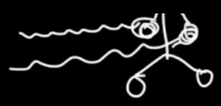

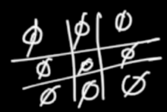

Measurements: [Sat, 21 NOV 2020]

① orthogonal basis to describe and measure quantum states.

② we chose orthogonal basis to describe and measure quantum states. ✓

This is also called projected measurements, and generalizations of those also exist;

③ If you have a measurement onto the basis $\{|0\rangle, |1\rangle\}$, the basis are orthogonal states, the state will collapse into either state $\boxed{|0\rangle \text{ or } |1\rangle}$ this is because, as those are the eigen states of the σ_z operator are poly operator | poly matrices. thus we call this Z-measurement, however in general there are infinitely many different bases, but there are some common ones, are

I $\boxed{\text{Plus - Minus Basis}}$

$$\left\{ |+\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right),\ |-\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \right\}$$

equal superposition of states $\underline{0}$ and $\underline{1}$

④ There's different amplitudes, and for

interference effect we can have different amplitudes and relative phases between different states. You can have superposition of 0 and 1. In fact, you can take dot product and find these 2 are orthogonal states.

II $\boxed{\text{Plus } i \text{ and minus } i \text{ Basis}}$ other names $\boxed{+y \text{ and } -y \text{ basis}}$ or $\boxed{\text{left right basis}}$

$$\left\{ |+i\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i\,|1\rangle\right) \text{ and } |-i\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - i\,|1\rangle\right) \right\}$$

Both the basis I and II corresponding to the eigenstates of σ_x and σ_y respectively.

So, when we do measurement on plus-minus basis, I can call it as X measurement and when doing measurement on plus i and minus i basis then we call it Y measurement

Now to determine the probabilities of diff. outcomes, we have $\boxed{\text{born rule}}$

Born Rule: The probability that a state $|\psi\rangle$ collapse during a projective measurement onto the basis $\{|x\rangle, |x^{\perp}\rangle\}$ to the state $|x\rangle$ is given by

$$P(x) = |\langle x|\psi\rangle|^2 , \quad \sum_i P(x_i) = 1$$

example ①

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}\,|1\rangle\right)$$

$\boxed{\text{measure it in the } 0\text{-}1 \text{ basis !}}$

determine the probability to measure state $|0\rangle$

$\boxed{P(0)}$

sol^n:

$$P(0) = \left|\left\langle 0 \,\middle|\, \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}\,|1\rangle\right)\right\rangle\right|^2$$

$$P(0) = \left\| \frac{1}{\sqrt{3}} \underbrace{\langle 0|0\rangle}_{1} + \sqrt{\frac{2}{3}} \underbrace{\langle 0|1\rangle}_{0} \right\|^2$$

normalized hence 1

orthogonal hence 0

$$= \left| \frac{1}{\sqrt{3}}(1) + \sqrt{\frac{2}{3}}(0) \right|^2$$

$$= \left[\frac{1}{\sqrt{3}}(1)\right]^2 = \frac{1}{3}$$

$$P(1) = 1 - P(0)$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

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example ②

is measured in plus minus

$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ is measured in $\{|+\rangle, |-\rangle\}$; basis. We are interested in getting the probability as output '+'

Soln

$$p(+) = |\langle + | \psi \rangle|^2 = \left|\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|) \cdot \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right|^2$$

$$= \left(\frac{1}{\sqrt{2}}\right)^{2^2} \left| \underbrace{\langle 0|0\rangle}_{1} - \cancel{\langle 0|1\rangle} + \cancel{\langle 1|0\rangle} - \underbrace{\langle 1|1\rangle}_{1} \right|^2$$

$$= \left(\frac{1}{4}\right)|1-1| = \underline{\underline{0}}$$

This is expected, not surprising, the inner product of ψ and $+$, ψ is exactly the -ve when we discussed bases

$$p(-) = 1 - 0 = \underline{\underline{1}}$$

{ as $\langle + | \psi \rangle = \langle + | - \rangle = 0$ orthogonal }

There are 2 ways to work on quiskit, you can either run on local machine or you can run virtually on IBM, quantum Experience!

BLOCH SPHERE :- If we consider any Norm-quantum state and restricts ourselves to pure quantum states as :-

| Pure states are the one where we know |

the state of the qubit, in other words, pure quantum states are the states that are in known state where the entropy is 0, so we know that which state that is in. which can be a superposition state. we know that its in state 0+1, but its not accurate, we will find the correct one after we complete measurement specifically.

$$\boxed{|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle}$$, where $\varphi \in [0, 2\pi]$ describes the relative phase

Relative phase v/s # Global phase

# relative phase	# global phase
what you have b/w states 0 and one, in this case, if I have 0+1, or 0−1, or (0+i 1) this $e^{i\varphi}$ is the term in the formula which determines the relative phase.	when we multiply whole state $\vert\psi\vert$ with (−1) and would still be normalize and still be a valid state, however physically it does not make any difference!

$\theta \in [0, \pi]$ determines the probability, to measure