FE ASSIGNMENT 2

(Sanyam Jain
P20QC001)

Sol 1: given :-

$S(0) = 50$, $K = 52$, $T = 1$, $\overset{20\%}{u = 1.2}$
$d = 0.$

$r = 0.05$, $\Delta t = 0.5$

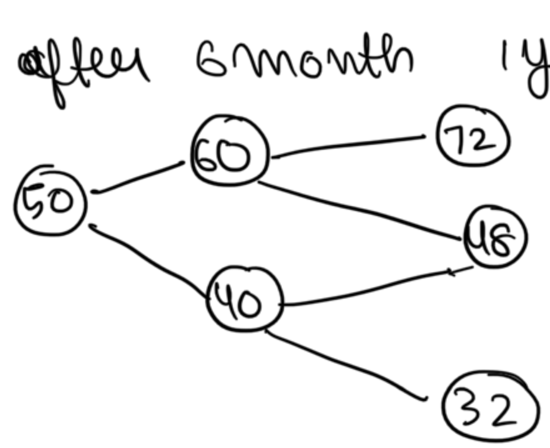
$$\tilde{p} = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \cdot 0.5} - 0.8}{1.2 - 0.8}$$

$$= 0.56328$$

$$\approx 0.56$$

$\tilde{p}$ denotes risk neutral probablity

S(0) after 6month 1years payoff



$(S(T) - K)^+$

$= 72 - 52$

$= \underline{\underline{20}}$ ✓

$48 - 52 < 0$

$= \underline{\underline{0}}$ ✓

∴ European call price is :-

$$C(0) = \left[\tilde{p}^2 \times 20 + 2\tilde{p}(1-\tilde{p}) \times 0 + (1-\tilde{p}) \times 0\right] e^{-2(0.05}$$

$$= (0.56^2 \times 20)(e^{-0.05})$$

$$= 5.966$$

<u>Sol 2</u> :

