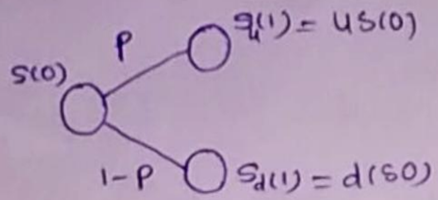
① ①

① Given: $S(0) = 50$, $u' = 10\%$, $d' = 10\%$.

$r = 0.05$, $T = 1$, $K = 52$



$$\tilde{p} = \frac{e^{rT} - d}{u - d}$$

$$= \frac{e^{0.05\times1} - d}{u - d}$$

$u = (1+u')$ & $d = (1-d')$

$= (1+0.1)$ $\quad = (1-0.1)$

$\boxed{u = 1.1}$ $\quad \boxed{d = 0.9}$

$$\tilde{p} = \frac{e^{0.05\times1} - 0.9}{1.1 - 0.9} = 0.7564$$

$S_u(1) = uS(0) = 1.1\times50 = 55$

$S_d(1) = dS(0) = 0.9\times50 = 45$

Price of European call option $= C^E(0) = E_*(e^{-rT}(S(T) - X)^+$

$$C(1) = \begin{cases} 3 \\ 0 \end{cases}$$

$$= \frac{3\times0.7564 + (0\times(1-p))}{e^{rT}}$$

$$= \frac{3\times0.7564 + 0}{e^{0.05\times1}}$$

$= 2.1585$

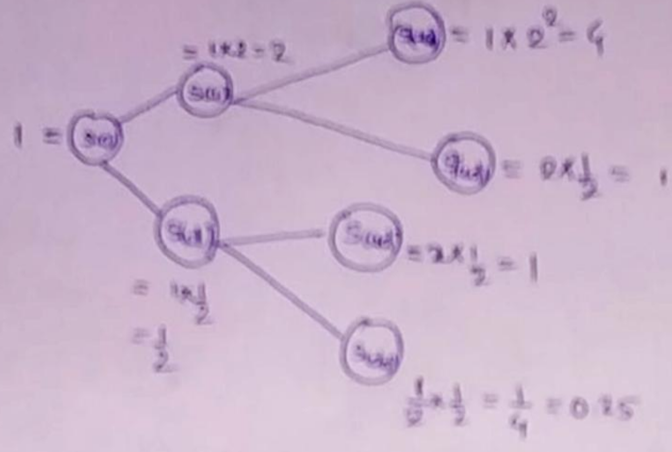
$\boxed{\text{price of call option} = 2.1585}$

② S(0)

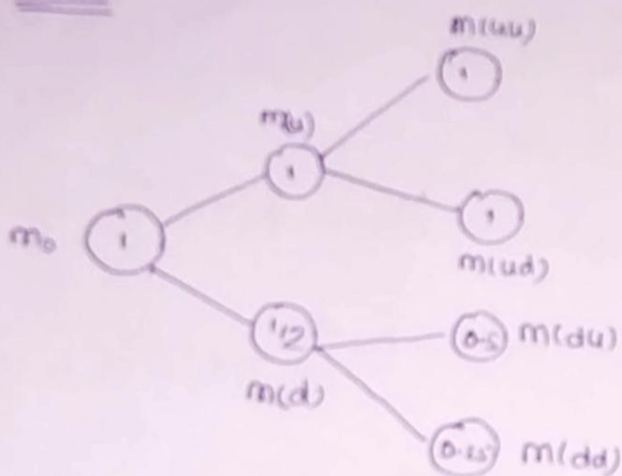
(2) $S(0) = 1$, $u = 2$, $d = \frac{1}{2}$, $r = 0$, $N = 2$

$X = (S_N - m_N)$ where $m_n = \min_{n \le N} S_n$

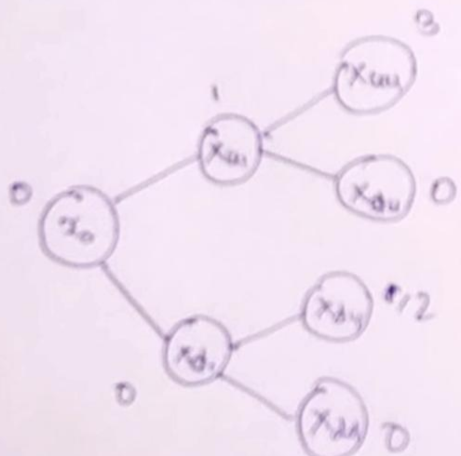
(a)



for m

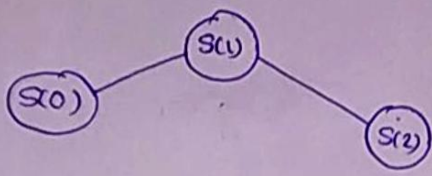


$X = (S_N - m_N)$



(b)

