If we have quantum algorithm we are looking for, and we are searching for an element from big database, for one single element, then we can use this INTERFERENCE EFFECT, where we say okay, we somehow design a quantum algorithm, very smartly in a way that, all elements we dont want, they somehow somehow destructively interfere while the elements that we want to get, we make sure that, we get some constructive Interference !

This way the wrong answers cancels itself and the correct answer that remains.

this way we can get right answer faster than classical computer, on a quantum computer !

! DIRAC'S NOTATION ! used to describe quantum states ✔ lets chose 2 strings a, b where $\boxed{a, b \in \mathbb{C}^2}$ where a, b are 2 dim array with complex entries.

- ket : $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

- bra : $\langle b| = \underbrace{|b\rangle'}_{\text{complex conjugate of ket}} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}' = (b_1 \ b_2)$

Ket

$|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

(column vector)

bra (Row vector)

$\langle b| = |b\rangle^T$

$= \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}^T$

$= (b_1^* \ b_2^*)$

if in general, $b = c + di$ is complex number then, b^* is the complex conjugate of the complex number b

This way,

$b = c + di$

then $b^* = c - di$

$\boxed{i = \sqrt{-1}}$

- (bra-ket) $= \langle b|a\rangle$ = INNER PRODUCT

$= a_1 b_1^* + a_2 b_2^*$ $\quad (1\times2)(2\times1) = (1\times1)$

$= \langle a|b\rangle^*$

solving this :

$\langle b|a\rangle$

$\langle b|$ $\quad$ $|a\rangle$

$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

a

bra ket

$$\underbrace{\begin{pmatrix} b_1^* & b_2^* \end{pmatrix}_{1\times 2} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{2\times 1}}$$

if we take inner product then,

$$(a_1 b_1^* + a_2 b_2^*)_{(1\times 1)} = \text{[illegible]} \quad \text{— (1)}$$

Now, consider $\langle a|b\rangle^*$ then,

$\langle a|$ and $|b\rangle$ and whole conjugate

$$\begin{pmatrix} a_1^* & a_2^* \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\left.\begin{aligned} &= (a_1^* b_1 + a_2^* b_2)^* \\ &= (a_1 b_1^* + a_2 b_2^*) \end{aligned}\right\}$$

can be verified if considered,

$$\left.\begin{aligned} a_1 &= c_1 + d_1 i \\ a_2 &= c_2 + d_2 i \\ b_1 &= e_1 + f_1 i \\ b_2 &= e_2 + f_2 i \end{aligned}\right\}$$

(2)

from (1) and (2) we can say

$$\text{bra-ket} = \langle b|a\rangle = \langle a|b\rangle^* \in \text{Complex Number}$$

bra = Row vector

ket = column vector

bra-ket = complex number

Now, same way we also calculate ket-bra.

$$\boxed{\begin{aligned} \text{bra-ket} &= \langle a|b\rangle \\ \text{ket bra} &= \boxed{|a\rangle\langle b|} \simeq |a \cancel{\rangle\langle} b| \end{aligned}}$$

IMP NOTATIONS

$$|a\rangle\langle b| = \begin{pmatrix} a_1 b_1^* & a_1 b_2^* \\ a_2 b_1^* & a_2 b_2^* \end{pmatrix}$$

ket column

bra row

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} b_1^* & b_2^* \end{pmatrix}$$

2×1 $\quad$ 1×2 $= 2 \times 2$

Dirac's

- ket
- bra
- bra-ket
- ket-bra

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we define states

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

both orthogonal and can be verified when we take inner product,

$$\langle 0|1\rangle$$

bra row

ket column

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 + 0 = \underline{0}$$

ORTHOGONAL

ORTHONORMAL!

(#) all quantum states are normalized. i.e., mathematically, $\boxed{\langle \psi | \psi \rangle = 1}$

eg $|\psi\rangle = \left(|0\rangle + |1\rangle \right) = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\underbrace{= \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}}_{\text{NORMALIZED}}$$

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why do we discriminate only orthogonal states? $\Rightarrow$ measurements, projective measurements

we consider PROJECTIVE MEASUREMENTS which are measurements on orthogonal states, with much advanced concepts, POVM we can discriminate non-orthogonal states.!

(#) To determine probabilities of measurements, of getting different outcomes, and for that one, its very important useful if we have NORMALIZED quantum states