$S(0) = 60$

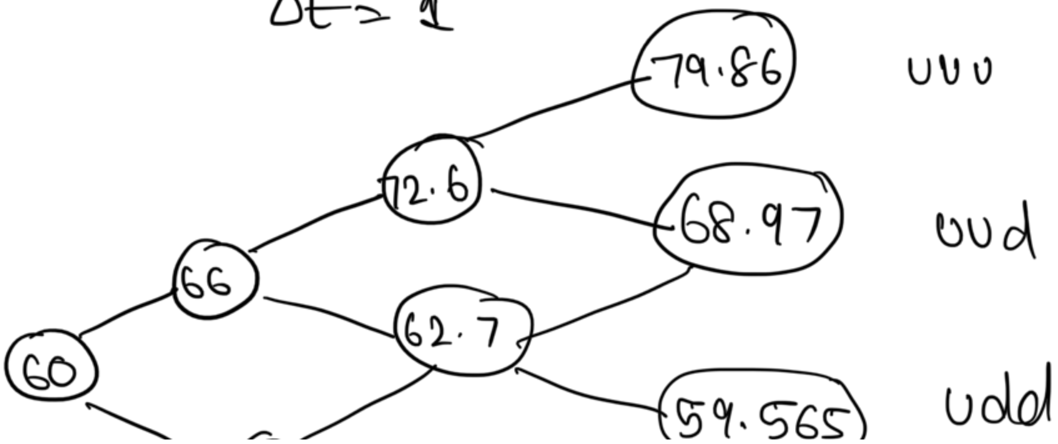
$K = 62$

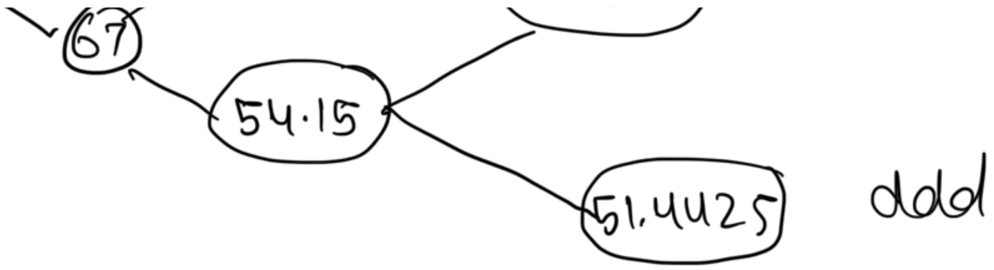
$u = 1.1$, $d = 0.95$

$r = 0.03$

$T = 3$ years

$\Delta t = 1$





European put option

payoff $\left(K - S(T)\right)^+$

$$\tilde{p} = \frac{e^r - d}{u - d} = \frac{e^{0.03} - 0.95}{1.1 - 0.95} = 0.5364$$

$f(S_{3uuu}) = 0$

$f(S_{3uud}) = 0$

$f(S_{3udd}) = 2.435$

$f(S_{3ddd}) = 10.5575$

$$\tilde{E} f(S(T)) = {}^3C_0\, \tilde{p}^3 f(S_{3uuu}) + {}^3C_1\, \tilde{p}^2 (1-\tilde{p}) f(S_{3uud}) + {}^3C_2\, \tilde{p} (1-\tilde{p})^2 f(S_{3udd}) + {}^3C_3\, (1-\tilde{p})^3 f(S_{3ddd})$$

$$= (0.5364)^3 \times 0 + 3 \times 0.4636 \times (0.5364)^2 \times$$

$+ 3 \times (0.5364)(0.4636) \times 2.435$

$+ (0.4636)^3 \times (10.5575)$

$= 0.8422 + 1.0519$

$= 1.8941$

Now $P(0) = e^{-rt} f(S(T))$

$= 1.731$

European Call option :

Payoff $(S(T) - K)^+$

$f(S_{3uuu}) = 17.86$

$f(S_{3uud}) = 6.99$

$f(S_{3udd}) = 0$ $\quad f(S_{3ddd}) = 0$

$$\tilde{E} f(S(T)) = {}^3C_0 \tilde{p}^3 f(S_{3uuu}) +$$

$${}^3C_1 \tilde{p}^2 (1-\tilde{p}) f(S_{3uud}) +$$

$${}^3C_2 \tilde{p} (1-\tilde{p})^2 f(S_{3udd}) +$$

$${}^3C_3 (1-\tilde{p})^3 f(S_{3ddd})$$

$$= (0.5364)^3 \times 17.86 + 3 \times (0.5364)^2 + (0.4636 \times 6.99) + 3(0.5364)(0.4636)$$