(3)



At time 0 ~~by~~ buy x

At time 1 sell x to get cash $A = 2x\,S(1)$

At time 2 don't buy x.

(4) $S_0 = 4,\ u = 2,\ d = \frac{1}{2},\ r = \frac{1}{4},\ n = 0, 1, 2, 3.$

$$X_n = \sum_{k=0}^{n} S_n.$$

(i) $$X_n = \sum_{k=0}^{n} S_n \quad \text{—— (1)}$$

$$x_{n+1} = \sum_{k=0}^{n+1} S_{n+1}$$

$$x_{n+1} = \left[\sum_{k=0}^{n} S_n\right] + S_{n+1} \quad \ldots \text{(from eqn (1))}$$

$$\boxed{x_{n+1} = x_n + S_{(n+1)}} \quad \text{—— (2)}$$

$$V_n = \left[\frac{1}{4} x_n - 4\right]^+ \quad \ldots \text{from given.}$$

$$V_{n+1} = \left[\frac{1}{4} x_{n+1} - 4\right]^+$$

$$V_{n+1} = \left[\frac{1}{4}\left(x_n + S_{n+1}\right) - 4\right]^+ \quad \ldots \text{(from eqn (2))}$$

$$V_{n+1} = \left[\frac{1}{4} x_n + \frac{S_{n+1}}{4} - 4\right]$$

on Rearranging.

$$V_{n+1} = \left[\frac{1}{4} x_n - 4\right] + \frac{S_{n+1}}{4} \quad \dots \quad \text{given.}$$

$$V_{n+1} = V_n + \frac{S_{n+1}}{4} \quad \dots \quad \text{from eq}^n \text{ (2)}$$

$$\boxed{V_n = \left[V_{n+1} - \frac{S_{n+1}}{4}\right]} \quad \dots \quad (3)$$

(ii) approch 1: $V_{n+1} = V_{n+2} - \frac{S_{n+2}}{4} \quad \dots$ from (3)

$$V_{n-1} = \left[V_n - \frac{S_n}{4}\right] \quad \dots$$

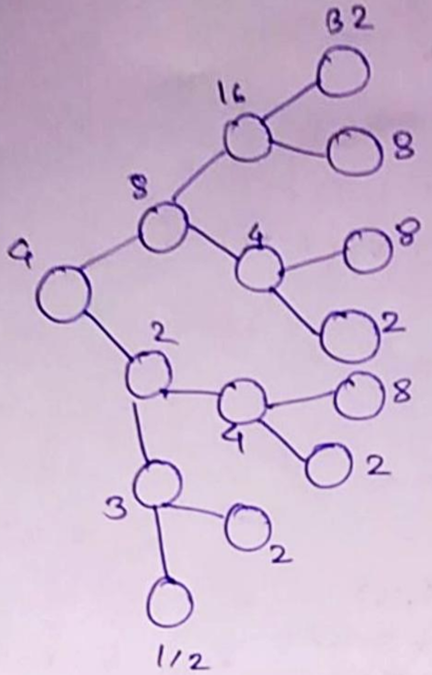
$$V_{n-1} = \left[V_{n+1} - \frac{(S_{n+1} + S_n)}{4}\right]$$

put $n = 0$

$$\boxed{V_0 = V_2 - \frac{\sum_{n=1}^{n+1} S_n}{4}}$$

$$\tilde{p} = \frac{e^{rT} - d}{u - d} = \frac{e^{\frac{1}{4}\times 1} - \frac{1}{2}}{4 - 0.5} = 0.522$$

$$1 - \tilde{p} = 1 - 0.522 = 0.478$$



X_3	$\frac{X_3}{4}$	Payoff. $(X_{3/4} - 4)^+$
$= 4+8+16+32 = 60$	15	11
$4+8+16+8 = 36$	9	5
$4+8+4+8 = 24$	6	2
$4+8+4+2 = 18$	4.5	0.5
$4+2+4+8 = 18$	4.5	0.5
$4+2+4+2 = 12$	3	0
$4+2+1+2 = 9$	2.25	0
$4+2+1+\frac{1}{2} = 7.5$	2.625	0

