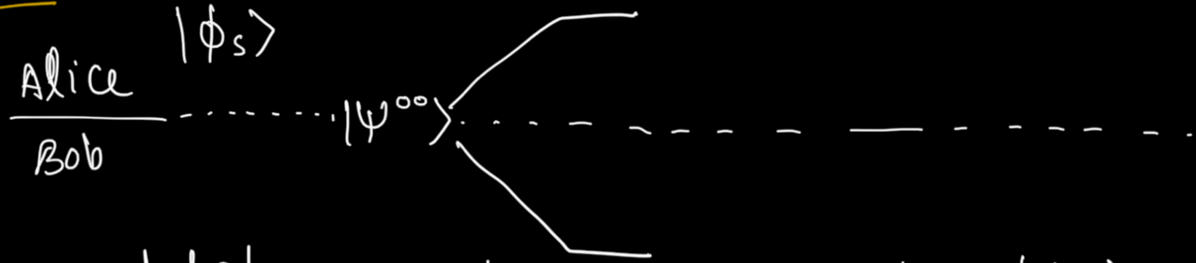
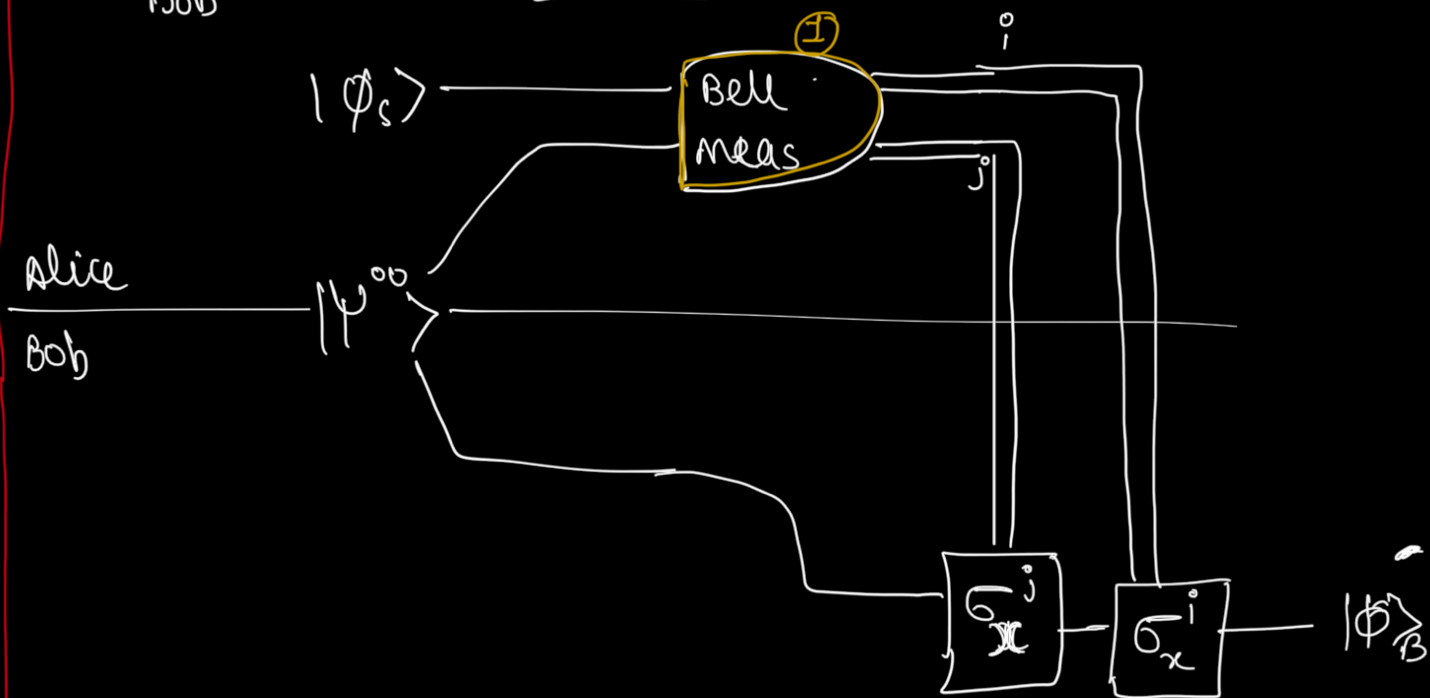
$+ |\psi\rangle_{SA} \otimes \sigma_z |\psi\rangle_B + |\psi\rangle_{SA} \ldots$

initial state of superposition → if we measure something it will collapse!

