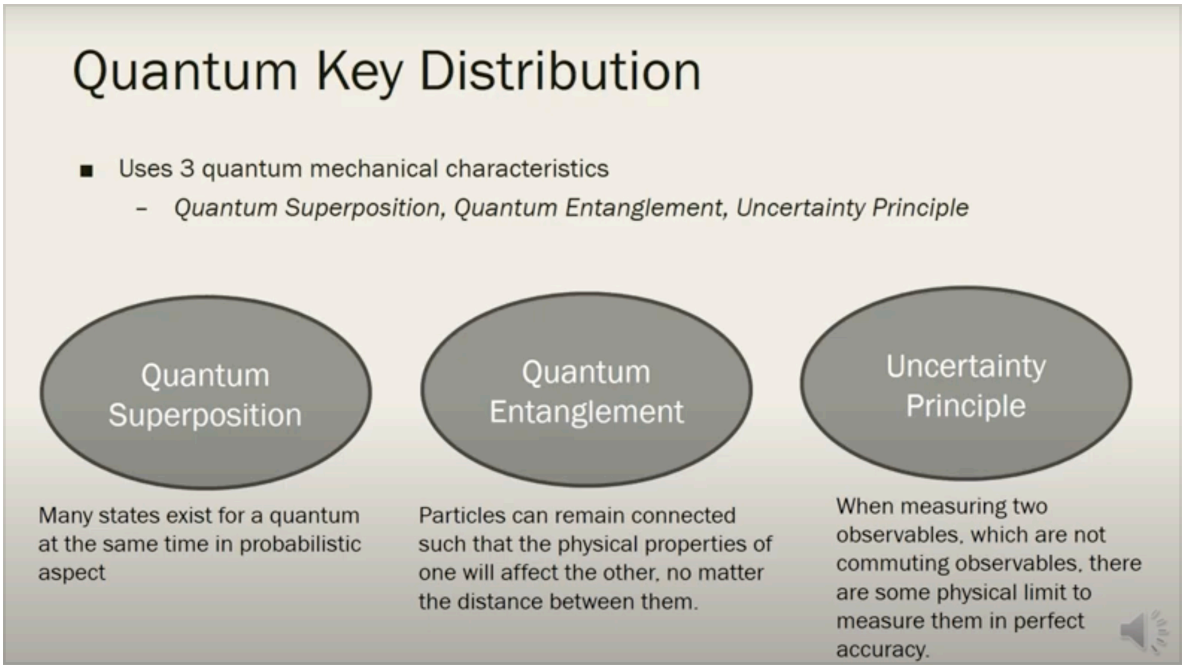
LECTURE NOTES

QKD: BB84



Classical cryptography can be divided into two major branches; secret or symmetric key cryptography and public key cryptography, which is also known as asymmetric cryptography. Secret key cryptography represents the most traditional form of cryptography in which two parties both encrypt and decrypt their messages using the same shared secret key. While some secret key schemes, such as one-time pads, are perfectly secure against an attacker with arbitrary computational power [Gisin02], they have the major practical disadvantage that before two parties can communicate securely they must somehow establish a secret key. In order to establish a secret key over an insecure channel, key distribution schemes basd on public key cryptography, such as Diffie-Hellman, are typically employed.

Paper Summary

(Imp Notes)

Cryptography

Public key cryptography

Assymetric.

traditional methodology

same key is used. Typically to share the key b/w parties is done via DHKE

Secret key cryptoge

Symmetric

No prior distri

one can use some one's publi key and then encrypt using public key of the reciever. Once reciever receives th encrypted message, then it can decrypt using its private key.

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while there is no requirement to exchange keys in such cases, however, all these crypto-systems are built upon the difficulty power or difficult computation power of attacke all based on unproven assumptions of difficulty of certain problems such as integer factorizati or discrete logarithm.

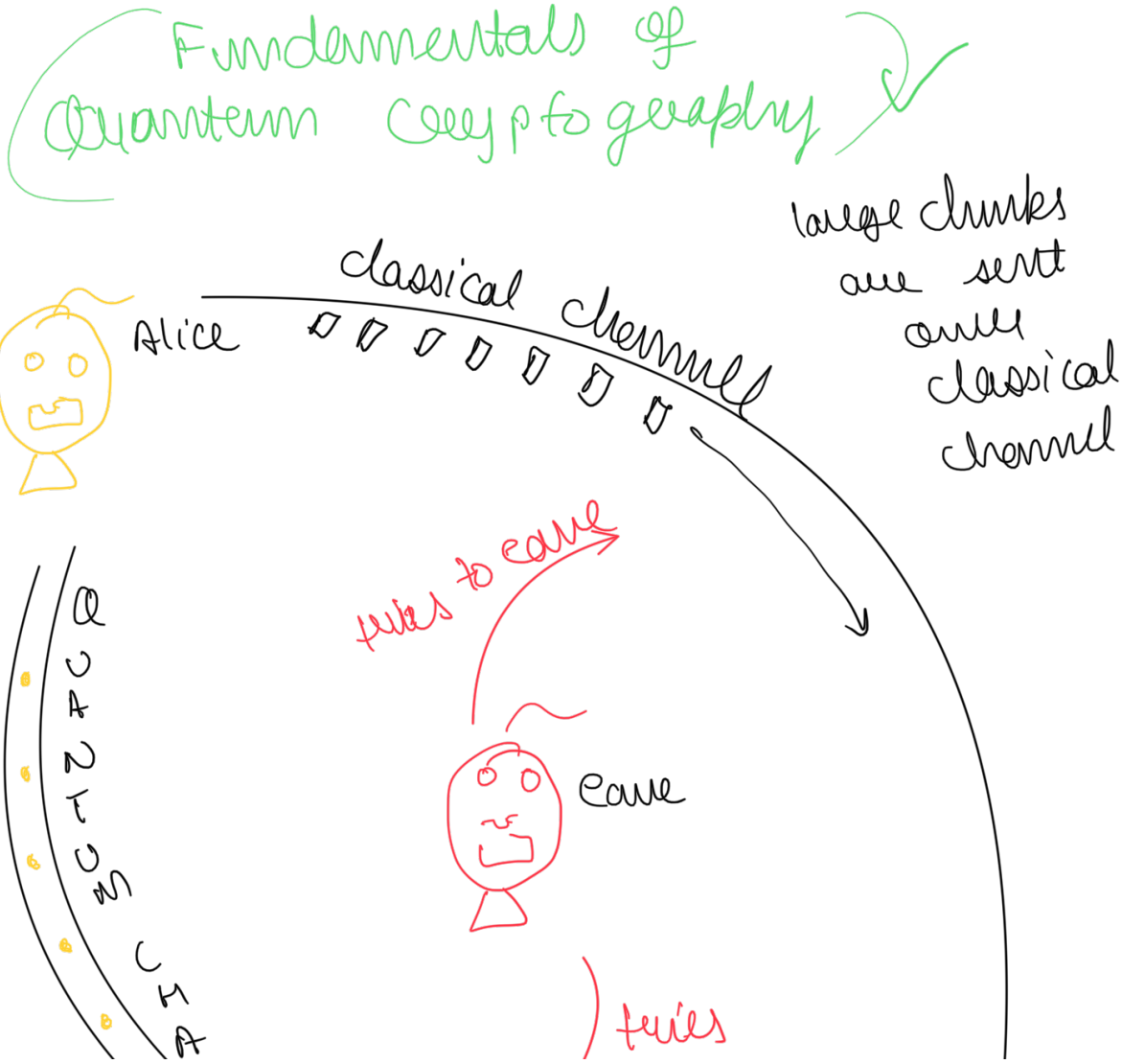
In contrast to secret key cryptography, a shared secret key does not need to be established prior to communication in public key cryptography. Instead each party has a private key, which remains secret, and a public key, which they may distribute freely. If one party, say Alice, wants to send a message to another party, Bob, she would encrypt her message with Bob's public key after which only Bob could decrypt the message using his private key. While there is no need for key exchange, the security of public key cryptography algorithms are currently all based on the unproven assumption of the difficulty of certain problems such as integer factorization or the discrete logarithm problem. This means that public key cryptography algorithms are potentially vulnerable to improvements in computational power or the discovery of efficient algorithms to solve their underlying problems. Indeed algorithms have already been proposed to perform both integer factorization and solve the discrete logarithm problem in polynomial time on a quantum computer [Shor97] [Bruss07].

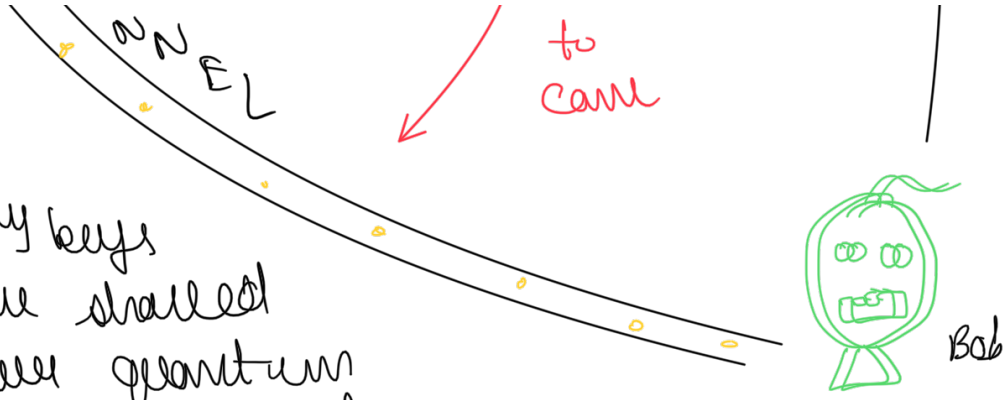
the major quantum advantage of introducing quantum mechanics is, having the entangled photons to get collapsed when tried to observe by eavesdropper. This is the inherent property of quantum photon particle. Entangled particles collapse on getting observed. Exploiting this property we can start QKD

While the advent of a feasible quantum computer would make current public key cryptosystems obsolete and threaten key distribution protocols such as Diffie-Hellman, some of the same principles that empower quantum computers also offer an unconditionally secure solution to the key distribution problem. Moreover, quantum mechanics also provides the ability to detect the presence of an eavesdropper who is attempting to learn the key, which is a new feature in the field of cryptography. Because the research community has been focused primarily on using quantum mechanics to enable secure key distribution, quantum cryptography and quantum key distribution (QKD) are generally synonymous in the literature.

