Evaluating and Mitigating the Effects of Job Fair Recruiter Bias

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Appendix: Proofs

Proof of Proposition 1

Deriving Case 1 Equilibria Decision Rules

I start by taking first order conditions of the workers' payoff functions with respect to their strategies.

(A1) $\frac{\delta \mathrm{vf1}}{\delta \mathrm{af}} = \left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{1}{\mathrm{c}}\right)\left(\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{eh+el}}\right) - \mathrm{t\,s}$

(A2) $\frac{\delta \mathrm{vd1}}{\delta \mathrm{ad}} = \left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{1}{\mathrm{c}}\right)\left(\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{eh+el}}\right)(1-\mathrm{zu}) - \mathrm{t\,s}$

I then add symmetry by substituting

(A3) $\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}$

in for c, as discussed in 2.4, which yields the following.

(A4) $\frac{\delta \mathrm{vf1}}{\delta \mathrm{af}}\Big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} = \left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{ad(uhd+uld)+af(uhf+ulf)}}\right) - \mathrm{t\,s}$

(A5) $\frac{\delta \mathrm{vd1}}{\delta \mathrm{ad}}\Big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} = \left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{ad(uhd+uld)+af(uhf+ulf)}}\right)(1-\mathrm{zu}) - \mathrm{t\,s}$

Favored workers will have incentive to go to the career fair if A4 is positive, will have incentive not to go if it is negative, and will be indifferent between going and not going if it is zero. Discriminated workers' incentives are similarly described by the sign of A5.

Since these equations are specific to case 1 equilibria, ad and af cannot both be zero. Hence the denominators on the first terms of A4 and A5 can be multiplied to both sides of the equations $\frac{\delta \mathrm{vf1}}{\delta \mathrm{af}}\Big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} > 0$ and $\frac{\delta \mathrm{vd1}}{\delta \mathrm{ad}}\Big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} > 0$, respectively.

Doing so yields

$\mathrm{eh\,xh + el\,xl > r\,s(af(uhf + ulf) + ad(uhd + uld))}$ and

$\mathrm{(1 - zu)(eh\,xh + el\,xl) > r\,s(af(uhf + ulf) + ad(uhd + uld))}$, which are the expressions for rules 1fa and 1da, respectively.

Simplifying the expressions

$\frac{\delta \mathrm{vf1}}{\delta \mathrm{af}}\big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} < 0$ and $\frac{\delta \mathrm{vd1}}{\delta \mathrm{ad}}\big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} < 0$

in a similar way yields the expressions for rules 1fn and 1dn, while simplifying the expressions

$\frac{\delta \mathrm{vf1}}{\delta \mathrm{af}}\big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} = 0$ and $\frac{\delta \mathrm{vd1}}{\delta \mathrm{ad}}\big|_{\mathrm{c}=\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld)}}{\mathrm{eh+el}}} = 0$

yields the expressions for rules 1fi and 1di.

Deriving Case 2 Equilibria Decision Rules

As in case 1, I start by taking the first order conditions of the workers' payoff functions with respect to their strategies.

(A6) $\frac{\delta \mathrm{vf2}}{\delta \mathrm{af}} = \left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{\mathrm{ehxh+elxl}}{\mathrm{eh+el}}\right) - \mathrm{t\,s}$

(A7) $\frac{\delta \mathrm{vd2}}{\delta \mathrm{ad}} = \left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{eh+el}}\right)(1-\mathrm{zu}) - \mathrm{t\,s}$

Note that these expressions are identical to A1 and A2, except that they lack the $\frac{1}{\mathrm{c}}$ term. As there is no need for further substitution, A6 and A7 describe worker incentives for case 2 equilibria as A4 and A5 did for case 1 equilibria. The case 2 decision rules 2fa−2di derive directly from the simplification of $\frac{\delta \mathrm{vf2}}{\delta \mathrm{af}}>0$, $\frac{\delta \mathrm{vd2}}{\delta \mathrm{ad}}=0$, etc., in the manner shown previously for the case 1 decision rules.

Proof of Lemma 1

Note that

(A8) $\mathrm{eh\,xh + el\,xl} \geq (1-\mathrm{zu})(\mathrm{eh\,xh + el\,xl})$

for all values of zu between 0 and 1, inclusive.

For all possible values in that range, A1 yields the following:

1. 1da $\Rightarrow$ 1fa, which proves L1 for case 1.
2. 1fn $\Rightarrow$ 1dn, which proves L2 for case 1.
3. 1fi $\Rightarrow$ 1di (if $\mathrm{zu} = 0$) or 1dn (if $\mathrm{zu} > 0$), which proves L3 for case 1.
4. 1di $\Rightarrow$ 1fa (if $\mathrm{zu} > 0$) or 1fi (if $\mathrm{zu} = 0$), which proves L4 for case 1.

