FE - Assignment - II

Md. Umar Faruk Ali

M19MA009

Q.1

Given,

Strike price, $K = 52$

Curren price, $S(0) = 50$.

Expiry time, $T = 1$ year.

and, $u = 1.2$

$d = 0.8$

$r = 5\% = 0.05$

$\Delta t = \frac{1}{2}$

now,

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u-d} = \frac{e^{0.05 \times \frac{1}{2}} - 0.8}{1.2 - 0.8}$$

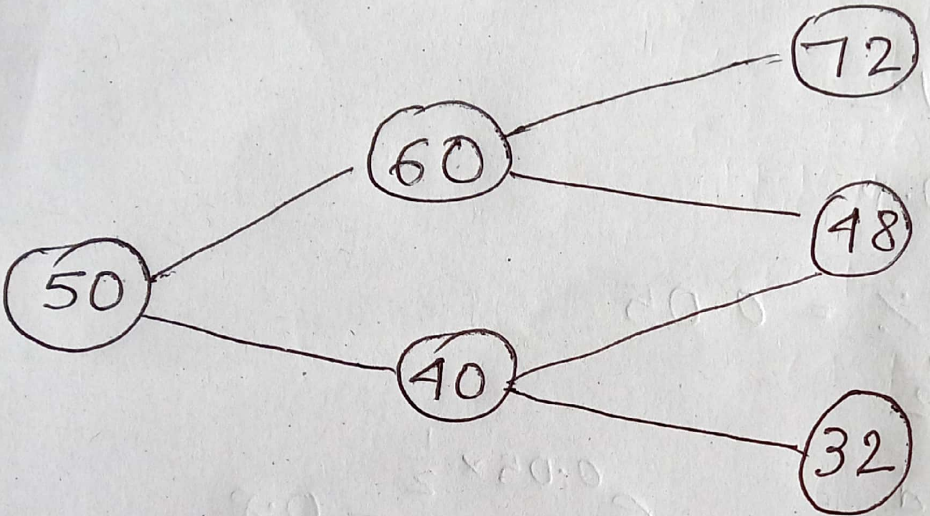
$$= \frac{1.02531 - 0.8}{0.4}$$

$$= 0.56328$$

$$\approx 0.56$$

$\tilde{p}$ denotes risk-neutral probability.

S(0) after 6-month 1 year payoff



$(S(T)-K)^+ = 72-52$
$\boxed{=20}$

$48-52<0$
$=\boxed{0}$

$\boxed{0}$

Therefore,

European call price is,

$$C(0) = e^{-2(0.05\times\frac{1}{2})}\left[\tilde{p}^2\times 20 + 2\tilde{p}(1-\tilde{p})\times 0 + (1-\tilde{p})\times 0\right]$$

$$= e^{-0.05}\times\left[0.56^2\times 20 + 0 + 0\right]$$

$$\cancel{5.68388} = 5.966$$

Q.3

Given,

$S(0) = Rs.\ 100$, $K = Rs.\ 80$

annual voltality, $\sigma = 20\% = 0.2$

$r = 5\% = 0.05$

$n = 2$, $T = 4$ years

so, $\Delta t = \frac{T}{n} = 2$ years

now,

By C-R-R Model,

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2 \times \sqrt{2}} = 1.326$$

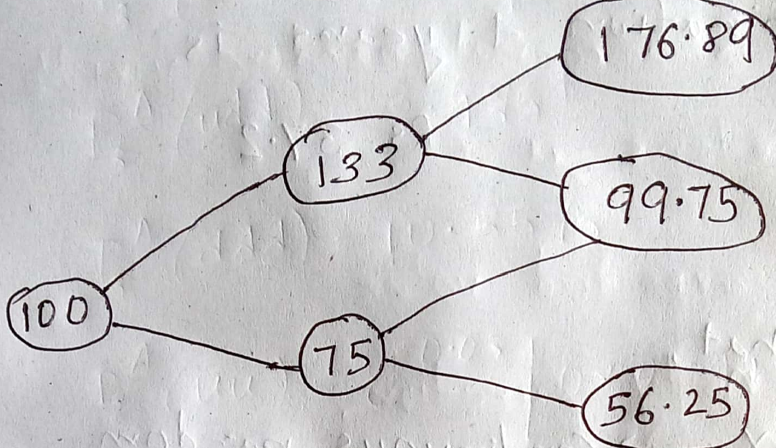
$$\approx 1.33$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{1.33} = 0.75$$

