Concepts in Parallel Programming

Arrows for Parallel Computation

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Arrows are a general functional interface for computation and an alternative to Monads for API design. In contrast to Monad-based parallelism, we explore the use of Arrows for specifying generalised parallelism. Specifically, we define an Arrow-based language and implement it using multiple parallel Haskells. As each parallel computation is an Arrow, such parallel Arrows (PArrows) can be readily composed and transformed. To allow for more sophisticated communication schemes between computation nodes in distributed systems, we utilise the concept of Futures to wrap direct communication. Addressing the recent trends in cloud computing, we also explore the possibility for a cloud based implementation of this newly created DSL. To show that PArrows have similar expressive power as existing parallel languages, we implement several algorithmic skeletons and four benchmarks. The Benchmarks show that our framework does not induce any notable performance overhead. We conclude that Arrows have considerable potential for composing parallel programs and more specifically for programs that have to be executed with multiple different parallel language implementations.

Konzepte in Paralleler Programmierung

Arrows für Parallele Berechnungen

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Arrows sind eine allgemeine funktionale Schnittstelle für die Berechnung und eine Alternative zu Monaden zum Design von APIs. Im Gegensatz zu auf Monaden basierender Parallelität untersuchen wir die Verwendung von Arrows zur Spezifizierung von Parallelität Konkret definieren wir eine Arrow-basierte Sprache und implementieren sie mit mehreren parallelen Haskells. Da jede parallele Berechnung ein Arrow ist, können solche parallelen Arrows (PArrows) leicht zusammengesetzt und transformiert werden. Um komplexere Kommunikationsschemata zwischen Berechnungsknoten in verteilten Systemen zu ermöglichen, verwenden wir das Konzept der Futures, um die direkte Kommunikation zu abstrahieren. Um den aktuellen Trends im Cloud Computing Rechnung zu tragen, untersuchen wir auch die Möglichkeit einer Cloud-basierten Implementierung dieser neu geschaffenen DSL. Um zu zeigen, dass PArrows eine ähnliche Ausdruckskraft wie bestehende parallele Sprachen haben, implementieren wir mehrere algorithmische Skelette und vier Benchmarks. Die Benchmarks zeigen, dass unser Framework keinen nennenswerten Performance-Overhead verursacht. Wir kommen zu dem Schluss, dass Arrows ein beträchtliches Potenzial für das schreiben paralleler Programme und speziell auch solcher, welche auf mehreren unterschiedlichen parallelen Sprachimplementierungen ausgeführt werden können sollen.

Contents

Introduction

In recent years, functional programming has been on the rise as functional languages like Haskell, Scala, Lisp or their derivatives have seen an increase in popularity. Furthermore, imperative languages are adopting features originally coming from functional languages. So, even Java, which is generally not associated with introducing new features quickly, has officially embraced at least some functional concepts such as Lambdas and the functional interfaces in the standard library. Many new concepts such as the streaming API rely heavily on these new-to-Java ideas. Other languages such as C++, C# or Python show an even greater influence of functional paradigms as they all have been seen improving their support for a functional style of programming, even though they can not be considered pure functional languages by any means.

This rise in popularity does not come from nowhere. The core benefit of functional programming, its modularity, allows programmers to write concise programs in a clear and structured way.

Functional languages, coming from an academic environment, historically also have a long history of being used for experimenting with novel programming paradigms. Among these is the use of functional languages for parallel programming.

In Haskell, which we focus on in this thesis, there already exist several ways to write parallel programs. One approach that has not been explored in depth yet, however, is to represent parallel computations with Arrows. Nonetheless, they seem a natural fit since they can be thought of as a more general function arrow ($\rightarrow$) and serve as a general interface to computations while not being as restrictive as Monads (Hughes, 2000).

This is why in this thesis we will explain how a parallel Haskell based on parallel Arrows (PArrows) can be achieved. We do not want to re-invent parallelism, though, as we only provide an Arrow based type class hosting an Arrow combinator $parEvalN :: [arr\ a\ b] \rightarrow arr\ [a]\ [b]$ that turns a list of Arrows into a singular parallel Arrow. We use this class as an interface to wrap around existing parallel Haskells instead of introducing yet another new low-level parallel backend.

For this thesis we have chosen three of the most important parallel Haskells: GpH, or more specifically – its Multicore SMP implementation, (Trinder et al., 1996, 1998) for its simplicity, the classic *Par* Monad (Foltzer et al., 2012; Marlow et al., 2011) to represent a monadic DSL, and Eden (Loogen, 2012; Loogen et al., 2005) as a distributed memory parallel Haskell targeting clusters running MPI or PVM. These languages represent orthogonal approaches. Some use a Monad, even if only for the internal representation while others introduce additional language constructs.

While alternatives such as HdpH (Maier et al., 2014; Stewart, 2016, a Template Haskell-based parallel Haskell for distributed memory) and LVish (Kuper et al., 2014, a *Par* extension with focus on communication) exist, these were not chosen. The former does not differ from the original *Par* Monad with regard to how we would have used it in this thesis, while the latter (at least in its current form) does not suit a wrapping approach such as the one presented here due to its heavy reliance on Template Haskell[1]. We will nonetheless experiment with a potential Cloud Haskell (Epstein et al., 2011) backend to someday enable the DSL to work in modern cloud based clusters.

With the shallow–embedded DSL based on Arrows we define in this thesis we do however not only aim to define a novel parallel programming interface that allows for arbitrary Arrow types to be parallelised – we also aim to tame the zoo of parallel Haskells. Even more so, we want to provide a common, low-penalty programming interface that is general by allowing to switch the parallel implementations at will.

Structure

This thesis is structured as follows. In Chapter 2, we discuss related work. Chapter 3 covers the background of this thesis including an introduction to functional programming, a Monad tutorial to finally introduce the concepts of Arrows. In Chapter 4 we define the shallow-embedded DSL based on Arrows (PArrows) together with some first basic extensions and *map*-based skeletons. Chapter 5 develops the PArrows API further by introducing the concept of *Future*s and by giving the definitions of some topology skeletons. We describe an experimental Cloud Haskell based backend in Chapter 6. Chapter 7 evaluates the performance of the PArrows DSL. Chapter 8 discusses our results and Chapter 9 concludes.

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[1]a Haskell extension that allows to generate code.

Related Work

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In this Chapter we will discuss work related to this thesis. We start with work on parallel Haskells in Chapter 2.1. Next, we discuss research on algorithmic skeletons in Chapter 2.2. Then, we go over previous work in the field of Arrows in Chapter 2.3. Finally, we explain how this thesis is related to previous work done on PArrows during the exploration of the subject.

2.1 Parallel Haskells

The non-strict semantics of Haskell, and the fact that reduction encapsulates computations as closures, makes it relatively easy to define alternate execution strategies. A range of approaches have been explored, including data parallelism (Chakravarty et al., 2007; Keller et al., 2010), GPU-based approaches (Mainland and Morrisett, 2010; Svensson, 2011) and software transactional memory (Harris et al., 2005; Perfumo et al., 2008). The Haskell–GPU bridge Accelerate (Chakravarty et al., 2011; Clifton-Everest et al., 2014; McDonell et al., 2015) is completely orthogonal to our approach as it is by nature focused on parallel execution of singular functions on "Arrays" of data instead of parallel execution of multiple functions with their respective inputs. A good survey of parallel Haskells can be found in Marlow (2013).

Our PArrow implementation uses three task parallel languages as backends: the GpH (Trinder et al., 1996, 1998) parallel Haskell dialect and its multicore version (Marlow et al., 2009), the Par Monad (Foltzer et al., 2012; Marlow et al., 2011), and Eden (Loogen, 2012; Loogen et al., 2005). These languages are under active development, for example a combined shared and distributed memory implementation of GpH is available (Aljabri et al., 2015, 2014). Research on Eden includes low-level implementation (Berthold, 2016, 2008), skeleton composition (Dieterle et al., 2016), communication (Dieterle et al., 2010b), and generation of process networks (Horstmeyer and Loogen, 2013). The definitions of new Eden skeletons is a specific focus (Berthold and Loogen, 2006; Berthold et al., 2009b, 2009c; Dieterle et al., 2010a, 2013; Encina et al., 2011; Hammond et al., 2003; Janjic et al., 2013).

Other task parallel Haskells related to Eden, GpH, and the Par Monad include: HdpH (Maier et al., 2014; Stewart, 2016) is an extension of Par Monad to heterogeneous

clusters. LVish (Kuper et al., 2014) is a communication-centred extension of the *Par* Monad and is based on monotonically growing data structures.

Cloud Haskell (Epstein et al., 2011) which we use for an experimental backend is a domain specific language for cloud computing but can also be used for parallel computation.

2.2 Algorithmic skeletons

Algorithmic skeletons as a concept to abstract the general idea of different classes of algorithms were introduced by Cole (1989). Early publications on this topic include Danelutto et al. (1992), Darlington et al. (1993), Botorog and Kuchen (1996), Lengauer et al. (1997) and Gorlatch (1998). Rabhi and Gorlatch (2003) consolidated early reports on high-level programming approaches. Types of algorithmic skeletons include *map*-, *fold*-, and *scan*-based parallel programming patterns, special applications such as divide-and-conquer or topological skeletons.

The *farm* skeleton (Hey, 1990; Peña and Rubio, 2001; Poldner and Kuchen, 2005) is a statically task-balanced parallel *map*. When tasks' durations cannot be foreseen, a dynamic load balancing (*workpool*) brings a lot of improvement [Rudolph et al. (1991); Hammond et al. (2003); Hippold and Rünger (2006); Berthold et al. (2008); Marlow2009]. For special tasks *workpool* skeletons can be extended with dynamic task creation (Brown and Hammond, 2010; Dinan et al., 2009; Priebe, 2006). Efficient load-balancing schemes for *workpool*s are subject of research (Acar et al., 2000; Blumofe and Leiserson, 1999; Chase and Lev, 2005; Michael et al., 2009; Nieuwpoort et al., 2001; Olivier and Prins, 2008).

The *fold* (or *reduce*) skeleton was implemented in various skeleton libraries (Buono et al., 2010; Dastgeer et al., 2011; Karasawa and Iwasaki, 2009; Kuchen, 2002), as also its inverse, *scan* (Bischof and Gorlatch, 2002; Harris et al., 2007). Google *map–reduce* (Dean and Ghemawat, 2008, 2010) is more special than just a composition of the two skeletons (Berthold et al., 2009b; Lämmel, 2008).

The effort is ongoing, including topological skeletons (Berthold and Loogen, 2006), special-purpose skeletons for computer algebra (Berthold et al., 2009c; Janjic et al., 2013; Lobachev, 2011, 2012), and iteration skeletons (Dieterle et al., 2013). The idea of Linton et al. (2010) is to use a parallel Haskell to orchestrate further software systems to run in parallel. Dieterle et al. (2016) compare the composition of skeletons to stable process networks.

We implement some of these algorithmic skeletons in Chapters 4.3 as well as 5.2, namely various parallel *map* variants as well as the topological skeletons *pipe*, *ring* and *torus*.

2.3 Arrows

Arrows were introduced by Hughes (2000) as a less restrictive alternative to Monads, in essence they are a generalised function arrow $\rightarrow$. Hughes (2005) presents a tutorial on Arrows. Jacobs (2009), Lindley et al. (2011), Atkey (2011) develop theoretical background of Arrows. (Paterson, 2001) introduced a new notation for Arrows. Arrows have applications in information flow research (Li and Zdancewic, 2006, 2010; Russo et al., 2008), invertible programming (Alimarine et al., 2005), and quantum computer simulation (Vizzotto, 2006). But the probably most prominent application of Arrows is Arrow-based functional reactive programming, AFRP (Czaplicki and Chong, 2013; Hudak et al., 2003; Nilsson et al., 2002). (Liu et al., 2009) formally define a more special kind of Arrows that encapsulate the computation more than regular Arrows do and thus enable optimisations. Their approach would allow parallel composition, as their special Arrows would not interfere with each other in concurrent execution. In contrast, we capture a whole parallel computation as a single entity: our main instantiation function *parEvalN* creates a single (parallel) Arrow out of a list of Arrows. Huang et al. (2007) utilise Arrows for parallelism, but strikingly different from our approach. They use Arrows to orchestrate several tasks in robotics. We, however, propose a general interface for parallel programming, while remaining completely in Haskell.

Arrows in other languages

Although this work is centred on Haskell implementation of Arrows, it is applicable to any functional programming language where parallel evaluation and Arrows can be defined. Basic definitions of PArrows are possible in the Frege language[1] – a Haskell-like language that compiles to Java code to then be compiled natively on the Java Virtual Machine (JVM). However, they are beyond the scope of this work, as are similar experiments with the Eta language[2], a new approach to Haskell on the JVM that compiles directly to JVM bytecode.

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[1]GitHub project page at `https://github.com/Frege/frege`
[2]Eta project page at `http://eta-lang.org`

Arrows have also been shown to be useful for better handling of typical GUI tasks in Clean (Achten et al., 2004, 2007). Dagand et al. (2009) used Arrows in OCaml in the implementation of a distributed system.

2.4 Earlier work

This thesis is based on the paper "Arrows for Parallel Computation" (Braun et al., 2018) written by me, Martin Braun, together with Oleg Lobachev and Phil Trinder. The original idea for this paper came up during a Masters Research Project done under the supervision of Oleg Lobachev.

While based on the ideas of the Master's research project, the work presented here is dramatically different. In this thesis, the results are a lot more sophisticated and detailed and only Chapters 3.2 – 4.4 are somewhat based on work done during the original project.

Similarities to the paper however, are not coincidental. Chapters 2, 3.1.4 – 5 as well as Chapter 7 were taken from the paper and embedded into the rest of the text. The introduction in Chapter 1 as well as the conclusion in 9 are also inspired by the paper. But as I, Martin Braun, am the primary author of the paper, this still means that the following work was done independently as required by studying regulations.

Background

Before we delve into our novel approach for parallel programming using Arrows, we give a short overview of all our main concepts and technologies. We start with an introduction to functional programming (Chapter 3.1) including a short tutorial on Monads (Chapter 3.1.3) before explaining the concept of Arrows (Chapter 3.1.4). Finally we give a short introduction to the main parallel Haskells used as backends for our DSL in this thesis (Chapter 3.2) - GpH, the Par Monad, and Eden.

3.1 Functional Programming

This chapter covers the basics of functional programming. We start by citing Hughes (1990) on why functional programming matters including a characterisation of the concept in general (Chapter 3.1.1). Then, we give a short introduction to functional programming with Haskell (Chapter 3.1.2) and also explain the concept of Monads (Chapter 3.1.3) which some parallel Haskells use. Finally, we introduce Arrows and explain their type class in Haskell (Chapter 3.1.4).

3.1.1 Why Functional Programming?

Hughes (1990) describes the fundamental idea of functional programming like this:

> " *Functional programming is so called because its fundamental operation is the application of functions to arguments. A main program itself is written as a function that receives the program's input as its argument and delivers the program's output as its result. Typically the main function is defined in terms of other functions, which in turn are defined in terms of still more functions, until at the bottom level the functions are language primitives.* "

Basically, functional programs only contain logic described in terms of functions and their compositions. Additionally, functional programming is also often characterized as follows (Hughes, 1990):

> *" Functional programs contain no assignment statements, so variables, once given a value, never change. More generally, functional programs contain no side-effects at all. A function call can have no effect other than to compute its result. This eliminates a major source of bugs, and also makes the order of execution irrelevant — since no side- effect can change an expression's value, it can be evaluated at any time. This relieves the programmer of the burden of prescribing the flow of control. Since expressions can be evaluated at any time, one can freely replace variables by their values and vice versa — that is, programs are "referentially transparent". This freedom helps make functional programs more tractable mathematically than their conventional counterparts. "*

While all these are all good arguments in favour of functional programming – because of the elimination of programming bottlenecks – these arguments only describe functional programming by means of what it can not do. Hughes (1990) describes his dissatisfaction with this argument as follows:

> *" Even a functional programmer should be dissatisfied with these so-called advantages, because they give no help in exploiting the power of functional languages. One cannot write a program that is particularly lacking in assignment statements, or particularly referentially transparent. There is no yardstick of program quality here, and therefore no ideal to aim at. "*

To argue that there is merit in functional programming besides having fewer error-prone features Hughes (1990) then goes into detail about one of the areas where functional programming shines and why it therefore matters – modularity. He argues that modularity is only possible with good glue code. This is where he sees functional programming to be better suited because of two powerful tools: higher-order functions and laziness.

Higher-order functions are functions that take other functions as arguments. They usually generalize a concept (e.g. mapping over a list, zipping two lists, etc.) and take the passed function(s) as their internal worker function. They provide the skeleton of the program. Laziness here means that values are only evaluated when required. This allows for programs to work in a producer/consumer pattern without having to write manual interweaving code. We will explain both concepts in greater details later in this Chapter.

The focus on modularity through the level of composability functional languages have, can be seen in all the definitions of Haskell functions in the following chapters of this thesis. Also, the main functional concept this thesis uses, Arrows, are by

nature a result of the desire to generalize modularity. We will show how to enable programs based on this concept to benefit from parallelism by using our combinator $parEvalN$.

3.1.2 A Short introduction to Haskell

Even though this thesis is called "Concepts in Parallel Programming" and focuses on Arrows for parallel (functional) computation, we have to first define the basic building blocks of our programming language and show how to use them in regular programs before we can explore *parallel* programming. Therefore, in the following chapter, we will give a short introduction to functional programming with Haskell. While this will give a good idea of how programming in Haskell works, this is not supposed to be a complete tutorial on Haskell, but merely a quick overview over the most relevant features of the language which are used in this thesis. The following is loosely based on the book "Learn you a haskell for great good!" (Michaelson, 2013).

From Imperative Programming to Functional Programming

In order to ease the introduction to functional programming, we will give a short introduction to functional programming in Haskell in this chapter by comparing the general style of imperative C code to functional Haskell using the example of the Fibonacci sequence.

To start off, we take a look at the following iterative implementation of the Fibonacci sequence:

```
int fib ( int n ) {
    int pre = 0;
    int cur = 1;
    int res = 0;
    for ( int i = 0; i < n; ++i ) {
        res = pre + cur;
        pre = cur;
        cur = res;
    }
    return cur;
}
```

It contains assignments and a loop, which in pure[1] functional programming we
we can not use[2]. If we translate this Fibonacci example into a recursive definition,
however, we get pure functional C code without any assignment statements:

```c
int fib ( int n ) {
    if ( n <= 0 )
        return 0;
    else if ( n == 1 )
        return 1;
    else
        return fib ( n − 2 ) + fib ( n − 1 );
}
```

This resembles the Haskell variant in Figure 3.1. We can see how the flow of the
programming works even without requiring any modifiable state. Furthermore, the
Haskell implementation uses guards (e.g. $n \leqslant 0 = 0$) which are equivalent to the
conditional statements in the C variant. They will be explained in more detail later
in this chapter.

$$
\begin{aligned}
&\textit{fib} :: \textit{Int} \rightarrow \textit{Int} \\
&\textit{fib } n \\
&\quad \mid n \leqslant 0 = 0 \\
&\quad \mid n \equiv 1 = 1 \\
&\quad \mid \textit{otherwise} = \\
&\qquad (\textit{fib } (n - 2)) \\
&\qquad\quad + (\textit{fib } (n - 1))
\end{aligned}
$$

Figure 3.1: Standard Fibonacci in Haskell.

In functional languages like Haskell we only express computations in this matter by
composition of functions (recursion in essence is also just a composition of a function
with itself). Because of this and since we can not change the state of any associated
variables, we usually also do not have to worry about the order of execution in
functional programs and let the compiler decide how to resolve the recursive formula.
In general, we can say that in functional programming we primarily focus on what
information is required and by which transformations to compute it instead of how
we perform them and how we track the changes in state.[3]

Haskell being a functional language does not mean, that we do not have the usual
problem of a too small call-stack size encountered when programming with recur-

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[1] Pure code is code without side-effects. Assignments are side-effects.

[2] It is however possible to introduce monadic DSLs in Haskell that mimic C style behaviour, see
https://hackage.haskell.org/package/ImperativeHaskell-2.0.0.1.

[3] From https://docs.microsoft.com/en-us/dotnet/visual-basic/programming-guide/concepts/linq/functional-programming-vs-imperative-programming

sion. While Haskell programs can naturally handle much bigger call-stacks without overflowing, at some point the limit will be reached and the program will crash. But since the class of tail-recursive programs which all have the form

```
f x = if < end >
   then s x
   else f (r x)
   -- s and r arbitrary, but not depending on f
s = ...
r = ...
```

is equivalent to the class of all recursive programs (which is in turn equivalent to all imperative programs), this is no big problem. We can just translate our *fib* definition into a tail-recursive variant (Figure 3.2) which Haskell's compiler is capable of automatically translating into looping machine code.

$$
\begin{aligned}
&\mathit{fib} :: Int \rightarrow Int \\
&\mathit{fib}\ n \\
&\quad |\ n \leqslant 0 = 0 \\
&\quad |\ otherwise = \mathit{fib}'\ n\ 0\ 1 \\
&\quad \textbf{where} \\
&\qquad \mathit{fib}' :: Int \rightarrow Int \rightarrow Int \rightarrow Int \\
&\qquad \mathit{fib}'\ n\ prev\ res \\
&\qquad\quad |\ n \equiv 0 = res \\
&\qquad\quad |\ otherwise = \mathit{fib}'\ (n-1)\ res\ (res+prev)
\end{aligned}
$$

Figure 3.2: Tail Recursive Fibonacci in Haskell.

Functions

As already mentioned above, the basic building blocks of a Haskell program are functions. We define them like this:

$$
\begin{aligned}
&f :: Int \rightarrow Int \rightarrow Int \\
&f\ x\ y = multiply\ x\ y
\end{aligned}
$$

Here, we declared a function f which takes two arguments of type Int and returns yet another Int. In the definition we say that f is the function $multiply$ applied to both its arguments x and y. We define $multiply$ as

$$
\begin{aligned}
&multiply :: Int \rightarrow Int \rightarrow Int \\
&multiply\ x\ y = x * y
\end{aligned}
$$

In Haskell, since f and *multiply* seem to be the same, we can even write this relationship directly:

$$f :: Int \rightarrow Int \rightarrow Int$$
$$f = multiply$$

We can do so because in Haskell functions can be treated just like any other type. For example, if we wanted to have another function g which applied f on two lists of integers, we can write

$$g :: [Int] \rightarrow [Int] \rightarrow [Int]$$
$$g = zipWith\ f$$

where $zipWith$ would be of type $(Int \rightarrow Int \rightarrow Int) \rightarrow [Int] \rightarrow [Int] \rightarrow [Int]$. It is common to express calculations in such a way using higher-order functions. We will see more of this later in this chapter.

Type inference

Taking the same example function g from above, it does not make sense to be so restrictive in terms of which type to allow in such a function since all it does is apply some function to zip two lists. Thankfully, in Haskell we can define functions in a completely generic way such that we can write the actual type of zipWith as $(a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$ as in meaning: Zip a list containing some as with a list containing a list of bs with a function $a \rightarrow b \rightarrow c$ to get a list of cs. Only because we use this function in the context of our function g it is specialized to the Int form.

Type inference even allows us to define g without writing down the type definition and let the compiler determine the actual type of g.

$$g = zipWith\ f$$

While this is possible, it is generally encouraged to always specify the type of top-level functions for better readability. Leaving out the type specification can, however, be useful when defining nested helper functions.

Function composition, higher-order functions, and function application

As we have seen, functions can be handled similar to data types in Haskell. This way, we can for example define a function that computes a number to the power of four as

$$toThePowerOfFour :: Int \rightarrow Int$$
$$toThePowerOfFour = toThePowerOfTwo \circ toThePowerOfTwo$$

with $\circ$ being the functional composition operator with type $(\circ) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$[4] and where $toThePowerOfTwo$ is defined simply as

$$toThePowerOfTwo :: Int \rightarrow Int$$
$$toThePowerOfTwo\ x = multiply\ x\ x$$

Another aspect of functions being similar to data types is that, in functional programming, we frequently use higher order functions to express calculations. We have seen this earlier with the use of $zipWith$. Other often used higher-order functions include mapping ($map :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$, i.e. convert a list of as into a list of bs with the given function $a \rightarrow b$) and folding (e.g. $foldLeft :: (b \rightarrow a \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$, i.e. reduce the list with the given function $b \rightarrow a \rightarrow b$ into a singular value given a starting value of type b). These are often used in some sort of composition like

$$euclidDistance :: [Int] \rightarrow [Int] \rightarrow Int$$
$$euclidDistance = sqrt \circ foldLeft\ (+)\ 0 \circ map\ (toThePowerOfTwo) \circ zipWith\ (-)$$

Note that while this could have easily been written shorter as something along the lines of $sqrt\ (foldLeft\ (+)\ 0\ (zipWith\ (\lambda a\ b \rightarrow toThePowerOfTwo\ (a-b))))$ it is easy to see that the above declaration is easier to understand because of the simple steps the computation takes. We first zip the list of inputs with element-wise subtraction and then square this difference, sum these results up and finally take the square root. This is something we see a lot in Haskell code: Complex computations can be expressed with the help of higher-order functions instead of having to write it manually. This is not only much shorter, but also easier to understand for other programmers if they have to read-up on the implementation for some reason.

Something which is also quite useful in Haskell is the function application operator $(\$) :: (a \rightarrow b) \rightarrow a \rightarrow b$ allowing for the application of a function $a \rightarrow b$ to a given argument a. It is simply defined as:

$$(\$) :: (a \rightarrow b) \rightarrow a \rightarrow b$$
$$f\ \$\ x = f\ x$$

While the use-case for such an operator might not be immediately clear, it will be, if we take a look at the following function $listApp :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$ where we take a list of functions $[a \rightarrow b]$ and apply them one-by-one with their respective input values from the input list $[a]$ to generate a list of results b:

$$listApp :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$$
$$listApp = zipWith\ (\$)$$

---

[4]Note the order of the arguments, $g \circ f$ means to first apply f and then g and not the other way around

Here, if we had not used the ($) operator, we would have to write $zipWith\ (\lambda f\ a \rightarrow f\ a)$ which certainly seems a bit redundant.

Something the ($) operator is also used quite often is to write shorter code. For example, code snippets like

$$someFunc = f1\ (f2\ param1\ (f3\ param2\ (f4\ param3)))$$

can also be written without the braces as

$$someFunc = f1\ \$\ f2\ param1\ \$\ f3\ param2\ \$\ f4\ param3$$

which is sometimes preferred to the brace-style, but is semantically identical.

Conditional Computation

Haskell has different styles of dealing with conditional evaluation. We will now show the most common variants to express conditional statements.

