Financial Derivatives
(Black Scholes and Greeks)

Financial Engineering

by

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Option Pricing – Black-Scholes formula

Call option price at t=0

$$\boxed{C(0) = S(0)\, N(d_1) - e^{-rT} K\, N(d_2)}$$

$$\text{Where } d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$\text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

Here, r – interest rate, σ – annual volatility

$S(0)$ – spot price, K – strike price, T – maturity time

Option Pricing – Black-Scholes formula

Call option price at time t

$$\boxed{C(t) = S(t)\, N(d_1) - e^{-r(T-t)} K\, N(d_2)}$$

$$Where\ \ d_1 = \frac{\ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$and \qquad d_2 = d_1 - \sigma\sqrt{(T-t)}$$

Here, r – interest rate, σ *– annual volatility*

S(0) – spot price, K – strike price, T – maturity time

Greek Parameters

- Note

$$\mathrm{N}(\mathrm{y}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} dx$$

$$\mathrm{N}'(\mathrm{y}) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$$

Delta –

$$\Delta = \frac{\partial C}{\partial S} = N(d_1)$$

A position with a total delta equal to zero is said to be Delta Neutral.

Greek Parameters

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Theta – $$\theta = -\frac{\partial c}{\partial t}$$

Rate of change of security value with respect to time.

Gamma – $$\gamma = \frac{\partial^2 c}{\partial S^2}$$

This measures the sensitivity of Delta. A portfolio with high gamma needs to be rebalanced more often to maintain Delta neutrality.

Greek Parameters

- *Vega –* $$\vartheta = \frac{\partial C}{\partial \sigma}$$

Vega is a unimodal function (first increasing, the decreasing, having single peak) of σ.

Rho – $$\rho = \frac{\partial C}{\partial r}$$

This measures the sensitivity of Delta. A portfolio with high gamma needs to be rebalanced more often to maintain Delta neutrality.