$V_3(32,60) = 1$

$V_2(16,8) = \dfrac{(11 \times 0.522 + 5 \times 0.478}{e^{1/4 \times 1}} = 6.4$

$V_3(8,36) = 5$

$V_1(8,12) = \dfrac{6.4 \times 0.522 + 1 \times 0.478}{e^{1/4 \times 1}} = 2.96$

$V_3(8,24) = 2$

$V_2(4,16) = \dfrac{2 \times 0.522 + 0.5 \times 0.478}{e^{1/4 \times 1}} = 1$

$V_3(2,18) = 0.5$

$V_0(4,4) = \dfrac{2.96 \times 0.522 + 0.08 \times 0.478}{e^{1/4 \times 1}}$

$= 1.216$

$V_3(8,18) = 0.5$

$V_2(4,10) = \dfrac{0.5 \times 0.522 + 0 \times 0.478}{e^{1/4 \times 1}} = 0.2$

$V_3(2,12) = 0$

$V_1(2,6) = \dfrac{0.2 \times 0.522 + 0 \times 0.478}{e^{1/4 \times 1}}$

$= 0.08$

$V_3(2,9) = 0$

$\dfrac{0 \times 0.577 + 0 \times 0.478}{e^{1/4 \times 1}}$

$= 0$

$V_3(0.5,7.5) = 0$

(iii) $V_n = x(S_n)$

$$\left(\frac{1}{4} x_n - 4\right) = (S_n)(Z)$$

$$\left(\frac{1}{4}\frac{x_n}{S_n} - \frac{4}{S_n}\right) = Z$$

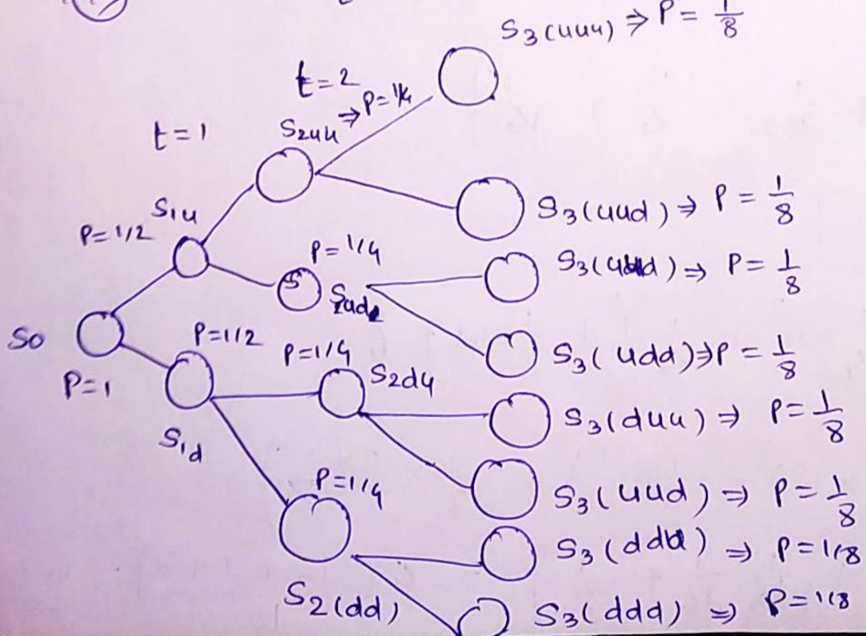
$$\frac{\frac{1}{4}\sum_{k=0}^{n} S_n}{S_n} - \frac{4}{S_n} = Z(n)$$

$$\boxed{\frac{S_0 + S_1 + \dots + S_n}{4(S_n)} - \frac{4}{S_n} = Z_n}$$

(5)

$$\tilde{P} = \frac{e^{rt} - d}{u - d} = \frac{e^{\frac{1}{a}\times 1} - 1/2}{2 - \frac{1}{2}} = 0.522 \approx 0.5$$