The most obvious one in terms of functionality is the **if** … **then** … **else** construct:

$$myFunc :: Int \rightarrow Int$$
$$myFunc\ x = \textbf{if}\ x < 10\ \textbf{then}\ x * 2\ \textbf{else}\ x * 4$$

While having the same well-known semantics of any **if** … **then** … **else** like they could be found in imperative languages like e.g. C, in Haskell, being a functional language, the **else** is non-optional as expressions are required to be total.[5]

An alternative to this are guards, which make expressions easier to read if many alternatives are involved in a function:

$$myFunc :: Int \rightarrow Int$$
$$myFunc\ x$$
$$\quad |\ x < 10 = x * 2$$
$$\quad |\ x < 12 = x * 3$$
$$\quad |\ x < 14 = x$$
$$\quad |\ x > 18 \wedge x < 20 = 42$$
$$\quad |\ otherwise = x * 4$$

Yet another technique for conditional computation is to use pattern matching. For conditional statements we can use it by writing definitions of the function for specific values, like

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[5]Total in terms of computation, unsuccessful calculations can still be expressed with constructs like *Maybe a*.

$$myFunc :: Int \rightarrow Int$$
$$myFunc\ 5 = 10$$
$$myFunc\ x\ @\ 10 = x * 10$$
$$myFunc\ x = x * 2$$

where the first matching definition is chosen during computation.[6] Alternatively, we can do pattern matching with the help of **case** expressions:

$$myFunc :: Int \rightarrow Int$$
$$myFunc\ x = \textbf{case}\ x\ \textbf{of}$$
$$5 \rightarrow 10$$
$$10 \rightarrow x * 10$$
$$\_ \rightarrow x * 2$$

These can be used just like ordinary expressions.

We can not, however, express boolean statements in this way. This is because pattern matching is done on the structure of the value that is being pattern matched. Later in this chapter we will see what other powerful things we can do with this technique.

where, let

While Haskell does not have variables and only works on values instead, it still allows the programmer to name sub-expressions so that either the code becomes more clear or that it can be reused more easily. Here, two different variants are available: **where** and **let**.

With the help of **where** we can write code like

$$whereReuse :: Double \rightarrow Double \rightarrow String$$
$$whereReuse\ a\ b$$
$$|\ divided > 10 = \texttt{"a is more than 10 times b"}$$
$$|\ divided \equiv 10 = \texttt{"a is 10 times b"}$$
$$|\ otherwise = \texttt{"a is less than 10 times b"}$$
$$\textbf{where}\ divided = a\ /\ b$$

It is important to note that **where** is just syntactic sugar, though, and can not used in expressions like $f\ (a * 2\ \textbf{where}\ a = 3)$. This is in turn possible with **let** which allows us to write expressions like $f\ (\textbf{let}\ a = 3\ \textbf{in}\ a * 2)$ or $\textbf{let}\ a = 3\ \textbf{in}\ f\ (a * 2)$. **let** can in contrast, however, not be used in conjunction with guards.

---

[6]Here, the @ allows us to bind the value to a variable while also pattern matching it.

Type safety

Haskell is a statically typed functional language. This means that during compilation all types are checked for compatibility and type declarations are not just treated as optional "hints" to the type-checker. Pairing this with the pure aspect of the language means that Haskell programs seem to be correct more often in practice if the program compiles than in imperative languages. The compiler essentially helps the programmer to write *semantically correct* instead of just syntactically correct code. It should be noted that this does not mean that testing can be omitted. It is still extremely important, but becomes less cumbersome because state is mostly a non-issue.

Type classes

The example function $multiply$ which we have seen earlier seems a bit restrictive as it only allows for the usage of Ints. Ints are obviously not the only type which can be multiplied. Haskell has a way to express this fact: type classes. We can express a type class $Multiplicable\ a$ that encapsulates the contract of $multiply$ on some type a as

> **class** $Multiplicable\ a$ **where**
> $multiply :: a \rightarrow a \rightarrow a$

With this class in place, we can then introduce instances – implementations of the contract – for specific types. For example the instance for Int, $Multiplicable\ Int$, can be defined as

> **instance** $Multiplicable\ Int$ **where**
> $multiply\ x\ y = x * y$

Now if we want to use this new contract on a generic function f, we require a *Multiplicable* instance for every type that we want to use $multiply$ on inside the function:

> $f :: Multiplicable\ a \Rightarrow a \rightarrow a \rightarrow a$
> $f\ x\ y = multiply\ (multiply\ x\ y)\ x$

Such a function f does work with the contract of *Multiplicable* instead of requiring some specific type. Using this technique allows many definitions in Haskell to be reused even though it is a statically typed language.

In Haskell we can also write type classes with more than one type parameter. This allows for encapsulation of contracts of arbitrary complexity. Furthermore type

classes can itself have constraints placed on what types are allowed. Both can be seen here:

class $(SomeClass\ a, SomeOtherClass\ b) \Rightarrow MyClass\ a\ b\ c$ **where**

...

Custom types

As in any mature programming language, Haskell programmers obviously do not have to represent everything with only some base-set of types. Types are usually defined in three different ways. For starters, we can give types aliases with the **type** keyword like

```
    -- Tuple of a and b
type Tuple a b = (a, b)
    -- Tuple of ints
type IntTuple = (Int, Int)
```

which are treated just like original (a, b) or Int, Int would. This means that we can use such types loosely and pass e.b. a $Tuple\ Int\ Int$ into a function $f :: (Int, Int) \to ....$ The same also holds for type classes. While not actually defining new types, **type** declarations are therefore often used to make function types easier to read.

The second way to declare types, **data**, declares new-types as in actual new types in the type system:

```
data Direction =
  North
   | NorthEast
   | East
   | SouthEast
   | South
   | SouthWest
   | West
   | NorthWest
```

The upper case words $North$ - $NorthWest$ in this example are called constructors.

data types are not limited to enum-style types, though. They can also hold values, like the $Maybe\ a$ type from Haskell. This type, which *may* hold a value a internally, is usually used as a return type for functions which not always return an actual result (e.g. in case of a failure). We can define this type as follows:

```
    -- unnamed field
data Maybe a = Just a | Nothing
```

Values are created by calling the constructors with the appropriate parameters (if any), i.e. when passed into a function: f $(Just\ 1)$. Furthermore, **data** constructors can have named fields defined like

```
    -- named field
data Maybe a =
    Just { theThing :: a }
    | Nothing
```

where values are created by calling the constructor and passing the appropriate parameters to the properties, i.e. f $(Maybe\ \{theThing = 1\})$ The final way to define custom types is via **newtype**:

```
    -- unnamed field
newtype MyNewType a = Constructor a
    -- named field
newtype MyOtherNewType a = Constructor { myOnlyThing :: a }
```

Types declared this way are similar to **data** types, but can only contain a single constructor with just a single field. Also, unlike **data**, constructors declared with **newtype** are strict instead of lazy, meaning the compiler can optimize away the surrounding declaration. Everything else is handled exactly like with **data** types. The specifics of what laziness or strictness means will be explained in the next section of this chapter. **newtype** types are also a useful tool if we were to write a wrapper for a type while not wanting to inherit all instances of type classes, but are also often used when declaring more complicated types.

Lazy Evaluation

One thing that is not obvious when looking at the definitions from this chapter is that Haskell is a lazy language[7]. This means that values are only evaluated when required. This has one major benefit: We get a Producer/Consumer pattern for free. For example if we have the lazy function $producer :: Int \rightarrow [Int]$ producing some list of integers and some consumer consuming $consumer :: [Int] \rightarrow Int$ this list. Then, in a program $consumer \circ producer$, $producer$ generates the elements of the result-list

---

[7]Haskell is actually defined as a non-strict language, meaning that only as much as required is evaluated, not when it is done. Laziness is just a way to achieve non-strictness. The same could be achieved with an eager, but non-strict evaluation mechanism. But as Haskell's main compilers all implement non-strictness via lazy evaluation, it is okay to call Haskell a lazy language here. See `https://wiki.haskell.org/Lazy_vs._non-strict`

as they are consumed. This also means that, if *consumer* only requires the first few elements of the list to compute the result, *producer* does not produce unneeded results.

Laziness even allows us to express infinite streams, which can be helpful in some cases. As an example, an infinite list of ones is defined with the help of the list constructor $(:) :: a \rightarrow [\,a\,] \rightarrow [\,a\,]$, which prepends a value to a list, as

$$ones :: [\,Int\,]$$
$$ones = 1 : ones$$

or, if we require a list of some value at least n times so that it can be consumed with some list of length n, we can just use an infinite list instead of computing the actual required amount (which would take n steps for a linked list). The helper function for this is called *repeat* and can be written as

$$repeat :: a \rightarrow [\,a\,]$$
$$repeat\ a = a : (repeat\ a)$$

Another good example where Laziness simplifies things is when branching is involved:

$$calculateStuff :: [\,Int\,] \rightarrow Int$$
$$calculateStuff = \textbf{if} < someCondition >$$
$$\quad \textbf{then}\ doStuff\ list1\ list2$$
$$\quad \textbf{else}\ doSomeOtherStuff\ list1$$
$$\qquad \textbf{where}$$
$$\qquad\quad list1 = ...$$
$$\qquad\quad list2 = ...$$

Here, *list2* is not required in both branches of the **if** statement. Thanks to laziness it is therefore only evaluated upon a successful if-check. While such a behaviour is obviously possible in non-lazy languages the elegance of the above definition is apparent. We can define as many variables in the same clear way without having unnecessary computations or code dealing with conditional computation like nested **wheres**.

Usually laziness is beneficial to programs and programmers as it allows for easy composition and better structure in code, but sometimes we require more control about when something is evaluated. Haskell therefore has several ways to control *when* and *how* values are evaluated.

The most basic primitive to force values to be *strict* instead of lazy is $seq :: a \rightarrow b \rightarrow b$, which is by nature part of the compiler and can not be expressed in Haskell directly.

It's semantics however, are as follows: We tell the compiler that the first argument (of type a) is to be evaluated before the second argument. For example, in an expression like

$$myFun :: Int \rightarrow (Int, Int)$$
$$myFun\ x = \textbf{let}\ y = f\ x\ \textbf{in}\ y\ \text{`}seq\text{`}\ g\ y$$
$$\quad \textbf{where}$$
$$\quad\quad f = ...$$
$$\quad\quad g = ...$$

we can then hint at the compiler that we want $y = f\ x$ evaluated before returning the (still non-evaluated) result of $g\ y$. This trick is usually used if during profiling a big chunk of non-evaluated values are noticed to aggregate before or in the process of evaluation of $f\ x$. As this is a common pattern seen in Haskell programs, there exists the strict function application operator ($!) :: (a \rightarrow b) \rightarrow a \rightarrow b$ to encapsulate it. It is straightforwardly defined as:

$$(\$!) :: (a \rightarrow b) \rightarrow a \rightarrow b$$
$$f\ \$!\ x = x\ \text{`}seq\text{`}\ f\ x$$

With it we can then write our example function as

$$myFun :: Int \rightarrow (Int, Int)$$
$$myFun\ x = g\ \$!\ f\ x$$
$$\quad \textbf{where}$$
$$\quad\quad f = ...$$
$$\quad\quad g = ...$$

Note that the strictness implied by these two operations does not equal *complete* evaluation. They only force to weak-head-normal-form (WHNF) meaning that evaluation is only forced until the outermost constructor in contrast to normal-form (NF) which stands for full evaluation. This means that if we were to evaluate some calculation $f\ (g\ (h\ (i\ x)))$ embedded in some lazy tuple (y, z) to WHNF, y and z would not be touched as the evaluation stops at the tuple constructor (for more about constructors see the chapter on custom types). All the computations that lead to this constructor however, are forced to be evaluated. Therefore, if we want to make the insides of a tuple strict, we would have to write something along the lines of

$$\textbf{let}\ tup\ @\ (y, z) = f\ (g\ (h\ (i\ x)))\ \textbf{in}\ y\ \text{`}seq\text{`}\ z\ \text{`}seq\text{`}\ tup$$

instead of just

$$\textbf{let}\ tup = f\ (g\ (h\ (i\ x)))\ \textbf{in}\ y\ \text{`}seq\text{`}\ y$$

But as *seq* and $! both only evaluate to WHNF, y and z might still not be completely evaluated, since they could be of some more complex type than just *Int* or any other primitive. This is the reason why in the Haskell eco system, there exists the library *deepseq*[8] which comes with the typeclass *NFData* defined as

> **class** *NFData a* **where**
>   *rnf* :: $a \rightarrow ()$

Instances of this typeclass for some type a are required to provide an appropriate implementation of *rnf* for *full* evaluation to normal-form, where *rnf* stands for "reduce-to-normal-form". With this we can then implement the NF equivalent to *seq*, *deepseq*, as

> *deepseq* :: $NFData\ a \Rightarrow a \rightarrow b \rightarrow b$
> *deepseq a* = *rnf a* `seq` *a*

A deep analogue to $! is then easily definable as well as

> ($!!) :: $NFData\ a \Rightarrow (a \rightarrow b) \rightarrow a \rightarrow b$
> *f* $!! *x* = *x* `deepseq` *f x*

When dealing with WHNF and NF, note that in all computations annotated with some forcing construct, be it *seq* or *deepseq*, laziness does not go away entirely. All forced values, even the ones forced to NF, can still be considered somewhat lazy as they are only forced when they are requested. However, in practice, this is usually a desired property.

Pattern Matching

While we have seen pattern matching as an alternative to **if** ... **then** ... **else** and guard statements, it can do more things. For example, if we have a datatype *MyType a* defined as

> **data** *MyType a* = *SomeConstructor a* | *SomeOtherConstructor a*

and we want to write a function *unwrap* :: $MyType\ a \rightarrow a$ to unwrap the a value, we use pattern matching like this:

> *unwrap* :: $MyType\ a \rightarrow a$
> *unwrap* (*SomeConstructor x*) = *x*
> *unwrap* (*SomeOtherConstructor x*) = *x*

---

[8]See `haskell.org/package/deepseq-1.4.3.0/docs/Control-DeepSeq.html`.

This kind of unwrapping can also be done with the help of **case** statements so that we do not require a new function definition:

$$someFunc :: MyType\ a \to a$$
$$someFunc\ t = \textbf{case}\ t\ \textbf{of}$$
$$(SomeConstructor\ a) \to ...$$
$$(SomeOtherConstructor\ a) \to ...$$

In Haskell programs we can also write unwrapping code with the help of the **let** notation for single constructor types like **let** $(x, y) = vec2d$ **in** $sqrt\ (x * x + y * y)$. Predictably, we can do this with **where** as well with constructs like **where** $(x, y) = vec2d$.

Sometimes we only care about some part of the value. For example, in a definition of $maybeHead :: [a] \to Maybe\ a$, which should return the first element of the list or $Nothing$ if it is an empty list $[\ ]$, we can write this with the help of wildcards (_) as:

$$maybeHead :: [a] \to Maybe\ a$$
$$maybeHead\ (x : \_) = Just\ x$$
$$maybeHead\ [\ ] = Nothing$$

We could even write

$$maybeHead :: [a] \to Maybe\ a$$
$$maybeHead\ (x : \_) = Just\ x$$
$$maybeHead\ \_ = Nothing$$

as the second equation will only ever match when the list is empty.[9] Furthermore, functions $isJust :: Maybe\ a \to Bool$, if they only care about the structure of the type, can be written with only wildcards:

$$isJust :: Maybe\ a \to Bool$$
$$isJust\ (Just\ \_) = True$$
$$isJust\ \_ = False$$

Additionally, if we want to preserve laziness and we are sure that a match will work (e.g. if we have called $isJust$), we can use irrefutable patterns like $\sim(Just\ a) = someMaybe$.

Lambdas and Partial application

As Functions are just another type that can be passed into higher-order functions it makes sense to have a short-hand to write anonymous functions – lambdas. In Haskell they look like this:

---

[9]A single element list has two forms in Haskell, $a : [\ ]$ and $[a]$ of which the latter is just syntactic sugar of the former.

$$\lambda(a, b) \rightarrow a + b$$

Lambdas can easily be passed into functions, like $zipWith$[10]:

$$someFunc :: [Int] \rightarrow [Int] \rightarrow [Int]$$
$$someFunc \ xs \ ys = zipWith \ (\lambda(a, b) \rightarrow a + b) \ xs \ ys$$

Here, we notice the reason for yet another feature in Haskell that is commonly used: Partial application. While the definition of $someFunc$ is definitely not wrong, we could have written it more elegantly as

$$someFunc :: [Int] \rightarrow [Int] \rightarrow [Int]$$
$$someFunc = zipWith \ (\lambda(a, b) \rightarrow a + b)$$

where we this means that $someFunc$ is defined as $zipWith :: (a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c])$ partially applied with the passed lambda to get a function with type $[a] \rightarrow [b] \rightarrow [c]$ which the compiler then automatically binds to the type of $someFunc :: [Int] \rightarrow [Int] \rightarrow [Int]$.

3.1.3 Monads

Functional programmers try to avoid mutable state at all cost, but programs that do not only just compute some function usually involve some sort of it. So, doesn't this make Haskell useless being be a pure functional language without *any* mutable state? No. Functional Programs generally just avoid *unnecessary* mutable state at all cost. The fact of the matter is that in functional programming, we can represent mutable state as well, but we do so in a meaningful and controlled manner.

While in most computations, we could represent state by passing it into every function that can possibly change it and returning it alongside of the actual returned value like

$$comp :: MyState \rightarrow Int \rightarrow (Int, MyState)$$
$$comp \ curState \ x = (x + 3, nextState)$$
$$\quad \textbf{where} \ nextState = changeState \ curState$$

this can become unnecessarily complicated to handle by hand. A better alternative is the use of Monads, which are the main concept generally used in computations involving some sort of mutable state. The type class for a $Monad$ can be defined as

$$\textbf{class} \ Monad \ m \ \textbf{where}$$
$$(\ggg) :: m \ a \rightarrow (a \rightarrow m \ b) \rightarrow m \ b$$

---

[10]While $(\lambda(a, b) \rightarrow a + b)$ is obviously the same as $(+)$, we just write it as a lambda here for demonstration purposes

$$(\gg) :: m\ a \to m\ b \to m\ b$$
$$m \gg k = m \ggeq \backslash\_ \to k$$
$$return :: a \to m\ a$$

Thinking of Monads as computations, we can come up with the following explanation: *return* is used to create a computation $m\ a$ just returning the given value a. Next, $(\ggeq)$ is used to compose some monadic computation $m\ a$ resulting in some a with a monadic function $a \to m\ b$ to return some computation $m\ b$ resulting in some b. Finally, $(\gg)$ is used to define the order of two monadic computations $m\ a$ and $m\ b$ so that $m\ a$ is computed before $m\ b$ while discarding the result of the first one as can also be seen in its default implementation above.

Given this definition of a Monad, we can now take a look at how we would implement a *State* Monad. Its type is defined as (Michaelson, 2013)

newtype $State\ s\ a = State\ \{\ runState :: s \to (a, s)\ \}$

State $s\ a$ encapsulates a stateful computation on some state type s yielding some value of type a. For easier understanding it is often useful to think of *State* $s\ a$ as a usability wrapper around a function $s \to (a, s)$ that returns some a and the final state s if we pass it some starting state s. The State Monad therefore merely contains the "blueprint" of the computation that can only be run if we start it by passing a state. The instance for the *Monad* type class can then be defined as

instance $Monad\ (State\ s)$ **where**
$\quad (State\ h) \ggeq f = State\ \$\ \lambda s \to$ **let** $(a, newState) = h\ s$
$\qquad (State\ g) = f\ a$
$\quad$ **in** $g\ newState$
$\quad return\ x = State\ \{\ runState = \lambda s \to (x, s)\ \}$

Here, we declare the instance deliberately on top of *State* s meaning that *State* itself is not a Monad, but it is a one together with some state representation s.[11] Note how the operations are defined here: *return* encapsulates the given value $x :: a$ inside the internal function and therefore is equal to the identity $id :: a \to a$ function on tuples with one parameter already applied.[12] In the composition operator $\ggeq$, the monadic computation $State\ h :: State\ s\ a$ is composed with the function $f :: a \to State\ s\ b$ into a new monadic computation of type $State\ s\ b$. The internal function of the state is essentially taken out of the first argument and composed with the second argument inside the returned Monad.

Additionally, we can define helper operations to use this construct with. The first one is $put :: s \to State\ s\ ()$. It overwrites the current state returning a unit () as result:

---

[11]We can't declare *State* a Monad anyways since *Monad* is a type class with just one type parameter
[12]With the help of $curry :: ((a, b) \to c) \to a \to b \to c$, we could have therefore also written $return\ x = State\ \{\ runState = (curry\ id)\ x\ \}$

$$put :: s \rightarrow State\ s\ ()$$
$$put\ newState = State\ \{\ runState = \lambda s \rightarrow ((),\ newState)\ \}$$

The second one is $get :: State\ s\ s$. It returns the current state, but does not change it:

$$get :: State\ s\ s$$
$$get = State\ \{\ runState = \lambda s \rightarrow (s,\ s)\ \}$$

With these operations, we can easily write stateful programs like this one:[13]

```
type Stack = [Int]
empty :: Stack
empty = []
pop :: State Stack Int
pop = get ≫= (λ(x : xs) → put xs ≫ return x)
push :: Int → State Stack ()
push a = State $ λxs → ((), a : xs)
peek :: State Stack Int
peek = get ≫= λ(x : xs) → return x
computeStateful :: State Stack Int
computeStateful = push 10 ≫
   push 20 ≫
   pop ≫= λa →
     (pop ≫= λb → push (a + b)) ≫
   peek
   -- main program inside the IO Monad
main :: IO ()
main = print (evalState computeStateful empty)
```

Here, *computeStateful* first pushes some values on top of a stack represented by a list $[Int]$ (the actual state inside of the *State* Monad) and then *pop*s these values and *push*es their sum back on the stack. Finally, we *peek* the top of our stack. This is then the result of the computation. To make writing such code easier, Haskell has syntactic sugar: The **do** notation. With it we can write the above method *computeStateful* in a way that resembles imperative-style code (but with side-effects clearly encapsulated) as:

```
computeStateful :: State Stack Int
computeStateful = do
   push 10
   push 20
```

---

[13]Inspired and adapted from `https://gist.github.com/sdiehl/8d991a718f7a9c80f54b`.

$$a \leftarrow pop$$
$$b \leftarrow pop$$
$$push \ (a + b)$$
$$peek$$

Here, we can also see the duality of ($\gg$) and simple new lines as well as the one between ($\ggg$) and the special $\leftarrow$ operator in **do** notation which facilitates the binding to a variable.[14]

Other often used Monads in the Haskell eco-system include the *Writer* Monad, which is useful for e.g. logging, or the *IO* Monad, which is used to encapsulate I/O computations as well as low level internal operations such as the usage of modifiable variables *IORef* or *MVar* among others. Furthermore, as one of many other applications, Monads are used in some parallel Haskells as we will see later in this thesis.

3.1.4 Arrows

Arrows were introduced by Hughes (2000) as a general interface for computation and a less restrictive generalisation of Monads. Hughes (2000) motivates the broader interface of Arrows with the example of a parser with added static meta-information that can not satisfy the monadic bind operator ($\ggg$) :: $Monad \ m \Rightarrow m \ a \rightarrow (a \rightarrow m \ b) \rightarrow m \ b$.[15]

An Arrow $arr \ a \ b$ represents a computation that converts an input a to an output b. The general concept is defined in the *Arrow* type class shown in Figure 3.3. To lift an ordinary function to an Arrow, arr is used, analogous to the monadic *return*. Similarly, the composition operator $\ggg$ is analogous to the monadic composition $\ggg$ and combines two Arrows $arr \ a \ b$ and $arr \ b \ c$ by "wiring" the outputs of the first to the inputs to the second to get a new Arrow $arr \ a \ c$. Lastly, the *first* operator takes the input Arrow $arr \ a \ b$ and converts it into an Arrow on pairs $arr \ (a, c) \ (b, c)$ that leaves the second argument untouched. It allows us to to save input across Arrows. Figure 3.4 shows a graphical representation of these basic Arrow combinators. The most prominent instances of this interface (Figure 3.3) are regular functions ($\rightarrow$) and the Kleisli type, which wraps monadic functions, e.g. $a \rightarrow m \ b$.[16]

---

[14]($\ggg$) is also often called *bind* in languages which do not support operator overloading.

[15]In the example a parser of the type *Parser s a* with static meta information s and result a is shown to not be able to use the static s without applying the monadic function $a \rightarrow m \ b$. With Arrows this is possible.

[16]In 2.3 we referenced further relevant Arrow types, especially ones that can be used for Arrow-based functional reactive programming.

```
class Arrow arr where
    arr :: (a → b) → arr a b
    (⋙) :: arr a b → arr b c → arr a c
    first :: arr a b → arr (a, c) (b, c)
instance Arrow (→) where
    arr f = f
    f ⋙ g = g ∘ f
    first f = λ(a, c) → (f a, c)
data Kleisli m a b = Kleisli { run :: a → m b }
instance Monad m ⇒ Arrow (Kleisli m) where
    arr f = Kleisli (return ∘ f)
    f ⋙ g = Kleisli (λa → f a ⋙= g)
    first f = Kleisli (λ(a, c) → f a ⋙= λb → return (b, c))
```

Figure 3.3: The *Arrow* type class and its two most typical instances.
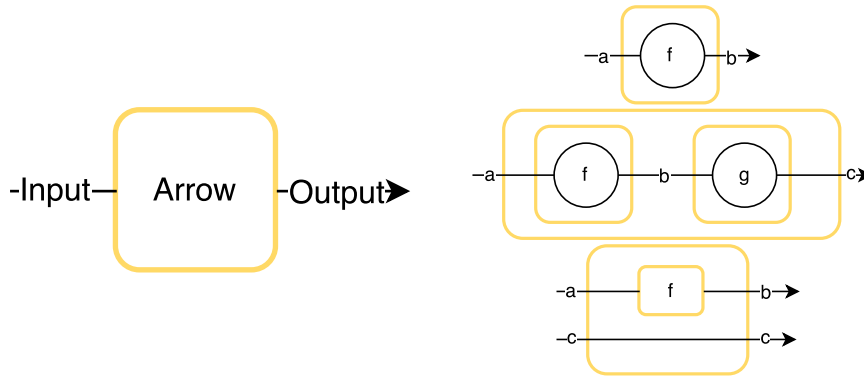
Figure 3.4: Schematic depiction of an Arrow (left) and its basic combinators arr, $⋙$ and *first* (right).