The proof that lemma 1 holds for the case 2 to rules is similar, with the only difference being that $\mathrm{r\,s(eh+el)}$ replaces $\mathrm{r\,s(af(uhf+ulf)+ad(uhd+uld))}$ as the intermediate expression with which the left and right hand sides of A8 are compared in rule pairings.

Proof of Proposition 2

Note that the number of workers in attendance is $\mathrm{af(uhf+ulf)+ad(uhd+uld)}$ while the number of recruiters is $\mathrm{eh+el}$. This observation is sufficient to prove (6) and (7).

The remainder of proposition 2 comes from determining which decision rules can be simultaneously applied by both favored and discriminated workers and then determining the conditions under which workers will not have incentive to deviate from the equilibrium results.

By lemma 1, the following combinations cannot occur:

1fn with 1da or 1di

2fn with 2da or 2di

1fi with 1da

2fi with 2da

Additionally, 1fn and 1dn cannot occur together since if no workers attended, the result would not be a case 1 equilibrium.

However, every other combination is possible since by observation the inequalities presented by the remaining rule pairings do not contradict.

Since the decision rules for the case 2 equilibria are already in terms of parameter values and since the strategies of each individual worker does not depend on the behavior of others, the decision rules are generally sufficient to describe conditions under which workers will have no incentive to deviate from equilibrium strategies. However, in the case of equilibria 2C and 2D additional restrictions for the range of possible randomized strategies can be determined by incorporating (7).

Each of the equilibria 1A, 1B, 1C, 1D, 1E, 2C, and 2D are examined below.

Equilibrium 1A

Suppose that rules 1fa and 1da occur simultaneously. Note that only 1da binds as zu must be between 0 and 1. Since attending favored workers will always get at least the payoffs of attending discriminated workers, it is sufficient to determine under which conditions discriminated workers will have no incentive to deviate.

If all workers attend, then A3 becomes

(A9) $\frac{\text{uhf}+\text{ulf}+\text{uhd}+\text{uld}}{\text{eh}+\text{el}}$

By substituting 1 for ad and A9 for c in vd1, I find that the benefit of attending for a discriminated worker in equilibrium 1A is $\frac{\text{t}(\text{eh}\,\text{xh}+\text{el}\,\text{xl})(1-\text{zu})}{\text{r}(\text{uhd}+\text{uhf}+\text{uld}+\text{ulf})}$, so that a discriminated worker will have no incentive to deviate as long as $\frac{\text{t}(\text{eh}\,\text{xh}+\text{el}\,\text{xl})(1-\text{zu})}{\text{r}(\text{uhd}+\text{uhf}+\text{uld}+\text{ulf})} > \text{t}\,\text{s}$, which simplifies to

(A10) $(\text{eh}\,\text{xh}+\text{elxl})(1-\text{zu}) > \text{r}\,\text{s}(\text{uhd}+\text{uhf}+\text{uld}+\text{ulf})$

A10 is referred to as (8) in proposition 2.

Equilibrium 1B

Suppose that rules 1fa and 1dn occur simultaneously.

If only favored workers attend, then A3 becomes

(A11) $\frac{\mathrm{uhf+ulf}}{\mathrm{eh+el}}$

Substituting 1 for af and A11 for c in vf1 yields the benefit of attending for a favored worker in equilibrium 1B, which is $\frac{\mathrm{t(eh\,xh+el\,xl)}}{\mathrm{r\,(uhf+ulf)}}$.

Hence favored workers will have no incentive to deviate unless $\frac{\mathrm{t(eh\,xh+el\,xl)}}{\mathrm{r(uhf+ulf)}} < \mathrm{t\,s}$, which simplifies to

(A12) $\mathrm{eh\,xh + el\,xl < r\,s(uhf + ulf)}$

On the other hand, if a discriminated worker decides to deviate by attending, A3 becomes

(A13) $\frac{\mathrm{uhf+ulf+1}}{\mathrm{eh+el}}$

Substituting 1 for ad and A13 for c in vd1 yields the benefit of attending for the deviating worker, which is $\frac{\mathrm{t(eh\,xh+el\,xl)(1-zu)}}{\mathrm{r(uhf+ulf+1)}}$.