hence, up by 33%

and down by 25%

$S(0)$ $\frac{T=2}{\Delta t_1}$ $\frac{T=4}{\Delta t_2}$ $\frac{\text{payoff}}{(S(T)-K)^+}$



96.86

19.75

0

now,

$$\tilde{p} = \frac{e^{0.05\times 2} - 0.75}{1.33 - 0.75}$$

$$\approx 0.61$$

hence, $1-\tilde{p} = 0.39$

Therefore, by two period CRR binomial option pricing model,

European call option,

$$C^E(0) = e^{-0.05\times 4}\left[\tilde{p}^2 \times 96.86 + 2\tilde{p}(1-\tilde{p}) \times 19.75 + (1-\tilde{p})^2 \times 0\right]$$

$$= e^{-0.2} \times \left[0.61^2 \times 96.86 + 2\times 0.61 \times 0.39 \times 19.75\right]$$

$$\approx 37.2$$

Hence price of European call option ~~order~~ with strike price Rs. 80 and time of expiration 4 years is

Rs. 37.2

Q.4

Given that,

$X \longrightarrow$ Continuous random variable.

$f(x) \longrightarrow$ Probability density function.

$K \rightarrow$ constant.

Now,

Expected value,

$$E(X-K)^{+} = \int_{-\infty}^{\infty} (x-K)^{+} f(x)\,dx$$

$$= \lim_{K\to\infty} \int_{K}^{\infty} (x-K)\, f(x)\, dx$$

Let, $x-K = u$, and $v = \int_{-\infty}^{K} f(t)\,dt$

$du = dx$, $dv = f(x)\,dx$

this gives,

$$E\left[(x-K)^+\right]$$

$$= \lim_{M\to\infty}\left(\left[\int_{-\infty}^{x} f(t)\,dt\right](x-K)\Big|_K^M - \int_K^M \int_{-\infty}^{x} f(t)\,dt\,dx\right)$$

$$= \lim_{M\to\infty}\left(\left[\int_{-\infty}^{M} f(t)\,dt\right](M-K) - \int_K^M \left[1-\int_x^{\infty} f(t)\,dt\right]dx\right)$$

$$= \lim_{M\to\infty}\left(\left[1-\int_M^{\infty} f(t)\,dt\right](M-K) - \int_K^M \left[1-\int_x^{\infty} f(t)\,dt\right]dx\right)$$

$$= \lim_{M\to\infty}\left(\int_K^M \int_x^{\infty} f(t)\,dt\,dx - (M-K)\int_K^{\infty} f(t)\,dt\right)$$

$$= \int_K^{\infty}\left(\int_x^{\infty} f(t)\,dt\right)dx.$$

Hence,

$$E\left[(x-K)^+\right] = \int_K^{\infty}\left(\int_x^{\infty} f(t)\,dt\right)dx$$

2nd part,

Given,

x is a normal random variable with parameters μ & σ^2.

Therefore, ~~expected value~~

density function,

$$f(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$-\infty < x < \infty$$

Hence, expected value of $(x-K)^+$ is

$$E\left[(x-K)^+\right] = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty}(x-K)^+ e^{-(x-\mu)^2/2\sigma^2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\int_{K}^{\infty}(x-K)\, e^{-(x-\mu)^2/2\sigma^2}\,dx.$$