Protocols are divided into 2 categories — Entanglement Based
↳ HUP (Heisenberg Uncertainty Principle)

From these principles the protocols are divided into two categories; those based primarily on the Heisenberg Uncertainty Principle, and those utilizing quantum entanglement. *While much of the recent research focus is on developing practical quantum cryptosystems [Bruss07]*

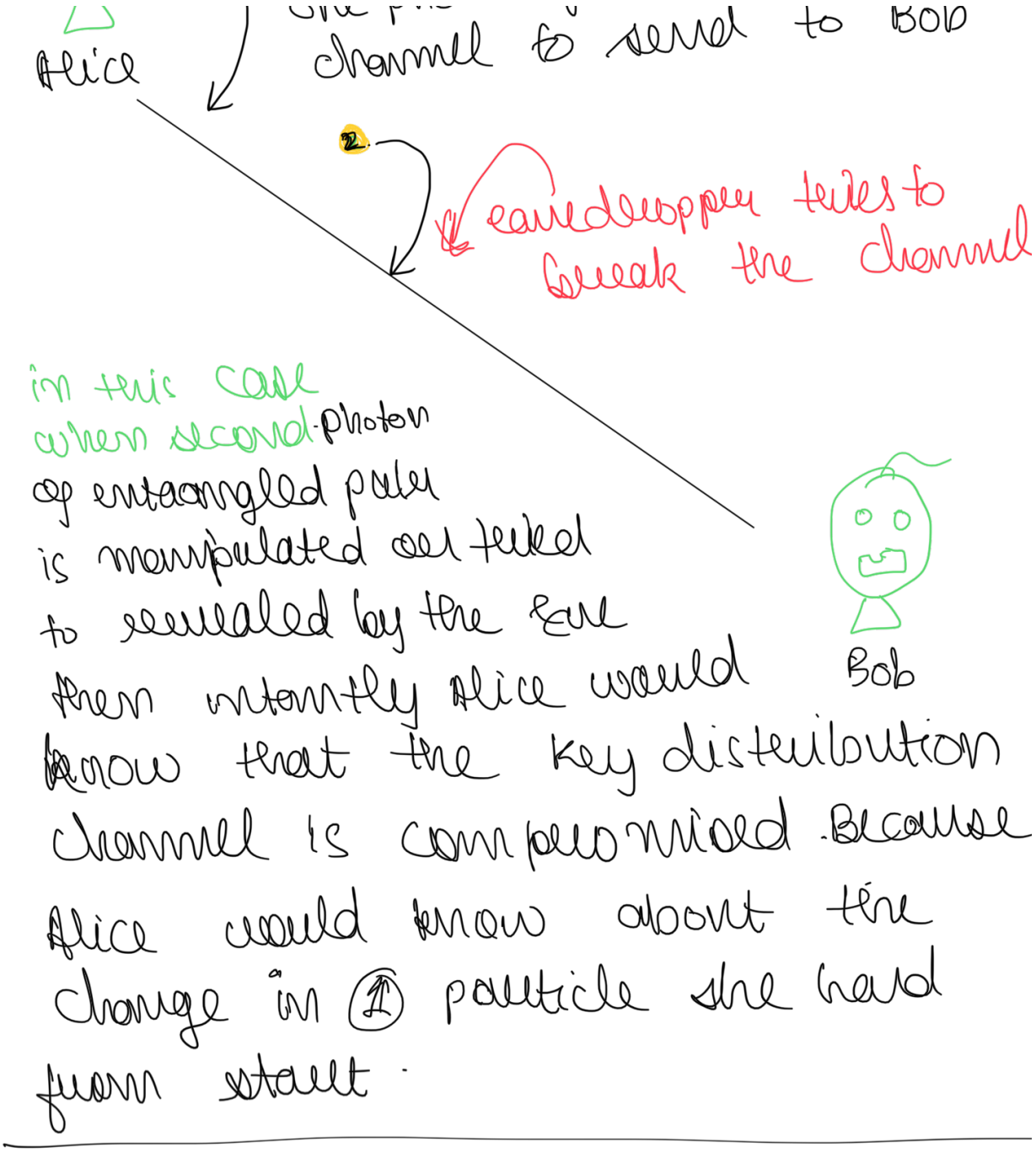




only keys are shared over quantum channel

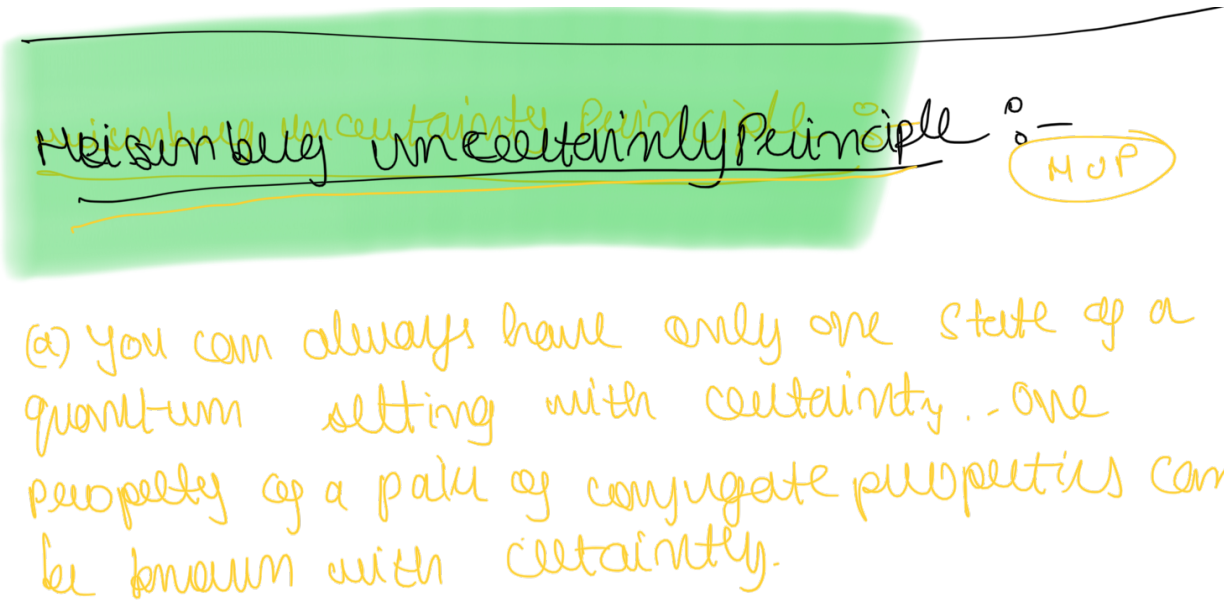
in perspective of eavesdropper, even, eavesdropper (Eve) is able to break the classical channel, the data packets will be of no use since the key remains unknown. whereas, the keys are shared over the quantum channel, which on interception will result in the detection of breach. As when Alice sends the quantum entangled pairs of photons to bob, if eve intercepts in between them, the entanglement will be collapse This can be shown as :-

one photon from pair is put on



The basic model for QKD protocols involves two parties, referred to as Alice and Bob, wishing to exchange a key both with access to a classical public communication channel and a quantum communication channel. This is shown in figure 1. An eavesdropper, called Eve, is assumed to have access to both channels and no assumptions are

made about the resources at her disposal. With this basic model established, we describe in layman's terms the necessary quantum principles needed to understand the QKD protocols.



Heisenberg Uncertainty Principle (HUP) which states that in a quantum system only one property of a pair of conjugate properties can be known with certainty. Heisenberg, who was initially referring to the positioas the conjugate properties in question. This is because photons can be exchanged over fiber optic links and are perhaps the most practical quantum systems for transmission between two parties wishing to perform key exchange.

No Cloning Theorem : It is impossible to create multiple copies of quantum state the no cloning theorem 1982. states that its impossible to create multip identical copies of an arbitrarily unknown quantum state. The importan of no cloning theorem is that, without NCT, one may think to create multiple copies of same quantum particle. That is, one can overcome heisenberg uncertainty principle, by creating multiple copies of quantum state & then measuring its polariza of each copy to create many copies. However this will violate HUP Because otherwise its collapsed already if you would be knowing quantum state of each photon already. This is against the fundamental Properties of quantum mechanic

One principle of quantum mechanics, the no cloning theorem, intuitively follows from Heisenberg's Uncertainty Principle. The no cloning theorem, published by Wooters, Zurek, and Dieks in 1982 stated that it is impossible to create identical copies of an arbitrary unknown quantum state [Bruss07] [Wooters82]. One could see that without the no cloning theorem, it would be possible to circumvent Heisenberg's uncertainty principle by creating multiple copies of a quantum state and measuring a different conjugate property on each copy. This would allow one to simultaneously know with certainty both conjugate properties of the original quantum particle which would violate HUP.

