Indian Institute of Technology Jodhpur

Financial Engineering

Semester I (2019-20)

Assignment (Option Pricing II)

1. If X is a log-normal random variable with parameters μ and σ^2, then show that $E(X) = e^{\mu+\sigma^2/2}$ and $Var(X) = e^{2\mu+\sigma^2/2}(e^{\sigma^2-1})$.

2. The ratio of selling prices of a security on consecutive days is a log-normally distributed random variable with parameters $\mu = 0.01$ and $\sigma = 0.05$. What is the probability of a two-day increase in the selling price? What is the probability of a one-day decrease in the selling price? What is the probability of a four-day decrease in the selling price?

3. Suppose a stock is currently selling at Rs 100 with annual volatility 24%. A European call option is offered on this stock with time to maturity 3 months and strike price Rs 125. Calculate the price of the block of 100 options in the Black-Scholes framework.

4. Let X be a continuous random variable with probability density $f(x)$. If K is a constant then

$$E(X-K)^+ = \int_K^\infty (\int_x^\infty f(t)dt)dx$$

Further, if X is a normal random variable with parameters μ and σ^2, then

$$E(X-K)^+ = \frac{\sigma}{2\pi} e^{-(\mu-K)^2/2\sigma^2} + (\mu - K)F_Z(\frac{\mu-K}{\sigma})$$

5. Find the price of European and American put options with the given data as

$$S(0) = 12, \quad K = 14, \quad u = 1.1, \quad d = 0.95, \quad r = 2\%(\text{per year})$$

and time to expiry is two years. Assume that a dividend of Rs 2 is paid at time 1.

6. Derive the formulae for all five Greeks.