Hughes also defined some syntactic sugar (Figure 3.5): *second*, $***$ and &&&. *second* is the mirrored version of *first*:

$$second :: Arrow\ arr ⇒ arr\ a\ b → arr\ (c, a)\ (c, b)$$
$$second\ f = arr\ swap ⋙ first\ f ⋙ arr\ swap$$
$$\textbf{where}\ swap\ (x, y) = (y, x)$$

The $***$ function combines *first* and *second* to handle two inputs in one Arrow, and is defined as follows:

$$(***) :: Arrow\ arr ⇒ arr\ a\ b → arr\ c\ d → arr\ (a, c)\ (b, d)$$
$$f *** g = first\ f ⋙ second\ g$$

The &&& combinator, which constructs an Arrow that outputs two different values like $***$, but takes only one input, is:
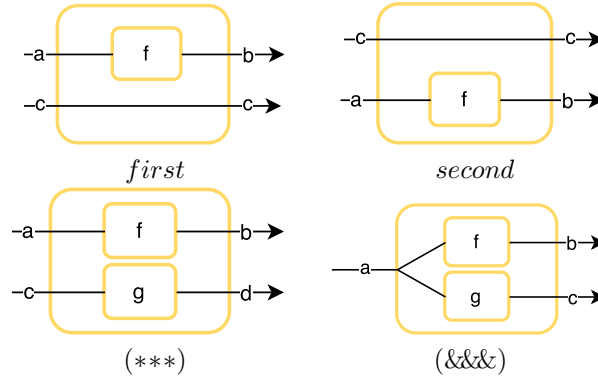
Figure 3.5: Visual depiction of syntactic sugar for Arrows.

$$(\&\&\&) :: Arrow\ arr \Rightarrow arr\ a\ b \to arr\ a\ c \to arr\ a\ (b, c)$$
$$f\ \&\&\&\ g = arr\ (\lambda a \to (a, a)) \ggg f *** g$$

A first short example given by Hughes on how to use the Arrow interface is the addition of results of two generic Arrows to get a new Arrow:

$$add :: Arrow\ arr \Rightarrow arr\ a\ Int \to arr\ a\ Int \to arr\ a\ Int$$
$$add\ f\ g = f\ \&\&\&\ g \ggg arr\ (\lambda(u, v) \to u + v)$$

As we can rewrite the monadic bind operation ($\ggg\!\!=$) with only the Kleisli type into $m\ a \to Kleisli\ m\ a\ b \to m\ b$, but not with a general Arrow $arr\ a\ b$, we can intuitively get an idea of why Arrows must be a generalisation of Monads. While this also means that a general Arrow can not express everything a Monad can, Hughes (2000) shows in his parser example that this trade-off is worth it in some cases.

Utility Combinators

In order to ease the use of Arrows, we will now define some utility Arrow combinators, namely $evalN$ as well as $mapArr$. $evalN$, which turns a list of Arrows $[arr\ a\ b]$ into a new Arrow $arr\ [a]\ [b]$ evaluating a list of inputs $[a]$ against these Arrows is defined in Figure 3.6

$$evalN :: (ArrowChoice\ arr) \Rightarrow [arr\ a\ b] \to arr\ [a]\ [b]$$
$$evalN\ (f : fs) = arr\ listcase \ggg$$
$$\quad arr\ (const\ [\,]) \parallel (f *** evalN\ fs \ggg arr\ (uncurry\ (:)))$$
$$\quad \textbf{where}\ listcase\ [\,] = Left\ ()$$
$$\quad\quad listcase\ (x : xs) = Right\ (x, xs)$$
$$evalN\ [\,] = arr\ (const\ [\,])$$

Figure 3.6: The definition of $evalN$.

Next, we have the $mapArr$ combinator (Figure 3.7). It lifts any Arrow $arr\ a\ b$ to an Arrow $arr\ [a]\ [b]$. The original inspiration was from Hughes (2005), but the definition was then unified with $evalN$.

$$mapArr :: ArrowChoice\ arr \Rightarrow arr\ a\ b \rightarrow arr\ [a]\ [b]$$
$$mapArr = evalN \circ repeat$$

Figure 3.7: The definition of map over Arrows.

These combinators make use of the $ArrowChoice$ type class providing the $\|$ combinator. This combinator takes two Arrows $arr\ a\ c$ and $arr\ b\ c$ and combines them into a new Arrow $arr\ (Either\ a\ b)\ c$ which pipes all $Left\ a$'s to the first Arrow and all $Right\ b$'s to the second Arrow:

$$(\|) :: ArrowChoice\ arr \Rightarrow arr\ a\ c \rightarrow arr\ b\ c \rightarrow arr\ (Either\ a\ b)\ c$$

One thing we can see from these utility Arrows is how easily we can define generic Arrow code that will work on any sufficient specific Arrow. This is exactly why we aim to allow for parallelisation of programs via Arrows. We want to empower such generic code to be easily parallelised so these generic types of APIs are better to use.

Notes

In this thesis we will show that parallel computations can be expressed with this more general interface of Arrows without requiring Monads (we will see an example of monadic parallelism in Chapter 3.2). We also do not restrict the compatible Arrows to ones which have $ArrowApply$ instances – as every Arrow that has a $ArrowApply$ instance gives rise to a Monad – but instead only require instances for $ArrowChoice$ (for the if-then-else construction in $evalN$ (Figure 3.6)) and $ArrowLoop$ (for the looping used in the topological skeletons in Chapter 5.2). Because of this, we have a truly more general interface when compared to a monadic one or a purely function $(\rightarrow)$ based one.

While we could have based our DSL on Profunctors[17] as well, we chose Arrows in this thesis since they allow for a more direct way of thinking about parallelism than general Profunctors because of their composability. However, they are a promising candidate for future improvements of our DSL. Some Profunctors, especially ones supporting a composition operation, choice, and looping, can already be adapted to our interface as shown in Appendix 10.1.

---

[17]See `http://hackage.haskell.org/package/profunctors-5.3/docs/Data-Profunctor.html` for more information as well as the Haskell interface.
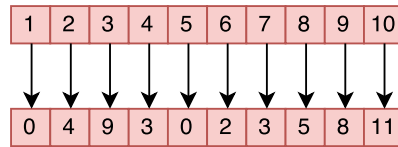
Figure 3.8: Schematic illustration of $parEvalN$. A list of inputs is transformed by different functions in parallel.

3.2 Short introduction to parallel Haskells

In Chapter 3.1, we cited Hughes (1990) saying that in functional programming, the order of evaluation is irrelevant. In parallel programs this is not the case, as at least some kind of structure of evaluation is required to have actual speedup in programs. In the following we will take a look at how parallelism can be achieved in Haskell programs in general. Now, one might think that we would want side effects (parallel evaluation is a side-effect) and require the need to think about order of evaluation in a pure functional program seems a bit odd. The fact of the matter is that functional programs only aim to avoid *unnecessary* side-effects and in the case of parallelism it is obvious that some amount of side-effects are required. Also, parallel Haskells generally aim to encapsulate all the necessary and complicated code in a way such that the room for code-breaking errors is almost impossible. If some parallel evaluation code is written in a sub-optimal way, only the performance is affected, but not the result, which will always be tractable no matter the order of evaluation.[18]

In its purest form, parallel computation (on functions) can be looked at as the execution of some functions $a \to b$ in parallel or $parEvalN :: [a \to b] \to [a] \to [b]$, as also Figure 3.8 symbolically shows. In this Chapter, we will implement this non-Arrow version which will later be adapted for usage in our Arrow-based parallel Haskell.

There exist several different parallel Haskells already. As already stated in the Introduction this is the reason why we base our efforts on existing work which we wrap as backends behind a common interface so that we do not re-implement yet another parallel runtime. As described earlier, we here focus on three of the most important ones – GpH (Trinder et al., 1996, 1998, based on par and $pseq$ "hints"), the Par Monad (Foltzer et al., 2012, a Monad for deterministic parallelism; Marlow et al., 2011) and Eden (Loogen, 2012, a parallel Haskell for distributed memory; Loogen et al., 2005).

---

[18] Some exceptions using unsafe and non-deterministic operations exist, though. These situations can however only be achieved if the programmer actively chooses to use these kinds of operations.

We will now go into some detail on these parallel Haskells, and also give their respective implementations of the non-Arrow version of $parEvalN$. Chapter 3.2.1 covers GpH, while Chapters 3.2.2 and Chapter 3.2.3 explain the Par Monad and Eden, respectively.

3.2.1 Glasgow parallel Haskell – GpH

GpH (Marlow et al., 2009; Trinder et al., 1998) is one of the simplest ways to do parallel processing found in standard GHC.[19] Besides some basic primitives,[20] it already ships with parallel evaluation strategies for several types which can be applied with $using :: a \rightarrow Strategy\ a \rightarrow a$. This is exactly what is required for an implementation of $parEvalN$.

$$parEvalN :: (NFData\ b) \Rightarrow [\,a \rightarrow b\,] \rightarrow [\,a\,] \rightarrow [\,b\,]$$
$$parEvalN\ fs\ as = \mathbf{let}\ bs = zipWith\ (\$)\ fs\ as$$
$$\mathbf{in}\ bs\ `using`\ parList\ rdeepseq$$

In the above definition of $parEvalN$ we just apply the list of functions $[\,a \rightarrow b\,]$ to the list of inputs $[\,a\,]$ by zipping them with the application operator $\$$. We then evaluate this lazy list $[\,b\,]$ according to a $Strategy\ [\,b\,]$ with the $using :: a \rightarrow Strategy\ a \rightarrow a$ operator. We construct this strategy with $parList :: Strategy\ a \rightarrow Strategy\ [\,a\,]$ and $rdeepseq :: NFData\ a \Rightarrow Strategy\ a$ where the latter is a strategy which evaluates to normal form. Other strategies like e.g. evaluation to weak head normal form are available as well. It also allows for custom $Strategy$ implementations to be used. Figure 3.9 shows a visual representation of this code.

---

[19]The Multicore implementation of GpH is available on Hackage under `https://hackage.haskell.org/package/parallel-3.2.1.0`, compiler support is integrated in the stock GHC.

[20]$par :: a \rightarrow b \rightarrow b$ to evaluate a and b in parallel and $pseq :: a \rightarrow b \rightarrow b$, a special variant of seq that allows enforcing of parallel evaluation.
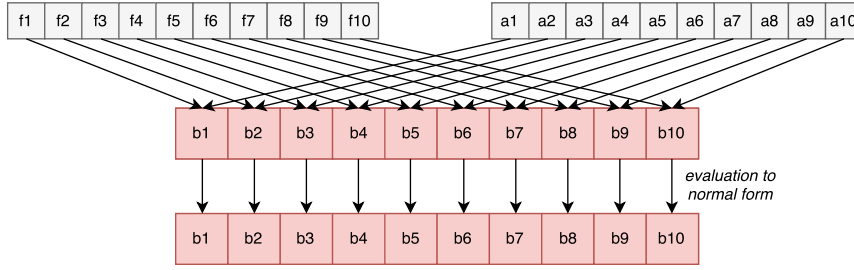
Figure 3.9: *parEvalN* (GpH).

3.2.2 *Par* Monad

The *Par* Monad[21] introduced by (Marlow et al., 2011), is a Monad designed for composition of parallel programs:

$$myComp :: Par\ (a, b)$$
$$myComp = \mathbf{do}$$
$$\quad fx \leftarrow spawn\ \$\ return\ (f\ x) \quad \text{-- start evaluating (f x)}$$
$$\quad gx \leftarrow spawnP\ \$\ return\ (g\ x) \quad \text{-- start evaluating (g x)}$$
$$\quad a \leftarrow get\ fx \quad \text{-- wait for fx}$$
$$\quad b \leftarrow get\ gx \quad \text{-- wait for gx}$$
$$\quad return\ (a, b) \quad \text{-- return results}$$

We, however, do not need its composition features and only use its parallel backend in our definition of *parEvalN*:

$$parEvalN :: (NFData\ b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$$
$$parEvalN\ fs\ as = runPar\ \$$$
$$\quad (sequenceA\ (map\ (return \circ spawn)\ (zipWith\ (\$)\ fs\ as))) \ggg mapM\ get$$

This *Par* Monad version of our parallel evaluation function *parEvalN* is defined by zipping the list of $[a \rightarrow b]$ with the list of inputs $[a]$ with the application operator $ just like with GpH. Then, we map over this not yet evaluated lazy list of results $[b]$ with $spawn :: NFData\ a \Rightarrow Par\ a \rightarrow Par\ (IVar\ a)$ to transform them to a list of not yet evaluated forked away computations $[Par\ (IVar\ b)]$, which we convert to $Par\ [IVar\ b]$ with *sequenceA*. We wait for the computations to finish by mapping over the *IVar b* values inside the *Par* Monad with *get*. This results in $Par\ [b]$. We execute this process with *runPar* to finally get the fully evaluated list of results$[b]$. While we used *spawn* in the definition above, a head-strict variant can easily be defined by replacing *spawn* with $spawn\_ :: Par\ a \rightarrow Par\ (IVar\ a)$. Figure 3.10 shows a graphical representation.

---

[21]The *Par* Monad can be found in the $monad-par$ package on Hackage under `https://hackage.haskell.org/package/monad-par-0.3.4.8/`.
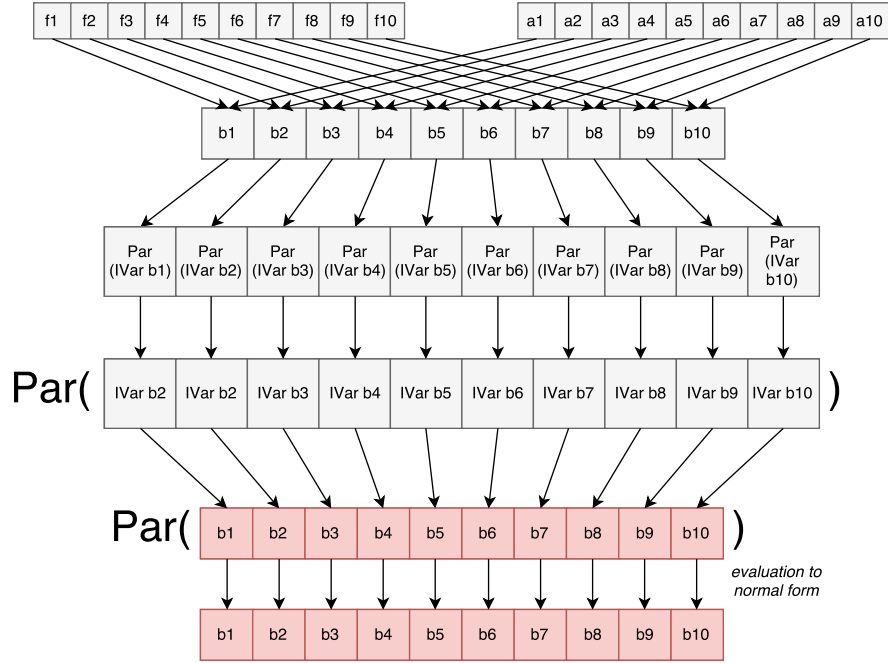
Figure 3.10: *parEvalN* (*Par* Monad).

3.2.3 Eden

Eden (Loogen, 2012; Loogen et al., 2005) is a parallel Haskell for distributed memory and allows for MPI and PVM as distributed backends.[22] It is targeted towards clusters, but also functions well in a shared-memory setting with a further simple backend. However, in contrast to many other parallel Haskells, in Eden each process has its own heap. This seems to be a waste of memory, but with distributed programming paradigm and individual GC per process, Eden yields good performance results on multicores, as well (Aswad et al., 2009; Berthold et al., 2009a).

While Eden comes with a Monad *PA* for parallel evaluation, it also ships with a completely functional interface that includes a $spawnF :: (Trans\ a, Trans\ b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$[23] function that allows us to define *parEvalN* directly:

$$parEvalN :: (Trans\ a, Trans\ b) \Rightarrow [a \rightarrow b] \rightarrow [a] \rightarrow [b]$$
$$parEvalN = spawnF$$

---

[22]The projects homepage can be found at `http://www.mathematik.uni-marburg.de/~eden/`. The Hackage page is at `https://hackage.haskell.org/package/edenmodules-1.2.0.0/`.

[23]The type class *Trans* stands for Transmissible. The Eden library already has instances for most common types and allows for easy construction of further instances.

Eden TraceViewer

To comprehend the efficiency or the lack thereof in a parallel program, an inspection of its execution is extremely helpful. While some large-scale solutions exist (Geimer et al., 2010), the parallel Haskell community mainly utilises the tools Threadscope (Wheeler and Thain, 2009) and Eden TraceViewer[24] (Berthold and Loogen, 2007). In the next Chapters we will present some *trace visualisations*, the post-mortem process diagrams of Eden processes and their activity.

The trace visualisations are colour-coded. In such a visualisation (Figure 5.1), the x axis shows the time, the y axis enumerates the machines and processes. The visualisation shows a running process in green, a blocked process is red. If the process is "runnable", i.e. it may run, but does not, it is yellow. The typical reason for this is garbage collection (GC). An inactive machine, where no processes are started yet, or all are already terminated, shows as a blue bar. A communication from one process to another is represented with a black arrow. A stream of communications, e.g. a transmitted list is shows as a dark shading between sender and receiver processes.

---

[24]See `http://hackage.haskell.org/package/edentv` on Hackage for the last available version of Eden TraceViewer.

Parallel Arrows

<div style="text-align: right; font-size: 3em;">4</div>

Having discussed the idea of Arrows as well as the basics of the APIs we wish to use as backends, we can now discuss the design and implementation of the actual PArrows DSL. We present the *ArrowParallel* type class and explain the reasoning behind it in Chapter 4.1 before discussing its implementations in GpH, the *Par* Monad and Eden in Chapter 4.2. Then, we give first basic extensions in Chapter 4.3. Finally, we explain basic *map*-based skeletons in Chapter 4.4.

4.1 The *ArrowParallel* type class

A parallel computation (on functions) can be seen as the execution of some functions $a \to b$ in parallel, as our *parEvalN* prototype shows (Chapter 3.2). Translating this into Arrow terms gives us a new operator *parEvalN* that lifts a list of Arrows $[arr\ a\ b]$ to a parallel Arrow $arr\ [a]\ [b]$. This combinator is similar to the evaluation combinator $evalN :: [arr\ a\ b] \to arr\ [a]\ [b]$ from Chapter 3.1.4, but does parallel instead of serial evaluation:

$$parEvalN :: (Arrow\ arr) \Rightarrow [arr\ a\ b] \to arr\ [a]\ [b]$$

With such a definition of *parEvalN*, parallel execution is yet another Arrow combinator. But since the implementation may differ depending on the actual type of the Arrow arr – or even the input a and output b – and we want this to be an interface for different backends that we should be able to switch between, we introduce a new type class *ArrowParallel arr a b*.

class $Arrow\ arr \Rightarrow ArrowParallel\ arr\ a\ b$ **where**
$\quad parEvalN :: [arr\ a\ b] \to arr\ [a]\ [b]$

Sometimes parallel Haskells require or allow additional configuration parameters, e.g. information about the execution environment or the level of evaluation (WHNF vs. NF, see the section on laziness in Chapter 3.1.2). For this reason we introduce an additional *conf* parameter as we do not want *conf* to be a fixed type, as the configuration parameters can differ for different instances of *ArrowParallel*.

class $Arrow\ arr \Rightarrow ArrowParallel\ arr\ a\ b\ conf$ **where**
$\quad parEvalN :: conf \to [arr\ a\ b] \to arr\ [a]\ [b]$

By restricting the implementations of our backends to a specific *conf* type, we also get interoperability between backends for free as it serves as a discriminator for which backend has to be used. We can therefore parallelise one part of a program using one backend, and do the same for the next but with another one by just passing a different configuration type.

4.2 *ArrowParallel* instances

With the *ArrowParallel* type class defined, we will now give implementations of it with GpH (Chapter 4.2.1), the *Par* Monad (Chapter 4.2.2) and Eden (Chapter 4.2.3). Finally, we explain default configuration instances in Chapter 4.2.4.

4.2.1 Glasgow parallel Haskell

The GpH instance of *ArrowParallel* is implemented in a straightforward manner in Figure 4.1, but a bit different compared to the variant from Chapter 3.2.1. We use $evalN :: [arr\ a\ b] \to arr\ [a]\ [b]$ (definition in Appendix 3.1.4, think *zipWith* ($) on Arrows) combined with $withStrategy :: Strategy\ a \to a \to a$ from GpH, where *withStrategy* is the same as $using :: a \to Strategy\ a \to a$, but with flipped parameters. Our *Conf a* datatype simply wraps a *Strategy a*, but could be extended in future versions of our DSL.

$$
\begin{aligned}
&\textbf{data}\ Conf\ a = Conf\ (Strategy\ a) \\
&\textbf{instance}\ (ArrowChoice\ arr) \Rightarrow \\
&\quad ArrowParallel\ arr\ a\ b\ (Conf\ b)\ \textbf{where} \\
&\quad\quad parEvalN\ (Conf\ strat)\ fs = \\
&\quad\quad\quad evalN\ fs \ggg \\
&\quad\quad\quad arr\ (withStrategy\ (parList\ strat))
\end{aligned}
$$

Figure 4.1: GpH *ArrowParallel* instance.

4.2.2 *Par* Monad

As for GpH we can easily lift the definition of *parEvalN* for the *Par* Monad to Arrows in Figure 4.2. To start off, we define the *Strategy a* and *Conf a* type so we can have a configurable instance of ArrowParallel:

$$
\begin{aligned}
&\textbf{type}\ Strategy\ a = a \to Par\ (IVar\ a) \\
&\textbf{data}\ Conf\ a = Conf\ (Strategy\ a)
\end{aligned}
$$

Now we can once again define our *ArrowParallel* instance as follows: First, we convert our Arrows [*arr a b*] with *evalN* (*map* (≫ *arr strat*) *fs*) into an Arrow *arr* [*a*] [(*Par* (*IVar b*))] that yields composable computations in the *Par* Monad. By combining the result of this Arrow with *arr sequenceA*, we get an Arrow *arr* [*a*] (*Par* [*IVar b*]). Then, in order to fetch the results of the different threads, we map over the *IVars* inside the *Par* Monad with *arr* (≫= *mapM get*) – our intermediary Arrow is of type *arr* [*a*] (*Par* [*b*]). Finally, we execute the computation *Par* [*b*] by composing with *arr runPar* and get the final Arrow *arr* [*a*] [*b*].

> **type** *Strategy a* = *a* → *Par* (*IVar a*)
> **data** *Conf a* = *Conf* (*Strategy a*)
> **instance** (*ArrowChoice arr*) ⇒ *ArrowParallel arr a b* (*Conf b*) **where**
>   *parEvalN* (*Conf strat*) *fs* =
>   *evalN* (*map* (≫ *arr strat*) *fs*) ≫
>   *arr sequenceA* ≫
>   *arr* (≫= *mapM Control.Monad.Par.get*) ≫
>   *arr runPar*

Figure 4.2: *Par* Monad *ArrowParallel* instance.

4.2.3 Eden

For both the GpH Haskell and *Par* Monad implementations we could use general instances of *ArrowParallel* that just require the *ArrowChoice* type class. With Eden this is not the case as we can only spawn a list of functions, which we cannot extract from general Arrows. While we could still manage to have only one instance in the module by introducing a type class

> **class** (*Arrow arr*) ⇒ *ArrowUnwrap arr* **where**
>   *unwrap* :: *arr a b* → (*a* → *b*)

we avoid doing so for aesthetic reasons. For now, we just implement *ArrowParallel* for normal functions and the Kleisli type in Figure 4.3, where *Conf* is simply defined as **data** *Conf* = *Nil* since Eden does not have a configurable *spawnF* variant.

Note that while writing this thesis, we found another solution that could be feasible: We could write the instance parEvalN as

> **instance** (*Trans b, ArrowChoice arr*) ⇒ *ArrowParallel arr a b Conf* **where**
>   *parEvalN* _ *fs* = *evalN fs* ≫ *arr* (*spawnF* (*repeat id*))

We were however, not able to prove that this behaves exactly the same as the variant presented above as this would have required re-running the whole test-suite. First tests suggest correct behaviour, though.

$$\textbf{instance } (\textit{Trans a, Trans b}) \Rightarrow \textit{ArrowParallel } (\rightarrow) \textit{ a b Conf } \textbf{where}$$
$$\qquad \textit{parEvalN } \_ = \textit{spawnF}$$
$$\textbf{instance } (\textit{ArrowParallel } (\rightarrow) \textit{ a } (m \textit{ b}) \textit{ Conf},$$
$$\quad \textit{Monad m, Trans a, Trans } (m \textit{ b})) \Rightarrow$$
$$\quad \textit{ArrowParallel } (\textit{Kleisli m}) \textit{ a b Conf } \textbf{where}$$
$$\qquad \textit{parEvalN conf fs} =$$
$$\qquad\quad \textit{arr } (\textit{parEvalN conf } (\textit{map } (\lambda(\textit{Kleisli f}) \rightarrow f) \textit{ fs})) \ggg$$
$$\qquad\quad \textit{Kleisli sequence}$$

Figure 4.3: Eden *ArrowParallel* instance.

4.2.4 Default configuration instances

While the configurability of the *ArrowParallel* instances above is nice, users probably would like to have proper default configurations for many parallel programs as well. These can also easily be defined as we can see by the example of the default implementation of *ArrowParallel* for GpH:

$$\textbf{instance } (\textit{NFData b, ArrowChoice arr, ArrowParallel arr a b } (\textit{Conf b})) \Rightarrow$$
$$\quad \textit{ArrowParallel arr a b } () \textbf{ where}$$
$$\qquad \textit{parEvalN } \_ \textit{ fs} = \textit{parEvalN } (\textit{defaultConf fs}) \textit{ fs}$$
$$\textit{defaultConf} :: (\textit{NFData b}) \Rightarrow [\textit{arr a b}] \rightarrow \textit{Conf b}$$
$$\textit{defaultConf fs} = \textit{stratToConf fs rdeepseq}$$
$$\textit{stratToConf} :: [\textit{arr a b}] \rightarrow \textit{Strategy b} \rightarrow \textit{Conf b}$$
$$\textit{stratToConf } \_ \textit{ strat} = \textit{Conf strat}$$

The other backends have similarly structured implementations which we do not discuss here for the sake of brevity. We can, however, only have one instance of *ArrowParallel arr a b* () present at a time, which should not be a problem, anyways.

Up until now we discussed Arrow operations more in detail, but in the following sections we focus more on the data-flow between the Arrows, now that we have seen that Arrows are capable of expressing parallelism. We nevertheless do explain new concepts in greater detail if required for better understanding.

4.3 Extending the interface

With the *ArrowParallel* type class in place, we can now define other parallel interface functions. These are basic algorithmic skeletons that are used to define more

sophisticated ones later in this thesis. Namely, these are a lazy variant of $parEvalN$ (Chapter 4.3.1) as well as a method to spawn heterogeneous tasks (Chapter 4.3.2).

4.3.1 Lazy $parEvalN$

The resulting Arrow of $parEvalN$ fully traverses the list of input Arrows as well as their inputs. Sometimes this might not be feasible, as it will not work on infinite lists of Arrows/functions like e.g. $map\ (arr \circ (+))\ [1\,..]$ or just because in case we need the Arrows evaluated in chunks. $parEvalNLazy$ (Figs. 4.4, 4.5) fixes this. It works by first chunking the input from $[a]$ to $[[a]]$ with the given $chunkSize$ in $arr\ (chunksOf\ chunkSize)$. These chunks are then fed into a list $[arr\ [a]\ [b]]$ of chunk-wise parallel Arrows with the help of our lazy and sequential $evalN$. The resulting $[[b]]$ is lastly converted into $[b]$ with $arr\ concat$.