2.2 Quantum Entanglement :-

Eckert 91 :

The other important principle on which QKD can be based is the principle of quantum entanglement. It is possible for two particles to become entangled such that when a particular property is measured in one particle, the opposite state will be observed on the entangled particle instantaneously. This is true regardless of the distance between the entangled particles. It is impossible, however, to predict prior to measurement what state will be observed thus it is not possible to communicate via entangled particles without discussing the observations over a classical channel. The process of communicating using entangled states,

aided by a classical information channel, is known as quantum teleportation and is the basis of Eckert's protocol as will be described in Section 4 [Eckert91] .

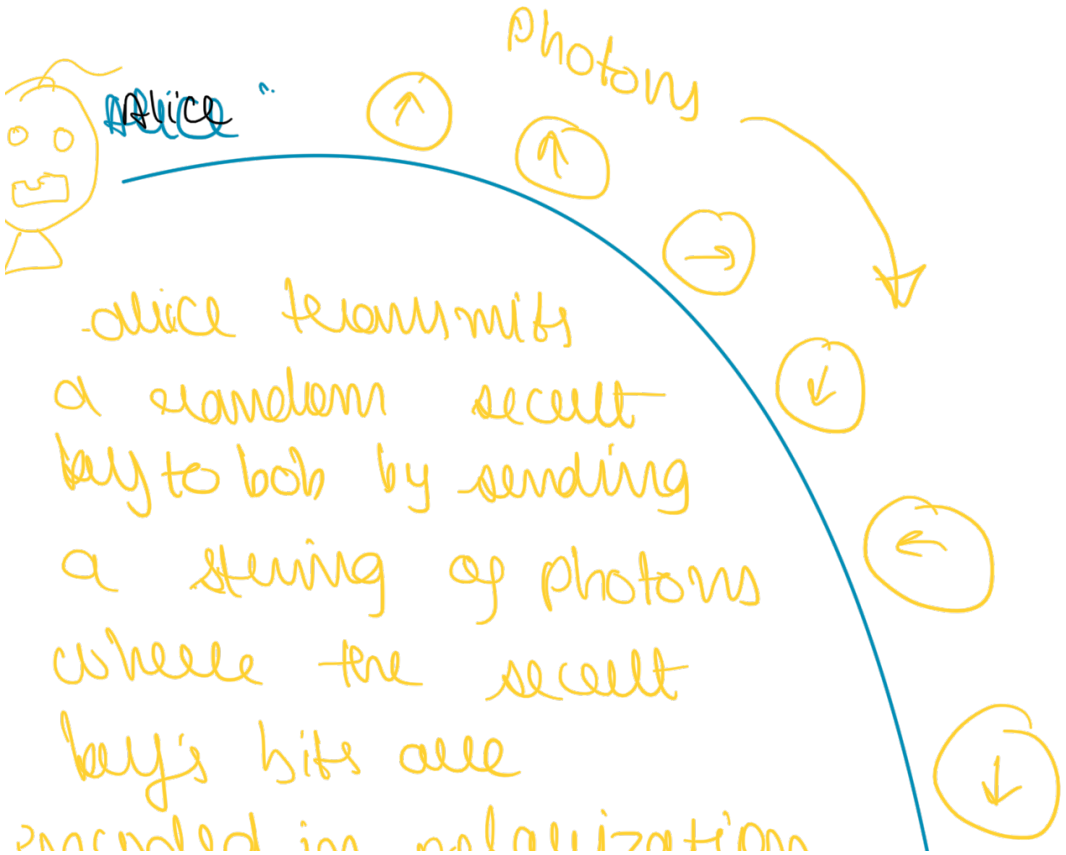
BB84: BB84 IMPORTANT :-

1984 = C. Bennett and G. Brassard

(a) most prominent quantum cryptography protocol.

(b) All other HUP based protocols are more or less variants of this protocol.

(c) Basic idea:



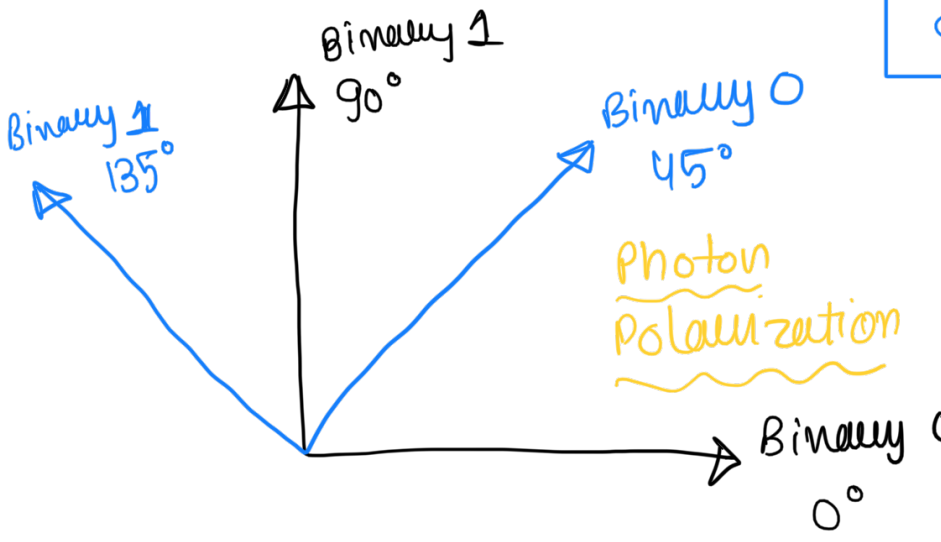
of the photons.

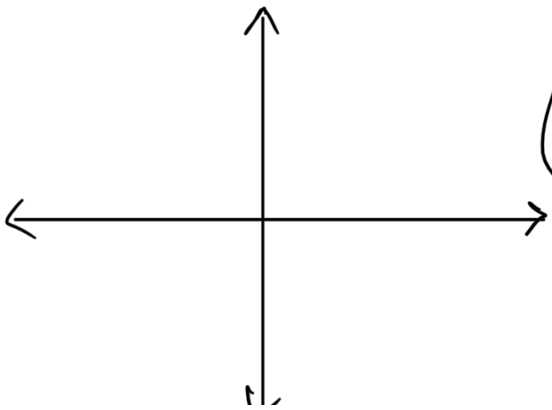
Bob

The HUP guarantees that eve cannot measure the photons and transmit them on to Bob without disturbing the photons state in detectable way. Thus revealing her presence.

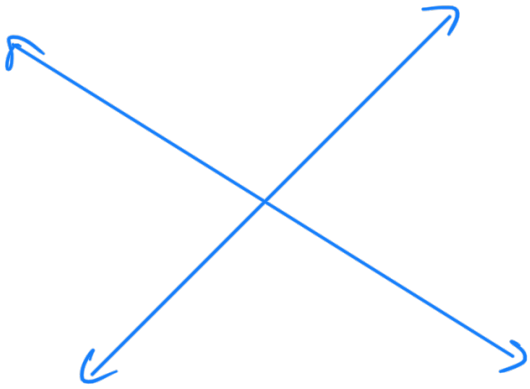
3.1 BB84 Protocol :-

Bit Encoding Schemes





(rectilinear basis)



(Diagonal Basis)

Phase 1 of communication :

Bit String = 00110100 1

(a) Alice → quantum channel → Bob

(b) bits $\xleftarrow{\$}$ $\{0,1\}^n$

(c) basis $\xleftarrow{\$}$ $\{X, +\}^n$

(d) Encode (bits) using basis

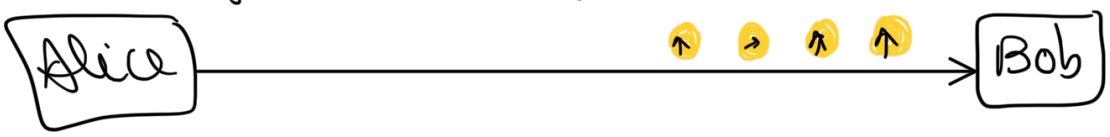
encoded bit(s) ← encode (bits, basis)

(e)

transmit photon corresponding to

each bit with corresponding polarization.

