

Appendix

In the following pages of the Appendix, we are presenting a systematic investigation of coarse-graining for:

- The 88 Minimum Equivalent ECA rules
- The Game of Life
- An evolved MNCA rule.

Results for ECA are also available digitally at the following link:

- <https://s4nyam.github.io/eca88>.

A portal which provides the pointers to all our code implementations is available at the following link:

- <https://s4nyam.github.io/mncaportal>.

Starting from the next page, we present one rule per page. Each page is structured as follows:

- The first row and first image displays the original CA simulation with 1% initialisation. The CA size is 100, and the rule is applied for 100 steps.
- Further, the second image shows the histogram plot for each of the blocks, containing three different thresholds. Note that the used block size is 2x1 for 1D CA and 2x2 for 2D CA.
- Further, the next three images show the coarse-grained versions of the original CA simulation with the corresponding thresholds applied in ascending order.
- Similarly, for the second and third rows, the density of initialisation is 20% and 50% respectively.

Note that for some rules, not all blocks are present in the space-time diagram and therefore there are less bars in the histogram and less coarse-graining results visualized.

The thresholds are obtained by adding a δ of 0.005 to the calculated probabilities. Please note that for some rules, the resulting threshold includes two histogram bars with almost identical values. This results in less coarse-graining to visualize. An example is Rule 43 (Third row with 50% initialisation in Table 38), where several bars have almost identical frequency.

The following Appendix in PDF is also available at the following link:

- <https://s4nyam.github.io/mncaportal/appendix.pdf>

Table 1: FHCG plots for ECA Rule 0.

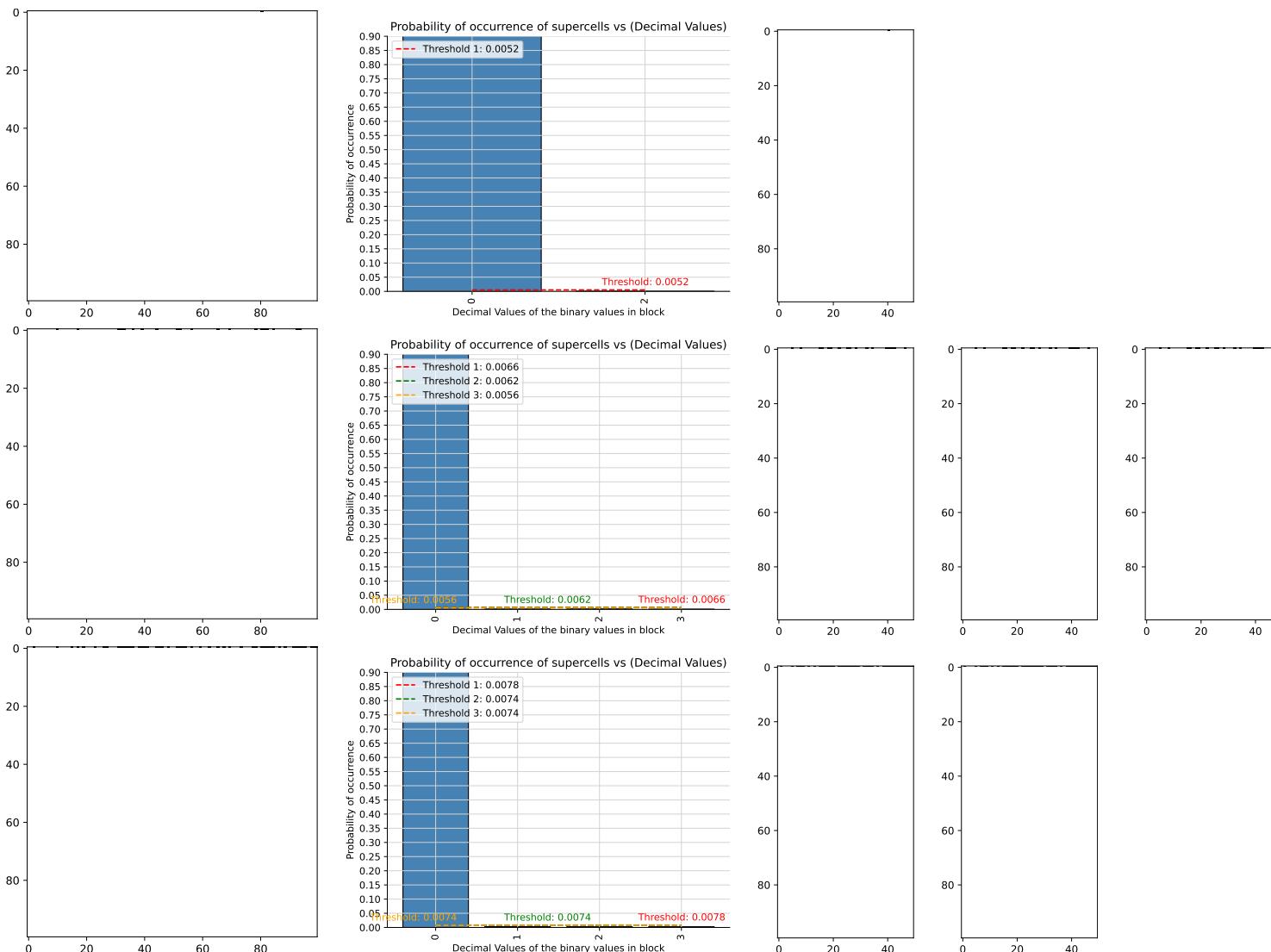


Table 2: FHCG plots for ECA Rule 1.

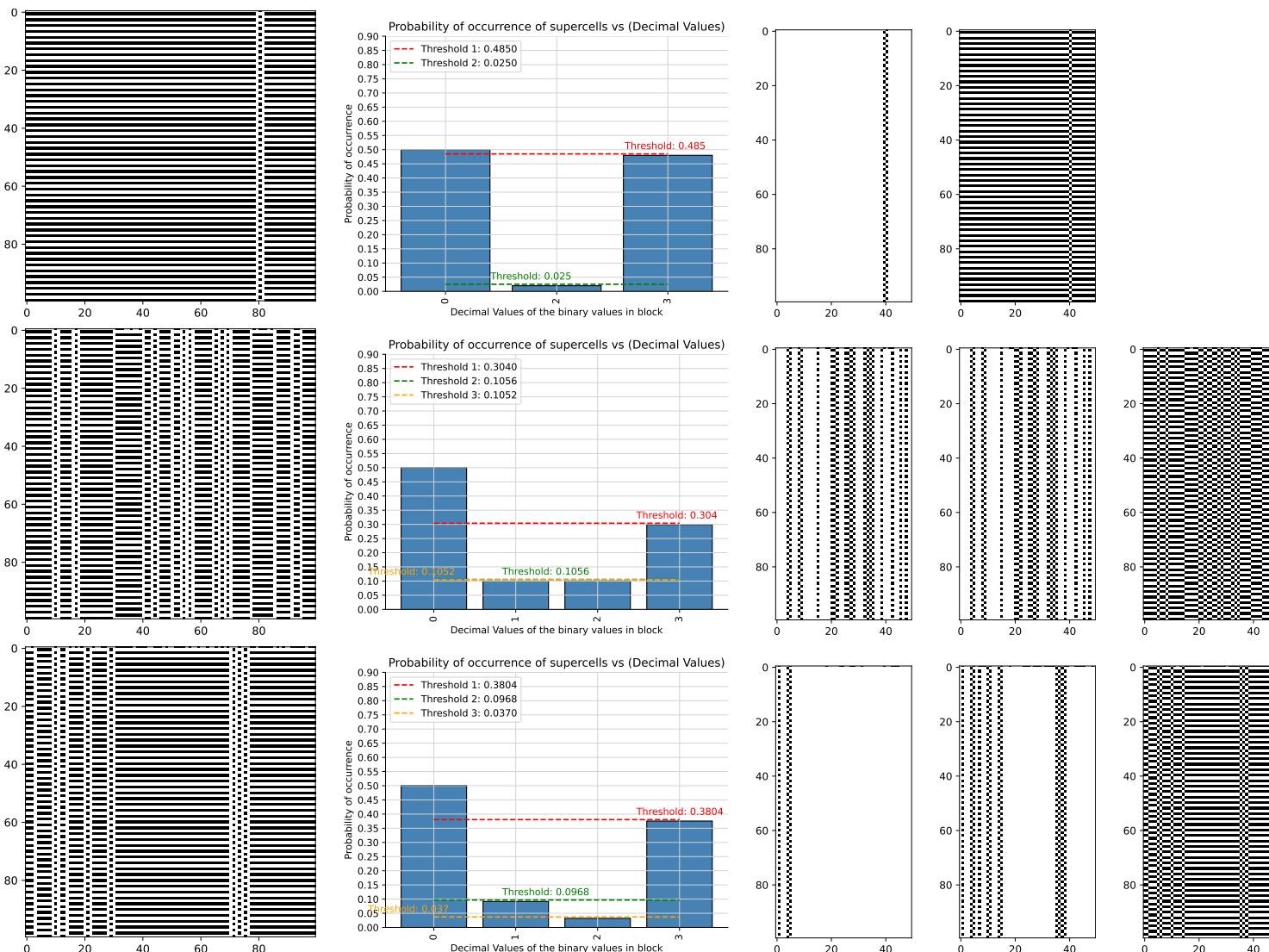


Table 3: FHCG plots for ECA Rule 2.

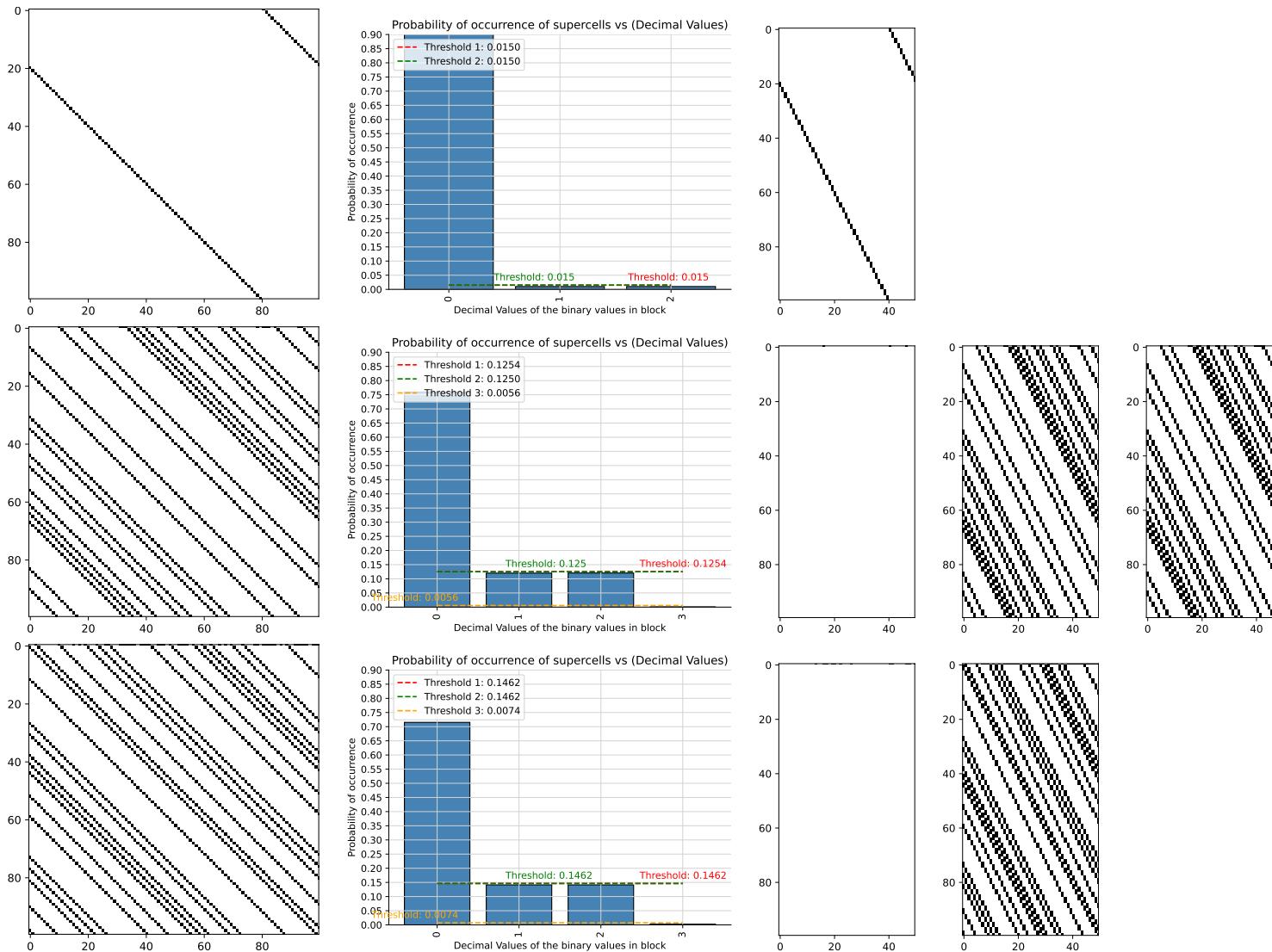


Table 4: FHCG plots for ECA Rule 3.

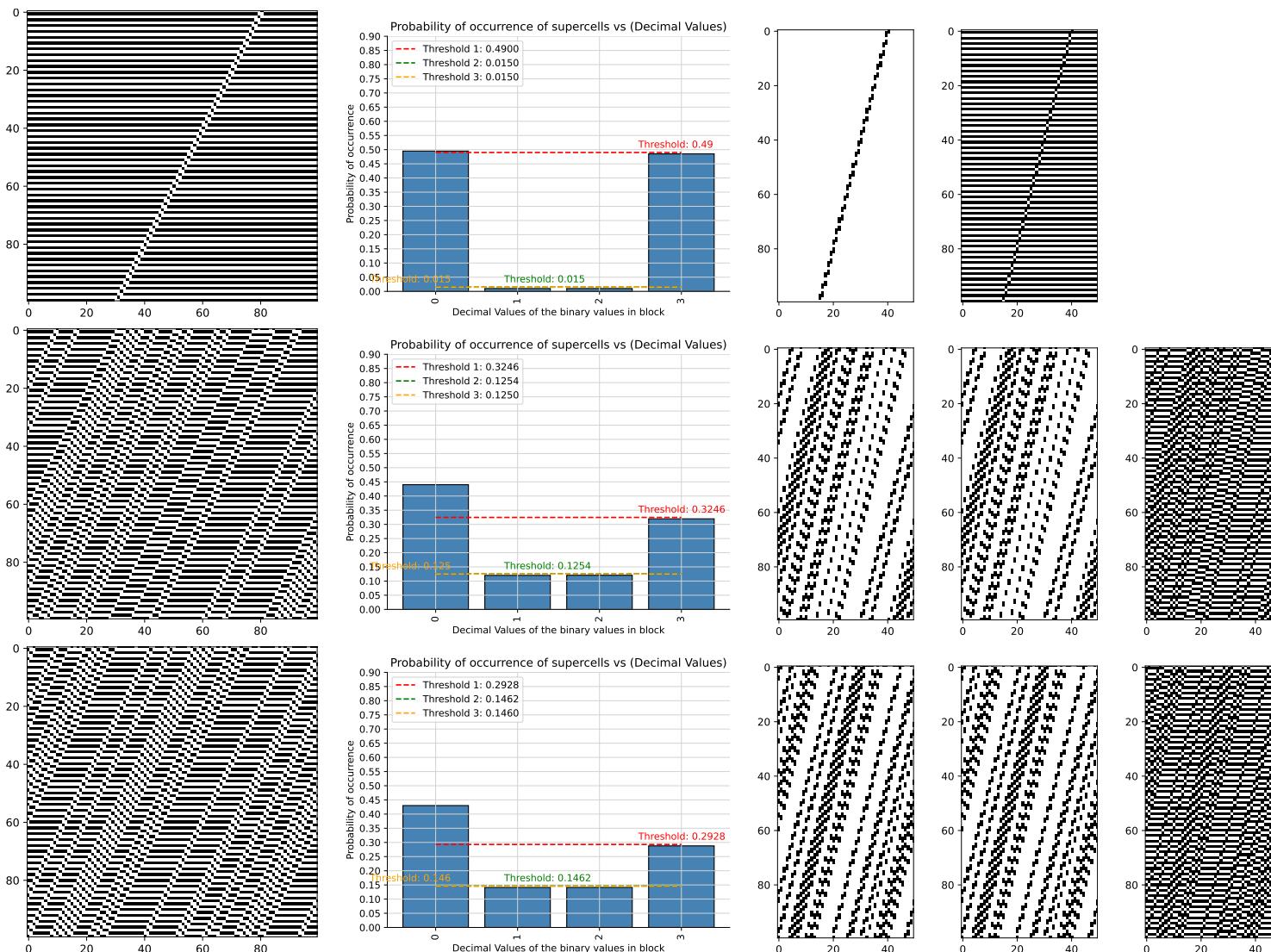


Table 5: FHCG plots for ECA Rule 4.

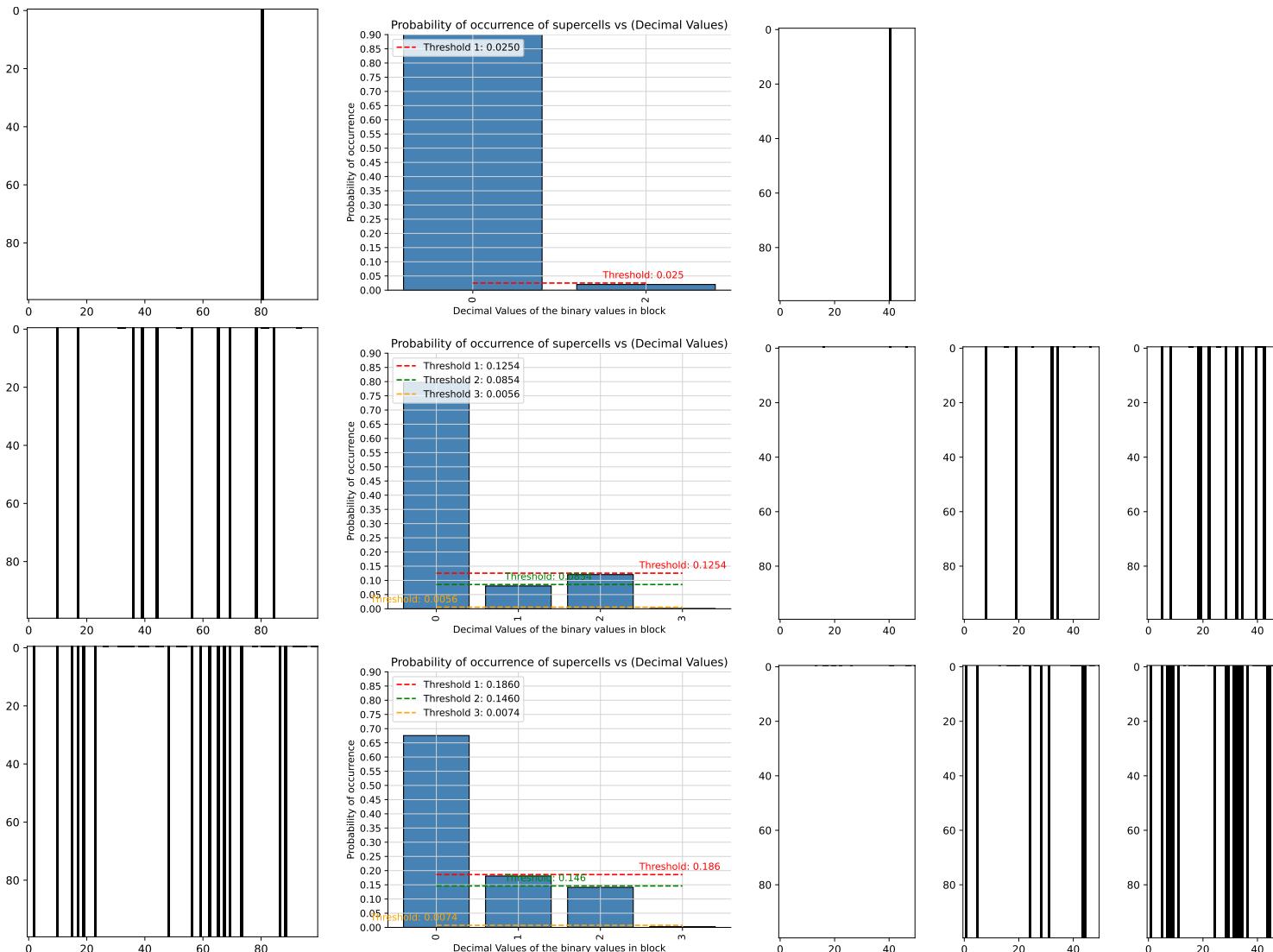


Table 6: FHCG plots for ECA Rule 5.

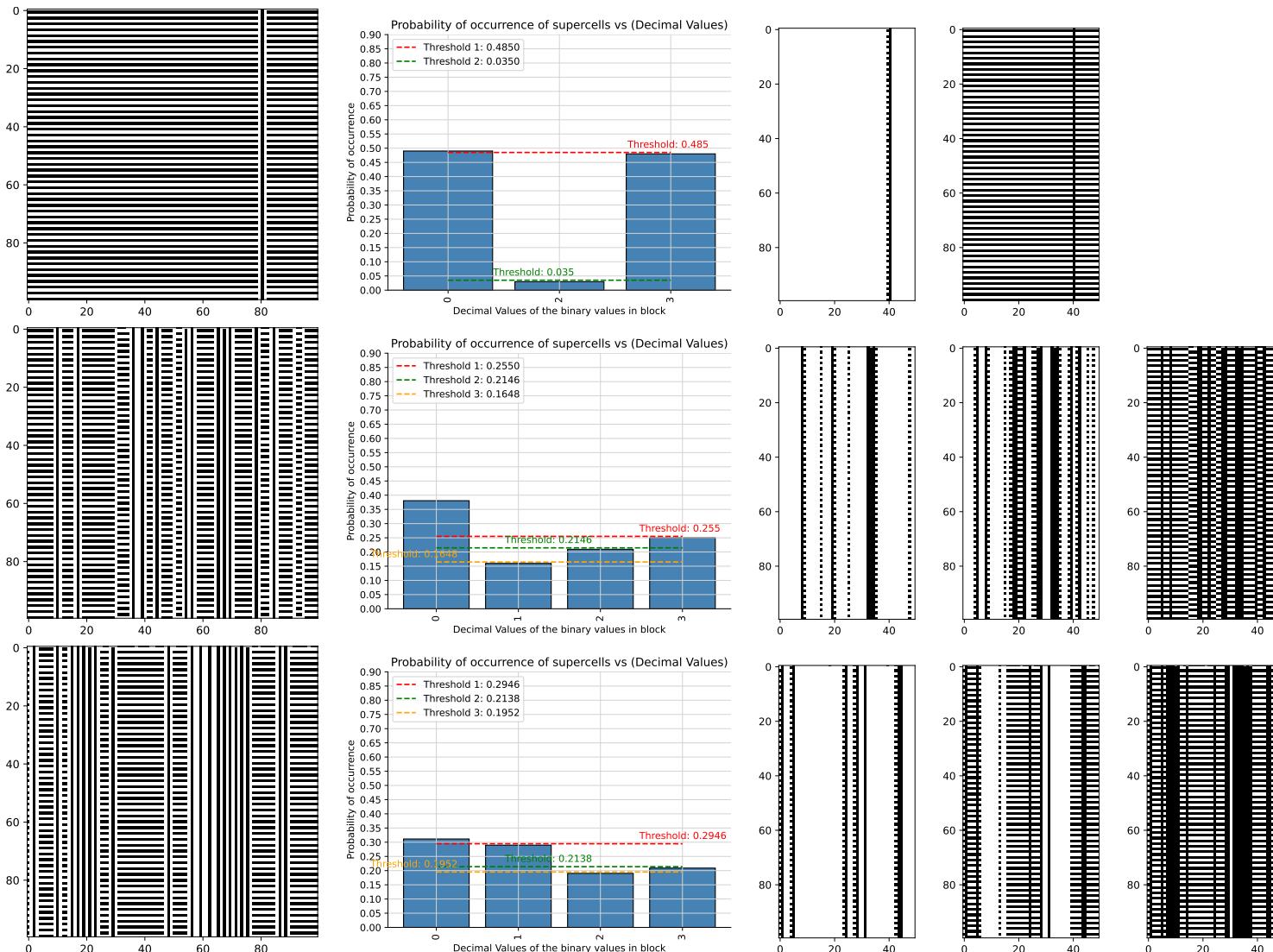
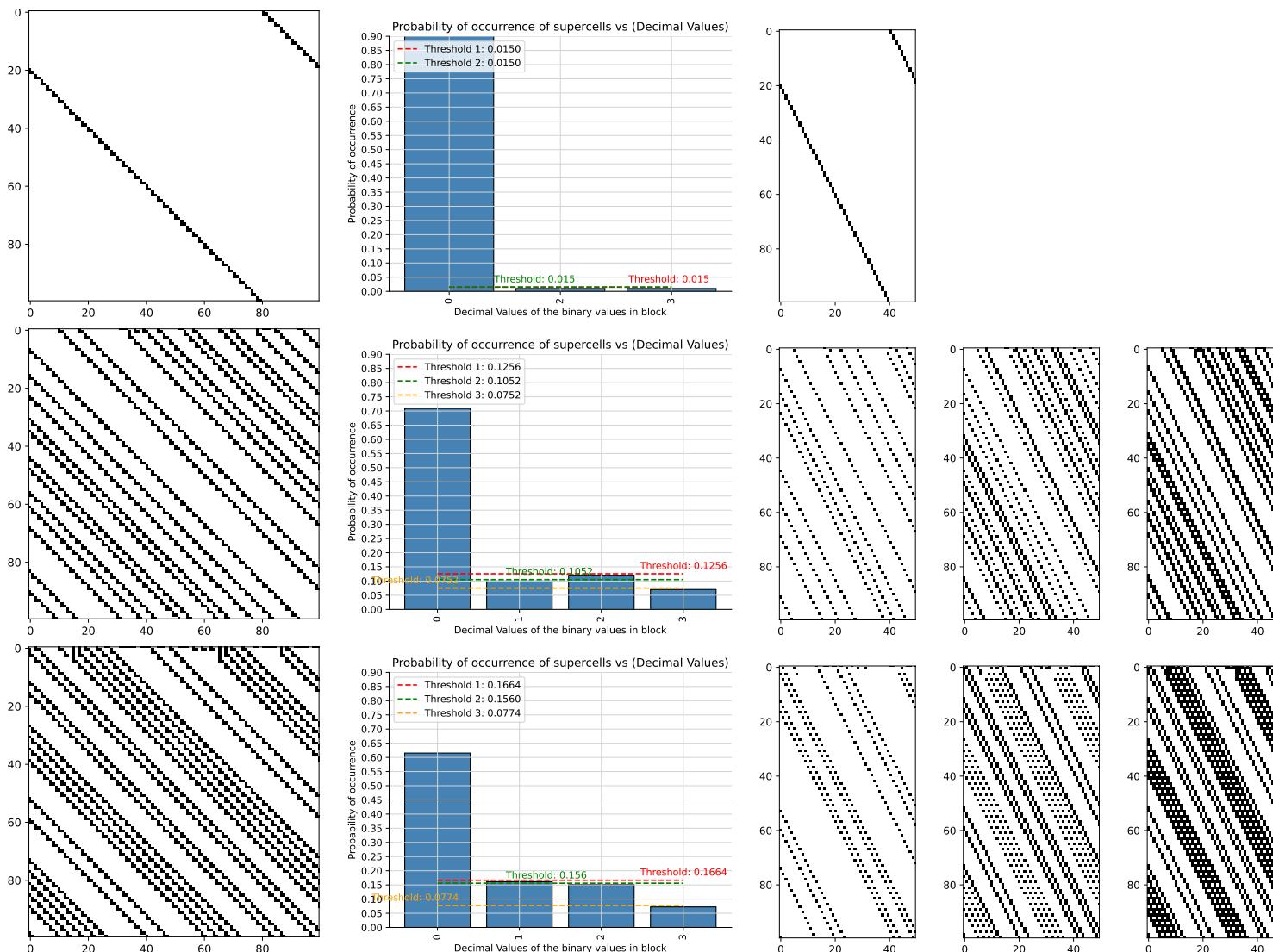


Table 7: FHCG plots for ECA Rule 6.



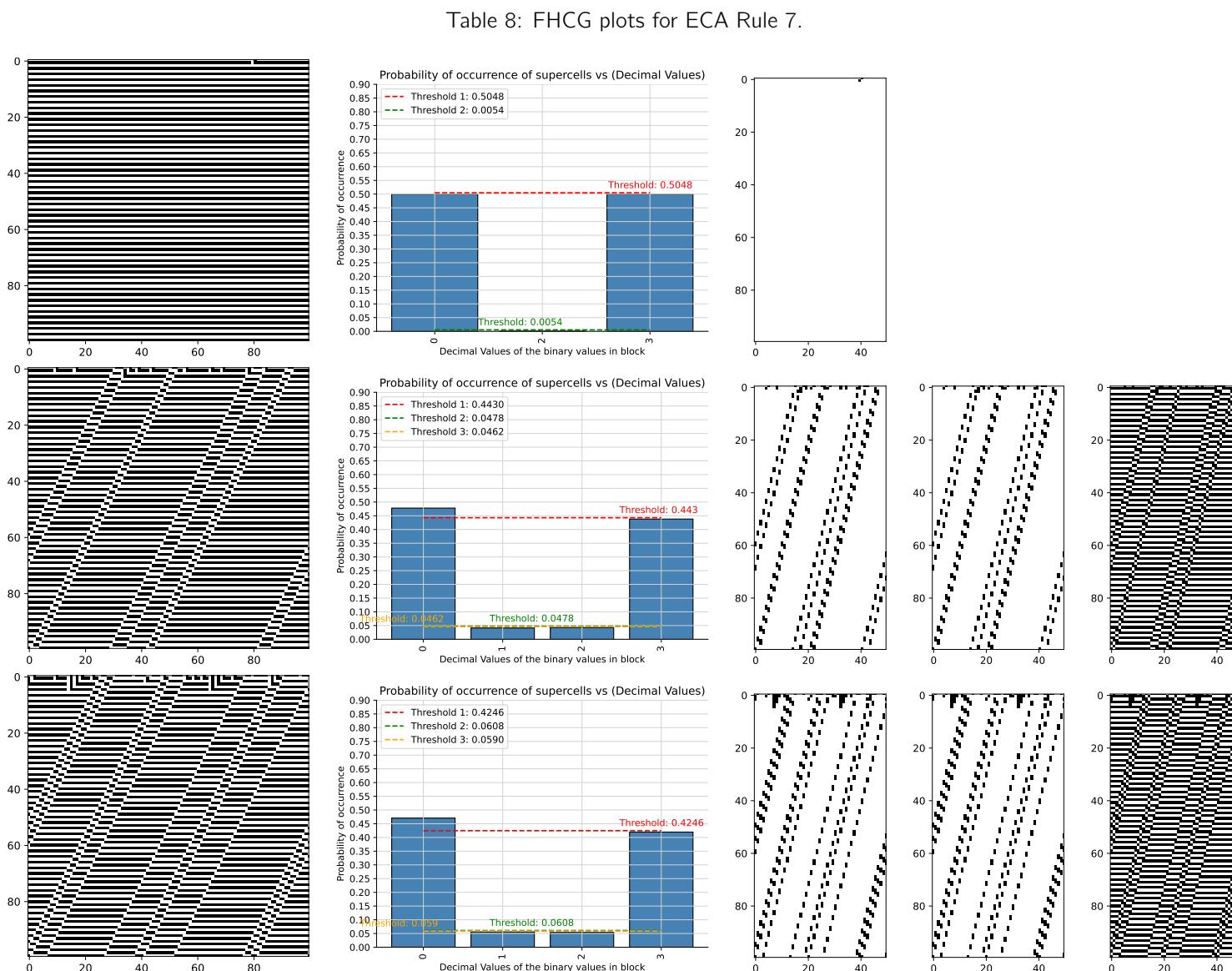


Table 9: FHCG plots for ECA Rule 8.

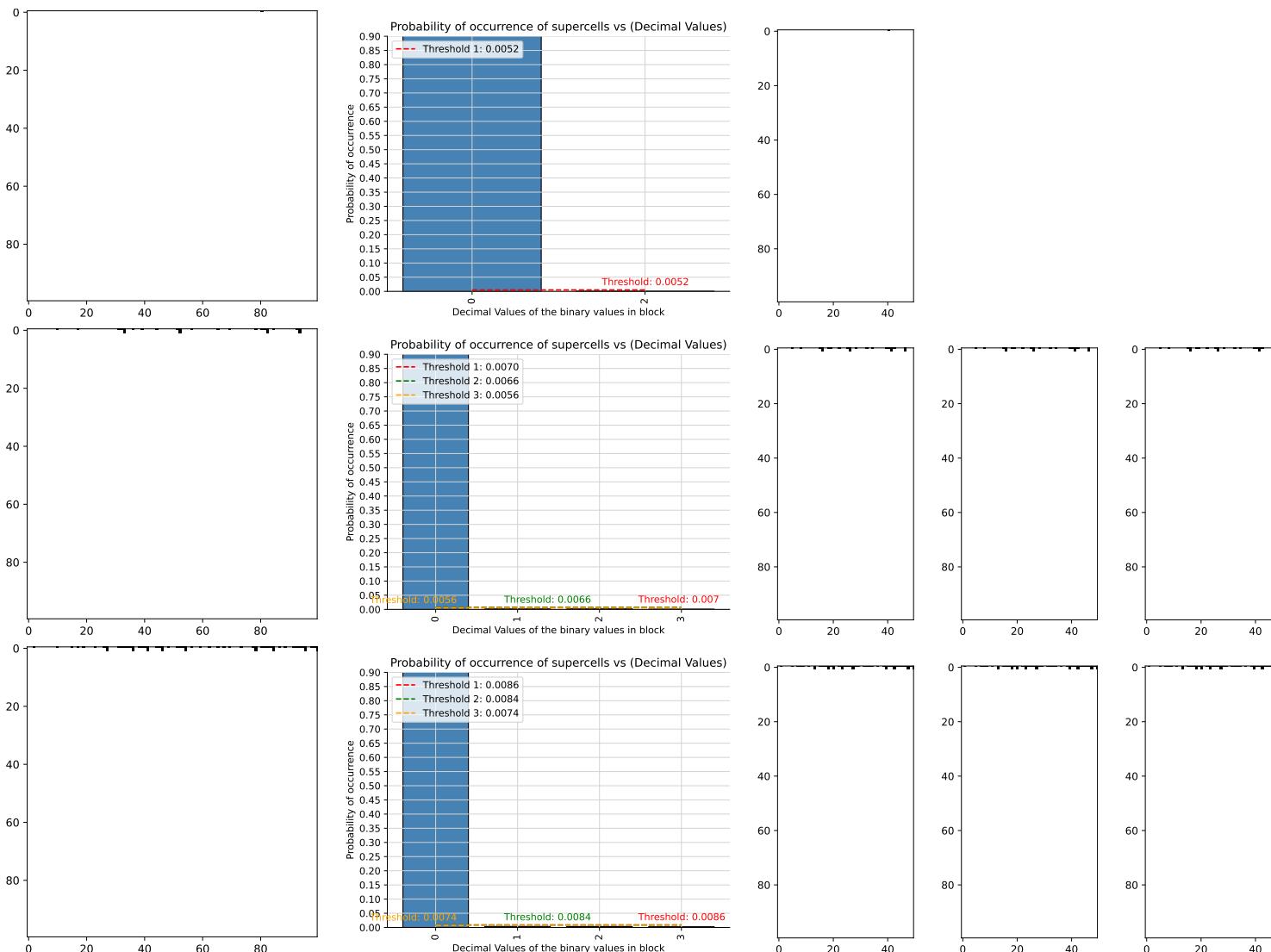


Table 10: FHCG plots for ECA Rule 9.

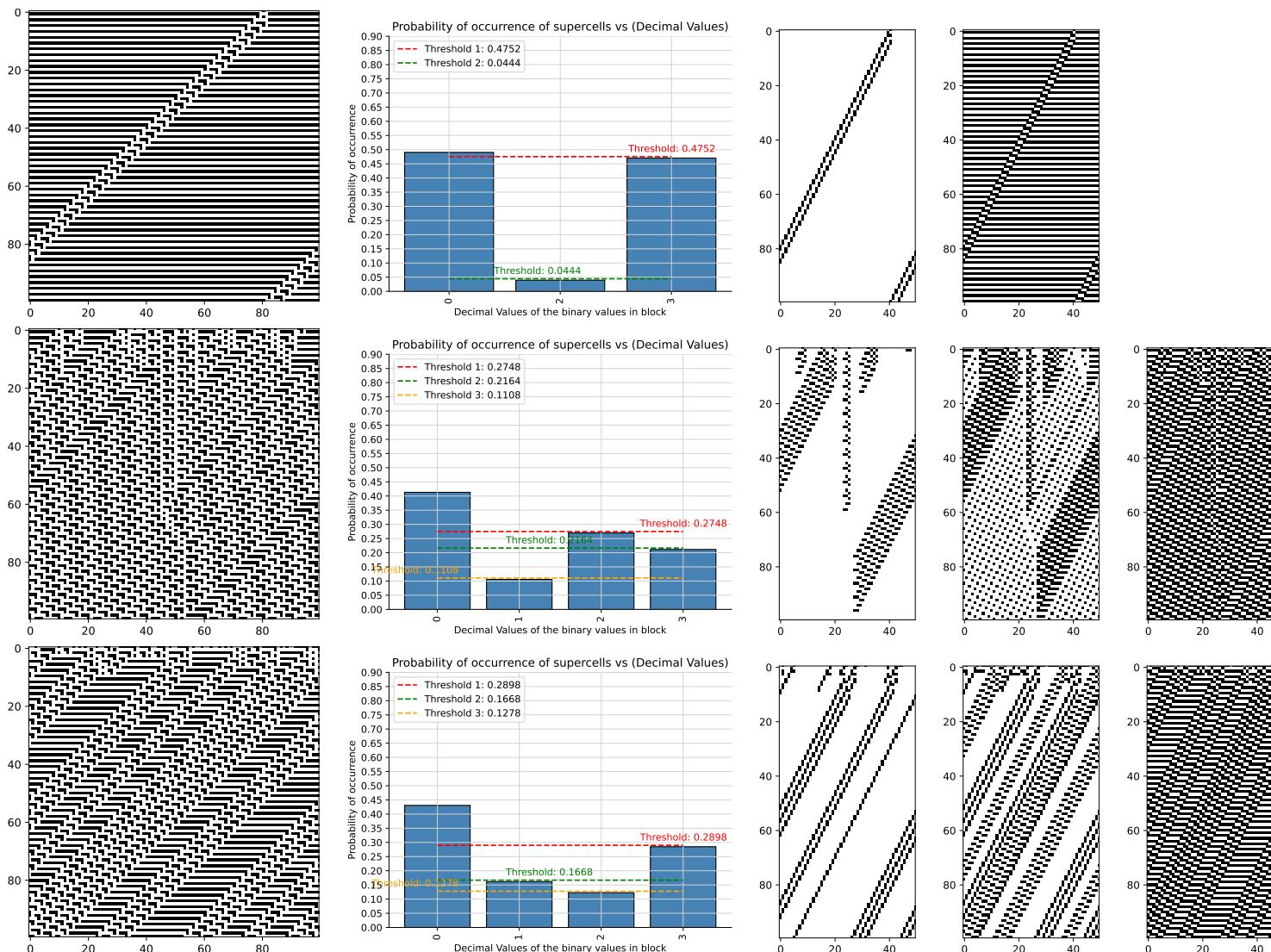


Table 11: FHCG plots for ECA Rule 10.

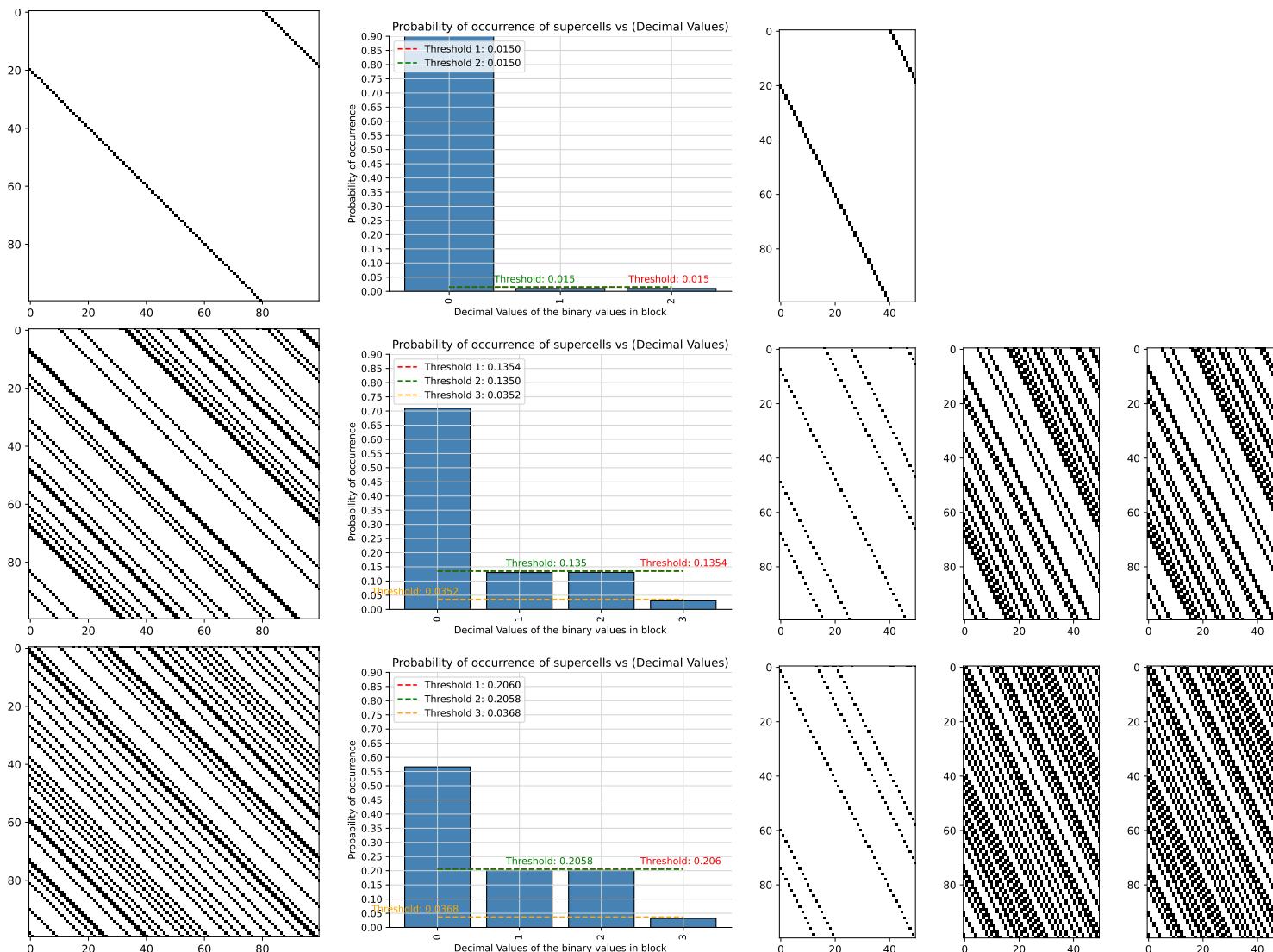


Table 12: FHCG plots for ECA Rule 11.