So (ii) $1 - \tilde{P} = \tilde{q} = 0.478 \approx 0.5$



$P(S_{uuu}) = u^3 S(0) = (2)^3 \times 4 = 32$

$P(S_{uud}) = u^2 d\, S(0) = (2)^2 \times \frac{1}{2} \times 4 = 8$

$P(S_{udu}) = P(S_{duu}) = 8$

$P(S_{ddd}) = d^3 S(0) = \left(\frac{1}{2}\right)^3 \times 4 = \frac{1}{2}$

$P(S_{ddu}) = P(S_{dud}) = P(S_{udd}) = \left(\frac{1}{2}\right)^2 \times 2 \times 4 = 2$

distribut^n of S_3 is given in foll. d. Table.

S_3	0.5	2	8	32
Prob.(S_3)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(b)

$$E(S_3) = S_0 \left[\frac{1}{8} \times u^3 + \frac{1}{8} d^2 + \frac{3}{8} \times u^2 d + \frac{3}{8} \times u d^2 \right]$$

$$= 4 \left[\frac{1}{8} \times (2)^3 + \frac{1}{8} \times \left(\frac{1}{2}\right)^2 + \frac{3}{8} \times (2)^2 \times \frac{1}{2} + \frac{3}{8} \times (2)\left(\frac{1}{2}\right)^2 \right]$$

$$= 4 \left[1 + \frac{1}{32} + \frac{3}{4} + \frac{3}{16} \right]$$

$$\boxed{E(S_3) = 7.875}$$

$$E(S_2) = S_0 \left[\frac{1}{4} \times u^2 + \frac{1}{4} d^2 + \frac{1}{2} u d \right]$$

$$= 4 \left[\frac{1}{4} \times (2)^2 + \frac{1}{4} \times \left(\frac{1}{2}\right)^2 + \frac{1}{2} \times 2 \times \frac{1}{2} \right]$$

$$= 4 \left[1 + \frac{1}{16} + \frac{1}{2} \right] = 4 \left(\frac{16 + 1 + 8}{16} \right) = \underline{\underline{6.25}}$$