(f) For every photon bob receives, Bob measures the photon's polarization by a randomly chosen basis



(g) encoded bit (from Alice) should match decoded bits (Bob)

That is :-

In the first phase, Alice will communicate to Bob over a quantum channel. Alice begins by choosing a random string of bits and for each bit, Alice will randomly choose a basis, rectilinear or diagonal, by which to encode the bit. She will transmit a photon for each bit with the corresponding polarization, as just described, to Bob. For every photon Bob receives, he will measure the photon's polarization by a randomly chosen basis. If, for a particular photon, Bob chose the same basis as Alice, then in principle, Bob should measure the same polarization and thus he can correctly infer the bit that Alice intended to send. If he chose the wrong basis, his result, and thus the bit he reads, will be random.
In the second phase, Bob will notify Alice over any insecure channel what basis he used to measure each photon. Alice will report back to Bob whether he chose the correct basis for each photon. At this point Alice and Bob will discard the bits corresponding to the photons which Bob measured with a different basis. Provided no errors occurred or no one manipulated the photons, Bob and Alice should now both have an identical string of bits which is called a sifted key. The example below shows the bits Alice chose, the bases she

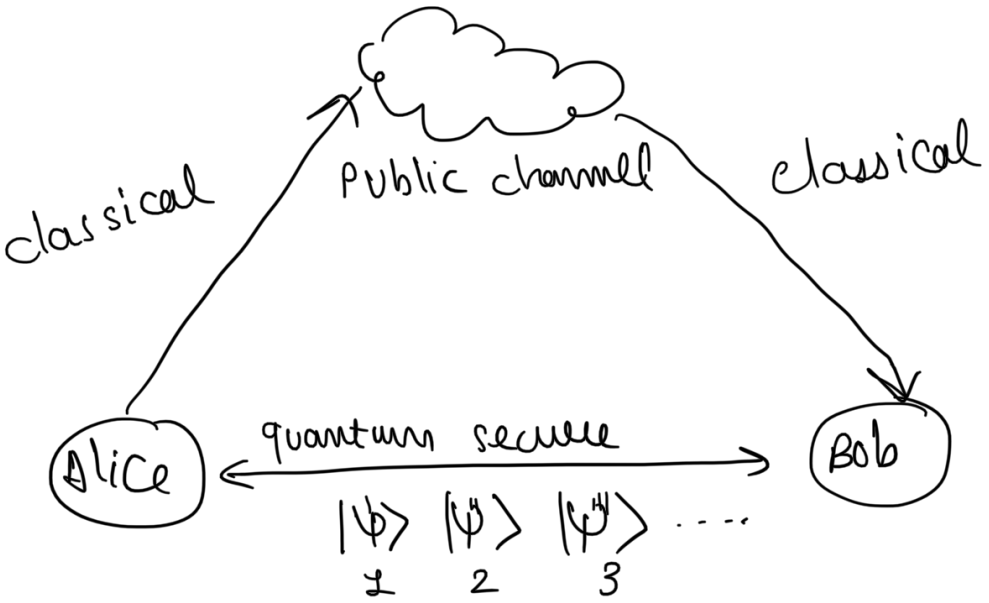
encoded them in, the bases Bob used for measurement, and the resulting sifted key after Bob and Alice discarded their bits as just mentioned [Wiki-SIFT].

Alice's bit	0	1	1	0	1	0	0	1
Alice's basis	+	+	X	+	X	X	X	+
Alice's polarization	↑	→	↖	↑	↖	↗	↗	→
Bob's basis	+	X	X	X	+	X	+	+
Bob's measurement	↑	↗	↖	↗	→	↗	→	→
Public discussion								
Shared Secret key	0		1			0		1

Before they are finished however, Alice and Bob agree upon a random subset of the bits to compare to ensure consistency. If the bits agree, they are discarded and the remaining bits form the shared secret key. In the absence of noise or any other measurement error, a disagreement in any of the bits compared would indicate the presence of an eavesdropper on the quantum channel. This is because the eavesdropper, Eve, were attempting to determine the key, she would have no choice but to measure the photons sent by Alice before sending them on to Bob. This is true because the no cloning theorem assures that she cannot replicate a particle of unknown state [Wooters82]. Since Eve will not know what bases Alice used to encoded the bit until after Alice and Bob discuss their measurements, Eve will be forced to guess. If she measures on the incorrect bases, the Heisenberg Uncertainty Principle ensures that the

information encoded on the other bases is now lost. Thus when the photon reaches Bob, his measurement will now be random and he will read a bit incorrectly 50% of the time. Given that Eve will choose the measurement basis incorrectly on average 50% of the time, 25% of Bob's measured bits will differ from Alice [Rieffel00]. If Eve has eavesdropped on all the bits then after n bit comparisons by Alice and Bob, they will reduce the probability that Eve will go undetected to $¾^n$ [Lomonaco98]. The chance that an eavesdropper learned the secret is thus negligible if a sufficiently long sequence of the bits are compared.

BB84 protocol in simpler words :—

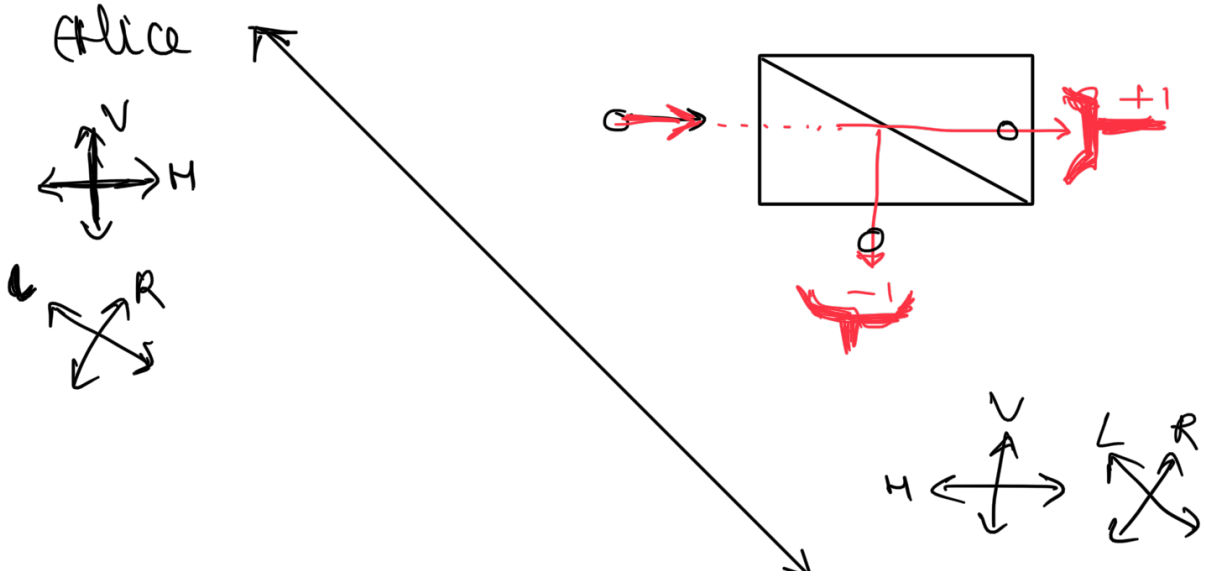


(#) any measurement on an individual quantum object leaves a trace.

In other words its impossible for Eve to read the Quantum information on the photon without perturbing it. This perturbation can be detected by Alice / Bob. who can communicate over classical public channel. which does not need to be secure.

The polarization of the photons turned out to be the privileged support for quantum key distr

The BB84 scheme with polarized one-photon wave packets :-



Bob

In order to generate a key, that will be sent to bo
Alice has a source of polarized photon and
she can choose a polarization of each photon
among 4 possibilities associated with 2 diff.
linear polarization basis. Two are along the
axis x and y and are often noted V and
vertical
The two other polarization are at 45° and a
usually noted R & L for Right & Left.
Both are orthogonal polarizations. That is to
say, the 2 output channels of polarizing beam
splitter conviniently oriented the 2 bits
corresponds to 2 polarizing 2 beam splitter
at 45° from each other. They are assoc
two non commuting observables. When bob
receives a photon, you analyzes its polarization
with a polarization beam splitter of which it
chooses randomly the orientation b/w 2
possibilities either vertical @ 45°, if its
choice corresponds to polarization sent by
Alice, he obtains a result corresponding
to the value chosen by Alice. This is case
for instance, if Alice sends a photon wi
(V) or (H) and if it puts it beam split
at (0°/90°) from x axis. But if we make
the wrong choice that is to say if Bob puts
polarizer/detector in 45°/135° from x when

Alice was sent [illegible] photon [illegible]
a random result un-correlated with the initial choice of Alice. so he gets the right information in often cases only. It may seem a poor result, but infact, there is a simple way for Bob and Alice to know what were the right choices and keep these only this is called

RECONCILIATION.

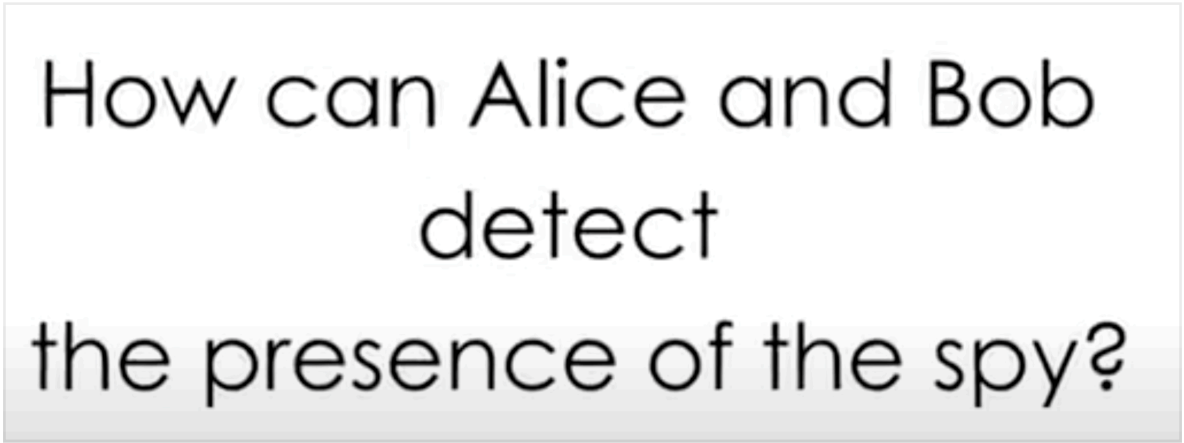
In order for Alice and Bob to know what were the right choices, its a faces that bob announces on the public channel what were the choices of basis, and then Alice replies which ones are the correct via the public channel. Alice and bob now knows what were the situation where both had right polarization & wrong ones. They keep only right cases.