Therefore, a discriminated worker will only have incentive to deviate if $\frac{\mathrm{t(eh\,xh+el\,xl)(1-zu)}}{\mathrm{r(uhf+ulf+1)}} > \mathrm{t\,s}$,

which simplifies to

(A14) $\mathrm{(eh\,xh + el\,xl)(1 - zu) - r\,s > r\,s(uhf + ulf)}$

A12 and A14 jointly show that equilibrium 1B requires

(A15) $\mathrm{(eh\,xh + el\,xl)(1 - zu) - r\,s \le r\,s(uhf + ulf) < eh\,xh + el\,xl}$

A15 is reported in proposition 2 as (9).

Equilibrium 1C

Suppose that rules 1fa and 1di occur simultaneously.

Then unless favored workers deviate and as long as discriminated workers behave symmetrically, A3 becomes

(A16) $\frac{\mathrm{uhf+ulf+ad(uhd+uld)}}{\mathrm{eh+el}}$

Substituting 1 for af and A16 for c in vf1 yields the benefit of attending for favored workers, which is $\frac{\mathrm{t(eh\,xh+el\,xl)(1-zu)}}{\mathrm{r(uhf+ulf+ad(uhd+uld))}}$.

Then favored workers will not have incentive to deviate unless $\frac{\mathrm{t(eh\,xh+el\,xl)(1-zu)}}{\mathrm{r(uhf+ulf+ad(uhd+uld))}} < \mathrm{t\,s}$, which simplifies to

$(\mathrm{eh\,xh} + \mathrm{el\,xl})(1 - \mathrm{zu}) < \mathrm{r\,s}(\mathrm{uhf} + \mathrm{ulf} + \mathrm{ad}(\mathrm{uhd} + \mathrm{uld}))$, so that this equilibrium holds only if

(A17) $\mathrm{ad} \leq \frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld})}$

Conversely, if a discriminated worker deviates by attending with probability 1, then A3 becomes

(A18) $\frac{\mathrm{uhf}+\mathrm{ulf}+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld}-1)+1}{\mathrm{eh}+\mathrm{el}}$

Substituting 1 for ad and A18 for c in vd1 yields the benefit of deviation for discriminated workers, which is $\frac{\mathrm{t}(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r}(1+\mathrm{uhf}+\mathrm{ulf}+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld}-1))}$.

Then discriminated workers will only incentive to deviate and attend unless

$\frac{\mathrm{t}(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r}(1+\mathrm{uhf}+\mathrm{ulf}+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld}-1))} \leq \mathrm{t\,s}$, which simplifies to

$\frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(1+\mathrm{uhf}+\mathrm{ulf})}{\mathrm{r\,s}} \leq \mathrm{ad}(\mathrm{uhd} + \mathrm{uld} - 1)$.

Since uhd + uld > 1 by initial assumptions, then this becomes

(A19) $\mathrm{ad} \geq \frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf}+1)}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld}-1)}$

An explicit expression for ad can be found by noticing that discriminated workers receive $\mathrm{t\,s}$ in this equilibrium. If a discriminated worker does not deviate from the symmetric strategy ad, then A3 becomes

(A20) $\frac{\mathrm{uhf}+\mathrm{ulf}+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld})}{\mathrm{eh}+\mathrm{el}}$

Substituting A20 in for c in vd1 yields $\mathrm{t\,s}(1 - \mathrm{ad}) + \frac{\mathrm{ad\,t}(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r}(\mathrm{uhf}+\mathrm{ulf}+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld}))}$.

Since discriminated workers employ a mixed strategy in this equilibrium, then it must be that

$\mathrm{t\,s}(1 - \mathrm{ad}) + \frac{\mathrm{ad\,t}(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r}(\mathrm{uhf}+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld})+\mathrm{ulf})} = \mathrm{t\,s}$, which simplifies to

(A21) $\mathrm{ad} = \frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld})}$

since ad > 0 or this equilibrium is not distinct from 1B.

Therefore, A17 holds with equality when 1C is distinct from 1B.

Combining A19 and A21 shows that $\frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf}+1)}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld}-1)} \leq \frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld})}$, which simplifies to

(A22) $(\mathrm{eh\,xh} + \mathrm{el\,xl})(1 - \mathrm{zu}) \leq \mathrm{r\,s}(\mathrm{uhd} + \mathrm{uhf} + \mathrm{uld} + \mathrm{ulf})$

since uhd + uld > 1.

This contradicts (8) in proposition 2, so that this result can only occur when 1C is distinct from 1A. Therefore, since A22 derives from A21, A22 occurs when 1C is distinct from both 1A and 1B.

Proposition 2 reports A21 as (10) and A22 as (11).

