7. $S(0) = 120, \ u = 1.2, \ d = 0.9, \ r = 0.01, \ K = 120, \ T = 2$

$$\begin{tikzcd}[row sep=small]
 & & 172.8 \\
 & 144 \arrow[ur] \arrow[dr] & \\
120 \arrow[ur] \arrow[dr] & & 129.6 \\
 & 108 \arrow[ur] \arrow[dr] & \\
 & & 97.2
\end{tikzcd}$$

$\hat{p} = 0.37$

$$\begin{tikzcd}[row sep=small]
 & & 52.8 \\
 & 25.32 \arrow[ur] \arrow[dr] & \\
\underline{\underline{11.47}} \arrow[ur] \arrow[dr] & & 9.6 \\
 & 3.52 \arrow[ur] \arrow[dr] & \\
 & & 0
\end{tikzcd}$$

$\Delta(0) = 0.61$

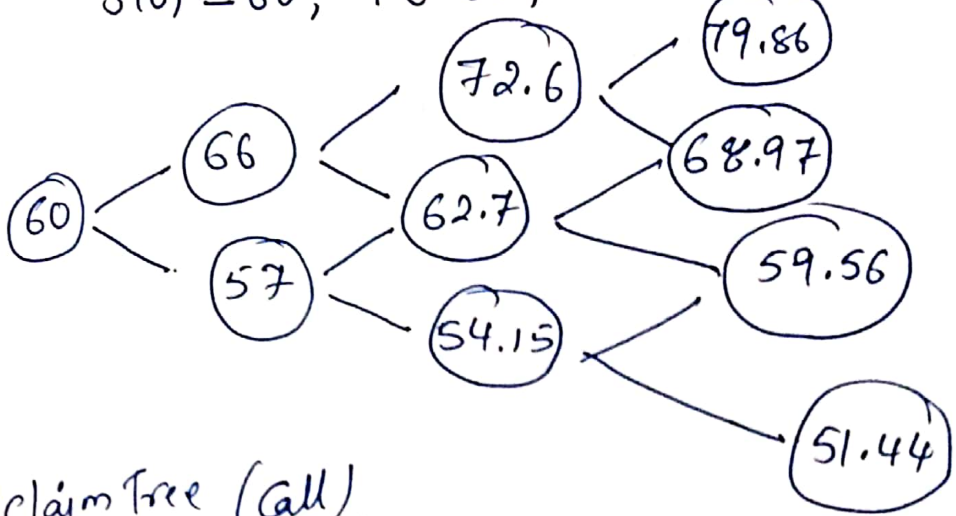
$\Delta_1(u) = 1$

$\Delta_1(d) = 0.30$

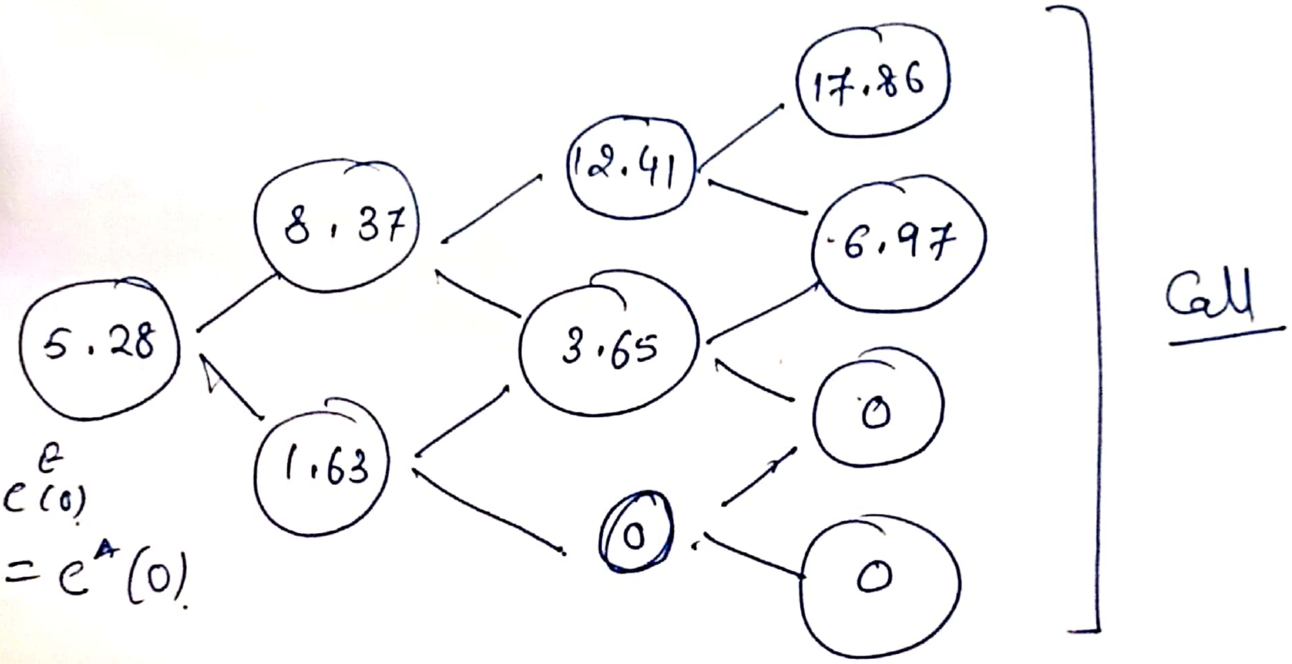
8, 9 & 10 are similar.