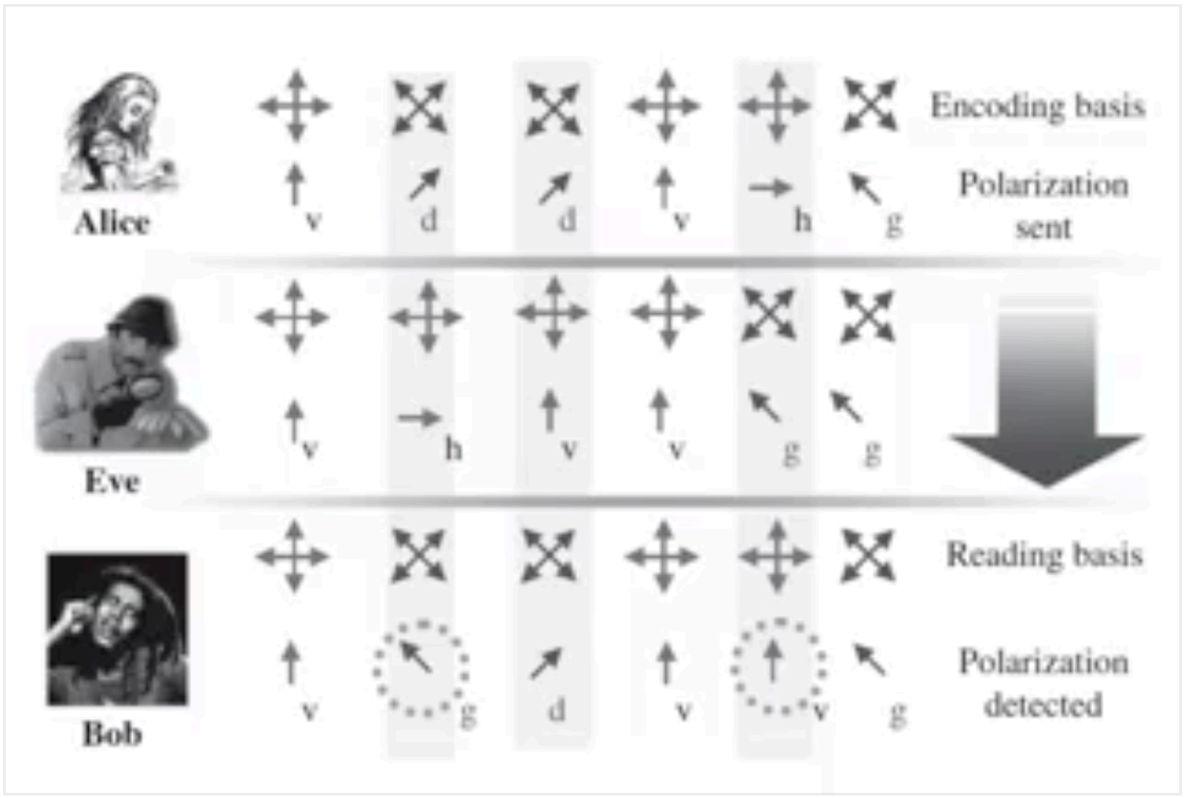
After elimination, they keep only identical cases and removes the error from final strings. The communication for chosen orientations b/w Alice and bob can happen over public channel. It can be listened to &

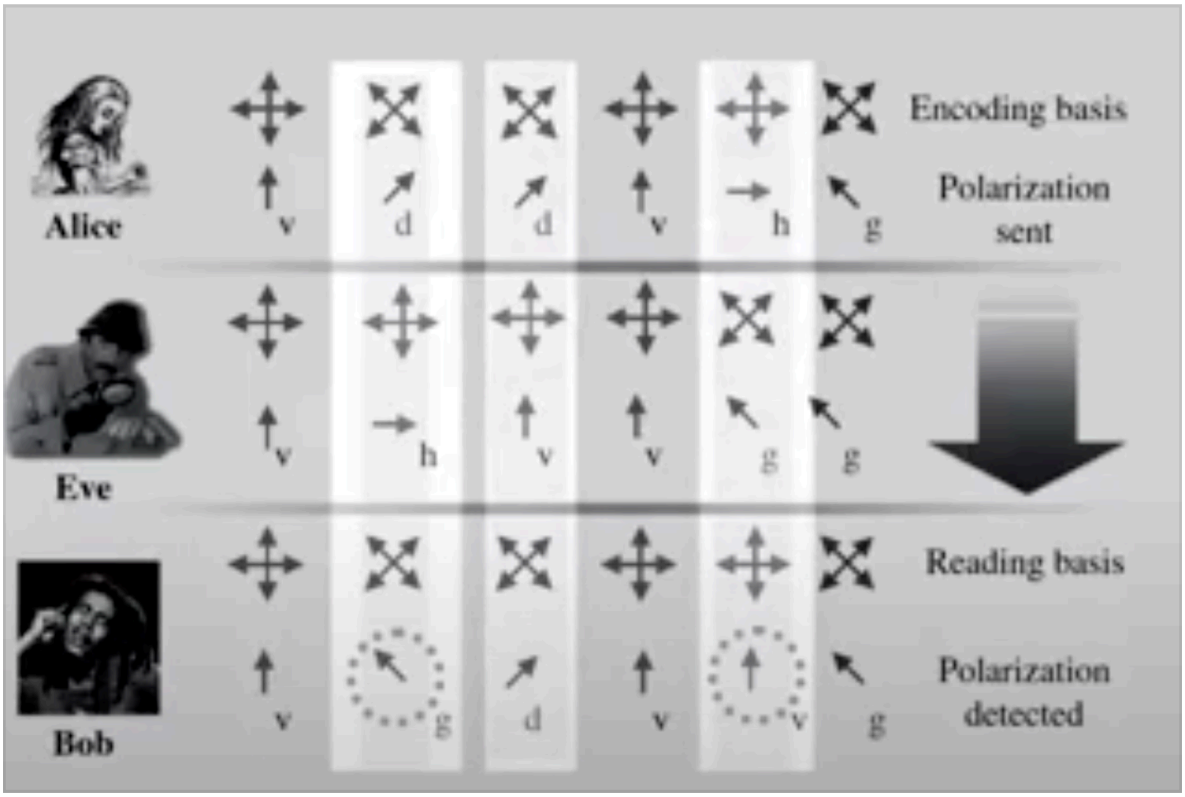
Tenjing to get copy of key 8

let us pretend we are Eve and try to obtain the same information as Bob without being detected to get information we must intercept the polarized photon and make a polarization measurement on it but that Bob will receive nothing and does this fit on this photon will not be used for the reconciliation procedure so we must be a smarter evil after doing the measurement we will use a 1 Photon Source to resend a photon towards Bob with the polarization we have just found ball will then receive a photo with a polarization that we know so if this photon is used for the key and

information we will get on the public channel we will know that value of a bit indicate let us now take the point of view of Alice and Bob were as smart and know as much quantum optics as if they understand that if you do what we have just been describing measuring a photon polarization and resending a photon with the results I have just found can they detect such maneuver the answer is yes can you find how you do not find yet I'll give you a hint what Alice and Bob can do is sacrifice a subset of the key they are established after reconciliation more precisely Bob will choose randomly some of the cases where they agreed and tell on the public channel what he found in disguises with this information Alice can tell that there is an eavesdropper is there is one can you tell why I am sure most of you I found the answer let us describe it in detail since it will allow us to fully appreciate where the quantum nature of the signal plays a role the series of cases shown here allow us to understand there are cases when it does not choose a simpler ization orientation at the one decided by ELISA here it is cases number 2 3 and 5 in these cases the result found by Eve is random but this is not the point the fact is that in these cases ball will find a random result which means that in half these cases he finds the wrong direction as shown here in cases 2 & 5 so when receiving the results found by Bob in cases 2 & 5 Alice will observe that they are wrong although Bob had chosen the right basis she will thus conclude that there is a spy on the channel and 1 Bob that this game must not be used can you tell what is a fraction of the cases where Bob gets a wrong result although we have chosen the right basis