Figure 4.4: $parEvalNLazy$ depiction.

$$\textbf{type}\ ChunkSize = Int$$

$$parEvalNLazy :: (ArrowParallel\ arr\ a\ b\ conf, ArrowChoice\ arr) \Rightarrow$$
$$conf \rightarrow ChunkSize \rightarrow [\,arr\ a\ b\,] \rightarrow (arr\ [\,a\,]\ [\,b\,])$$
$$parEvalNLazy\ conf\ chunkSize\ fs =$$
$$arr\ (chunksOf\ chunkSize) \ggg$$
$$evalN\ fchunks \ggg$$
$$arr\ concat$$
$$\textbf{where}$$
$$fchunks = map\ (parEvalN\ conf)\ (chunksOf\ chunkSize\ fs)$$

Figure 4.5: Definition of $parEvalNLazy$.

4.3.2 Heterogeneous tasks

We have only talked about the parallelization of Arrows of the same input and output types until now. But sometimes we want to parallelise heterogeneous types as well. We can implement such a $parEval2$ combinator (Figs. 4.6, 4.7) which combines two Arrows $arr\ a\ b$ and $arr\ c\ d$ into a new parallel Arrow $arr\ (a, c)\ (b, d)$ quite easily with the help of the $ArrowChoice$ type class. Here, the general idea is to use the $+\!+\!+$ combinator which combines two Arrows $arr\ a\ b$ and $arr\ c\ d$ and transforms them into $arr\ (Either\ a\ c)\ (Either\ b\ d)$ to get a common Arrow type that we can then feed into $parEvalN$.

We can implement this idea as follows: Starting off, we transform the (a, c) input into a two-element list $[\,Either\ a\ c\,]$ by first tagging the two inputs with $Left$ and $Right$ and wrapping the right element in a singleton list with $return$ so that we can combine them with $arr\ (uncurry\ (:))$. Next, we feed this list into a parallel Arrow running on two instances of $f +\!+\!+ g$. After the calculation is finished, we convert the resulting $[\,Either\ b\ d\,]$ into $([\,b\,], [\,d\,])$ with $arr\ partitionEithers$. The two lists in this tuple contain only one element each by construction, so we can finally just convert the tuple to (b, d) in the last step.

Figure 4.6: $parEval2$ depiction.

$$
\begin{aligned}
&parEval2 :: (ArrowChoice\ arr, \\
&\quad ArrowParallel\ arr\ (Either\ a\ c)\ (Either\ b\ d)\ conf) \Rightarrow \\
&\quad conf \rightarrow arr\ a\ b \rightarrow arr\ c\ d \rightarrow arr\ (a, c)\ (b, d) \\
&parEval2\ conf\ f\ g = \\
&\quad arr\ Left *\!*\!* (arr\ Right \ggg arr\ return) \ggg \\
&\quad arr\ (uncurry\ (:)) \ggg \\
&\quad parEvalN\ conf\ (replicate\ 2\ (f +\!\!+\!\!+ g)) \ggg \\
&\quad arr\ partitionEithers \ggg \\
&\quad arr\ head *\!*\!* arr\ head
\end{aligned}
$$

Figure 4.7: *parEval2* definition.

4.4 Basic *map*-based skeletons

Now we have developed Parallel Arrows far enough to define some basic, yet useful, algorithmic skeletons that abstract typical parallel computations – parallel *map*s. The essential differences between the skeletons presented here are in terms of order of evaluation and work distribution. They nevertheless still provide the same semantics as a sequential *map*. We discuss a basic parallel *map* and a lazy variant thereof (Chapter 4.4.1) as well as a statically load balancing parallel *map* (Chapter 4.4.2).

4.4.1 Parallel *map* and laziness

The *parMap* skeleton (Figs. 4.8, 4.10) is probably the most common skeleton for parallel programs. We can implement it with *ArrowParallel* by repeating an Arrow *arr a b* and then passing it into *parEvalN* to obtain an Arrow *arr* [*a*] [*b*].

$$
\begin{aligned}
&parMap :: (ArrowParallel\ arr\ a\ b\ conf) \Rightarrow \\
&\quad conf \rightarrow (arr\ a\ b) \rightarrow (arr\ [a]\ [b]) \\
&parMap\ conf\ f = parEvalN\ conf\ (repeat\ f)
\end{aligned}
$$

Figure 4.8: *parMap* definition.

Just like *parEvalN*, *parMap* traverses all input Arrows as well as the inputs. Because of this, it has the same restrictions as *parEvalN* as compared to *parEvalNLazy*. So it makes sense to also have a *parMapStream* (Figs. 4.9, 4.11) which behaves like *parMap*, but uses *parEvalNLazy* instead of *parEvalN*.

type *ChunkSize* = *Int*

parMapStream :: (*ArrowParallel arr a b conf*,
   ArrowChoice arr,
   ArrowApply arr) ⇒
   conf → *ChunkSize* → *arr a b* → *arr* [*a*] [*b*]
parMapStream conf chunkSize f = *parEvalNLazy conf chunkSize* (*repeat f*)

Figure 4.9: *parMapStream* definition.
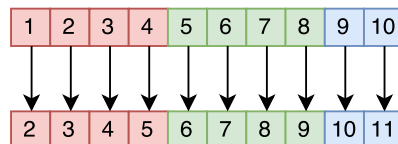

Figure 4.10: *parMap* depiction.

Figure 4.11: *parMapStream* depiction.

4.4.2 Statically load-balancing parallel *map*

Our *parMap* spawns every single computation in a new thread (at least for the instances of *ArrowParallel* we presented in this thesis). This can be quite wasteful and a statically load-balancing *farm* (Figs. 4.12, 4.14) that equally distributes the workload over *numCores* workers seems useful. The definitions of the helper functions *unshuffle*, *takeEach*, *shuffle* (Figure 10.2) originate from an Eden skeleton[1].

$$
\begin{aligned}
&\textbf{type } \mathit{NumCores} = \mathit{Int} \\
&\mathit{farm} :: (\mathit{ArrowParallel\ arr\ a\ b\ conf}, \\
&\quad \mathit{ArrowParallel\ arr\ [a]\ [b]\ conf}, \\
&\quad \mathit{ArrowChoice\ arr}) \Rightarrow \\
&\quad \mathit{conf} \rightarrow \mathit{NumCores} \rightarrow \mathit{arr\ a\ b} \rightarrow \mathit{arr\ [a]\ [b]} \\
&\mathit{farm\ conf\ numCores\ f} = \\
&\quad \mathit{unshuffle\ numCores} \ggg \\
&\quad \mathit{parEvalN\ conf\ (repeat\ (mapArr\ f))} \ggg \\
&\quad \mathit{shuffle}
\end{aligned}
$$

Figure 4.12: *farm* definition.

Since a *farm* is basically just *parMap* with a different work distribution, it has the same restrictions as *parEvalN* and *parMap*. We can, however, define *farmChunk* (Figs. 4.13, 4.15) which uses *parEvalNLazy* instead of *parEvalN*. Its definition is identical to the one for *farm* apart from the use of *parEvalNLazy* instead of *parEvalN*.

$$
\begin{aligned}
&\textbf{type } \mathit{ChunkSize} = \mathit{Int} \\
&\textbf{type } \mathit{NumCores} = \mathit{Int} \\
&\mathit{farmChunk} :: (\mathit{ArrowParallel\ arr\ a\ b\ conf}, \\
&\quad \mathit{ArrowParallel\ arr\ [a]\ [b]\ conf}, \\
&\quad \mathit{ArrowChoice\ arr}, \\
&\quad \mathit{ArrowApply\ arr}) \Rightarrow \\
&\quad \mathit{conf} \rightarrow \mathit{ChunkSize} \rightarrow \mathit{NumCores} \rightarrow \mathit{arr\ a\ b} \rightarrow \mathit{arr\ [a]\ [b]} \\
&\mathit{farmChunk\ conf\ chunkSize\ numCores\ f} = \\
&\quad \mathit{unshuffle\ numCores} \ggg \\
&\quad \mathit{parEvalNLazy\ conf\ chunkSize\ (repeat\ (mapArr\ f))} \ggg \\
&\quad \mathit{shuffle}
\end{aligned}
$$

Figure 4.13: *farmChunk* definition.

---

[1]Available on Hackage under `https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/src/Control-Parallel-Eden-Map.html`.

Figure 4.14: *farm* depiction.

Figure 4.15: *farmChunk* depiction.

Further development of Parallel Arrows

<div align="right">

5

</div>

With the basic PArrows API in place, we will in this Chapter develop the API even further. In Chapter 5.1 we introduce the concept of Futures that allows for direct inter-process communication. Then, in Chapter 5.2 we use this concept to define more sophisticated topological skeletons, namely a *pipe*, a *ring* and a *torus*.

5.1 Futures

Consider the following outline parallel Arrow combinator:

$$outlineCombinator :: ($$
$$ArrowParallel\ arr\ a\ b\ (),$$
$$ArrowParallel\ arr\ b\ c\ ()) \Rightarrow$$
$$[\,arr\ a\ b\,] \rightarrow [\,arr\ b\ c\,] \rightarrow arr\ [\,a\,]\ [\,c\,]$$
$$outlineCombinator\ fs1\ fs2 =$$
$$parEvalN\ ()\ fs1 \ggg$$
$$rightRotate \ggg$$
$$parEvalN\ ()\ fs2$$

In a distributed environment this first evaluates all $[\,arr\ a\ b\,]$ in parallel, sends the results back to the master node, rotates the input once and then evaluates the $[\,arr\ b\ c\,]$ in parallel to then gather the input once again on the master node. Such situations arise, e.g. in scientific computations when data distributed across the nodes needs to be transposed. A concrete example is 2D FFT computation (Berthold et al., 2009c; Gorlatch and Bischof, 1998).

While the example could be rewritten into a single *parEvalN* call by directly wiring the Arrows together before spawning, it illustrates an important problem. When using a *ArrowParallel* backend that resides on multiple computers, all communication between the nodes is done via the master node, as shown in the Eden trace in Figure 5.1. This can become a serious bottleneck for a larger amount of data and number of processes as e.g. Berthold et al. (2009c) showcases.

Figure 5.1: Communication between 4 Eden processes without Futures. All communication goes through the master node. Each bar represents one process. Black lines represent communication. Colours: blue $\hat{=}$ idle, green $\hat{=}$ running, red $\hat{=}$ blocked, yellow $\hat{=}$ suspended.

This is usually only a problem in distributed memory and we should allow nodes to communicate directly with each other. Eden already provides "remote data" that enable this (Alt and Gorlatch, 2003; Dieterle et al., 2010b). But as we want code using our DSL to be agnostic in terms of which backend is used, we have to wrap this concept. We do this with the *Future* type class to abstract the idea of handles on data that can be passed between nodes:

$$
\begin{aligned}
&\textbf{class } \textit{Future fut a conf} \mid a \; conf \rightarrow fut \; \textbf{where} \\
&\quad put :: (Arrow \; arr) \Rightarrow conf \rightarrow arr \; a \; (fut \; a) \\
&\quad get :: (Arrow \; arr) \Rightarrow conf \rightarrow arr \; (fut \; a) \; a
\end{aligned}
$$

A *conf* parameter is required here as well, but only so that Haskells type system allows us to have multiple Future implementations imported at once without breaking any dependencies similar to what we did with the *ArrowParallel* type class earlier. However, we can obviously yet again define default utility instances *Future fut a* () for each backend similar to how *ArrowParallel arr a b* () was defined in Chapter 4 as we will shortly see in the implementations for the backends.

Maybe even more interestingly, we use a functional dependency *a conf → fut* in the definition. This means that the type of *fut* can always fully be determined from the actual types of *a* and *conf*. We need this because we do not want users of our DSL to have to rely on a specific type of Future in their code. They only have to declare that they require a compatible Future type and do not need to worry about any specifics. This can be seen in the Future version of *outlineCombinator* we will define soon.

In order to implement this type class for Eden and since *RD* is only a type synonym for a communication type that is used internally in their library, we have to use some wrapper classes to fit the type class, as the following code showcases[1]:

```
data RemoteData a = RD { rd :: RD a }
put' :: (Arrow arr) ⇒ arr a (BasicFuture a)
put' = arr BF
get' :: (Arrow arr) ⇒ arr (BasicFuture a) a
get' = arr (λ(∼(BF a)) → a)
instance NFData (RemoteData a) where
    rnf = rnf ∘ rd
instance Trans (RemoteData a)
instance (Trans a) ⇒ Future RemoteData a Conf where
    put _ = put'
    get _ = get'
instance (Trans a) ⇒ Future RemoteData a () where
    put _ = put'
    get _ = get'
```

For GpH and *Par* Monad, we can simply use *BasicFuture*s, which are just simple wrappers around the actual data with boiler-plate logic so that the type class is satisfied. This is because the concept of a *Future* does not change anything for shared-memory execution as there are no communication problems to fix. Nevertheless, we require a common interface so the parallel Arrows are portable across backends. Here, the implementation is:

```
data BasicFuture a = BF a
put' :: (Arrow arr) ⇒ arr a (BasicFuture a)
put' = arr BF
get' :: (Arrow arr) ⇒ arr (BasicFuture a) a
get' = arr (λ(∼(BF a)) → a)
instance NFData a ⇒ NFData (BasicFuture a) where
    rnf (BF a) = rnf a
instance Future BasicFuture a (Conf a) where
    put _ = put'
    get _ = get'
instance Future BasicFuture a () where
    put _ = put'
    get _ = get'
```

---

[1]Instances of type classes can not be declared on type synonyms

Figure 5.2: Communication between 4 Eden processes with Futures. Unlike in Figure 5.1, processes communicate directly (one example message is highlighted) instead of always going through the master node (bottom bar).

Now, we can use this *Future* concept in our communication example for direct communication between nodes:

$$outlineCombinator :: ($$
$$\quad ArrowParallel\ arr\ a\ (fut\ b)\ (),$$
$$\quad ArrowParallel\ arr\ (fut\ b)\ c\ (),$$
$$\quad Future\ fut\ b\ ())\ \Rightarrow$$
$$\quad [arr\ a\ b] \rightarrow [arr\ b\ c] \rightarrow arr\ [a]\ [c]$$
$$outlineCombinator\ fs1\ fs2 =$$
$$\quad parEvalN\ ()\ (map\ (\ggg put\ ())\ fs1) \ggg$$
$$\quad rightRotate \ggg$$
$$\quad parEvalN\ ()\ (map\ (get\ ()\ggg)\ fs2)$$

In a distributed environment, this gives us a communication scheme with messages going through the master node only if it is needed – similar to what is shown in the trace visualisation in Figure 5.2. This is because only the handles to the data that are passed through the master node, while all communication of actual data can happen between the actual nodes. We will build upon this concept in more complicated combinators in the next chapter.

5.2 Advanced topological skeletons

Even though many algorithms can be expressed by *parMap*s, some problems require more sophisticated skeletons. The Eden library resolves this problem and already comes with more predefined skeletons[2], among them a *pipe*, a *ring*, and a *torus*

---

[2]Available on Hackage: `https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/Control-Parallel-Eden-Topology.html`.

implementation (Loogen et al., 2003). These seem like reasonable candidates to be ported to our Arrow-based parallel Haskell. By doing so, we aim to showcase that we can express more sophisticated skeletons with parallel Arrows as well.

If we were to use the original definition of *parEvalN*, however, these skeletons would produce an infinite loop with the GpH and *Par* Monad which during runtime would result in the program crashing. This materialises with the usage of *loop* of the *ArrowLoop* type class and we think that this is due to difference of how evaluation is done in these backends compared to Eden. An investigation of why this difference exists is beyond the scope of this work – the results of the experimental Cloud Haskell backend in Chapter 6.3.3 touch on the likely root cause of this problem, though. We only provide a workaround for these types of skeletons as such they probably are not of much importance outside of a distributed memory environment. Nevertheless, our workaround enables users of the DSL to test their code within a shared memory setting.

The idea of the fix is to provide a *ArrowLoopParallel* type class that has two functions – *loopParEvalN* and *postLoopParEvalN*. The first is to be used inside an *loop* construct while the latter will be used right outside of the *loop*. This way we can delegate to the actual *parEvalN* in the spot where the backend supports it.

```
class ArrowParallel arr a b conf ⇒
    ArrowLoopParallel arr a b conf where
    loopParEvalN :: conf → [ arr a b ] → arr [ a ] [ b ]
    postLoopParEvalN :: conf → [ arr a b ] → arr [ a ] [ b ]
```

Because Eden has no problems with the looping skeletons, we use this instance:

```
instance (ArrowChoice arr, ArrowParallel arr a b Conf) ⇒
    ArrowLoopParallel arr a b Conf where
    loopParEvalN = parEvalN
    postLoopParEvalN _ = evalN
```

Par Monad and GpH have problems with *parEvalN* inside of *loop*. Their respective instances for *ArrowLoopParallel* look like this:

```
instance (ArrowChoice arr, ArrowParallel arr a b (Conf b)) ⇒
    ArrowLoopParallel arr a b (Conf b) where
    loopParEvalN _ = evalN
    postLoopParEvalN = parEvalN
```

Chapter 5.2.1 explains how to achieve a parallel *pipe* with our DSL. Then, Chapter 5.2.2 goes into detail on how to achieve a *ring* skeleton, which we then build upon to achieve a *torus* in Chapter 5.2.3.

$$pipeSimple :: (ArrowLoop\ arr,\ ArrowLoopParallel\ arr\ a\ a\ conf) \Rightarrow$$
$$conf \rightarrow [\,arr\ a\ a\,] \rightarrow arr\ a\ a$$
$$pipeSimple\ conf\ fs =$$
$$loop\ (arr\ snd\ \&\&\&$$
$$(arr\ (uncurry\ (:) \ggg lazy) \ggg loopParEvalN\ conf\ fs)) \ggg$$
$$arr\ last$$

Figure 5.3: Simple *pipe* skeleton. The use of *lazy* (Figure 10.3) is essential as without it programs using this definition would never halt. We need to ensure that the evaluation of the input $[\,a\,]$ is not forced fully before passing it into *loopParEvalN*.

5.2.1 Parallel pipe

We start with the parallel *pipe* skeleton, which is semantically equivalent to folding over a list $[\,arr\ a\ a\,]$ of Arrows with $\ggg$, but in parallel, meaning that the Arrows do not have to reside on the same thread/machine. We implement this skeleton using the *ArrowLoop* type class which provides us with the $loop :: arr\ (a,b)\ (c,b) \rightarrow arr\ a\ c$ combinator allowing us to express recursive fix-point computations in which output values are fed back as input. For example

$$loop\ (arr\ (\lambda(a,b) \rightarrow (b,a:b)))$$

which is the same as

$$loop\ (arr\ snd\ \&\&\&\ arr\ (uncurry\ (:)))$$

defines an Arrow that takes its input a and converts it into an infinite stream $[\,a\,]$ of it. Using *loop* to our advantage gives us a first draft of a pipe implementation (Figure 5.3) by plugging in the parallel evaluation call *loopParEvalN conf fs* inside the second argument of $\&\&\&$ and then only picking the first element of the resulting list with *arr last* outside of the *loop*.

However, using this definition directly will make the master node a potential bottleneck in distributed environments as described in Chapter 5.1. Therefore, we introduce a more sophisticated version that internally uses Futures and obtain the final definition of *pipe* in Figure 5.4.

Sometimes, this *pipe* definition can be a bit inconvenient, especially if we want to pipe Arrows of mixed types together, i.e. *arr a b* and *arr b c*. By wrapping these two Arrows inside a bigger Arrow *arr* $(([\,a\,],[\,b\,]),[\,c\,])$ $(([\,a\,],[\,b\,]),[\,c\,])$ suitable for *pipe*, we can define *pipe2* as in Figure 5.5.

Extensive use of *pipe2* over *pipe* with a hand-written combination data type will probably result in worse performance because of more communication overhead

$$pipe :: (ArrowLoop\ arr,$$
$$ArrowLoopParallel\ arr\ (fut\ a)\ (fut\ a)\ conf,$$
$$Future\ fut\ a\ conf) \Rightarrow$$
$$conf \rightarrow [\,arr\ a\ a\,] \rightarrow arr\ a\ a$$
$$pipe\ conf\ fs = unliftFut\ conf\ (pipeSimple\ conf\ (map\ (liftFut\ conf)\ fs))$$

$$liftFut :: (Arrow\ arr, Future\ fut\ a\ conf, Future\ fut\ b\ conf) \Rightarrow$$
$$conf \rightarrow arr\ a\ b \rightarrow arr\ (fut\ a)\ (fut\ b)$$
$$liftFut\ conf\ f = get\ conf \ggg f \ggg put\ conf$$

$$unliftFut :: (Arrow\ arr, Future\ fut\ a\ conf, Future\ fut\ b\ conf) \Rightarrow$$
$$conf \rightarrow arr\ (fut\ a)\ (fut\ b) \rightarrow arr\ a\ b$$
$$unliftFut\ conf\ f = put\ conf \ggg f \ggg get\ conf$$

Figure 5.4: *pipe* skeleton definition with Futures.

from the many calls to *parEvalN* inside of *evalN*. Nonetheless, we can define a parallel piping operator $|\ggg|$, which is semantically equivalent to $\ggg$ similarly to other parallel syntactic sugar from Appendix 10.3.

$$
\begin{aligned}
&pipe2 :: (ArrowLoop\ arr,\ ArrowChoice\ arr, \\
&\quad ArrowLoopParallel\ arr\ (fut\ (([a],[b]),[c]))\ (fut\ (([a],[b]),[c]))\ conf, \\
&\quad Future\ fut\ (([a],[b]),[c])\ conf) \Rightarrow \\
&\quad conf \to arr\ a\ b \to arr\ b\ c \to arr\ a\ c \\
&pipe2\ conf\ f\ g = \\
&\quad (arr\ return\ \&\&\&\ arr\ (const\ []))\ \&\&\&\ arr\ (const\ [])\ \ggg \\
&\quad pipe\ conf\ (replicate\ 2\ (unify\ f\ g))\ \ggg \\
&\quad arr\ snd\ \ggg \\
&\quad arr\ head \\
&\quad \textbf{where} \\
&\qquad unify :: (ArrowChoice\ arr) \Rightarrow \\
&\qquad\quad arr\ a\ b \to arr\ b\ c \to arr\ (([a],[b]),[c])\ (([a],[b]),[c]) \\
&\qquad unify\ f'\ g' = \\
&\qquad\quad (mapArr\ f'\ *\!*\!*\ mapArr\ g')\ *\!*\!*\ arr\ (const\ [])\ \ggg \\
&\qquad\quad arr\ (\lambda((b,c),a) \to ((a,b),c)) \\
&(|\!\ggg\!|) :: (ArrowLoop\ arr,\ ArrowChoice\ arr, \\
&\quad ArrowLoopParallel\ arr\ (fut\ (([a],[b]),[c]))\ (fut\ (([a],[b]),[c]))\ (), \\
&\quad Future\ fut\ (([a],[b]),[c])\ ()) \Rightarrow \\
&\quad arr\ a\ b \to arr\ b\ c \to arr\ a\ c \\
&(|\!\ggg\!|) = pipe2\ ()
\end{aligned}
$$

Figure 5.5: Definition of $pipe2$ and $(|\!\ggg\!|)$, a parallel $\ggg$.

5.2.2 Ring skeleton

Eden comes with a ring skeleton[3] (Figure 5.6) implementation that allows the computation of a function $[i] \to [o]$ with a ring of nodes that communicate with each other. Its input is a node function $i \to r \to (o,r)$ in which r serves as the intermediary output that is sent to the neighbour of each node. It uses the direct "remote data" communication channels that were already mentioned in Chapter 5.1. We depict it in the Appendix, Figure 10.5.

Figure 5.6: $ring$ skeleton depiction.

---

[3]Available on Hackage: `https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/Control-Parallel-Eden-Topology.html`.

$$ring :: (Future\ fut\ r\ conf,$$
$$ArrowLoop\ arr,$$
$$ArrowLoopParallel\ arr\ (i, fut\ r)\ (o, fut\ r)\ conf,$$
$$ArrowLoopParallel\ arr\ o\ o\ conf) \Rightarrow$$
$$conf \rightarrow arr\ (i, r)\ (o, r) \rightarrow arr\ [i]\ [o]$$
$$ring\ conf\ f =$$
$$loop\ (second\ (rightRotate \ggg lazy) \ggg$$
$$arr\ (uncurry\ zip) \ggg$$
$$loopParEvalN\ conf$$
$$(repeat\ (second\ (get\ conf) \ggg f \ggg second\ (put\ conf))) \ggg$$
$$arr\ unzip) \ggg$$
$$postLoopParEvalN\ conf\ (repeat\ (arr\ id))$$

Figure 5.7: $ring$ skeleton definition.

We can rewrite this functionality easily with the use of $loop$ as the definition of the node function, $arr\ (i, r)\ (o, r)$, after being transformed into an Arrow, already fits quite neatly into $loop$'s signature: $arr\ (a, b)\ (c, b) \rightarrow arr\ a\ c$. In each iteration we start by rotating the intermediary input from the nodes $[fut\ r]$ with $second\ (rightRotate \ggg lazy)$ (Figure 10.3). Similarly to the $pipe$ from Chapter 5.2.1 (Figure 5.3), we have to feed the intermediary input into our $lazy$ (Figure 10.3) Arrow here, or the evaluation would fail to terminate. The reasoning is explained by Loogen (2012) as a demand problem.

Next, we zip the resulting $([i], [fut\ r])$ to $[(i, fut\ r)]$ with $arr\ (uncurry\ zip)$. We then feed this into our parallel Arrow $arr\ [(i, fut\ r)]\ [(o, fut\ r)]$ obtained by transforming our input Arrow $f :: arr\ (i, r)\ (o, r)$ into $arr\ (i, fut\ r)\ (o, fut\ r)$ before $repeat$ing and lifting it with $loopParEvalN$. Finally we unzip the output list $[(o, fut\ r)]$ into $([o], [fut\ r])$.

Plugging this Arrow $arr\ ([i], [fut\ r])\ ([o], fut\ r)$ into the definition of $loop$ from earlier gives us $arr\ [i]\ [o]$, our ring Arrow (Figure 5.7). To make sure this algorithm has speedup on shared-memory machines as well, we pass the result of this Arrow to $postLoopParEvalN\ conf\ (repeat\ (arr\ id))$. This combinator can, for example, be used to calculate the shortest paths in a graph using Warshall's algorithm.

5.2.3 Torus skeleton

If we take the concept of a $ring$ from Chapter 5.2.2 one dimension further, we obtain a $torus$ skeleton (Figure 5.8, 5.9). In a $torus$, every node sends and receives data from horizontal and vertical neighbours in each communication round. With our

Figure 5.8: *torus* skeleton depiction.