6. $S(0) = 60$, $K = 62$, $u = 1.1$, $d = 0.95$, $r = 0.03$, $T = 3$

$\tilde{p} = 0.54$




claim Tree (Call)




$c^E(0)$
$= c^A(0)$


$p^E(0)$

$\boxed{2.98}$ $\boxed{4.08}$ → $\boxed{5}$ $\boxed{7.85}$ (for American)

$p^A(0)$.

5.

(i)

$$\tilde{p} = \frac{e^{0.25} - 0.5}{2 - \frac{1}{2}} = 0.52$$

Distribution of $S_3$

$S_3$ : 32   8   2   $\frac{1}{2}$

$\tilde{P}$ : $(0.52)^3$   ${}^3C_1 (0.52)^2 (0.48)$   ${}^3C_2 (0.52)(0.42)^2$   $(0.42)^3$

(ii)

$\tilde{E}(S_1) = 5.14$

$\tilde{E}(S_2) = 6.5536$   27.5%

$\tilde{E}(S_3) = 8.39.$   28%

$$\text{Average Growth Rate} = \frac{1}{3}\left(\frac{5.14}{4} + \frac{6.55}{5.14} + \frac{8.39}{6.555}\right) = \underline{1.27}$$

(iii) Under actual probability distribution

$S_3$ : 32   8   2   $\frac{1}{2}$

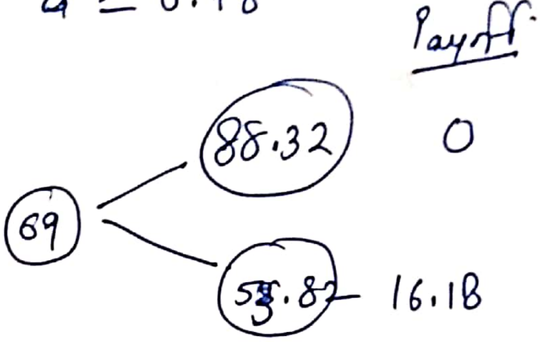
$P$ : $\left(\frac{2}{3}\right)^3$   $3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)$   $3\cdot\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)$   $\left(\frac{1}{3}\right)^3.$

3. $S(0)=69 \quad K=70, \quad r=0.05, \quad \sigma=0.35, \quad T=\tfrac{1}{2}$

$$u = e^{0.35\sqrt{\frac{1}{2}}} = 1.28 \qquad d = 0.78$$

$\tilde{p} = \cancel{0.54}\, 0.49$

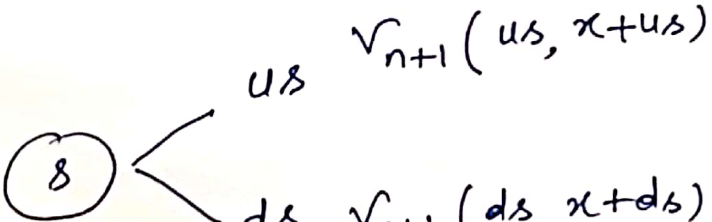
$\underline{p(0) = \cancel{7.08}\, 8.05}$




4. $S(0)=4, \; u=2, \; d=\tfrac{1}{2}, \; r=\tfrac{1}{4}, \quad n=0,1,2,3.$

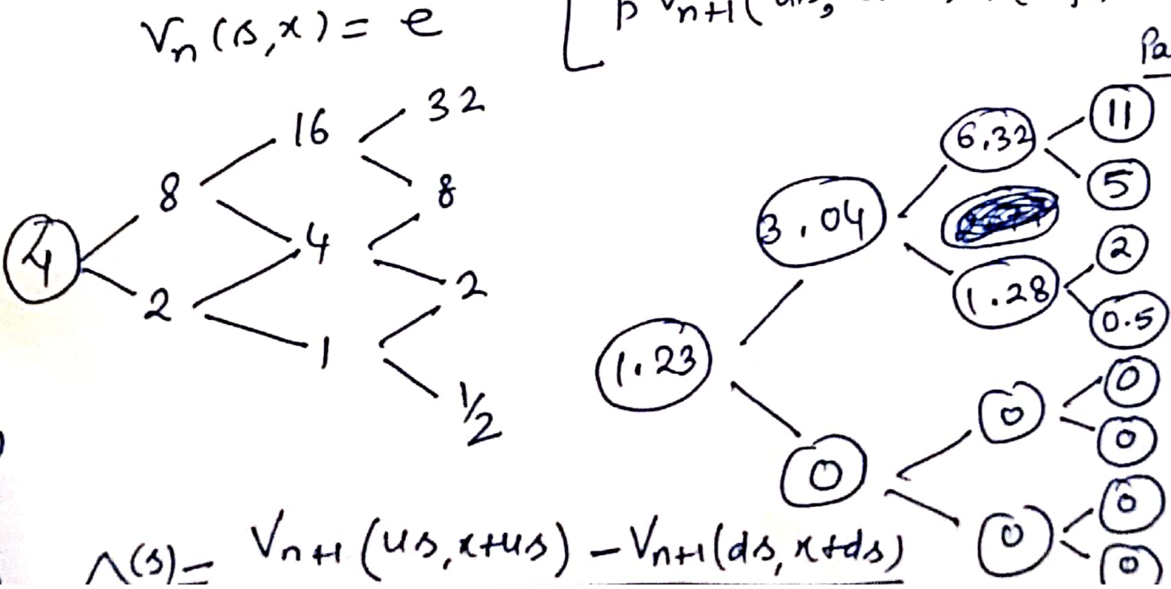
$K=4. \qquad X_n = \sum_{k=0}^{n} S_k \qquad$ Payoff $\left(\frac{1}{4}X_3 - 4\right)^+$