Equilibrium 1D

Suppose that rules 1fi and 1dn are simultaneously applied.

By 1fi, any case 1 equilibrium where favored workers are indifferent requires

$\mathrm{eh\,xh + el\,xl = r\,s(af(uhf + ulf) + ad(uhd + uld))}$, which means that for equilibrium 1D to exist,

it must be that

(A23) $\mathrm{eh\,xh + el\,xl = r\,s(af(uhf + ulf))}$

since no discriminated workers attend in that case.

Solving A23 for af yields shows that af must equal

(A24) $\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{r\,s(uhf+ulf)}}$

in equilibrium.

A24 is reported as (12) in proposition 2.

Also, since af is strictly a mixed strategy, then af < 1, so that $\mathrm{r\,s(af(uhf + ulf)) < r\,s(uhf + ulf)}$.

By A24, the above implies

(A25) $\mathrm{eh\,xh + el\,xl < r\,s(uhf + ulf)}$

if this equilibrium exists.

A25 is reported as (13) in proposition 2.

Suppose a discriminated worker decided to deviate by attending. Then, substituting A24 for af, A3 becomes

(A26) $\frac{\mathrm{r\,s+eh\,xh+el\,xl}}{\mathrm{r\,s(eh+el)}}$

Substituting 1 for ad and A26 for c in vd1 yields the discriminated worker's benefit for deviating, which is

$\frac{\mathrm{t\,s(eh\,xh+el\,xl)(1-zu)}}{\mathrm{r\,s+eh\,xh+el\,xl}}$.

Hence, a discriminated worker will have incentive to deviate unless

$\frac{\mathrm{t\,s(eh\,xh+el\,xl)(1-zu)}}{\mathrm{r\,s+eh\,xh+el\,xl}} \leq \mathrm{t\,s}$, which simplifies to $\mathrm{zu} \geq -\frac{\mathrm{r\,s}}{\mathrm{eh\,xh+el\,xl}}$, which is always true.

Therefore, discriminated workers will never have incentive to deviate by attending in this case.

Suppose instead that a favored worker deviates by always attending.

Then substituting A24 in for the non-deviating favored worker's strategy in A3 yields

(A27) $\frac{\mathrm{r\,s(uhf+ulf)+(uhf+ulf-1)(eh\,xh+el\,xl)}}{\mathrm{r\,s(uhf+ulf)(eh+el)}}$

Substituting 1 for af and A27 for c in vf1 yields the favored worker's benefit for deviating, which is

$\frac{\mathrm{t\,s(uhf+ulf)(eh\,xh+el\,xl)}}{\mathrm{r\,s(uhf+ulf)+(uhf+ulf-1)(eh\,xh+el\,xl)}}$.

Thus, a favored worker will deviate by attending unless $\frac{\mathrm{t\,s(uhf+ulf)(eh\,xh+el\,xl)}}{\mathrm{r\,s(uhf+ulf)+(uhf+ulf-1)(eh\,xh+el\,xl)}} \leq \mathrm{t\,s}$,

which simplifies to

(A28) $\mathrm{eh\,xh + el\,xl \leq r\,s(uhf + ulf)}$

since $\mathrm{uhf + ulf} > 1$.

Note that the restriction presented by A28 is consistent with that of A25.

The application of rule 1dn does not add additional information about the range of possible worker strategies, but it does show that this equilibrium requires recruiter bias to exist (see corollary 1 for more information on the existence of bias for each equilibrium).

Equilibrium 1E

Suppose that rules 1fi and 1di are simultaneously applied.

By 1fi, $\mathrm{eh\,xh + el\,xl = r\,s(af(uhf + ulf) + ad(uhd + uld))}$.

However, obtaining expressions for af and ad in terms of exogenous parameters is not possible in this case since there are multiple combinations of the strategies that could satisfy this relationship. This rule is therefore left as is for proposition 2.

1di provides no additional information on the possible values of af and ad. However, it does jointly show with 1fi that 1E cannot exist in the presence of recruiter bias (see corollary 1 for more information on the existence of bias for each equilibrium).

Equilibrium 2C

Suppose rules 2fa and 2di are simultaneously satisfied.

Since case 2 rules are independent of worker behavior, workers will never have incentive to deviate.

Note also that all case 2 equilibria must satisfy $\mathrm{af(uhf + ulf) + ad(uhd + uld) \leq eh + el}$ by (7).

Then, substituting 1 for af, solving the above for ad yields

(A29) $\mathrm{ad} \leq \frac{\mathrm{eh+el-(uhf+ulf)}}{\mathrm{uhf+ulf}}$

A29 is given as (14) in proposition 2.

