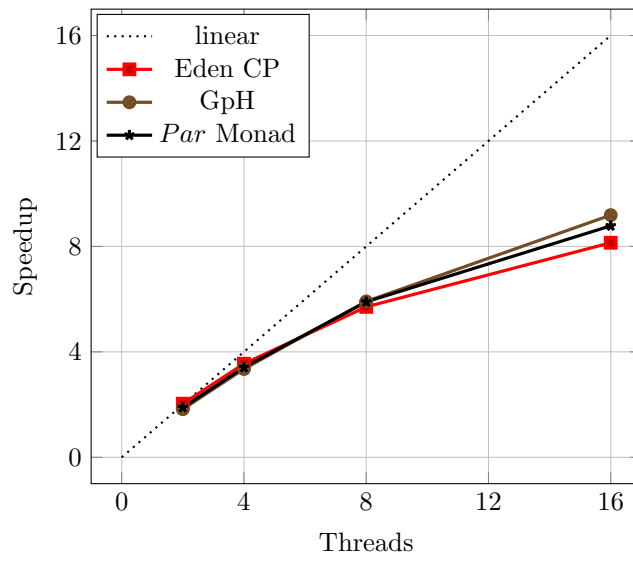
**Figure 10.25:** Mean overhead for shared-memory Rabin—Miller test „11213 64“ vs Eden CP



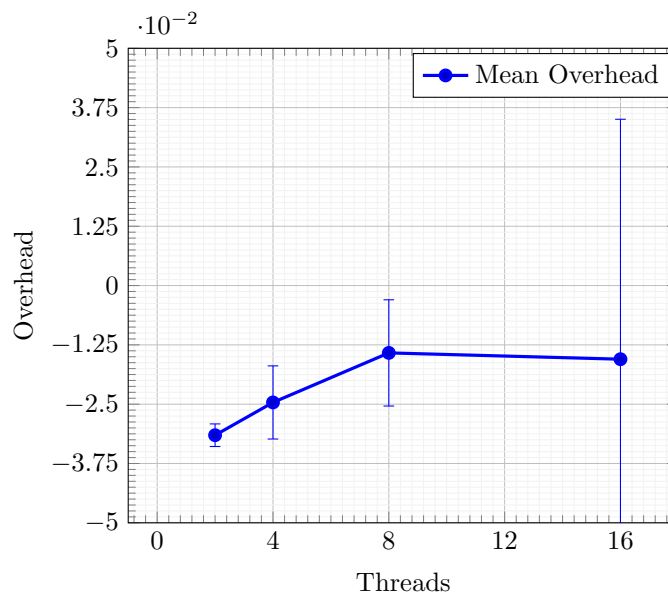
**Figure 10.26:** Mean overhead for shared-memory Rabin—Miller test „11213 64“ vs GpH



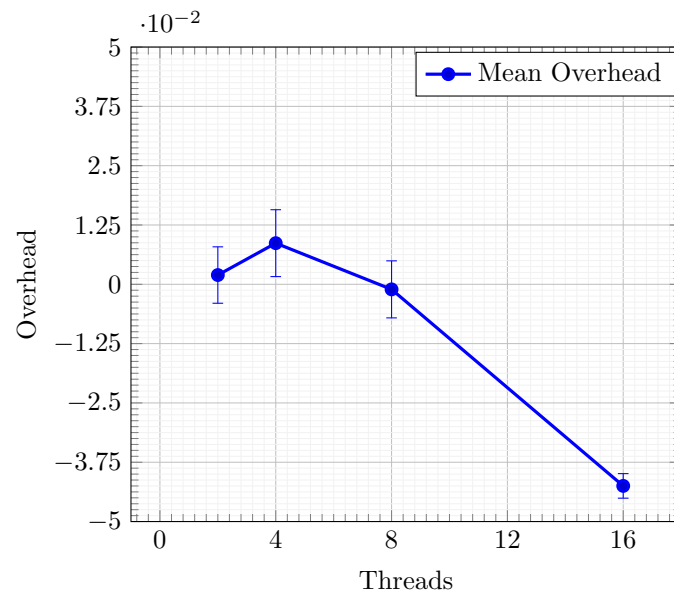
**Figure 10.27:** Mean overhead for shared-memory Rabin—Miller test „11213 64“ vs *Parmonad*