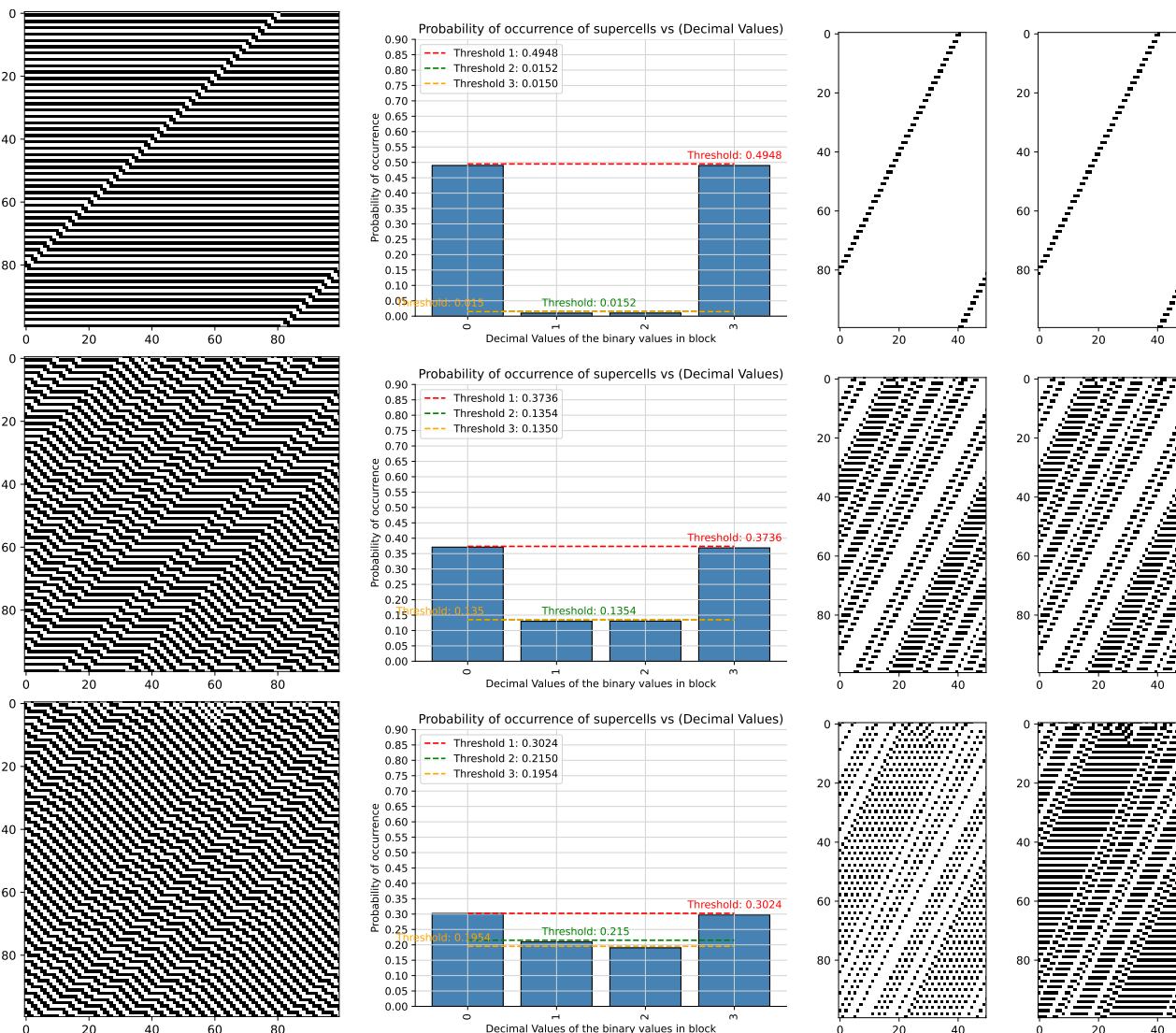


Table 13: FHCG plots for ECA Rule 12.

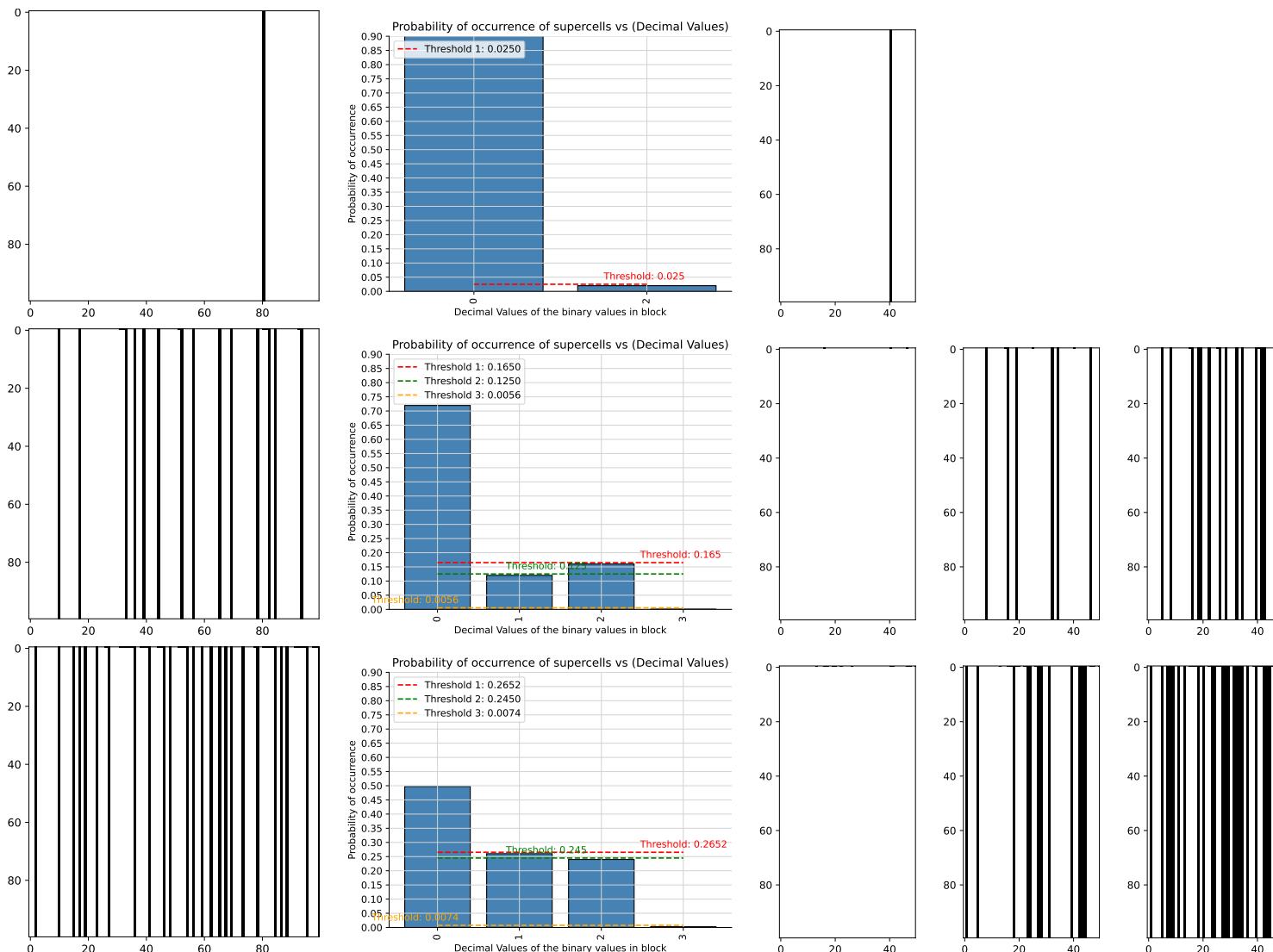


Table 14: FHCG plots for ECA Rule 13.

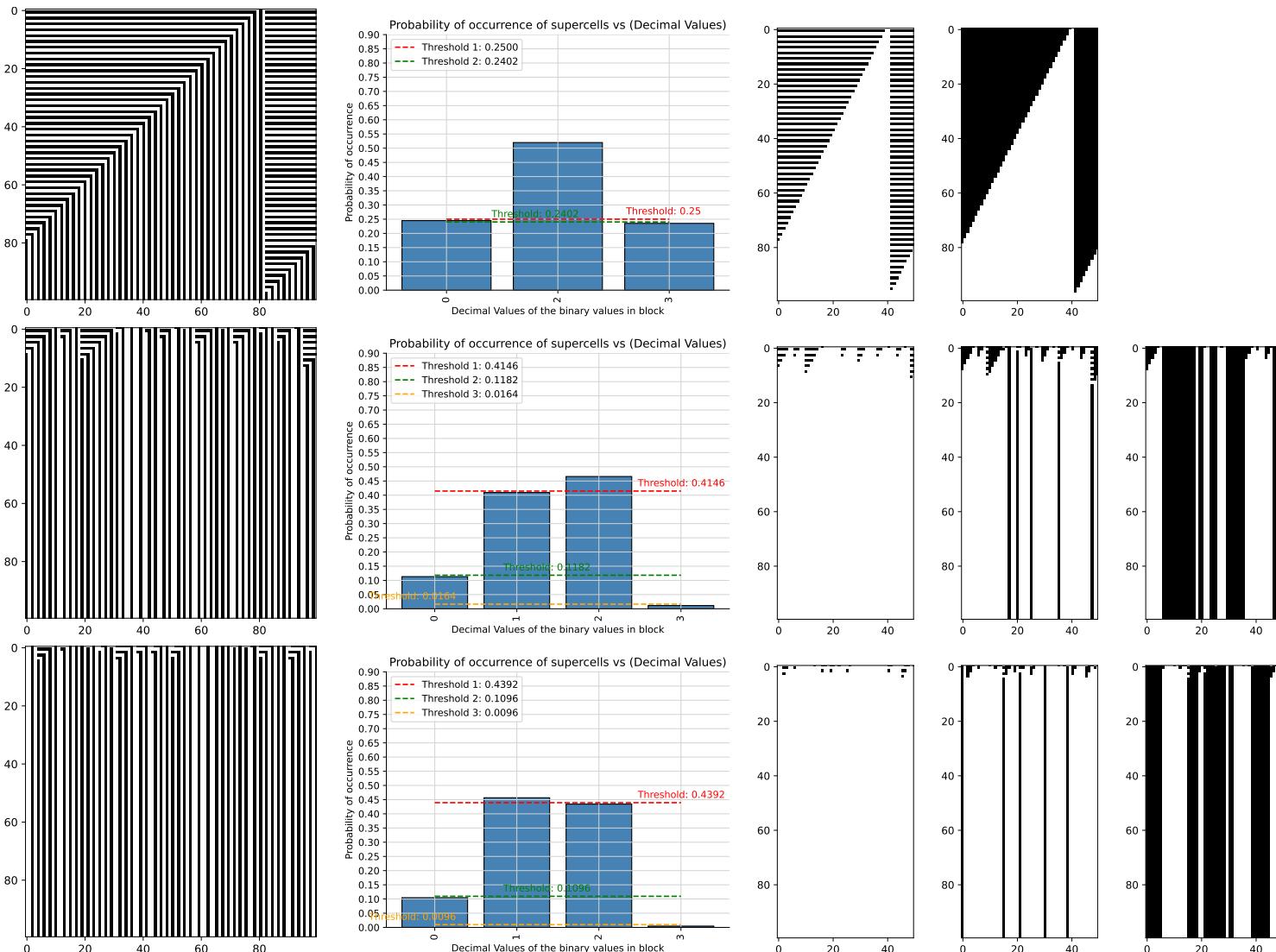


Table 15: FHCG plots for ECA Rule 14.

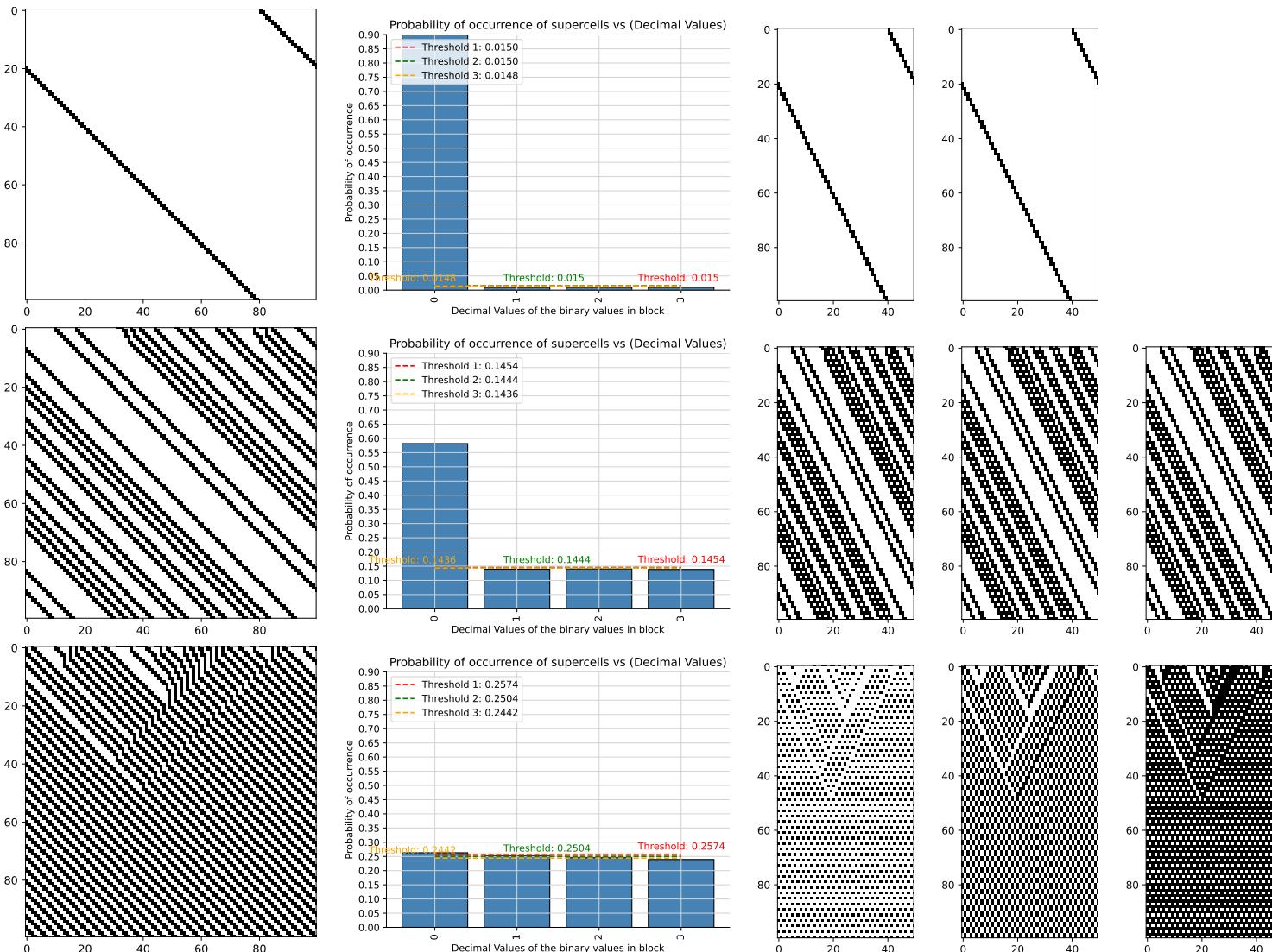


Table 16: FHCG plots for ECA Rule 15.

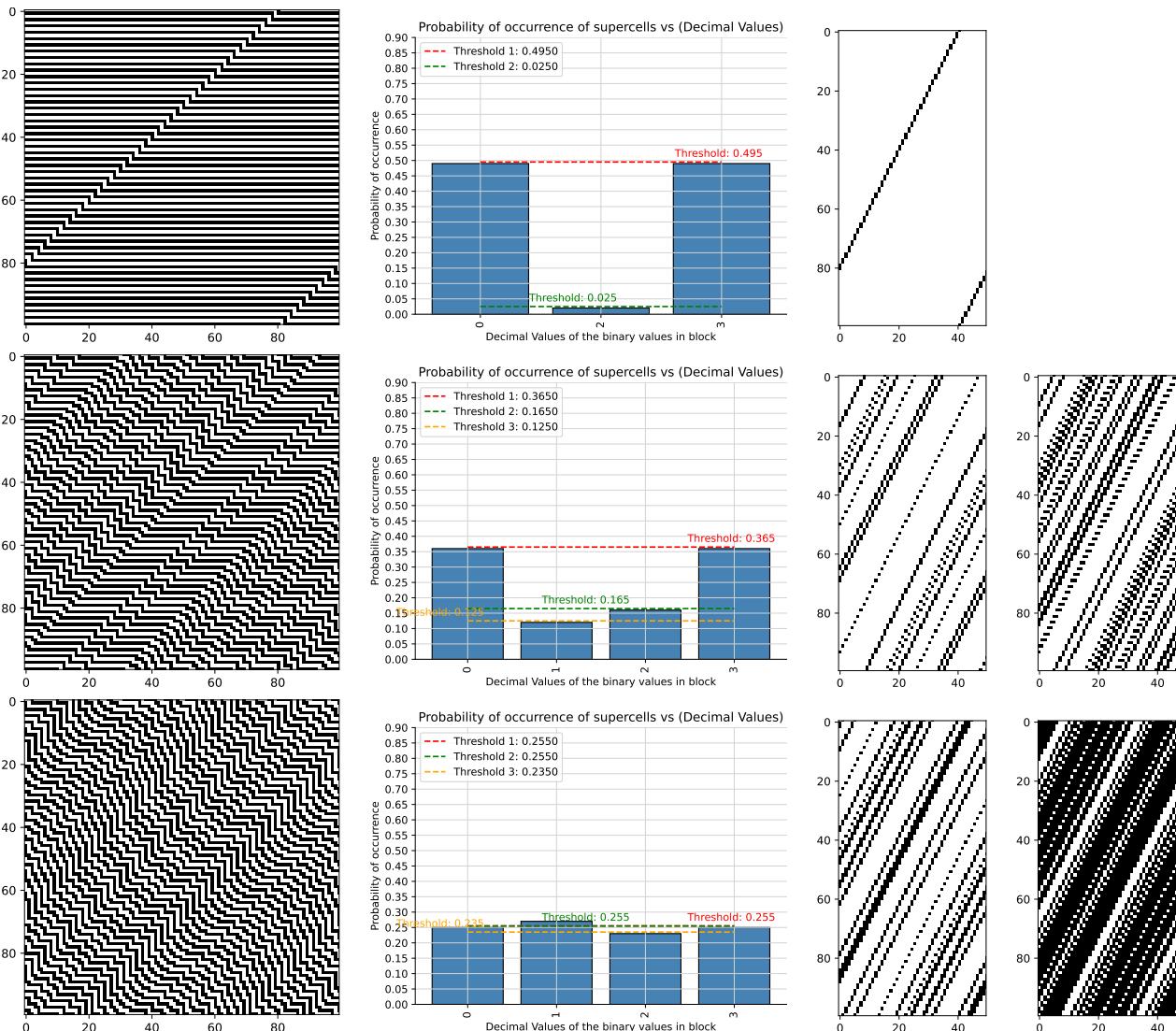


Table 17: FHCG plots for ECA Rule 18.

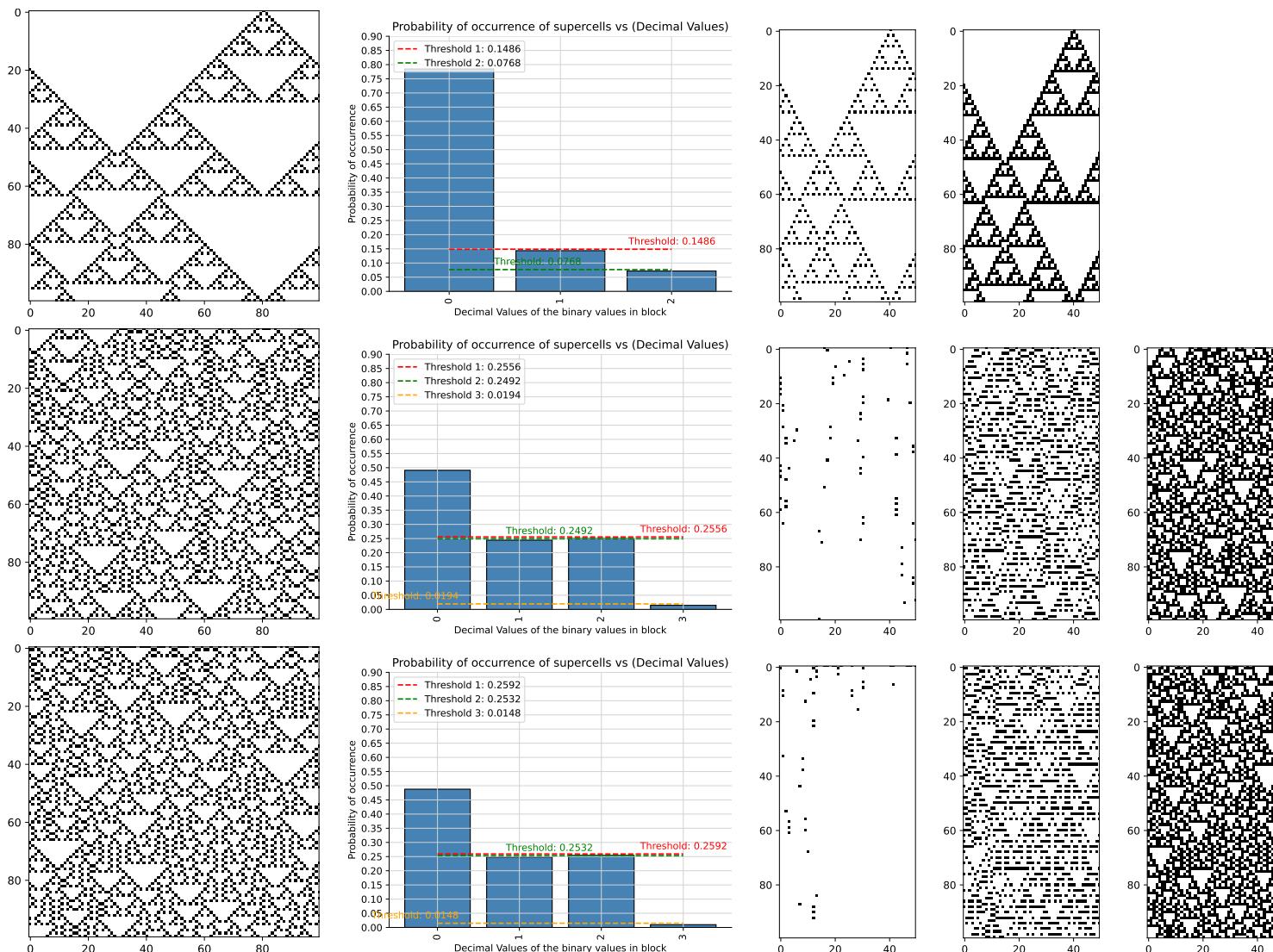


Table 18: FHCG plots for ECA Rule 19.

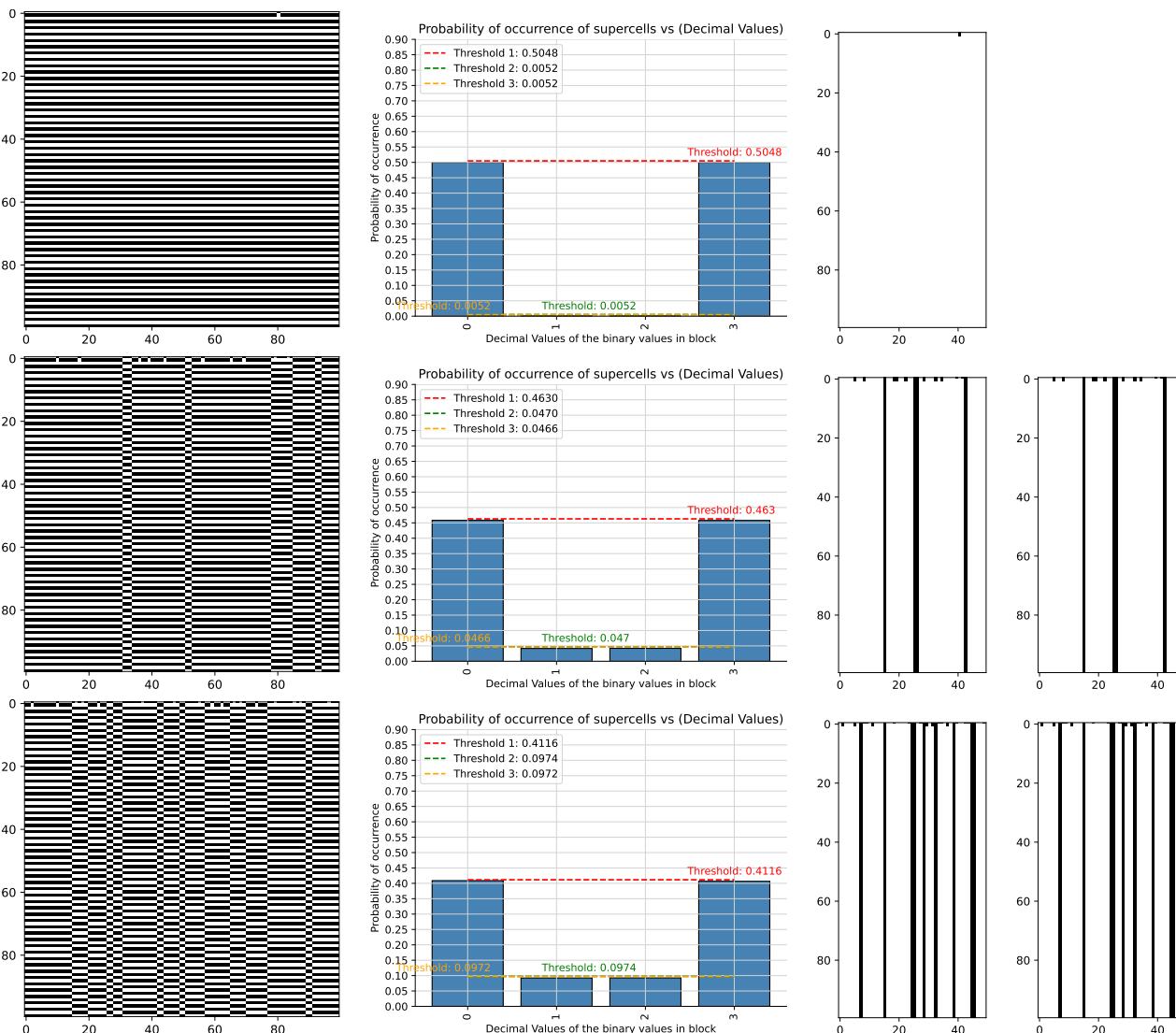


Table 19: FHCG plots for ECA Rule 22.

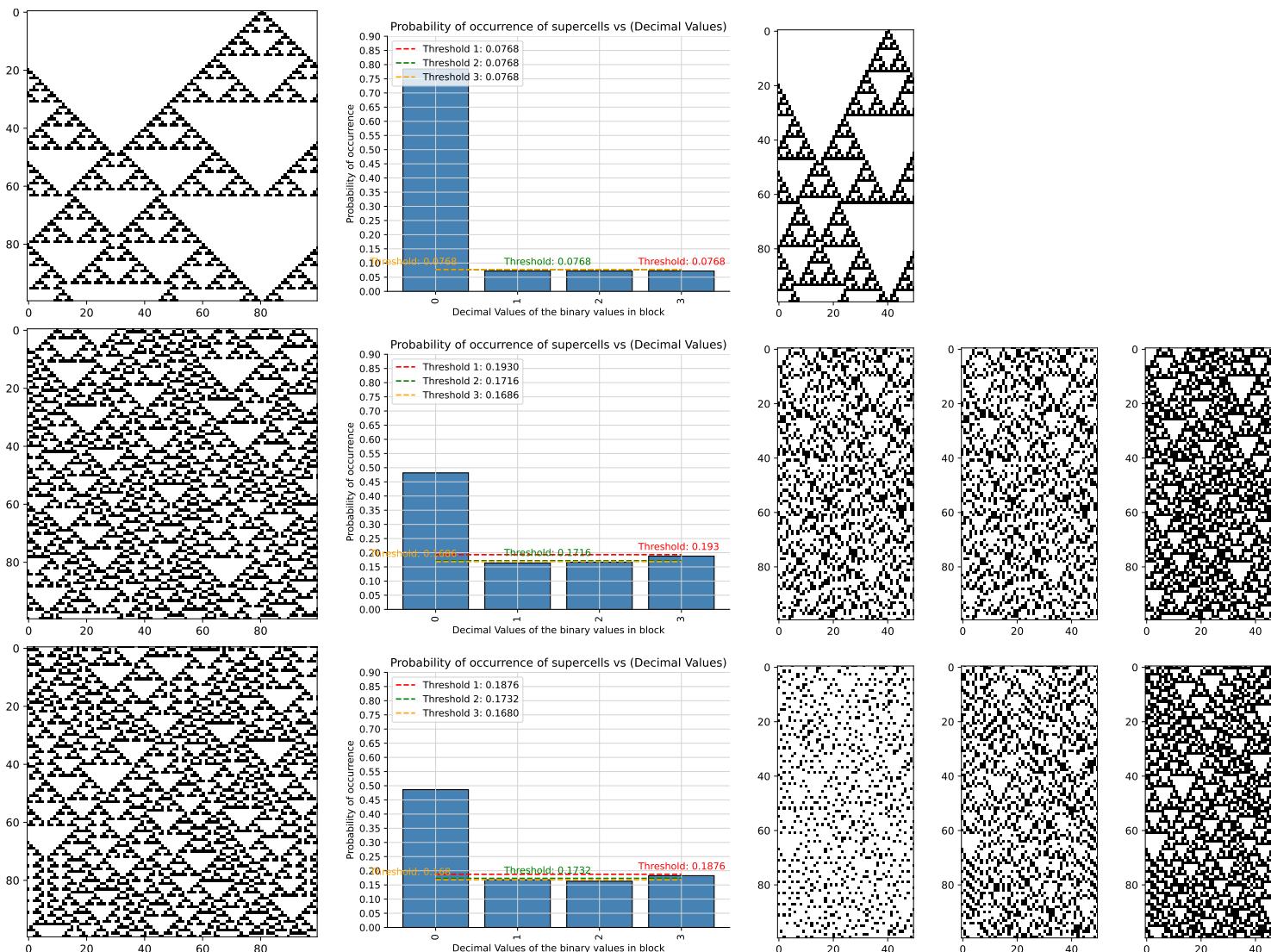


Table 20: FHCG plots for ECA Rule 23.

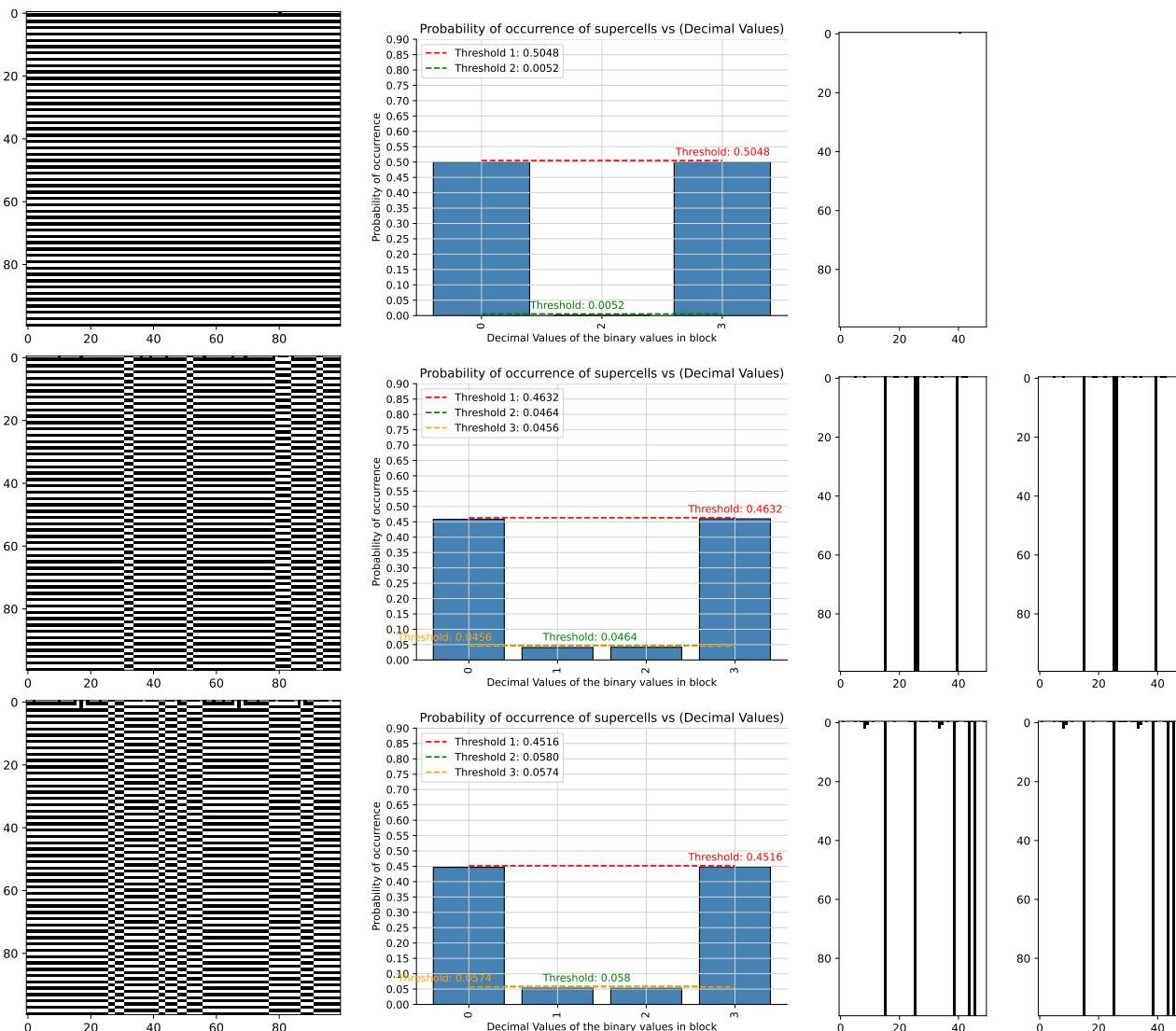


Table 21: FHCG plots for ECA Rule 24.

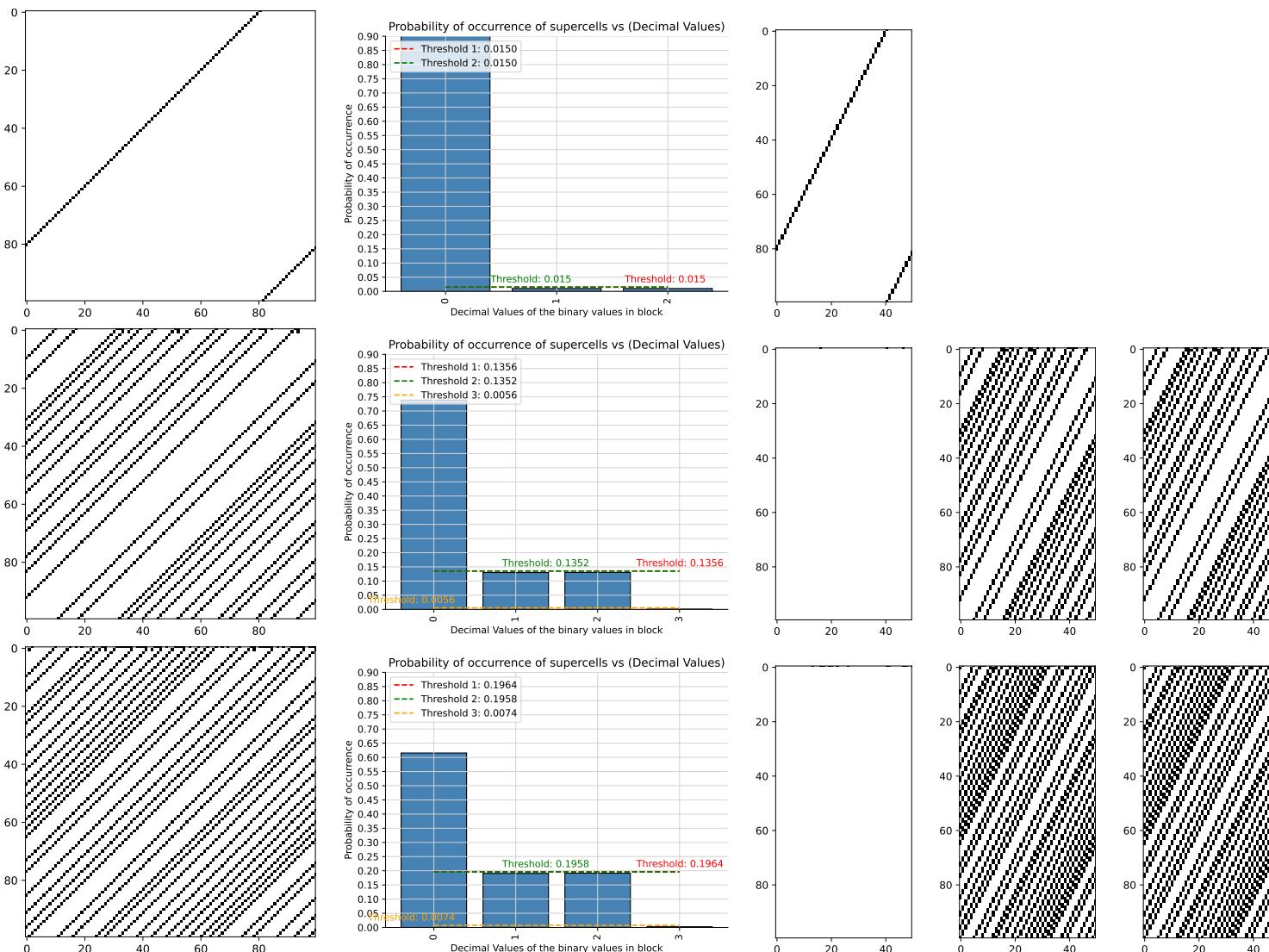


Table 22: FHCG plots for ECA Rule 25.

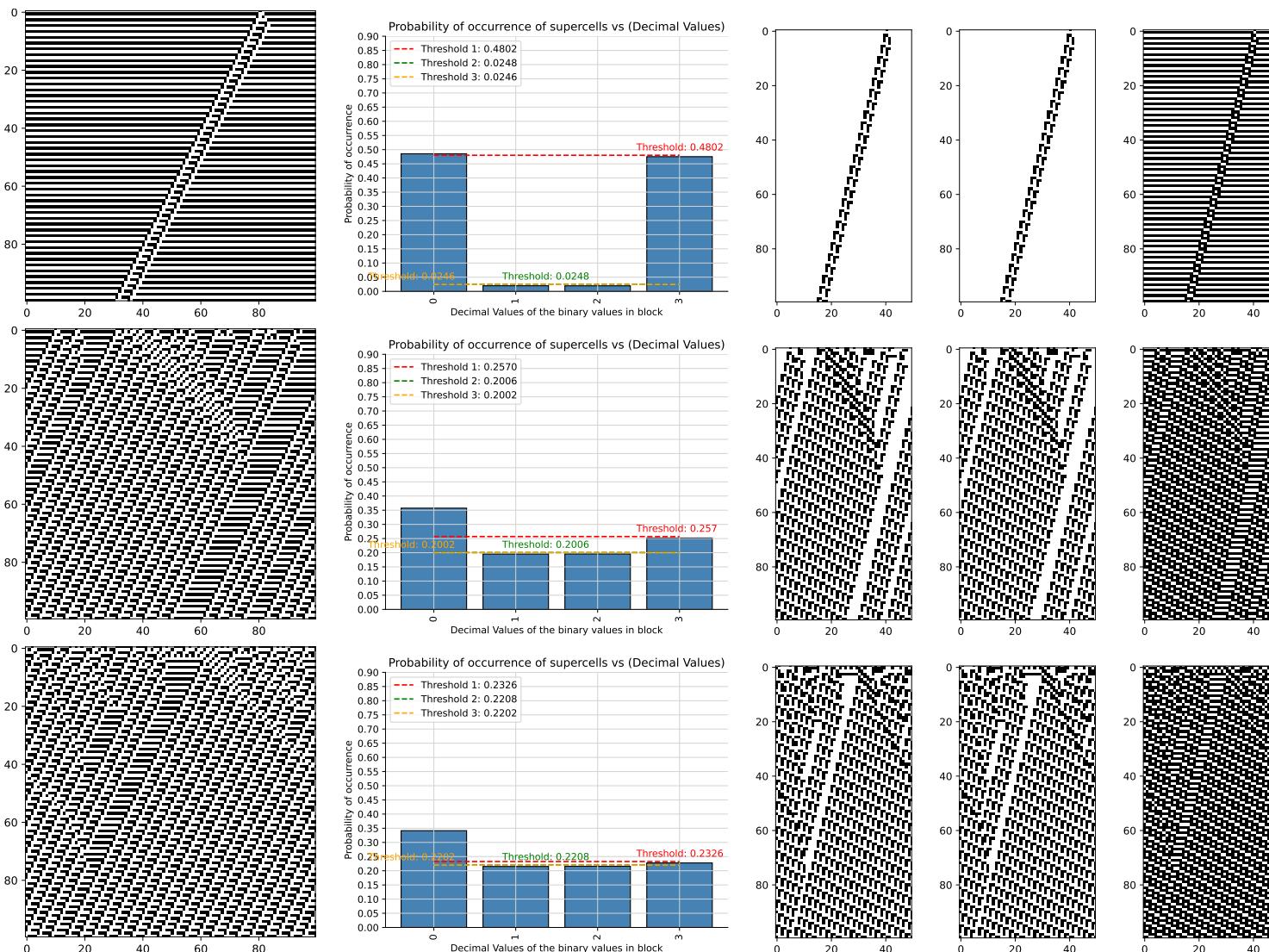


Table 23: FHCG plots for ECA Rule 26.