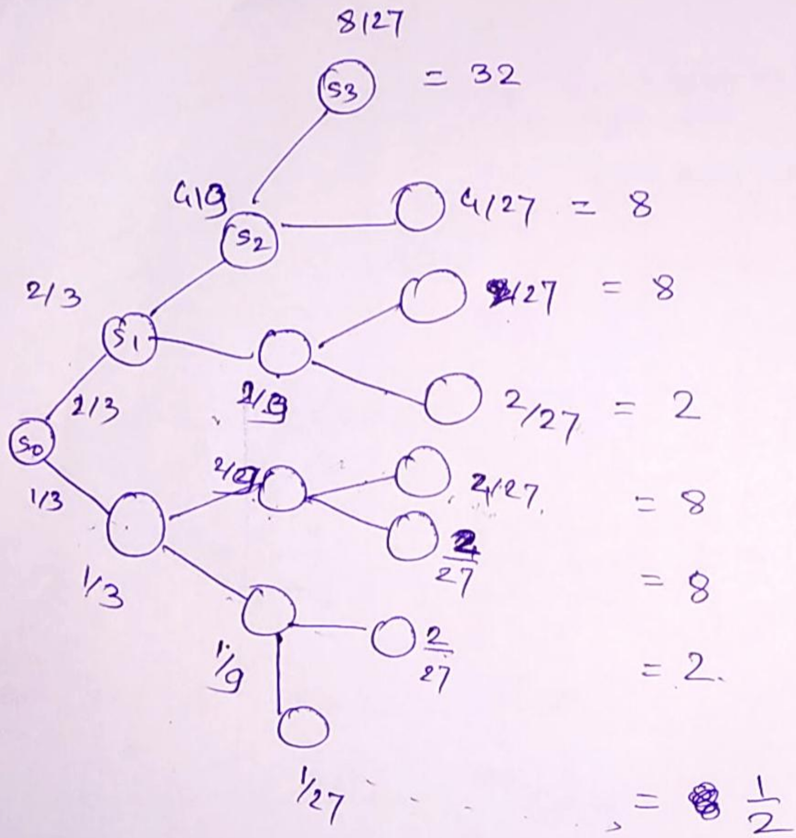
$E(S_1) = \frac{1}{2}u + \frac{1}{2}d$

$= \frac{1}{2} \times 2 + \frac{1}{2} \times \frac{1}{2}$

$= 1 + \frac{1}{4} = \frac{5}{4} = \underline{1.25}$

(iii) $p = \frac{2}{3}$, $q = \frac{1}{3}$

$P(S_{uuu}) = u^3 S(0) = (2)^3 \times 4 = 32$



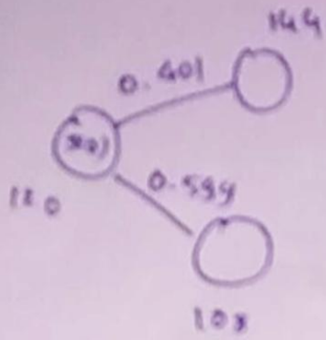
S_3	0.5	2	8	32
Probability	$\frac{1}{27}$	$\frac{8}{27}$	$\frac{10}{27}$	$\frac{8}{27}$

(7) call option

(10)

$u = 1.2, \quad d = 0.9$

$K = 120, \quad r = 1\%, \quad T = 2\,yrs.$



$$\tilde{p} = \frac{e^{rT} - d}{u - d}$$

$$= \frac{e^{0.01 \times 2} - 0.9}{1.2 - 0.9} = 0.401$$

$$c(0) = \frac{24 \times 0.401 + 0 \times 0.599}{e^{0.01 \times 2}}$$

$\boxed{c(0) = 9.4334}$ option price.

replicating strategy

$$\Delta = \frac{P_u - P_d}{S_u - S_d} = \frac{24 - 0}{144 - 108} = 0.666$$

As if $S_T = S_d$ then

$$\Delta S_d + B e^{rt} = P_d$$

$$0.666 \times 108 + B e^{0.01 \times 2} = 0$$

$$B = -70.57$$

$$c(0) = S_0 \Delta + B$$

$$= 120 \times 0.666 - 70.57$$

$\boxed{c(0) = 9.422}$

(8) $S(0) = 100$, $r = 5\%$, $T = 4$ yrs. (1)

$\sigma = 20\%$. $K = 80$

2 period Binomial model

→ CRR

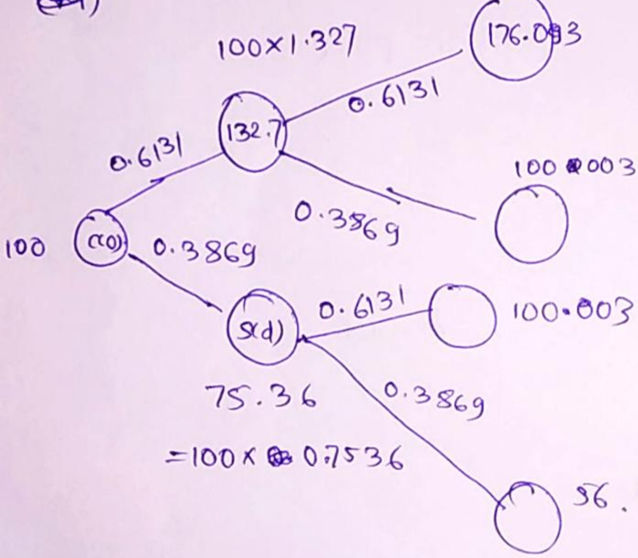
→ European call option.

$\Delta t = \frac{T}{n} = \frac{4}{2} = 2$

$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2 \times \sqrt{2}} = 1.327$

(i) $d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.2 \times \sqrt{2}} = 0.7536$

(ii)