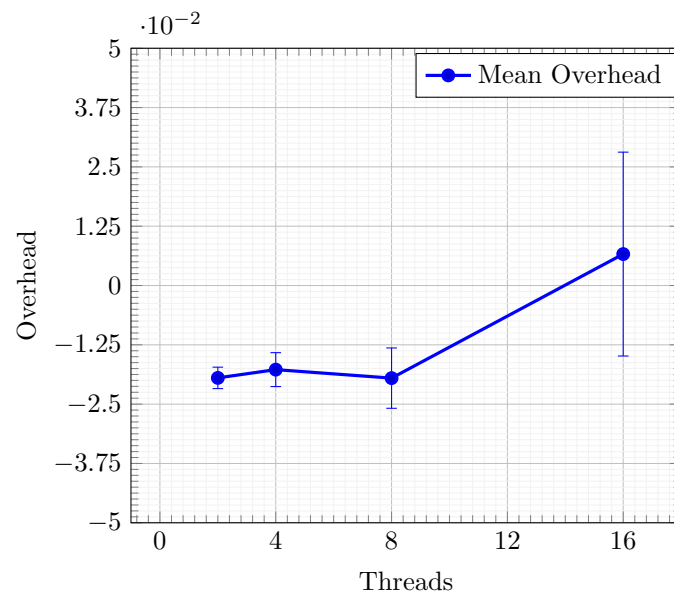
**Figure 10.28:** Parallel speedup of shared-memory Sudoku „1000“



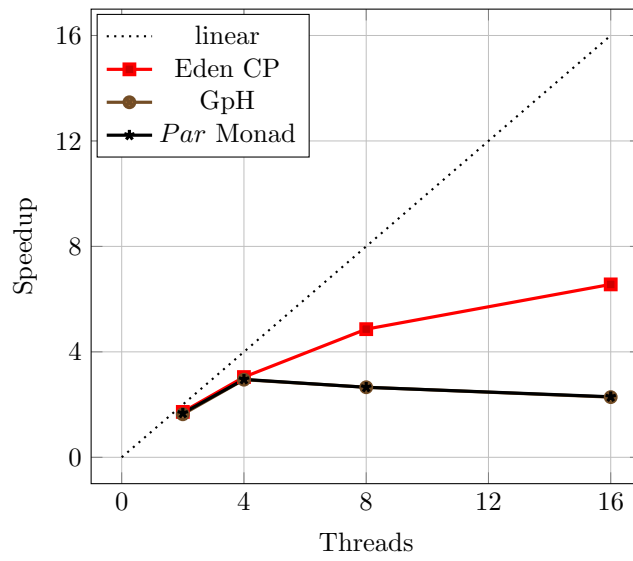
**Figure 10.29:** Mean overhead for shared-memory Sudoku „1000“ vs Eden CP



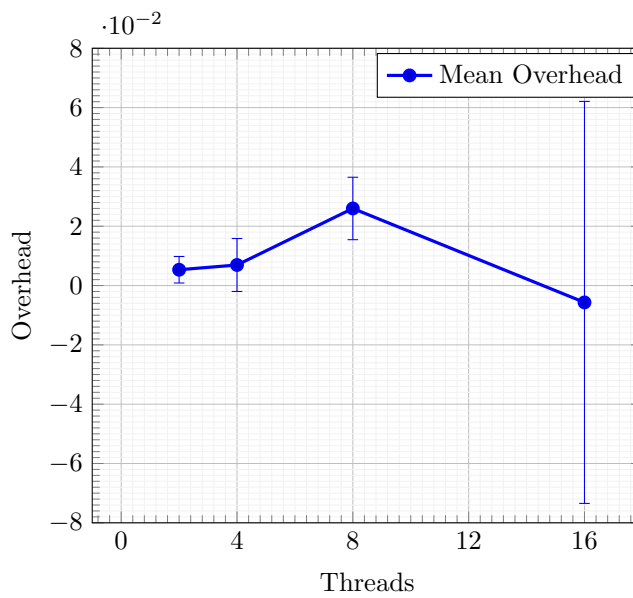
**Figure 10.30:** Mean overhead for shared-memory Sudoku „1000“ vs GpH