Parallel Arrows we re-implement this combinator from Eden[4] – yet again with the help of the *ArrowLoop* type class.

Similar to the *ring*, we start by rotating the input (Figure 10.3), but this time not only in one direction, but in two. This means that the intermediary input from the neighbour nodes has to be stored in a tuple $([[fut\ a]], [[fut\ b]])$ in the second argument (*loop* only allows for two arguments) of our looped Arrow of type

$$arr\ ([[c]], ([[fut\ a]], [[fut\ b]]))\ ([[d]], ([[fut\ a]], [[fut\ b]]))$$

and our rotation Arrow becomes

$$second\ ((mapArr\ rightRotate \ggg lazy) *** (arr\ rightRotate \ggg lazy))$$

instead of the singular rotation in the ring as we rotate $[[fut\ a]]$ horizontally and $[[fut\ b]]$ vertically. Then, we zip the inputs for the input Arrow with

$$arr\ (uncurry3\ zipWith3\ lazyzip3)$$

from $([[c]], ([[fut\ a]], [[fut\ b]]))$ to $[[(c, fut\ a, fut\ b)]]$, which we then evaluate in parallel.

This, however, is more complicated than in the *ring* case as we have one more dimension of inputs that needs to be transformed. We first have to *shuffle* all the inputs and then pass them into *loopParEvalN conf* (*repeat* (*ptorus conf f*)) to get an output of $[(d, fut\ a, fut\ b)]$. We then *unshuffle* this list back to its original ordering by feeding it into *arr* (*uncurry unshuffle*) which takes the input length we saved one step earlier as additional input to get a result matrix $[[[(d, fut\ a, fut\ b)]]]$. Finally, we unpack this matrix with *arr* (*map unzip3*) $\ggg$ *arr unzip3* $\ggg$ *threetotwo* to get $([[d]], ([[fut\ a]], [[fut\ b]]))$.

This internal looping computation is once again fed into *loop* and we also compose a final *postLoopParEvalN conf* (*repeat* (*arr id*)) due to the same problem with *loop* as explained for the *ring* skeleton.

---

[4]Available on Hackage: `https://hackage.haskell.org/package/edenskel-2.1.0.0/docs/Control-Parallel-Eden-Topology.html`.

$$torus :: (Future\ fut\ a\ conf, Future\ fut\ b\ conf,$$
$$ArrowLoop\ arr, ArrowChoice\ arr,$$
$$ArrowLoopParallel\ arr\ (c, fut\ a, fut\ b)\ (d, fut\ a, fut\ b)\ conf,$$
$$ArrowLoopParallel\ arr\ [d]\ [d]\ conf) \Rightarrow$$
$$conf \rightarrow arr\ (c, a, b)\ (d, a, b) \rightarrow arr\ [[c]]\ [[d]]$$
$$torus\ conf\ f =$$
$$loop\ (second\ ((mapArr\ rightRotate \ggg lazy)$$
$$*** (arr\ rightRotate \ggg lazy)) \ggg$$
$$arr\ (uncurry3\ (zipWith3\ lazyzip3)) \ggg$$
$$arr\ length\ \&\&\& (shuffle \ggg$$
$$loopParEvalN\ conf\ (repeat\ (ptorus\ conf\ f))) \ggg$$
$$arr\ (uncurry\ unshuffle) \ggg$$
$$arr\ (map\ unzip3) \ggg arr\ unzip3 \ggg threetotwo) \ggg$$
$$postLoopParEvalN\ conf\ (repeat\ (arr\ id))$$

$$ptorus :: (Arrow\ arr, Future\ fut\ a\ conf, Future\ fut\ b\ conf) \Rightarrow$$
$$conf \rightarrow$$
$$arr\ (c, a, b)\ (d, a, b) \rightarrow$$
$$arr\ (c, fut\ a, fut\ b)\ (d, fut\ a, fut\ b)$$
$$ptorus\ conf\ f =$$
$$arr\ (\lambda{\sim}(c, a, b) \rightarrow (c, get\ conf\ a, get\ conf\ b)) \ggg$$
$$f \ggg$$
$$arr\ (\lambda{\sim}(d, a, b) \rightarrow (d, put\ conf\ a, put\ conf\ b))$$

Figure 5.9: *torus* skeleton definition. *lazyzip3*, *uncurry3* and *threetotwo* definitions are in Figure 10.4.

As an example of using this skeleton, Loogen et al. (2003) showed the matrix multiplication using Gentleman's algorithm (Gentleman, 1978). An adapted version can be found in Figure 5.10.

If we compare the trace from a call using our Arrow definition of the *torus* (Figure 5.11) with the Eden version (Figure 5.12) we can see that the behaviour of the Arrow version and execution times are comparable – our port was successful. We discuss further benchmarks on larger clusters in more detail in Chapter 7.

type $Matrix = [[Int]]$

$prMM\_torus :: Int \rightarrow Int \rightarrow Matrix \rightarrow Matrix \rightarrow Matrix$
$prMM\_torus\ numCores\ problemSizeVal\ m1\ m2 =$
   $combine\ \$\ torus\ ()\ (mult\ torusSize)\ \$\ zipWith\ zip\ (split1\ m1)\ (split2\ m2)$
   where $torusSize = (floor \circ sqrt)\ \$\ fromIntegral\ \$\ numCoreCalc\ numCores$
       $combine\ x = concat\ (map\ ((map\ (concat)) \circ transpose)\ x)$
       $split1\ \ x = staggerHorizontally$
        $(splitMatrix\ (problemSizeVal\ `div`\ torusSize)\ x)$
       $split2\ \ x = staggerVertically$
        $(splitMatrix\ (problemSizeVal\ `div`\ torusSize)\ x)$

  -- Function performed by each worker
$mult :: Int \rightarrow$
  $((Matrix, Matrix), [Matrix], [Matrix]) \rightarrow$
  $(Matrix, [Matrix], [Matrix])$
$mult\ size\ ((sm1, sm2), sm1s, sm2s) = (result, toRight, toBottom)$
  where $toRight = take\ (size - 1)\ (sm1 : sm1s)$
      $toBottom = take\ (size - 1)\ (sm2 : sm2s)$
      $sms = zipWith\ prMM\ (sm1 : sm1s)\ (sm2 : sm2s)$
      $result = foldl1'\ matAdd\ sms$

Figure 5.10: Adapted matrix multiplication in Eden using the *torus* skeleton. *prMM_torus* is the parallel matrix multiplication. *mult* is the function performed by each worker. *prMM* is the sequential matrix multiplication in the chunks. *splitMatrix* splits the Matrix into chunks. *staggerHorizontally* and *staggerVertically* pre-rotate the matrices. *matAdd* calculates $A + B$. Omitted definitions can be found in 10.6.

Figure 5.11: Communication trace of a matrix multiplication with *torus* (PArrows).

Figure 5.12: Communication trace of a matrix multiplication with *torus* (Eden).

Experiment: Cloud Haskell Backend

6

Cloud Computing has become more and more prevalent in the recent years. Servers are replaced with virtual ones positioned all around the globe. These can easily be brought up when required and shut down when not in use. This trend in computing has also been embraced by the Haskell community, and therefore libraries such as Cloud Haskell were born. Cloud Haskell is described on the project's website[1] as:

> *" Cloud Haskell: Erlang-style concurrent and distributed programming in Haskell. The Cloud Haskell Platform consists of a generic network transport API, libraries for sending static closures to remote nodes, a rich API for distributed programming and a set of platform libraries modelled after Erlang's Open Telecom Platform.*
>
> *Generic network transport backends have been developed for TCP and in-memory messaging, and several other implementations are available including a transport for Windows Azure.[...] "*

It is basically a set of APIs and libraries for communication between networks of nodes in a cloud environment. With it programmers can write fully-featured Haskell based cloud solutions targeting a wide range of architectures.

While users can already write concurrent applications with the help of Cloud Haskell using some of its libraries or even with the bare communication API, it seems like a good idea to enable writing parallel programs requiring less involvement from the user. This way, they can focus on parallel algorithms instead of manual communication. In the following Chapter we will therefore explore the possibility of a Cloud Haskell based backend for the *ArrowParallel* interface given in this thesis while explaining all the necessary parts of Cloud Haskell's API. For easier testing and as this is only meant as a proof of concept, we only work with a local-net Cloud Haskell backend in this thesis. The results of this experiment, however, are transferable to other architectures as well when building upon the results presented here.[2]

---

[1]See `http://haskell-distributed.github.io/`.
[2]With the help of virtual private networks one could even use this local-net variant.

The following is structured as follows. We start by explaining how to discover nodes with a master-slave structure while also defining a program startup harness that can be used with this scheme in Chapter 6.1. Then, we explain how parallel evaluation of arbitrary data is possible with Cloud Haskell in Chapter 6.2 and also discuss how we can implement the PArrows DSL with this knowledge in Chapter 6.3.

6.1 Node discovery and program harness

In cloud services it is more common that the architecture of the running network changes than in ordinary computing clusters where the participating nodes are usually known at startup. In the SimpleLocalNet[3] Cloud Haskell backend we are using for this experiment, this is reflected in the fact that there already exists a pre-implemented master-slave structure. The master node – the node that starts the computation is considered the master node here – has to keep track of all the available slave nodes. The slave nodes wait for tasks and handle them as required.

We will now first go into detail on the data-structure (Chapter 6.1.1) we use in order to handle this information to then explain how to start slave (Chapter 6.1.2) and master nodes (Chapter 6.1.3). We also explain how to create a startup harness (Chapter 6.1.4) to wrap all of this.

6.1.1 The $State$ data-structure

The data-structure containing all relevant information about the state of the network as well as the computation in general, $State$, is defined as

```
data State = State {
    workers :: MVar [NodeId],
    shutdown :: MVar Bool,
    started :: MVar Bool,
    localNode :: LocalNode,
    serializeBufferSize :: Int
}
```

Notice that $workers :: MVar\ [NodeId]$, $shutdown :: MVar\ Bool$ and $started :: MVar\ ()$ are all low level mutable locations instead of regular fields. This is because we pass this $State$ around between functions, but want to update it with new information on-the-fly. These modifiable variables can be created empty with $newEmptyMVar ::$ $IO\ (MVar\ a)$ or already with contents with $newMVar :: a \rightarrow IO\ (MVar\ a)$. They can

---

[3]See http://hackage.haskell.org/package/distributed-process-simplelocalnet.

be read with $readMVar :: MVar\ a \rightarrow IO\ a$ or emptied with $takeMVar :: MVar\ a \rightarrow IO\ a$. Values can be placed inside with $putMVar :: MVar\ a \rightarrow a \rightarrow IO\ ()$. $MVar$s are thread-safe and all reading operations block until some content is placed in them. We will see them used in other places of this backend as well.

In the $State$ type, $workers :: MVar\ [NodeId]$ holds information about all available slave nodes, $shutdown :: MVar\ Bool$ determines whether the backend is to be shut down, $started :: MVar\ ()$ returns a signalling () if the backend has properly started when accessed with $readMVar$. $localNode :: LocalNode$ and $serializeBufferSize :: Int$ store information about all Cloud Haskell internals for the master node and the buffer size for serialization (we will discuss serialization system itself separately), respectively.

Note that as we will later use the $State$ type as the $conf$ parameter in the $ArrowParallel$ instance, we use the type synonym **type** $Conf = State$ in the following code Chapters. Furthermore, an initial config can be created with the function $initialConf ::$ $Int \rightarrow LocalNode \rightarrow IO\ Conf$ where the resulting config contains a $serializeBufferSize$ as specified by the first parameter and the $LocalNode$ specified by the second parameter. Additionally, the list of workers $workers :: MVar\ [NodeId]$ is initialized with an empty list, $shutdown :: MVar\ Bool$ is set to $False$ and $started :: MVar\ ()$ is created as an empty $MVar$ so that it can be populated with the signalling () when the startup is finished. The complete code for $initialConf$ is the following:

```
initialConf :: Int → LocalNode → IO Conf
initialConf serializeBufferSize localNode = do
  workersMVar ← newMVar [ ]
  shutdownMVar ← newMVar False
  startedMVar ← newEmptyMVar
  return State {
    workers = workersMVar,
    shutdown = shutdownMVar,
    started = startedMVar,
    localNode = localNode,
    serializeBufferSize = serializeBufferSize
  }
```

A utility function $defaultInitConf$ using a default serialization buffer size of 10MB is also defined as:

```
defaultBufSize :: Int
defaultBufSize = 10 * 2 ↑ 20    -- 10 MB

defaultInitConf :: LocalNode → IO Conf
defaultInitConf = initialConf defaultBufSize
```

6.1.2 Starting Slave nodes

With the *State/Conf* data structure we can now implement our node-discovery scheme. Starting with the slave nodes, we can just use the basic utilities for a slave backend in the SimpleLocalNet library. The code to start a node for the *Slave* backend is then:

```
type Host = String
type Port = String

initializeSlave :: RemoteTable → Host → Port → IO ()
initializeSlave remoteTable host port = do
    backend ← initializeBackend host port remoteTable
    startSlave backend
```

We here start by initializing the Cloud Haskell backend with a given *host*, *port* and *remoteTable* via *initializeBackend* :: *String* → *String* → *RemoteTable* to then delegate the logic completely to the library function *startSlave* :: *Backend* → *IO* () which does not return unless the slave is shutdown manually from the master node. The *RemoteTable* contains all serialization information about static values required by Cloud Haskell. We will later see how we can automatically generate such a table.

6.1.3 Starting Master nodes

For master nodes, the implementation is a bit more involved. The actual *startMaster* :: *Backend* → *Process* → *IO* () supplied by SimpleLocalNet is meant to start a computation represented by a *Process* Monad and then return. In our use-case we want to be able to spawn functions outside of the *Process* Monad, though. We therefore use the following *Process* passed into this startup function only for slave-node discovery and management:

```
master :: Conf → Backend → [NodeId] → Process ()
master conf backend slaves = do
  forever $ do
    shutdown ← liftIO $ readMVar $ shutdown conf
    if shutdown
      then do
        terminateAllSlaves backend
        die "terminated"
      else do
        slaveProcesses ← findSlaves backend
        redirectLogsHere backend slaveProcesses
```

```
      let slaveNodes = map processNodeId slaveProcesses
      liftIO $ do
        modifyMVar_ (workers conf) (\_ → return slaveNodes)
        isEmpty ← isEmptyMVar $ started conf
        if (isEmpty ∧ length slaveNodes > 0) then
          putMVar (started conf) ()
        else
          return ()
```

Basically, this continuously updates the list of slaves inside the configuration by first querying for all slave processes with *findSlaves backend* and redirecting the log output to the master node with *redirectLogsHere backend slaveProcesses* to then finally update *workers* :: *MVar* [*NodeId*] inside the configuration. Additionally, as soon as one slave is found, *started* :: *MVar* () is supplied with the signalling () so that any thread waiting for node discovery can start its actual computation.[4] All of this is embedded in a check whether a shutdown is requested with *liftIO* $ *readMVar* $ *shutdown conf*. If instructed to do so, the program does the necessary cleanup - terminating all slaves with *terminateAllSlaves backend* and shutting itself down with *die* "terminated" - otherwise continuing with the updating process.

With this *master* function, we can now define our initialization function *initializeMaster* :: *RemoteTable* → *Host* → *Port* → *IO Conf*:

```
      initializeMaster :: RemoteTable → Host → Port → IO Conf
      initializeMaster remoteTable host port = do
        backend ← initializeBackend host port remoteTable
        localNode ← newLocalNode backend
        conf ← defaultInitConf localNode
        forkIO $ startMaster backend (master conf backend)
        waitForStartup conf
        return conf
```

Similar to the slave code, we initialize the Cloud Haskell backend via *initializeBackend* :: *String* → *String* → *RemoteTable*, but also create a new local node that is used to start computations outside of the initialization logic. With this node we can create a default initial config via *defaultInitConf* :: *LocalNode* → *Conf* which is then passed into the discovery function with *startMaster backend* (*master conf backend*). We have to fork this *IO* action away with *forkIO*, because the *IO* action will run forever as long as the program has not been manually shutdown via the corresponding variable in the *State*. Finally, we wait for the startup to finish via

---

[4]Notice that while we could add an additional sleep here to not generate too much network noise in this function, we leave it out for the sake of simplicity.

$waitForStartup :: Conf \rightarrow IO$ () to end with returning a *IO Conf* action containing the initial config/state. *waitForStartup* can simply be defined as

$$waitForStartup :: Conf \rightarrow IO\ ()$$
$$waitForStartup\ conf = readMVar\ (started\ conf)$$

because of the blocking behaviour of empty *MVar*s and the fact that we are signalling the startup with a simple dummy value () as described earlier.

6.1.4 Startup harness

If we put together all of the startup logic we have discussed until now, we can easily write a startup harness in which we simply delegate to the proper initialization code depending on the command line arguments:

$$myRemoteTable :: RemoteTable$$
$$myRemoteTable = Main.\_\_remoteTable\ initRemoteTable$$
$$main :: IO\ ()$$
$$main = \textbf{do}$$
$$\quad args \leftarrow getArgs$$
$$\quad \textbf{case}\ args\ \textbf{of}$$
$$\quad\quad [\texttt{"master"}, host, port] \rightarrow \textbf{do}$$
$$\quad\quad\quad conf \leftarrow initializeMaster\ myRemoteTable\ host\ port$$
$$\quad\quad\quad\quad \text{-- read and print the list of available workers}$$
$$\quad\quad\quad readMVar\ (workers\ conf) \ggg print$$
$$\quad\quad\quad\quad \text{-- TODO: parallel computation here}$$
$$\quad\quad [\texttt{"slave"}, host, port] \rightarrow \textbf{do}$$
$$\quad\quad\quad initializeSlave\ myRemoteTable\ host\ port$$
$$\quad\quad\quad print\ \texttt{"slave shutdown."}$$

In order to launch a program using this harness, we have to start slave nodes for each cpu core with commands like "<executable> slave 127.0.0.1 8000" where the last parameter determines the port the slave will listen to and wait for requests on. Similarly a single master node can be started with "<executable> master 127.0.0.1 7999" where, once again, the last parameter determines the communication port.

This example also shows how a *RemoteTable* is obtained so that it can be used inside $main :: IO$ (). Note, that the definition of $Main.\_\_remoteTable :: RemoteTable \rightarrow RemoteTable$ used in $myRemoteTable :: RemoteTable$ is a function automatically generated by Template-Haskell[5] building a *RemoteTable* on top of the *initRemoteTable*

---

[5]Template-Haskell is a code generator for Haskell written in Haskell, and can be enabled with a language pragma   {-# LANGUAGE TemplateHaskell #-}   at the top of the source file.

supplied by Cloud Haskell by adding all relevant static declarations of the program. In Cloud Haskell, we can for example generate such a declaration for some function $f :: Int \rightarrow Int$, with a call to $remotable$ inside a Template-Haskell splice as $\$(remotable\ ['f])$.

As can be seen from this, any function passed to $remotable$ must have a top-level declaration. Furthermore, we must also add any function manually. This is usually okay for basic applications where the user usually knows which functions/values need to be serialized statically at compile time, but not in our use case as we want to be able to evaluate arbitrary functions/Arrows on remote nodes. In Chapter 6.2 we will see how to resolve this problem.

6.2 Parallel Evaluation with Cloud Haskell

As we have seen in the previous chapter, we can not send arbitrary functions or Arrows to the slave nodes. Thankfully, there is an alternative: Eden's serialization mechanism has been made available separately in a package called "packman".[6] This allows values to be serialized in the exact evaluation state they are currently in.

We can use this to our advantage. Instead of sending inputs and functions/Arrows to the slave nodes and sending the result back (which does not work with the current Cloud Haskell API), we can instead apply the function, serialize this unevaluated thunk, send it to the evaluating slave, and communicate the fully evaluated value back to the master.

With this idea in mind we will now explain how to achieve parallel evaluation of Arrows with Cloud Haskell. We start by explaining the communication basics in Chapter 6.2.1. Next, we describe how to achieve evaluation of single values on slave nodes in Chapter 6.2.2. Finally, we use these results to implement a parallel evaluation scheme in Chapter 6.2.3.

6.2.1 Communication basics

We will now go over the communication basics we require in the later parts of this chapter. This includes a quick introduction on how we actually send the data in Cloud Haskell and also a quick definition of our serialized data wrapper we use to send unevaluated data between nodes.

---

[6]See https://hackage.haskell.org/package/packman.

Sending and Receiving data

Cloud Haskell uses typed channels to send and receive data between nodes. A typed channel consists of a $SendPort\ a$ and a $ReceivePort\ a$. We can create a new typed cannel with the help of $newChan :: Serializable\ a \Rightarrow Process\ (SendPort\ a, ReceivePort\ a)$:

$$myProc :: Process\ ()$$
$$myProc = \mathbf{do}$$
$$\quad (sendPort, receivePort) \leftarrow newChan$$
$$\qquad \text{-- do stuff}$$

Data can be sent with $sendChan :: Serializable\ a \Rightarrow SendPort\ a \rightarrow a \rightarrow Process\ ()$:

$$sendTen :: SendPort\ Int \rightarrow Process\ ()$$
$$sendTen\ sendPort = sendChan\ sendPort\ 10$$

Values are received in a blocking manner with $receiveChan :: Serializable\ a \Rightarrow ReceivePort\ a \rightarrow Process\ a$:

$$receiveVal :: ReceivePort\ Int \rightarrow Process\ Int$$
$$receiveVal\ receivePort = receivechan\ receivePort$$

One thing to keep in mind is that only $SendPort$ are serializable. So in order to have a two way communication where the master sends some input to the slave and awaits its result, like we require in our use case, we have to first receive a $SendPort\ a$ in the master via the $ReceivePort\ (SendPort\ a)$ of some channel $(SendPort\ (SendPort\ a), ReceivePort\ (SendPort\ a))$. This $SendPort\ a$ is sent the slave and belongs to the channel $(SendPort\ a, ReceivePort\ a)$ where it expects its input to come through the corresponding $ReceivePort\ a$. Additionally, we also require a channel $(SendPort\ b, ReceivePort\ b)$ on which the slave sends its result through the $SendPort\ b$ and the master awaits its result on the $ReceivePort\ b$. We depict this process with schematically in Figure 6.1.

This idea is executed in the following code example. The master looks like

$$master :: ReceivePort\ (SendPort\ a) \rightarrow ReceivePort\ b \rightarrow Process\ ()$$
$$master\ aSenderReceiver\ bReceiver = \mathbf{do}$$
$$\quad aSender \leftarrow receiveChan\ aSenderReceiver$$
$$\quad \mathbf{let}\ someA = ...$$
$$\quad sendChan\ aSender\ someA$$
$$\quad someB \leftarrow receiveChan\ bReceiver$$
$$\quad ...$$
$$\quad return\ ()$$

Figure 6.1: Required communication scheme for our Cloud Haskell backend. Actions corresponding to specific channels are marked with their respective colour.

while the slave is schematically defined as

$$slave :: SendPort\ (SendPort\ a) \rightarrow SendPort\ b \rightarrow Process\ ()$$
$$slave\ aSenderSender\ bSender = \textbf{do}$$
$$\quad (aSender, aReceiver) \leftarrow newChan$$
$$\quad sendChan\ aSenderSender\ aSender$$
$$\quad someA \leftarrow receiveChan\ aReceiver$$
$$\quad \textbf{let}\ someB = useAToMakeB\ someA$$
$$\quad sendChan\ bSender\ someB$$

Serialized data type

The packman package comes with a serialization function $trySerializeWith :: a \rightarrow Int \rightarrow IO\ (Serialized\ a)$ (the second parameter is the buffer size) and a deserialization function $deserialize :: Serialized\ a \rightarrow IO\ a$. Here, $Serialized\ a$ is the type containing the serialized value of a.

In order to have a clean slate in terms of type class instances, we define a wrapper type $Thunk\ a$ around $Serialized\ a$ as

```
    -- Wrapper for the packman type Serialized
```
$$\textbf{newtype}\ Thunk\ a = Thunk\ \{fromThunk :: Serialized\ a\}\ \textbf{deriving}\ (Typeable)$$
$$toThunk\ a = Thunk\ \{fromThunk = a\}$$

Additionally, we require a *Binary* for our wrapper in order to be able to send it with Cloud Haskell. This only delegates to the implementation of the actual *Serialized* we wrap:

$$\textbf{instance } (Typeable\ a) \Rightarrow Binary\ (Thunk\ a)\ \textbf{where}$$
$$put = Data.Binary.put \circ fromThunk$$
$$get = \textbf{do}$$
$$(ser :: Serialized\ a) \leftarrow Data.Binary.get$$
$$return\ \$\ Thunk\ \{fromThunk = ser\}$$

6.2.2 Evaluation of values on slave nodes

Having discussed the general communication scheme and serialization mechanism we want to use, we can explain how the evaluation of values on slave nodes works with Cloud Haskell, next. We give the master node's code for evaluation of a single value on a slave node and also the slave nodes' code.

Master node

The following function $forceSingle :: NodeId \rightarrow MVar\ a \rightarrow a \rightarrow Process\ ()$ is used to evaluate a single value a. It returns a monadic action $Process\ ()$ that evaluates a value of type a on the node with the given $NodeId$ and stores the evaluated result in the given $MVar\ a$.

Unlike the master from Chapter 6.2.1, it starts by creating the top level communication channels $(SendPort\ a, ReceivePort\ a)$[7] and $(SendPort\ (SendPort\ (Thunk\ a)), ReceivePort\ (SendPort\ (Th$ as we have to spawn the slave node from the master. This is different from the schematic depiction we have seen earlier where the two main channels were created outside of both functions. The communication scheme is the same, nevertheless. Then, it spawns the actual evaluation task (the slave from Chapter 6.2.1)

$$evalTask :: (SendPort\ (SendPort\ (Thunk\ a)), SendPort\ a) \rightarrow Process\ ()$$

with the necessary $SendPort$s for input communication $(SendPort\ (SendPort\ (Thunk\ a)))$ and result communication $(SendPort\ a$[8]$)$ on the given node via

$$spawn\ node\ (evalTask\ (inputSenderSender, outputSender))$$

where $spawn$ is of type

---

[7]Here we only evaluate some a. Therefore, we intentionally do not have $(SendPort\ b, ReceivePort\ b)$.