Protocol :



Alice and bob are two parties with $|\phi_s\rangle$ as alice's state which she want to teleport to bob.



Stepwise breakdown :

1. Alice measures on S and A in Bell basis
2. Alice sends her classical output i, j to Bob

Alice measurement ⟶ Bob's state

Alice measurement	i	j	$\xrightarrow{\text{classically}}$	
$\|\psi^{00}\rangle$	0	0		$\|\phi\rangle_B$
$\|\psi^{01}\rangle$	0	1		$\sigma_x \|\phi\rangle_B$
$\|\psi^{10}\rangle$	1	0		$\sigma_z \|\phi\rangle_B$
$\|\psi^{11}\rangle$	1	1		$\sigma_x \sigma_z \|\phi\rangle_B$

Step 1. If Alice does a bell measurement, that means she knows either ψ^{00}, ψ^{10}, ψ^{01}, ψ^{11}

→ superposition of the four states!

This is the superposition of these four bell states times something similar to ϕ. These four bell states are orthogonal, so if we measure one, then we know that whole system collapses, it collapses to that state times whatever we have then on B.

Then Alice sends her classical outputs i and j to Bob, because bob, sofar he does not know even if Alice does the measurement, he doesn't know which state he is in Because he does not know what the outputs of Alice were. So in the second step she sends the message to him. Alice sends $[i, j\ (00, 01, 10, 11)]$.

3. Third step: Bob applied $\sigma_z^i\ \sigma_x^j$ to his qubit and gets $|\phi\rangle$!

This way teleportation takes place :-

Alice's Measurement / Bob's State | Alice sends | Bob applies →

		ij	Bob's final state
ψ^{00}	$\lvert\phi\rangle_B$	0 0	$\mathbb{1} \to \lvert\phi\rangle_B$
ψ^{01}	$\sigma_x \lvert\phi\rangle_B$	0 1	$\sigma_x \to \lvert\phi\rangle_B$
ψ^{10}	$\sigma_z \lvert\phi\rangle_B$	1 0	$\sigma_z \to \lvert\phi\rangle_B$
ψ^{11}	$\sigma_x \sigma_z \lvert\phi\rangle_B$	1 1	$\sigma_z \sigma_x \to \lvert\phi\rangle_B$

Note that Alice's state collapses during the measurement so she does not have initial $\lvert\phi\rangle$ state anymore

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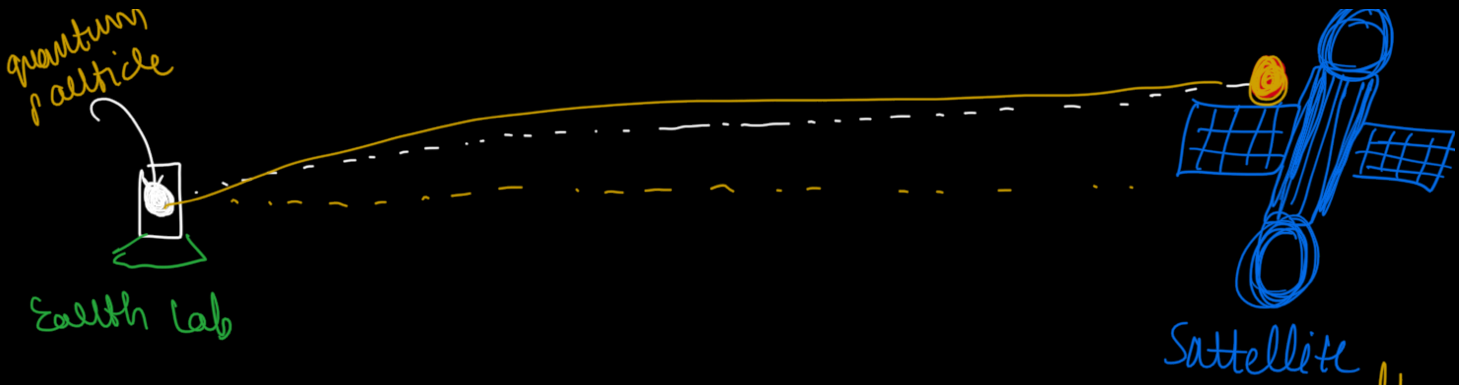
In simple words: quantum teleportation is same as quantum communication as we are transferring quantum information without any medium.