**Figure 10.31:** Mean overhead for shared-memory Sudoku „1000“ vs *Par* monad



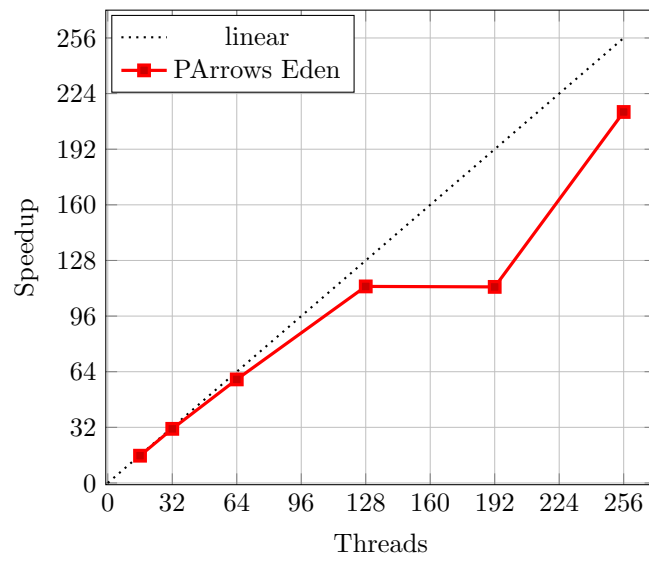
**Figure 10.32:** Parallel speedup of shared-memory Gentleman „512“



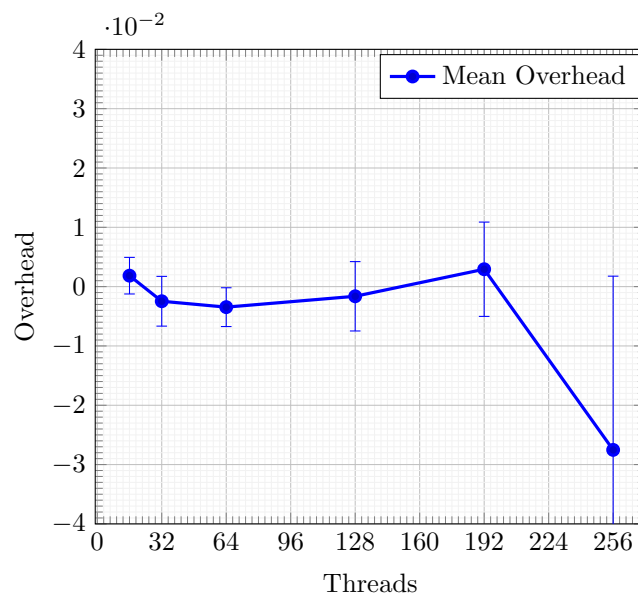
**Figure 10.33:** Mean overhead for shared-memory speedup of Gentleman „512“ vs Eden CP



