# Financial Derivatives
## (Binomial Model)

Financial Engineering

by

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# Financial Derivative

Derivative assets are assets whose values are determined by the value of some other assets, called the **underlying**.

There are two common types of derivatives; **Contracts and Options**

**Contracts** – Agreement to buy or to sell something at a **pre-specified time** (Maturity Time) at some **prescribed amount** (Strike Price)

Long Position – The party which agrees to buy

Short Position – The party which agrees to sell

* No Exchange of money  ** Both are obliged

# Contracts

- **Forward contract –** Over the counter

**Futures contract –** Traded at exchange

**Forward Price** $\boldsymbol{F(0,T) = S(0)e^{rT}}$

**Forward price under dividends** $\boldsymbol{F(0,T) = [S(0) - de^{-rt}]e^{rT}}$

**Marking to Market – Daily settlement of margins**

**Note** – $\boldsymbol{F(0,T) < S(0)e^{rT}}$ or $\boldsymbol{F(0,T) > S(0)e^{rT}}$ implies arbitrage

**Notation**

$S(0)$ - Spot price, K – Strike price, T – Time to maturity, r – Rate of interest

# Contract - Example

Suppose that the risk-free rate is 10%. Is there any arbitrage profit if $F(0, 1) = 89$ and $S(0) = 83$ Rs, and a Rs 2 dividend is paid in the middle of the year, that is, at time 1/2?

# Options

A **call option** is a contract giving the owner the right, but not the obligation, to purchase, at expiration, an asset at a specified price called the **strike price**.

A **put option** is a contract giving the owner the right, but not the obligation, to sell, at expiration, an asset at the strike (exercise) price.

The amount of the underlying asset is called the **notional principal** or **underlying amount**.

The price of the option contract is called the **option premium**.

The issuer of the option (call or put) is called the **writer.**

# Options

There are many types of options.

A **European option** can be exercised only at expiration (T).

An **American option** can be exercised at any time between initiation of the contract and expiration.

A **standard** or **plain vanilla option** has no additional contractual features.

An **exotic option** has additional features affecting the payoff.

# Put-Call parity

- Call Option Payoff $\mathrm{Max}((S(T) - K), 0)$

Put Option Payoff $\mathrm{Max}((K - S(T)), 0)$

Put-call parity

$$\mathrm{C}(0) - \mathrm{P}(0) = \mathrm{S}(0) - e^{-rT}K$$

**Note** – $\mathrm{C}(0) - \mathrm{P}(0) < \mathrm{S}(0) - e^{-rT}K$ or

$\mathrm{C}(0) - \mathrm{P}(0) > \mathrm{S}(0) - e^{-rT}K$ implies arbitrage

# Option Pricing – Binomial Model

**Required Ingredients**

Spot Price – S(0)

Strike Price – K

Maturity Time – T

Interest Rate – r

Up factor – u

Down factor – d

**Assumptions**

- No arbitrage
- Volatility is constant
- Constant rate of interest
- Only two possibilities

# Option Pricing – One Period Binomial Model

- A European call option price is

$$C(0) = e^{-rT}\tilde{E}\Big((S(T) - K)^{+}\Big),$$

where, the *Expectation* is taken with respect to the **risk neutral probability measure** given by

$$\tilde{p} = \frac{e^{rT} - d}{u - d},$$

where, $u \; and \; d$ are up and down factors respectively.

**Note – For no arbitrage, d < (1+r) < u**

# Example

Let $S(0)$ = 100 Rs, $u$ = 1.1, $d$ = $0.90$ and $r$ = 0.05. Consider a European call option with strike price $K$ = 105 Rs and exercise time $T$ = 1. Find the option price and the replicating strategy.

What changes do you see if we use two periods?

# Option Pricing – Multi Period Binomial Model

- A European call option price is

$$C(0) = e^{-rT}\tilde{E}\big((S(T) - K)^+\big),$$

where, the *Expectation* is taken with respect to the **risk neutral probability measure** given by

$$\tilde{p} = \frac{e^{r\Delta t} - d}{u - d},$$

where, $\Delta t, u\ and\ d$ are time duration, up and down factors respectively for one period.

The risk neutral expectation will be calculated by using the Binomial distribution.

# Example

Let $S(0)$ = 100 Rs, $u$ = 1.05, $d$ = 0.95 and $r$ = 0.05. Consider a European call option with strike price $K$ = 105 Rs and exercise time $T$ = 1. Find the option price and the replicating strategy using two period Binomial model.