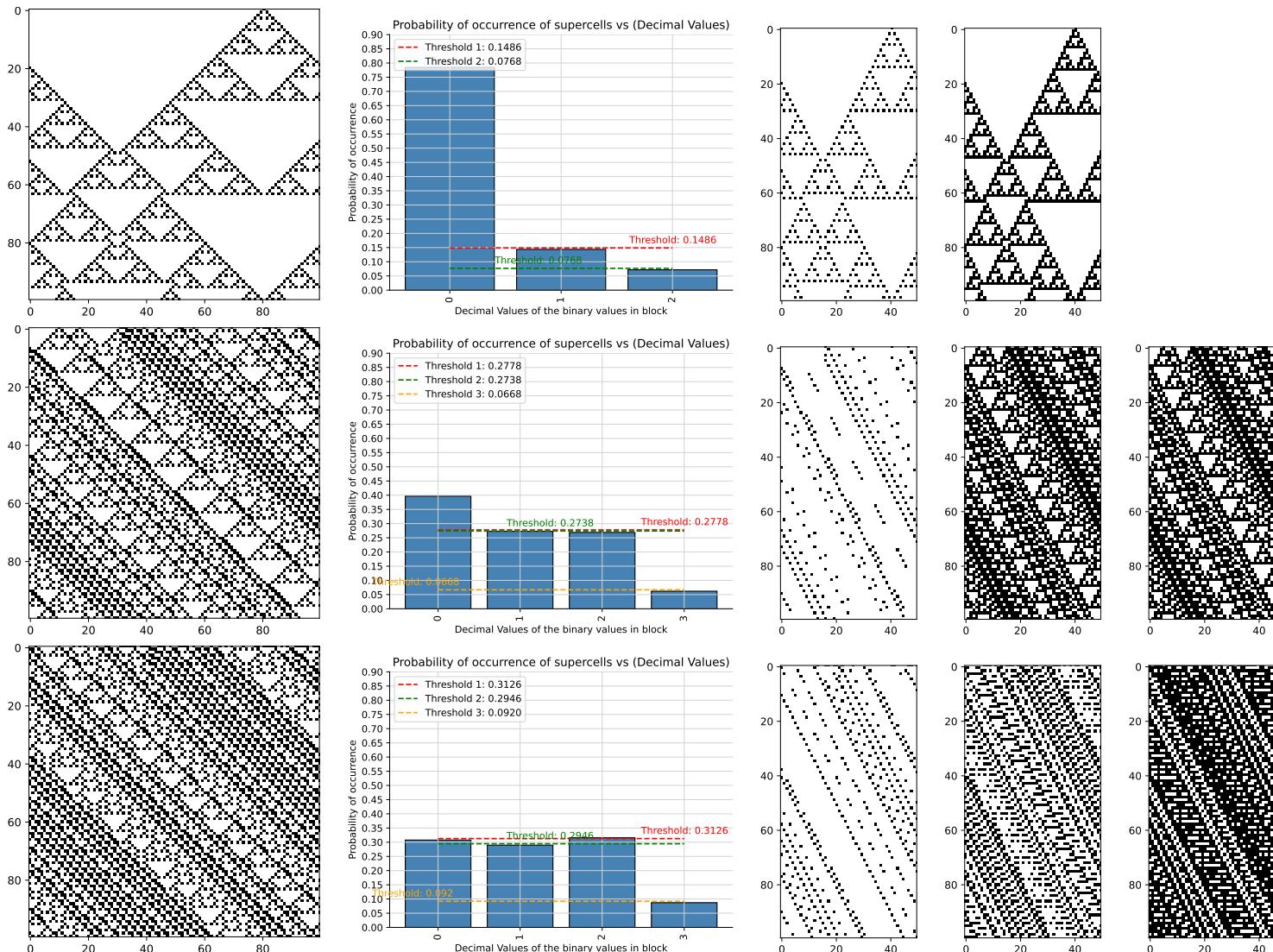


Table 24: FHCG plots for ECA Rule 27.

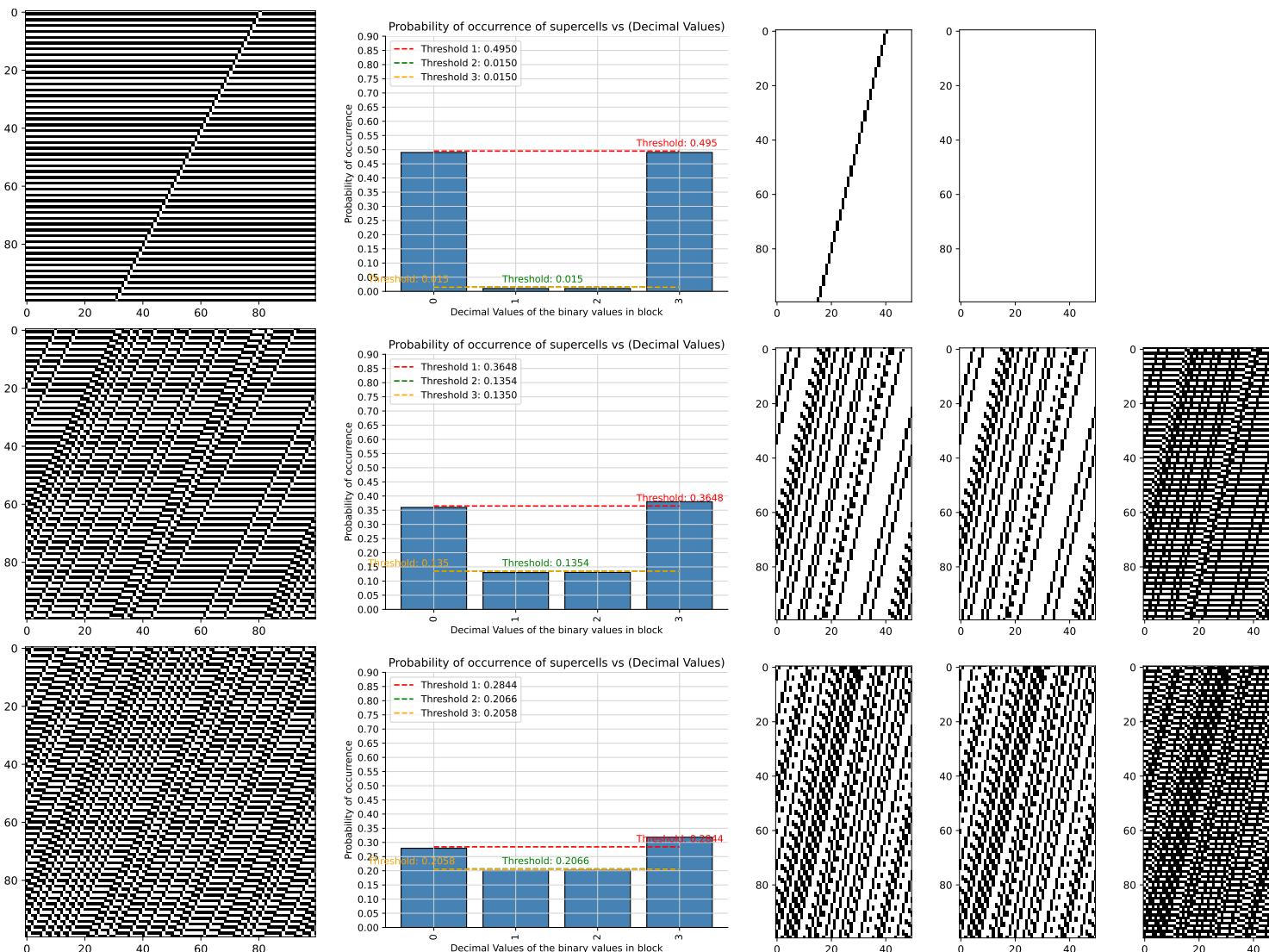


Table 25: FHCG plots for ECA Rule 28.

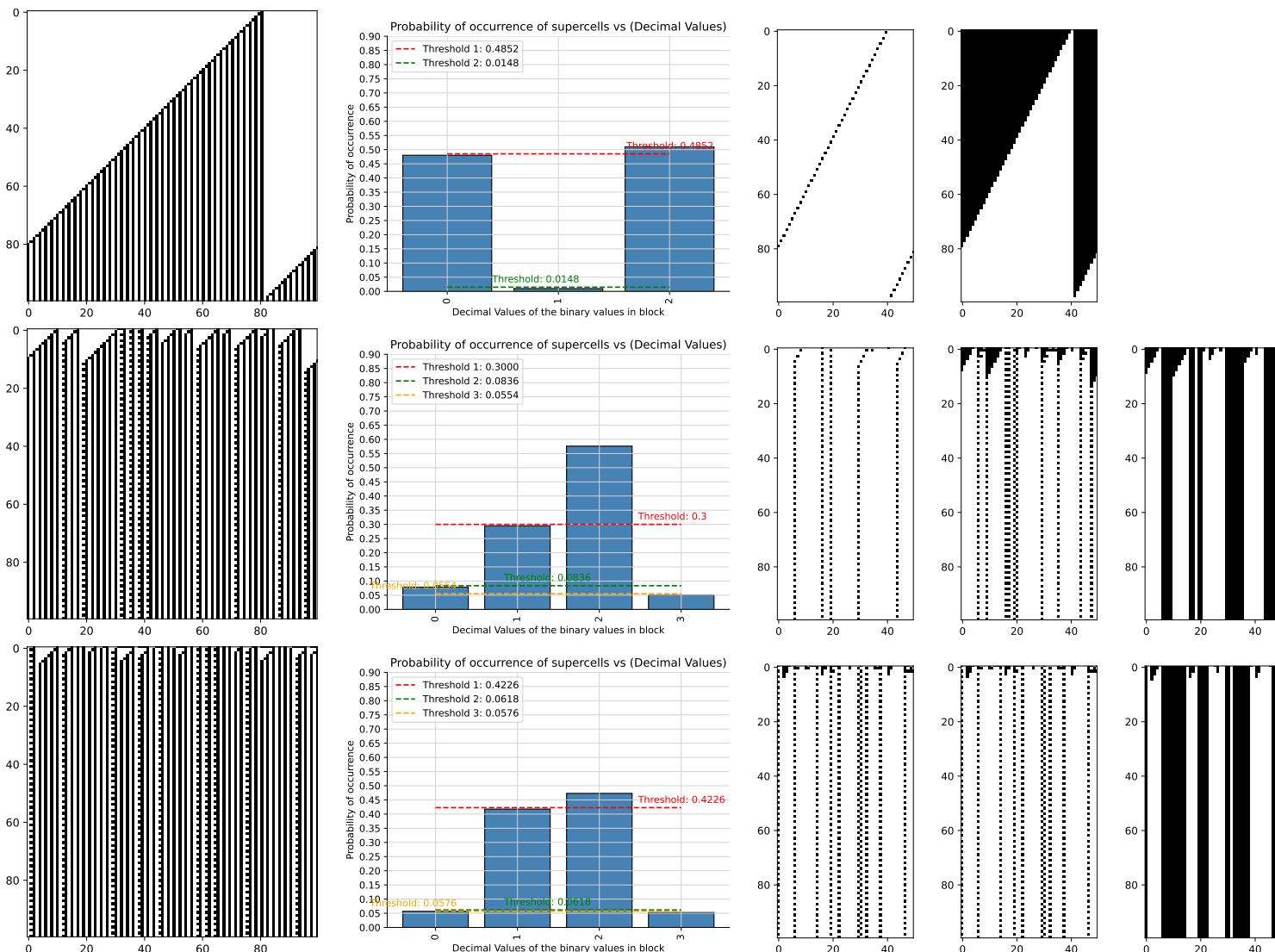


Table 26: FHCG plots for ECA Rule 29.

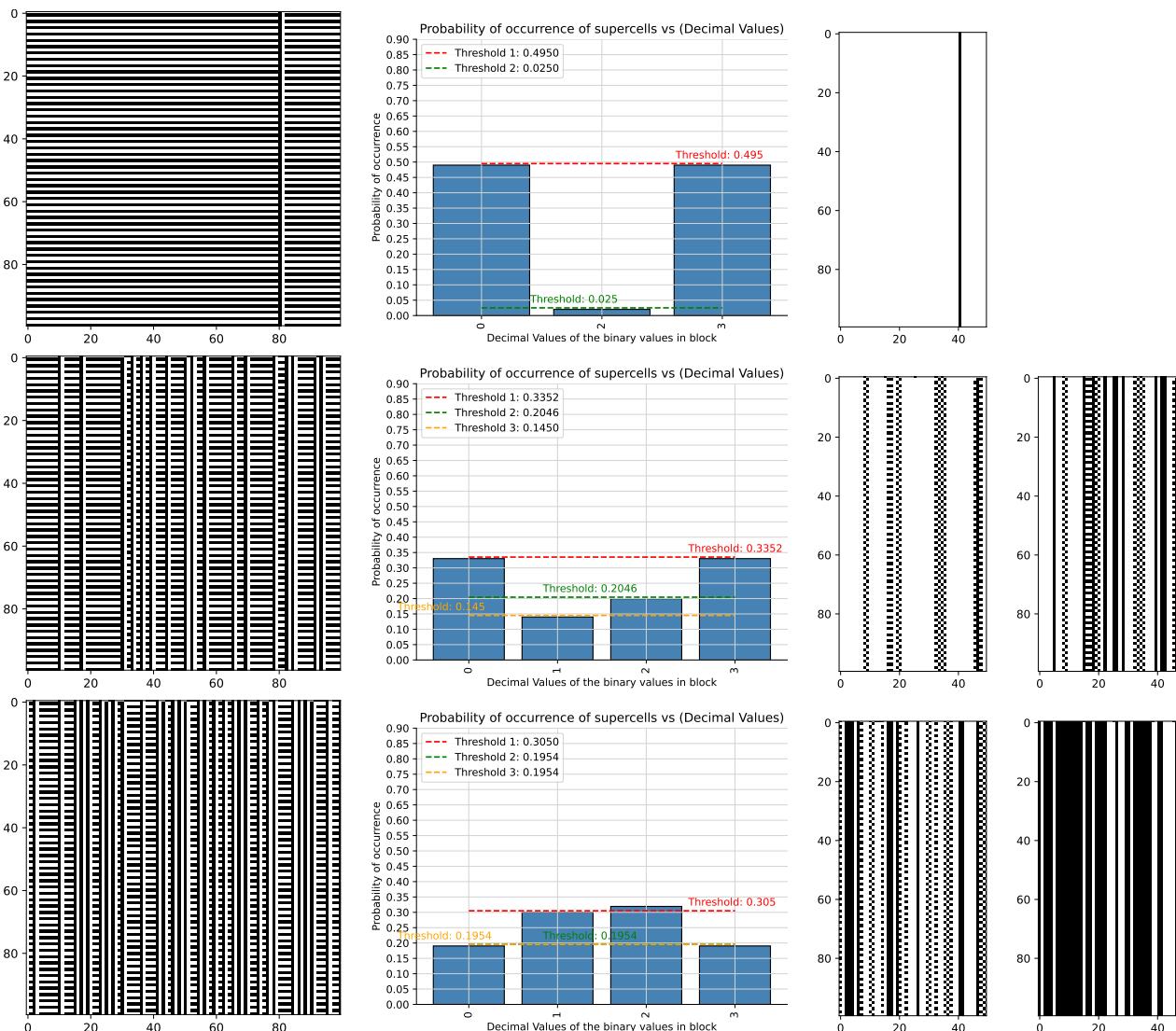


Table 27: FHCG plots for ECA Rule 30.

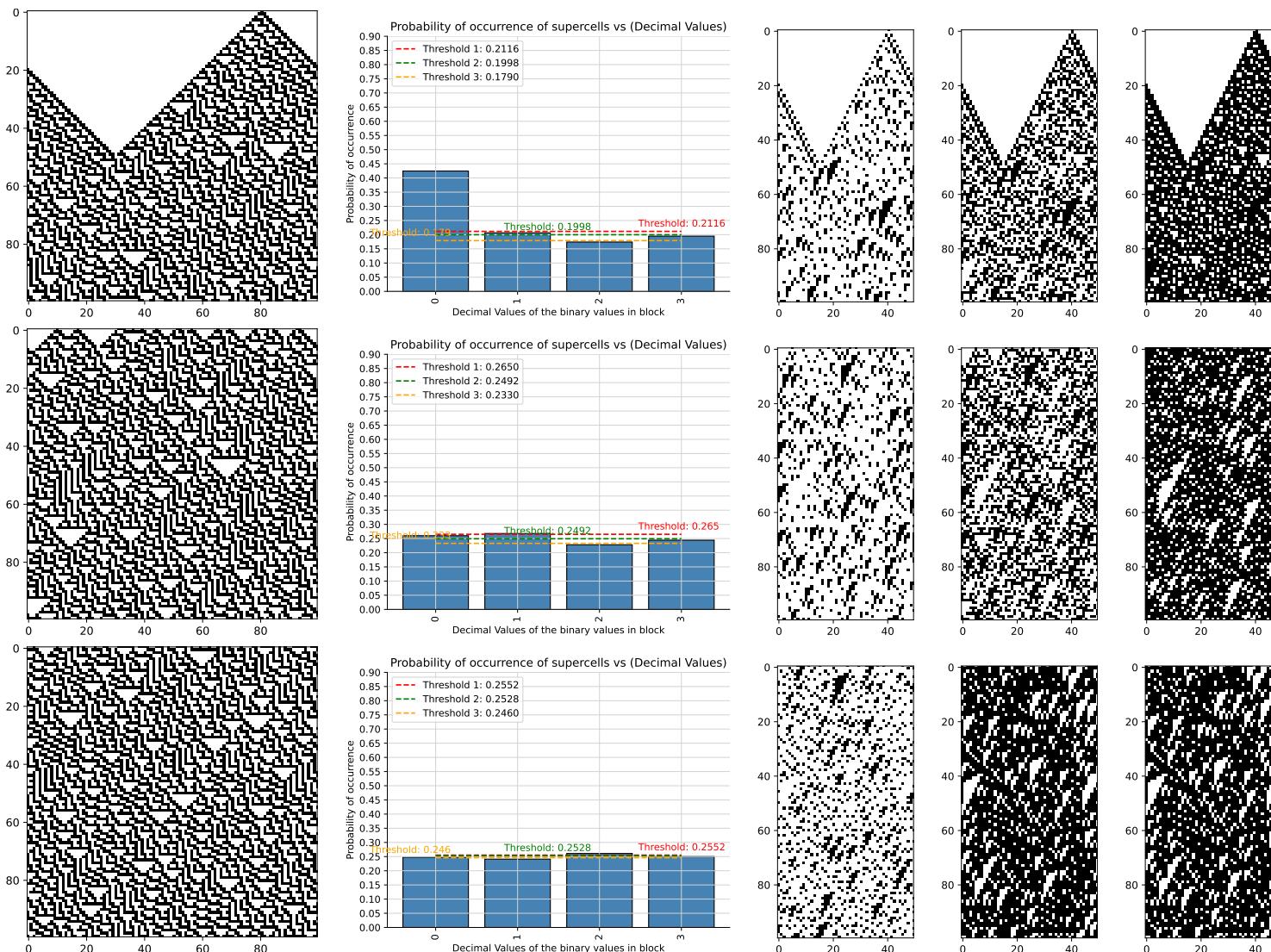


Table 28: FHCG plots for ECA Rule 32.

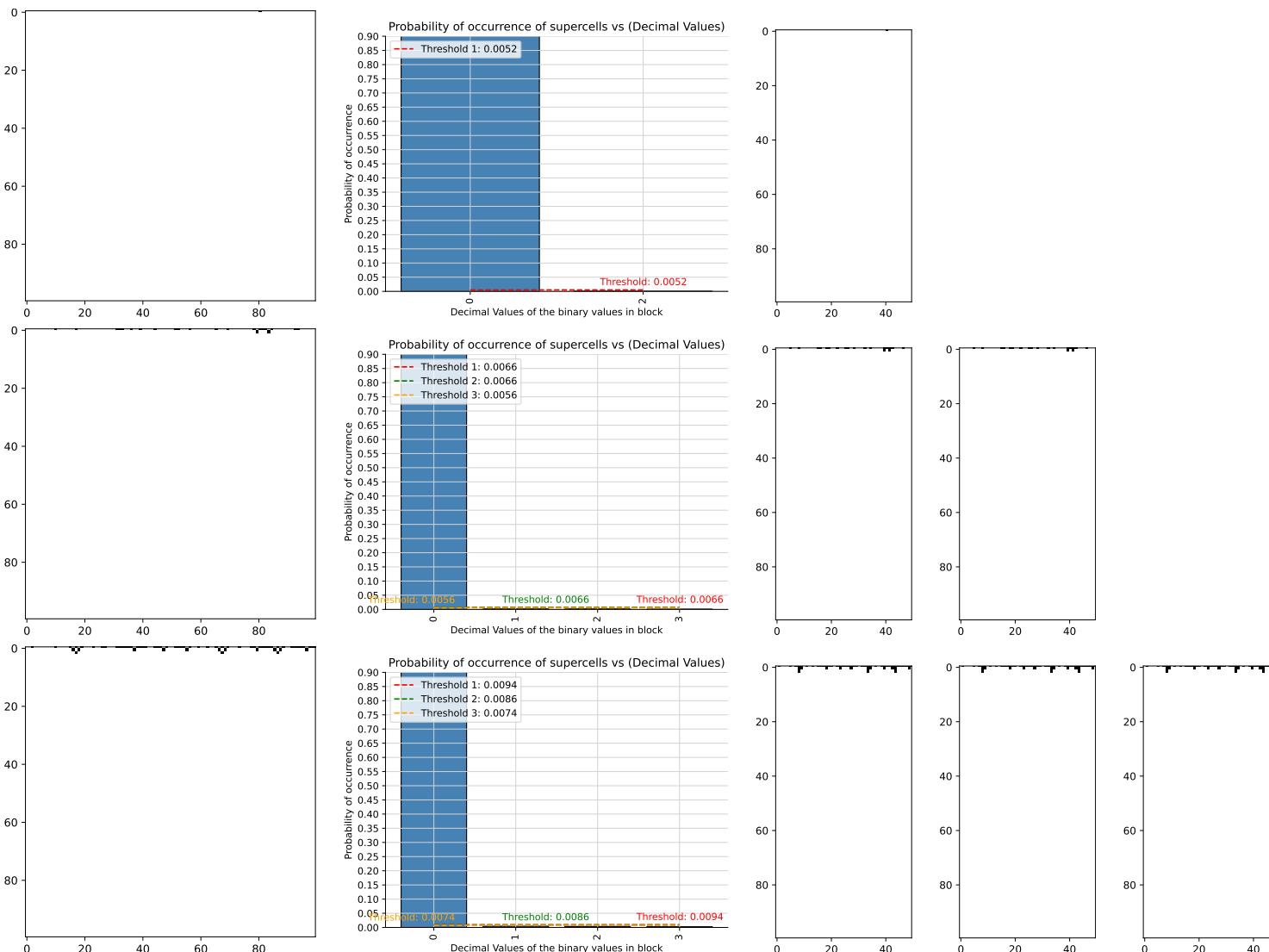


Table 29: FHCG plots for ECA Rule 33.

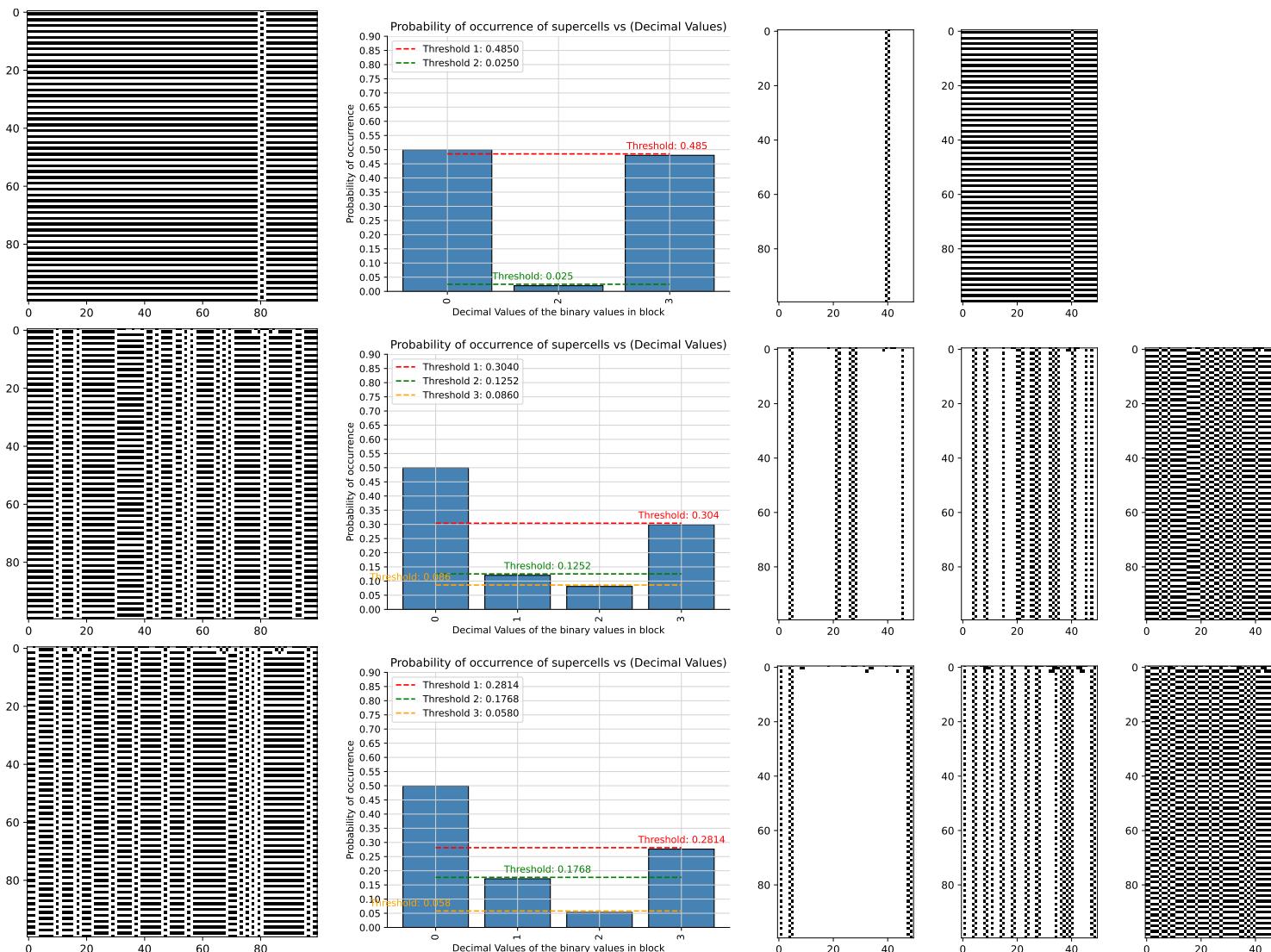


Table 30: FHCG plots for ECA Rule 34.

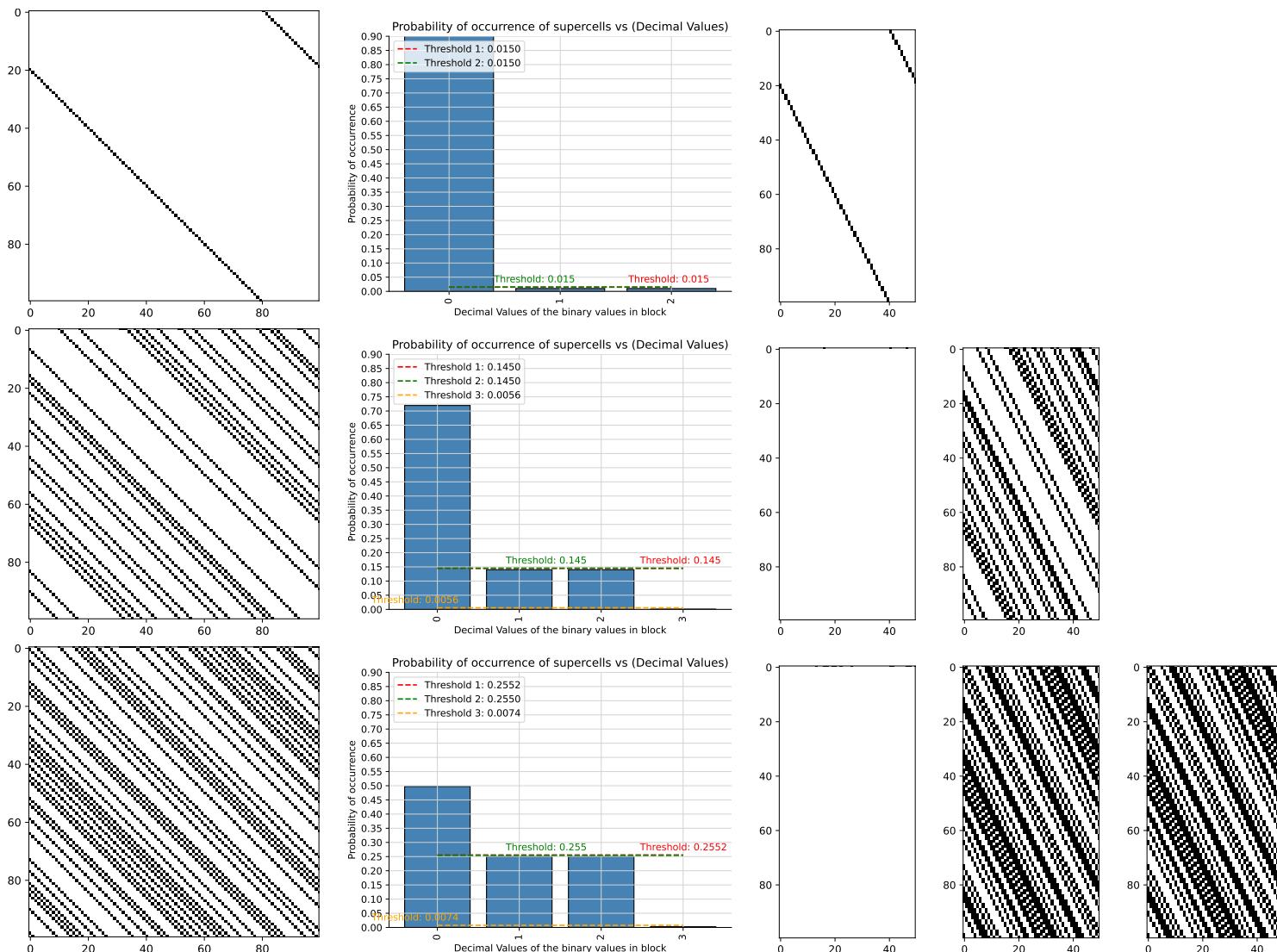


Table 31: FHCG plots for ECA Rule 35.

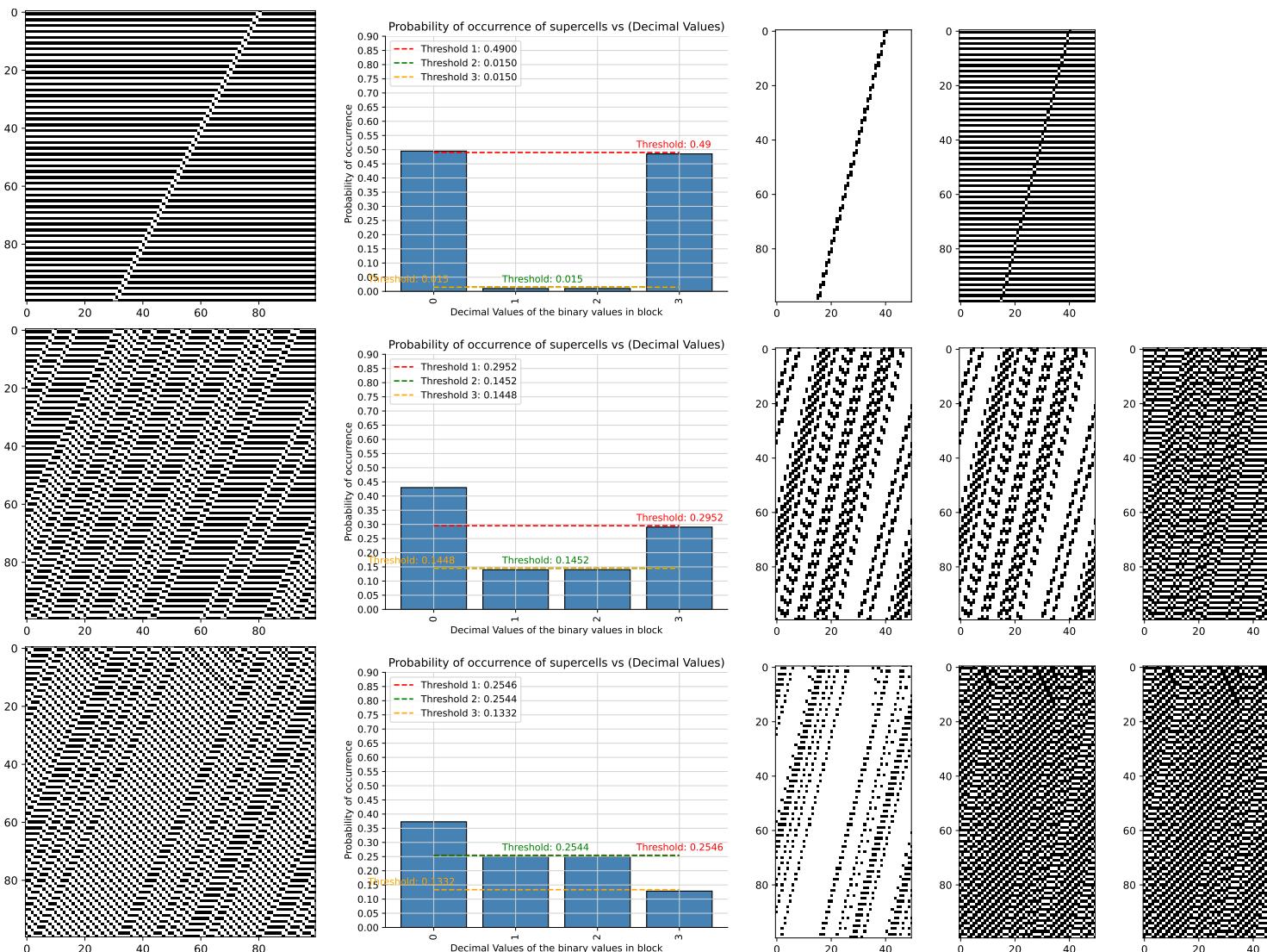


Table 32: FHCG plots for ECA Rule 36.

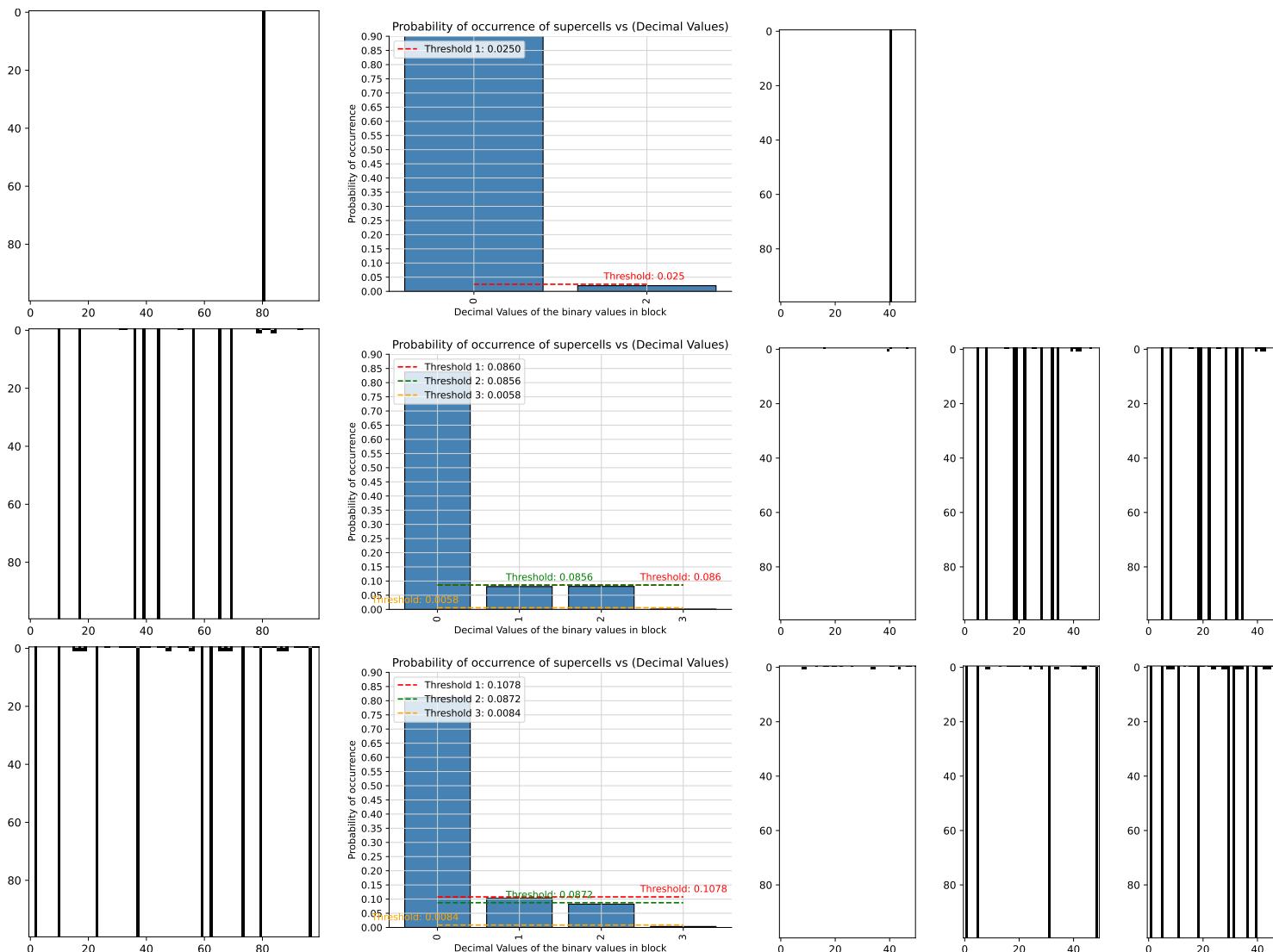


Table 33: FHCG plots for ECA Rule 37.

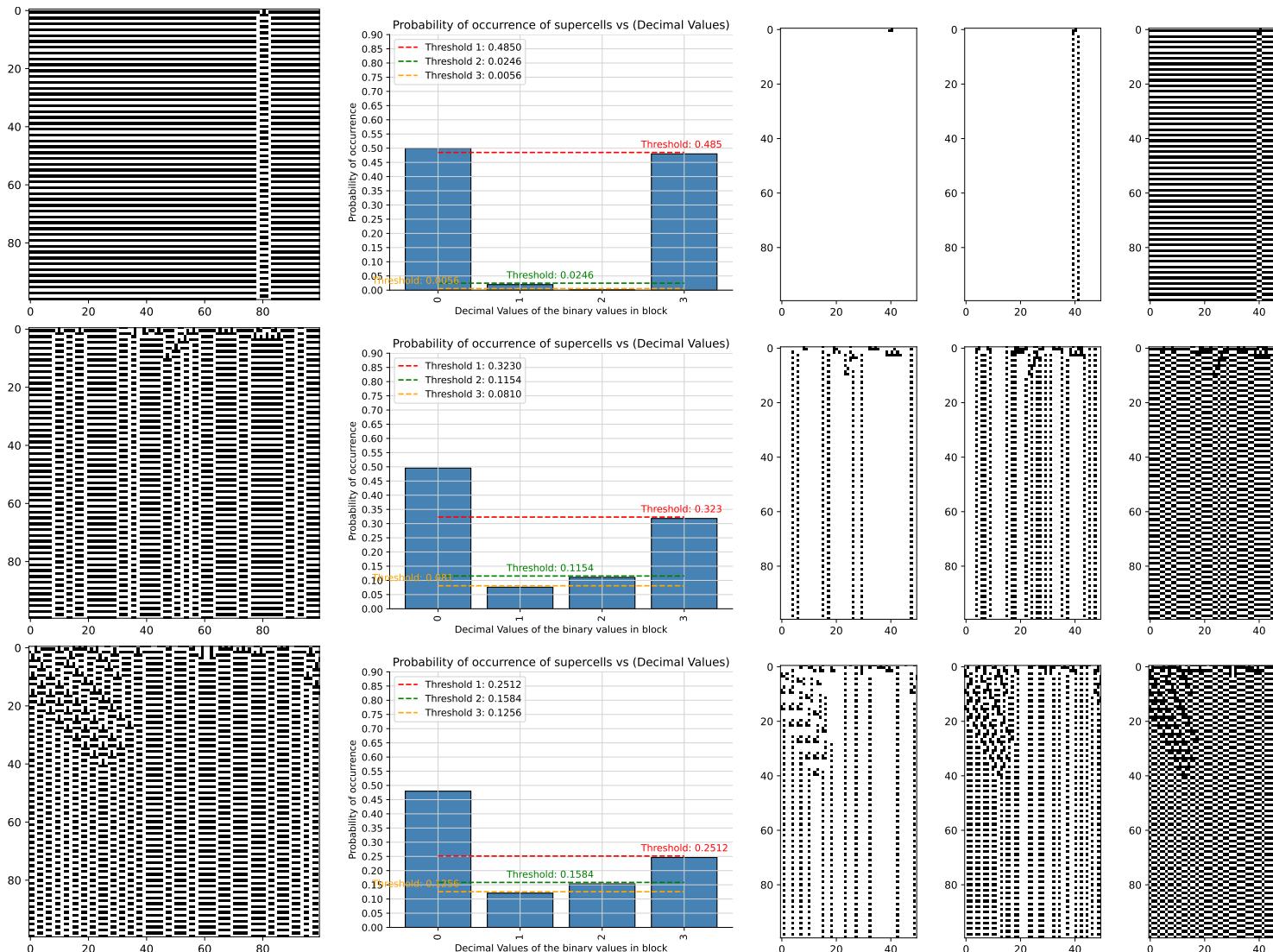


Table 34: FHCG plots for ECA Rule 38.