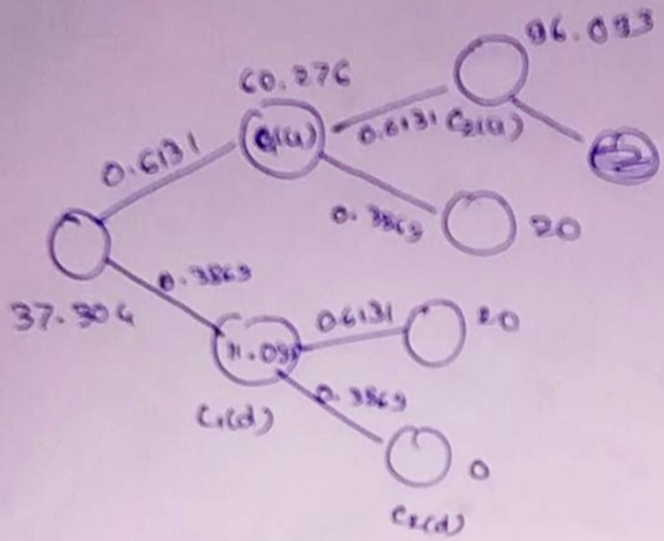
$(S(T) - K)^+$

$176.093 - 80 = 90$

$100.083 - 80 = 20$

$= 0$

$\tilde{p} = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \times 2} - 0.7536}{1.327 - 0.7536} = 0.6131$



$$C_1(u) = \frac{(0.6131 \times 96.093) + (0.3869) \times 20)}{e^{0.05 \times 2}} = \cancel{37.322}\ 18$$

$$C_1(d) = \frac{(0.6131 \times 20) + (0 \times 0.3869)}{e^{0.05 \times 2}} = 11.080$$

$$C(0) = \frac{(0.6131 \times 60.276) + (0.3869 \times 11.095)}{e^{0.05 \times 2}} = 37.322$$

(9) $S_0 = Rs\ 1500$ $K = Rs\ 1470$

$\sigma = 22\%$ $T = 44$ days

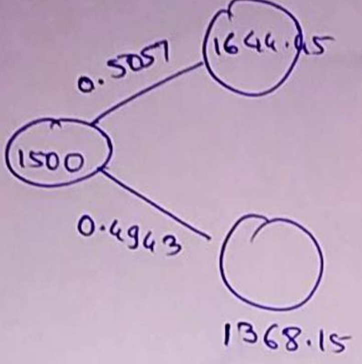
$r = 3\%$

No. of trading days $= 252$

$$\frac{44}{252} = 0.1746 \text{ yrs.}$$

$$u = e^{\sigma\sqrt{t}} = e^{0.22\sqrt{0.1746}} = 1.0963$$

$$d = e^{-\sigma\sqrt{t}} = \frac{1}{1.0963} = 0.9121$$



payoff.

$(S(T)-K)^+$

174.45

0

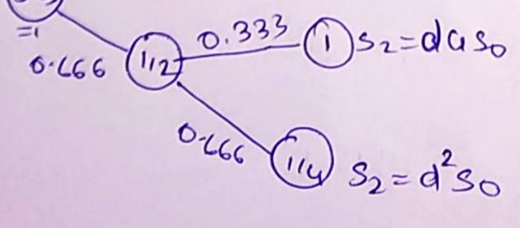
$$\tilde{p} = \frac{e^{rt} - d}{u-d} = \frac{e^{0.03 \times 0.1746} - 0.9121}{1.0963 - 0.9121}$$

$$= 0.50571$$

$$c(0) = \frac{(0.5057)(1644.45) + (0.4943)(1368.15)}{e^{0.03 \times 0.1947}}$$

$$\boxed{c(0) = 1498.22}$$

(10) $S_0 = 1,\ u = 2,\ d = \frac{1}{2},\ r = 0,\ N = 2$

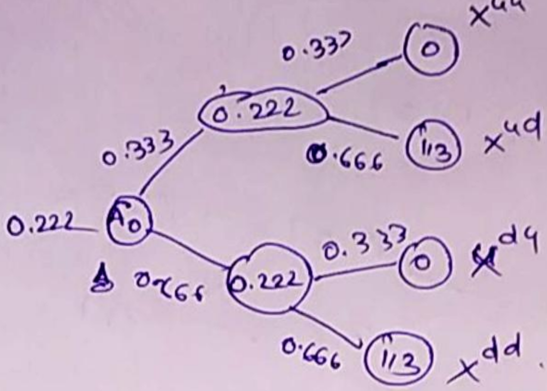


Avg.	X
$\frac{4+2+1}{3} = \frac{7}{3}$	0
$\frac{1+2+1}{3} = \frac{4}{3}$	1/3
$\frac{1+1/2+1}{3} = \frac{5}{6}$	0
$\frac{1+\frac{1}{2}+\frac{1}{4}}{3} = \frac{7}{12}$	$\frac{1}{3}$