Equilibrium 2D

Suppose rules 2fi and 2dn are simultaneously satisfied.

Again, workers will never have incentive to deviate since the rules hold independent of worker strategies and all case 2 equilibria must satisfy $\mathrm{af(uhf+ulf)+ad(uhd+uld)} \leq \mathrm{eh+el}$ by (7).

Then substituting 0 for ad, solving the above for af yields

(A30) $\mathrm{af} \leq \frac{\mathrm{eh+el}}{\mathrm{uhf+ulf}}$

A30 is given as (15) in proposition 2.

Proof of Corollary 1

Note that a discriminated worker will only have less incentive to attend than a favored worker if zu is positive. Hence equilibria 1B, 1C, 1D, 2B, 2C, and 2D all require the presence of recruiter bias to exist (when distinct from 1A and 2A). This is also directly observable in the decision rules, which will only be equivalent across types if zu is 0. Directly verifying that the decision rules confirm these bias-dependent incentives is straightforward, and is thus left to the interested reader.

Note also that for equilibrium 1E, requiring 1fi and 1di to hold simultaneously gives the result $\mathrm{eh\,xh+el\,xl = r\,s(af(uhf+ulf)+ad(uhd+uld)) = (eh\,xh+el\,xl)(1-zu)}$, so that zu must be 0. The case of equilibrium 2E is similar, so the existence of either of these two equilibria requires the absence of recruiter bias.

As long as rule 1da (or rule 2da) holds, so will rule 1fa (or rule 2fa). Similarly, if 2fn holds, then so will 2dn. Hence, the value of zu is unrelated to the validity of equilibria 1A, 2A, and 2F, so they can hold with or without the presence of recruiter bias.

Proof of Lemma 2

Note that this lemma is trivially true for any case 2 equilibrium since each is trivially trembling hand perfect.

In a case 1 equilibrium, a worker's expected payoff from attending is inversely proportional to c, the average number of workers in attendance per recruiter. Since workers playing mixed strategies are able to do so in the presence of trembles as long as the trembles are sufficiently small, then the expected number of such workers at the event will not be affected by the introduction of trembles. Also, a worker who has incentive to attend in the absence of trembles will have at least as much incentive to do so if trembles cause one of her fellow workers not to attend who would have otherwise done so.

Therefore, the introduction of trembles can only have a negative effect on a worker's incentive to attend by causing some workers who would otherwise never attend to deviate by attending with some positive probability.

Proof of Proposition 3

Note first that all case 2 equilibria are trembling hand perfect since in those equilibria workers strategies do not affect each other. Therefore, it remains to prove the proposition for the case 1 equilibria.

I will assume that trembles will eventually be sufficiently small so as to not change whether there are more or fewer workers than recruiters in attendance at the career fair (i.e. eventually trembles will not cause a change from case 1 to case 2, although an equilibrium may be described by both cases if one worker attends for each recruiter).

To begin, note that the benefit a favored worker gets from attending is

(A31) $\left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{1}{\mathrm{c}}\right)\left(\frac{\mathrm{eh}}{\mathrm{eh+el}}\mathrm{xh}+\frac{\mathrm{el}}{\mathrm{eh+el}}\mathrm{xl}\right)$

while the benefit a discriminated worker gets from attending is

(A32) $\left(\frac{\mathrm{t}}{\mathrm{r}}\right)\left(\frac{1}{\mathrm{c}}\right)\left(\frac{\mathrm{eh}}{\mathrm{eh+el}}\mathrm{xh}+\frac{\mathrm{el}}{\mathrm{eh+el}}\mathrm{xl}\right)(1-\mathrm{zu})$

These expressions are the first terms in vf1 and vd1, respectively, when individual workers always attend.

The ratio of attending unemployed workers to recruiters can be represented as

(A33) $\frac{\mathrm{af(uhf+ulf)+ad(uhd+uld-1)+adj}}{\mathrm{eh+el}}$

or as

(A34) $\frac{\mathrm{af(uhf+ulf-1)+ad(uhd+uld)+afj}}{\mathrm{eh+el}}$

where afj and adj represent the strategies of individual favored and discriminated workers.

Suppose any worker deciding on a strategy considers that every favored worker plays attend with at most probability 1−k and plays don't attend with at least probability k, while every discriminated worker plays attend with at most probability 1−p and plays don't attend with at least probability p.

Then equilibrium 1A is trembling hand perfect by lemma 2. Also, 1E is trembling hand perfect when distinct from the other equilibria because, for small enough trembles, the workers can each play their mixed strategies.

It remains to determine which of the equilibria 1B−1D are trembling hand perfect, and under which conditions (if any).