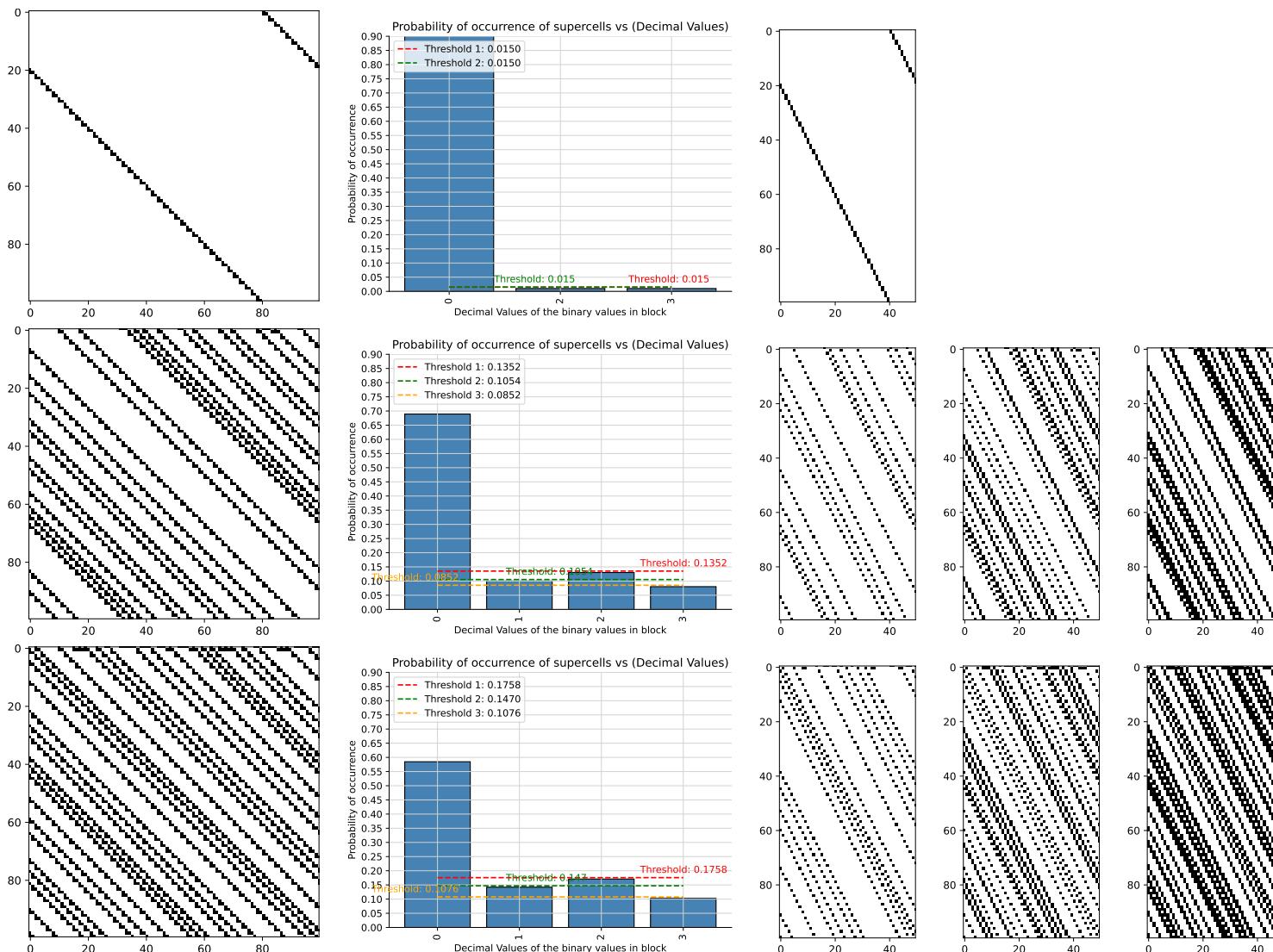


Table 35: FHCG plots for ECA Rule 40.

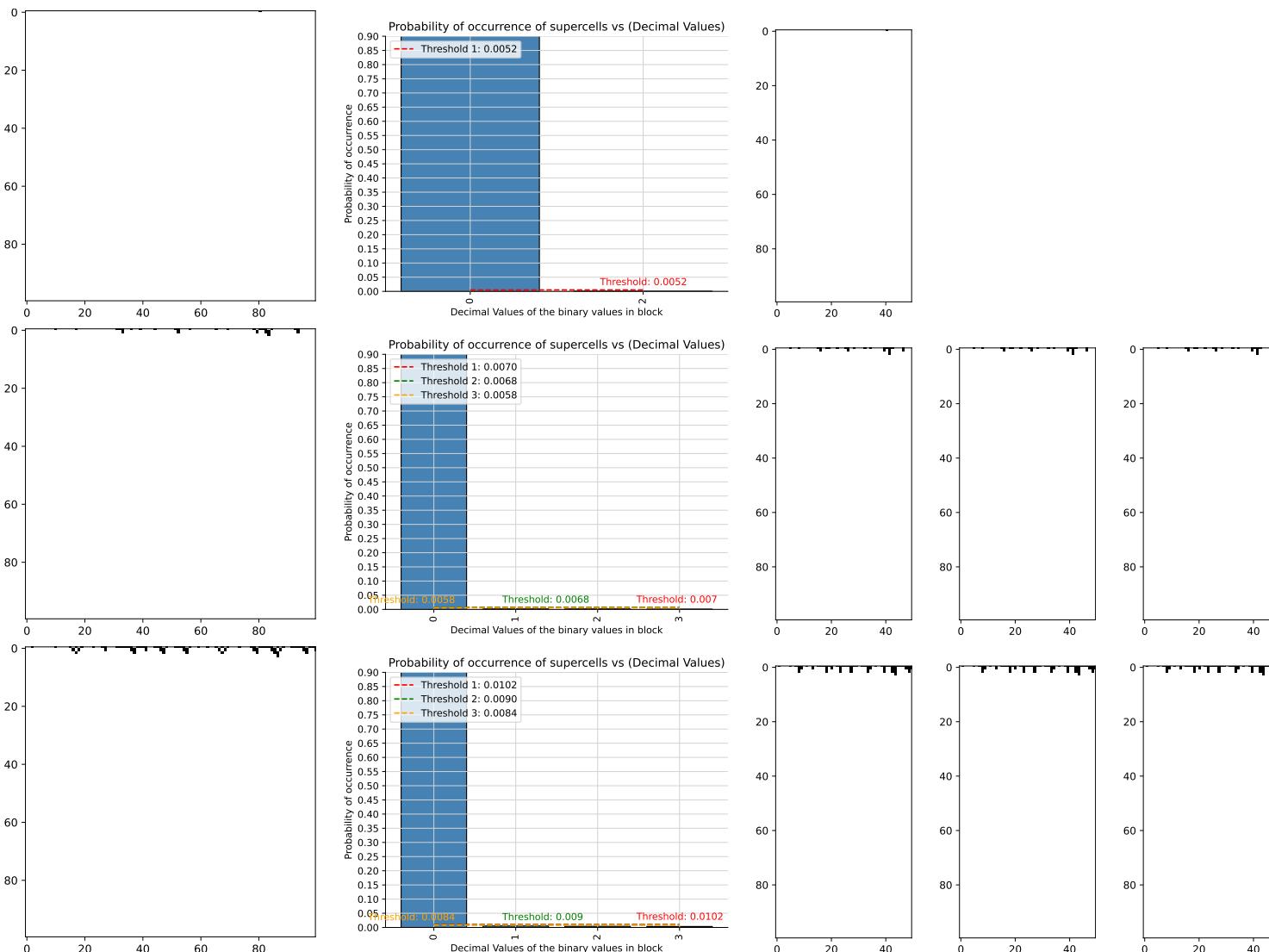


Table 36: FHCG plots for ECA Rule 41.

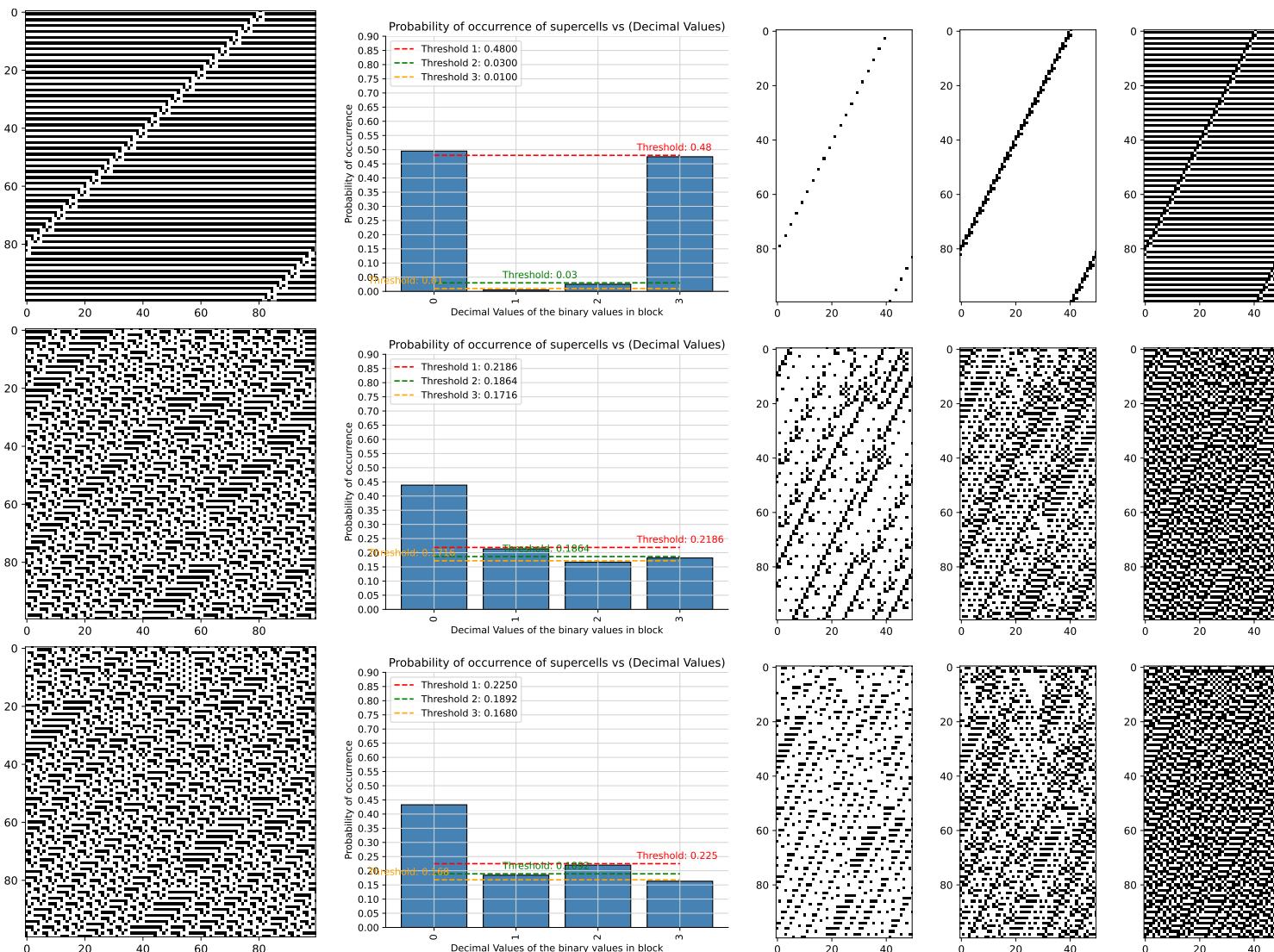


Table 37: FHCG plots for ECA Rule 42.

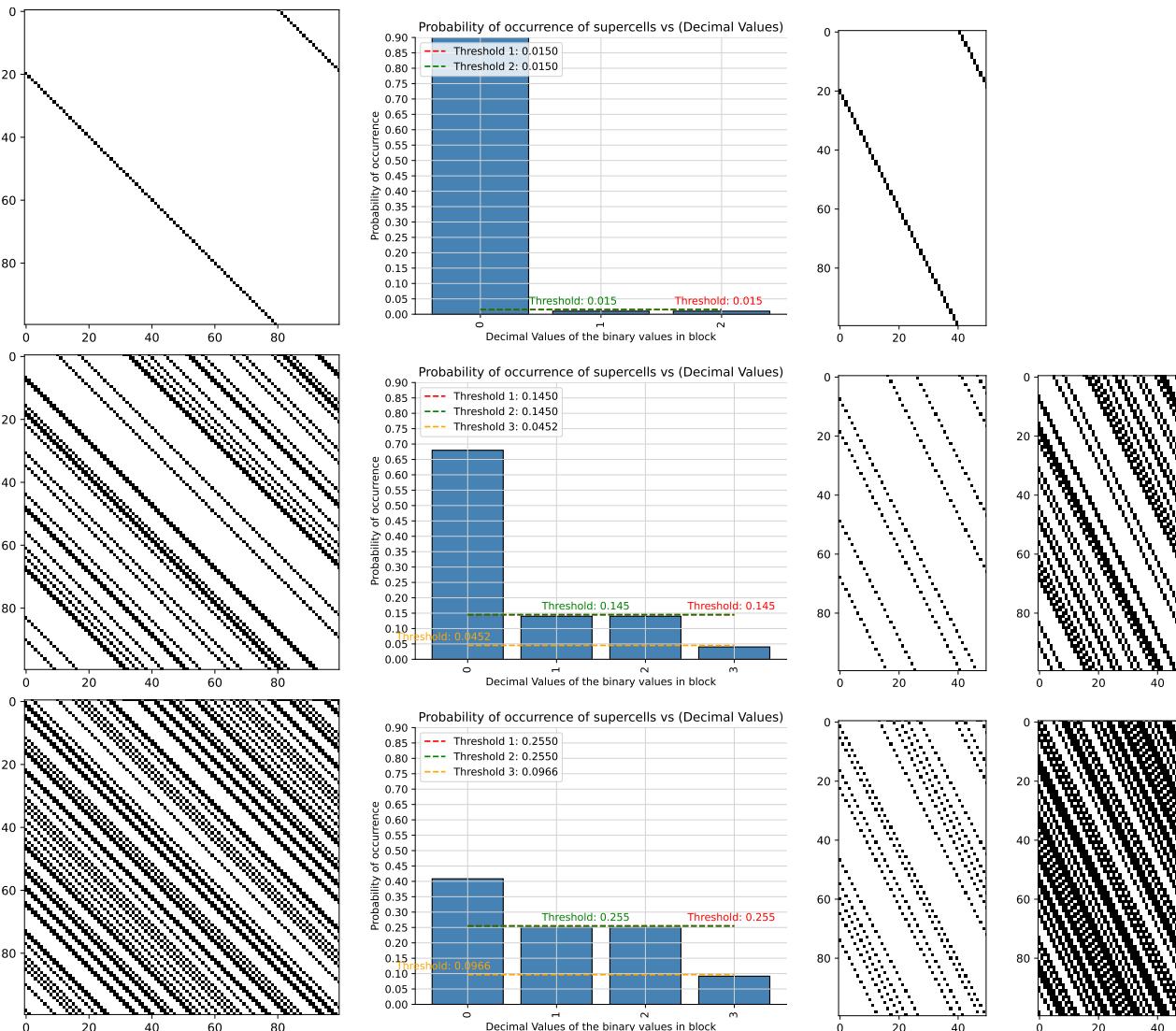


Table 38: FHCG plots for ECA Rule 43.

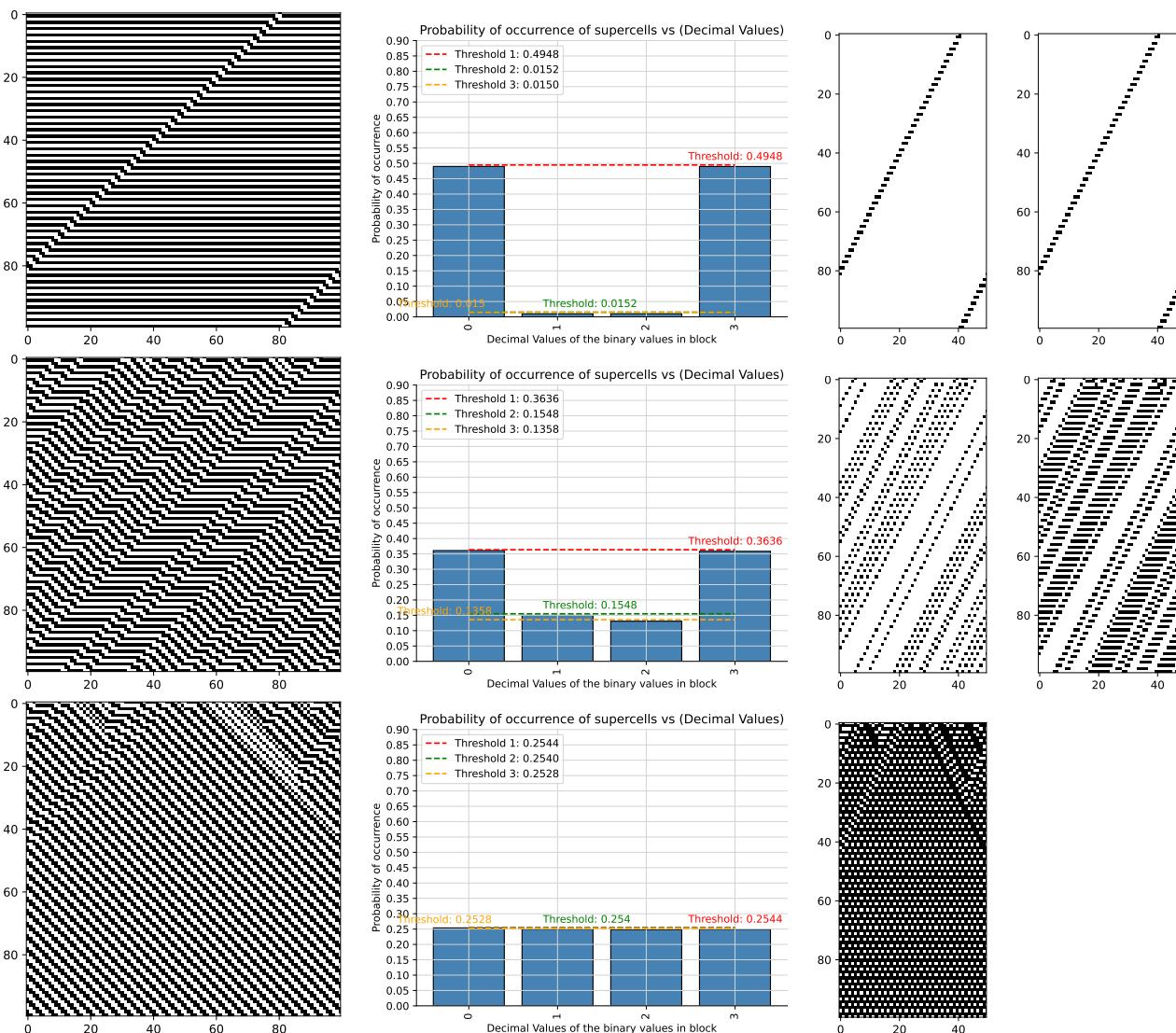


Table 39: FHCG plots for ECA Rule 44.

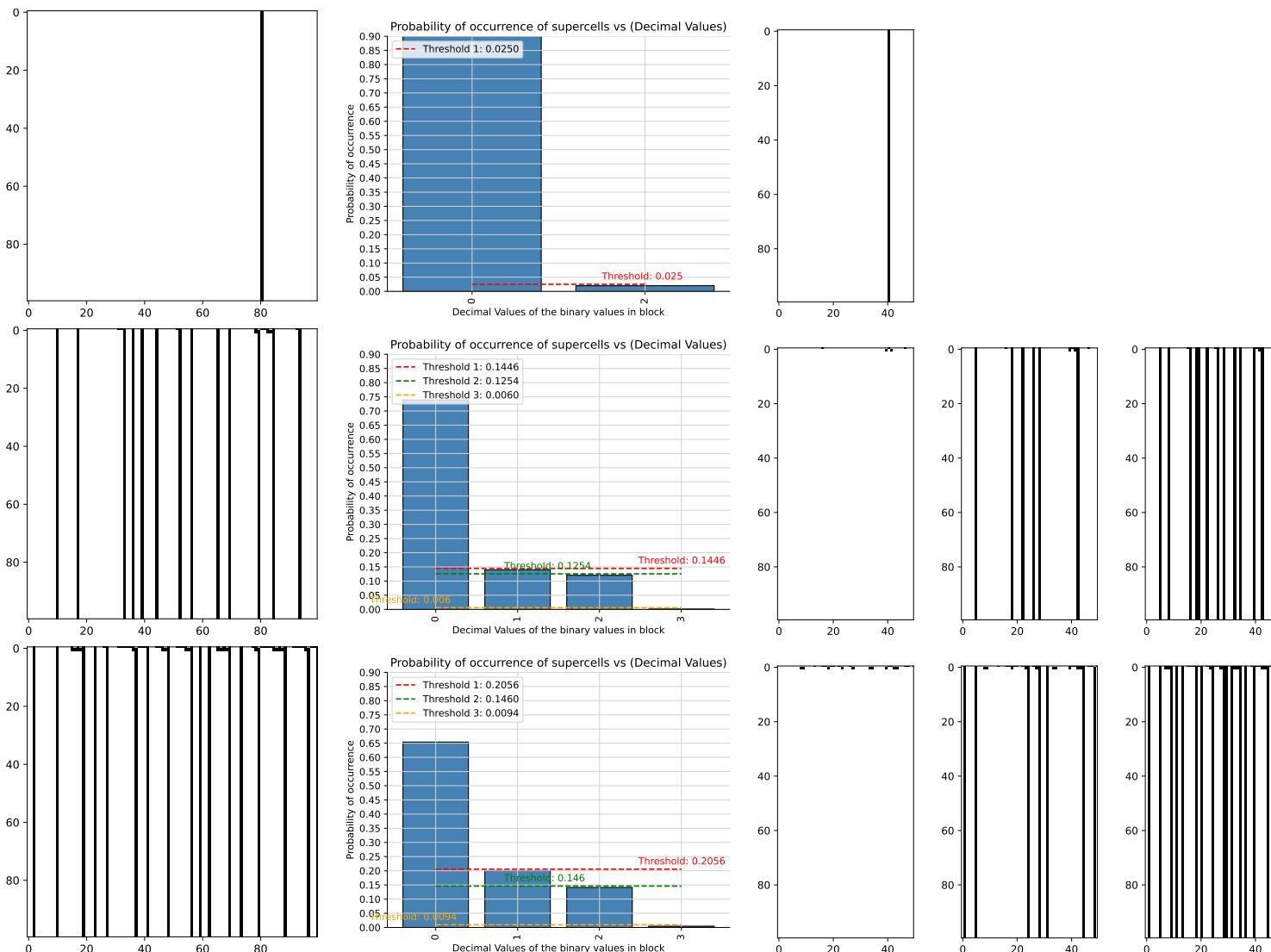


Table 40: FHCG plots for ECA Rule 45.

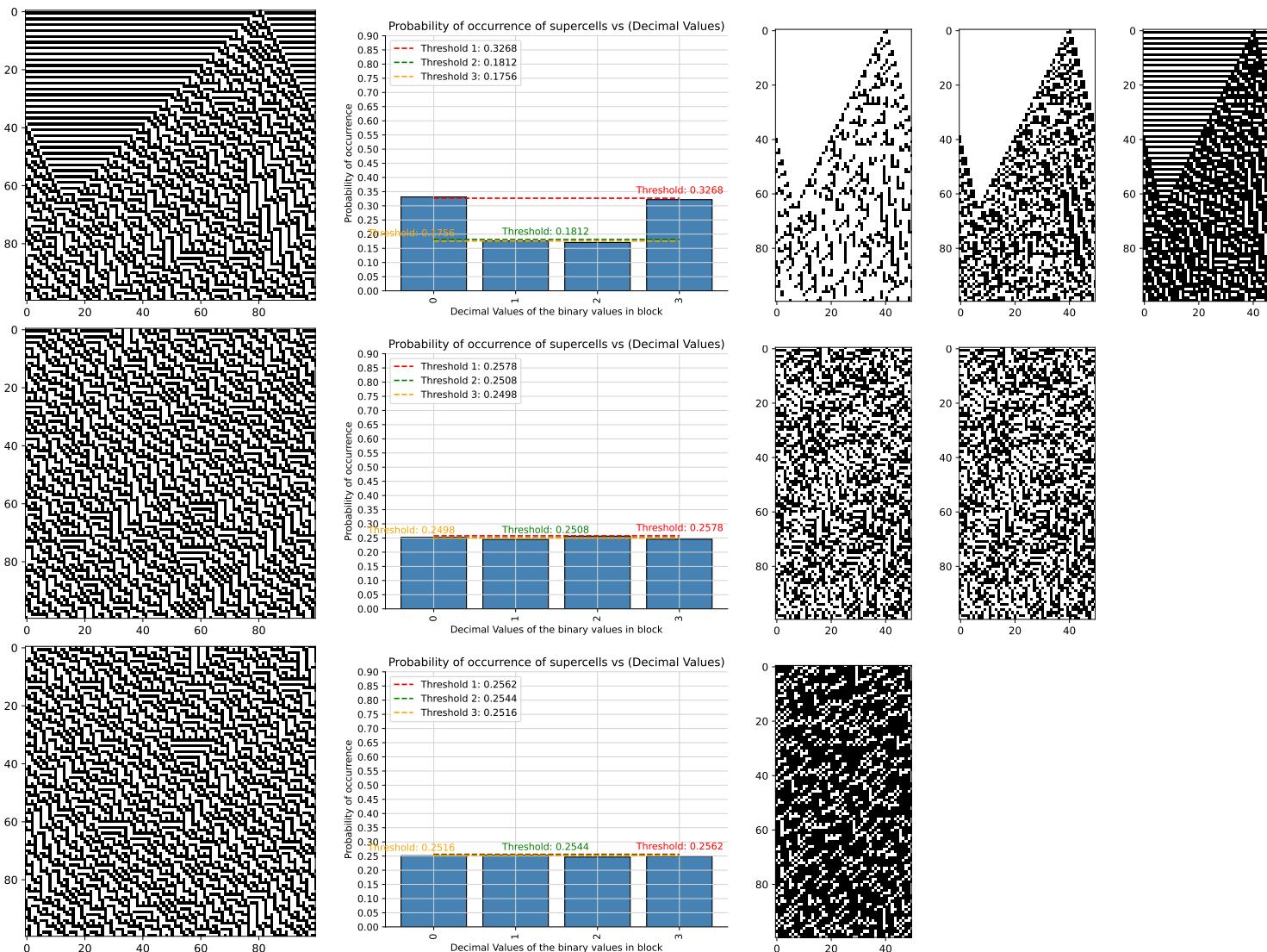


Table 41: FHCG plots for ECA Rule 46.

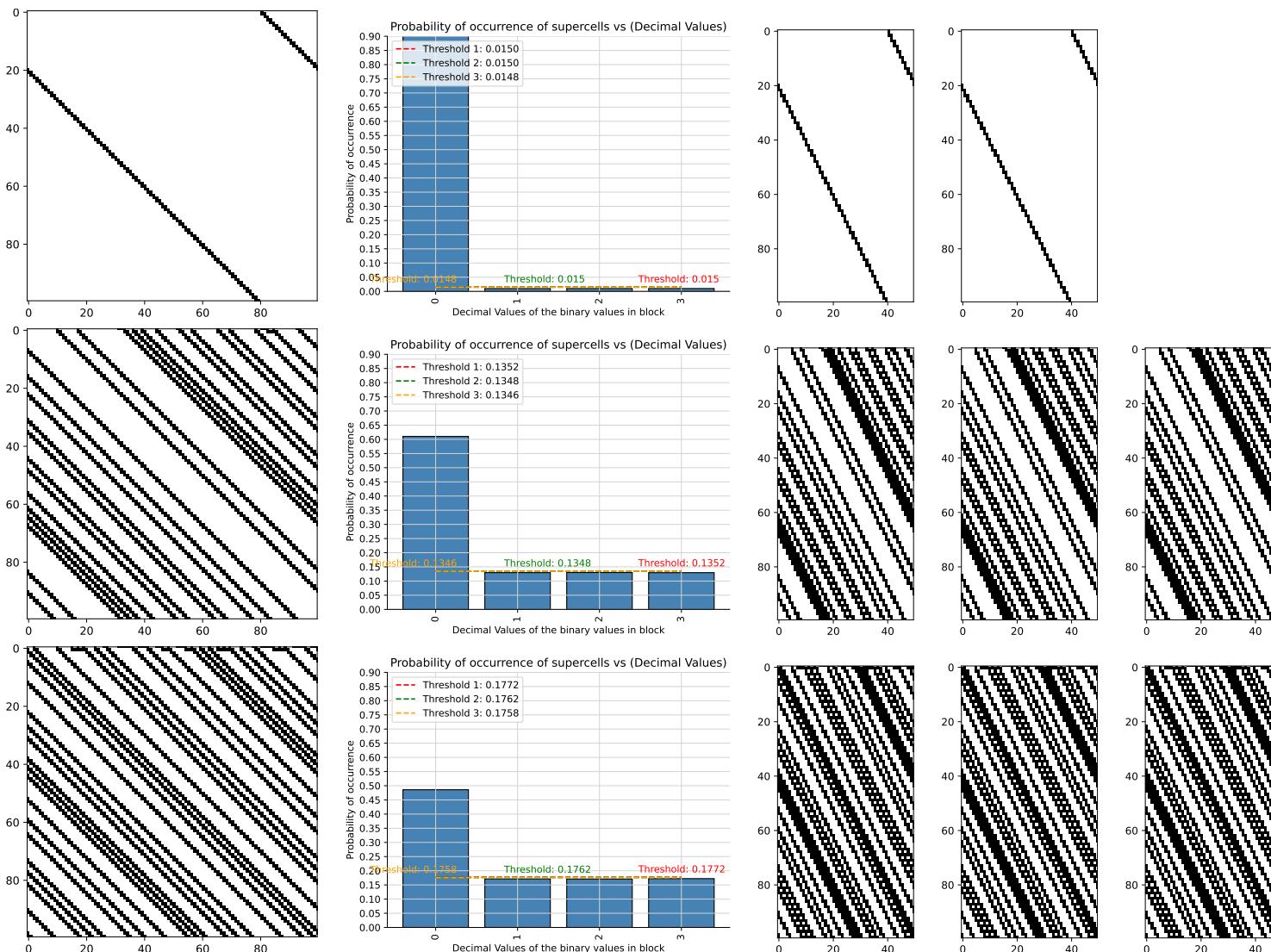


Table 42: FHCG plots for ECA Rule 50.

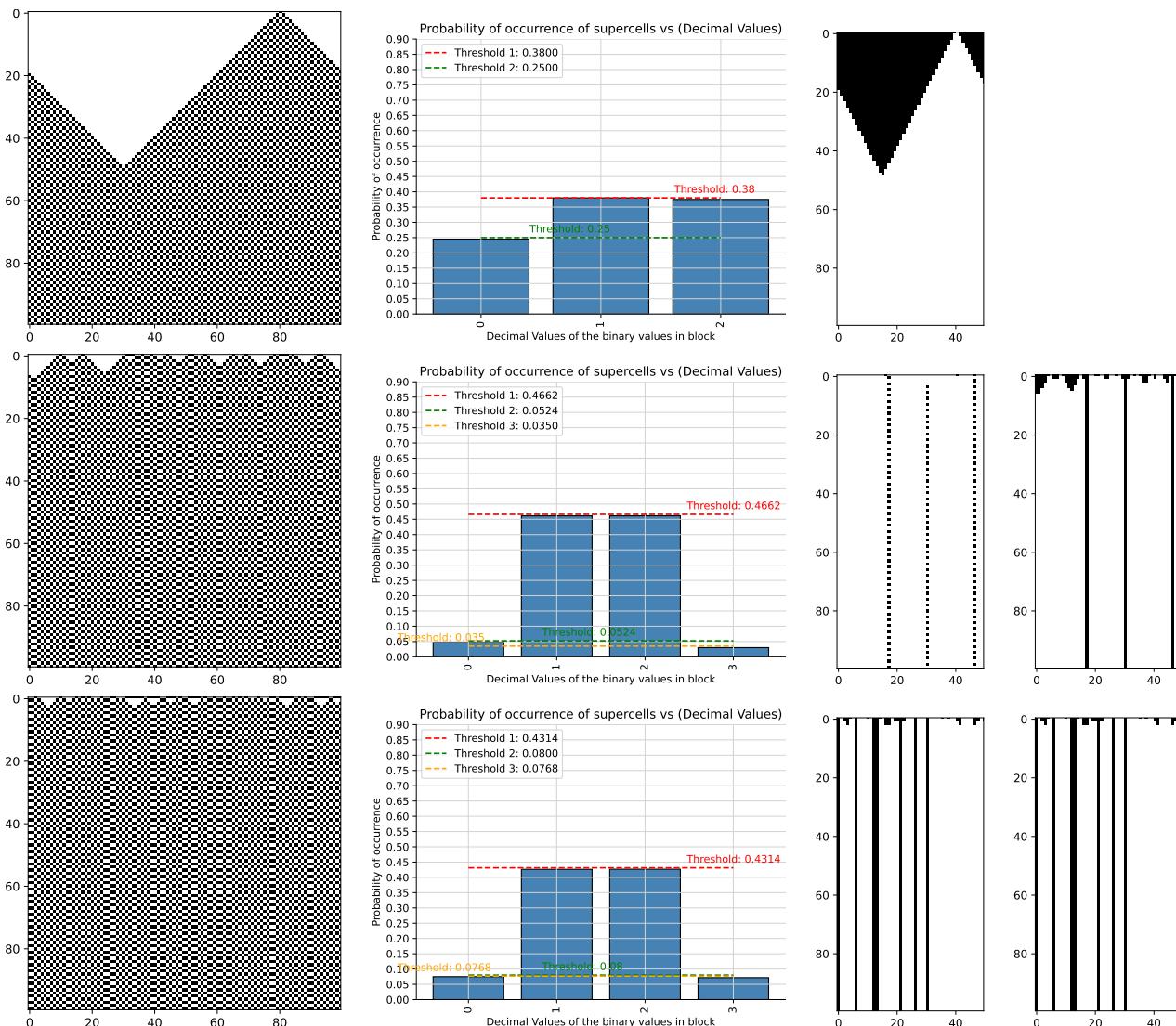


Table 43: FHCG plots for ECA Rule 51.

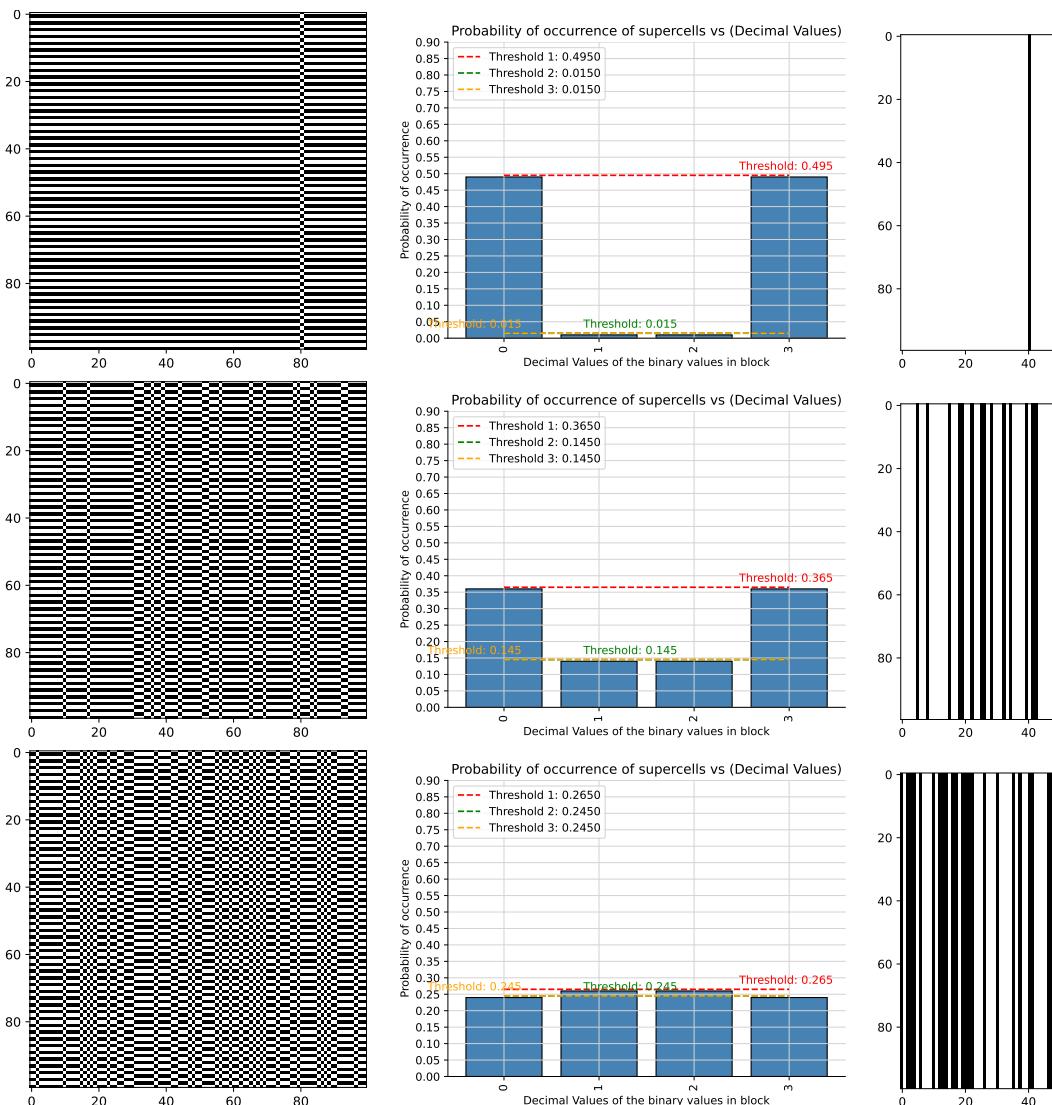


Table 44: FHCG plots for ECA Rule 54.

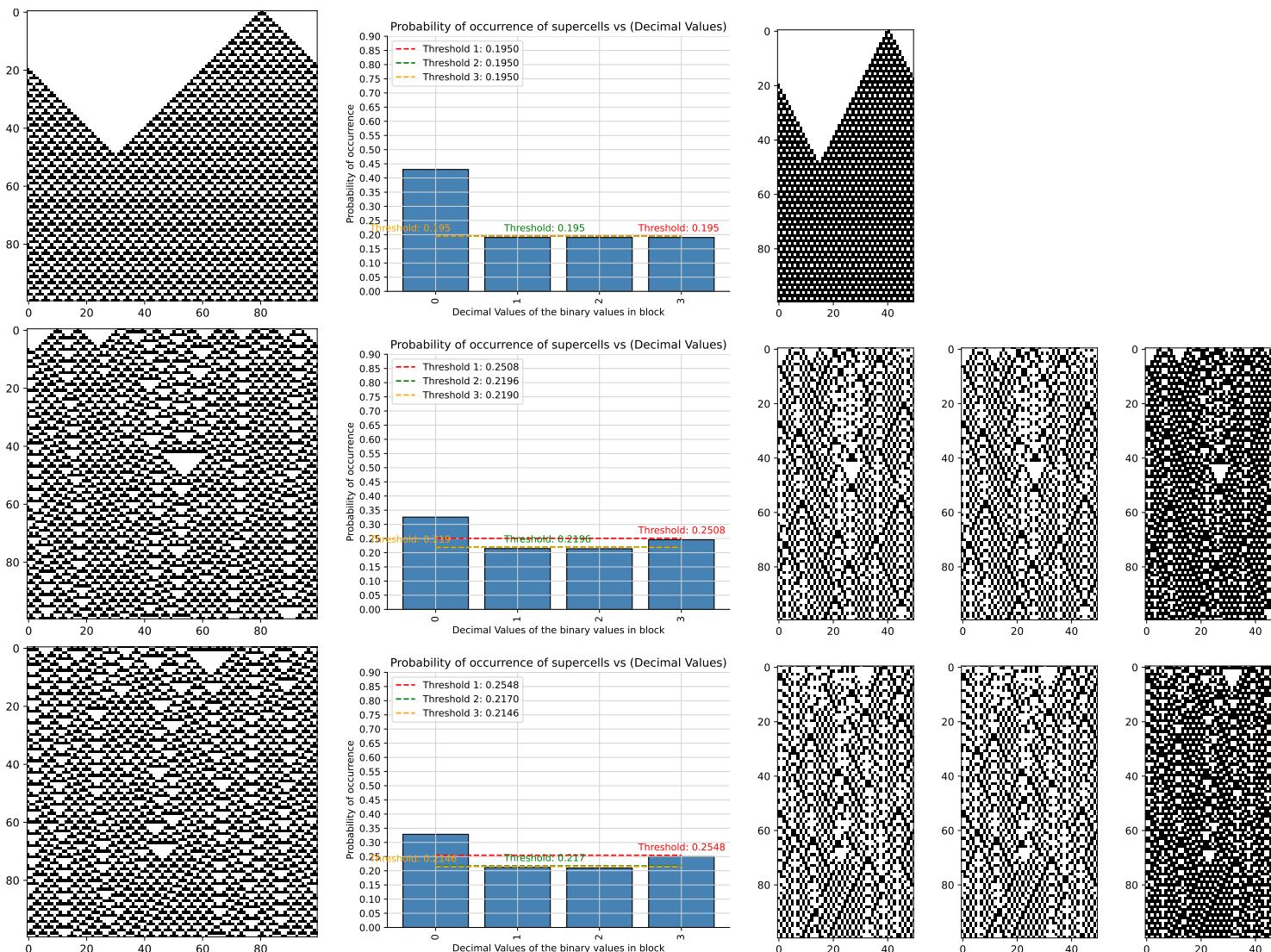


Table 45: FHCG plots for ECA Rule 56.