$$\tilde{p} = \frac{e^{rt} - d}{u - d}$$

$$= \frac{e^{0\times 2} - 1/2}{2 - \frac{1}{2}}$$

$$= \frac{1}{3} = 0.333$$



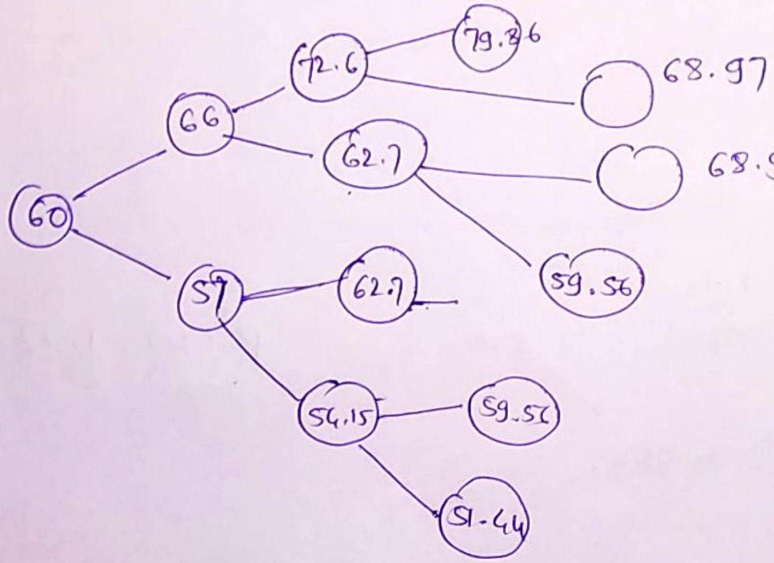
$$C_1(u) \text{ or } A_1^u = \frac{0 \times 0.333 + 0.666 \times 1/3}{e^{0\times t}} \simeq \frac{2}{9} = 0.222$$

$$C_1(d) \text{ or } A_1^d = \frac{0.333 \times 0 + 0.666 \times \frac{1}{3}}{e^{0\times t}} = 0.222 = \frac{2}{9}$$

$$C_1(0) \text{ or } A_0 = \frac{(0.333 \times 0.222) + (0.666 \times 0.222)}{e^{0\times t}}$$

$$= 0.222 = \frac{2}{9}$$

⑥



$$p = \frac{e^{0.03} - 0.95}{1.1 - 0.95} = 0.53$$

$$1 - p = 0.47$$

$C^{E}(0)$.

17.86

12.47

8.16

6.97

3.65

5.13

1.91

0 — 0

0

$C^{E}(0) = 5.13$

$C^{A}(0)$

17.86

12.47 — 6.97

8.16

max(12.47, 10.6)

5.13

max(8.16,4) 3.69

0

1.91

max(0.7, 3.65)

max(1.91,0)

0 — 0

0

$P^{E}(0)$

0

0.50 — 0

0

1.11

1.77

2.44

$P^{E}(0) = 1.77$

3.33

6.068

10.56

C^E should be exercised at year 1

for max. profit

put option

$$E(T=3) = 3P(1-P)^2 \times 2.44 + (1-P)^3 \times 10.56$$

$$= 0.857 + 1.096 = 1.05$$

$$E(T=2) = 2P(1-P) \times 1.11 + (1-P)^2 \times 7.85$$

$$= 0.553 + 1.734 = 2.287$$

$$E(T=1) = 2.615$$

$$E(T=3)e^{-3r\tau} = 1.779$$

$$E(T=2)e^{-2r\tau} = 2.151$$

$$E(T=1)e^{-r\tau} = 2.531$$

put should ~~also~~ be excersised at 1

as expectation is max. share

$C^E \rightarrow$ 1 year

$P^E \rightarrow$ 1 year.

(3) $S_0 = 69$, $K = 70$, $r = 0.05$, $\sigma = 0.35$ & $T = 0.5$

$$d_1 = \frac{\ln(69/70) + (0.05 + 0.35^2/2) \times 0.5}{0.35 \times \sqrt{0.5}} = 0.1666$$

$$d_2 = d_1 - 0.35\sqrt{0.5} = -0.0809$$

The price of the European put is,

$$70 e^{-0.05 \times 0.5} N(0.0809) - 69 N(-0.1666)$$

$$= 70 e^{-0.0125} \times 0.5323 - (69 \times 0.4338)$$

$$= \underline{\underline{6.40}}$$