$$+ 0$$

$$= 2.7564 + 2.7893$$

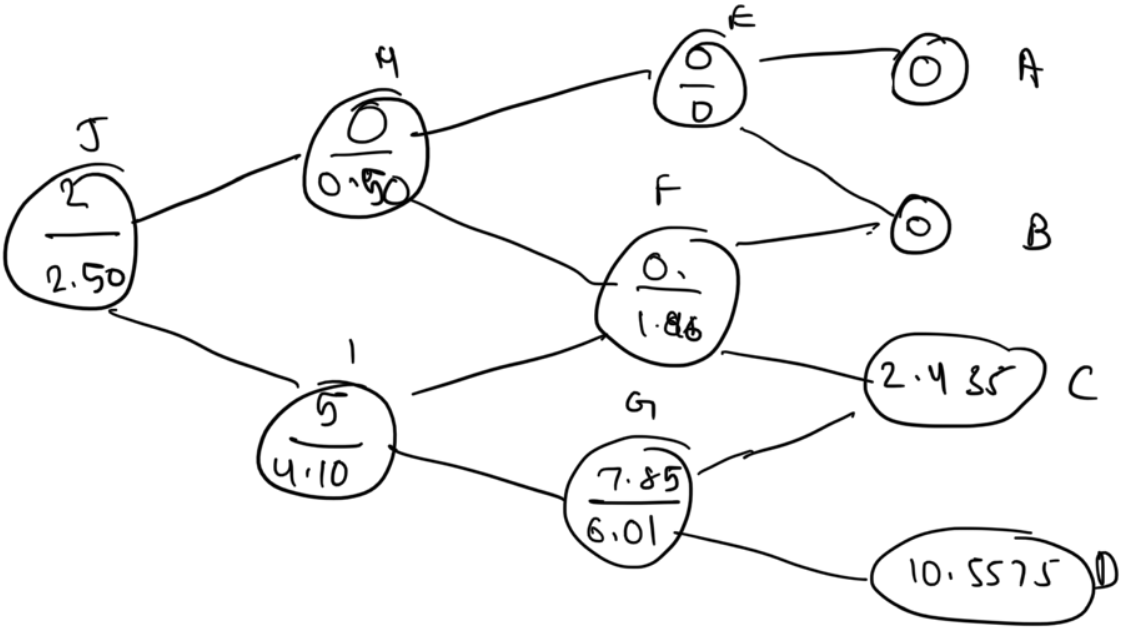
$$\approx 5.54$$

$$c(0) = e^{-3h} \tilde{E}(f(S(T)))$$

$$= e^{-0.09} \times 5.54$$

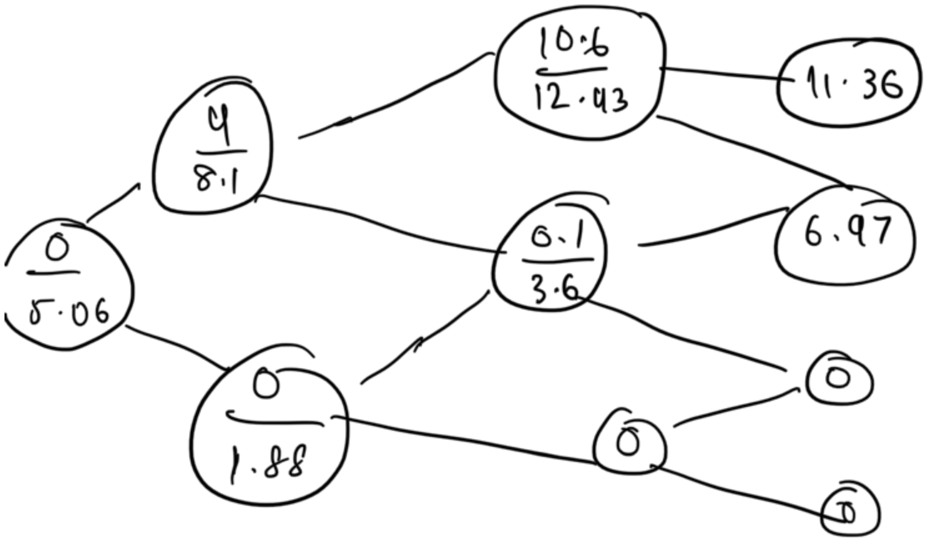
$$= 5.06$$

American Put option



Put option can be exercised at node G and I since payoff is greater.

American Call option



$C^A(0) = 5.068$, $\underline{T = 3 \text{ years}}$

C^A should be exercised at time of expiry

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sol 3 :