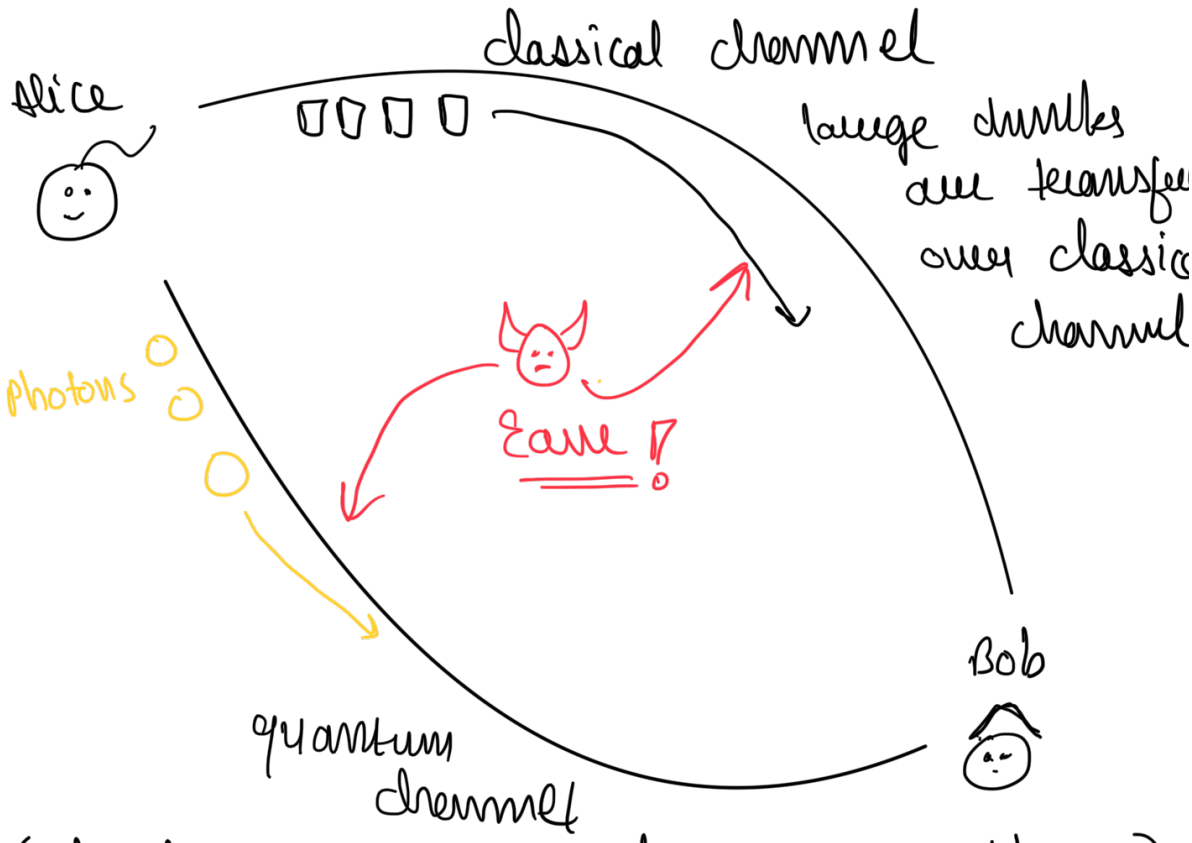


Notes:

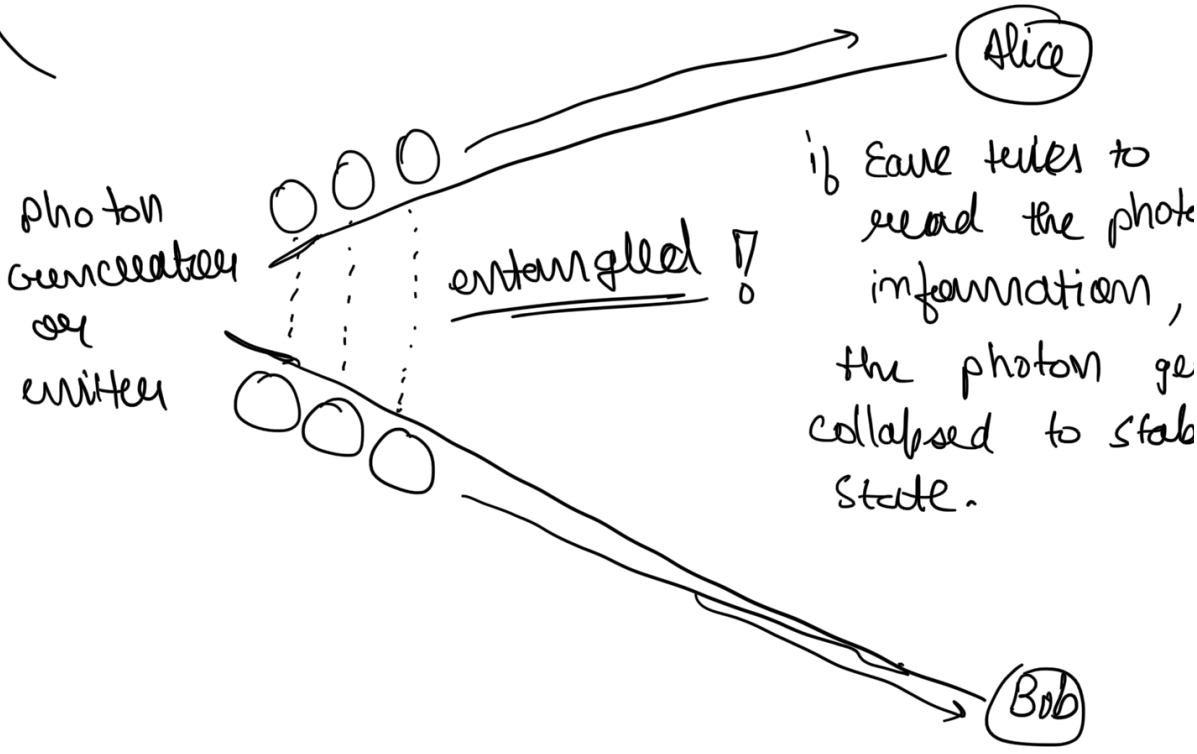
April 21, 2021 :-

(I) Fundamentals of Quantum

cryptography



only keys are agreed over quantum channel.



Quantum key Distribution :-

Quantum Key Distribution :-

Exploits 3 properties of Quantum System :-

(a) Quantum Entanglement
(b) Quantum Superposition
(c) Uncertainty Principle

∴ protocols are also designed considering the properties.

HUP :-

(a) You can only have one state of a quantum subsystem with full certainty. That is, a photon can be 0 or 1 with certainty.

(b) No cloning theorem :- for one case, its impossible to look a photon particle and measure its properties and replace it with a copy of that photon however this violates HUP, otherwise the particle would have collapsed already if you would be knowing Quantum State of each photon. which is against how these particles work in nature.

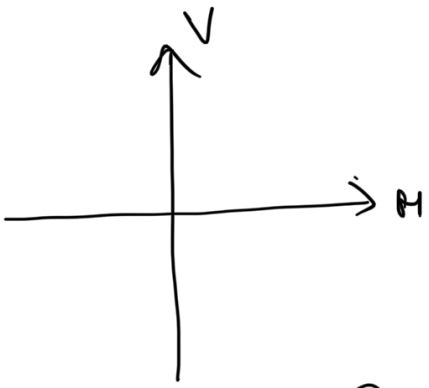
BB84

(a) most celebrated QKD protocol

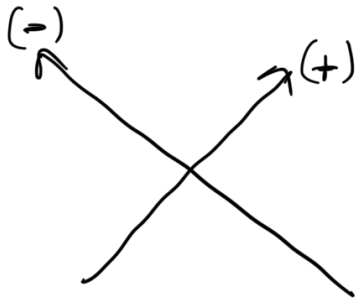
(b) all other QKD based protocols are more or less dependent on it.

Alice transmits a random secret key by sending a string of photons whose secret key's qbits are encoded via polarization / basis of photons.

Basis / Encoding / Polarization :-

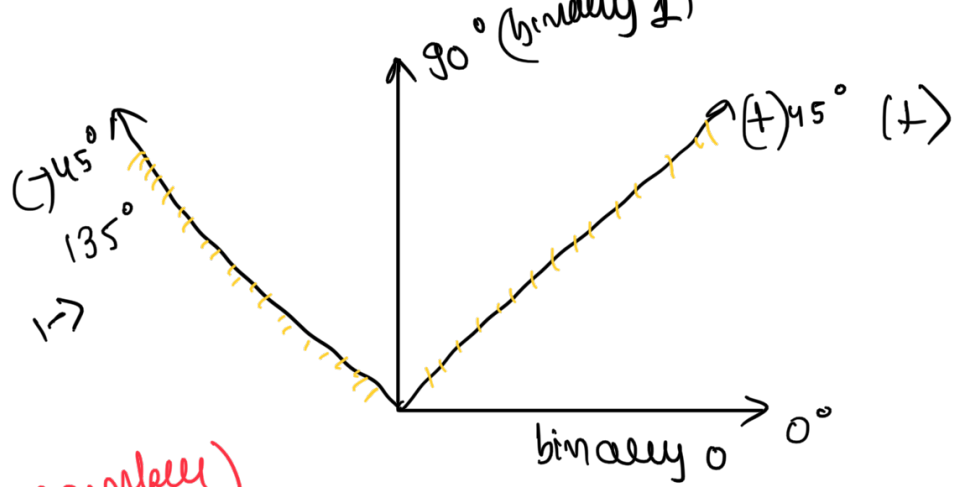


(Rectilinear basis)



(Diagonal basis)

Bit encoding scheme



(remember that)

$|\psi_{00}\rangle = |0\rangle$ $|\psi_{01}\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|\psi_{10}\rangle = |1\rangle$ $|\psi_{11}\rangle = |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Communication

(a) Alice —— Q channel ——→ Bob

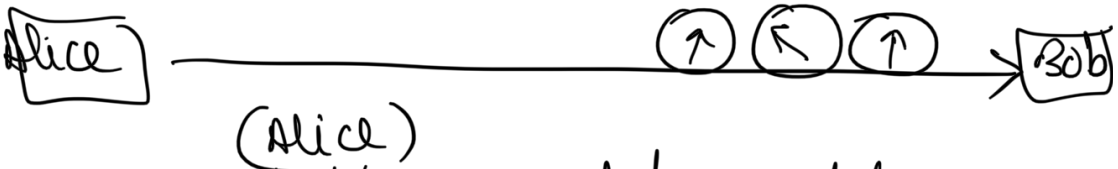
(b) bits ←—— $ (CoinToss / Randomness) ——— $\{0,1\}^n$

(c) Encode (bits) using basis

(d) encoded bits (S) ⟵ encode (bits, basis)

(e) Alice ——————→ Bob
transmit photon corresponding to each bit with corresponding polarization.