[8]Here the type is a instead of some potentially other b because we only evaluate some a.

$$spawn :: NodeId \rightarrow Closure\ (Process\ ()) \rightarrow Process\ ProcessId$$

Then, like the master from Chapter 6.2.1, *forceSingle* waits for the input *SendPort a* of the evaluation task with *receiveChan inputSenderReceiver*. It then sends the not yet evaluated, serialized version of *a*, *serialized* $\leftarrow$ *liftIO* $ *trySerialize a* over that *SendPort* with *sendChan inputSender* $ *toThunk serialized* to the evaluating slave node. Then, it awaits the result of the evaluation with *forcedA* $\leftarrow$ *receiveChan outputReceiver* to finally put it inside the passed *MVar a* with *liftIO* $ *putMVar out forcedA*.

$$forceSingle :: (Evaluatable\ a) \Rightarrow NodeId \rightarrow MVar\ a \rightarrow a \rightarrow Process\ ()$$
$$forceSingle\ node\ out\ a = \mathbf{do}$$

     -- create the Channel that we use to send the

     -- sender of the input from the slave node from

$$(inputSenderSender, inputSenderReceiver) \leftarrow newChan$$

     -- create the channel to receive the output from

$$(outputSender, outputReceiver) \leftarrow newChan$$

     -- spawn the actual evaluation task on the given node

     -- and pass the two sender objects we created above

$$spawn\ node\ (evalTask\ (inputSenderSender, outputSender))$$

     -- wait for the slave to send the input sender

$$inputSender \leftarrow receiveChan\ inputSenderReceiver$$

$$serialized \leftarrow liftIO\ \$\ trySerialize\ a$$

     -- send the input to the slave

$$sendChan\ inputSender\ \$\ toThunk\ serialized$$

     -- wait for the result from the slave

$$forcedA \leftarrow receiveChan\ outputReceiver$$

     -- put the output back into the passed MVar

$$liftIO\ \$\ putMVar\ out\ forcedA$$

Slave node

In the definition of *forceSingle* we use a function

$$evalTask :: (SendPort\ (SendPort\ (Thunk\ a)), SendPort\ a) \rightarrow Closure\ (Process\ ())$$

As indicated by the *Evaluatable a* in the type signature, this function is hosted on a *Evaluatable a* type class:

$$\mathbf{class}\ (Binary\ a, Typeable\ a, NFData\ a) \Rightarrow Evaluatable\ a\ \mathbf{where}$$
$$\quad evalTask :: (SendPort\ (SendPort\ (Thunk\ a)), SendPort\ a) \rightarrow$$
$$\qquad Closure\ (Process\ ())$$

This abstraction is required because of the way Cloud Haskell does serialization. We can not write a single definition *evalTask* and expect it to work even though it would be a valid definition. This is because for Cloud Haskell to be able to create the required serialization code, at least in our tests, we require a fixed type like for example for *Int*s:

‘$evalTaskInt :: (SendPort\ (SendPort\ (Thunk\ Int)), SendPort\ Int) \rightarrow Closure\ (Process\ ())$‘

This function can be made remotable with $\$(remotable\ ['evalTaskInt])$. We can now write a valid Cloud Haskell compatible instance *Evaluatable Int* simply as

instance *Evaluatable Int* **where**
$\quad evalTask = evalTaskInt$

We do not have to write these manually, though, as as they can easily be generated with the Template Haskell code generator in Figure 10.7 from the Appendix via calls to the following three Template Haskell functions:

$\$\,(mkEvalTasks\ [''Int])$
$\$\,(mkRemotables\ [''Int])$
$\$\,(mkEvaluatables\ [''Int])$

This is possible because *evalTaskInt*, just like any other function on types that have instances for *Binary a*, *Typeable a*, and *NFData a*, can be just delegated to *evalTaskBase*, which behaves as follows: It starty by creating the channel, that it wants to receive its input from, with $(sendMaster, rec) \leftarrow newChan$. Then it sends the *SendPort (Thunk a)* of this channel back to the master process via *sendChan inputPipe sendMaster* to then receive its actual input on the *ReceivePort (Thunk a)* end with $thunkA \leftarrow receiveChan\ rec$. It then deserializes this thunk with $a \leftarrow liftIO\ \$\ deserialize\ \$\ fromThunk\ thunkA$ and sends the fully evaluated result back with $sendChan\ output\ (seq\ (rnf\ a)\ a)$. Its complete definition is

$evalTaskBase :: (Binary\ a, Typeable\ a, NFData\ a) \Rightarrow$
$\quad (SendPort\ (SendPort\ (Thunk\ a)), SendPort\ a) \rightarrow Process\ ()$
$evalTaskBase\ (inputPipe, output) = \textbf{do}$
$\quad (sendMaster, rec) \leftarrow newChan$

$\qquad$-- send the master the SendPort, that we
$\qquad$-- want to listen the other end on for the input
$\quad sendChan\ inputPipe\ sendMaster$

$\qquad$-- receive the actual input
$\quad thunkA \leftarrow receiveChan\ rec$

$\qquad$-- and deserialize
$\quad a \leftarrow liftIO\ \$\ deserialize\ \$\ fromThunk\ thunkA$

```
      -- force the input and send it back to master
    sendChan output (seq (rnf a) a)
```

6.2.3 Parallel Evaluation Scheme

Since we now know how to evaluate a value on slave nodes via *forceSingle* :: (*Evaluatable a*) $\Rightarrow$ *NodeId* $\rightarrow$ *MVar a* $\rightarrow$ *a* $\rightarrow$ *Process* (), we can use this to build up an internal parallel evaluation scheme. For this we start by defining an abstraction of a computation as

```
    data Computation a = Comp {
      computation :: IO (),
      result :: IO a
    }
```

where *computation* :: *IO* () is the *IO* () action that has to be evaluated in order to get a result from *result* :: *IO a*.

Next is the definition of *evalSingle* :: *Evaluatable* $\Rightarrow$ *Conf* $\rightarrow$ *NodeId* $\rightarrow$ *a* $\rightarrow$ *IO* (*Computation a*). Its resulting *IO* action starts by creating an empty *MVar a* with *mvar* $\leftarrow$ *newEmptyMVar*. Then it creates an *IO* action that forks away the evaluation process of *forceSingle* on the single passed value *a* by means of *forkProcess* :: *LocalNode* $\rightarrow$ *Process* () $\rightarrow$ *IO ProcessId* on the the master node with

```
    forkProcess (localNode conf) $ forceSingle node mvar a
```

The action concludes by returning a *Computation a* encapsulating the evaluation *IO* () and the result communication action *takeMVar mvar* :: *IO a*:

```
    evalSingle :: Evaluatable a ⇒ Conf → NodeId → a → IO (Computation a)
    evalSingle conf node a = do
      mvar ← newEmptyMVar
      let comp = forkProcess (localNode conf) $ forceSingle node mvar a
      return $ Comp {
        computation = comp ≫ return ()
        result = takeMVar mvar
        }
```

With this we can easily define a function *evalParallel* :: *Evaluatable a* $\Rightarrow$ *Conf* $\rightarrow$ [*a*] $\rightarrow$ *IO* (*Computation* [*a*]) that builds an *IO* action containing a parallel *Computation* [*a*] from an input list [*a*]. This *IO* action starts by retrieving the current list of workers with *workers* $\leftarrow$ *readMVar* $ *workers conf*. It then continues by shuffling

this list of workers with $shuffledWorkers \leftarrow randomShuffle\ workers^9$ to ensure at least some level of equal work distribution between multiple calls to $evalParallel$. Then, the input values a are assigned to their corresponding workers to finally build the list of parallel computations $[Computation\ a]$ with $comps \leftarrow sequence\ \$$ $map\ (uncurry\ \$\ evalSingle\ conf)\ workAssignment$. The action concludes by turning this list $[Computation\ a]$ into a computation of a list $Computation\ [a]$ with $return\ \$\ sequenceComp\ comps$.

$evalParallel :: Evaluatable\ a \Rightarrow Conf \rightarrow [a] \rightarrow IO\ (Computation\ [a])$
$evalParallel\ conf\ as = \textbf{do}$
　　$workers \leftarrow readMVar\ \$\ workers\ conf$

　　　-- shuffle the list of workers, so we don not end up spawning
　　　-- all tasks in the same order everytime
　　$shuffledWorkers \leftarrow randomShuffle\ workers$

　　　-- complete the work assignment node to task (NodeId, a)
　　$\textbf{let}\ workAssignment = zipWith\ (,)\ (cycle\ shuffledWorkers)\ as$

　　　-- build the parallel computation with sequence
　　$comps \leftarrow sequence\ \$\ map\ (uncurry\ \$\ evalSingle\ conf)\ workAssignment$

　　$return\ \$\ sequenceComp\ comps$

Here, the definition of $sequenceComp :: [Computation\ a] \rightarrow Computation\ [a]$ is

$sequenceComp :: [Computation\ a] \rightarrow Computation\ [a]$
$sequenceComp\ comps = Comp\ \{\ computation = newComp, result = newRes\ \}$
　　$\textbf{where}\ newComp = sequence\_\ \$\ map\ computation\ comps$
　　　　$newRes = sequence\ \$\ map\ result\ comps$

Now, in order to start the actual computation from a blueprint in $Computation\ a$ and get the result back as a pure value a, we have to use the function $runComputation ::$ $IO\ (Computation\ a) \rightarrow a$ defined as follows:

Internally it uses an $IO\ a$ action that starts by unwrapping $Computation\ a$ from the input $IO\ (Computation\ a)$ with $comp \leftarrow x$ to then launch the actual evaluation with $computation\ comp$. The IO action then returns the result with $result\ comp$. Now, in order to turn the $IO\ a$ action into a, we have to use $unsafePerformIO :: IO\ a \rightarrow a$ which allows us to unwrap the pure values "contained" in IO actions. Generally, the use of this function is discouraged, because it can introduce severe bugs if not handled with utmost care. Here, its use is necessary and absolutely fine, though, since we only do evaluation inside the IO Monad and if this were to fail, the computation would be wrong anyways. Also in order to force the compiler to not inline the result – which is generally okay in pure functions but not here as we do

---

[9]$randomShuffle :: [a] \rightarrow IO\ [a]$ is from `https://wiki.haskell.org/Random_shuffle`.

not want to spawn the computation multiple times – we protect the definition of *runComputation* with a *NOINLINE* pragma:

```
{-# NOINLINE runComputation #-}
runComputation :: IO (Computation a) → a
runComputation x = unsafePerformIO $ do
    comp ← x
    computation comp
    result comp
```

6.3 Implementing the PArrows API

Finally, we describe in this Chapter how to implement the PArrows API with Cloud Haskell and evaluate our results.

We start by explaining how to implement *ArrowParallel* in Chapter 6.3.1. Then, we discuss the limits of the current code and explain why we can not yet give a proper instance for *ArrowLoopParallel* or a proper *Future* implementation in Chapter 6.3.2. Finally, we lay out a possible solution to this which could be implemented in the future in Chapter 6.3.3.

6.3.1 *ArrowParallel* instance

We will now give an experimental implementation of the *ArrowParallel* type class with Cloud Haskell. Obviously, as already mentioned earlier, here the additional conf parameter is the *State* / *Conf* type.

We implement *parEvalN* of our *ArrowParallel arr a b Conf* instance as follows: We start off by forcing the input [*a*] into normal form with *arr force* as during testing this was found necessary because a not fully evaluated value *a* can still have attached things like a file handle which may be not serializable. Then, the parallel Arrow goes on to feed the now fully forced input list [*a*] into the evaluation Arrow obtained by applying *evalN* :: [*arr a b*] → *arr* [*a*] [*b*] to the list of Arrows to be parallelized [*arr a b*] with *evalN fs*. This results in a not yet evaluated list of results [*b*] which is prepared to be forked away with *arr* (*evalParallel conf*) :: *arr* [*a*] (*Computation* [*b*]). The resulting computation blueprint is then executed with *arr runComputation* :: *arr* (*Computation* [*b*]) [*b*].

```
instance (NFData a, Evaluatable b, ArrowChoice arr) ⇒
    ArrowParallel arr a b Conf where
        parEvalN conf fs =
```

```
arr force ⋙
evalN fs ⋙
arr (evalParallel conf) ⋙
arr runComputation
```

6.3.2 Limits of the current implementation

Similar to the GpH and *Par* Monad backends, our experimental Cloud Haskell
backend suffers from the problem that it does not work in conjunction with the
looping skeletons *pipe/ring/torus* described in this thesis. All testing programs would
refuse to compute anything and hang indefinitely. While this is no big problem for
the shared-memory backends where we could just implement a workaround with
the help of an *ArrowLoopParallel* instance

```
instance (ArrowChoice arr, ArrowParallel arr a b Conf) ⇒
    ArrowLoopParallel arr a b Conf where
    loopParEvalN _ = evalN
    postLoopParEvalN = parEvalN
```

A similar solution would not be feasible here because we are in a distributed-memory
setting. The topology skeletons would become meaningless as all benefits of using a
sophisticated distributed evaluation scheme would be lost.

Since it would not make sense to have a *Future* instance without proper support for
skeletons that could make use of it, we also do not give an implementation for a
CloudFuture in this thesis.

6.3.3 Possible mitigation of the limits

While investigating the problem with the looping skeletons, we noticed a difference
in behaviour between Eden and all other backends including our experimental Cloud
Haskell backend: Eden streams lists of data $[a]$ instead of sending the complete list
as one big serialized chunk. Another difference is that tuples of data $(a, b, ...)$ are
sent in parallel on n threads for a tuple of n entries. When investigating the *torus*
or *ring* skeletons we ported from Eden, we notice how these two specialities are
important. For example, in the *ring* skeleton we build up the resulting Arrow so that
it calculates the result in multiple rounds:

```
ring conf f =
    loop (second (rightRotate ⋙ lazy) ⋙
            -- convert the current input into a form we can process in this round
        arr (uncurry zip) ⋙
```

```
        -- here, we evaluate the current round
     loopParEvalN conf
         (repeat (second (get conf) ⋙ f ⋙ second (put conf))) ⋙
         -- put the current result back into the original input form
     arr unzip) ⋙
   postLoopParEvalN conf (repeat (arr id))
```

Here, anything other than the exact same behaviour as Eden will result in a dead-lock when using $loopParEvalN = parEvalN$. We therefore believe that it is crucial for a proper Cloud Haskell backend to have the same streaming behaviour as Eden does. While we are confident that this is definitely possible to achieve with Cloud Haskell as early experiments on this suggest. We have to date not been able to achieve proper streaming behaviour, though. We stopped further development here, as this would have bursted the scope of this thesis.

Nevertheless, the most promising idea to implement the correct behaviour is to use a more sophisticated mechanism to stream data back to the master node by using pipes along the lines of

type $PipeIn\ a = SendPort\ (SendPort\ (Maybe\ (SendPort\ (Maybe\ a))))$
type $PipeOut\ a = ReceivePort\ (SendPort\ (Maybe\ (SendPort\ (Maybe\ a))))$

where $PipeIn\ a$ would be the port where the evaluating process on the slave node send its result through to the corresponding $PipeOut\ a$ on the master node. Note that we here encode a "stream" of some a with $SendPort\ (Maybe\ a)$: For types with singular values, we just request one value. And on types like e.g. a list $[a]$ we expect multiple singleton lists $Just\ [a]$, on the $SendPort$ and the end of the input with $Nothing$. For other multi-valued types, this would work similar even if some hacks would be required.

Then in order to communicate the result from the slave node, we would first send $SendPort\ (Maybe\ (SendPort\ (Maybe\ a)))$ on which the slave-node would want to receive the stream of $SendPort\ (Maybe\ a)$. This stream of $SendPort$s is required instead of a singular $SendPort$ because of types that have to be sent by multiple threads like e.g. tuples. Via these $SendPort\ (Maybe\ a)$s the slave can then finally communicate the stream of evaluated results back to the master node. A corresponding communication scheme doing the necessary opposite tasks would obviously be required on the master node.

During testing, as already mentioned, we were not successful in making this idea work with the looping skeletons.[10] We still believe that this path is worth exploring further in the future, though.

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[10]It did however still work with non-looping skeletons.

Experimental performance results | 7

The preceding chapters have shown what PArrows are and how expressive they are. In this Chapter we will now evaluate the performance overhead of our compositional abstraction in comparison to GpH and the Par Monad on shared memory architectures and Eden on a distributed memory cluster.[1] We describe our measurement platform (Chapter 7.1), the benchmark results (Chapter 7.2) – the shared-memory variants (GpH, Par Monad and Eden CP) followed by Eden in a distributed-memory setting, and conclude that PArrows hold up in terms of performance when compared to the original parallel Haskells (Chapter 7.3).

7.1 Measurement platform

We will now explain our measurement platform. We start by explaining the hardware and software stack in Chapter 7.1.1 and outline the benchmark programs and motivation for choosing them in Chapter 7.1.2. Chapter 7.1.3 explains the specifics of where each benchmark was run. We also shortly address hyper-threading and why we do not use it in our benchmarks in Chapter 7.1.4.

7.1.1 Hardware and software

The benchmarks are executed both in a shared and in a distributed memory setting using the Glasgow GPG Beowulf cluster, consisting of 16 machines with 2 Intel® Xeon® E5-2640 v2 and 64 GB of DDR3 RAM each. Each processor has 8 cores and 16 (hyper-threaded) threads with a base frequency of 2 GHz and a turbo frequency of 2.50 GHz. This results in a total of 256 cores and 512 threads for the whole cluster. The operating system was Ubuntu 14.04 LTS with Kernel 3.19.0-33. We found that hyper-threading does not provide any particular interesting insight over using real 16 cores in terms of performance in our benchmarks (numbers here for a single machine) discussed in Chapter 7.1.4. We therefore disregard the hyper-threading ability in most of the cases.

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[1]We do not include the Cloud Haskell backend here, as it is still a work-in-progress.

Apart from Eden, all benchmarks and libraries were compiled with Stack's[2] lts-7.1 GHC compiler which is equivalent to a standard GHC 8.0.1 with the base package in version 4.9.0.0. Stack itself was used in version 1.3.2. For GpH in its Multicore variant we used the *parallel* package in version 3.2.1.0[3], while for the Par Monad we used *monad-par* in version 0.3.4.8[4]. For all Eden tests, we used a manually built GHC-Eden compiler in version 7.8.2[5] together with OpenMPI 1.6.5[6].

Furthermore, all benchmarks were done with help of the bench[7] tool in version 1.0.2 which uses criterion ($>=$1.1.1.0 && $<$ 1.2)[8] internally. All runtime data (mean runtime, max stddev, etc.) was collected with this tool.

We used a single node with 16 real cores as a shared memory test-bed and the whole grid with 256 real cores as a device to test our distributed memory software.

7.1.2 Benchmarks

We measure four benchmarks from different sources. Most of them are parallel mathematical computations, initially implemented in Eden. Table 7.1 summarises.

Table 7.1: The benchmarks we use in this thesis.

Name	Area	Type	Origin	Source
Rabin–Miller test	Mathematics	$parMap + reduce$	Eden	Lobachev (2012)
Jacobi sum test	Mathematics	$workpool + reduce$	Eden	Lobachev (2012)
Gentleman	Mathematics	$torus$	Eden	Loogen et al. (2003)
Sudoku	Puzzle	$parMap$	Par Monad	Marlow et al. (2011)[9]

Rabin–Miller test is a probabilistic primality test that iterates multiple (here: 32–256) "subtests". Should a subtest fail, the input is definitely not a prime. If all n subtest pass, the input is composite with the probability of $1/4^n$.

Jacobi sum test or APRCL is also a primality test, that however, guarantees the correctness of the result. It is probabilistic in the sense that its run time is not certain. Unlike Rabin–Miller test, the subtests of Jacobi sum test have very different

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[2]See `https://www.haskellstack.org/`
[3]See `https://hackage.haskell.org/package/parallel-3.2.1.0`
[4]See `https://hackage.haskell.org/package/monad-par-0.3.4.8`
[5]See `http://www.mathematik.uni-marburg.de/~eden/?content=build_eden_7_&navi=build`
[6]See `https://www.open-mpi.org/software/ompi/v1.6/`
[7]See `https://hackage.haskell.org/package/bench`
[8]See `https://hackage.haskell.org/package/criterion-1.1.1.0`
[9]actual code from: `http://community.haskell.org/~simonmar/par-tutorial.pdf` and `https://github.com/simonmar/parconc-examples`

durations. Lobachev (2011) discusses some optimisations of parallel APRCL. Generic parallel implementations of Rabin–Miller test and APRCL were presented in Lobachev (2012).

"Gentleman" is a standard Eden test program, developed for their *torus* skeleton. It implements a Gentleman's algorithm for parallel matrix multiplication (Gentleman, 1978). We ported an Eden-based version (Loogen et al., 2003) to PArrows and completed the necessary implementations for all sequential parts that were left out.

A parallel Sudoku solver was used by Marlow et al. (2011) to compare *Par* Monad to GpH, we ported it to PArrows.

7.1.3 Which parallel Haskells run where

The *Par* Monad and GpH – in its multicore version (Marlow et al., 2009) – can be executed on shared memory machines only. Although GpH is available on distributed memory clusters, and newer distributed memory Haskells such as HdpH exist, current support of distributed memory in PArrows is limited to Eden. We used the MPI backend of Eden in a distributed memory setting. However, for shared memory Eden features a "CP" backend that merely copies the memory blocks between disjoint heaps. In this mode, Eden still operates in the "nothing shared" setting, but is adapted better to multicore machines. We call this version of Eden "Eden CP".

7.1.4 Effect of hyper-threading

In preliminary tests, the PArrows version of the Rabin-Miller test on a single node of the Glasgow cluster showed almost linear speedup on up to 16 shared-memory cores (as supplementary materials show). The speedup of 64-task PArrows/Eden at 16 real cores version was 13.65 giving a parallel efficiency of 85.3%. However, if we increased the number of requested cores to 32 – i.e. if we use hyper-threading on 16 real cores – the speedup did not increase that well. It was merely 15.99 for 32 tasks with PArrows/Eden. This was worse for other implementations. As for 64 tasks, we obtained a speedup of 16.12 with PArrows/Eden at 32 hyper-threaded cores and only 13.55 with PArrows/GpH.

While this shows that hyper-threading can be of benefit in real-world scenarios running similar workloads to the ones presented in the benchmarks, we only use real cores for the performance measurements in Chapter 7.2 as the purpose of

this chapter is to show the performance of PArrows and not to investigate parallel behaviour with hyper-threading.

7.2 Benchmark results

We compare the PArrow performance with direct implementations of the benchmarks in Eden, GpH and the Par Monad. We start with the definition of speedup (Chapter 7.2.1) and mean overhead (Chapter **??**) to evaluate PArrows-enabled and standard benchmark implementations. We continue by comparing speedups and overheads for the shared memory implementations in Chapter 7.2.3 and then study OpenMPI variants of the Eden-enabled PArrows as a representative of a distributed memory backend in Chapter 7.2.4. We plot all speedup curves and all overhead values in the Appendix in 10.5 and 10.6 for the shared memory and distributed memory benchmarks, respectively.

7.2.1 Defining speedup

In the following, when we talk about speedup, we use the common definition

$$S = \frac{T_1}{T_p}$$

where T_1 denotes the sequential and T_p the parallel runtime of the program. Note that we do not use a separate sequential program, though, instead we simply use the same binary with only 1 computation thread enabled.

7.2.2 Defining overhead

We compare the mean overhead, i.e. the difference of mean relative wall-clock run time between the PArrow and direct benchmark implementations executed multiple times with the same settings. The error margins of the time measurements, supplied by criterion package[10], yield the error margin of the mean overhead.

Quite often the zero value lies in the error margin of the mean overhead. This means that even though we have measured some difference (against or even in favour of PArrows), it could be merely the error margin of the measurement and the difference might not be existent. We are mostly interested in the cases where above issue does

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[10]See `https://hackage.haskell.org/package/criterion-1.1.1.0`

not persist, we call them *significant*. We usually denote the error margin with $\pm$ after the mean overhead value.

7.2.3 Shared memory

Speedup

The Rabin–Miller benchmark showed almost linear speedup for both 32 and 64 tasks, the performance is slightly better in the latter case: 13.7 at 16 cores for input $2^{11213} - 1$ and 64 tasks in the best case scenario with Eden CP. The performance of the Sudoku benchmark merely reaches a speedup of 9.19 (GpH), 8.78 (Par Monad), 8.14 (Eden CP) for 16 cores and 1000 Sudokus. In contrast to Rabin–Miller, here the GpH seems to be the best of all, while Rabin–Miller profited most from Eden CP (i.e. Eden with direct memory copy) implementation of PArrows. Gentleman on shared memory has a plummeting speedup curve with GpH and Par Monad and logarithmically increasing speedup for the Eden-based version. The latter reached a speedup of 6.56 at 16 cores.

Overhead

For the shared memory Rabin–Miller benchmark, implemented with PArrows using Eden CP, GpH, and Par Monad, the overhead values are within single percents range, but also negative overhead (i.e. PArrows are better) and larger error margins happen. To give a few examples, the overhead for Eden CP with input value $2^{11213}-1$, 32 tasks, and 16 cores is 1.5%, but the error margin is around 5.2%! Same implementation in the same setting with 64 tasks reaches -0.2% overhead, PArrows apparently fare better than Eden – but the error margin of 1.9% disallows this interpretation.

We focus now on significant overhead values. To name a few: $0.41\% \pm 7 \cdot 10^{-2}\%$ for Eden CP and 64 tasks at 4 cores; $4.7\% \pm 0.72\%$ for GpH, 32 tasks, 8 cores; $0.34\% \pm 0.31\%$ for Par Monad at 4 cores with 64 tasks. The worst significant overhead was in case of GpH with $8\% \pm 6.9\%$ at 16 cores with 32 tasks and input value $2^{11213} - 1$. In other words, we notice no major slow-down through PArrows here.

For Sudoku the situation is slightly different. There is a minimal significant ($-1.4\% \pm 1.2\%$ at 8 cores) speed *improvement* with PArrows Eden CP version when compared with the base Eden CP benchmark. However, with increasing number of cores the error margin contains zero again: $-1.6\% \pm 5.0\%$ at 16 cores. The Par Monad shows

a similar development, e.g. with $-1.95\% \pm 0.64\%$ at 8 cores. The GpH version shows both a significant speed improvement of $-4.2\% \pm 0.26\%$ (for 16 cores) with PArrows and a minor overhead of $0.87\% \pm 0.70\%$ (4 cores).

The Gentleman multiplication with Eden CP shows a minor significant overhead of $2.6\% \pm 1.0\%$ at 8 cores and an insignificant improvement at 16 cores.

Summarising, we observe a low (if significant at all) overhead, induced by PArrows in the shared memory setting.

7.2.4 Distributed memory

Speedup

The speedup of distributed memory Rabin–Miller benchmark with PArrows and Eden showed an almost linear speedup excepting around 192 cores where an unfortunate task distribution reduces performance. As seen in Figure 7.1, we reached a speedup of 213.4 with PArrrows at 256 cores (vs. 207.7 with pure Eden). Because of memory limitations, the speedup of Jacobi sum test for large inputs (such as $2^{4253} - 1$) could be measured only in a massively distributed setting. PArrows improved there from $9193s$ (at 128 cores) to $1649s$ (at 256 cores). A scaled-down version with input $2^{3217} - 1$ stagnates the speedup at about 11 for both PArrows and Eden for more than 64 cores. There is apparently not enough work for that many cores. The Gentleman test with input 4096 had an almost linear speedup first, then plummeted between 128 and 224 cores, and recovered at 256 cores with speedup of 129.