Let, $t = \frac{x-\mu}{\sigma}$, then we have,

$$E\left[(x-K)^+\right] = \frac{1}{\sqrt{2\pi}}\int_{(K-\mu)/\sigma}^{\infty}(\mu-K+t\sigma)\, e^{-t^2/2}\,dt$$

$$= \frac{\mu - K}{\sqrt{2\pi}} \int_{(K-\mu)/\sigma}^{\infty} e^{-t^2/2} dt + \frac{\sigma}{\sqrt{2\pi}} \int_{(K-\mu)/\sigma}^{\infty} t e^{-t^2/2} dt$$

$$\frac{1}{\sqrt{2\pi}} \int_{(K-\mu)/\sigma}^{\infty} e^{-t^2/2} dt = F_Z\left(\frac{\mu - K}{\sigma}\right)$$

is definition of cumulative distribution function for the standard normal random variable

now,

$$\int_{(K-\mu)/\sigma}^{\infty} t e^{-t^2/2} dt$$

let,

$t^2/2 = \theta$

$t\,dt = d\theta.$

$$\int t e^{-t^2/2} dt$$

$$= \int e^{-\theta} d\theta$$

$$= -e^{-\theta} + C.$$

$$= -e^{-\frac{t^2}{2}} + C.$$

Therefore,

$$\int_{(k-\mu)/\sigma}^{\infty} t e^{-\frac{t^2}{2}} \, dt$$

$$= \left[- e^{-\frac{t^2}{2}} \right]_{\frac{(k-\mu)}{\sigma}}^{\infty}$$

$$= 0 + e^{-\frac{(k-\mu)^2}{2\sigma^2}}$$

So,

$$E\left[(X-k)^+\right] = \frac{\mu - k}{\sqrt{2\pi}} \int_{(k-\mu)/\sigma}^{\infty} e^{-t^2/2} \, dt$$

$$+ \frac{\sigma}{\sqrt{2\pi}} \int_{(k-\mu)/\sigma}^{\infty} t e^{-t^2/2} \, dt$$

$$= (\mu - k) F_Z\left(\frac{\mu - k}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi}} \cdot e^{-\frac{(k-\mu)^2}{2\sigma^2}}$$

(proved)

8.5

Given,

$S(0) = 1500$,

$K = 1470$

$r = 0.03$

voltality, $\sigma = 22\% = 0.22$

total number of trading days between the given period is 44 days and number of trading days in year 2010 is 252 days.

So,
$$T = \frac{44}{252} = 0.175$$

therefore, for one period binomial model, $\Delta t = T = 0.175$

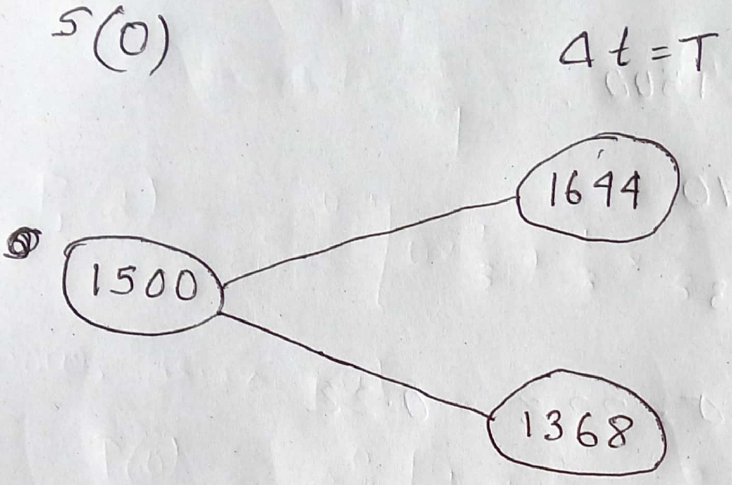
now,
$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.22 \times \sqrt{0.175}}$$
$$= 1.096$$

