Indian Institute of Technology Jodhpur

Financial Engineering

Semester II (2019-20)

Assignment (Option Picing - Discrete)

1. Price a European call option of 1 year with a strike price of Rs 52 on a stock whose current price is Rs 50. The stock price either moves up by 10% or moves down by 10%. We also suppose that the risk free interest rate is 5%.

2. Consider a Binomial market model where $S(0) = 1, u = 2, d = 1/2, r = 0$ and $N = 2$. For a "look-back call" option with payoff

$$X = (S_N - m_N), \quad \text{where } m_n = \min_{n \leq N} S_n$$

   Determine

   (a) The claim-tree (payoff).

   (b) The investment strategy if the stock moves from $S(0) = 1$ to $S(1) = 2$ to $S(2) = 1$.

3. What is the price of a European put option on a stock when the stock price is 69 and strike price is 70, the interest rate is 5%, the stock's volatility is 35%, and the exercise time is six months.

4. Consider a three period model with $S_0 = 4, u = 2, d = 1/2$ and take the interest rate $r = 1/4$. For $n = 0, 1, 2, 3$, define $X_n = \sum_{k=0}^{n} S_n$. Consider an Asian call option that expires at time three and has strike price $K = 4$, that is, whose payoff at time three is $(\frac{1}{4}X_3 - 4)^+$. Let $v_n(s, x)$ denote the price of this option at time n if $S_n = s$ and $X_n = x$. In particular, $v_3(s, x) = (\frac{1}{4}x - 4)^+$.

   (i) Develop an algorithm for computing v_n recursively. In particular, write a formula for v_n in terms of v_{n+1}.

   (ii) Apply the above algorithm to compute the price of the Asian option at time zero.

   (iii) Provide a formula for the number of shares of stock that should be held by the replicating portfolio at time n if $S_n = s$ and $X_n = x$.

5. Consider the above three period model

   (i) What is the probability distribution of S_3 under the risk neutral probabilities?

   (ii) Compute $\tilde{E}(S_1), \tilde{E}(S_2)$ and $\tilde{E}(S_3)$. What is the average growth rate of the stock price?

   (iii) What is the probability distribution of S_3 under the actual probabilities $p = 2/3$ and $q = 1/3$?

6. Consider the data $S(0) = 60, K = 62, u = 1.1, d = 0.95, r = 0.03$ and $T = 3$. Find $C^E(0), P^E(0), C^A(0)$ and $P^A(0)$. Identify the time instants when P^A and C^A will be exercised.

7. Let $S(0) = 120, u = 1.2, d = 0.9, r = 1\%$. Consider a call option with strike price $K = 120$ and $T = 2$. Find the option price and the replicating strategy.

8. A stock is currently selling at Rs 100 with annual volatility 20%. Assume that the continuously compounded risk-free interest rate is 5%. Using a two period CRR binomial option pricing model, find the price of a European call option with strike price Rs 80 and time of expiration 4 years.

9. A non-dividend paying stock is selling at Rs 1500 on March 1, 2010 with annual volatility of 22%. Assume that the continuously compounded risk free interest rate is 3%. Use one period Binomial model to compute the price of a European call option written on this stock with strike price Rs 1470 expiring on 29 April 2010 (The total number of trading days between the given period is 44 and number of trading days in year 2010 is 252).

10. In a Binomial market model with parameters $S_0 = 1, u = 2, d = 1/2, r = 0$ and $N = 2$, consider an Asian Put option with "floating strike" and payoff

$$X = (M - S_N)^+, \quad \text{where} \quad M = \frac{S_0 + S_1 + S_2}{3}$$

Determine the price process, denoted by (A_n) and the hedging strategy for $n = 0, 1$.