Overhead

We use our mean overhead quality measure and the notion of significance also for distributed memory benchmarks. The mean overhead of the Rabin–Miller test in here ranges from 0.29% to -2.8% (last value in favour of PArrows), but these values are not significant with error margins $\pm 0.8\%$ and $\pm 2.9\%$ correspondingly. A sole significant (by a very low margin) overhead is $0.35\% \pm 0.33\%$ at 64 cores. We measured the mean overhead for Jacobi benchmark for an input of $2^{3217} - 1$ for up to 256 cores. We reach the flattering value $-3.8\% \pm 0.93\%$ at 16 cores in favour of PArrows, it was the sole significant overhead value. The value for 256 cores was $0.31\% \pm 0.39\%$. Mean overhead for distributed Gentleman multiplication was also low. Significant values include $1.23\% \pm 1.20\%$ at 64 cores and $2.4\% \pm 0.97\%$ at 256 cores. It took PArrows 64.2 seconds at 256 cores to complete the benchmark.

Speedup of distributed Rabin–Miller test 44497 256
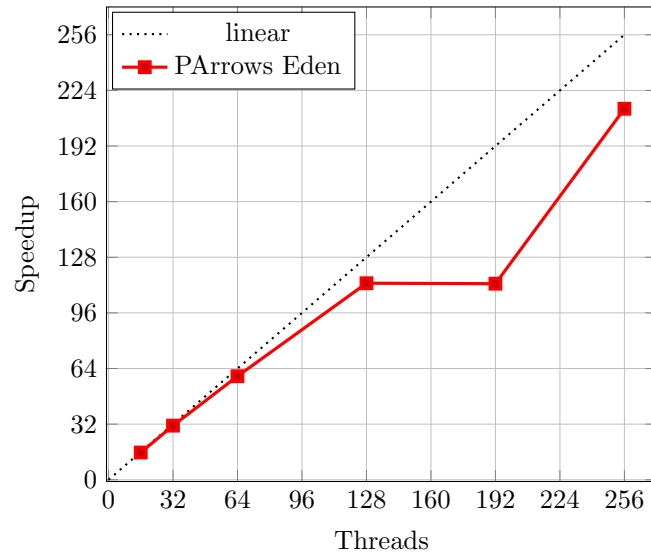
Figure 7.1: Speedup of the distributed Rabin–Miller benchmark using PArrows with Eden.

Similar to the shared memory setting, PArrows only imply a very low penalty with distributed memory that lies in lower single-percent digits at most.

7.3 Evaluation of results

Table 7.2: Overhead in the shared memory benchmarks. Bold marks values in favour of PArrows.

Benchmark	Base	Mean of mean overheads	Maximum normalised stdDev	Runtime for 16 cores (s)
Sudoku 1000	Eden CP	**-2.1%**	5.1%	1.17
	GpH	**-0.82%**	0.7%	1.11
	Par Monad	**-1.3%**	2.1%	1.14
Gentleman 512	Eden CP	*0.81%*	6.8%	1.66
Rabin–Miller 11213 32	Eden CP	*0.79%*	5.2%	5.16
	GpH	*3.5%*	6.9%	5.28
	Par Monad	**-2.5%**	19.0%	5.84
Rabin–Miller 11213 64	Eden CP	*0.21%*	1.9%	10.3
	GpH	*1.6%*	1.3%	10.6
	Par Monad	**-4.0%**	17.0%	11.4

Table 7.3: Overhead in the distributed memory benchmarks. Bold marks values in favour of PArrows.

Benchmark	Base	Mean of mean overheads	Maximum normalised stdDev	Runtime for 256 cores (s)
Gentleman 4096	Eden	*0.67%*	1.5%	110.0
Rabin–Miller 44497 256	Eden	**-0.5%**	2.9%	165.0
Jacobi Test 3217	Eden	**-0.74%**	1.6%	635.0

PArrows performed in our benchmarks with little to no overhead. Tables 7.2 and 7.3 clarify this once more: The PArrows-enabled versions trade blows with their vanilla counterparts when comparing the means of the mean overheads over all different core counts. If we combine these findings with the benefits of our DSL, the minor

overhead induced by PArrows is outweighed by their convenience and usefulness to the user.

Discussion

We will now discuss whether we have achieved the goals for our Arrow based parallel Haskells that we have set ourselves in the introduction in Chapter 1. We there described that we wanted

- a DSL that allows us to parallelize arbitrary Arrow types
- to tame the zoo of parallel Haskells.
- low performance penalty
- generality by being able to switch implementations at will

We will now discuss whether we have met these requirements in the results presented by this thesis.

Parallelizing arbitrary Arrow Types

Our PArrows API generally succeeded in providing parallelism for arbitrary Arrow types, but only those which fulfill certain type classes. There are two restrictions to that, though:

- Most of our API as well as the actual instances for our type class *ArrowParallel* in the case of GpH and the *Par* Monad require an *ArrowChoice* instance. Additionally, we require *ArrowLoop* for the looping skeletons. While this indeed restricts the set of suitable Arrows, it is not a restriction in which Arrows we can parallelize specifically, but more of a restriction on what type of Arrow can be parallelized. Furthermore, both type classes are an integral part in the code that uses them. For example, without *ArrowChoice* we could not express the recursion with Arrows as without it we can not express the recursion anchor in e.g. the definition of *evalN* (Figure 3.6). The same goes for *ArrowLoop* as it is required in order to have the know-tying fix-point semantics we require for our topology skeletons. In fact, Hughes (2005) mentions this as well in writing "there is little interesting that can be done without more operations on Arrows than just composition".

- The Eden backend currently has no general implementation for *ArrowParallel*. As explained earlier in Chapter 4.2.3, the reason for this is that Eden's spawn function *spawnF* only works on function $((\rightarrow))$ and therefore resorted to having manual implementations of *ArrowParallel* for every type. As noted in the same Chapter however, this seems to be no real issue as a possible general implementation is possible. This has to be evaluated with more involved tests in the future, though.

Summarizing we can say that we succeeded in our goal to provide support for parallelizing arbitrary Arrows as these two restrictions are no big issues at all as we have explained here.

Taming the zoo of parallel Haskells

In this thesis, we at least showed how we can tame at least the three parallel Haskells we used as backends – GpH, the *Par* Monad and Eden. We even included the blue print for a new backend based upon Cloud Haskell. Therefore, we are confident that other parallel Haskells can be used as backends in our DSL even if they require some special care.[1] With the PArrows DSL we are therefore able to tame the zoo of parallel Haskells.

Low Penalty interface

As our benchmarks in Chapter 7 show, our PArrows DSL did only induce very low overhead – i.e. never more than $8\% \pm 6.9\%$ and typically under 2%. Typically, the mean over all cores of relative mean overhead was less than 3.5% and less than 0.8% for all benchmarks with GpH and Eden, respectively. As for the *Par* Monad, the mean of mean overheads was in favour of PArrows in all benchmarks (Chapter 7). The PArrows DSL is therefore a very low penalty interface when compared to the native backends.

Generality – Switching Implementations at will

Because of the way we designed our central *ArrowParallel* type class in Chapter 4.1, we can truly switch between backend implementations at will and therefore

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[1]Like we have talked about in the case of HdpH, which heavily relied on Template Haskell to work. This Template Haskell code was however incompatible with PArrows and would need replacing were HdpH used as a backend

programs are portable between parallel Haskell implementations. The only thing that has to be done when switching between implementations, is changing the import statement. Implementation specifics such as different config types are well hidden away from the user with the help of default configuration instances, but can be accessed if required. The only thing that is not covered by our implementations is the need for specific transport logic for distributed backends for non standard data types (like Eden or in the experimental Cloud Haskell backend). These are easily implemented with default instances (*Trans* in Eden) or Template Haskell (Cloud Haskell).

Summary

Summarizing we can say that we have fulfilled the requirements set in the Introduction of this thesis. PArrows is a DSL that allows us to parallelize arbitrary Arrow types that allows us to tame the zoo of parallel Haskells while having a low performance penalty and is general by allowing to switch implementations at will.

By proving these properties, we have shown that for a generic and extensible parallel Haskell we do not have to restrict ourselves to a monadic interface.

We believe that Arrows are a better fit to parallelise pure code than a monadic solution as regular functions are already Arrows and can be used with our DSL in a more natural way while retaining all the composability. The benefit of being able to parallelize arbitrary Arrows that have the proper functionality (*ArrowChoice*/*ArrowLoop*) in a similar manner is also obvious. Additionally, the DSL still allows for a direct parallelisation of monadic code via the Kleisli type.

Conclusion

Arrows are a generic concept that allows for powerful composition combinators. To our knowledge we are first to represent *parallel* computation with Arrows, and hence to show their usefulness for composing parallel programs.

In this thesis, we proposed an Arrow-based encoding for parallelism based on a new Arrow combinator $parEvalN :: [arr\ a\ b] \rightarrow arr\ [a]\ [b]$. A parallel Arrow is still an Arrow, hence the resulting parallel Arrow can still be used in the same way as a potential sequential version. We evaluated the expressive power of such a formalism in the context of parallel programming.

We introduced a parallel evaluation formalism using Arrows. One big advantage of this specific approach is that we do not have to introduce any new types, facilitating composability (Chapter 4). These PArrow programs can readily exploit multiple parallel language implementations. We demonstrated the use of GpH, a Par Monad, and Eden. We did not re-implement all the parallel internals, as this functionality is hosted in the $ArrowParallel$ type class, which abstracts all parallel implementation logic. The implementations can easily be swapped, so we are not bound to any specific one.

This has many practical advantages. For example, during development we can run the program in a simple GHC-compiled variant using GpH and afterwards deploy it on a cluster by converting it into an Eden program, by just replacing the $ArrowParallel$ instance and compiling with Eden's GHC variant (Chapter 4).

Next, we extended the PArrows formalism with *Future*s to enable direct communication of data between nodes in a distributed memory setting similar to Eden's Remote Data (Dieterle et al., 2010b). Direct communication is useful in a distributed memory setting because it allows for inter-node communication without blocking the master-node. (Chapter 5.1)

Subsequently, we demonstrated the expressiveness of PArrows by using them to define common algorithmic skeletons (Chapters 4.4, 5.2), and by using these skeletons to implement four benchmarks (Chapter 7).

We also developed an experimental Cloud Haskell backend in Chapter 6 as a possible PArrows backend with support for the recent trends in cloud computing. This in an early proof of concept stage at the moment.

Then, we practically demonstrated that Arrow parallelism has a low performance overhead compared with existing approaches, with only some negligible performance hits (Chapter 7).

Finally, we discussed in Chapter 8 how we accomplished in fulfilling the requirements we deemed important in the introduction, namely that we wanted a DSL that allows us to parallelize arbitrary Arrow types while taming the zoo of parallel Haskells and having a low performance penalty all while being general and allowing to switch implementations at will.

9.1 Future work

Our PArrows DSL can be expanded to other task parallel Haskells, and a specific target is HdpH (Maier et al., 2014). Further Future-aware versions of Arrow combinators can be defined. Existing combinators could also be improved, for example more specialised versions of $\ggg$ and $***$ combinators are viable.

In ongoing work we are expanding both our skeleton library and the number of skeleton-based parallel programs that use our DSL. It would also be interesting to see a hybrid of PArrows and Accelerate (McDonell et al., 2015). Ports of our approach to other languages such as Frege, Eta, or Java directly are at an early development stage.

Appendix

<div style="text-align: right; font-size: 3em;">10</div>

Following are additional chapters with supplementary information for this thesis. Next, Chapter 10.1 explains how specific Profunctors fit the Arrow type class. Chapter 10.2 covers omitted function definitions. Then, Chapter 10.3 explains syntactic sugar for PArrows. We give additional definitions for the experimental Cloud Haskell backend in Chapter 10.4 and end with the plots for the shared memory backends and distributed memory backends in Chapters 10.5 and 10.6, respectively.

10.1 Profunctor Arrows

In Figure 10.1 we show how specific Profunctors can it into the Arrow type class. This works because Arrows are strong Monads in the bicategory $Prof$ of Profunctors as shown by Asada (2010). In Standard GHC $(\ggg)$ has the type $(\ggg) :: Category\ cat \Rightarrow cat\ a\ b \rightarrow cat\ b\ c \rightarrow cat\ a\ c$ and is therefore not part of the $Arrow$ type class like presented in this thesis.[1]

> **instance** $(Category\ p, Strong\ p) \Rightarrow Arrow\ p$ **where**
> $arr\ f = dimap\ id\ f\ id$
> $first = first'$
> **instance** $(Category\ p, Strong\ p, Costrong\ p) \Rightarrow ArrowLoop\ p$ **where**
> $loop = loop'$
> **instance** $(Category\ p, Strong\ p, Choice\ p) \Rightarrow ArrowChoice\ p$ **where**
> $left = left'$

Figure 10.1: Profunctors as Arrows.

10.2 Additional function definitions

We have omitted some function definitions in the main text for brevity, and redeem this here.

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[1]For additional information on the type classes used, see: `https://hackage.haskell.org/package/profunctors-5.2.1/docs/Data-Profunctor.html` and `https://hackage.haskell.org/package/base-4.9.1.0/docs/Control-Category.html`.

$$shuffle :: (Arrow\ arr) \Rightarrow arr\ [[a]]\ [a]$$
$$shuffle = arr\ (concat \circ transpose)$$
$$unshuffle :: (Arrow\ arr) \Rightarrow Int \rightarrow arr\ [a]\ [[a]]$$
$$unshuffle\ n = arr\ (\lambda xs \rightarrow [takeEach\ n\ (drop\ i\ xs)\ |\ i \leftarrow [0 \mathinner{..} n-1]])$$
$$takeEach :: Int \rightarrow [a] \rightarrow [a]$$
$$takeEach\ n\ [] = []$$
$$takeEach\ n\ (x:xs) = x : takeEach\ n\ (drop\ (n-1)\ xs)$$

Figure 10.2: *shuffle*, *unshuffle*, *takeEach* definition.

$$lazy :: (Arrow\ arr) \Rightarrow arr\ [a]\ [a]$$
$$lazy = arr\ (\lambda{\sim}(x:xs) \rightarrow x : lazy\ xs)$$
$$rightRotate :: (Arrow\ arr) \Rightarrow arr\ [a]\ [a]$$
$$rightRotate = arr\ \$\ \lambda list \rightarrow \textbf{case}\ list\ \textbf{of}$$
$$\quad [] \rightarrow []$$
$$\quad xs \rightarrow last\ xs : init\ xs$$

Figure 10.3: *lazy* and *rightRotate* definitions.

We begin with Arrow versions of Eden's *shuffle*, *unshuffle* and the definition of *takeEach* can be found in Figure 10.2. Similarly, Figure 10.3 contains the definition of Arrow versions of Eden's *lazy* and *rightRotate* utility functions. Figure 10.4 contains Eden's definition of *lazyzip3* together with the utility functions *uncurry3* and *threetotwo*. The full definition of *farmChunk* is in Figure 4.13. Eden definition of *ring* skeleton is in Figure 10.5. It follows Loogen (2012).

Furthermore, Figure 10.6 contains the omitted definitions required for parallel matrix multiplication with the *torus* skeleton. They are: *prMM* (sequential matrix multiplication), *splitMatrix* (which splits a matrix into chunks), *staggerHorizontally* and *staggerVertically* (to pre-rotate the matrices), and lastly *matAdd*, which calculates $A + B$ for two matrices A and B.

$$lazyzip3 :: [\,a\,] \to [\,b\,] \to [\,c\,] \to [\,(a, b, c)\,]$$
$$lazyzip3 \; as \; bs \; cs = zip3 \; as \; (lazy \; bs) \; (lazy \; cs)$$
$$uncurry3 :: (a \to b \to c \to d) \to (a, (b, c)) \to d$$
$$uncurry3 \; f \; (a, (b, c)) = f \; a \; b \; c$$
$$threetotwo :: (Arrow \; arr) \Rightarrow arr \; (a, b, c) \; (a, (b, c))$$
$$threetotwo = arr \; \$ \; \lambda{\sim}(a, b, c) \to (a, (b, c))$$

Figure 10.4: *lazyzip3*, *uncurry3* and *threetotwo* definitions.

$$ringSimple :: (Trans \; i, Trans \; o, Trans \; r) \Rightarrow (i \to r \to (o, r)) \to [\,i\,] \to [\,o\,]$$
$$ringSimple \; f \; is = os$$
$$\quad \textbf{where} \; (os, ringOuts) = unzip \; (parMap \; (toRD \; \$ \; uncurry \; f) \; (zip \; is \; \$ \; lazy \; ringIns))$$
$$\quad\quad ringIns = rightRotate \; ringOuts$$

$$toRD :: (Trans \; i, Trans \; o, Trans \; r) \Rightarrow ((i, r) \to (o, r)) \to ((i, RD \; r) \to (o, RD \; r))$$
$$toRD \; f \; (i, ringIn) = (o, release \; ringOut)$$
$$\quad \textbf{where} \; (o, ringOut) = f \; (i, fetch \; ringIn)$$

$$rightRotate :: [\,a\,] \to [\,a\,]$$
$$rightRotate \; [\,] = [\,]$$
$$rightRotate \; xs = last \; xs : init \; xs$$

$$lazy :: [\,a\,] \to [\,a\,]$$
$$lazy{\sim}(x : xs) = x : lazy \; xs$$

Figure 10.5: Eden's definition of the *ring* skeleton.

$$prMM :: Matrix \to Matrix \to Matrix$$
$$prMM \; m1 \; m2 = prMMTr \; m1 \; (transpose \; m2)$$
$$\quad \textbf{where}$$
$$\quad\quad prMMTr \; m1' \; m2' = [\,[\,sum \; (zipWith \; (*) \; row \; col) \mid col \leftarrow m2'\,] \mid row \leftarrow m1'\,]$$
$$splitMatrix :: Int \to Matrix \to [\,[\,Matrix\,]\,]$$
$$splitMatrix \; size \; matrix = map \; (transpose \circ map \; (chunksOf \; size)) \; \$ \; chunksOf \; size \; \$ \; matrix$$
$$staggerHorizontally :: [\,[\,a\,]\,] \to [\,[\,a\,]\,]$$
$$staggerHorizontally \; matrix = zipWith \; leftRotate \; [\,0 \,..\,] \; matrix$$
$$staggerVertically :: [\,[\,a\,]\,] \to [\,[\,a\,]\,]$$
$$staggerVertically \; matrix = transpose \; \$ \; zipWith \; leftRotate \; [\,0 \,..\,] \; (transpose \; matrix)$$
$$leftRotate :: Int \to [\,a\,] \to [\,a\,]$$
$$leftRotate \; i \; xs = xs2 \; {+}\!\!{+} \; xs1 \; \textbf{where}$$
$$\quad (xs1, xs2) = splitAt \; i \; xs$$
$$matAdd = chunksOf \; (dimX \; x) \; \$ \; zipWith \; (+) \; (concat \; x) \; (concat \; y)$$

Figure 10.6: *prMMTr*, *splitMatrix*, *staggerHorizontally*, *staggerVertically* and *matAdd* definition.

10.3 Syntactic sugar

Next, we also give the definitions for some syntactic sugar for PArrows, namely
$|***|$ and $|\&\&\&|$. For basic Arrows, we have the $***$ combinator (Figure 3.5) which
allows us to combine two Arrows $arr\ a\ b$ and $arr\ c\ d$ into an Arrow $arr\ (a, c)\ (b, d)$
which does both computations at once. This can easily be translated into a parallel
version $|***|$ with the use of $parEval2$, but for this we require a backend which has
an implementation that does not require any configuration (hence the $()$ as the $conf$
parameter:

$$(|***|) :: (ArrowChoice\ arr, ArrowParallel\ arr\ (Either\ a\ c)\ (Either\ b\ d)\ ())) \Rightarrow$$
$$arr\ a\ b \rightarrow arr\ c\ d \rightarrow arr\ (a, c)\ (b, d)$$
$$(|***|) = parEval2\ ()$$

We define the parallel $|\&\&\&|$ in a similar manner to its sequential pendant $\&\&\&$
(Figure 3.5):

$$(|\&\&\&|) :: (ArrowChoice\ arr, ArrowParallel\ arr\ (Either\ a\ a)\ (Either\ b\ c)\ ()) \Rightarrow$$
$$arr\ a\ b \rightarrow arr\ a\ c \rightarrow arr\ a\ (b, c)$$
$$(|\&\&\&|)\ f\ g = (arr\ \$\ \lambda a \rightarrow (a, a)) \ggg f\ |***|\ g$$

10.4 Experimental Cloud Haskell backend code

Finally, we include the Template Haskell based code generator to make the experimental Cloud Haskell backend easier to use and a version of the main Sudoku benchmark program as an example.

The code generator can be found in Figure 10.7. Here, if we enclose this in a Haskell module, the functions $mkEvalTasks$ (to generate the $evalTask$s for the specific types), $mkRemotables$ (to mark the evaluation tasks as remotable in Cloud Haskell) and $mkEvaluatables$ (to create the $Evaluatable$ instance) are the ones exposed to the user.

The Template Haskell version of the main Sudoku benchmark program can be found in Figure 10.8[2]. We have to write type aliases for $Maybe\ Grid$ ($MaybeGrid$) and $[Maybe\ Grid]$ ($MaybeGridList$). We can then use these to generate the code required to to evaluate these types in the Cloud Haskell backend with. In the $main$ program we have two cases: a) the program is started in master mode and starts the computation, b) the program is started in slave mode and waits for computation requests. In order to launch this program and have speedup as well, we have to start slave nodes for

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[2]For the full code, see the GitHub repository at `https://github.com/s4ke/Parrows/blob/ e1ab76018448d9d4ca3ed48ef1f0c5be26ae34ab/CloudHaskell/testing/Test.hs`

each cpu core with commands like "<executable> slave 127.0.0.1 8000" where the last parameter determines the port the slave will listen to and wait for requests on. Similarly a single master node can be started with "<executable> master 127.0.0.1 7999" where, once again, the last parameter determines the communication port.

$$nested :: Type \rightarrow Type \rightarrow Type$$
$$nested\ a\ b = a\ `AppT`\ (ParensT\ b)$$
$$tuple2 :: Type \rightarrow Type \rightarrow Type$$
$$tuple2\ a\ b = (TupleT\ 2\ `AppT`\ a)\ `AppT`\ b$$
$$fn :: Type \rightarrow Type \rightarrow Type$$
$$fn\ a\ b = (ArrowT\ `AppT`\ a)\ `AppT`\ b$$
$$nameToFnName :: Name \rightarrow Name$$
$$nameToFnName\ (Name\ (OccName\ str)\ \_) = mkName\ \$\ ("\_\_"\ +\!\!\!+\ str\ +\!\!\!+\ "\_evalTaskImpl")$$
$$evalTaskFn :: Name \rightarrow Name \rightarrow Q\ [Dec]$$
$$evalTaskFn\ typeName\ fnName = \textbf{do}$$
  $\textbf{let}\ sendPort = ConT\ \$\ mkName\ \text{"SendPort"}$
    $thunk = ConT\ \$\ mkName\ \text{"Thunk"}$
    $process = ConT\ \$\ mkName\ \text{"Process"}$
    $firstTup = (sendPort\ `nested`\ (sendPort\ `nested`\ (thunk\ `nested`\ (ConT\ typeName))))$
    $secondTup = sendPort\ `nested`\ (ConT\ typeName)$
    $procNil = process\ `AppT`\ (TupleT\ 0)$
  $return\ [$
    $SigD\ fnName\ ((firstTup\ `tuple2`\ secondTup)\ `fn`\ procNil),$
    $FunD\ fnName\ [Clause\ []\ (NormalB\ (VarE\ \$\ mkName\ \text{"evalTaskBase"}))\ []]$
    $]$
$$evaluatableInstance :: Name \rightarrow Name \rightarrow Q\ [Dec]$$
$$evaluatableInstance\ typeName\ fnName = \textbf{do}$$
  $\textbf{let}\ evaluatable = ConT\ \$\ mkName\ \text{"Evaluatable"}$
  $closure \leftarrow mkClosure\ fnName$
  $return\ [$
    $InstanceD\ (Nothing)\ []\ (evaluatable\ `nested`\ ConT\ typeName)\ [$
      $FunD\ (mkName\ \text{"evalTask"})\ [Clause\ []\ (NormalB\ closure)\ []]$
    $]$
    $]$
$$mkEvalTasks :: [Name] \rightarrow Q\ [Dec]$$
$$mkEvalTasks\ names = \textbf{do}$$
  $\textbf{let}\ fnNames = map\ nameToFnName\ names$
  $(mapM\ (uncurry\ evalTaskFn)\ (zipWith\ (,)\ names\ fnNames)) \ggg (return \circ concat)$
$$mkRemotables :: [Name] \rightarrow Q\ [Dec]$$
$$mkRemotables\ names = \textbf{do}$$
  $\textbf{let}\ fnNames = map\ nameToFnName\ names$
  $remotable\ fnNames$
$$mkEvaluatables :: [Name] \rightarrow Q\ [Dec]$$
$$mkEvaluatables\ names = \textbf{do}$$
  $\textbf{let}\ fnNames = map\ nameToFnName\ names$
  $(mapM\ (uncurry\ evaluatableInstance)\ (zipWith\ (,)\ names\ fnNames)) \ggg (return \circ concat)$

Figure 10.7: The Template Haskell code generator for the Cloud Haskell backend.

```haskell
type MaybeGrid = Maybe Grid
type MaybeGridList = [Maybe Grid]
  -- remotable declaration for all eval tasks
$ (mkEvalTasks ["MaybeGrid,"MaybeGridList])
$ (mkRemotables ["MaybeGrid,"MaybeGridList])
$ (mkEvaluatables ["MaybeGrid,"MaybeGridList])

myRemoteTable :: RemoteTable
myRemoteTable = Main.__remoteTable initRemoteTable

main :: IO ()
main = do
  args ← getArgs
  case args of
    ["master", host, port] → do
      conf ← startBackend myRemoteTable Master host port
      readMVar (workers conf) ≫ print
      grids ← fmap lines $ readFile "sudoku.txt"
      print (length (filter isJust (farm conf 4 solve grids)))
    ["slave", host, port] → do
      startBackend myRemoteTable Slave host port
      print "slave shutdown."
```

Figure 10.8: The Template Haskell version of the Sudoku benchmark program.