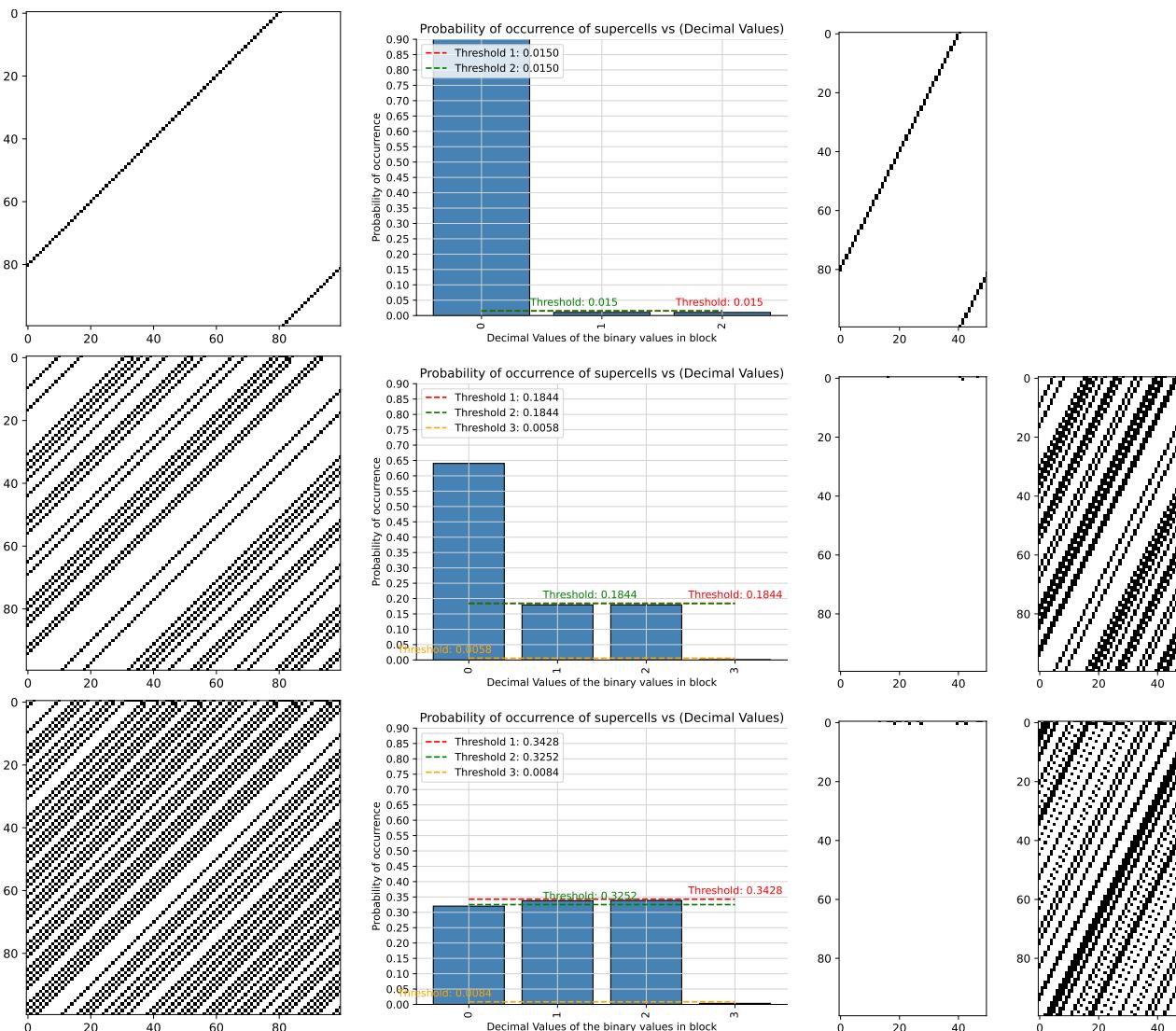


Table 46: FHCG plots for ECA Rule 57.

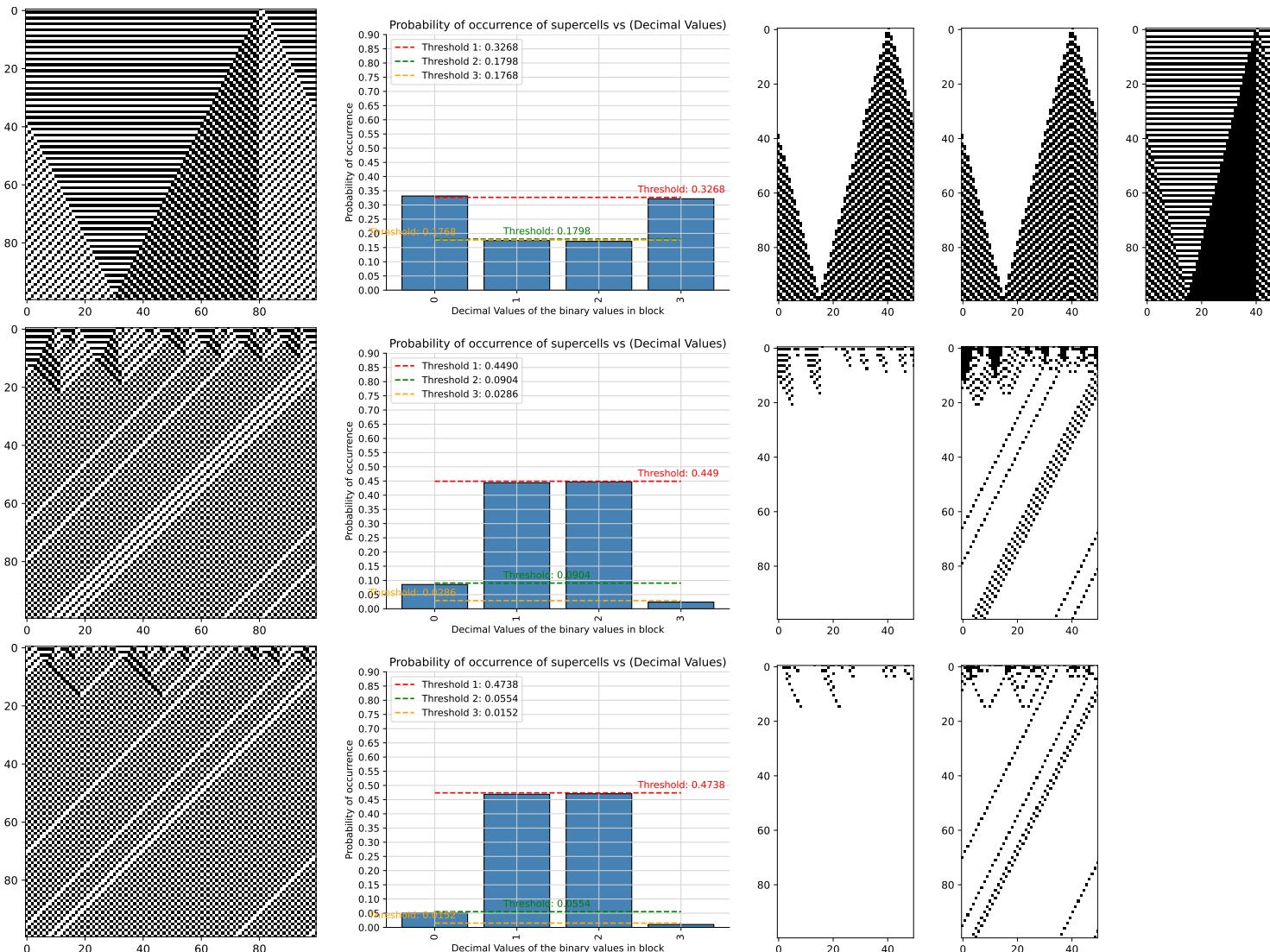


Table 47: FHCG plots for ECA Rule 58.

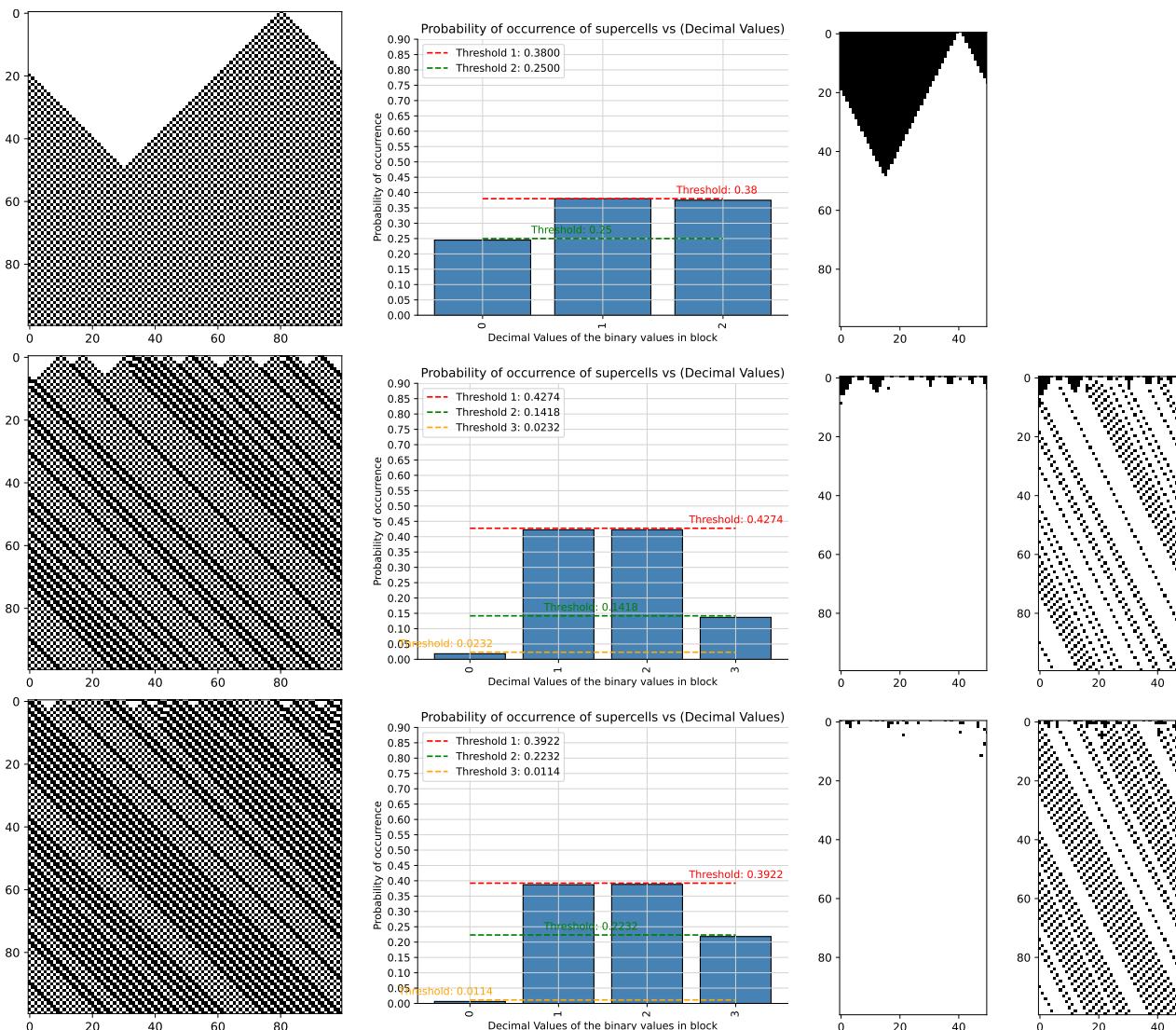


Table 48: FHCG plots for ECA Rule 60.

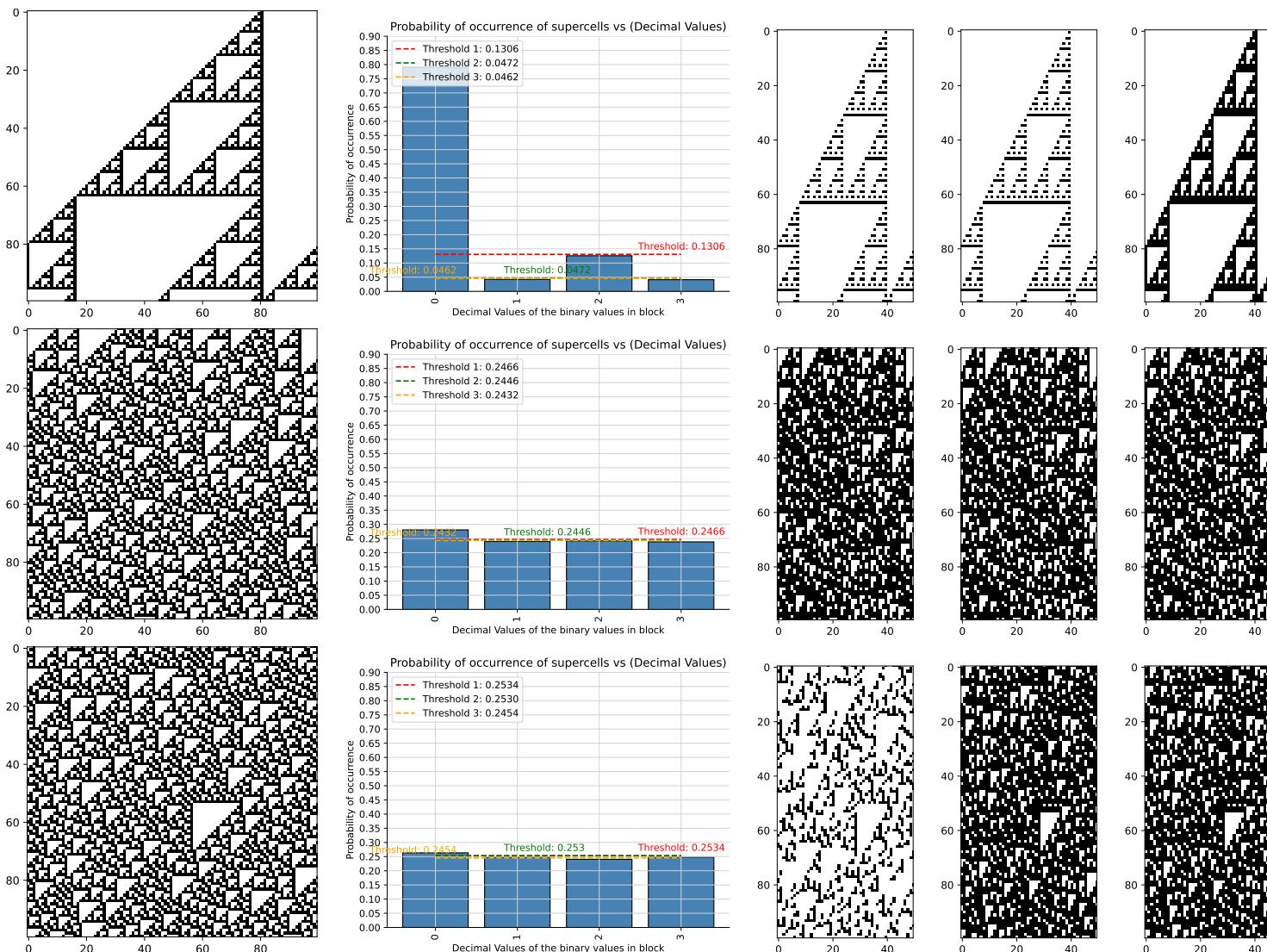


Table 49: FHCG plots for ECA Rule 62.

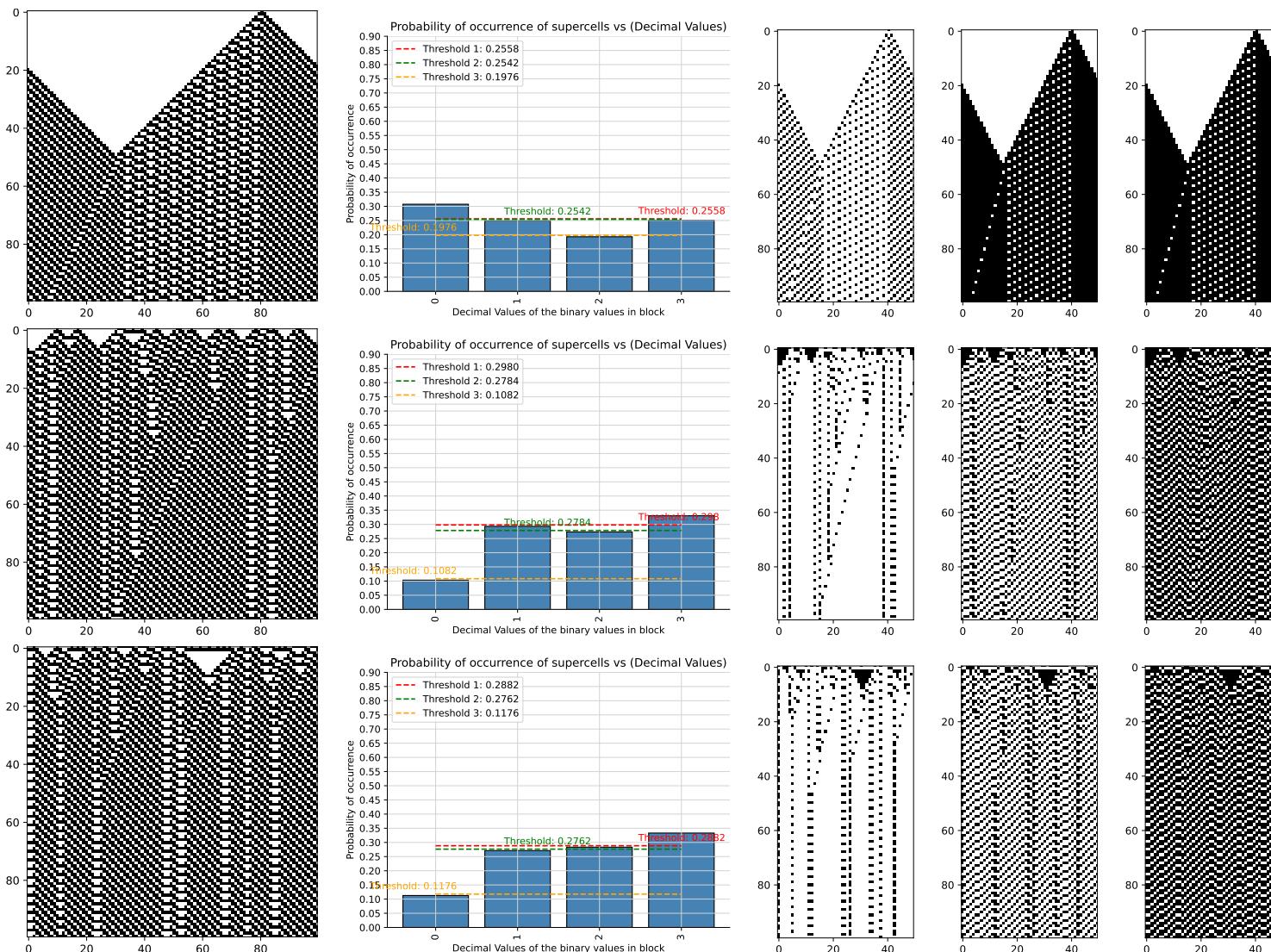


Table 50: FHCG plots for ECA Rule 72.

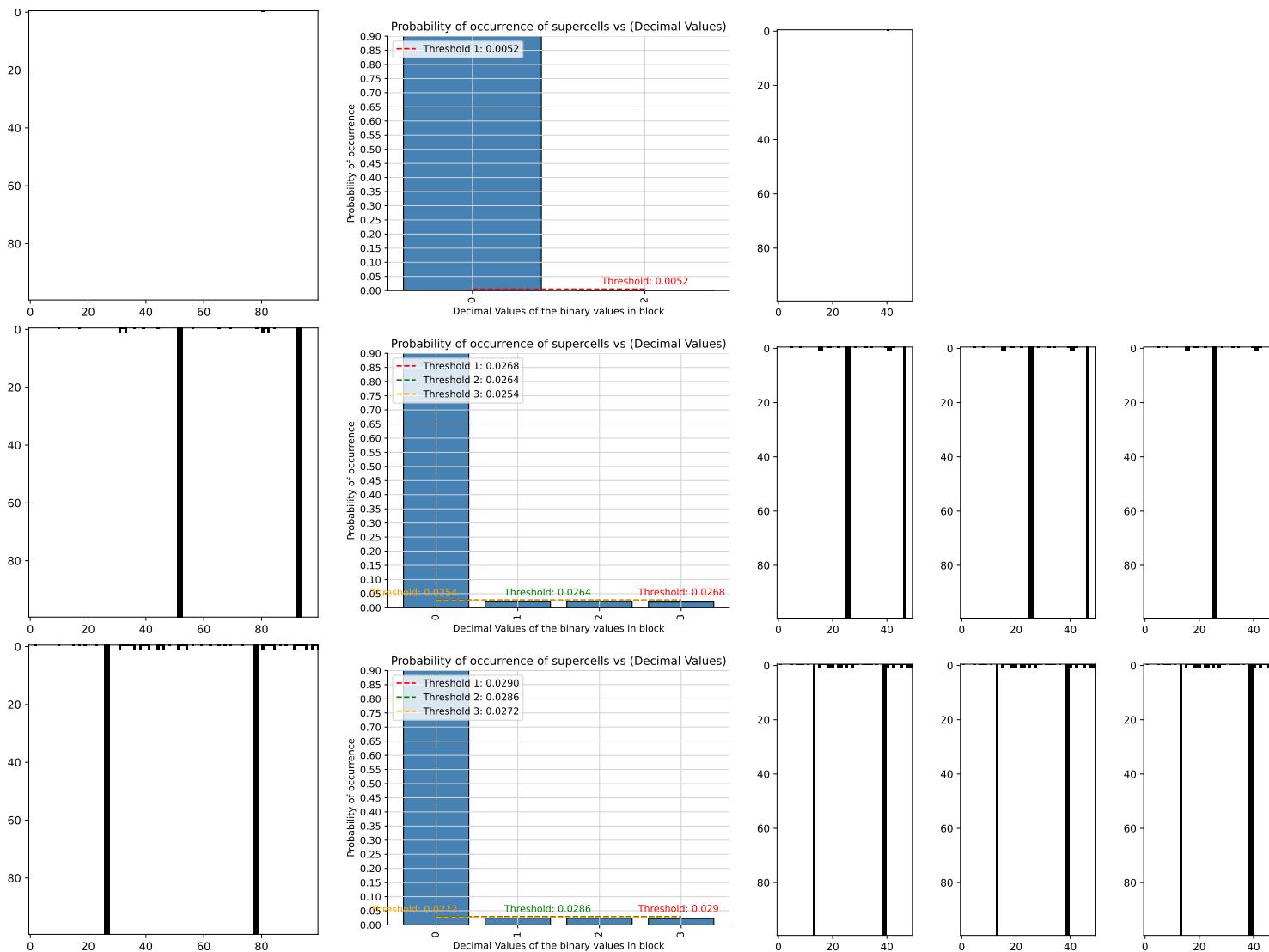


Table 51: FHCG plots for ECA Rule 73.

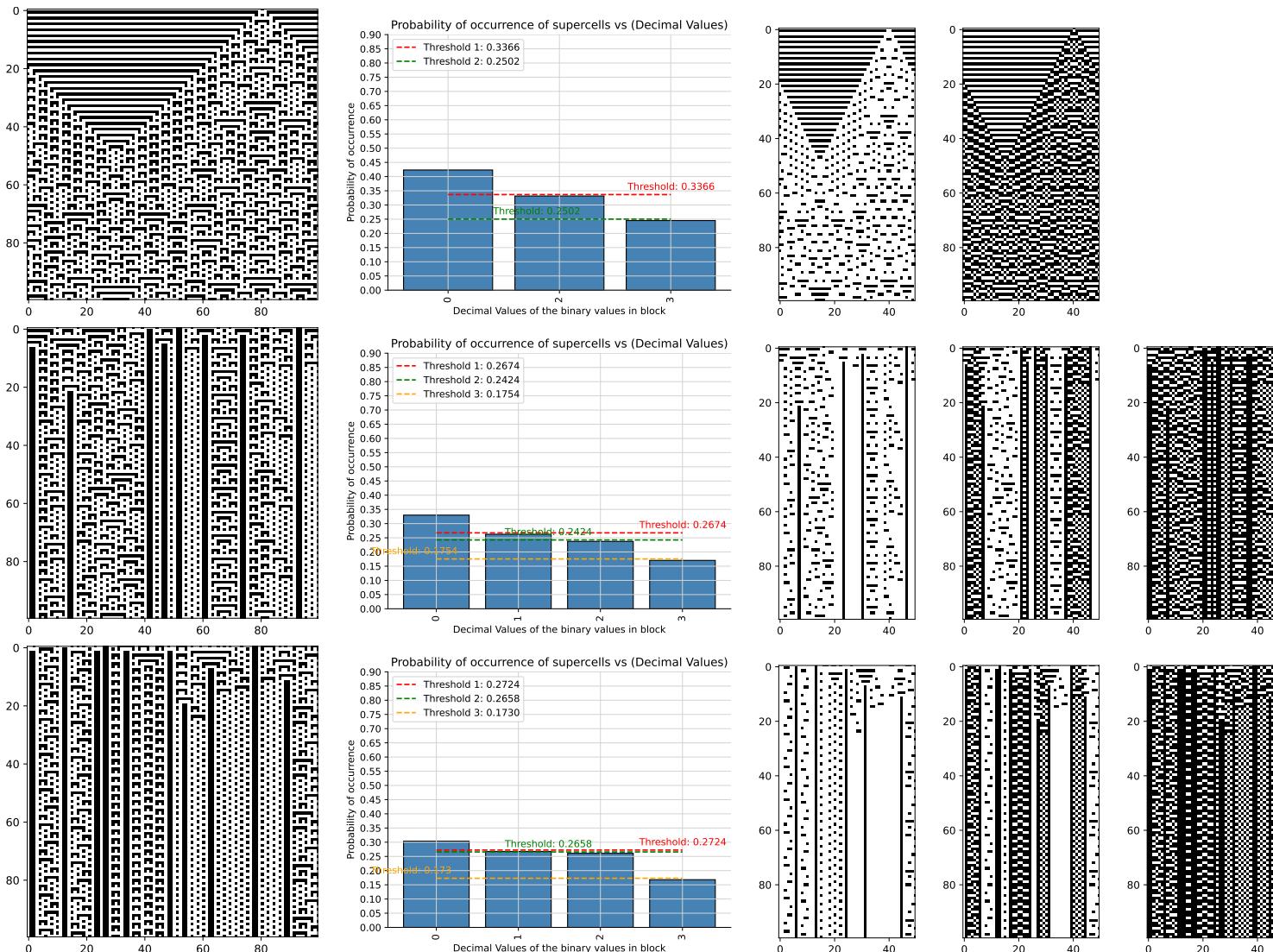


Table 52: FHCG plots for ECA Rule 74.

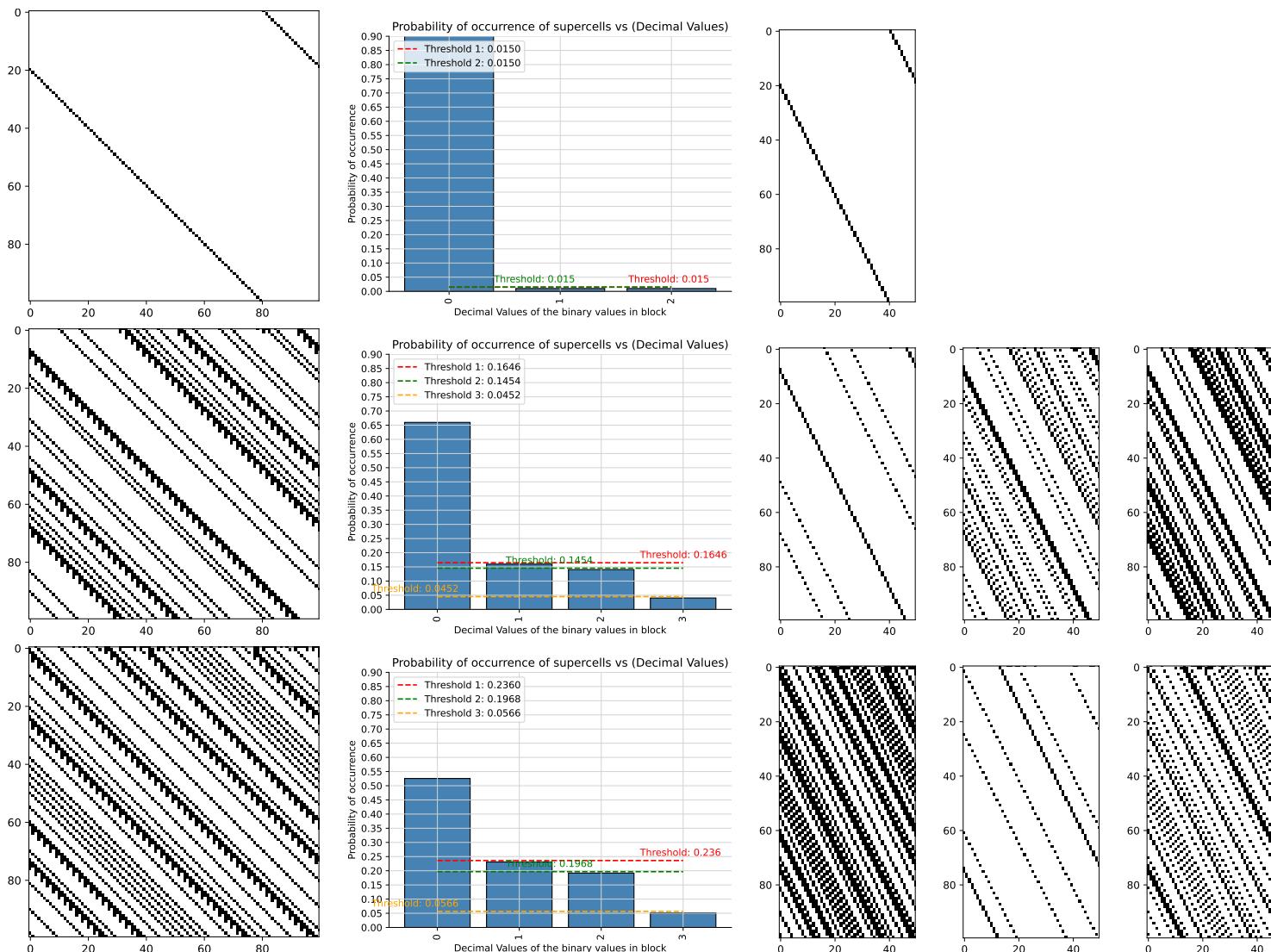


Table 53: FHCG plots for ECA Rule 76.

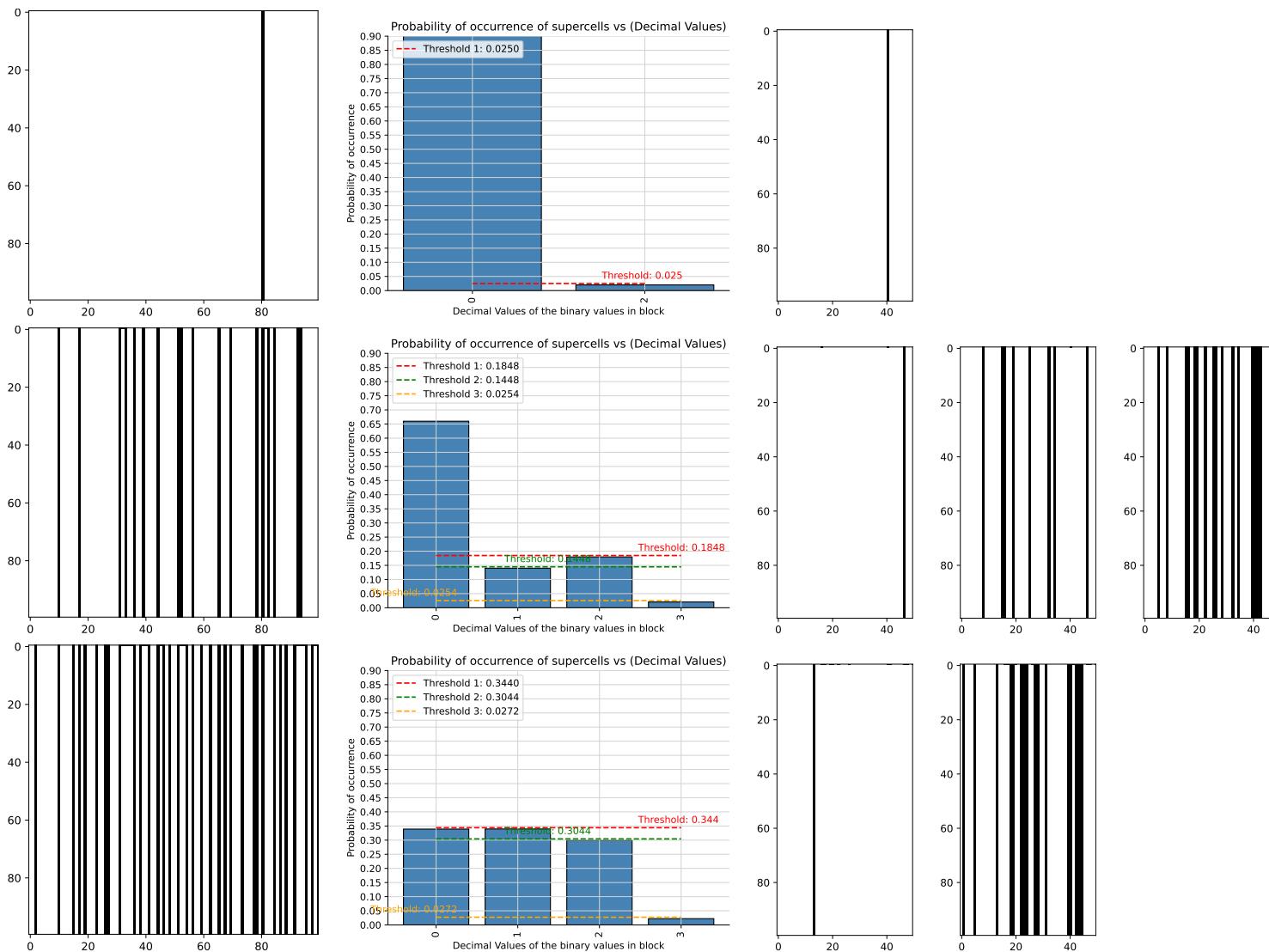


Table 54: FHCG plots for ECA Rule 77.

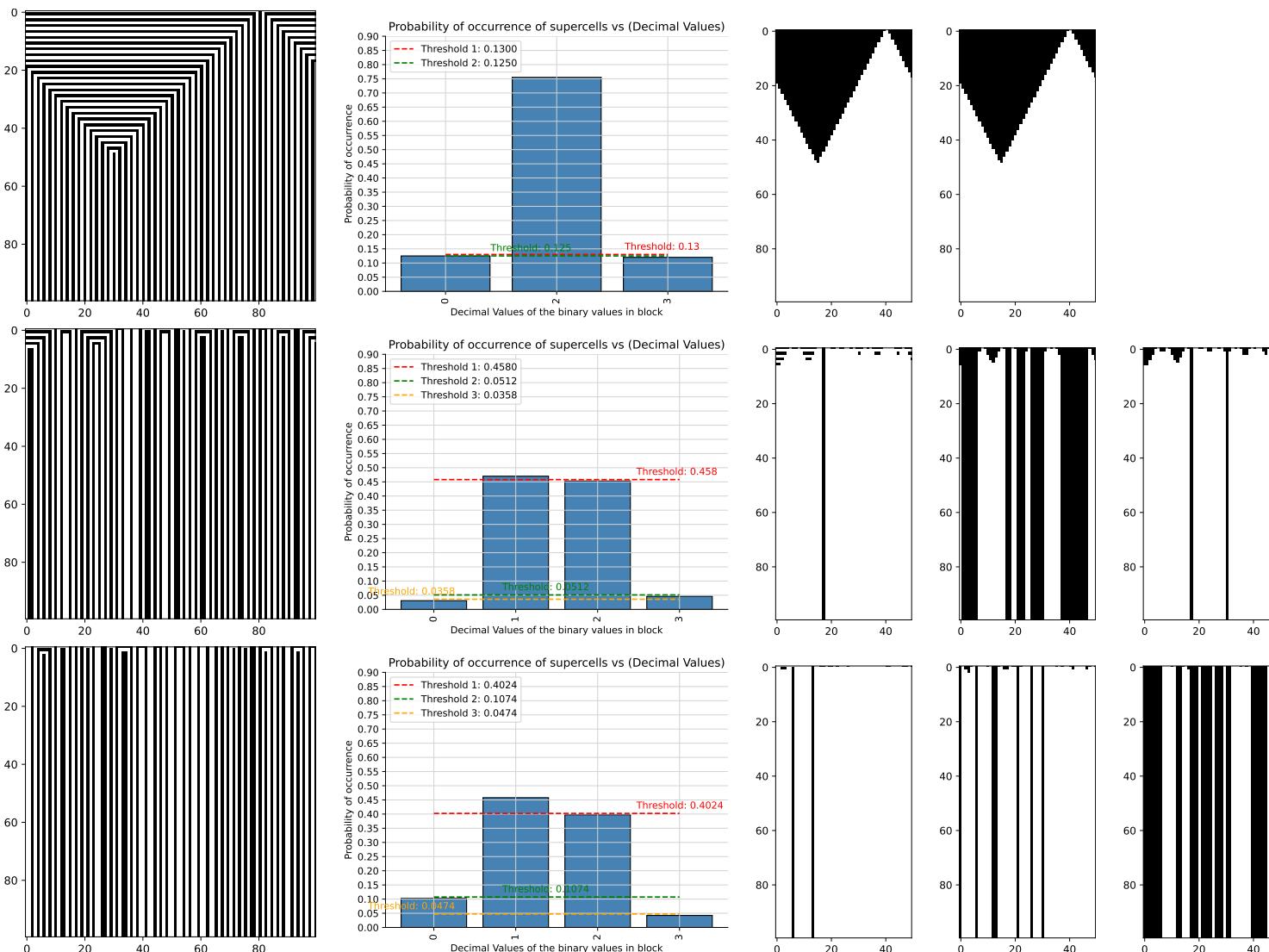


Table 55: FHCG plots for ECA Rule 78.

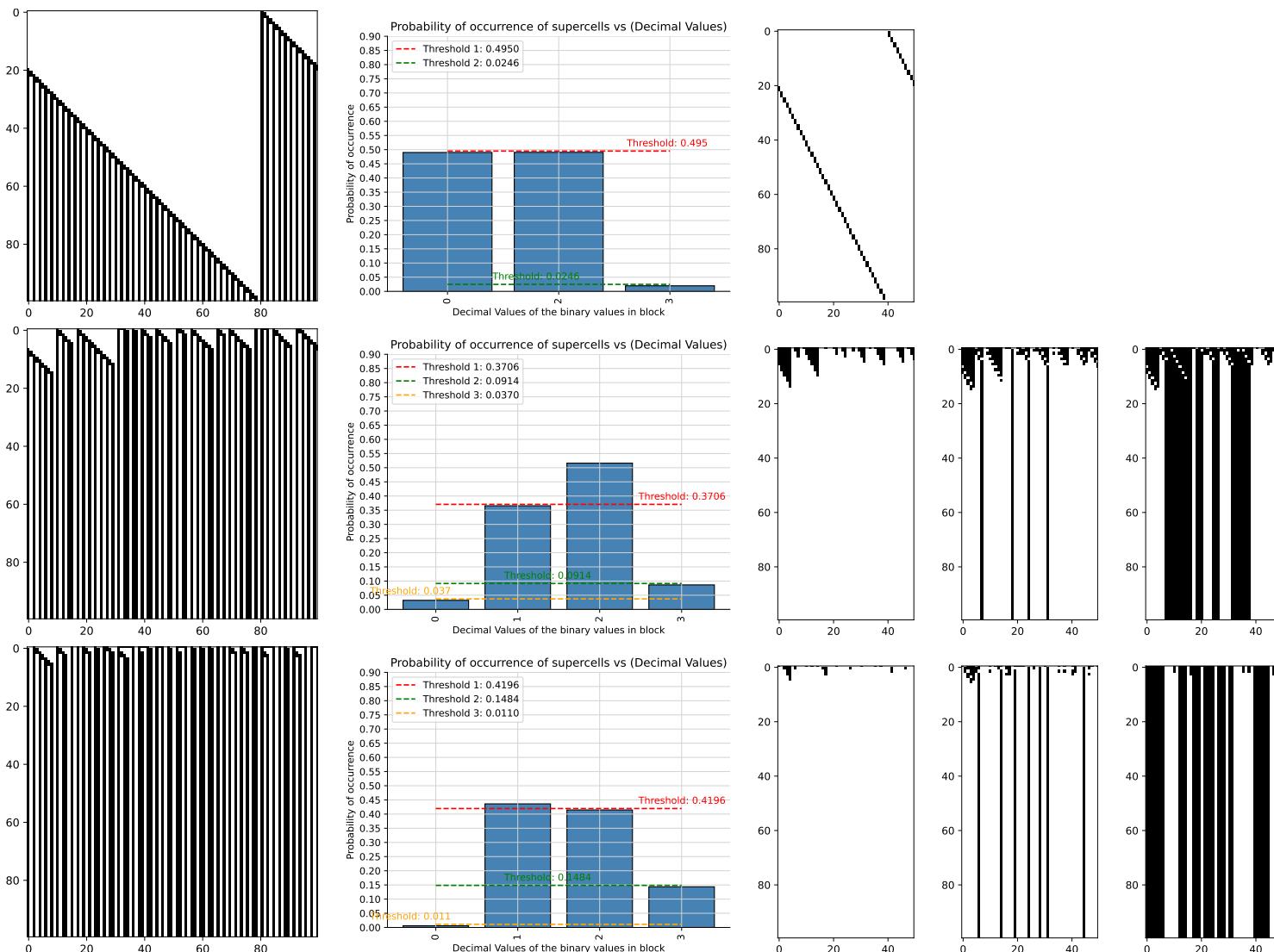


Table 56: FHCG plots for ECA Rule 90.

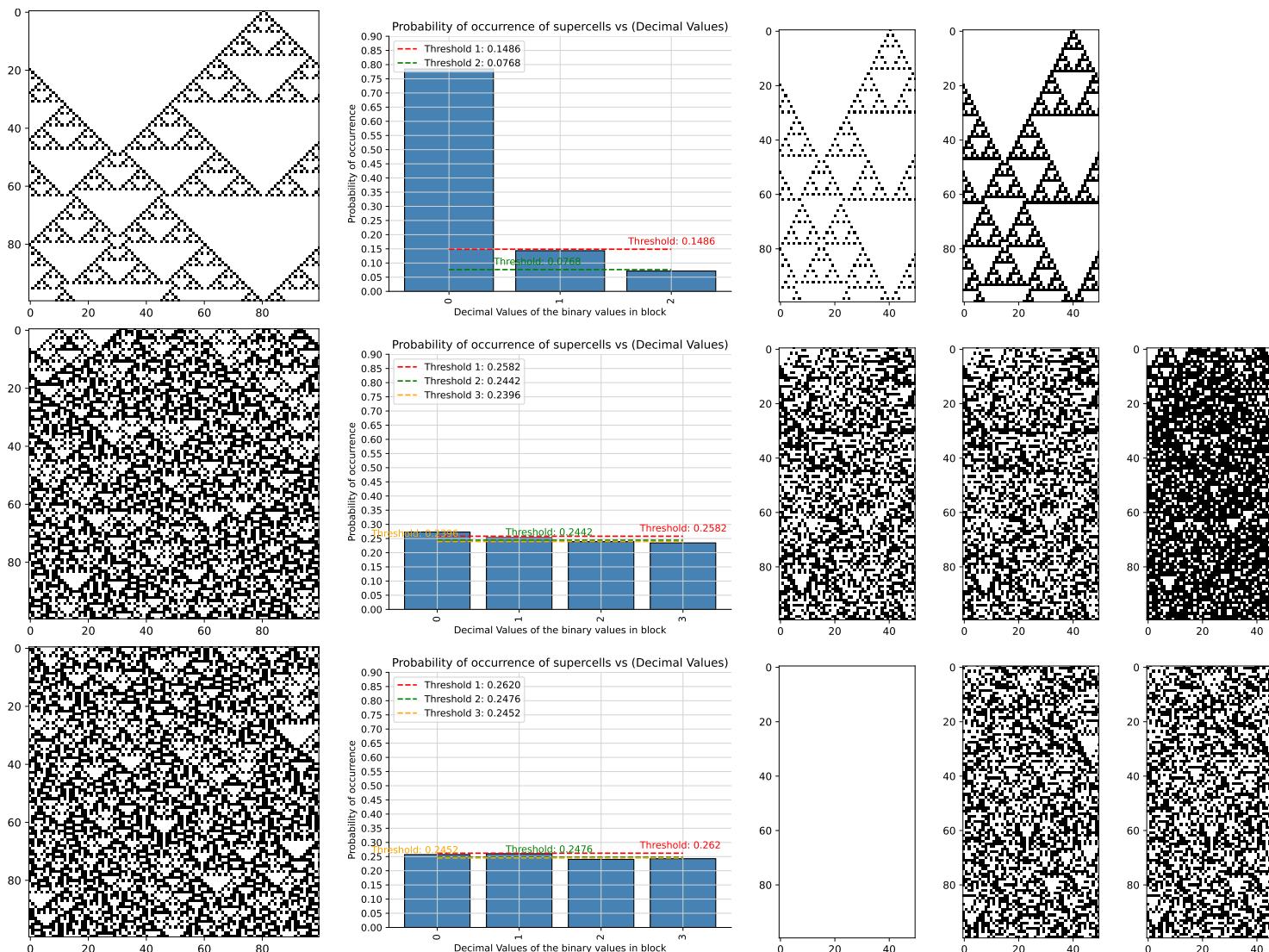


Table 57: FHCG plots for ECA Rule 94.