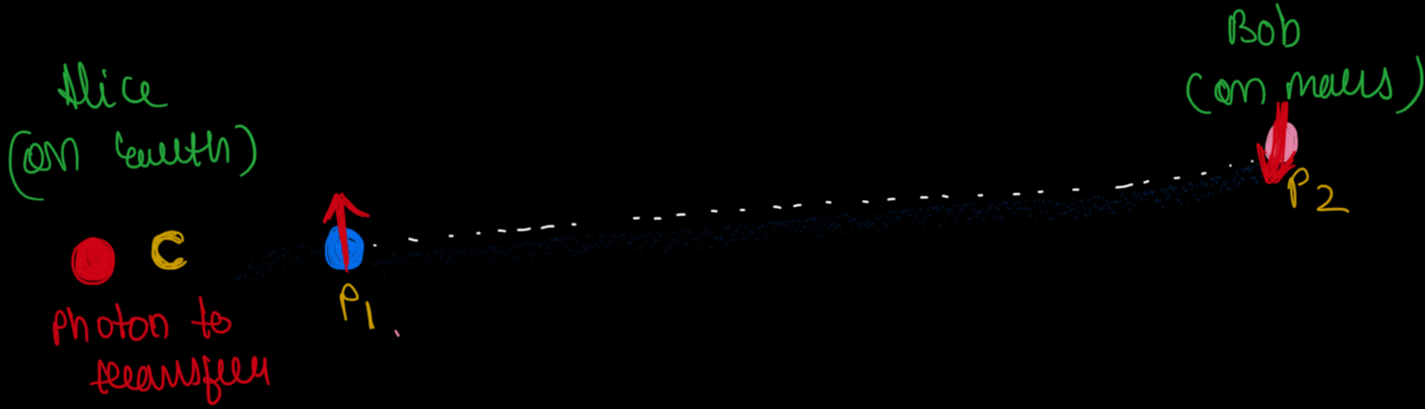
we know [No Cloning Theorem, No deleting theorem] Both of these theorem tells that

quantum information cannot be copied and cannot be deleted / destroyed.



Notice the particle which was present originally on Earth lab has now been shifted to the satellite

simple terms : Alice wants to send photon to Bob.



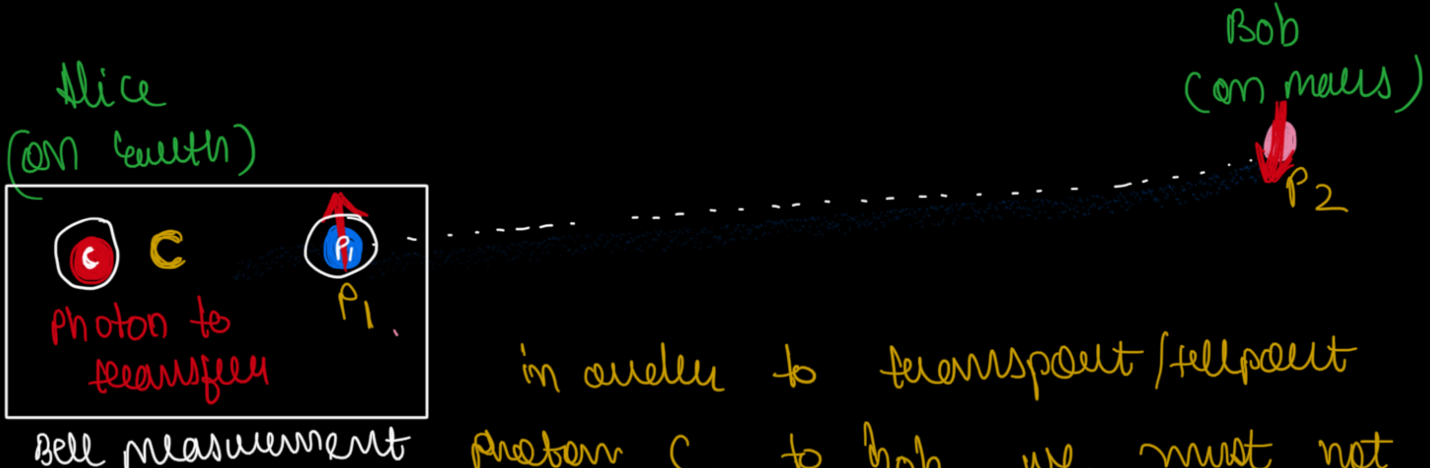
to make the teleportation/communication happen, Alice will need one pair of entangled particle.

one particle from the entangled pair ● is kept with Alice, and the other particle of the entangled pair with bob ●

once measured when we find the spin of the entangled particle P_1 as up then definitely the spin of particle P_2 will be down. And vice versa

This collapses their wave function bcz of super position principle. However at Alice end, and Bob's end either of them are not measuring the spin state of either particle.