10.5 Plots for the shared memory benchmarks

Following are all remaining plots for the shared memory benchmarks we conducted for this thesis.
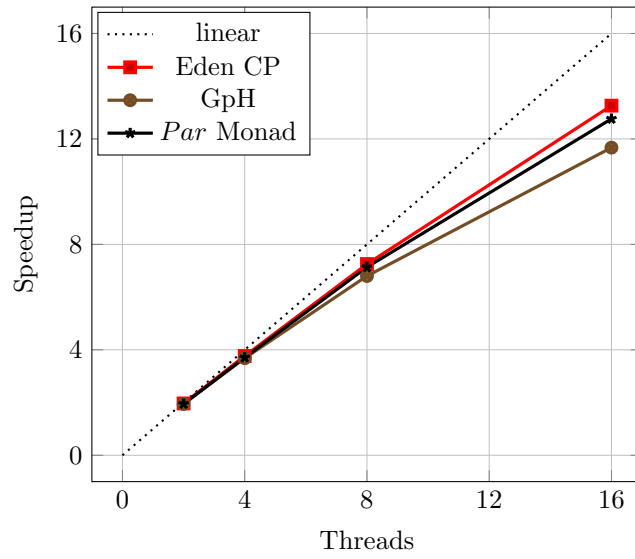
Figure 10.9: Parallel speedup of shared-memory Rabin–Miller test "11213 32".
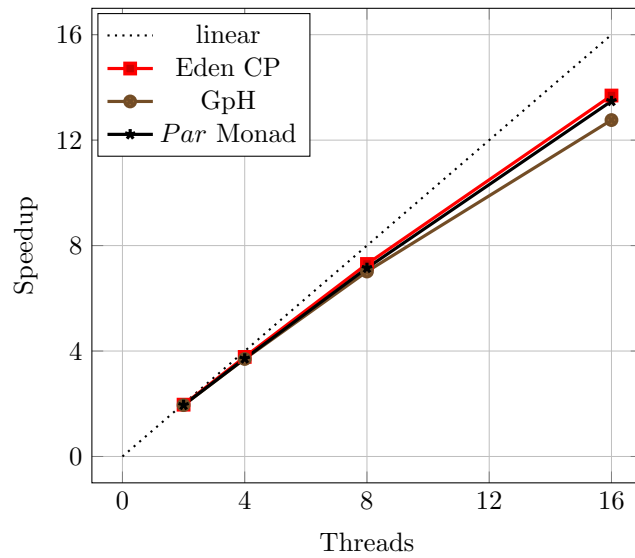
Figure 10.10: Parallel speedup of shared-memory Rabin–Miller test "11213 64".
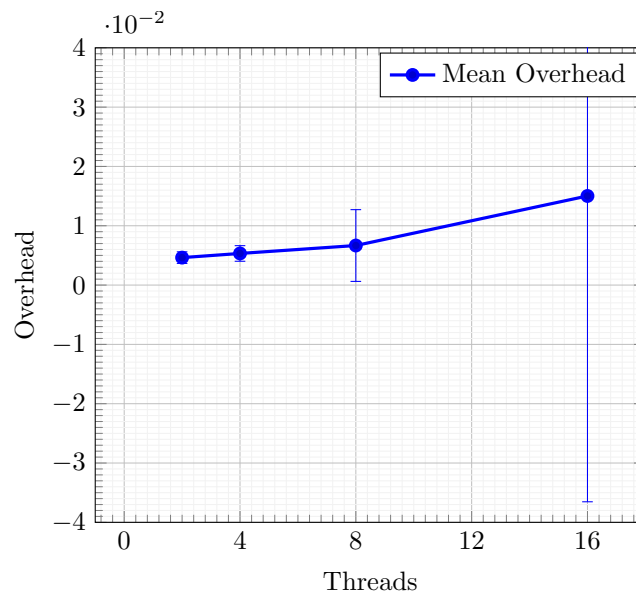
Figure 10.11: Mean overhead for shared-memory Rabin—Miller test "11213 32" vs Eden CP.
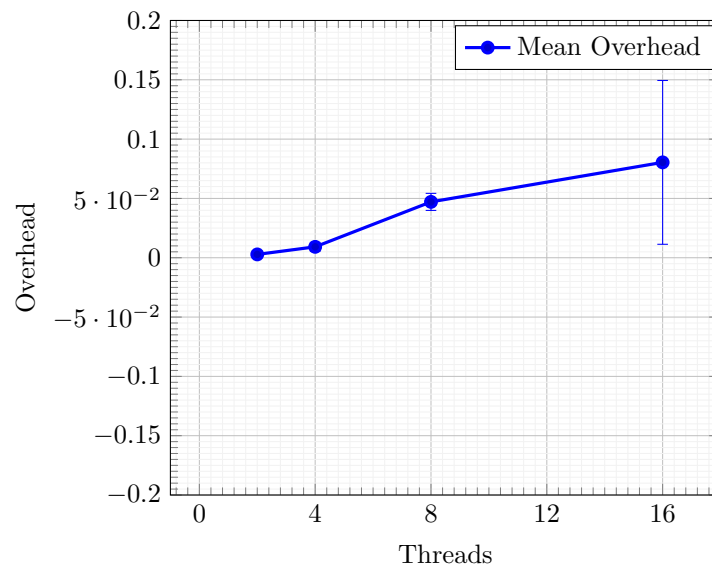
Figure 10.12: Mean overhead for shared-memory Rabin—Miller test "11213 32" vs GpH.
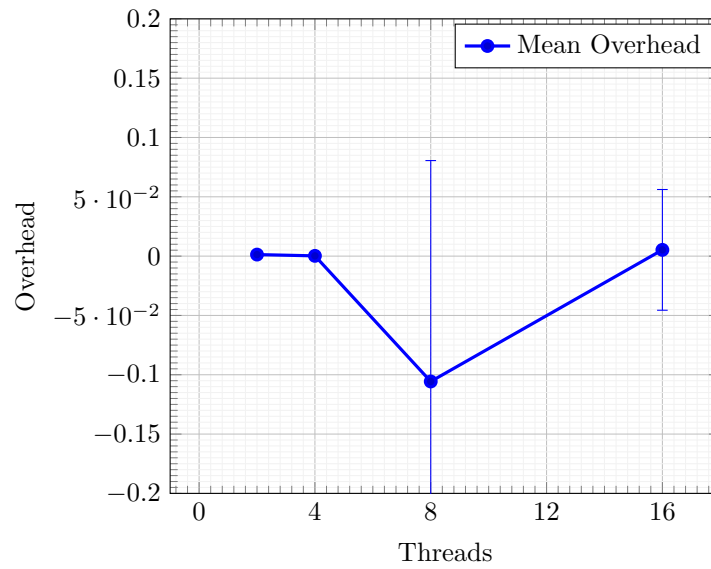
Figure 10.13: Mean overhead for shared-memory Rabin—Miller test "11213 32" vs *Par* monad.
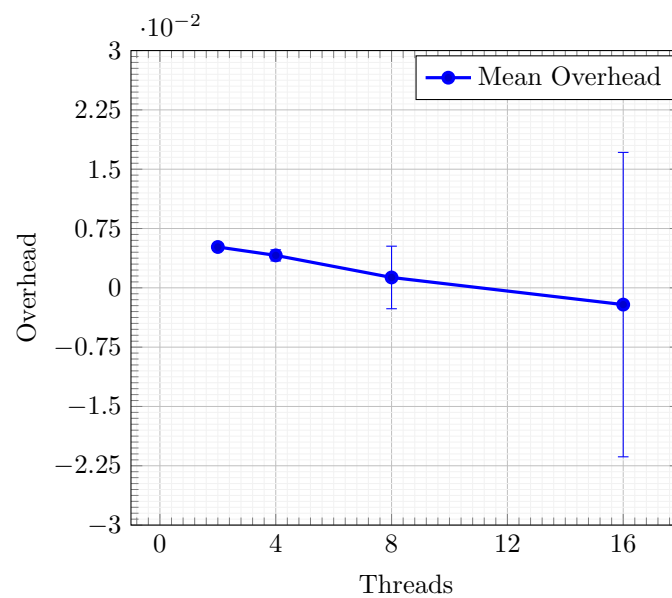
Figure 10.14: Mean overhead for shared-memory Rabin—Miller test "11213 64" vs Eden CP.
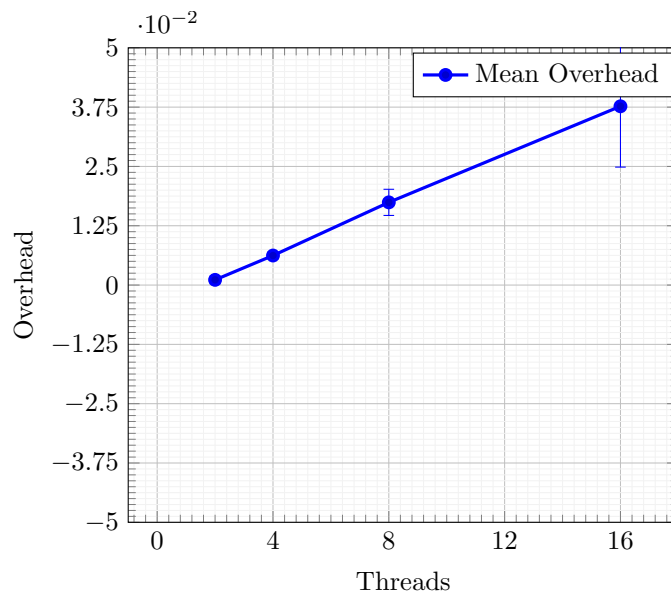
Figure 10.15: Mean overhead for shared-memory Rabin—Miller test "11213 64" vs GpH.
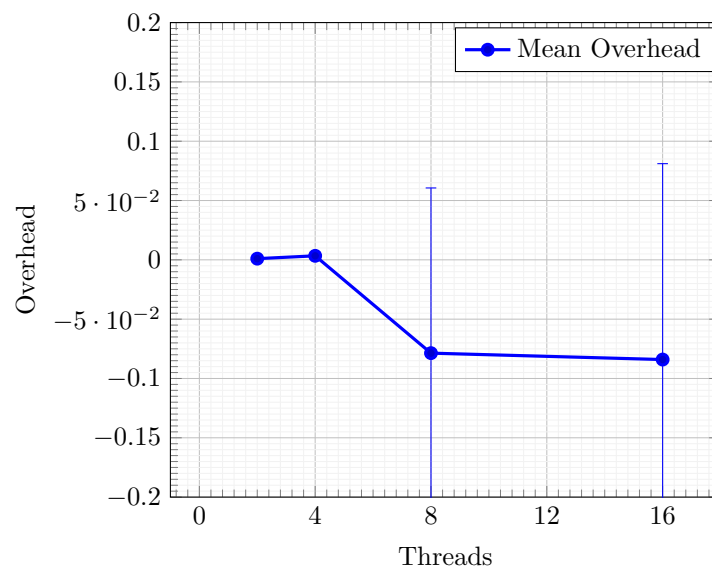
Figure 10.16: Mean overhead for shared-memory Rabin—Miller test "11213 64" vs *Par* Monad.
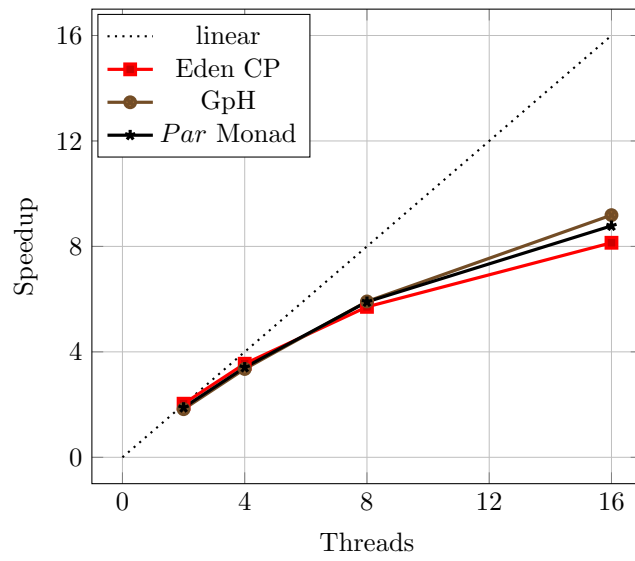
Figure 10.17: Parallel speedup of shared-memory Sudoku "1000".
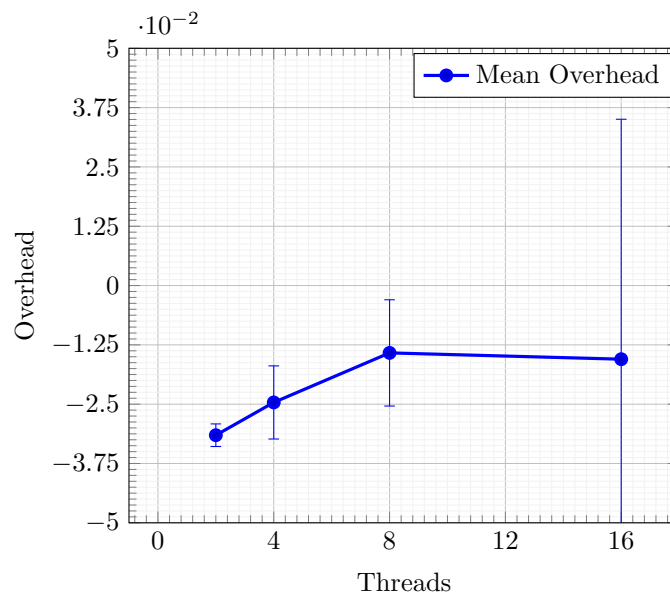
Figure 10.18: Mean overhead for shared-memory Sudoku "1000" vs Eden CP.
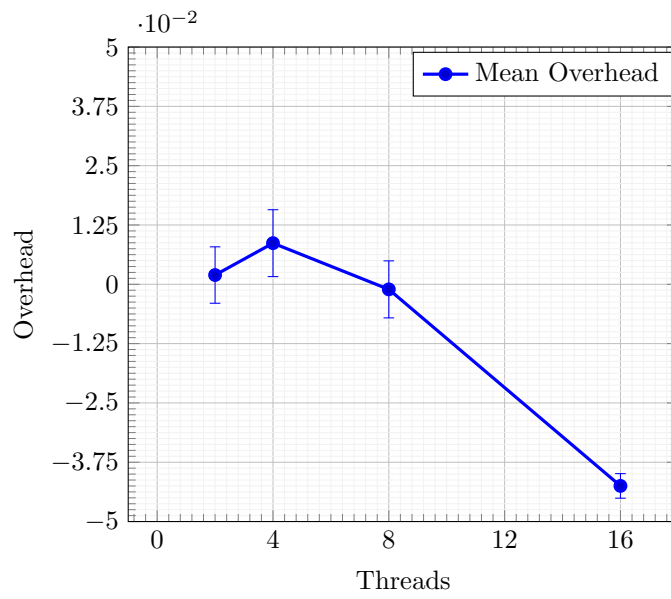
Figure 10.19: Mean overhead for shared-memory Sudoku "1000" vs GpH.
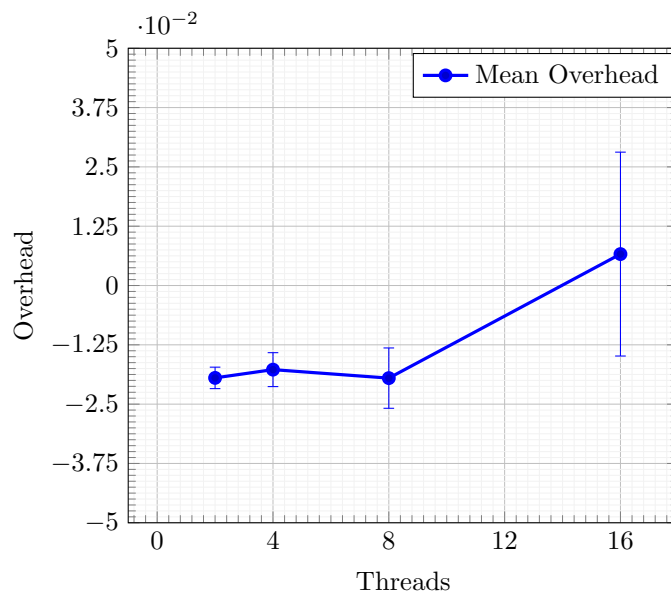
Figure 10.20: Mean overhead for shared-memory Sudoku "1000" vs Par Monad.
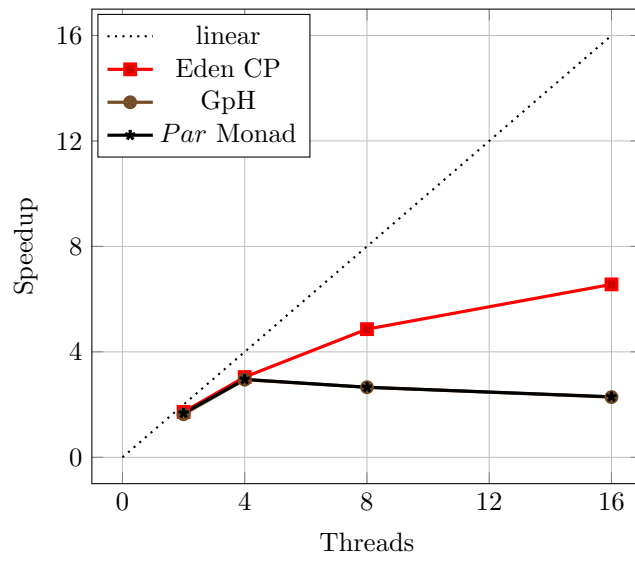
Figure 10.21: Parallel speedup of shared-memory Gentleman "512".
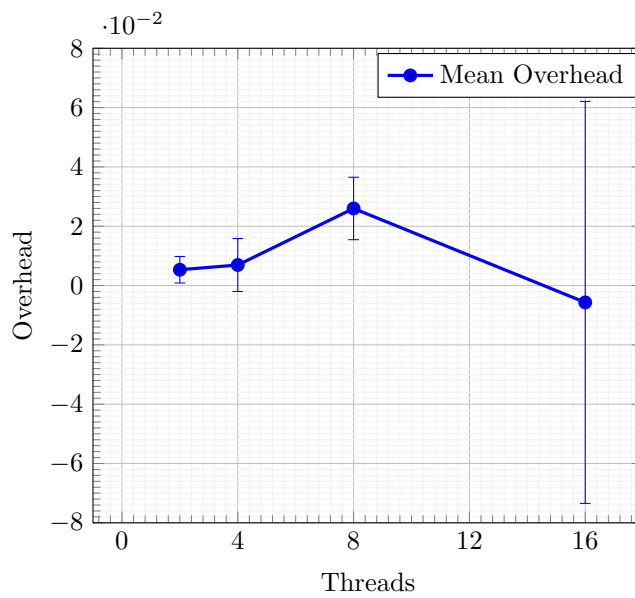
Figure 10.22: Mean overhead for shared-memory speedup of Gentleman "512" vs Eden CP.

10.6 Plots for the distributed memory benchmarks

Just like for the shared memory benchmarks, we here depict all the remaining plots for the distributed memory benchmarks.
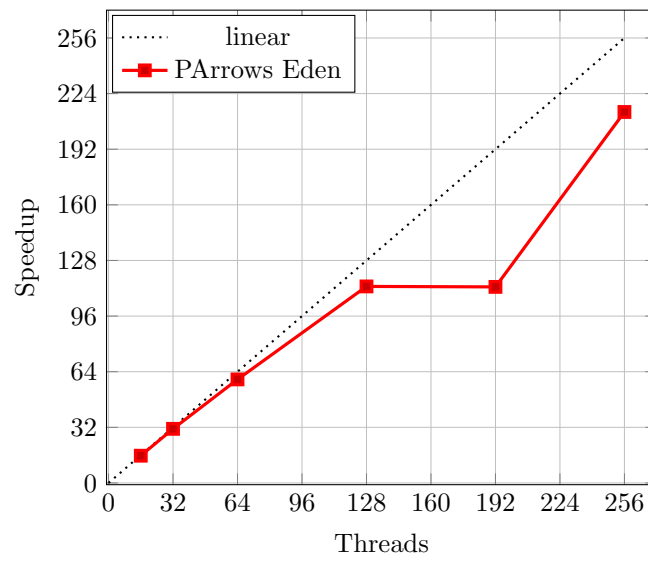
Figure 10.23: Parallel speedup of distributed-memory Rabin—Miller test "44497 256".
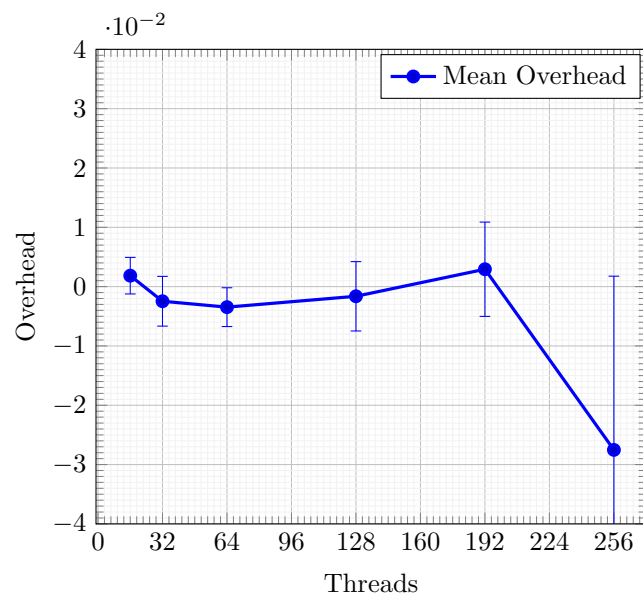
Figure 10.24: Mean overhead for distributed-memory Rabin—Miller test "44497 256" vs Eden.
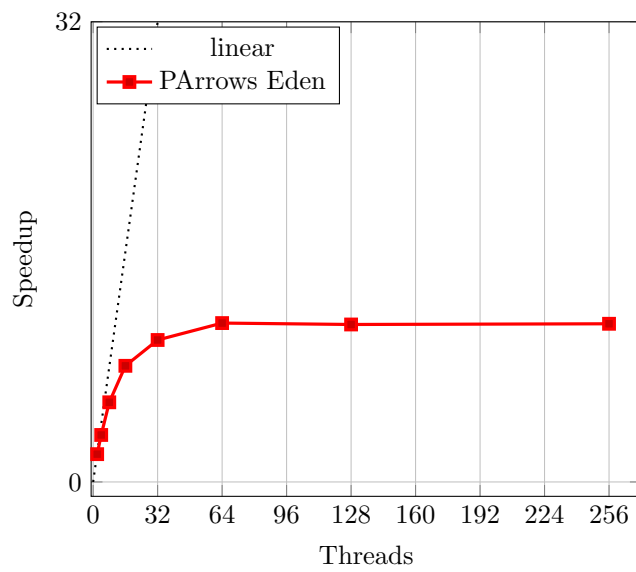
Figure 10.25: Parallel speedup of distributed-memory Jacobi sum test "3217".
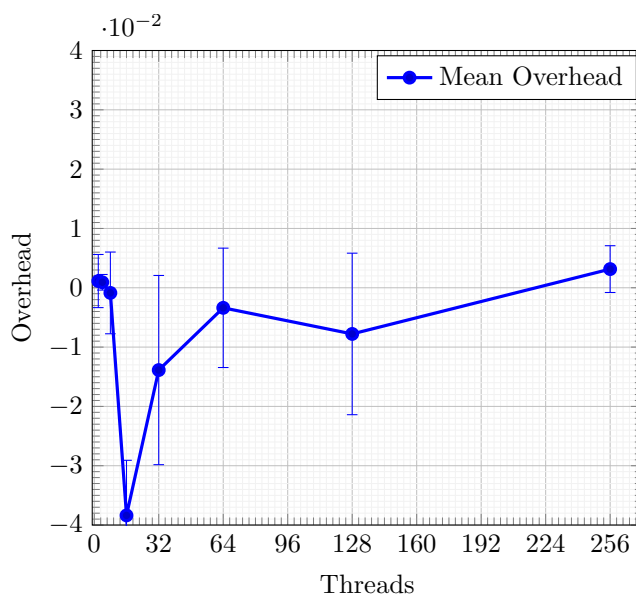
Figure 10.26: Mean overhead for distributed-memory Jacobi sum test "3217" vs Eden.
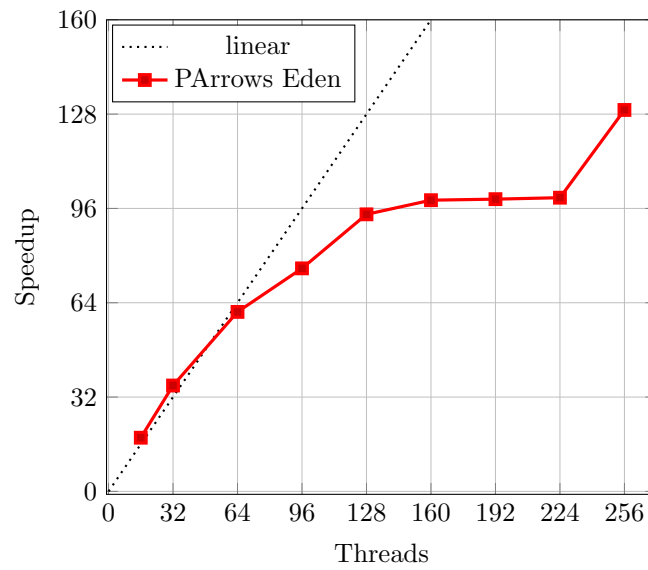
Figure 10.27: Parallel speedup of distributed-memory Gentleman "4096".
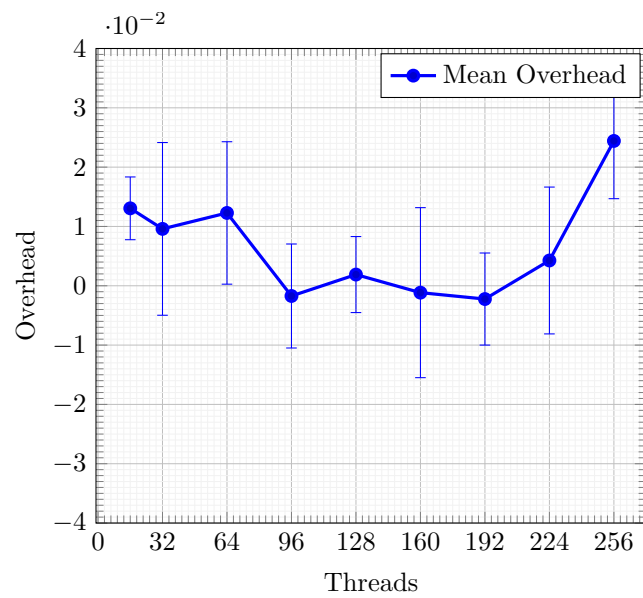
Figure 10.28: Mean overhead for distributed-memory Gentleman "4096" vs Eden.

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Declaration of independence

Hiermit versichere ich, Martin Braun (1249080), dass ich die von mir vorgelegte Arbeit *Concepts in Parallel Programming* selbstständig verfasst, keine anderen als die angegebenen Quelle und Hilfsmittel verwendet und die Arbeit nicht bereits zur Erlangung eines akademischen Grades eingereicht habe.

Bayreuth, 07.09.2018

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Martin Braun (1249080)

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