[0, 1] the state $|0\rangle$ or state $|1\rangle$ in this case (0-1 measurement)

$$P(|0\rangle) = \cos^2\frac{\theta}{2} \qquad P(|1\rangle) = \sin^2\frac{\theta}{2}$$

all normalized pure states can be illustrated on BLOCH SPHERE, they can be illustrated on the surface of the sphere ! with radius 1

$|\vec{r}| = 1$ which we call Bloch Sphere

mix states are actually then be illustrated as some states inside the bloch sphere however pure states are / all pure states will be on the surface of the Sphere

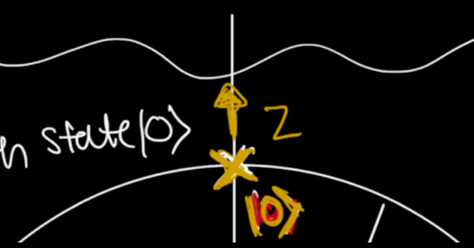
now we are interested in the coordinates of any given state such state are given by "Bloch vector"

BLOCH VECTOR

$$\vec{r} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

BLOCH VECTOR

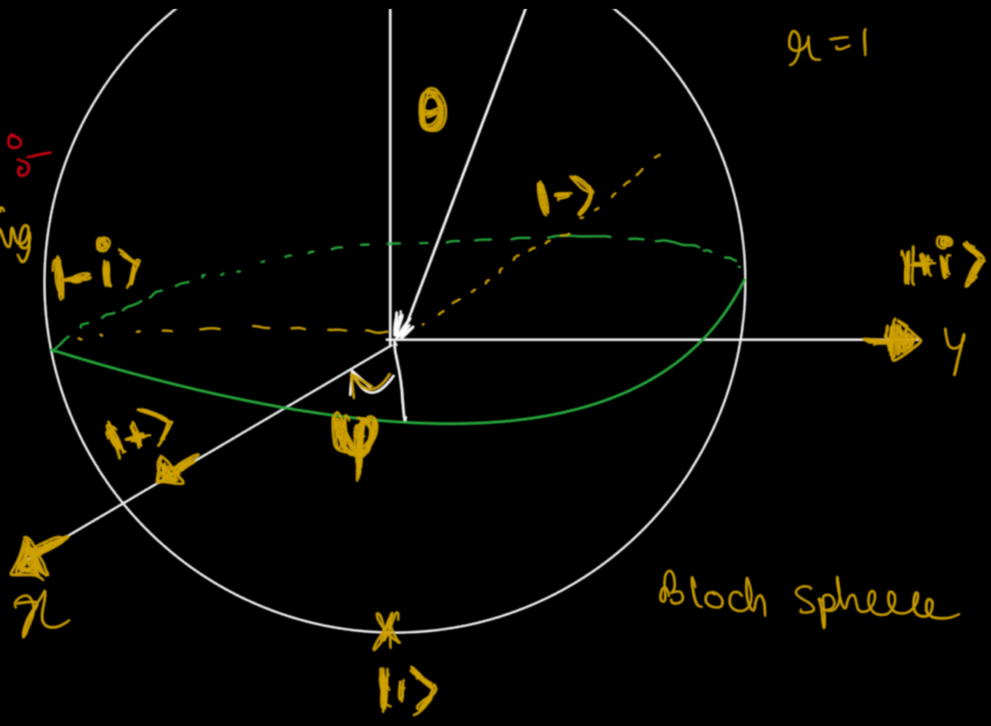
example : (I) Start with state $|0\rangle$



For $|0\rangle$ we can find out the Bloch vector as :-

$\theta = 0$, φ anything

φ can be arbitrary



$$\vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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for $\theta = 0$, φ arbitrary

$$\vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$

as $\theta = 0 \Rightarrow \sin\theta = 0$

$= 0$

$\sin(\theta = 0) = 0$

0 in x direction
0 in y direction
1 in z direction

by this we come to know $|0\rangle$ lies on the North Pole

In similar way we can understand where $|1\rangle$ lies ....