∴ all 3 are in superposition states C, P_1 and P_2



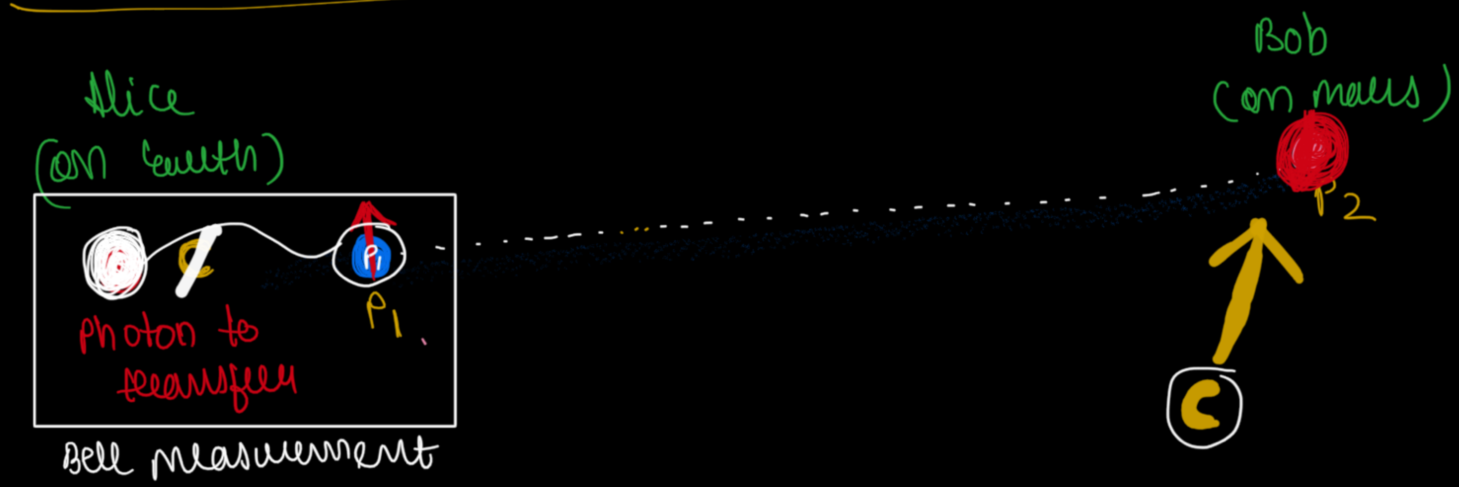
in order to transport/teleport photon C to bob we must not measure anything which will lead to collapse.

Thus we measure combined state of C and P_1. This is called Bell measurement, this does not reveal the measurements of individual particle but tells the relationship b/w both particles (here C and P_1). When we measure C and P_1 we will get information from photon C transferred to particle P_1 as they are measured combinely. However P_1 and P_2 are entangled, which transfers results in transfer of information from C to P_2

However Bob does not know in real that the information has been recieved from Alice

So, even if he has recieved the information, Bob cannot manipulate P_2 and make C. Additionally, he does not know the actual process by which Alice had made the photon

... by which ...
as C particle. Thus through any of the classical communication Alice has to send one or 2 bit information to Bob, the information which is required to transfer from Alice to Bob includes the results from bell measurement i.e., the combined state output of bell measure measurement. This will tell Bob which action to be taken on photon P_2 or new C at bob side by Bob. Once suitable action is taken, this will break the entanglement b/w P_1 and P_2 and properties of P_2 is changed to C #



from Brilliant :- brilliant.org

as a review, recall pauli matrices discussed in previous lecture.

$$\sigma_0 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The spin operators along each axis are defined as $\frac{\hbar}{2}$ times each of $(\sigma_1, \sigma_2, \sigma_3, \cancel{\sigma_4})$ for (x, y, z) axis

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

respectively. these pauli matrices are used to construct $\boxed{\text{Bell States}}$ an orthonormal basis of entangled states

$\uparrow = 1$
$\downarrow = 0$

$$|\phi_0\rangle = I \otimes \sigma_0 |\phi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle\right)$$

$$|\phi_1\rangle = I \otimes \sigma_1 |\phi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle\right)$$

$$|\phi_2\rangle = I \otimes \sigma_2 |\phi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\right)$$

$$|\phi_3\rangle = I \otimes \sigma_3 |\phi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\uparrow\rangle - |\downarrow\rangle \otimes |\downarrow\rangle\right)$$

Quiz : (1) select the correct options :-

Ans 5

✓ (1) Quantum information is transferred b/w states

✗ (2) The teleported particle is physically transferred b/w locations

✗ (3) A quantum state is cloned b/w observers

✗ (4) Quantum information is permanently removed from the system. (No Delete Theorem)