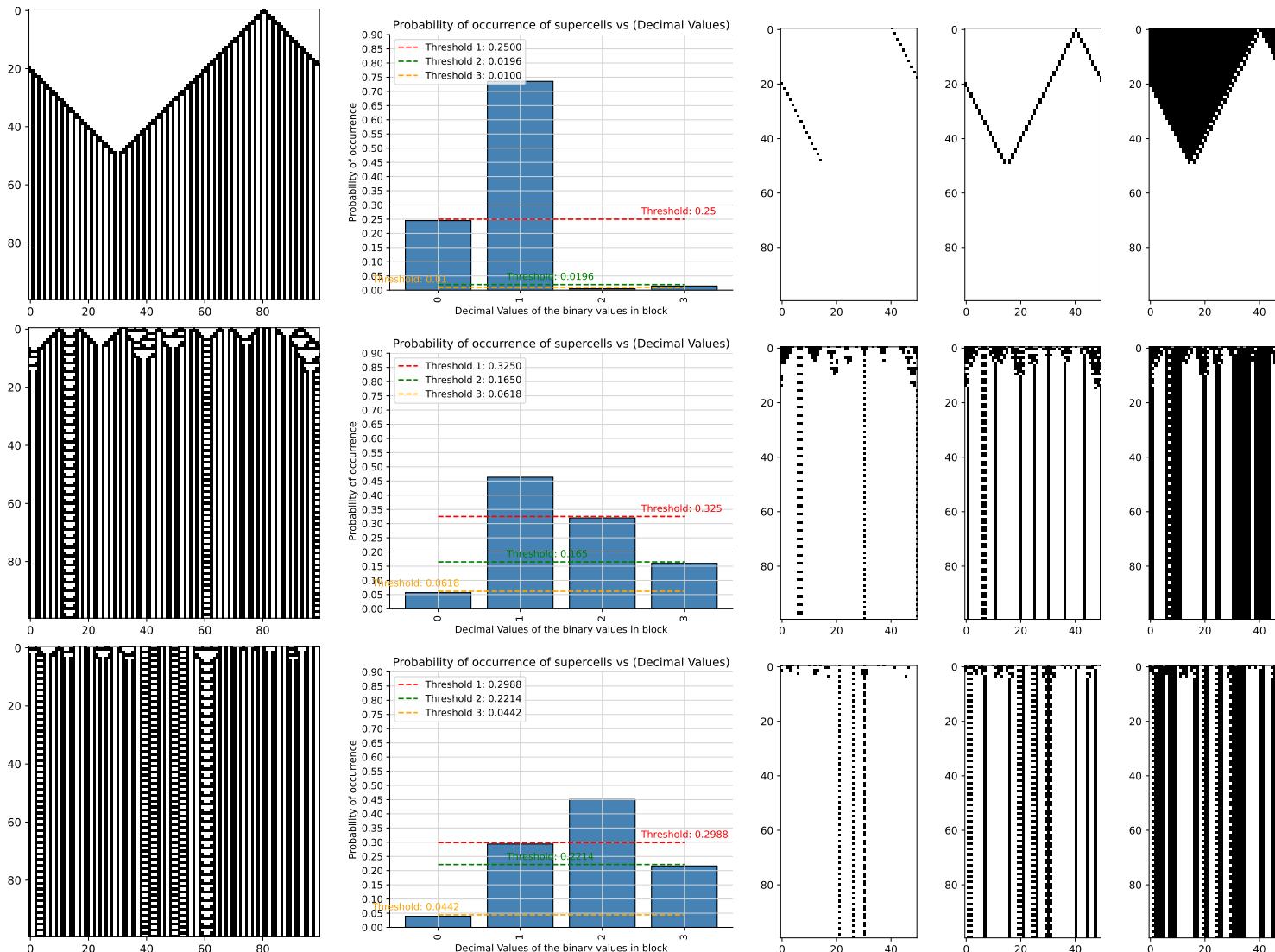


Table 58: FHCG plots for ECA Rule 104.

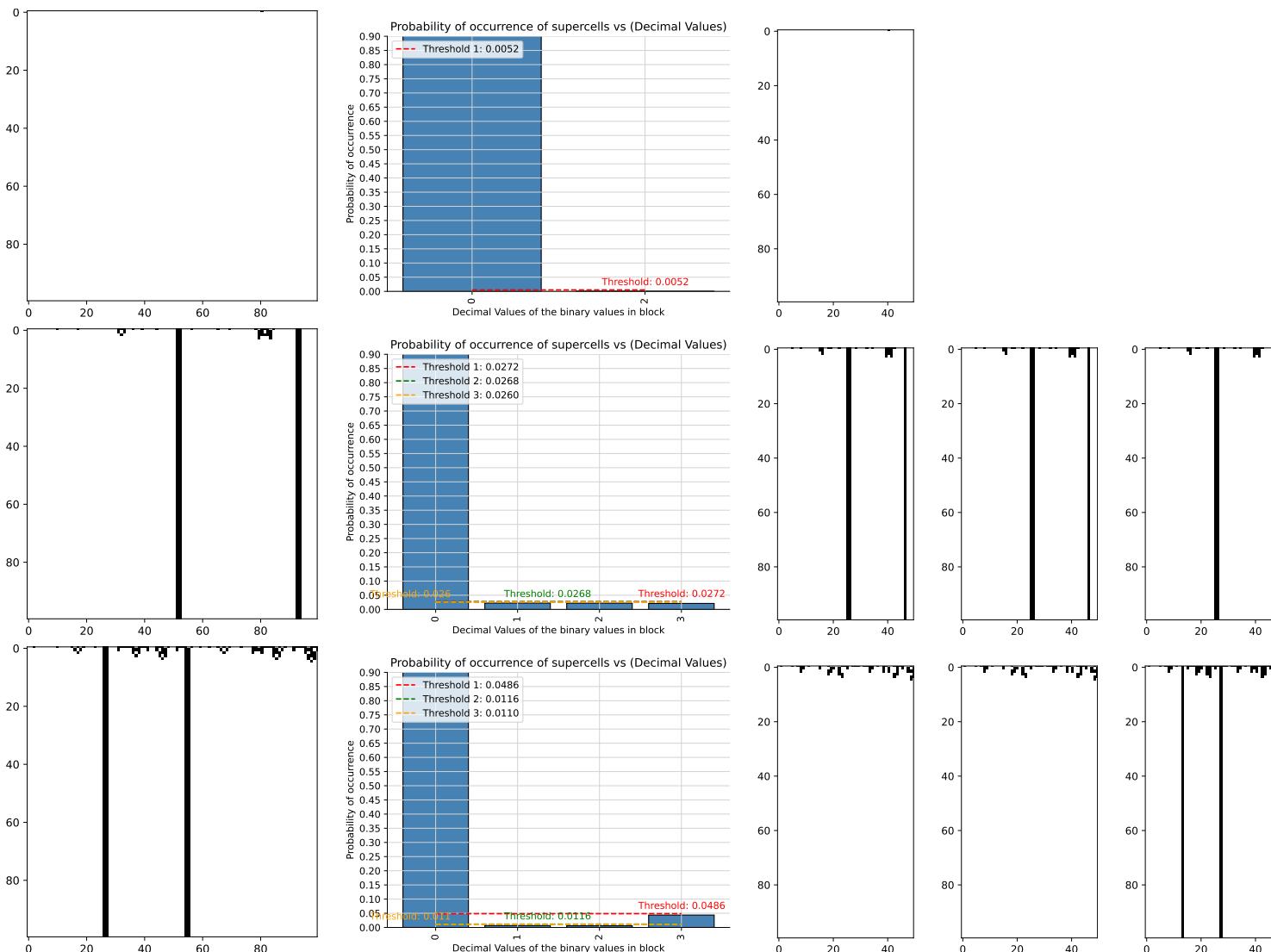


Table 59: FHCG plots for ECA Rule 105.

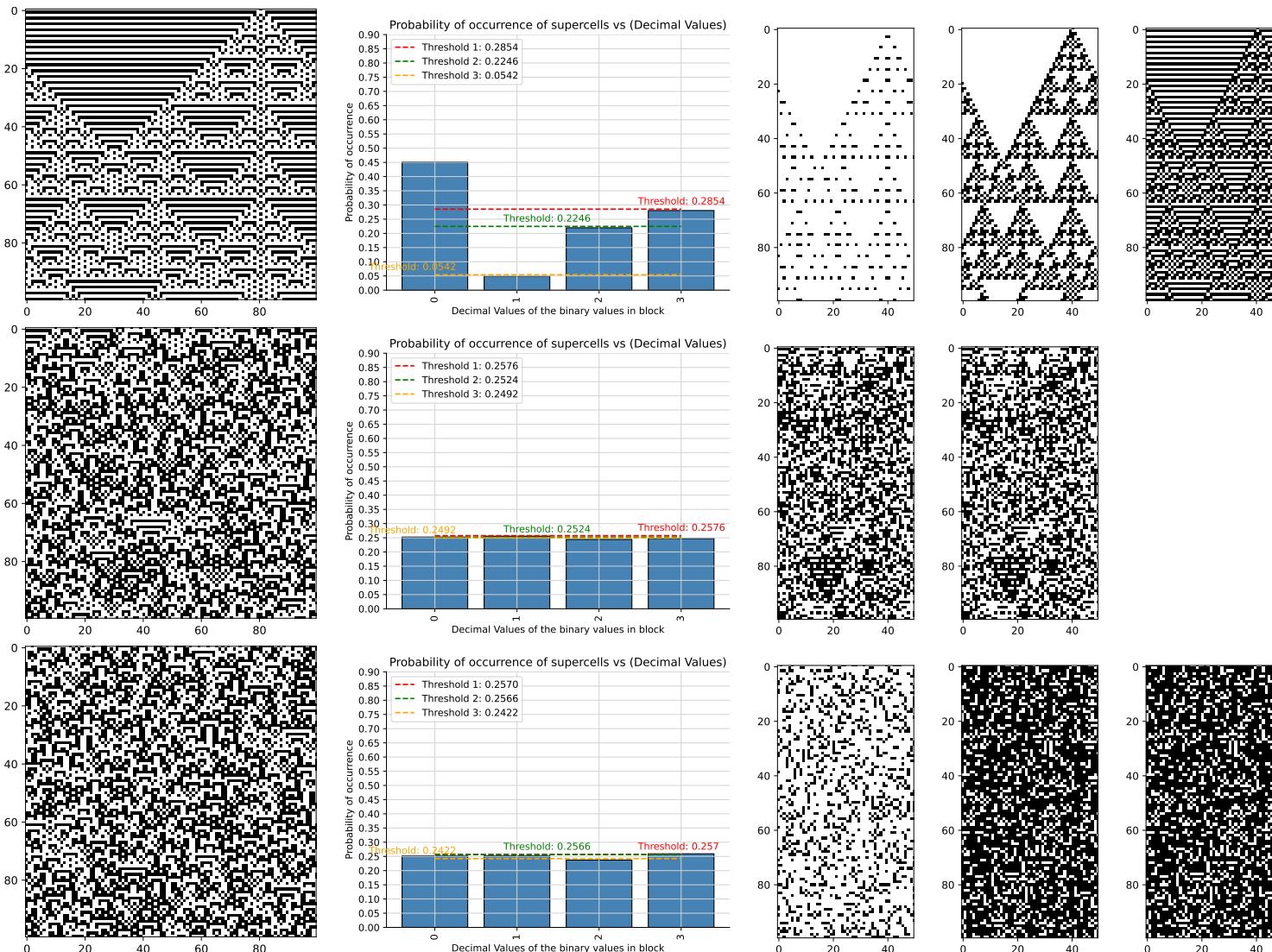


Table 60: FHCG plots for ECA Rule 106.

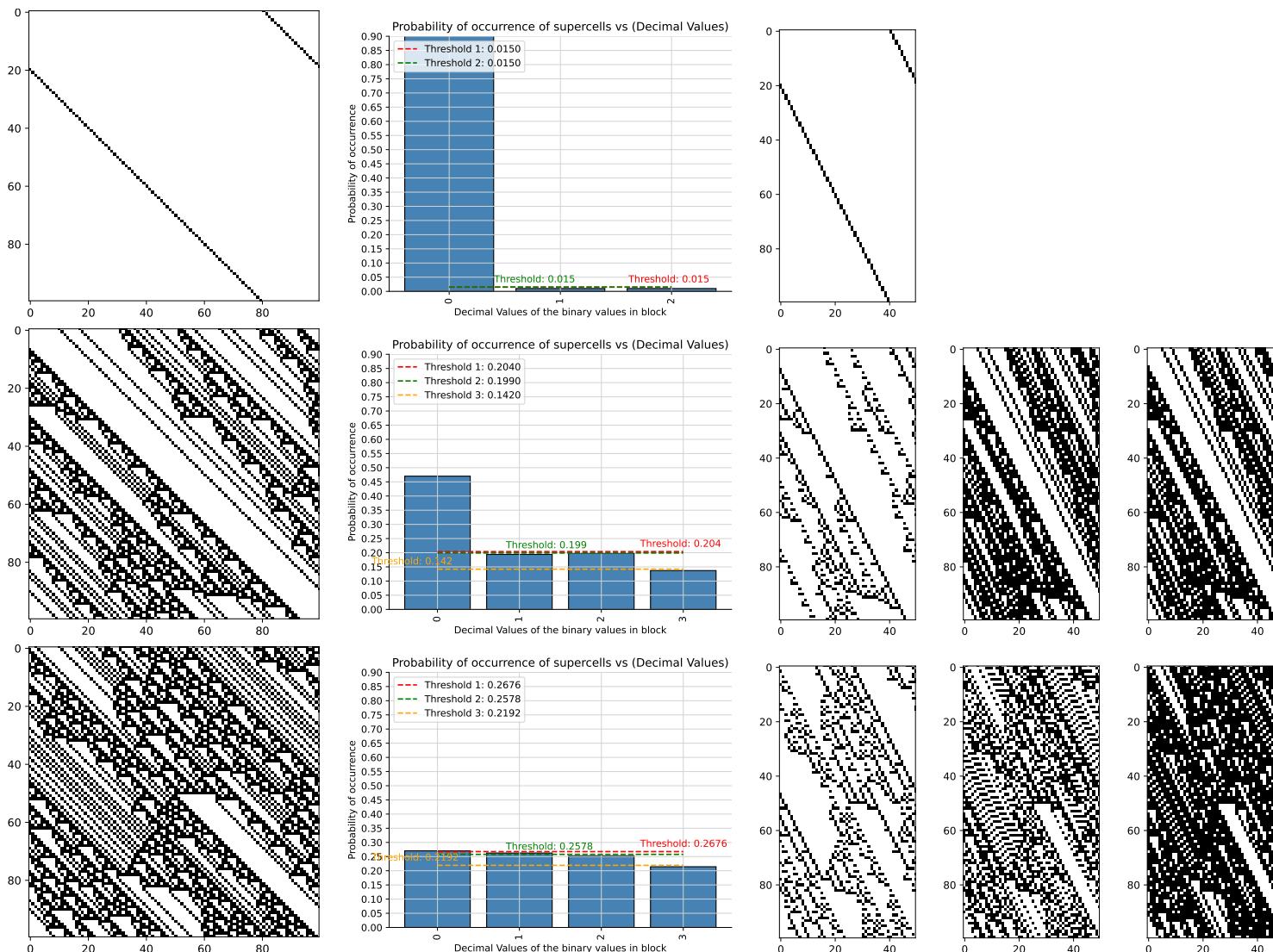


Table 61: FHCG plots for ECA Rule 108.

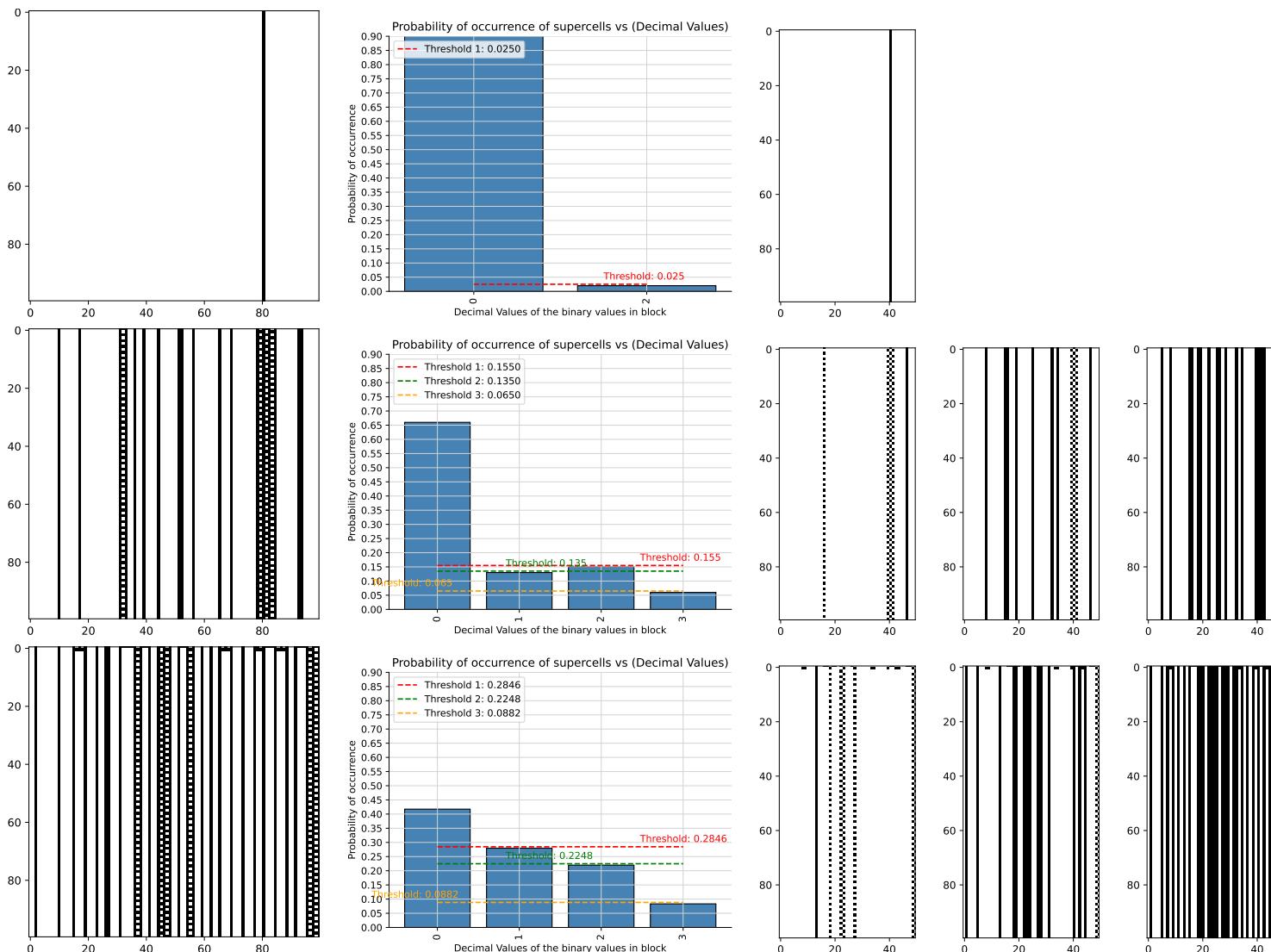


Table 62: FHCG plots for ECA Rule 110.

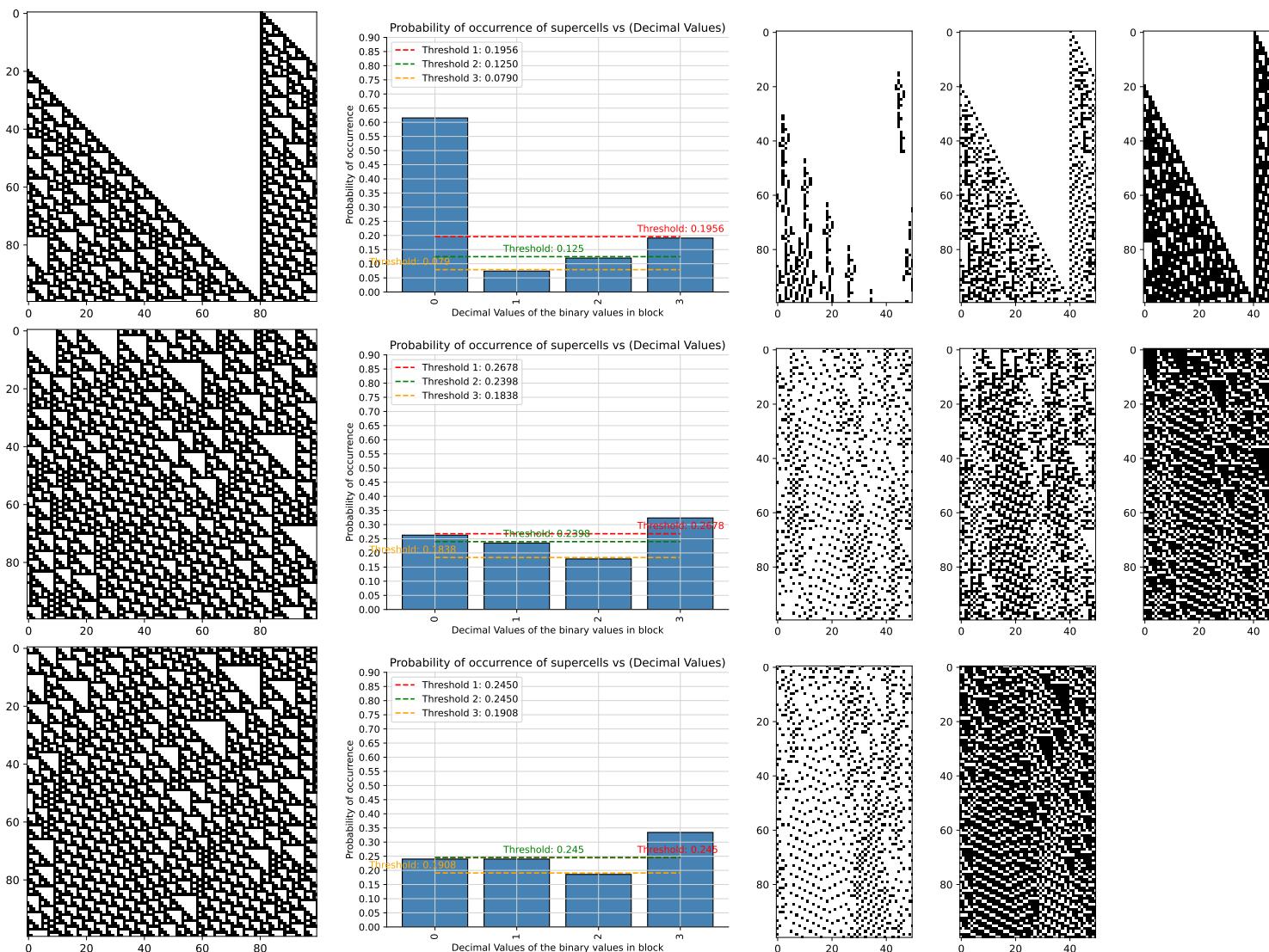


Table 63: FHCG plots for ECA Rule 122.

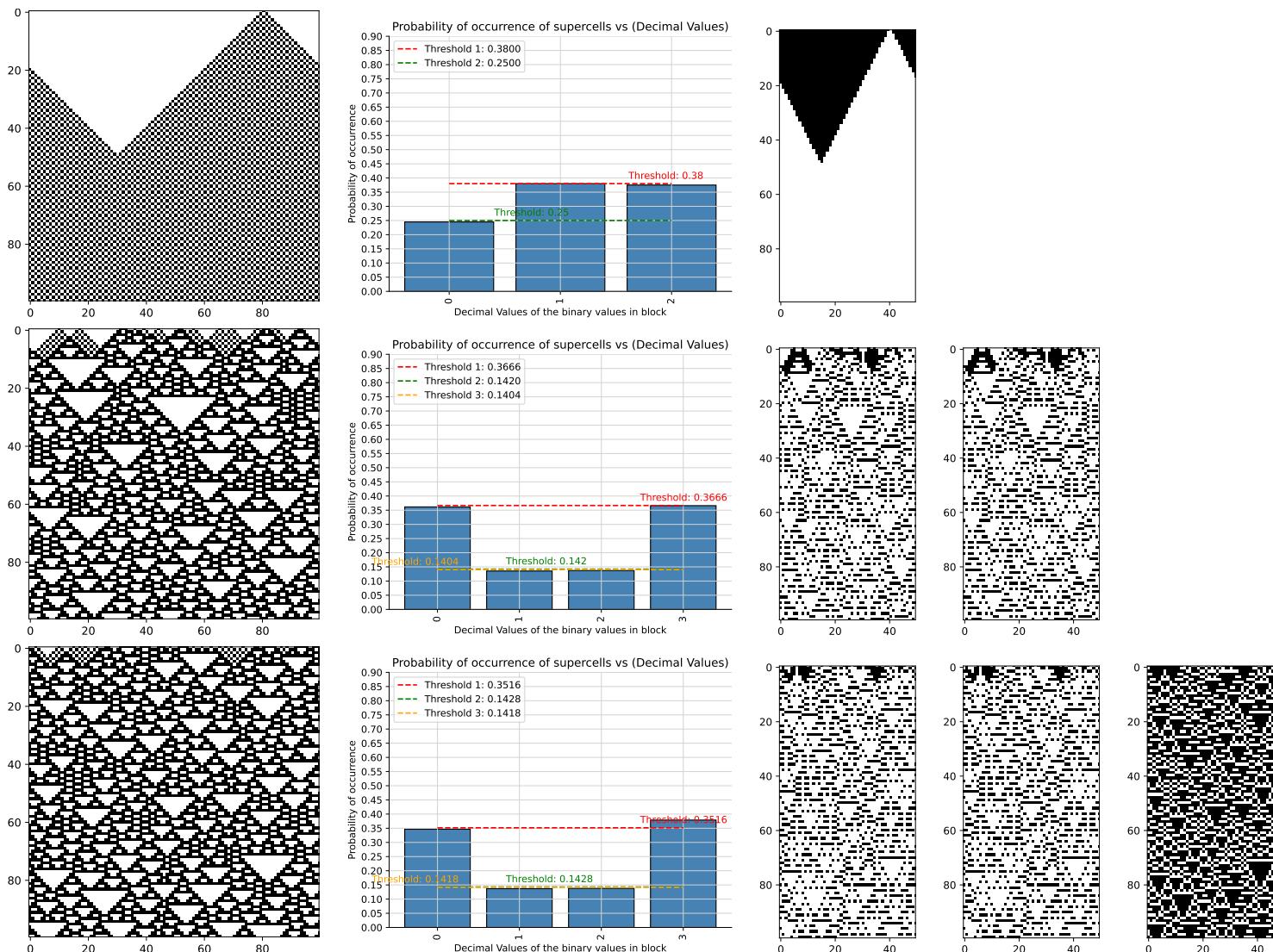


Table 64: FHCG plots for ECA Rule 126.

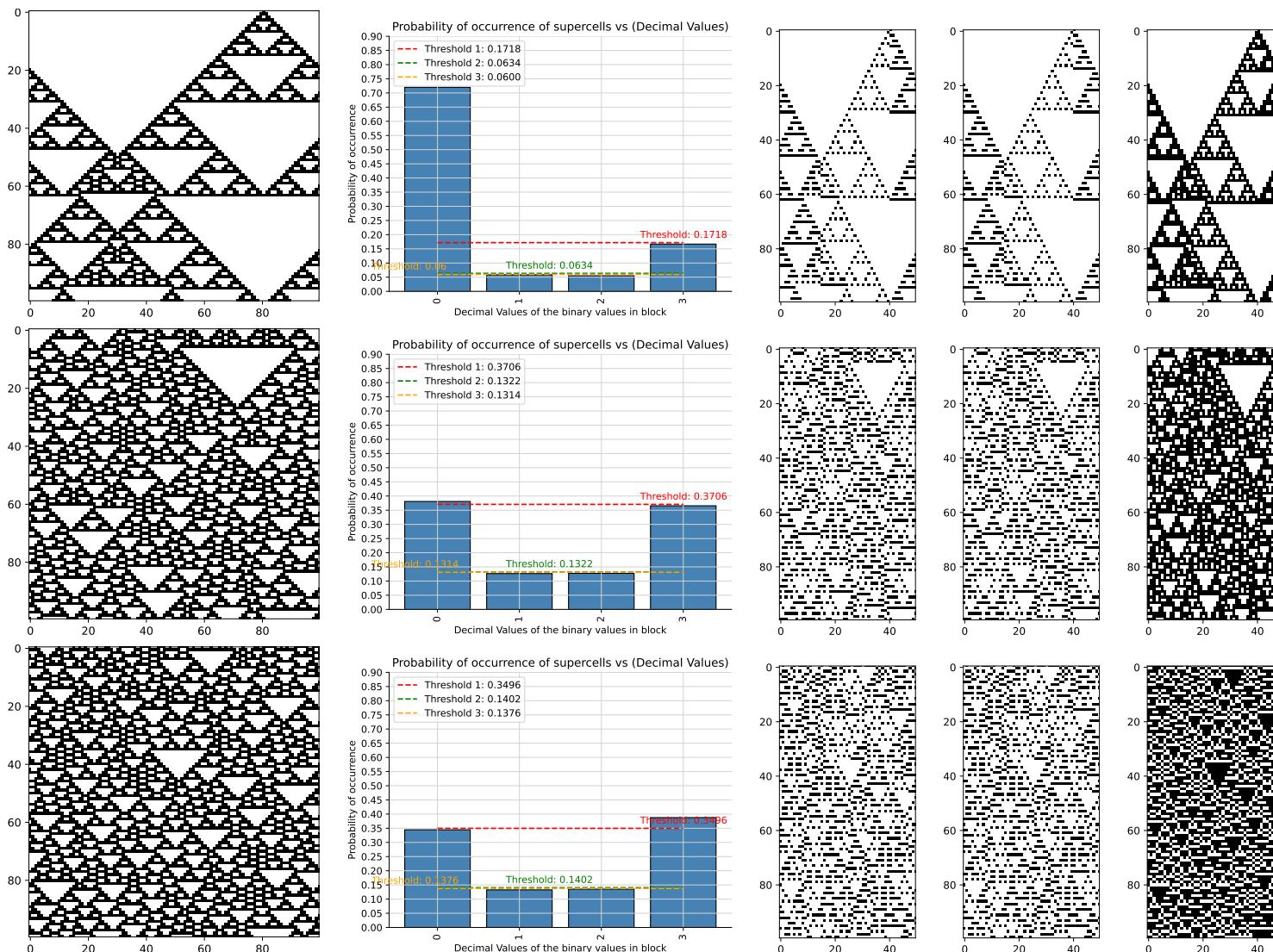


Table 65: FHCG plots for ECA Rule 128.

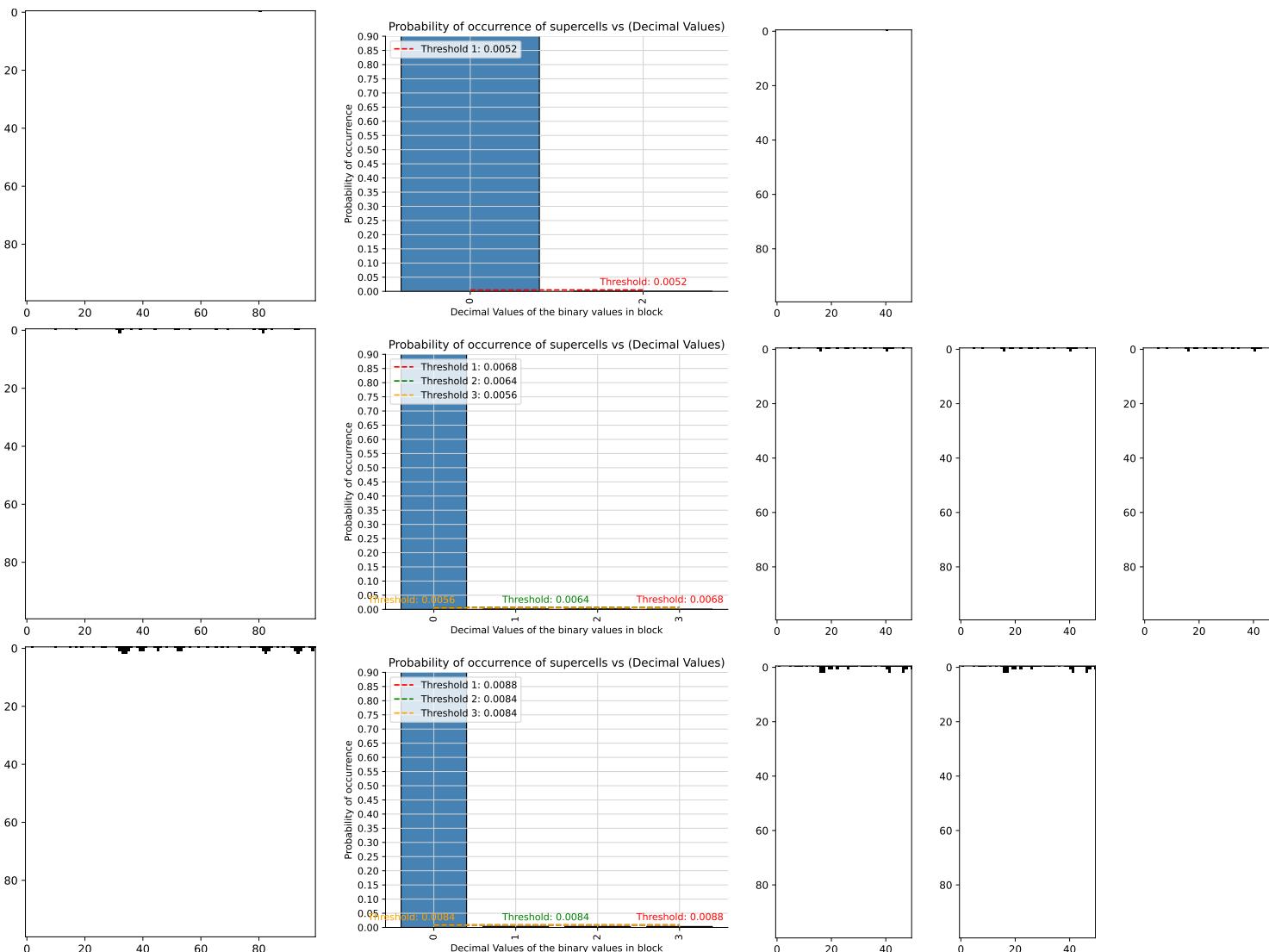


Table 66: FHCG plots for ECA Rule 130.

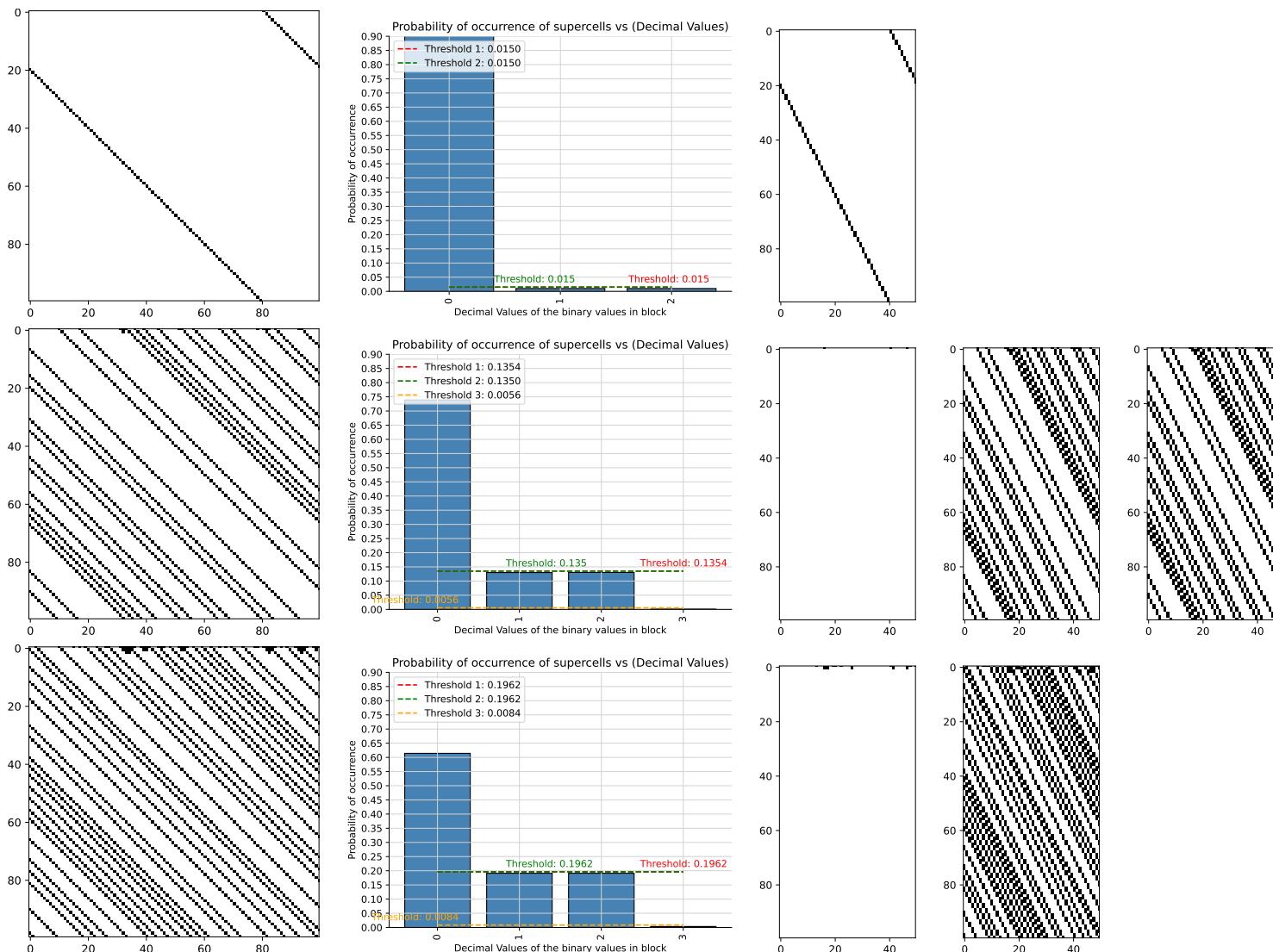
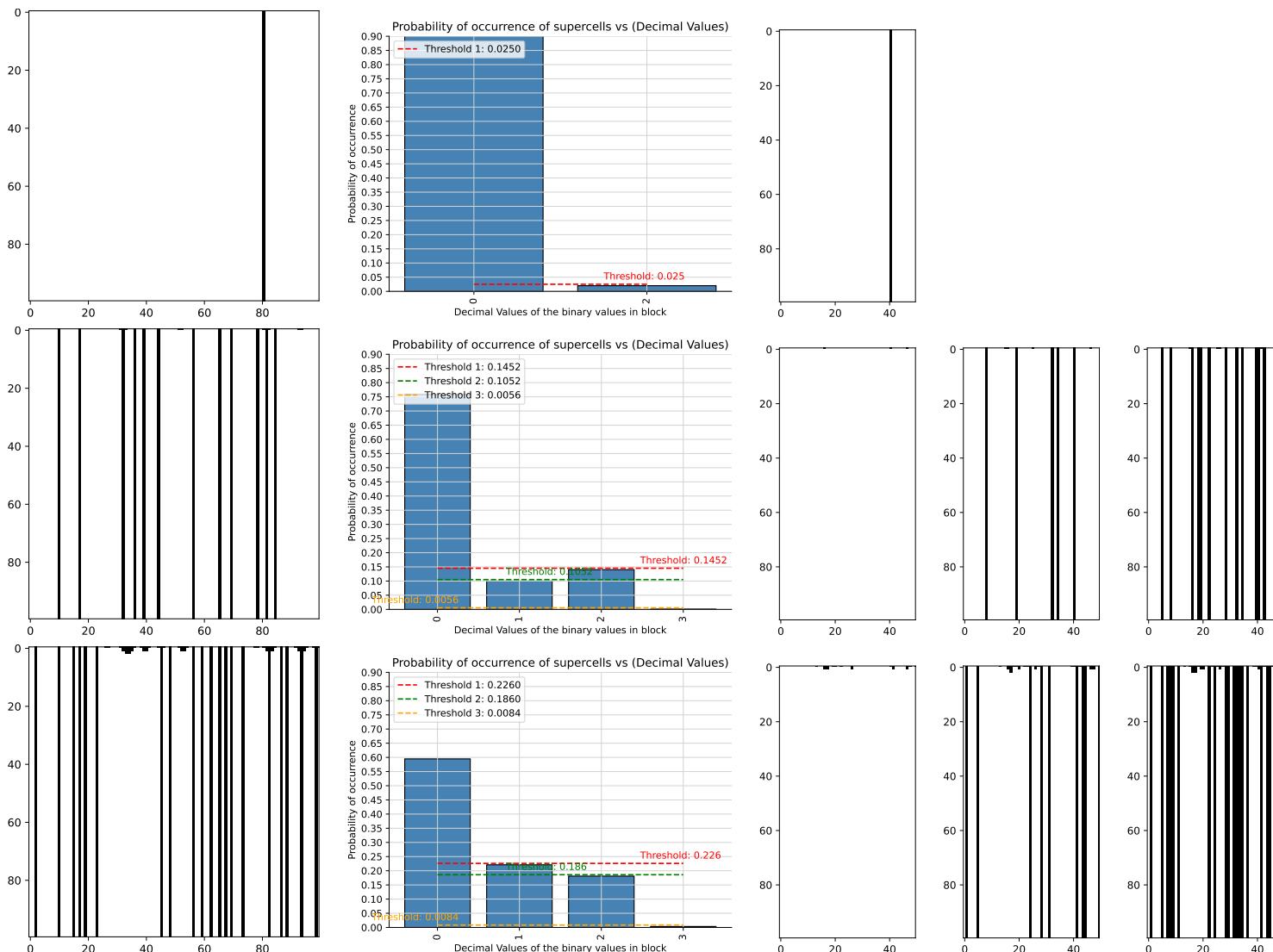


Table 67: FHCG plots for ECA Rule 132.



