Survival probability

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The survival probability is defined as

$$S_p(t) = |\langle\psi_0|\psi(t)\rangle|^2, \tag{1}$$

Consider that $H|\phi_n\rangle = E_n|\phi_n\rangle$, then the survival probability S_p yields

$$S_p(t) = \left|\sum_{k,l} |\langle\psi_0|\phi_k\rangle|^2 |\langle\psi_0|\phi_l\rangle|^2 e^{-i(E_k-E_l)t}\right|^2. \tag{2}$$

Let us consider a symmetry of H under an arbitrary operator Π such that $H = H_1 \oplus H_2$—for instance, you may consider Π to be the reflection operator. Then, we can relabel the energies E_n such that $\{E_n\}_{n=1}^{p}$ and $\{E_n\}_{n=p+1}^{N}$ are the spectra of H_1 and H_2, respectively. N denotes the dimension of H, p the dimension of H_1 and $N-p$ is the dimension of H_2. After this relabeling, $S_p(t)$ takes the form:

$$S_p(t) = \left|\sum_{k,l=1}^{p} |\langle\psi_0|\phi_k\rangle|^2 |\langle\psi_0|\phi_l\rangle|^2 e^{-i(E_k-E_l)t} + \sum_{k,l=p+1}^{N} |\langle\psi_0|\phi_k\rangle|^2 |\langle\psi_0|\phi_l\rangle|^2 e^{-i(E_k-E_l)t}\right|^2 \tag{3}$$

$$= \left|\sum_{k,l=1}^{p} |\langle\psi_0|\phi_k\rangle|^2 |\langle\psi_0|\phi_l\rangle|^2 e^{-i(E_k-E_l)t}\right|^2 \tag{4}$$

$$+ \left|\sum_{k,l=p+1}^{N} |\langle\psi_0|\phi_k\rangle|^2 |\langle\psi_0|\phi_l\rangle|^2 e^{-i(E_k-E_l)t}\right|^2 \tag{5}$$

$$+ 2\,\mathrm{Re}\left\{\sum_{k=1}^{p+1} \sum_{l=p+1}^{N} |\langle\psi_0|\phi_k\rangle|^2 |\langle\psi_0|\phi_l\rangle|^2 e^{-i(E_k-E_l)t}\right\}. \tag{6}$$

The first two terms are recognized as the survival probability in the two symmetric subspaces. The third term God knows what the hell it is, but it is the term explaining why the correlation whole survives (or not).