i.e, moves up by 9.6%

and,
$$d = \frac{1}{u} = 0.912$$

i.e, moves down by, 8.8%

So, pricing tree for call :-



payoff

$(S(T)-K)^+$

$1644-1470$

$= 174$

0

now,

$$\tilde{p} = \frac{e^{r \times \Delta t} - d}{u-d}$$

$$= \frac{e^{0.03 \times 0.175} - 0.912}{1.096 - 0.912}$$

$$\approx 0.506$$

$$\approx \frac{1}{2}$$

so, $1-\tilde{p} = \frac{1}{2}$

now,

Price of European call option is,

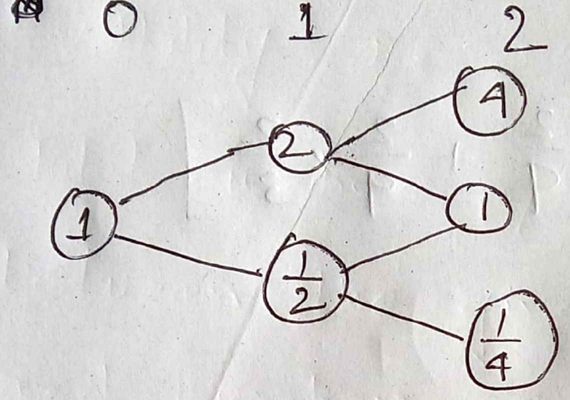
$$c^E(0) = e^{-0.03 \times 0.175} \times \left[\frac{1}{2} \times 174 + \frac{1}{2} \times 0\right]$$

$$= \boxed{86.54 \text{ Rupees}}.$$

Q.6

Given,

$S(0) = 1$, $u = 2$, $d = \frac{1}{2}$, $r = 0$, $N = 2$,



$M = \frac{S_0 + S_1 + S_2}{3}$

Payoff $(M - S_N)^+$

$\frac{7}{3}$ $\qquad f(S_2(uu)) = 0$

$\frac{4}{3}$ $\qquad f(S_2(ud)) = \frac{1}{3}$

$\frac{5}{6}$ $\qquad f(S_2(du)) = 0$

$\frac{7}{12}$ $\qquad f(S_2(dd)) = \frac{1}{3}$

now, $\tilde{p} = \frac{e^{0\times 1} - \frac{1}{2}}{2 - \frac{1}{2}} = \frac{1 - \frac{1}{2}}{3/2} = \frac{1}{3}$

so, $\tilde{p} = \frac{1}{3}$, $(1 - \tilde{p}) = \frac{2}{3}$

$$P_1(u) = e^{-0}\left[0 \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3}\right]$$

$$= \frac{2}{9}$$

$$P_1(d) = e^{-0}\left[0 \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3}\right] = \frac{2}{9}$$

now, asian put,

$$P(0) = e^{-0}\left[\frac{1}{3} \times \frac{2}{9} + \frac{2}{9} \times \frac{2}{3}\right]$$

$$= \frac{6}{27} = \frac{2}{9}$$

2nd part,

Hedging strategy (α_i, β_i):-

For, $n = 0$ (First period)

