Lecture 3

26th November, 2020, (11 pm IST) | Part 3 III

ENTANGLEMENT

Pure states which are correlated is called entanglement.

In other words, If a pure state $|\psi_{AB}\rangle$ on $\boxed{|\psi\rangle_{AB}}$ on systems A, B cannot be written as $|\psi\rangle_A \otimes |\Phi\rangle_B$ (some state A and some state B) then its entangled.

Bell states entangled states are called as so called Bell States, There are 4 so-called Bell states that are maximally entangled and build an orthonormal basis.

(i) $|\psi^{00}\rangle := \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

(ii) $|\psi^{01}\rangle := \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

(iii) $|\psi^{10}\rangle := \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

(iv) $|\psi^{11}\rangle := \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

Bell States!

in general (useful in describing teleportation protocol) we can write,

$$|\psi^{ij}\rangle = (1 \otimes \sigma_x^j \sigma_z^i)|\psi^{00}\rangle$$

$$\boxed{|\psi^{00}_{ij}\rangle = (\underbrace{\mathbb{1}}_{\text{identity}} \otimes \sigma_x^j \sigma_z^i)|\psi^{00}\rangle}$$

Example, in First bell state, whenever we do measurement, we observe first qubit is in the state 0, this means that state COLLAPSES and output will be $|00\rangle$ and however if we measure **1** then, immediately we can say state collapses to $|11\rangle$ so, immediately we can say, output at second qubit will also be 1. This correlation is inherent to entanglement, thats, the bits remain entangled until we observe them, the wave function gets collapsed immediately we observe **it**. So one part always knows what would be the value at other party even if its far away. we can also understand this classically with the Einstein's explanation, however, this quantum thing is mixed state, we would be either in $|01\rangle$ or $|10\rangle$, but we would not be in superposition

$$|\psi^{00}\rangle = |00\rangle + |11\rangle$$

so whenever we measure in the 0-1

can also be written as:-

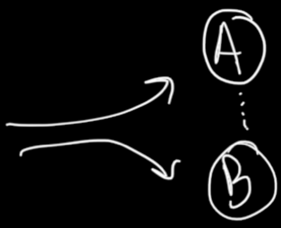
$$|\varphi^{o\varsigma}\rangle = |++\rangle + |--\rangle$$

so the strong correlation holds for every basis that we wanna write the state in., we will always have

basis, and that gets 0 for 1, it will get 0 for other 1 as well.

strong correlation.

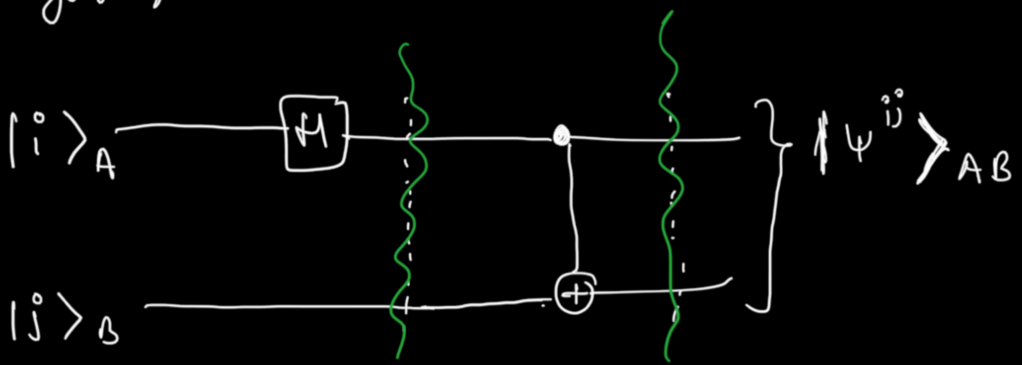
Even if we do X measurement,



Quantum Entanglement

Creation of Bell States : Friday 27th NOV

we shall write down a circuit, and we claim this circuit will give a bell state, we will apply, Hadamard gate and CNOT gate,



~~Table~~ Truth Table Analogy :-

initial state :	(intermediate states)	(final states)
$\|ij\rangle_{AB}$	$(H_A \otimes I_B)\|ij\rangle_{AB}$	$\|\psi_{ij}\rangle_{AB}$
$\|00\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle + \|10\rangle)$	$(\|00\rangle + \|11\rangle)/\sqrt{2}$

$$|01\rangle \xrightarrow{H_A} \frac{1}{\sqrt{2}}\left(|01\rangle + |11\rangle\right) \xrightarrow{CNOT\ A\ B} \left(|01\rangle + |10\rangle\right)/\sqrt{2}$$

$$|10\rangle \qquad \frac{1}{\sqrt{2}}\left(|00\rangle - |10\rangle\right) \qquad \left(|00\rangle - |11\rangle\right)/\sqrt{2}$$

$$|11\rangle \qquad \frac{1}{\sqrt{2}}\left(|01\rangle - |11\rangle\right) \qquad \left(|01\rangle - |10\rangle\right)/\sqrt{2}$$

TELEPORTATION

Actually a thing Not just science fiction!

Goal of teleportation protocol :-

Alice wants to send her (unknown) state $|\phi\rangle_s$

$$\boxed{|\phi\rangle_s := \alpha\,|0\rangle_s + \beta\,|1\rangle_s}$$

to bob, however she can send only 2 classical bits though.

Alice wants to communicate by sending her state to Bob, But have only 2 classical bits. Her state is unknown,

$$|\phi_s\rangle := \alpha\,|0\rangle_s + \beta\,|1\rangle_s$$

Infact, even if she'd known what her state is it would not be enough because alpha can be any number between 0 and 1 hence 2 bits won't be enough to characterize any number. # There is another way to do it, Because they both share the maximally entangled state, $|\psi^{00}\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$

initial state of the total system :

$|\phi\rangle_s$ (my system which I want to send)

$|\psi^{00}\rangle_{AB}$ (The bell state which Alice and Bob share)

$$|\phi\rangle_s \otimes |\psi^{00}\rangle_{AB} = \frac{1}{\sqrt{2}}\big(\alpha|000\rangle_{SAB} + \alpha|011\rangle_{SAB} + \beta|100\rangle_{SAB} + \beta|111\rangle_{SAB}\big)$$

$$= \frac{1}{2\sqrt{2}}\Big[\big[(|00\rangle_{SA} + |11\rangle_{SA}) \otimes (\alpha|0\rangle + \beta|1\rangle) + (|01\rangle_{SA} + |10\rangle_{SA}) \otimes (\alpha|1\rangle_B + \beta|0\rangle_B)\big]$$

$$+ \big[(|00\rangle_{SA} - |11\rangle_{SA}) \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) + (|01\rangle_{SA} - |10\rangle_{SA}) \otimes (\alpha|1\rangle_B - \beta|0\rangle_B)\big]\Big]$$

some terms gets cancel out, with remaining terms as

if → then → if → then

$$= \frac{1}{2}\Big[\underline{|\psi^{00}\rangle_{SA}} \otimes \underline{|\phi\rangle_B} + \underline{|\psi^{01}\rangle_{SA}} \otimes \underline{\sigma_x|\phi\rangle_B}$$

if → then if → then

$$+ |\psi^{10}\rangle \otimes \ldots + |\psi^{11}\rangle \otimes \sigma_x\sigma_z|\phi\rangle_B\Big]$$