$S(0) = 100/-$

$K = 80/-$

$\sigma = 0.20$

$r = 5\% = 0.05$

$n = 2$, $T = 4$

$\Delta t = \frac{T}{n} = 2$ years

now, by CRR model,

$u = e^{\sigma\sqrt{\Delta t}}$

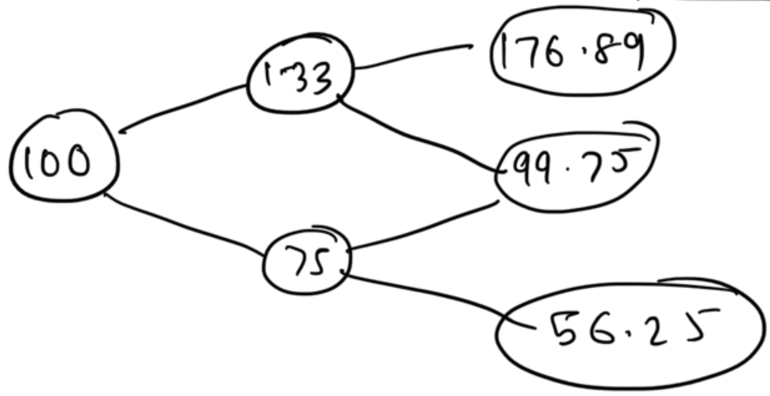
$= e^{0.2\sqrt{2}} = 1.33$

$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{1.33} = 0.75$

$\therefore$ $u = 1.33$ | up by 33%

$d = 0.75$ | down by 25%

$S(0)$ $\quad \dfrac{T=2}{\Delta t_1}$ $\quad \dfrac{T=4}{\Delta t_2}$ $\quad$ pay off $(S(T)-K)^+$



96.86

19.75

0

$$\tilde{p} = \frac{e^{0.05\times2} - 0.75}{1.33 - 0.75} = 0.61$$

$$1 - \tilde{p} = 0.39$$

$\therefore$ by 2 period CRR binomial option pricing model,

European call option,

$$C^E(0) = e^{-0.05\times4}\left[\tilde{p}^2 \times 96.86 + 2\tilde{p}(1-\tilde{p}) \times 19.75 + (1-\tilde{p})^2(0)\right]$$

$$= e^{-0.2}\left[(0.61)^2 \times 96.86 + 2 \times 0.61 \times 0.39 \times 19.75\right]$$

$$= 37.2$$

Hence price of European call option with strike price Rs. 80 and time of Expiration 4 years is 37.2/-

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Sol 4:

Given that,

$X \rightarrow$ continuous random variable

$f(x) =$ PDF

$K \rightarrow$ constant

Now, $E(X)$

$$E(X-K)^+ = \int_{-\infty}^{\infty} (x-K)^+ f(x)\, d(x)$$

$$= \lim_{K \to \infty} \int_{K}^{\infty} (x-K) f(x)\, d(x)$$

K

let $x - K = u$ and $v = \int_{-\infty} f(t)\, d(t)$

$du = dx \quad , \quad dv = f(x)\, d(x)$

$$E\left[(X-K)^+\right] = \lim_{M\to\infty}\left[\left[\int_{-\infty}^{x} f(t)\, d(t)\right](x-K)\Big|_K^M - \int_K^M\int_{-\infty}^{x} f(t)\, dt\, dx\right]$$

$$= \lim_{M\to\infty}\left(\left[\int_{-\infty}^{M} f(t)\, d(t)\right](M-K) - \int_K^M \left[1-\int_x^{\infty} f(t)\, d(t)\right] d(x)\right)$$

$$= \lim_{M\to\infty}\left(\left[1-\int_M^{\infty} f(t)\, d\cancel{x}t\right](M-K) - \int_K^M 1-\left[\int \cancel{f}(t)\, dt\right] dx\right)$$

$$= \int_K^{\infty}\left(\int_x^{\infty} f(t)\, dt\right) dx$$

hence,

$$E\left[(x-k)^+\right] = \int_K^{\infty}\left(\int_x^{\infty}\left(f(t)\, dt\right)\right) dx$$

2^{nd} Part :

Given,

x is a normal random variable with parameters μ and σ^2.

Density function $\quad -\infty < x < \infty$

$$f(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

hence $E(x)$ value $(x-k)^+$ is

$$E\left[(x-k)^+\right] = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty}(x-k)^+ e^{-(x-\mu)^2/2\sigma^2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\int_{k}^{\infty}(x-k)e^{-(x-\mu)^2/2\sigma^2}\,dx$$

let $t = \frac{x-\mu}{\sigma}$ then, we get,

$$E\left[(x-k)^+\right] = \frac{1}{\sqrt{2\pi}}\int_{\frac{k-\mu}{\sigma}}^{\infty}(\mu-k+t\sigma)\,e^{-t^2/2}\,dt$$

