Financial Derivatives
(Binomial Model II)

Financial Engineering

by

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Option Pricing – Multi Period Binomial Model

- Assuming n period between time 0 and T, price of a European option is given by

$$C(0) = e^{-rT}\tilde{E}(f(S(T)), \qquad \Delta t = T/n$$

where,

$f(S(T))$ is payoff of the option at time T (Time of Maturity),

the **return** of $S(T)$ is assumed to follow binomial distribution $B(n, \tilde{p})$,

$\tilde{p}$ is the risk neutral probability measure given by

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u - d},$$

$u\ and\ d$ are one period up and down factors respectively.

Advantages of Multi Period Binomial Model

- Replicating strategy

- Approximation in small interval of time

- Pricing American options

- Pricing under periodic dividends

Example

Let $S(0)$ = 50 Rs, u = 1.2, d = 0.80 and r = 0.05. Consider a European put option with strike price K = 52 Rs and exercise time T = 2 years. Find the option price using two period model and the replicating strategy.

CRR (Cox-Ross-Rubinstein Model)

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- For one period (Δt) model, the return of $S(T)$ follows $B(1, \tilde{p})$,
- Simple returns are $(u - 1)$ and $(d - 1)$ while the log-returns are u and d,
- In any case the variance will be same,
- For one-period model, the expected return (under RNPM) is

$$e^{r\Delta t} = \tilde{p}u + (1 - \tilde{p})d$$

- The variance is

$$\sigma^2 \Delta t = \tilde{p}u^2 + (1 - \tilde{p})d^2 - \{\tilde{p}u + (1 - \tilde{p})d\}^2$$

CRR (Cox-Ross-Rubinstien Model)

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- Now using $\tilde{p} = \frac{e^{r\Delta t} - d}{u - d}$, we get

$$\sigma^2 \Delta t \ = e^{r\Delta t}(u + d) - ud - e^{2r\Delta t}$$

- To further simplify, we expand and neglect the higher powers of Δt

- It can be verified that $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$ satisfy the above equation

- Thus, we have $u = \frac{1}{d} = e^{\sigma\sqrt{\Delta t}}$

Components of Option Pricing

- - Spot Price (S(0))

- Strike Price (K)

- Maturity Time (T)

- Interest Rate (r)

- Volatility (σ)

Dividend Paying Stocks

- If the dividend is periodic
  - The dividend will be subtracted from the stock price at that time
- If the dividend is continuous
  - The dividend will act as discounting

Example

Let $S(0)$ = 100 Rs, u = 1.2, d = 0.80 and r = 0.10. Consider a European call option with strike price K = 95 Rs and exercise time T = 2 years. The stock pays a dividend of Rs 10 after every year. Find the option price.

American Option - Early exercise

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- Every node of the tree will be checked to see if it is stopping node
- Stopping criteria: If the payoff of that node is more than that of the discounting value of the expectation evaluated at the last node,
- that is, at any node at time t, we use

$$Max(f\left(S(t)\right), e^{-r\Delta t}\tilde{E}\left(f\left(S(t+\Delta t)\right)\right))$$

Example

Let $S(0)$ = 50 Rs, u = 1.2, d = 0.80 and r = 0.01. Consider an American put option with strike price K = 52 Rs and exercise time T = 2 years. Find the option price using two period model. Is it better to exercise it early?