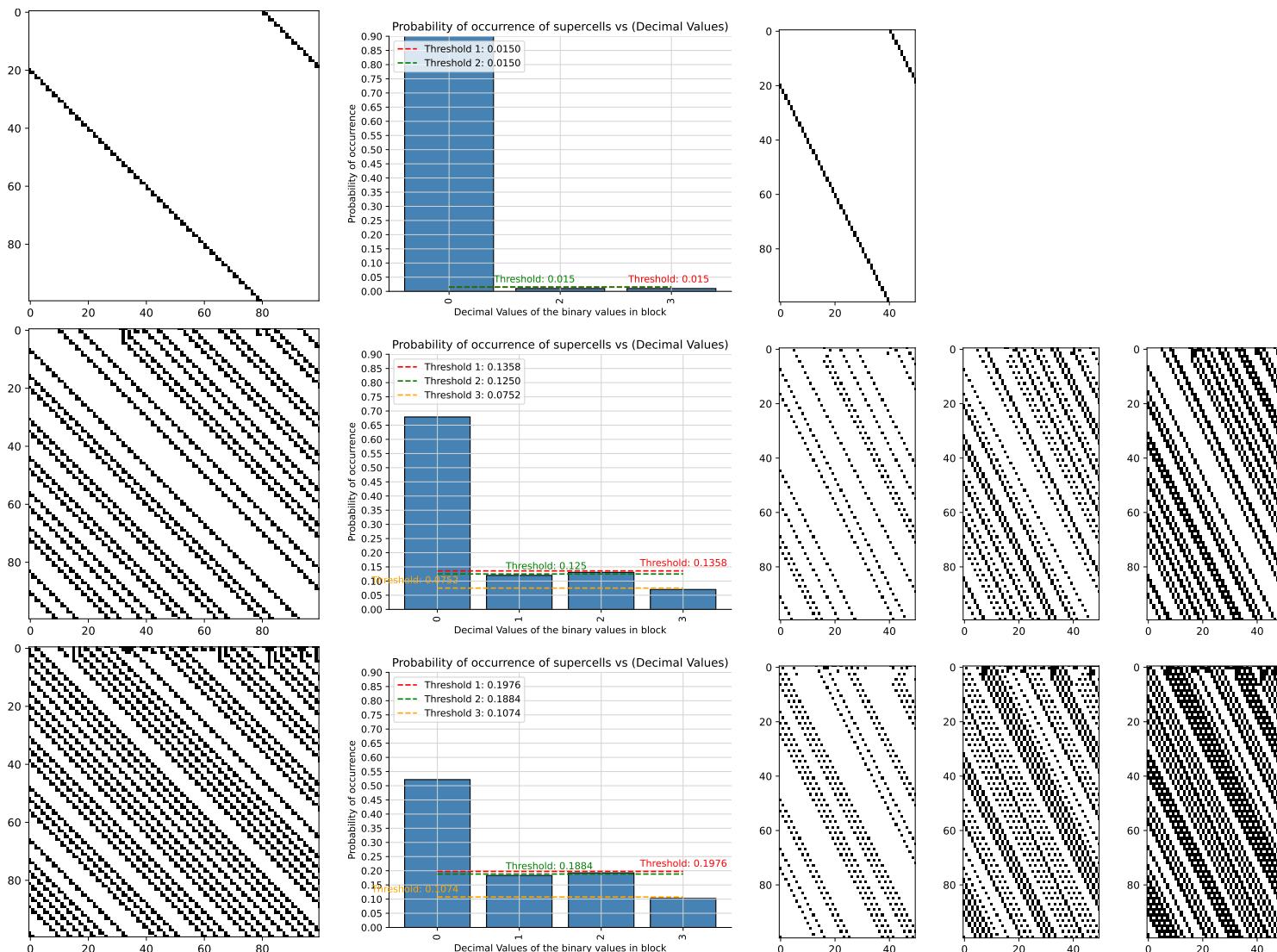


Table 69: FHCG plots for ECA Rule 136.

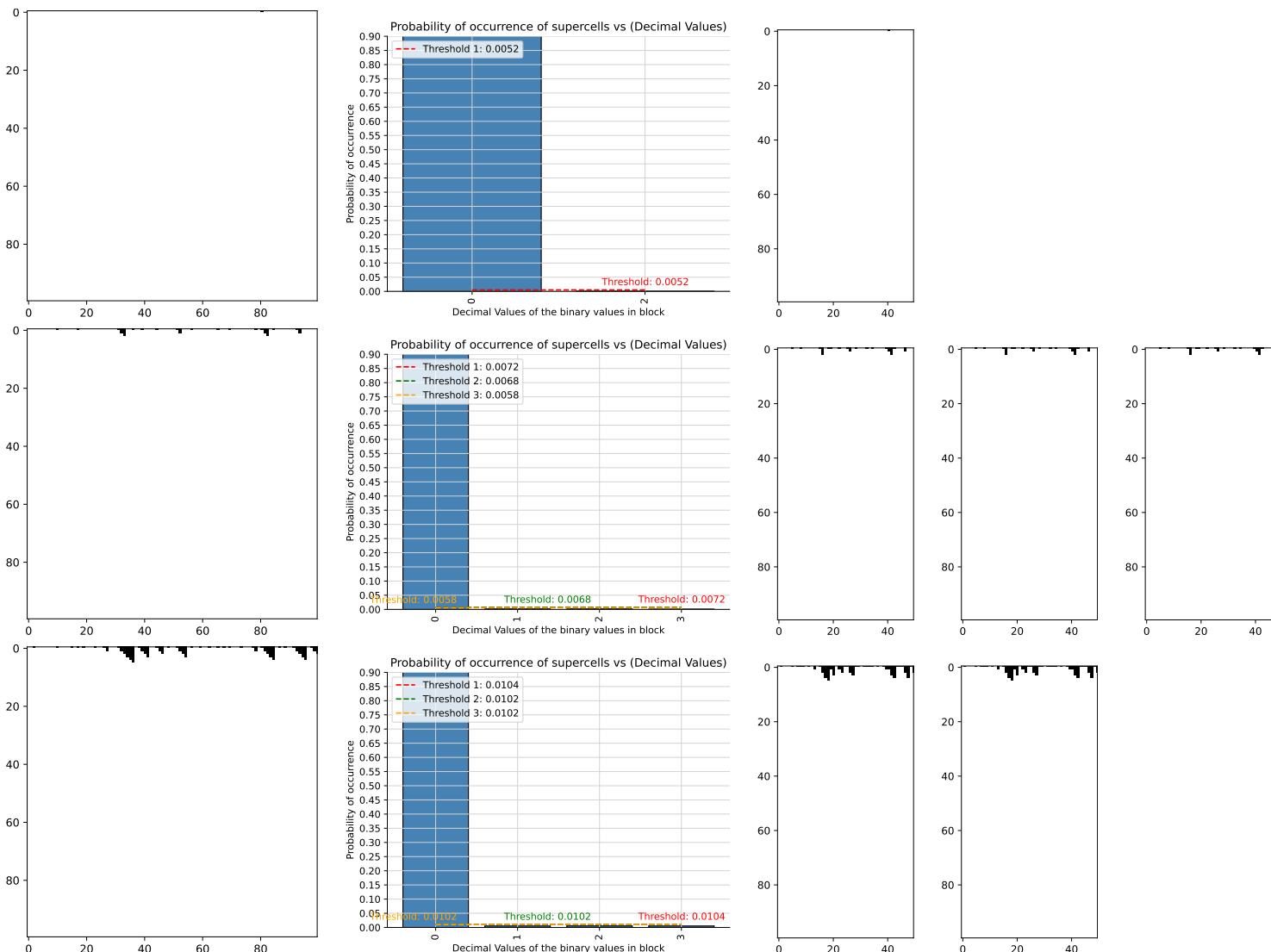


Table 70: FHCG plots for ECA Rule 138.

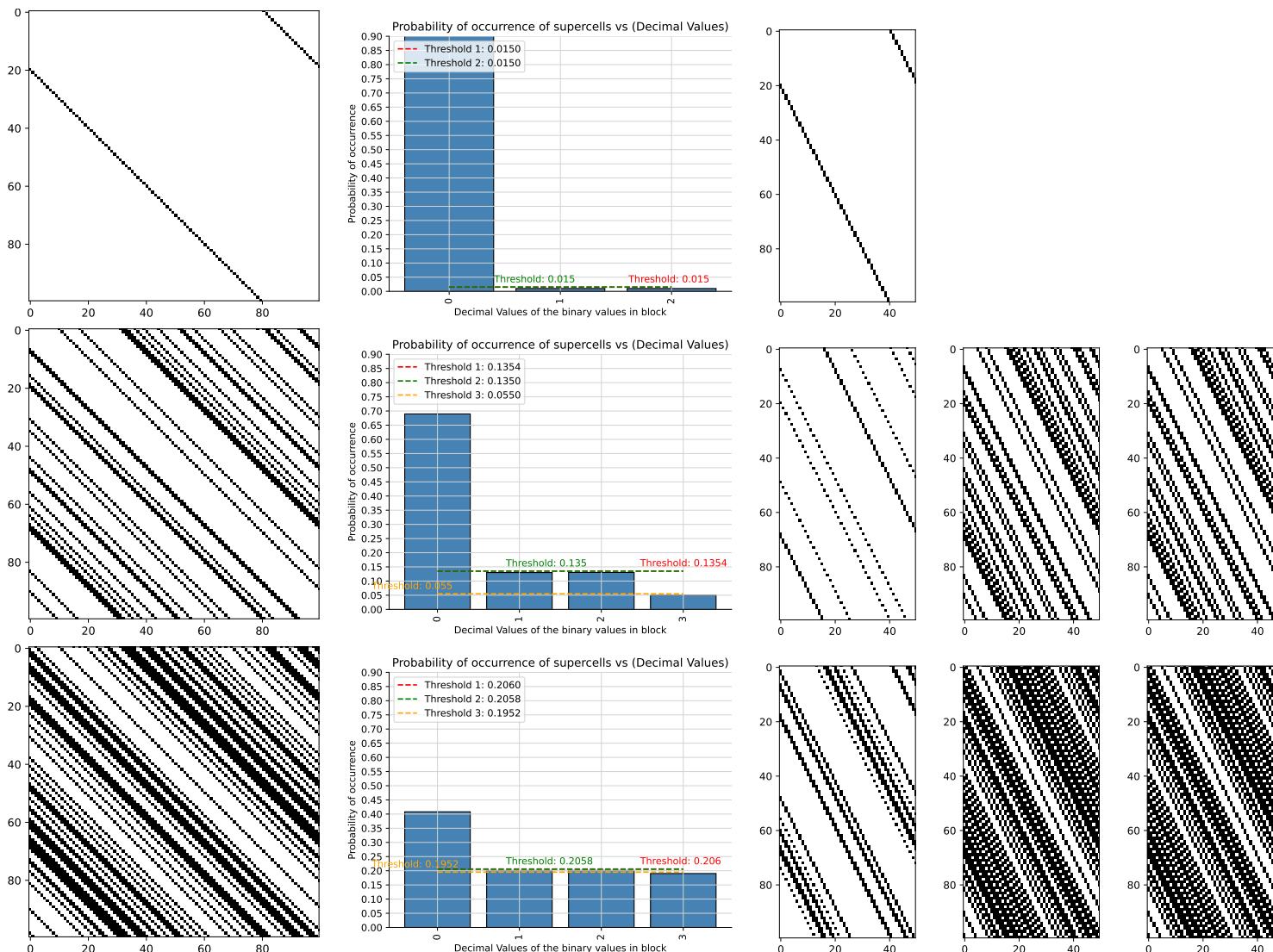


Table 71: FHCG plots for ECA Rule 140.

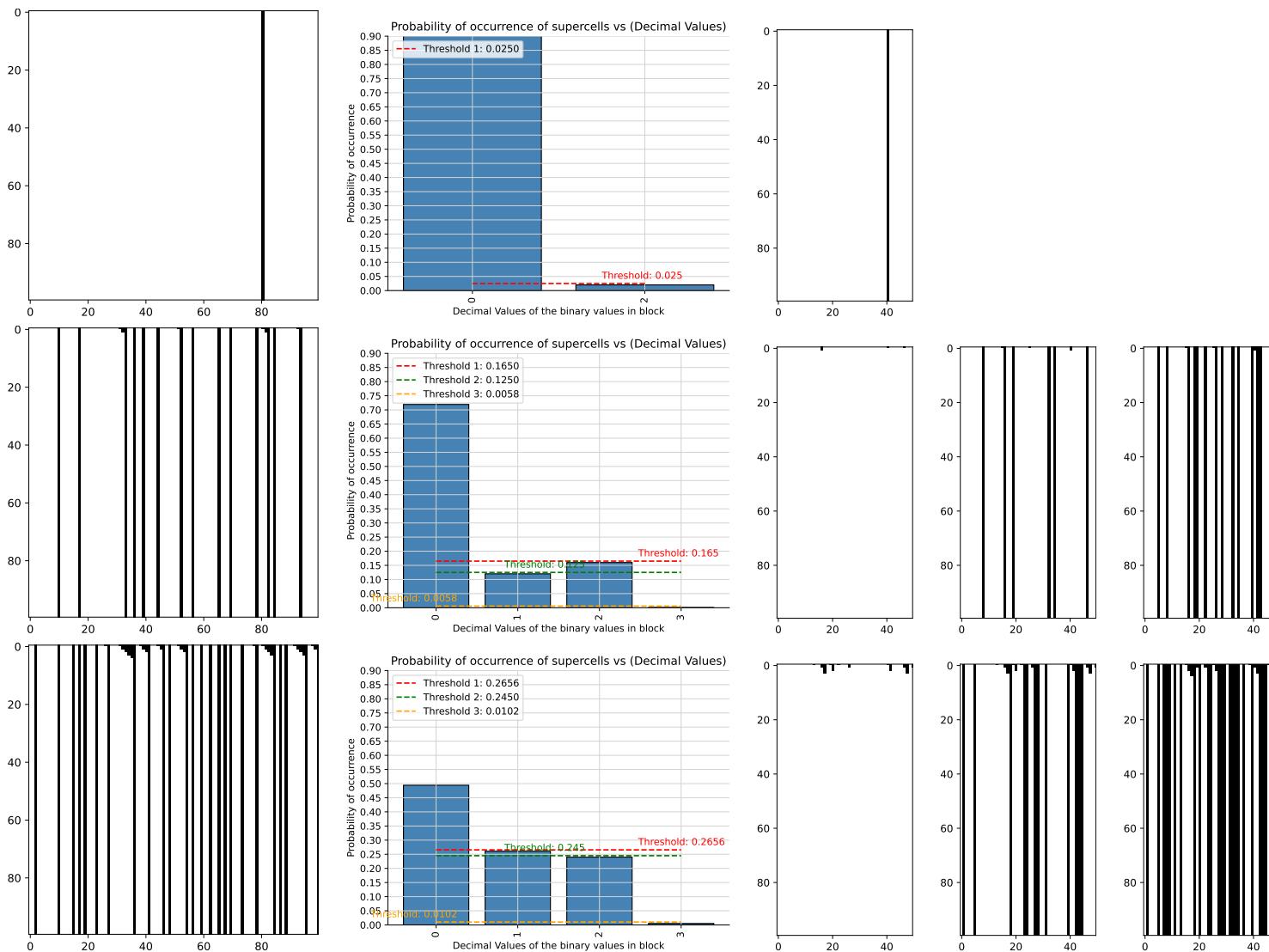


Table 72: FHCG plots for ECA Rule 142.

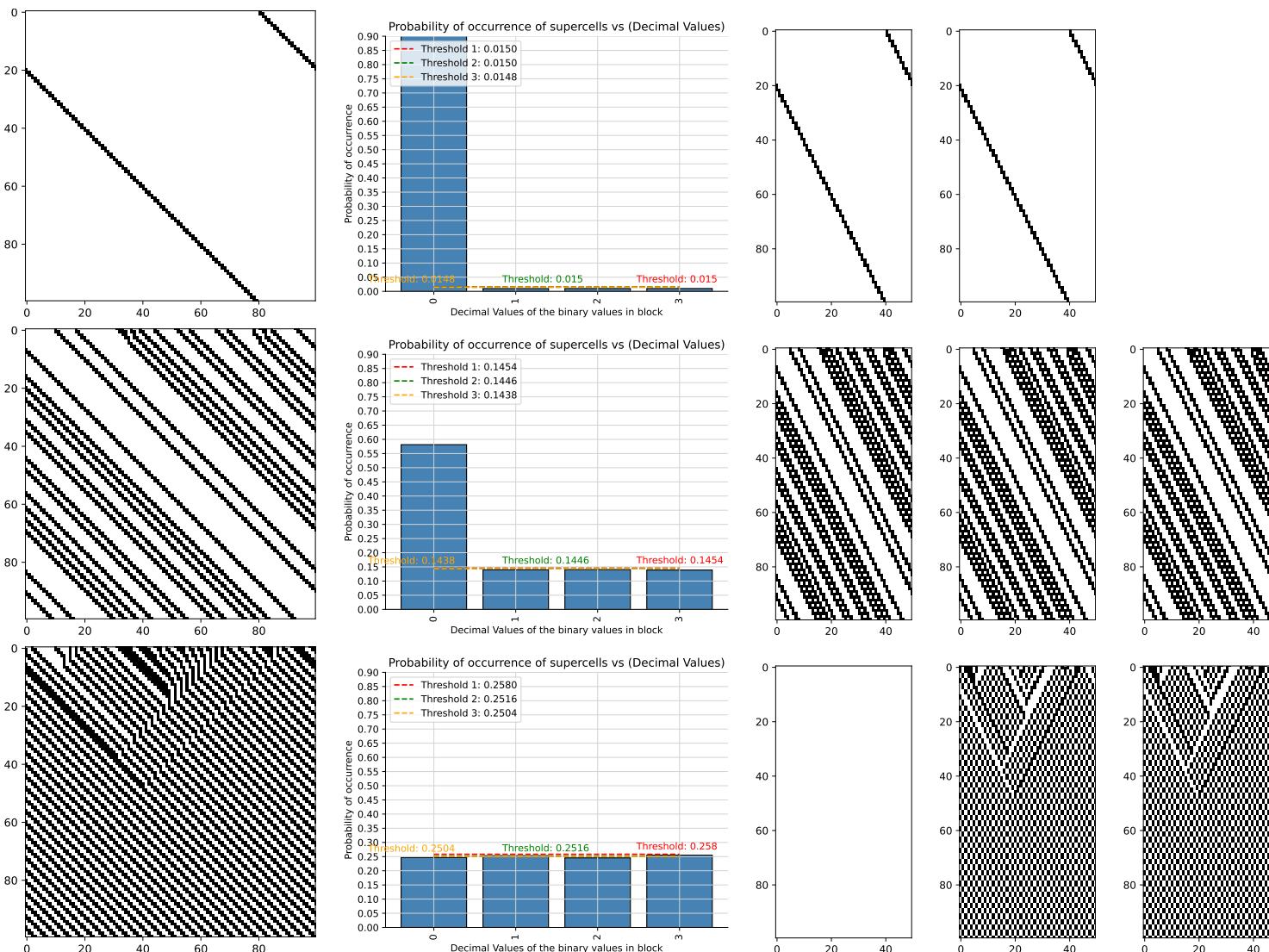


Table 73: FHCG plots for ECA Rule 146.

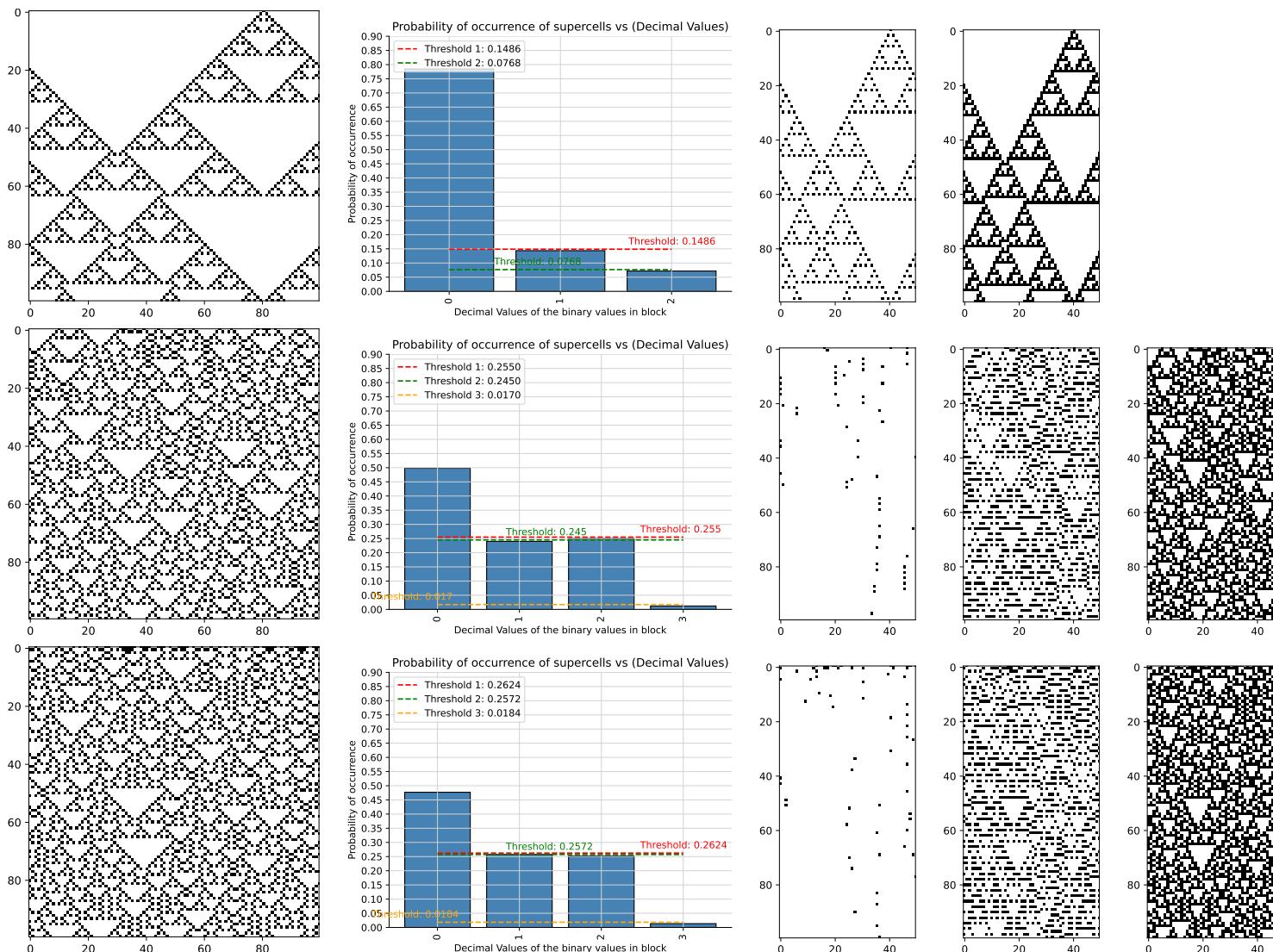


Table 74: FHCG plots for ECA Rule 150.

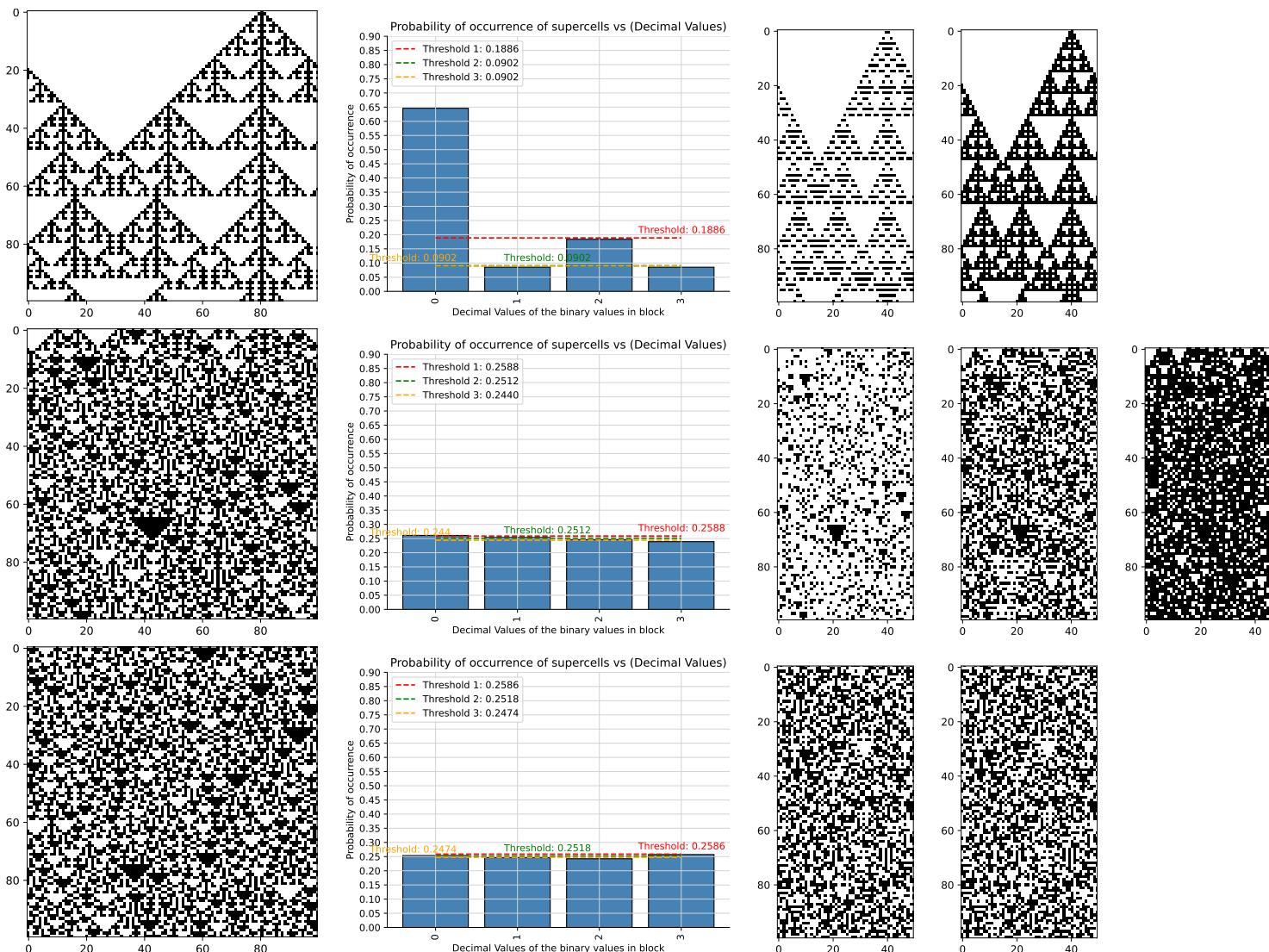


Table 75: FHCG plots for ECA Rule 152.

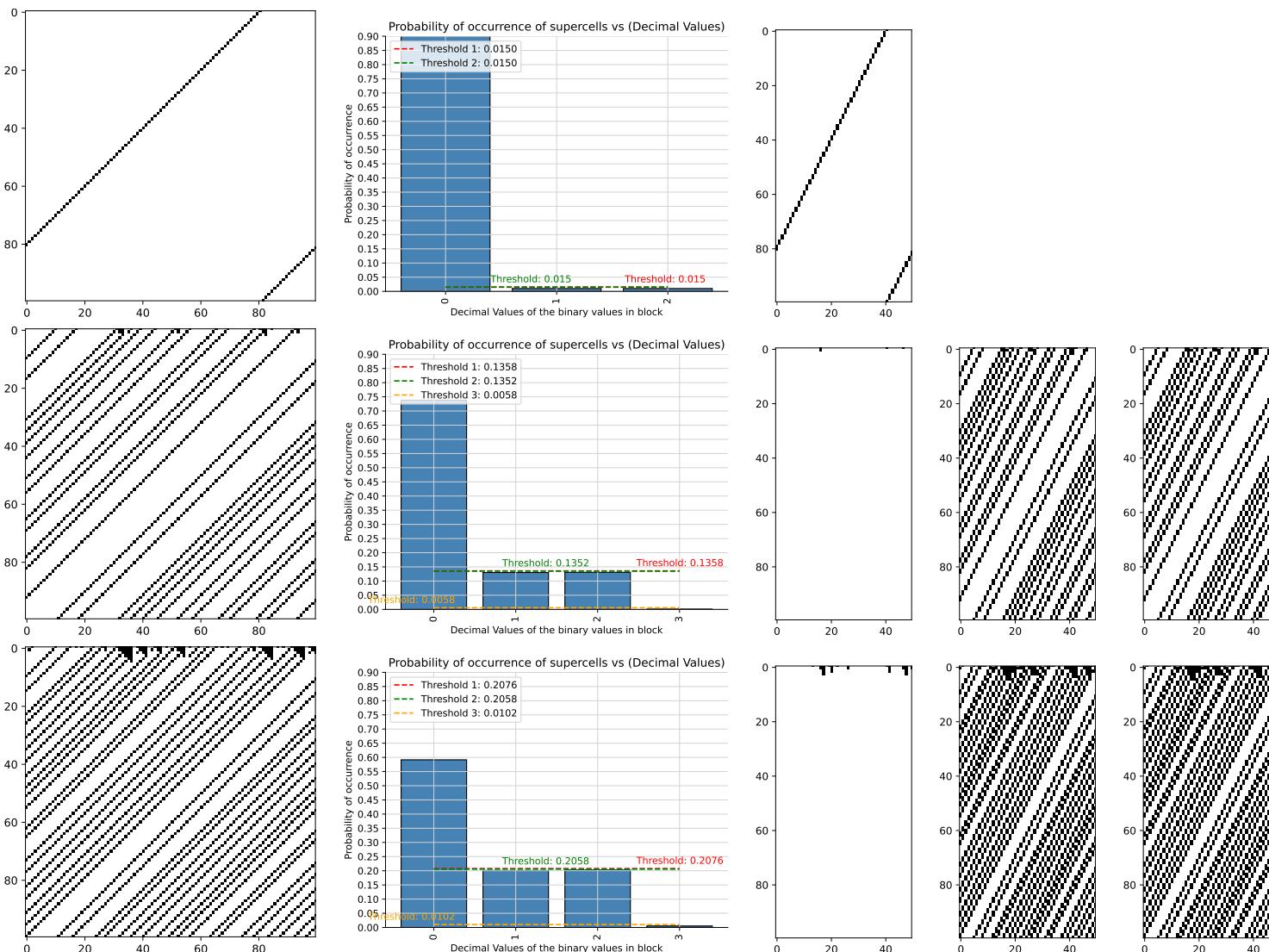


Table 76: FHCG plots for ECA Rule 154.

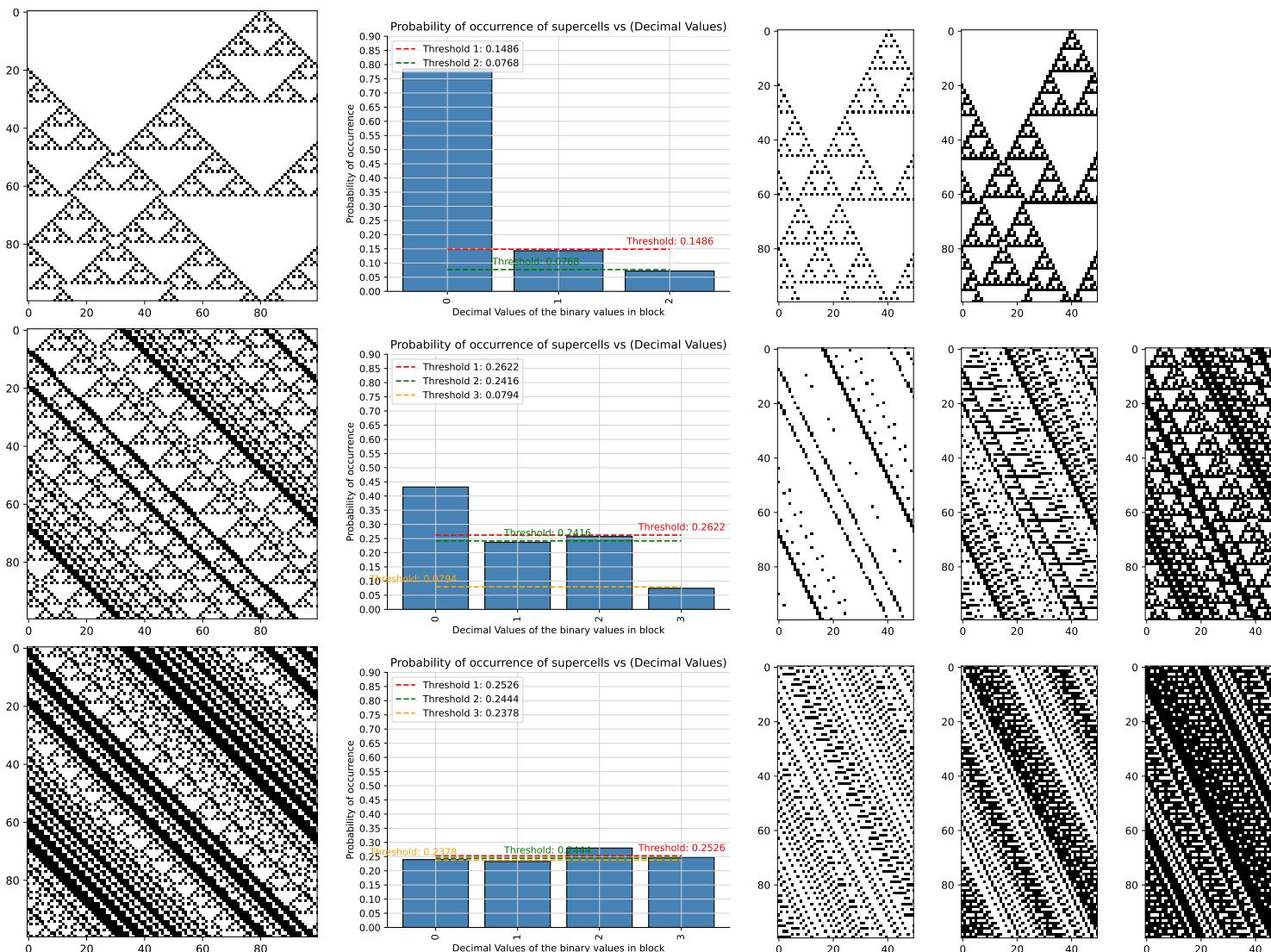


Table 77: FHCG plots for ECA Rule 156.

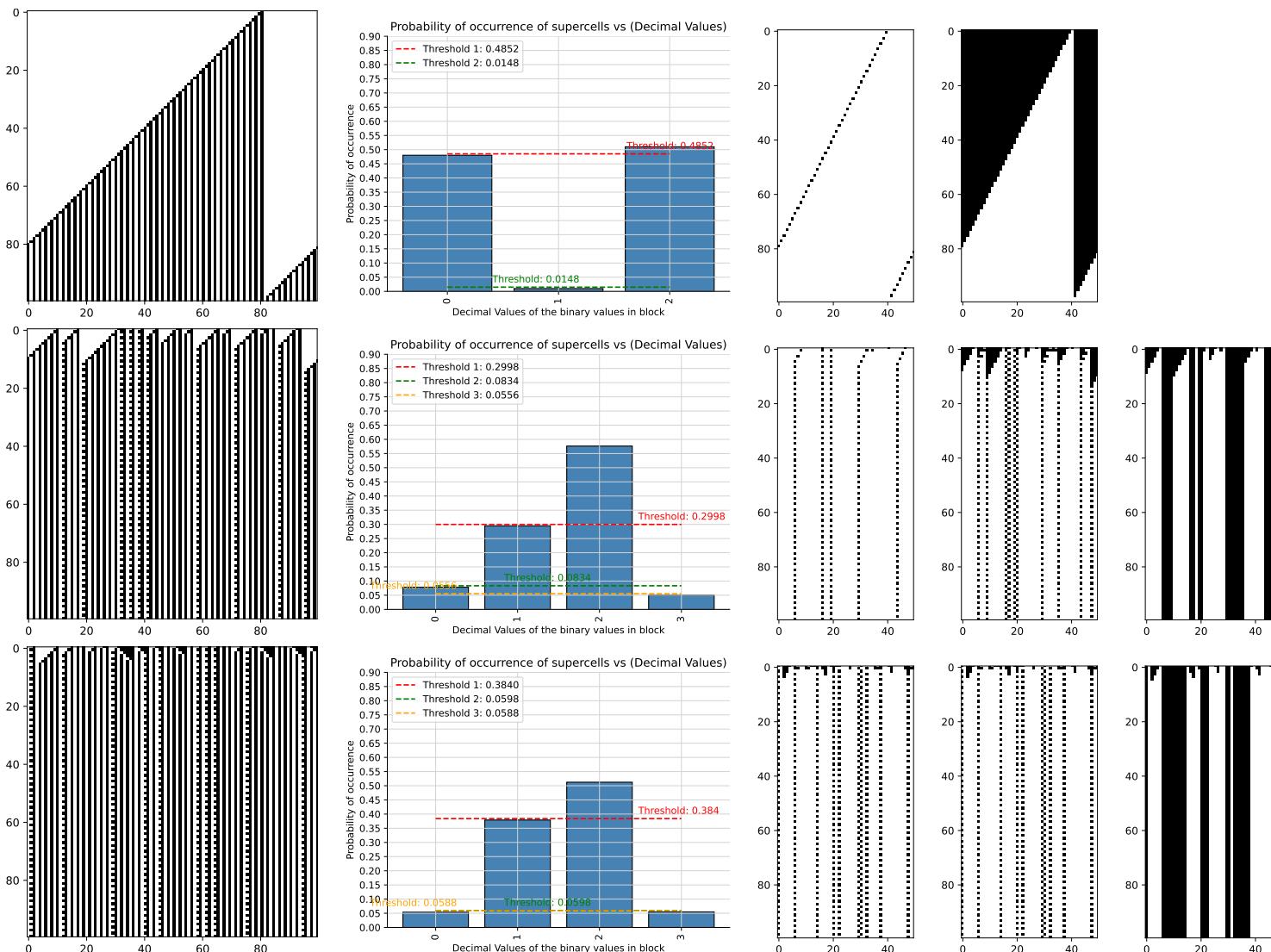


Table 78: FHCG plots for ECA Rule 160.

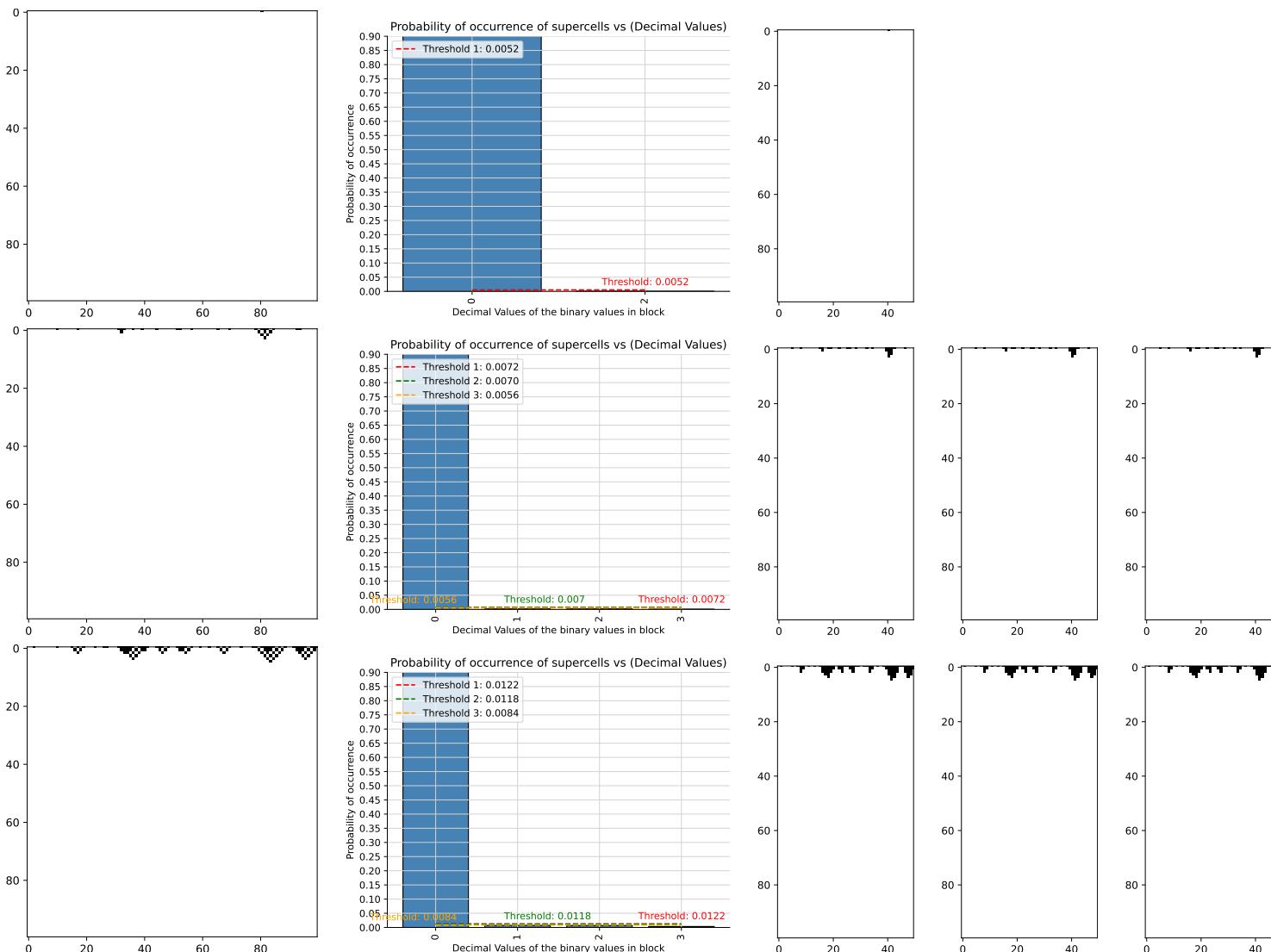


Table 79: FHCG plots for ECA Rule 162.