(f) for every photon bob receives, bob measures the photon's polarization by a randomly chosen basis



(Alice)

(g) Encoded bits should match

decoded bits (Bob)

(without care) (Let us analyze this a little more :-

BB84 :-

(a) Table/ works on one qubit at a time single line denotes qubit and double line means classical bit.

$\$$ = coin flip random

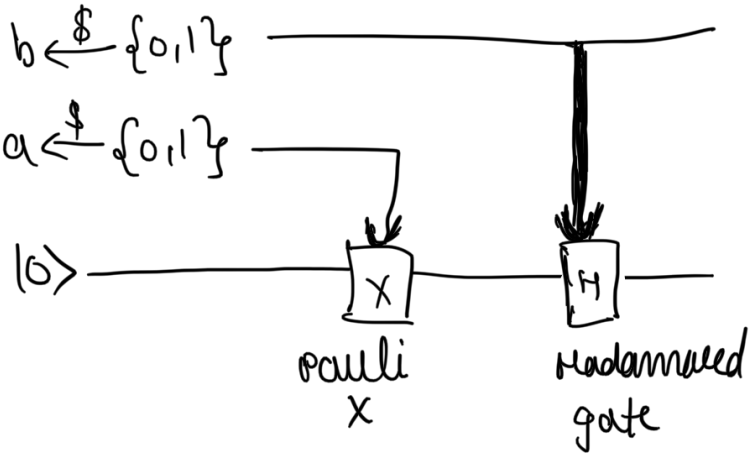
$b \xleftarrow{\$} \{0,1\}$

$a \xleftarrow{\$} \{0,1\}$

$|0\rangle$ ——

Alice

This will proceed to the communical channell as :-



we call pauli x = classical NOT

Hadamard gate = takes input and places it in

superposition.

Alice

$H: \{0,1\} \rightarrow \{+,-\}$

$b \xleftarrow{\$} \{0,1\}$

$a \xleftarrow{\$} \{0,1\}$

$|0\rangle$ — X — H_1 — state till now

X: pauli X

H_1: Hadamard gate

a	b	$\|\psi\rangle$ final state
0	0	$\|0\rangle$
0	1	$\|+\rangle$
1	0	$\|1\rangle$
1	1	$\|-\rangle$

(Hadamard gate)

$\{ |1\rangle \rightarrow |-\rangle$, $|0\rangle \rightarrow |+\rangle$, $|+\rangle \rightarrow |0\rangle$, $|-\rangle \rightarrow |1\rangle \}$

Bob $b' \xleftarrow{\$} \{0,1\}$

H_2

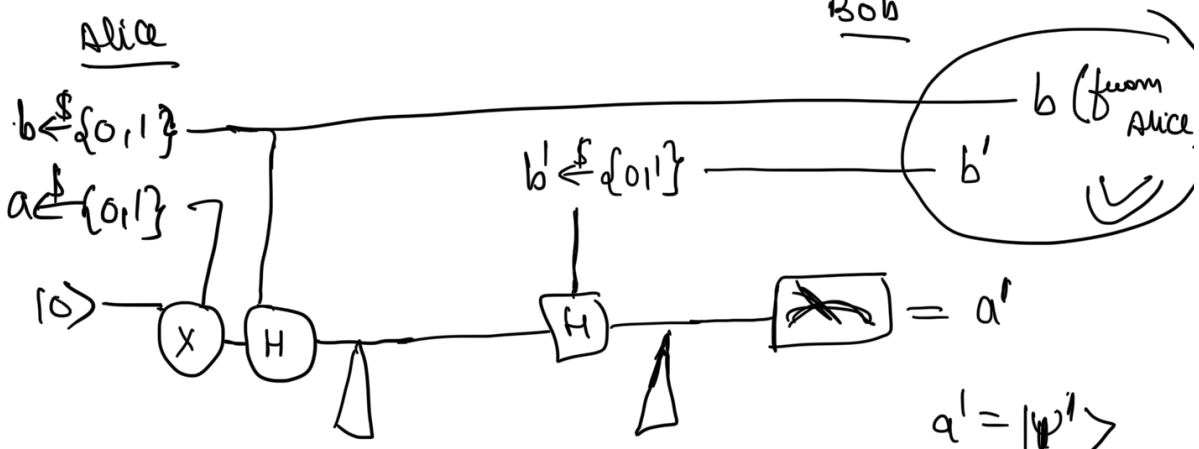
a	b	b'	$\|\psi'\rangle$ final state after H_2
0	0	0	$\|0\rangle$

$\|0\rangle$	0	0	1	$\|+\rangle$
$\|+\rangle$	0	1	0	$\|+\rangle$
	0	1	1	$\|1\rangle$
$\|-\rangle$	1	0	0	$\|-\rangle$
	1	0	1	$\|-\rangle$
$\|1\rangle$	1	1	0	$\|1\rangle$
	1	1	1	$\|-\rangle$

measured?

Bob

Alice



$(b' = b)$? ✓ :

$b' = b \mid b' \neq b$

can be done via classical channel

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one observation can be done :-

when $b' = b$, we get $a \Rightarrow |\psi'\rangle$

$|\psi'\rangle \longleftarrow a$ ✓

[illegible] by bob in his side are identical at that place where $b' = b \Rightarrow |\psi'\rangle = a$ the classical bit

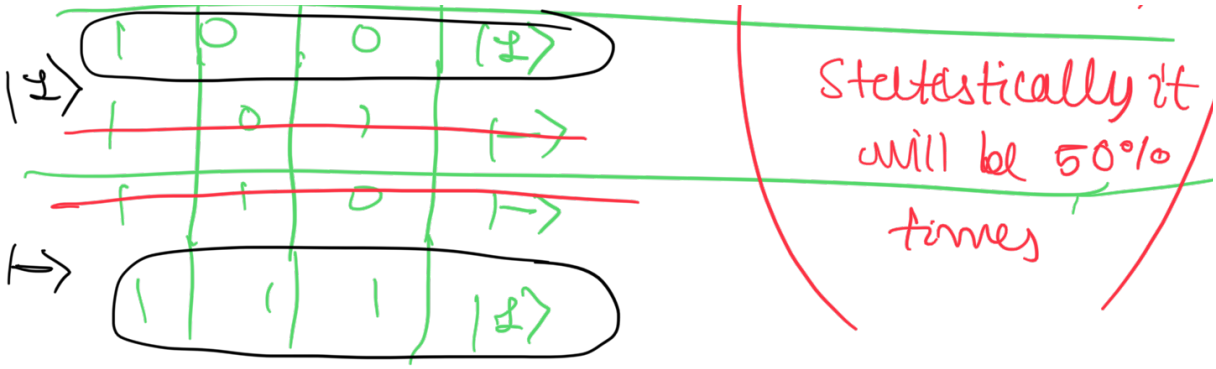
In other words, when we have same filters at both Alice & bob sides we get identical matches to the Alice's arrangement. when bob finds different (ie., $b' \neq b$) it will be discarded. Because bob will commu over classical channel to confirm, whether his 22nd bit or 76th bit or any numbered bit matches or not. whatever matches will be use of the private key.

Now :

a	b	b'	$\|\psi'\rangle$ final state after H_2
0	0	0	$\|0\rangle$
0	0	1	$\|+\rangle$
0	1	0	$\|+\rangle$
0	1	1	$\|0\rangle$

$|0\rangle$ $|+\rangle$

Now reject ones which are not matched,



with sitting of eavesdropper, we let eavesdropper to get 50% advantage, that is, eavesdropper can detect classica $|0\rangle$ and $|1\rangle$ without collapsing the quantum state of photon, as its deterministic however, rest of the 50% of the times we will have $|+\rangle$ and $|-\rangle$ MUP states because of applying [H] gate. hence, MUP will collapse & eave tries to intercept comm^n.

That is, the crux of the formulation is, for

a	b	[H]$\|\psi\rangle$	
0	0	$\|0\rangle$	}
0	1	$\|+\rangle$	
1	0	$\|1\rangle$	
1	1	$\|-\rangle$	}
X	H		

(0)

for $|0\rangle$ and $|1\rangle$ the eave can measure simply in standard basis, this measurement won disturb the Quantum state, however, the remaining 2 states + and

so if the eave [illegible]
basis, but the qubit happens to be in
$|+\rangle$ or $|-\rangle$ state, then the measurem
will get disturbed. Because in standard
basis the qubit will be in $|0\rangle$ or $|1\rangle$.
So after measurement by eave it will be
0 or 1 & vice versa !

If eave tries to measure in $|+\rangle|-$
basis then it will ~~[illegible]~~ sacrifice
$|0\rangle$ and $|1\rangle$ states.
Thus eave can get only 50%
advantage at a time - (at most)

So, whatever basis eave choses, it will happen that state of qubit gets disturbed. ✓ There is no way for eavesdropper to perform measurem without disturbing the state of th qubit in any case.

Now when eave intercept, the condition that whenever $b = b'$, it will not hold true that $a = a'$ always, now because eave have intercepted comm.