Equilibrium 1B

Now suppose instead that each favored worker still decides to maximize her probability of attending, but that each discriminated worker decides to minimize her probability of attending, as in equilibrium 1B.

From the perspective of any discriminated worker, all favored workers would attend with probability 1−k and all other discriminated workers would attend with probability p, while the individual discriminated worker would attend with probability 0 if not deviating.

If the discriminated worker did deviate by choosing to attend, then A33 would become

(A35) $\frac{(1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld}-1)+1}{\mathrm{eh}+\mathrm{el}}$

Substituting A35 for c in A32 yields the discriminated worker's benefit for deviating, which is $\frac{\mathrm{t}(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r}\,(1+(1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld}-1))}$, so that the discriminated worker might have incentive to deviate by attending if $\frac{\mathrm{t}(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r}(1+(1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld}-1))} \geq \mathrm{t\,s}$, which simplifies to

(A36) $(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}) \geq \mathrm{r\,s}((1-\mathrm{p})+(1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld}))$

By 1dn, the discriminated worker's choice not to attend in the absence of trembles requires $(1-\mathrm{zu})(\mathrm{eh\,xh}+\mathrm{el\,xl}) < \mathrm{r\,s}(\mathrm{af}(\mathrm{uhf}+\mathrm{ulf})+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld}))$, which in the case of equilibrium 1B is $(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}) < \mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})$.

Hence, for small enough k and p, A36 will fail to hold. Thus, discriminated workers will not have incentive to deviate in the presence of trembles.

Taken from the perspective of a favored worker who decides to attend, A34 would become

(A37) $\frac{(1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf}-1)+\mathrm{p}(\mathrm{uhd}+\mathrm{uld})+1}{\mathrm{eh}+\mathrm{el}}$

Substituting A37 for c in A31 yields the favored worker's benefit for attending, which is $\frac{\mathrm{t}(\mathrm{eh\,xh}+\mathrm{el\,xl})}{\mathrm{r}((1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld})+\mathrm{k})}$. Then the favored worker will have no incentive to deviate as long as $\frac{\mathrm{t}(\mathrm{eh\,xh}+\mathrm{el\,xl})}{\mathrm{r}((1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld})+\mathrm{k})} > \mathrm{t\,s}$, which simplifies to

(A38) $\mathrm{eh\,xh}+\mathrm{el\,xl} > \mathrm{r\,s}((1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})+\mathrm{p}(\mathrm{uhd}+\mathrm{uld})+\mathrm{k})$.

By 1fa, the favored worker's choice to attend in the absence of trembles requires $\mathrm{eh\,xh}+\mathrm{el\,xl} > \mathrm{r\,s}(\mathrm{af}(\mathrm{uhf}+\mathrm{ulf})+\mathrm{ad}(\mathrm{uhd}+\mathrm{uld}))$, which in the case of equilibrium 1B is $\mathrm{eh\,xh}+\mathrm{el\,xl} > \mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})$.

Hence, for small enough k, A38 will hold. Thus, favored workers will still have incentive to attend in the presence of trembles, and so equilibrium 1B is trembling hand perfect.

Equilibrium 1C

Now suppose instead that each favored worker still attends with probability $1-\mathrm{k}$, but also that each discriminated worker plays the strategy $\frac{(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})-\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld})}$ as in (10). Suppose further that (10) is between 0 and 1, so that the equilibrium considered is distinct from equilibria 1A and 1B. Then for small enough trembles, discriminated workers will be able to play the strategy symmetrically in perturbed games.

If a discriminated worker decided to deviate by attending with probability 1, then A33 would become

(A39) $\left(\frac{1}{\mathrm{eh}+\mathrm{el}}\right)\left(1+(1-\mathrm{k})(\mathrm{uhf}+\mathrm{ulf})-\frac{(\mathrm{uhd}+\mathrm{uld}-1)(\mathrm{r\,s}(\mathrm{uhf}+\mathrm{ulf})-(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}))}{\mathrm{r\,s}(\mathrm{uhd}+\mathrm{uld})}\right)$

Substituting A39 for c in A32 yields the benefit of such deviation, which is

$\frac{\mathrm{t\,s}(\mathrm{uhd}+\mathrm{uld})(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r\,s}(\mathrm{uhf}+\mathrm{uhd}+\mathrm{uld}+\mathrm{ulf}-\mathrm{k}(\mathrm{uhd}+\mathrm{uld})(\mathrm{uhf}+\mathrm{ulf}))+(\mathrm{uhd}+\mathrm{uld}-1)(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}.$