$$= \frac{\mu-k}{\sqrt{2\pi}}\int^{\infty} e^{-\frac{t^2}{2}}dt + \frac{\sigma}{\sqrt{2\pi}}\int t e^{-t^2/2}\,d$$

$$\sqrt{2\pi} \quad \frac{k-\mu}{\sigma} \qquad\qquad \sqrt{2\pi}\sigma \quad \frac{k-\mu}{\sigma}$$

$$\frac{1}{\sqrt{2\pi}}\int_{\frac{k-\mu}{\sigma}}^{\infty} e^{-t^2/2}\,dt = F_2\left(\frac{\mu-k}{\sigma}\right)$$

$$\int_{\frac{k-\mu}{\sigma}}^{\infty} t\, e^{-t^2/2}\,dt$$

which is standard form :-

$$= \int e^{-\theta}\,d\theta$$

where
$\theta = t^2/2$

$t\,dt = d\theta$

$$= -e^{-\theta} + c$$

$$= -e^{-\frac{t^2}{2}} + c$$

$$\therefore \int_{\frac{k-\mu}{\sigma}}^{\infty} t\, e^{-t^2/2}\,dt$$

$$= \left[-e^{-\frac{t^2}{2}}\right]_{\frac{k-\mu}{\sigma}}^{\infty} = \underline{\underline{e^{-\frac{(k-\mu)^2}{2\sigma^2}}}}$$

Thus proved !!

Sol 5   Given

$S(0) = 1500$ $\sigma = 22\%$

$K = 1470$ $= 0.22$

$D = 0.03$

Total number of trading days between the given period is 44 days and number of trading days in year 2010 is 252 days

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lets try again ...

$S(0) = 1500$

$K = 1470$

$r = 0.03$

$T = 0.22$

$T = \frac{44}{252} = 0.175$

$\therefore$ one period binomial model

$$\Delta t = T = 0.175$$

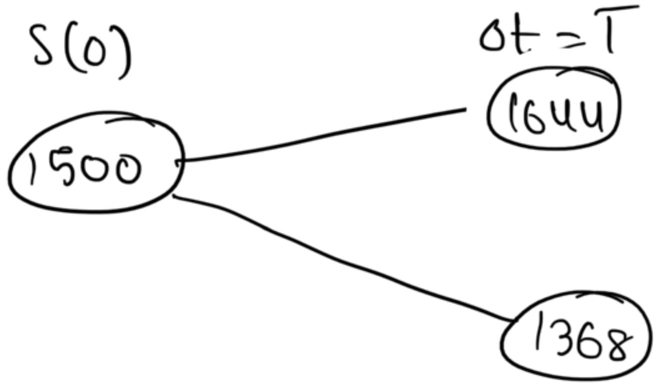
$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.22\sqrt{0.175}}$$

$$= 1.096$$

ie., move by 9.6% ↑

$d = \frac{1}{u} = 0.912$, 8.8% ↓

<u>Pricing tree:</u>



Payoff
$(S(t) - K)^+$

$1644 - 1470$
$= 174$

0

Now,

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u - d}$$

$$= \frac{e^{0.03 \times 0.175} - 0.912}{1.096 - 0.912}$$

≈ 0.506 $= \frac{1}{2}$ also, $1 - \tilde{p} = \frac{1}{2}$

$$C^{E}(0) = e^{-0.03\times 0.175} \times \left[\frac{1}{2}\times 174 + 0\right]$$

