LECTURE_2

Github.com/sanny280497/qiskit

(Part -II) (Video 2) Lecture 2

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[Qubits and Quantum states, Quantum circuits, measurements]

circuit model = sequence of some building blocks, which carry out our computations and those we call gates,

(Left)

single qubit gates : gates that act on single qubit. classical example :

classical example	NOT GATE 
quantum example	as quantum theory is unitary, quantum gates are represented by unitary matrices

$$\boxed{U^\dagger U = \mathbb{1}}$$

$[U^\dagger][U] = \mathbb{1}$

(recall dirac's Notation)

$U = (u_{00} , u_{01}$

$\Big(u_{00} \quad u_{01}$

$$\begin{pmatrix} U_{10}, & U_{11} \end{pmatrix} \qquad \begin{pmatrix} U_{10} & U_{11} \end{pmatrix}$$

<u>Quantum Example</u>: as quantum theory is unitary, quantum gates are represented as unitary matrices,

$U^T U = 1$, in Dirac notation,

$$U = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix} = \left\{ \boxed{U_{00}} + \boxed{U_{01}} + \boxed{U_{10}} + \boxed{U_{11}} \right.$$

<u>Now</u>, the matrix U can be represented as:

$$U = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix} = \left\{ \begin{array}{l} 1.\ U_{00} \cdot (|0\rangle \cdot \langle 0|) \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ + \\ 2.\ U_{01} \cdot (|0\rangle\langle 1|) \rightarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ + \\ 3.\ U_{10} (|1\rangle \cdot \langle 0|) \rightarrow \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\ + \\ 4.\ U_{11} (|1\rangle\langle 1|) \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{array} \right\}$$

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$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix} \longrightarrow |0\rangle \cdot \langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \end{pmatrix}$$

2.

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$

$$|0\rangle . \langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

3.

$|1\rangle . \langle 0|$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$

$$|1\rangle . \langle 0| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

4.

$|1\rangle . \langle 1|$

$|1\rangle$ = Ket = $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\langle 1|$ = Bra $\begin{pmatrix} 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Now lets look to some Gates :-

BIT FLIP

① $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ PAULI X

σ_x

Dirac Notation :- $\left[|0\rangle . \langle 1| + |1\rangle . \langle 0| \right]$

multiplication example :- $\sigma_x . |0\rangle = \begin{pmatrix} 0 & 1 \\ \end{pmatrix} \begin{pmatrix} 1 \\ \end{pmatrix} = \begin{pmatrix} 0 \\ \end{pmatrix} = |1\rangle$

① Example 1 $\begin{pmatrix} 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix} = |1\rangle$

$2\times 2 \quad 2\times 1$

(complete in Dirac's Notation) ② Example 2

$$\sigma_x |1\rangle = \Big[\, |0\rangle\langle 1| + |1\rangle\langle 0| \,\Big]\Big[\, |1\rangle \,\Big]$$

$$= |0\rangle\underbrace{\langle 1|1\rangle}_{\text{Braket}\,\to\,1} + |1\rangle\underbrace{\langle 0|1\rangle}_{\text{Braket}\,\to\,0} \quad ✓$$

$$= |0\rangle\,(1) = \underline{|0\rangle} \quad \underline{\underline{\text{Ans}}}$$

⇒ $\boxed{\text{Bit flip}}$: ≅ quantum equivalent for Not Gate!

example : $|0\rangle \xrightarrow{\text{wire}} \boxed{\sigma_x} \xrightarrow{\text{wire}} |1\rangle$

rotation around x-axis by π

NOTE : for pure states, it's easier to describe quantum states in ket-form

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quantum equivalent of not gate,

because what it does is flip the gate, if we input $|0\rangle$ it gives us $|1\rangle$ and if we input $|1\rangle$ it gives back $|0\rangle$, for example, $|0\rangle \to |1\rangle$

$$|0\rangle \longrightarrow \boxed{\sigma_x} \longrightarrow |1\rangle$$

what the state in general, I could not only input, So this is what means, it can act on quantum states, which furthere implies, it can not only act on 0/1 but also on superpositions, so, in general, it gives rotation around the x-axis by pie. (π) $\underline{180°}$ in bloch sphere

II PHASE FLIP σ_z

next gate: σ_z gate **Pauli Z**

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$$

Dirac's Notation,

ex: $\sigma_z . |+\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$

$\sigma_z |-\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle$

$\sigma_z |-\rangle = \big(|0\rangle\langle 0| - |1\rangle\langle 1|\big) \cdot \frac{1}{\sqrt{2}} \big(|0\rangle - |1\rangle\big)$

$= \frac{1}{\sqrt{2}} \big(|0\rangle + |1\rangle\big)$

$= |+\rangle$

phase flip!

In general rotation around Z axis, by π

III σ_y

Pauli Y

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i\,\sigma_x \sigma_z$$

$\big(\sigma_y = i.\sigma_x \sigma_z\big) \Rightarrow$ same as we apply bit and phase flip

In general, it's the rotation around y

{validate with bloch sphere covered in L1}

σ_x, σ_y and σ_z are the so called Pauli - matrices and $\sigma_i^2 = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

since we know, Pauli rotations are rotation by 180°, we illustrate that, applying any flip operation twice, we will have 360° rotation, that's the same state back, that is if we apply twice the same gates, we will basically do nothing. Together, combining Pauli matrices, with identity $\mathbb{1}$ they form a basis of 2x2 matrices, which means that any one qubit rotation, can be written as linear combination of these gates. Imp. for error correction.