- $|0\rangle$: $\theta = 0$, φ arbitrary $\rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ North pole

- $|1\rangle$: $\theta = \pi$, φ arbitrary $\Rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ South Pole

- $|+\rangle$: $\theta = \frac{\pi}{2}$, $\varphi = 0$ so that we do not get any face $\Rightarrow \vec{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ x-axis

- $|-\rangle$: $\theta = \frac{\pi}{2}$, $\varphi = \pi$ $\Rightarrow \vec{r} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} -\text{ve} \\ \text{x-axis} \\ -\text{ve} \end{pmatrix}$

- $|+i\rangle$: $\theta = \frac{\pi}{2}$ $\varphi = \pi/2$ $\vec{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} +\text{ve} \\ \text{y axis} \end{pmatrix}$

- $|-i\rangle$: $\theta = \frac{\pi}{2}$ $\varphi = 3\pi/2$ $\vec{r} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} -\text{ve} \\ \text{y axis} \end{pmatrix}$

<u>Be careful!</u> Now, one needs to be careful, Because if we look on the states at bloch sphere, the angles are twice as large, as in Hilbert Space.

<u>eg</u> $|0\rangle$ and $|1\rangle$ are orthogonal.

i.e., they have right angle (90°) but, on the BLOCH Sphere their angle is 180° degrees.

For a general state $|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + \cdots$

this $\frac{\theta}{2}$ is exactly why we have, θ = angle in Bloch sphere toward south pole.

$\frac{\theta}{2}$ is the angle that we have in Hilbert Space

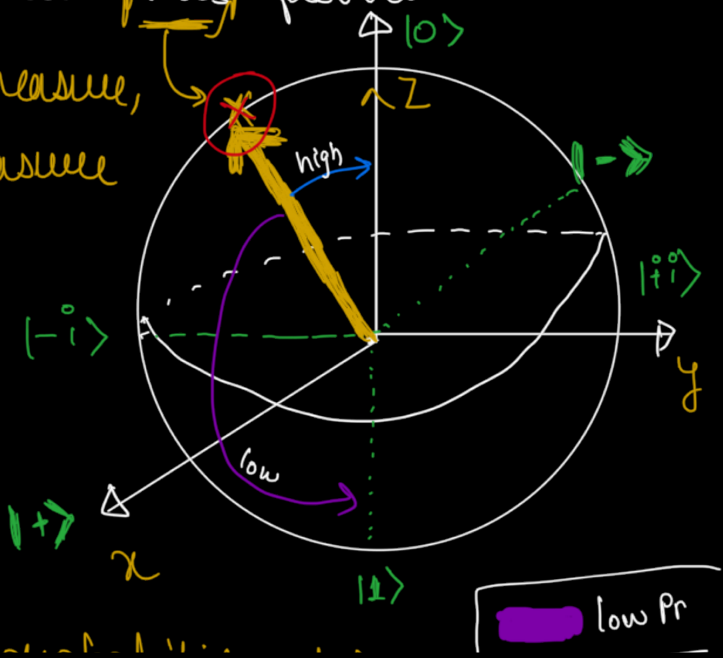
By the description, mathematically look at it, and write it as a vector, we can tell that $\frac{\theta}{2}$, is the actual angle we have in hilbert space. !

⇒ If States are orthogonal on the Bloch sphere, that means they are not orthogonal in hilbert space, but only if they are on opposite sides on the Bloch Sphere, that means they are orthogonal.

LEARNING OUTCOME :- we can nicely illustrate in the block sphere, Z- measurement, for example, corresponds to a projection on Z-axis,

Suppose we have a state at this point

then, the probability to measure, this, considering Z - measure that means, projecting on the Z-axis, so with a high probability we can get output $|0\rangle$ and with a small probability ...



get probability $|1\rangle$

high Pr

same way we can understand for $|-i\rangle$ and $|+i\rangle$ when we project on y-axis and $|+\rangle$ and $|-\rangle$ for x-axis!

we have so many basis, bcuz we can project onto any given orthogonal state and its state, we can do projective measurement, theoretical convinience we use C-measurements and standard axes.