$$= \underline{\underline{86.54/-}}$$

Sol^n 6

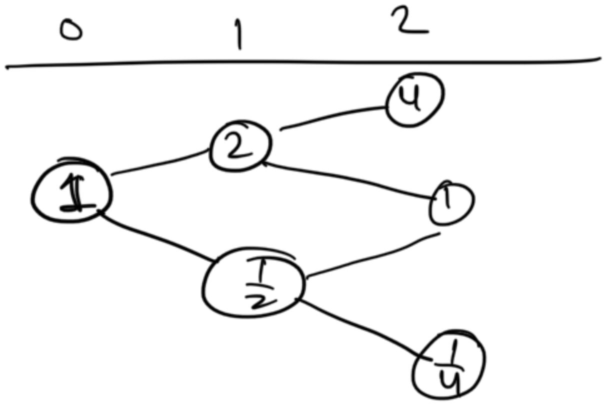
$S(0) = 1$

$u = 2$

$d = \frac{1}{2}$

$r = 0$

$N = 2$



$m = \dfrac{S_0 + S_1 + S_2}{3}$	payoff $(m - S_n$
$7/3$	$f(S_2(uu)) = 0$
$4/3$	$f(S_2(ud)) = \frac{1}{3}$
$5/6$	$f(S_2(du)) = 0$
$7/12$	$f(S_2(dd)) = \frac{1}{3}$

$$\tilde{p} = \frac{e^{0(1)} - \frac{1}{2}}{\quad} = \frac{1}{2}$$

$2 - \frac{1}{2}$

$\tilde{p} = \frac{1}{3}, \quad 1 - \tilde{p} = \frac{2}{3}$

$$P_1(u) = e^{-0}\left[0 + \frac{2}{3} \times \frac{1}{3}\right] = 2/9$$

$$P_1(d) = e^{-0}\left[0 + \frac{2}{3} \times \frac{1}{3}\right] = 2/9$$

Asian Put

$$P(0) = e^{-0}\left[\left(\frac{1}{3} \times \frac{2}{9}\right) + \left(\frac{2}{9} \times \frac{2}{3}\right)\right]$$

$$= 2/9$$

2^{nd} Part :

hedging Strategy (α_i, β_i)

for $n = 0$ (first period)

$$\alpha_1 S_1(u) + \beta_1 B_1 = P_1(u)$$

$$\alpha_1 S_1(d) + \beta_1 B_1 = P_1(d)$$

where,

$$B_n = e^{rn}$$

so,

$$2\alpha_1 + \beta_1 = \frac{2}{9} \quad \Big\} \quad 2\alpha_1 - \frac{\alpha_1}{2} = 0$$

$$\frac{\alpha_1}{2} + \beta_1 = \frac{2}{9} \Bigg\} \quad \boxed{\alpha_1 = 0}$$

$$\boxed{\beta_1 = \frac{2}{9}}$$

for $n=1$ (second period)

for $S_1 = 2$

$$\alpha_2 S_2(uu) + \beta_2 B_2 = P_2(uu)$$

$$\alpha_2 S_2(ud) + \beta_2 B_2 = P_2(ud)$$

$$\begin{aligned} 4\alpha_2 + \beta_2 &= 0 \\ \alpha_2 + \beta_2 &= \frac{1}{3} \end{aligned} \quad \Bigg| \quad \begin{aligned} \alpha_2 &= -\frac{1}{9} \\ \beta_2 &= \frac{4}{9} \end{aligned}$$

for $S_1 = \frac{1}{2}$

$$\alpha_3 S_2(du) + \beta_2 B_2 = P_2(du)$$

$$\alpha_3 S_2(dd) + \beta_2 B_2 = P_2(dd)$$

$$\begin{aligned} \alpha_3 + \beta_3 &= 0 \\ \frac{\alpha_3}{4} + \beta_3 &= \frac{1}{3} \end{aligned} \quad \Bigg| \quad \boxed{\begin{aligned} \alpha_3 &= -\frac{4}{9} \\ \beta_3 &= \frac{4}{9} \end{aligned}}$$

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sol 7 : given

$\mu = 0.0165$

$\sigma = 0.0730$

suppose price ratio of n^{th} week to that of $(n-1)^{th}$ week is $X_{n,n-1}$ (log Normal) $(0.0165, 0.0730)$