MOST IMP GATE : single qubit Gate,

IV **HADAMARD GATE** (H)

you will find this Gate in any quantum circuit,

$H = \frac{1}{\ } \begin{pmatrix} 1 & 1 \end{pmatrix}$ unitary matrix.

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Dirac's notation :-

$$= \frac{1}{\sqrt{2}} \left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1| \right)$$

Now if we apply Hadamard Gate to the state 0,

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle$$

$$H|1\rangle = |-\rangle$$

creates Superposition!

$$\boxed{\begin{aligned} H(|0\rangle) &= |+\rangle \\ H(|1\rangle) &= |-\rangle \end{aligned}}$$

$$\boxed{\begin{aligned} H|+\rangle &= 0 \\ H|-\rangle &= 1 \end{aligned}}$$

used to change between the different basis (X and Z)

So we can used this to change between different basis, So, for example, we usually cannot do X measurement, what we can do, is we measure in Z basis, we measure whether its $|0\rangle$ or $|1\rangle$ state and, if you would like to do an X measurement, what we do is first apply Hadamard Gate, and then do Z measurement;

applying first hadamard gate and then doing Z measure, is same as doing X- measurement, mathematically.

V S-GATE

we have S-Gate,

$S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ adds 90° to the phase φ :

so if we apply $\boxed{\begin{aligned} S|+\rangle &= |+i\rangle \\ S|-\rangle &= |-i\rangle \end{aligned}}$ if we visualize in bloch sphere :-

If we see, x axis, $|+\rangle$ and $|-\rangle$ then, we can check from ~~x~~ axis to y axis. (90°)

on applying again we will go to $|-\rangle$ state (180°)

Its used for is

SH is applied to change from Z to Y, so if you wanna do a Y measurement, we would apply S and H and then do Z measurement.

TOOK A BREAK !
(35min)

KET ✓ } same for
Bra ✗ } pure states!

Multipartite Quantum States To describe quantum states on multiple qubits.

we use TENSOR PRODUCTS to describe multipartite states.

$$|a\rangle \otimes |b\rangle = \underset{2\times 1}{\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}} \otimes \underset{2\times 1}{\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}} = \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix}$$

example :

System A is in state $|1\rangle_A$

Notation $\boxed{|1\rangle_A}$ denotes that qubit A is in state $|1\rangle$

System B is in state $|0\rangle_B$

The total bipartite states is given by $|10\rangle_{AB}$

or

$$|1\rangle_A \otimes |0\rangle_B = \boxed{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$$

So, if we have 2, n-dim matrix products, we will have 2^n sized matrix for the end product.

So if we have $n=10$, then we will have 2^{10} dimensional matrix, This is the reason why, simulating quantum computing is sometimes where the few qubits you will get at some point you will get, you to the limits

Remark : States of this form are called un-correlated but there are also bi-partite states that cannot be written as $\boxed{|\psi\rangle_A \otimes |\psi\rangle_B}$

these states are correlated and sometimes entangled, eg: $|\psi^{00}\rangle_{AB} = \frac{1}{\sqrt{2}}\left[|00\rangle_{AB} + |11\rangle_{AB}\right]$

$= \frac{1}{} \Big(1 \Big)$ 00 $\{$Bell$\}$ used for Teleportation.

$\sqrt{2}$ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\begin{matrix} 01 \\ 10 \\ 11 \end{matrix}$ $\underbrace{\{\text{state}\}}$

protocol, Bell Test, Cryptography and other use cases.

Correlated states are not entangled. Are only mixed states, So if we have pure states, that is correlated, that is also entangled, so for classical states if they are correlated that means they are always mixed states.

TWO QUBIT GATES:

classical example: XOR gate $\oplus$ (irreversible)

But quantum theory is unitary, that means we only consider unitary gates, and unitary gates are always reversible. $\boxed{U^\dagger U = 1}$

$= x\bar{y} + \bar{x}y$

$U^\dagger U = 1 \iff U^{-1} = U^\dagger$

CNOT gate $= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

In Dirac's Notation:

$$\left[|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10| \right]$$

So, if we perform

$$CNOT\ |00\rangle_{xy} = CNOT \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle_{xy}$$

Controlled NOT

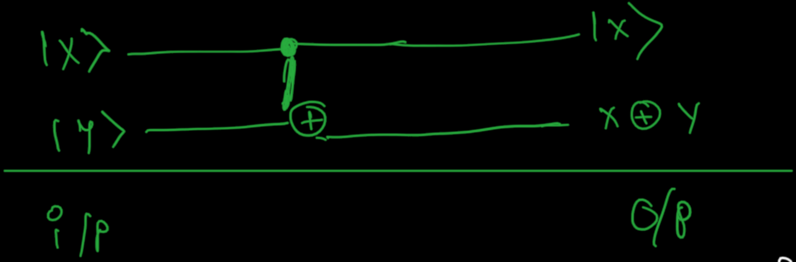
$\text{CNOT}\,|10\rangle_{xy} = |11\rangle_{xy}$

Input and output Table :- Truth Table

i/p		o/p	
x	y	x	$x \oplus y$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

It computes XOR Gate, but its also reversible ! because you also get the x back, ∴ if we have x and $x \oplus y$, we can always calculate y from $(x, x \oplus y)$

circuit :



(#) we can show that every function f can be described by a reversible circuit.

(#) Quantum circuits can perform all functions that can be calculated classically.