$$\alpha_1 S_1(u) + \beta_1 B_1 = P_1(u)$$

$$\alpha_1 S_1(d) + \beta_1 B_1 = P_1(d)$$

where,

$$B_n = e^{rn}$$

so,

$$2\alpha_1 + \beta_1 = \frac{2}{9} \quad \text{——} (1)$$

$$\frac{\alpha_1}{2} + \beta_1 = \frac{2}{9} \quad \text{——} (2)$$

therefore, by ① & ② we get,

$$2\alpha_1 - \frac{\alpha_1}{2} = 0$$

$$\Rightarrow \boxed{\alpha_1 = 0}$$

and, $\boxed{\beta_1 = \frac{2}{9}}$

<u>For n = 1 (second period)</u>

For, $S_1 = 2$

$$\alpha_2 S_2(uu) + \beta_2 B_2 = P_2(uu)$$

$$\alpha_2 S_2(ud) + \beta_2 B_2 = P_2(ud)$$

$\Rightarrow$

$$4\alpha_2 + \beta_2 = 0$$

$$\alpha_2 + \beta_2 = \frac{1}{3}$$

Hence,

$$\boxed{\alpha_2 = -\frac{1}{9}}, \quad \boxed{\beta_2 = \frac{4}{9}}$$

For, $S_1 = \frac{1}{2}$

$$\alpha_3 S_2(du) + \beta_3 B_2 = P_2(du)$$

$$\alpha_3 S_2(dd) + \beta_3 B_2 = P_2(dd)$$

$\Rightarrow$ $\alpha_3 + \beta_3 = 0$

& $\frac{\alpha_3}{4} + \beta_3 = \frac{1}{3}$ $\Rightarrow$ $\boxed{\alpha_3 = -\frac{4}{9}}, \quad \boxed{\beta_3 = \frac{4}{9}}$

Q.9

Given,

$S(0) = 69$

$K = 70$

$r = 0.05$

$\sigma = 0.35$

$T = \frac{1}{2}$ year.

now,

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{69}{70}\right) + \left(0.05 + \frac{0.35^2}{2}\right) \times 0.5}{0.35\sqrt{0.5}}$$

$$= 0.1666$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 0.1666 - 0.35\sqrt{0.5}$$

$$= -0.0809$$

∴ The European put,

$$= 70e^{-0.05 \times 0.5} N(+0.0809)$$

$$- 69 \times N(-0.1666)$$

$$= 6.4$$

$\underline{\underline{Q.10}}$

Given,

$S(0) = 110$

$K = 105$

$r = 0.08$

$T = \frac{1}{4} = 0.25$ year.

$\sigma = 0.2$

now, $d_1 = \frac{1}{\sigma\sqrt{T-t}} \left\{ \ln(S_t/K) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right\}$

$= \frac{1}{\sigma\sqrt{T}} \left\{ \ln(S_0/K) + \left(r + \frac{\sigma^2}{2}\right)T \right\}$

$= \frac{\ln\left(\frac{110}{105}\right) + \left(0.08 + \frac{0.2^2}{2}\right) \times 0.25}{0.2\sqrt{0.25}}$

$= \frac{0.0715}{0.1}$

$= 0.715$

$d_2 = d_1 - \sigma\sqrt{T-t}$

$= 0.715 - 0.2 \times \sqrt{0.25}$

$= 0.615$

$\therefore$ Price of call option,

$$C^E(0) = S(0)N(d_1) - e^{-rT} K N(d_2)$$

$$= 110 \times N(0.715) - e^{-0.02} \times 105 \times N(0.615)$$

$$= 110 \times 0.76115 - e^{-0.02} \times 105 \times 0.72907$$

$$= 8.689$$

2nd part,

given,

$$\sigma = 0.2$$

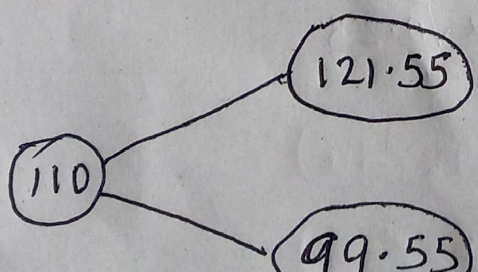
so, $u = e^{\sigma\sqrt{\Delta T}} = e^{0.2 \times \sqrt{0.25}}$

$$= e^{0.1}$$

$$= 1.105$$

and $d = \frac{1}{u} = 0.905$

so,



$G(u) = 121.55 - 105$

$= 16.55$

$G(d) = 0.$

$$\therefore \Delta = \frac{C_1(u) - C_1(d)}{(u-d)(S(0))}$$

$$= \frac{16.55 - 0}{(1.105 - 0.905) \times 110}$$

$$= \frac{16.55}{22}$$

$$= 0.752$$

Now,

By, Put-call parity,

European put,

$$P^E(0) = C(0) - S(0) + e^{-rT} K$$

$$= 8.689 - 110 + e^{-0.08 \times 0.25} \times 105$$

$$= 111.6098 - 110$$

$$= 1.6098$$

$$\approx 1.61$$

$$\therefore \boxed{P^E(0) = 1.61}$$