$\tilde{p} = 0.52$




$V_n(s,x)$

(i) $V_n(s,x) = e^{-r\Delta t}\left[\tilde{p}\, V_{n+1}(us, x+us) + (1-\tilde{p})\, V_{n+1}(ds, x+ds)\right]$



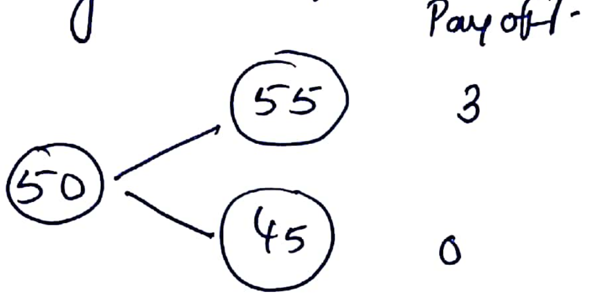

(ii)

$\Delta(s) = \dfrac{V_{n+1}(us, x+us) - V_{n+1}(ds, x+ds)}{}$

# Assignment-1

1. $S(0) = 50,\ K = 52,\ T = 1,\ u = 1.1,\ d = 0.9\ r = 0.05$

Assuming one-step tree




$$\hat{p} = \frac{e^{0.05\times 1} - 0.9}{1.1 - 0.9}$$

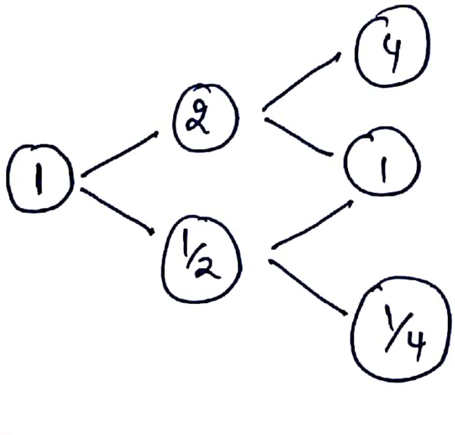
$$= 0.76$$

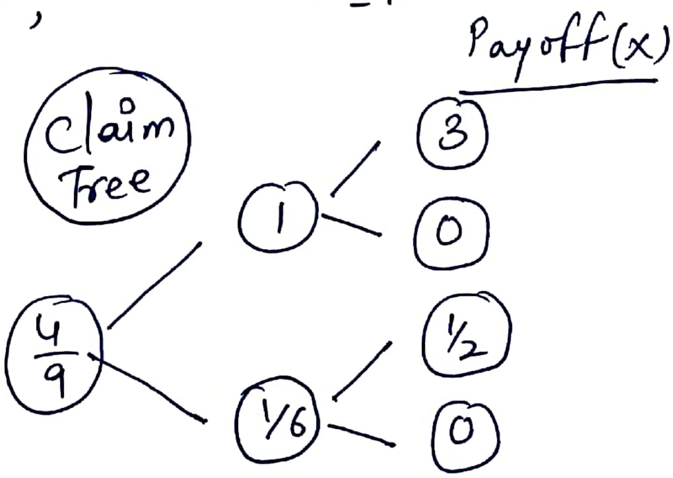
$$c(0) = e^{-0.05}\left[0.76\times 3 + 0.24\times 0\right]$$

$$\boxed{c(0) = 2.16}$$

2. $S(0) = 1,\ u = 2,\ d = \frac{1}{2},\ r = 0,\ N = 2$

Payoff $X = (S_N - m_N)$, $m_n = \min\limits_{n \le N}(S_n)$




$$\hat{p} = \frac{1 - \frac{1}{2}}{2 - \frac{1}{2}} = 0.33$$

$$\Delta(0) = \frac{1 - \frac{1}{6}}{2 - \frac{1}{2}} = 5/9$$

$$\Delta_1(u) = \frac{3 - 0}{4 - 1} = 1$$