(a) when price increases over of the next 2 weeks

$X_{01} > 1$, $X_{12} > 1$ ie;

probablity that price increases over each of the next 2 weeks

$$P(X_{01} > 1)\, P(X_{12} > 1) = P(X_{01} > 1)^2$$

$$= P(\ln X_{01} > 0)^2$$

$$= P(Z > -0.23)^2$$

$$= (0.59)^2$$

$$= 0.3481$$

(b) Probablity that the price at the end of 2 weeks is larger than it is today. we know P of the iid RV's of 2 log normal variables is log normal

$\therefore\ X_{01} X_{12} \sim \text{lognormal}\ (0.033, 0.103)$

$$P(X_{01} X_{12} > 1) = P(\ln X_{01} X_{12} > 0)$$

$$= P\left(\frac{\ln X_{01} X_{12} - 0.0333}{0.103} > -0.32\right)$$

$$= P(Z > -0.32) = \underline{\underline{0.6255}}$$

<u>soln 9</u> :

Given

$S(0) = 69$

$K = 70$

$r = 0.05$

$\sigma = 0.35$

$T = \frac{1}{2}$ years

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \ln\left(\frac{69}{70}\right) + \left(0.05 + \frac{0.35^2}{2}\right) \times 0.5$$

$$\overline{0.35\sqrt{0.5}}$$

$$= 0.1666 \ldots$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= -0.0809$$

The european Put,

$$= 70\, e^{-0.05\times 0.5}\, N(0.0809) - 69 \times N(0.166\ldots)$$

$$= \underline{\underline{6.4}}$$

Sol 10 : Given,

$$S(0) = 110$$

$$K = 105$$

$$r = 0.08$$

$$T = 1/4 = 0.25 \text{ years}$$

$$\sigma = 0.2$$

now,

$$d = \frac{1}{\sigma\sqrt{\ }}\left\{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(\Delta t)\right\}$$

$$= \frac{1}{\sigma\sqrt{T}}\left\{\ln\left(\frac{S_0}{K}\right) + \left(\mu + \frac{\sigma^2}{2}\right)T\right\}$$

$$= \frac{\ln\left(\frac{110}{105}\right) + \left(0.08 + \frac{0.2^2}{2}\right)0.25}{0.2\sqrt{0.25}}$$

$$= \underline{\underline{0.715}}$$

$$d_2 = d_1 - \sigma\sqrt{\Delta t}$$

$$= \underline{\underline{0.615}}$$

Price of call option

$$C^E(0) = S(0)\,N(d_1) - e^{-rT}K\,N(d_2)$$

$$= 110 \times N(0.715) - e^{-0.02} \times 105 \times N(0.6$$

$$= 110 \times 0.761 - e^{0.02} \times 105 \times 0.73$$

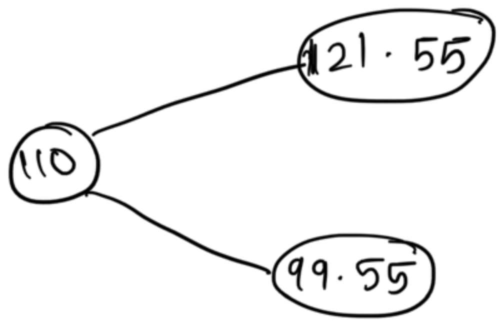
$$= \underline{\underline{8.69}}$$

Now,

given $\sigma = 0.2$

so, $u = e^{\sigma\sqrt{\Delta t}} = 1.10$

and $d = \frac{1}{u} = 0.905$



$$C_1(u) = 121.55 - 105 = 16.55$$

$$C_1(d) = 0$$

$$\Delta = \frac{C_1(u) - C_1(d)}{(u-d)[S(0)]} = \frac{16.55 - 0}{(1.105 - 0.905) \times 110}$$

$$= \frac{16.55}{22} = 0.75$$

By Put - Call Parity

European Put,

$$p^E(0) = C(0) - S(0) + e^{-rT} K$$

$$= 8.68 - 110 + e^{-0.08 \times 0.25} \times 105$$

$$= 1.61$$

$$\underline{p^E(0) = 1.61}$$