Hence, the discriminated worker will have incentive to deviate if

$\frac{\mathrm{t\,s}(\mathrm{uhd}+\mathrm{uld})(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})}{\mathrm{r\,s}(\mathrm{uhf}+\mathrm{uhd}+\mathrm{uld}+\mathrm{ulf}-\mathrm{k}(\mathrm{uhd}+\mathrm{uld})(\mathrm{uhf}+\mathrm{ulf}))+(\mathrm{uhd}+\mathrm{uld}-1)(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})} > \mathrm{t\,s},$

which simplifies to

(A40) $(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}) > \mathrm{r\,s}(\mathrm{uhf}+\mathrm{uhd}+\mathrm{uld}+\mathrm{ulf}-\mathrm{k}(\mathrm{uhd}+\mathrm{uld})(\mathrm{uhf}+\mathrm{ulf}))$

as long as

(A41) $\mathrm{r\,s}(\mathrm{uhf}+\mathrm{uhd}+\mathrm{uld}+\mathrm{ulf}-\mathrm{k}(\mathrm{uhd}+\mathrm{uld})(\mathrm{uhf}+\mathrm{ulf})) > (1-\mathrm{uhd}-\mathrm{uld})(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu})$

Since k approaches 0 as favored workers' trembles become smaller and since $\mathrm{uhd}+\mathrm{uld}\geq 1$, A41 will eventually hold. As such, I will hereafter use A40 as the condition under which a discriminated worker will deviate by choosing to attend.

As indicated by (11), this equilibrium only holds in the absence of trembles if $(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}) \leq \mathrm{r\,s}(\mathrm{uhd}+\mathrm{uhf}+\mathrm{uld}+\mathrm{ulf})$.

Note that A40 does not hold if

(A42) $(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}) = \mathrm{r\,s}(\mathrm{uhd}+\mathrm{uhf}+\mathrm{uld}+\mathrm{ulf})$

but will eventually hold if

(A43) $(\mathrm{eh\,xh}+\mathrm{el\,xl})(1-\mathrm{zu}) < \mathrm{r\,s}(\mathrm{uhd}+\mathrm{uhf}+\mathrm{uld}+\mathrm{ulf})$

By lemma 2, favored workers will not deviate in the presence of trembles. Hence equilibrium 1C is trembling hand perfect when distinct from 1A and 1B, conditioned on the validity of A43.

A42 is reported as (29) in proposition 3.

Equilibrium 1D

Suppose now that discriminated workers attend with probability p and that other favored workers play the strategy $\frac{\mathrm{eh\,xh+el\,xl}}{\mathrm{r\,s(uhf+ulf)}}$ as in (12). Suppose further that (12) is less than 1, so that the equilibrium considered in distinct from 1B. Since the strategy is then a mixed strategy, it may be played by all favored workers in the presence of trembles as long as they are sufficiently small.

Replacing af and afj by (12) and ad by p in A34 yields the ratio of workers to recruiters in the presence of trembles if all favored workers play (12). This is

(A44) $\frac{\mathrm{r\,s\,p(uhd+uld)+eh\,xh+el\,xl}}{\mathrm{r\,s(eh+el)}}$

Substituting A44 for c and (12) for af in vf1 yields the expected payoff for an individual favored worker if choosing not to deviate from playing (12). This is

$\frac{\mathrm{t\,s(p\,r\,s(uhf+ulf)(uhd+uld)+(uhf+ulf)(eh\,xh+el\,xl)-p(eh\,xh+el\,xl)(uhd+uld))}}{\mathrm{p\,r\,s(uhf+ulf)(uhd+uld)+(uhf+ulf)(eh\,xh+el\,xl)}}$, which simplifies to

(A45) $\frac{\mathrm{t\,s(1-p(eh\,xh+el\,xl)(uhd+uld))}}{\mathrm{p\,r\,s(uhf+ulf)(uhd+uld)+(uhf+ulf)(eh\,xh+el\,xl)}}$

Note that A45 will be less than t s for all positive values of p. Hence, favored workers could always do better by not attending than by continuing to play the mixed strategy of 1D in the presence of trembles. 1D is therefore not trembling hand perfect.