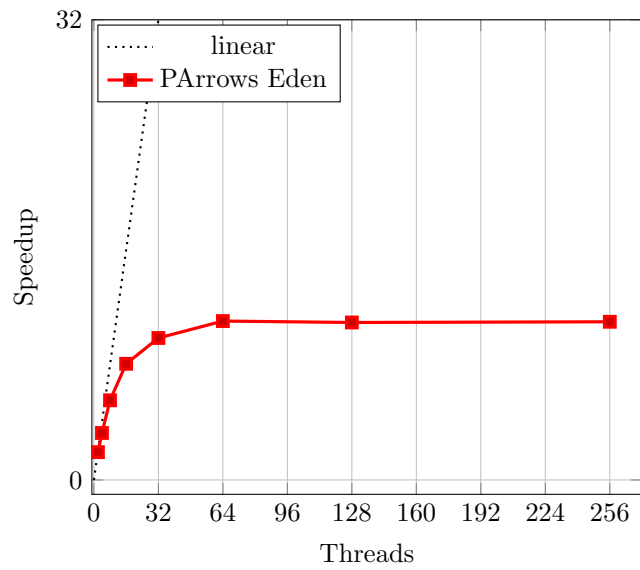
## 10.7 Distributed Memory Benchmarks



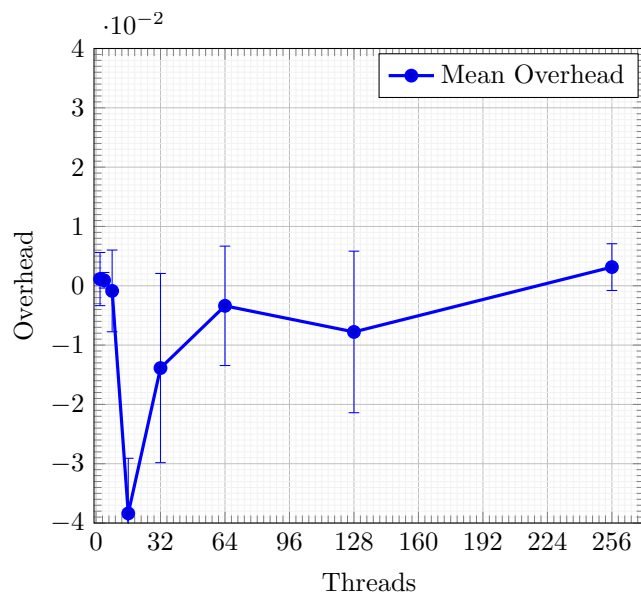
**Figure 10.34:** Parallel speedup of distributed-memory Rabin—Miller test „44497 256“



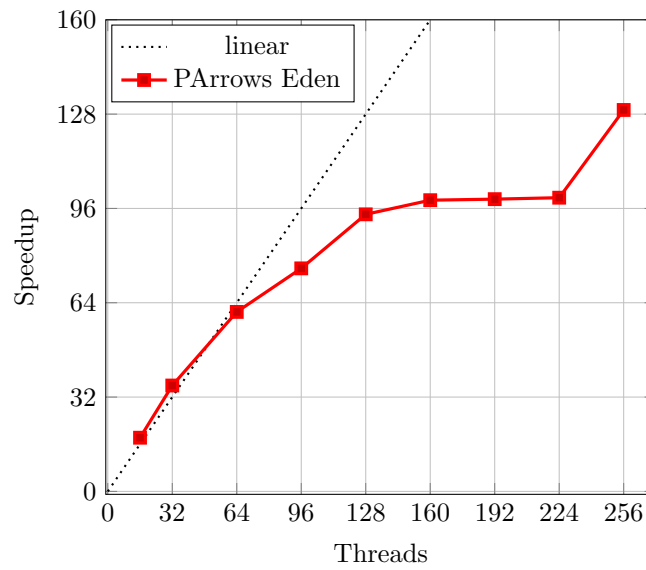
**Figure 10.35:** Mean overhead for distributed-memory Rabin—Miller test „44497 256“ vs Eden



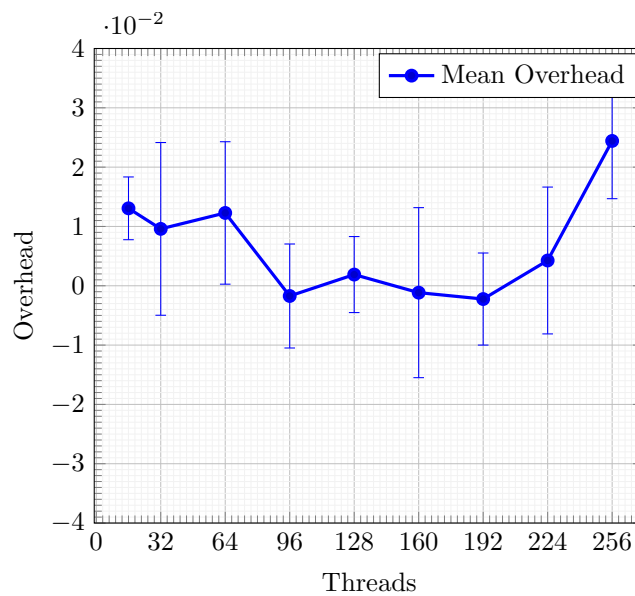
**Figure 10.36:** Parallel speedup of distributed-memory Jacobi sum test „3217“



**Figure 10.37:** Mean overhead for distributed-memory Jacobi sum test „3217“ vs Eden



**Figure 10.38:** Parallel speedup of distributed-memory Gentleman „4096“



**Figure 10.39:** Mean overhead for distributed-memory Gentleman „4096“ vs Eden

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