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last example : $a = 0$, $b = 1$

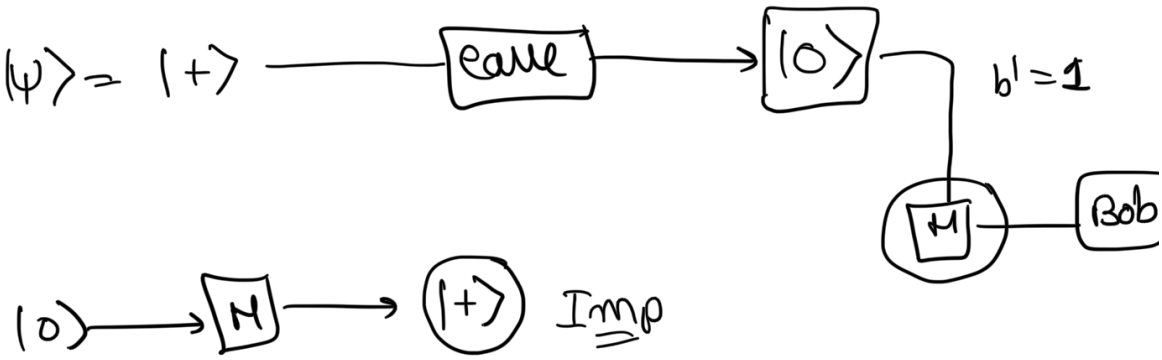
$\Rightarrow |\psi\rangle = |+\rangle$

② eave measures

now, after measurement, the resultant will be of standard basis, the qubit will be in lets say $|0\rangle$ then, assume $b' = 1$

That is :-

$$\left.\begin{matrix} a = 0 \\ b = 1 \end{matrix}\right\} \Rightarrow |\psi\rangle = |+\rangle$$



Now, the $|\psi'\rangle = |+\rangle$ & bob measure $|+\rangle$

Now, here observation is for $b' = b = a$ we must have got classical output either $|0\rangle$ or $|1\rangle$ or $a = a'$ which as $a = 0$ and $a' = |+\rangle$ that means channel is no more secure

- make keys longer than required
- Agree / Disagree four positions randomly from the key. by picking the digits
- not secure lines are discarded
- decide how much noise you can bear.

Analysis I came up with * Also other researchers have contributed to this work.

Alice
Eave
Bob

have the eave / eaves can easily guess the $|0\rangle$ and $|1\rangle$ as told before it will not affect the basis on measuring

a	b	$\|\psi\rangle$ final state
0	0	$\|0\rangle$
0	1	$\|+\rangle$
1	0	$\|1\rangle$
1	1	$\|-\rangle$

such states. ($|0\rangle$ and $|1\rangle$) ✓

This tells about the eave have at least 50% of the chances have success probability.

For addition, remaining 2 states $|+\rangle$ and $|-\rangle$ are the coin toss probability of 50 % again.

- Thus by analysis Eave will have 75% chances where she can succeed to guess the single qubit sent by Alice. Or, she can guess correct qubit 75% of the times.

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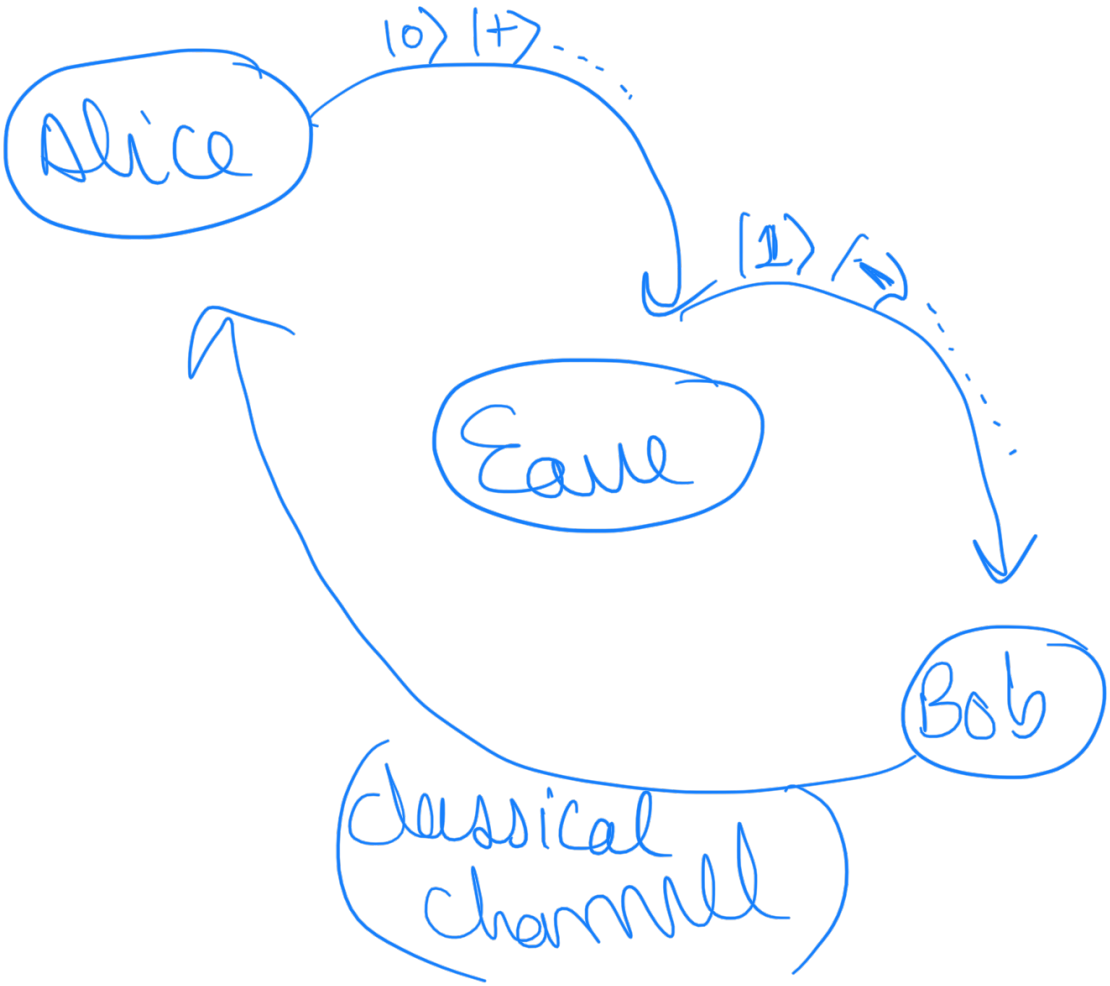
observation :-



Alice	Eave	Bob
a	$\bar{a}$	a (correct) 50%
a	$\bar{a}$	$\bar{a}$ (random)
a	a	a (undetected)
a	a	$\bar{a}$ (random)

I **Eve chooses the wrong basis:** $e_i = \overline{a_i} \longrightarrow$ qubit is altered.

Bob chooses correctly: $b_i = a_i \longrightarrow$ Eve introduces error with 50% probability.

Bob chooses incorrectly: $b_i = \overline{a_i} \longrightarrow$ random outcome, Eve is undetected.

II **Eve chooses the correct basis:** $e_i = a_i \longrightarrow$ qubit is unaltered.

Bob chooses correctly: $b_i = a_i \longrightarrow$ Eve is undetected and has one bit of the key.

Bob chooses incorrectly: $b_i = \overline{a_i} \longrightarrow$ random outcome, Eve is undetected.

These possibilities reveal that for each transmitted qubit, there is 75% probability that Eve's action goes undetected. The remaining 25% probability is due to Bob's correct choice when Eve chooses incorrectly: he obtains the wrong state from his basis and therefore decodes the wrong bit. Considering that Alice's and Bob's sequences of bits do not match exactly in such situation, they take an additional step to test against eavesdropping. They decide to select a subset of the remaining bits and compare them. If they don't match, they know for sure that Eve interfered. Of course, there is a compromise between the number of bits they want to "sacrifice" to discover Eve with a high probability and the length of the shared key, which decreases as they discard those bits that were compared.

The security of BB84 relies on only one imp. fact that if anyone wants to measure the quantum state then she must sacrifice of losing the spin of the photon particle / qubit if eave choses wrong filter than the spin in which photon / qubit was already

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Now, one paper I would like to quote is 2011, Arpita Maitra, have worked on calculating attacker's advantage :-

2011....

ISI Calc.

What is eave's target ?

To maximize the calculations / on measurements wrt the Alice / sender

So, whenever Alice and Bob tally these measurements by letting know their bases / filters over

public channel, Eave can intercept and understand that she has also used some of the bases and hence can confirm that some part of key she made is correct.

Remember Eave only can guess about the key and that guess is the attacker's advantage.

As we have seen, the reciever Bob recieves only 0/1 as on quantum channel, the Eave will always have 50% advantage to guess 0 or 1.

She only needs to decide whether the i^{th} bit was 0 or 1.

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if (Alice bit == Eave's bit)
    Success
else "Discard"
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Interesting Observation :-

We have seen the probability of guessing the qubit is 75% or 0.75, then, that is, the probability of Alice & Bob's bit getting same bit is 0.75

That done by some :-

Now, we know,

$$p_d + p_e = 1$$

and $p_d = 1 - p_e$

$$= 1 - (3/4)^n$$

for a very large n,

$$p_d = 1 - neg(n)$$

$$\simeq 1$$

Thus detection is very easy for Alice & Bob.