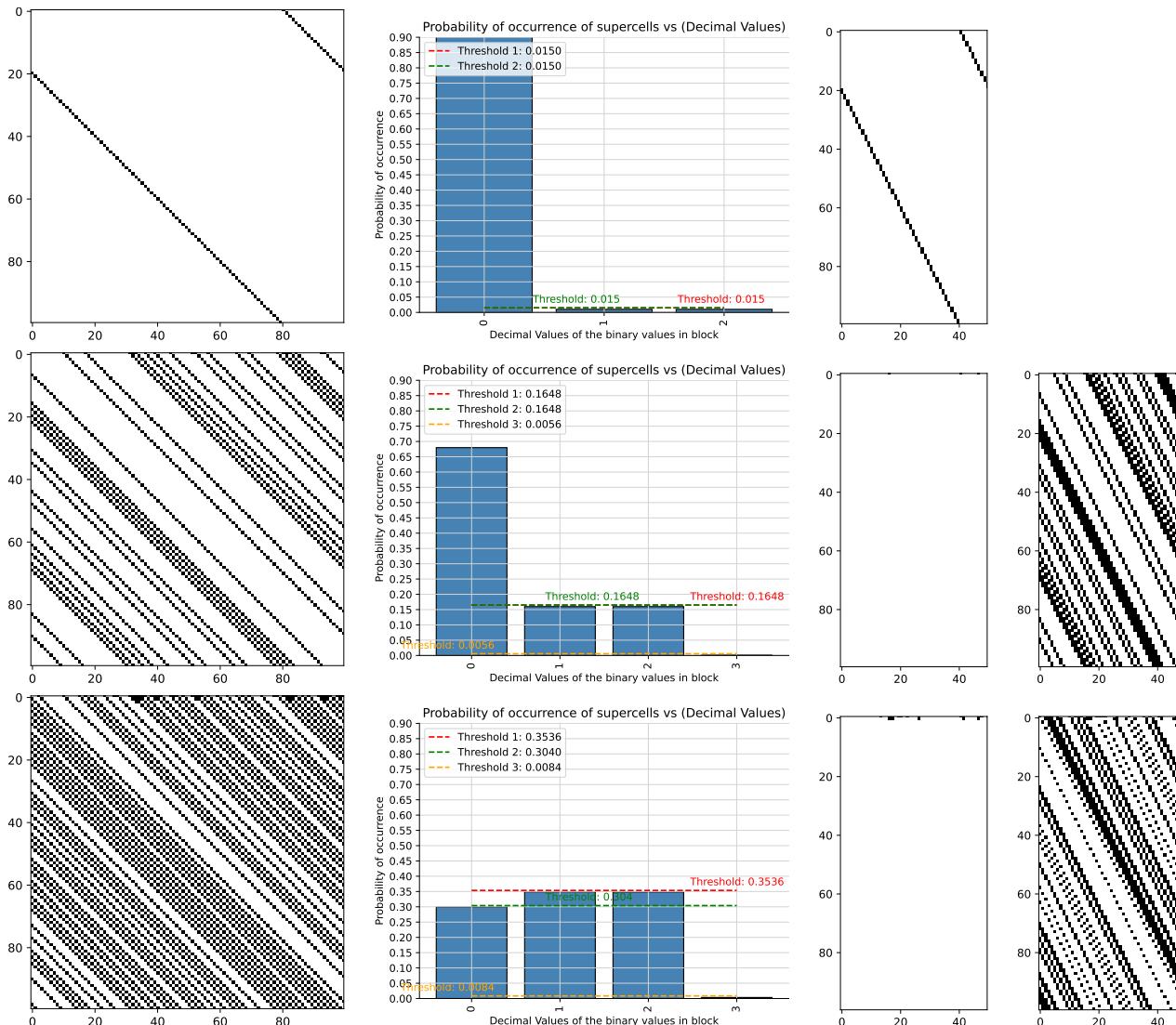


Table 80: FHCG plots for ECA Rule 164.

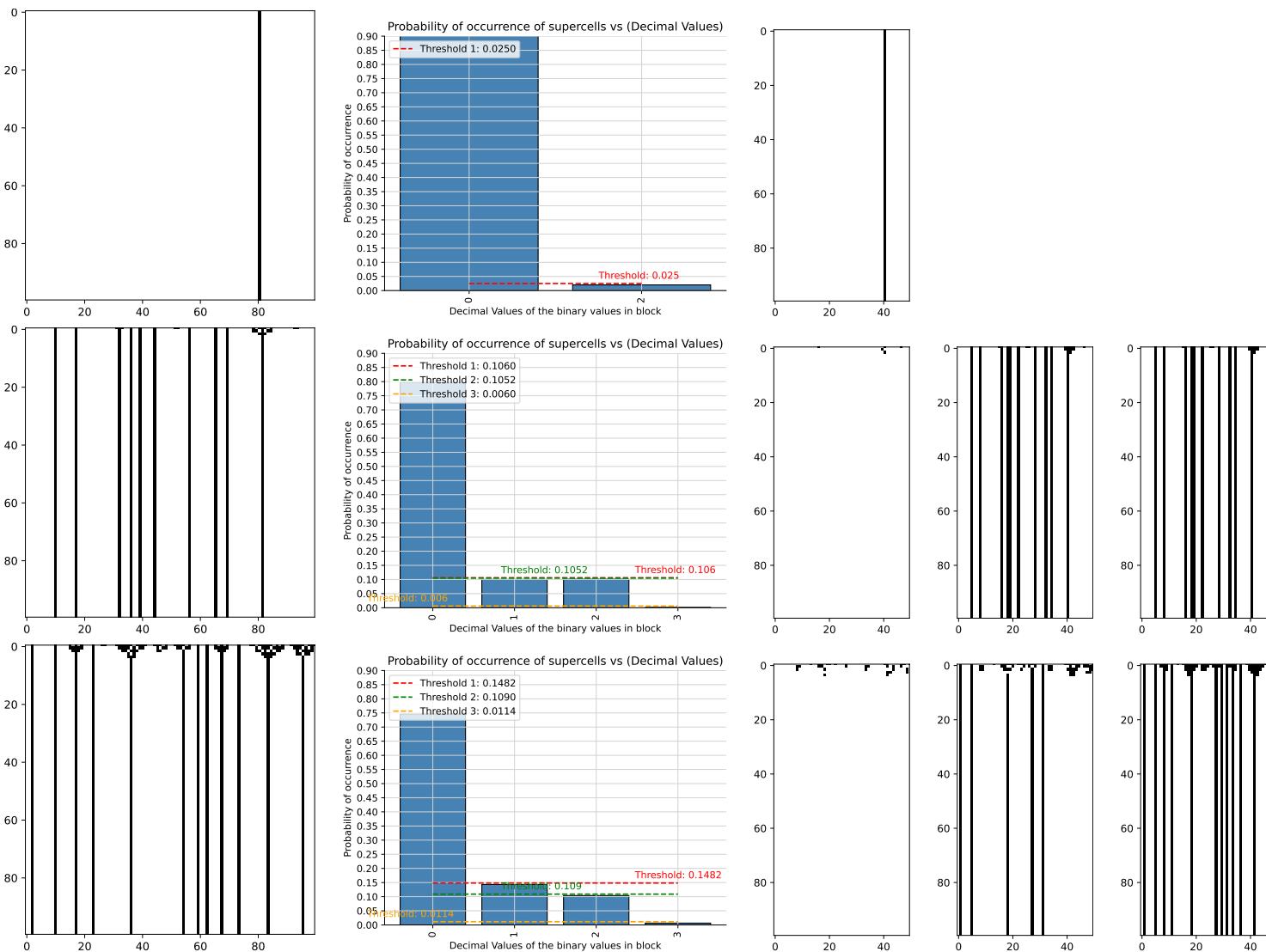


Table 81: FHCG plots for ECA Rule 168.

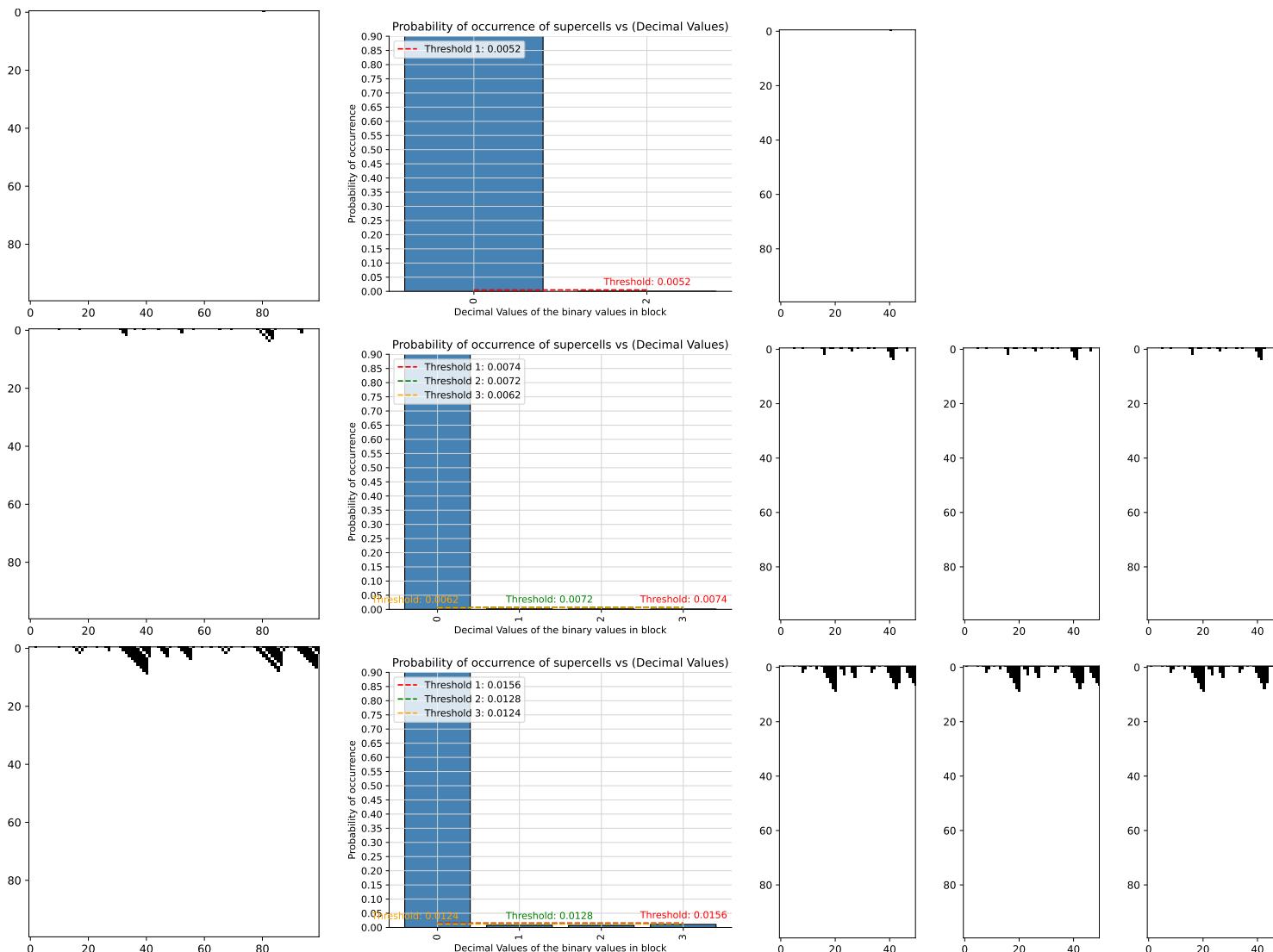


Table 82: FHCG plots for ECA Rule 170.

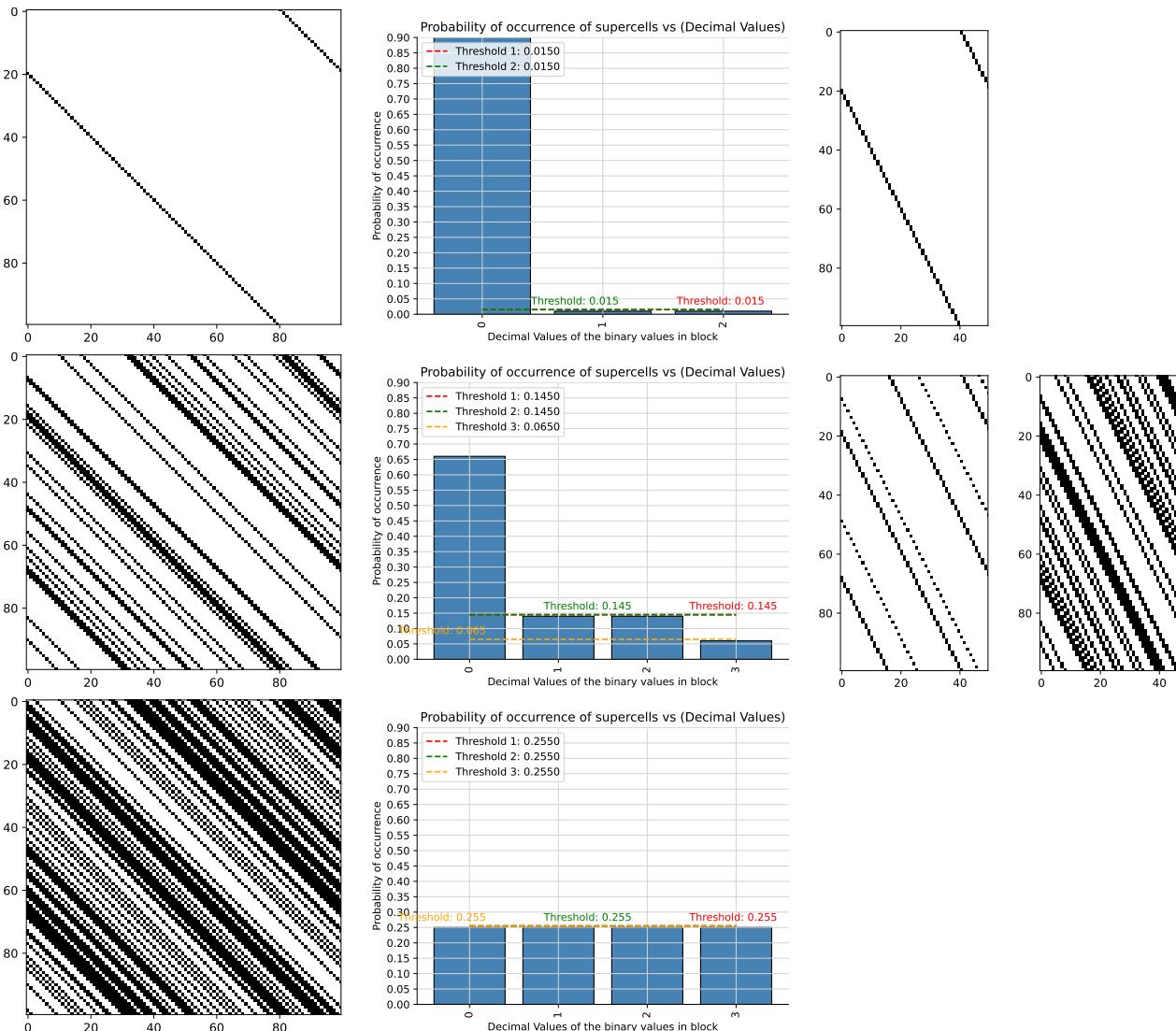


Table 83: FHCG plots for ECA Rule 172.

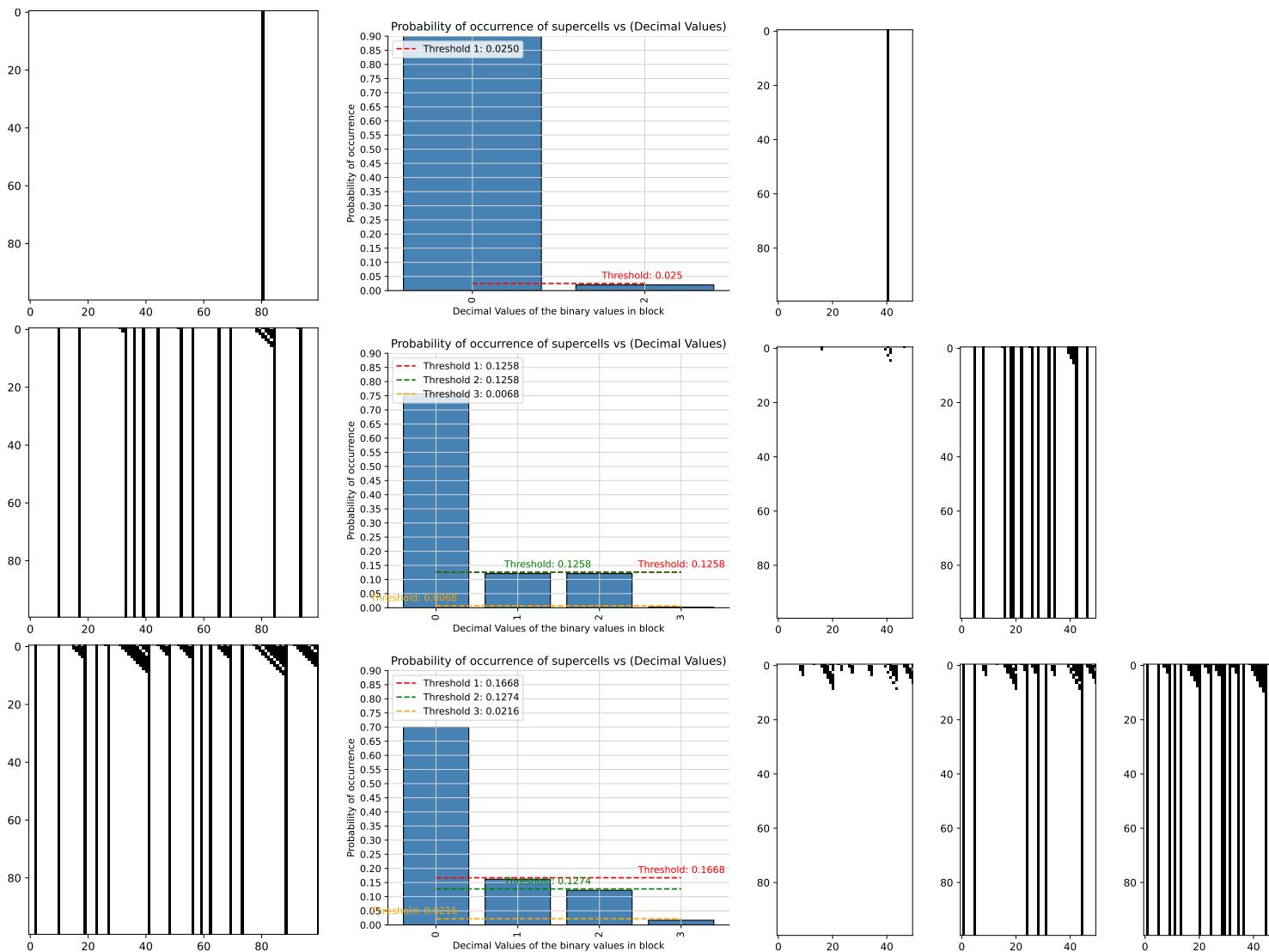


Table 84: FHCG plots for ECA Rule 178.

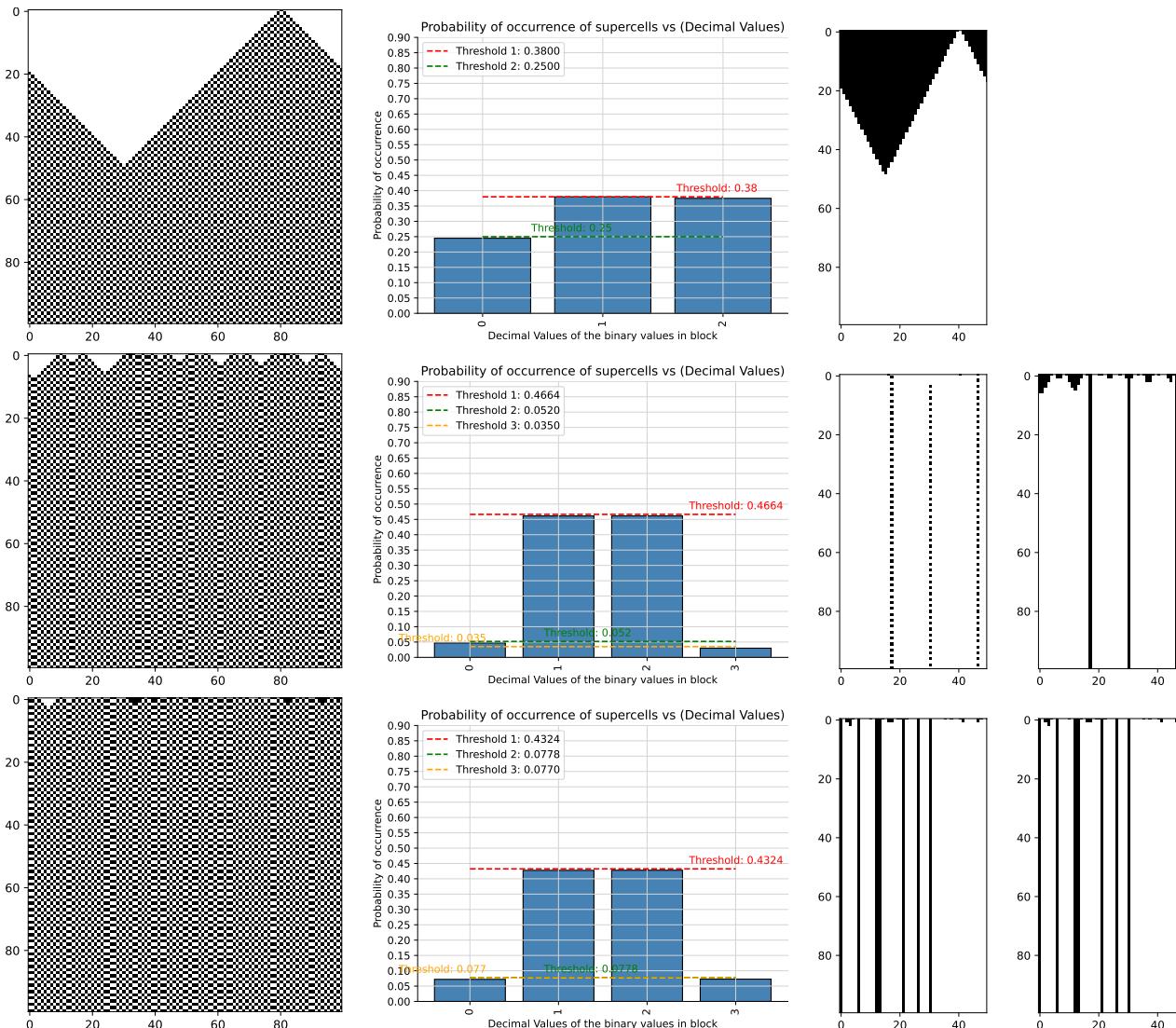


Table 85: FHCG plots for ECA Rule 184.

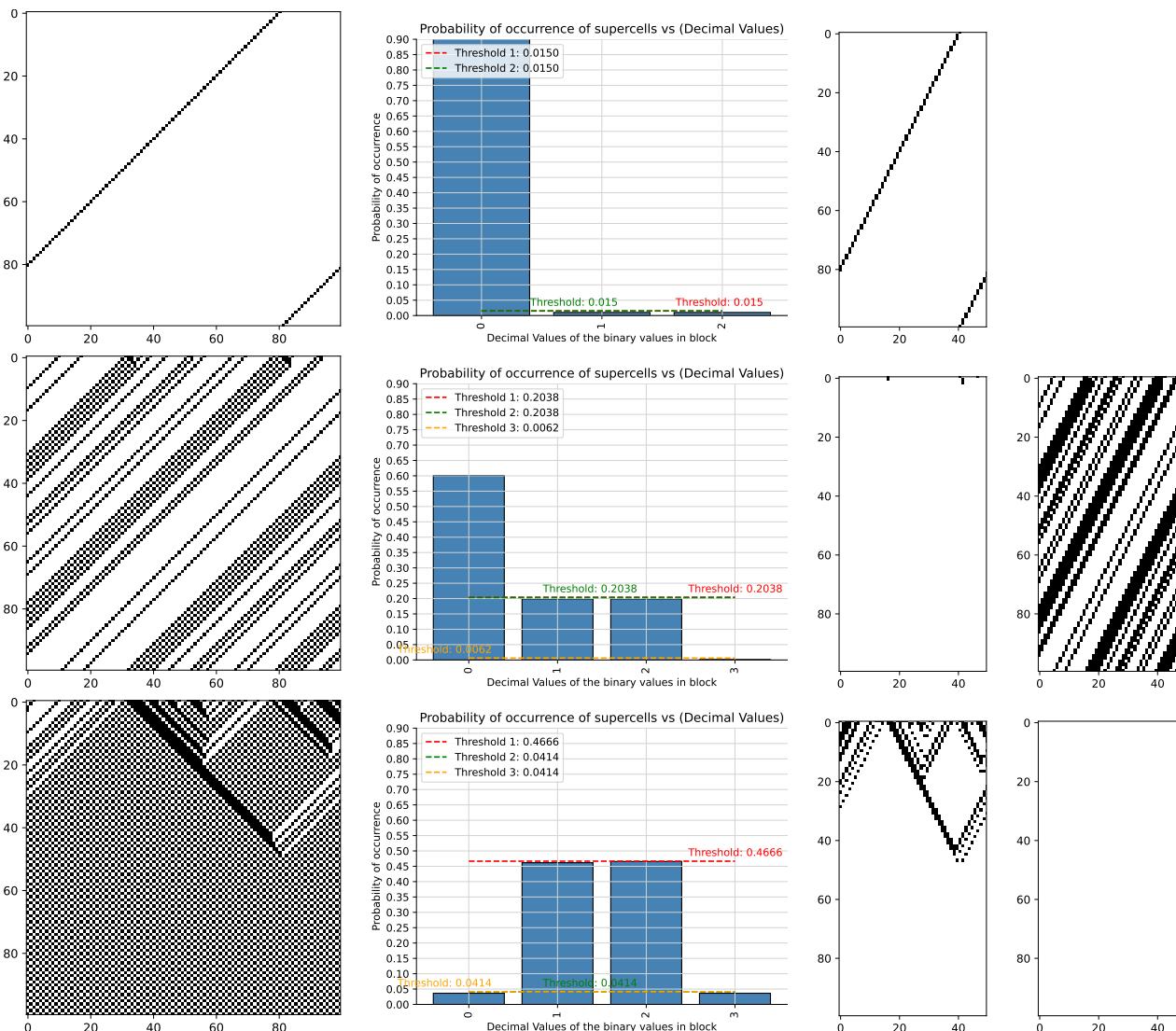


Table 86: FHCG plots for ECA Rule 200.

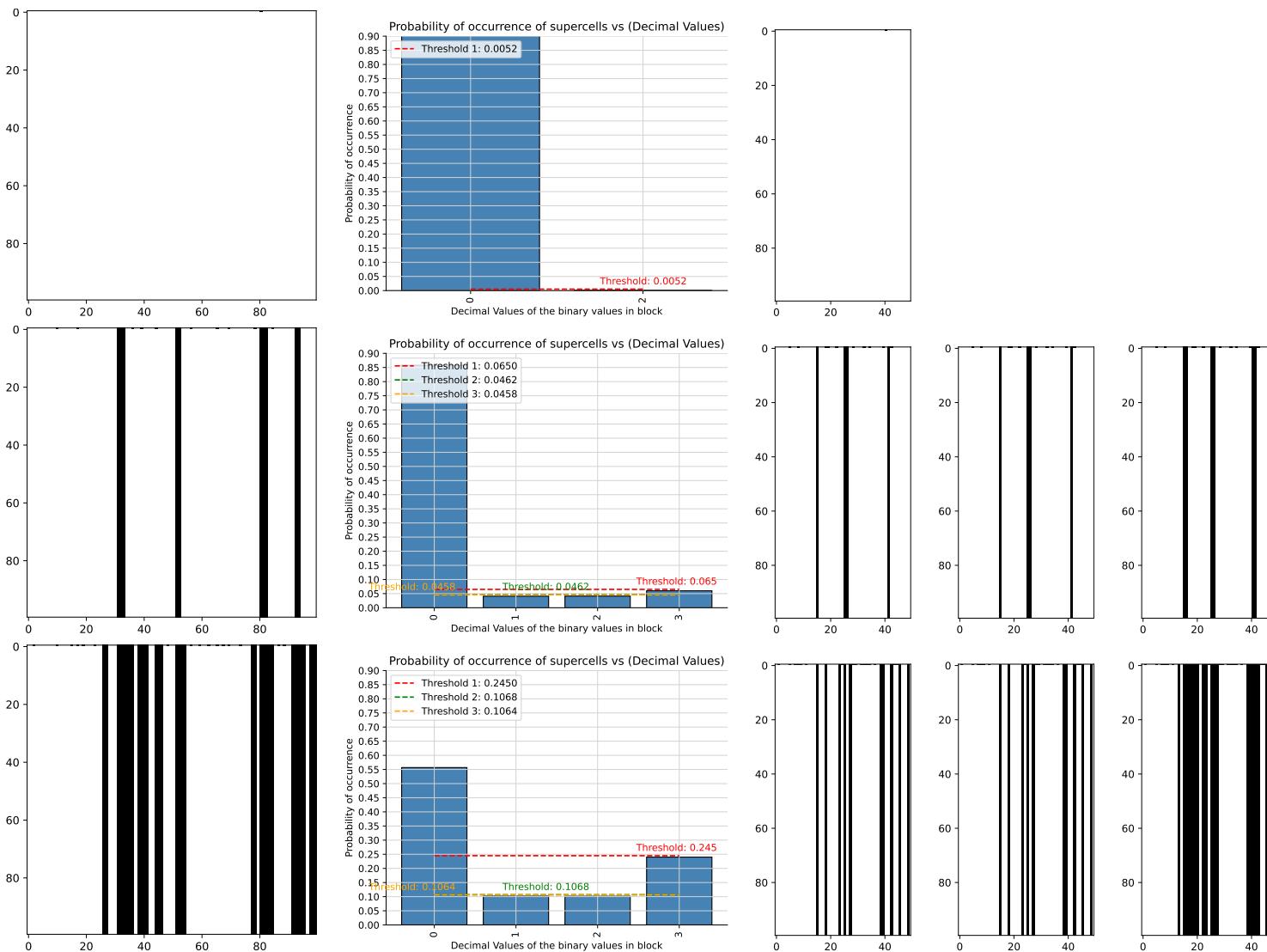


Table 87: FHCG plots for ECA Rule 204.

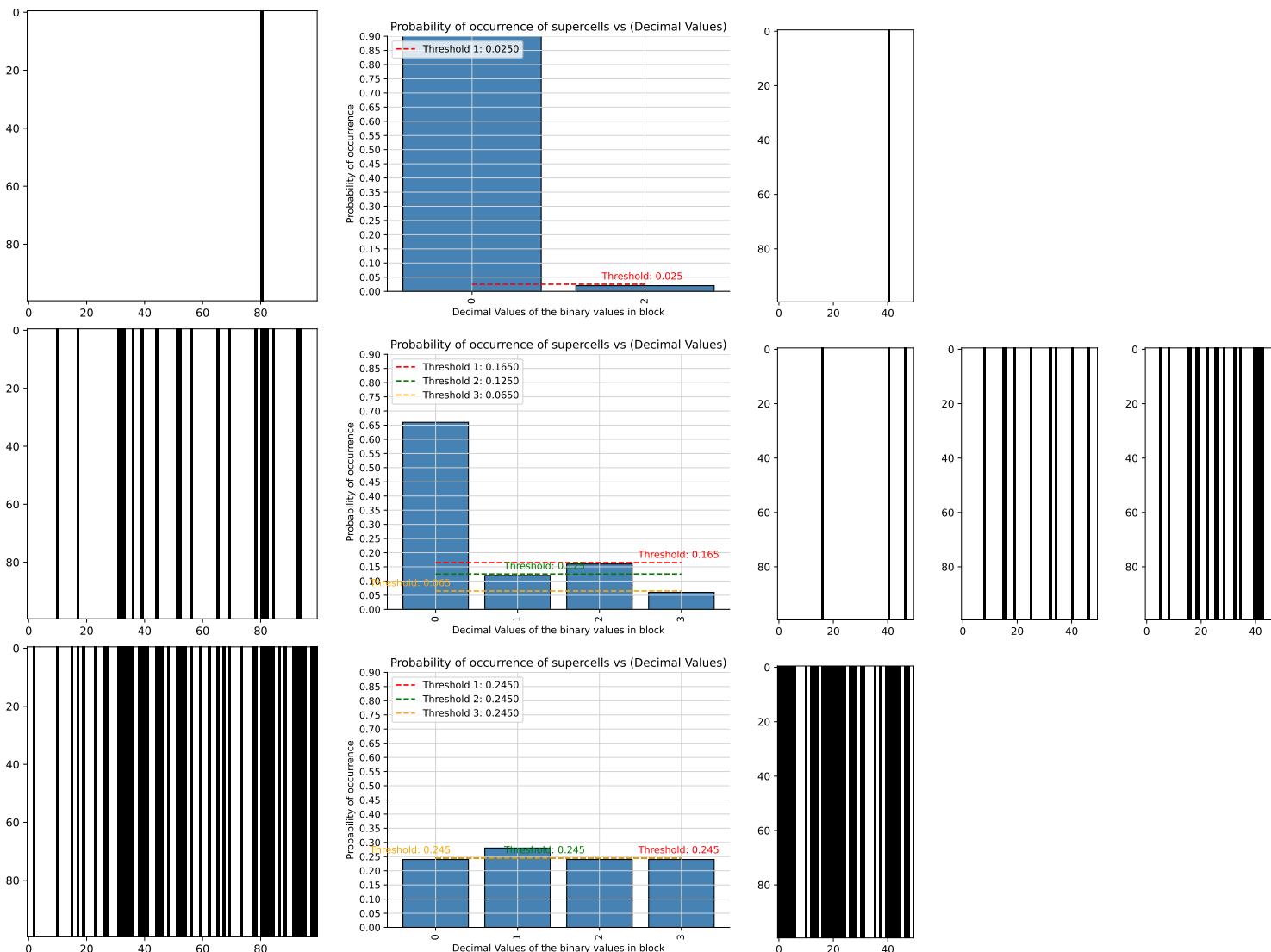


Table 88: FHCG plots for ECA Rule 232.

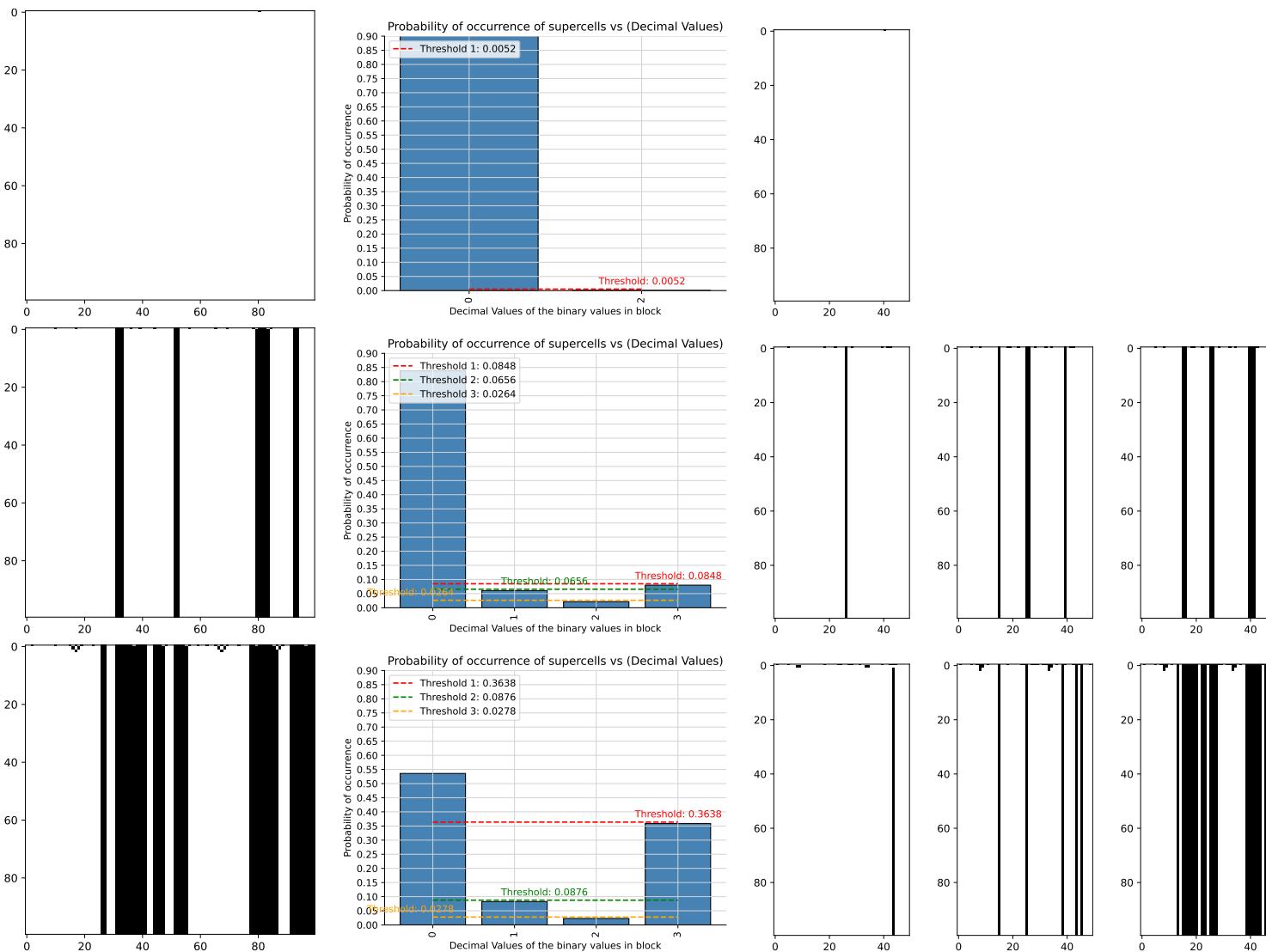
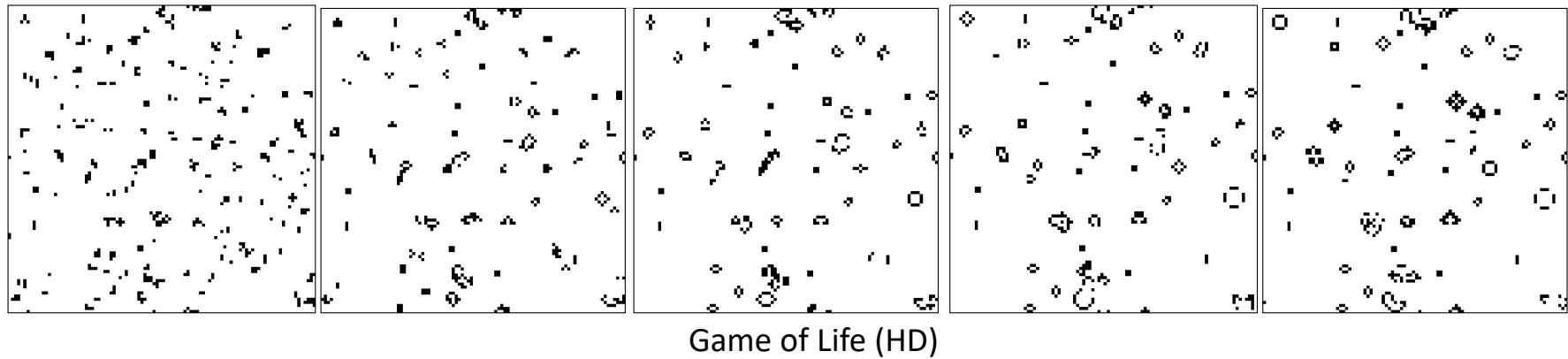


Table 89: Game of Life from MNCA with standard B3/S23 rule. The figure shows high definition (above) and FHCG (below). The chosen handpicked threshold for FHCG-Evo-MNCA is 0.011



Game of Life (HD)

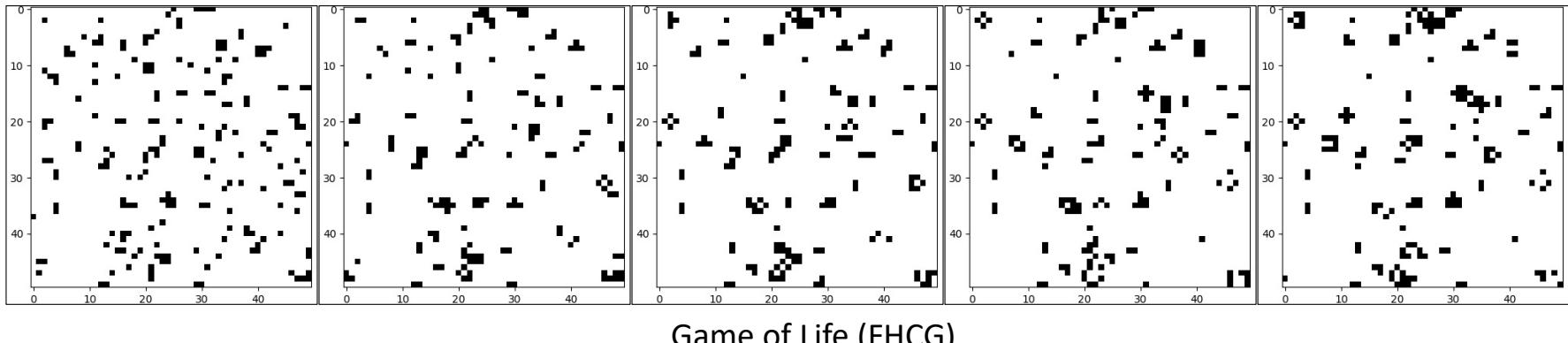


Table 90: Example of evolved rule from Evo-MNCA. The values of the evolved genotype are shown on top of the image. The figure shows high definition (above) and FHCG (below). The chosen handpicked threshold for FHCG-Evo-MNCA is 0.025