Proof of Comparative Statics

All of the comparative statics are easily observable except for those relating to (22), which relate to recruiter payoffs in equilibrium 1C. The first-order derivatives of (22) are given below.

$$\frac{\delta}{\delta\mathrm{eh}} = \frac{\mathrm{el\,t((xh-xl)(uhd\,yh+uld\,yl)(1-ze)(1-zu)+r\,s(uhf\,uld\,yl+uhd\,ulf\,yh))}}{\mathrm{r^2 s(eh+el)^2(uhd+uld)}} - \frac{\mathrm{el\,t(r\,s(uhf\,uld\,yh+uhd\,ulf\,yl+ze(uhf\,uhd\,yh+uhf\,uld\,yl+ulf\,uhd\,yh+ulf\,uld\,yl)))}}{\mathrm{r^2 s(eh+el)^2(uhd+uld)}} \leq 0$$

$$\frac{\delta}{\delta\mathrm{el}} = -\frac{\mathrm{t(eh(xh-xl)(uhd\,yh+uld\,yl)(1-ze)(1-zu)+r\,s(uhf\,uld\,yh+uhd\,ulf\,yl+ze(uhf\,uhd\,yh+uhf\,uld\,yl+ulf\,uhd\,yh+ulf\,uld\,yl)))}}{\mathrm{r^2\,s(eh+el)^2(uhd+uld))}} + \frac{\mathrm{t\,r\,s(uhf\,uld\,yl+uhd\,ulf\,yh)}}{\mathrm{r^2\,s(eh+el)^2(uhd+uld)}} \leq 0$$

$$\frac{\delta}{\delta\mathrm{uhf}} = \frac{\mathrm{t(uld(yh-yl(1-ze))+uhd\,yh\,ze)}}{\mathrm{r(eh+el)(uhd+uld)}} \geq 0$$

$$\frac{\delta}{\delta\mathrm{ulf}} = \frac{\mathrm{t(uhd(yl-yh(1-ze))+uld\,yl\,ze)}}{\mathrm{r(eh+el)(uhd+uld)}} \geq 0$$

$$\frac{\delta}{\delta\mathrm{uhd}} = \frac{\mathrm{t\,uld(yh-yl)(1-ze)((eh\,xh+el\,xl)(1-zu)-r\,s(uhf+ulf))}}{\mathrm{r^2\,s(eh+el)(uhd+uld)^2}} \leq 0$$

$$\frac{\delta}{\delta\mathrm{uld}} = \frac{\mathrm{t\,uhd(yh-yl)(1-ze)(r\,s(uhf+ulf)-(eh\,xh+el\,xl)(1-zu))}}{\mathrm{r^2\,s(eh+el)(uhd+uld)^2}} \geq 0$$

$$\frac{\delta}{\delta\mathrm{zu}} = -\frac{t\mathrm{(eh\,xh+el\,xl)(uhd\,yh+uld\,yl)(1-ze)}}{\mathrm{r^2\,s(eh+el)(uhd+uld)}} \leq 0$$

$$\frac{\delta}{\delta\mathrm{ze}} = \frac{\mathrm{t(uhd\,yh+uld\,yl)(r\,s(uhf+ulf)-(eh\,xh+el\,xl)(1-zu))}}{\mathrm{r^2\,s(eh+el)(uhd+uld)}} \geq 0$$

Note that several of these derivatives will be 0 if specific conditions are met (e.g. $\mathrm{yh} = \mathrm{yl}$ and $\mathrm{ze} = 0$ for $\frac{\delta}{\delta \mathrm{uhf}}$).

The signs of the expressions reflect the following assumptions:

(A46) $\mathrm{r\,s(uhd + uhf) \geq (ehxh + elxl)(1 - zu)}$

(A47) $\mathrm{r\,s(uhf\,uld\,yh + uhd\,ulf\,yl + ze(uhf\,uhd\,yh + uhf\,uld\,yl + ulf\,uhd\,yh + ulf\,uld\,yl)) \geq}$

$\mathrm{(xh - xl)(uhd\,yh + uld\,yl)(1 - ze)(1 - zu) + r\,s(uhf\,uld\,yl + uhd\,ulf\,yh)}$

(A48) $\mathrm{eh(xh - xl)(uhd\,yh + uld\,yl)(1 - ze)(1 - zu) + r\,s(uhf\,uld\,yh + uhd\,ulf\,yl}$

$\mathrm{+ze(uhf\,uhd\,yh + uhf\,uld\,yl + ulf\,uhd\,yh + ulf\,uld\,yl)) \geq r\,s(uhf\,uld\,yl + uhd\,ulf\,yh)}$

(A49) $\mathrm{yl(uhd + uld\,ze) \geq yh(1 - ze)}$

The signs of $\frac{\delta}{\delta \mathrm{uhd}}$, $\frac{\delta}{\delta \mathrm{uld}}$, and $\frac{\delta}{\delta \mathrm{ze}}$ reverse with reversal of A46. Similarly, $\frac{\delta}{\delta \mathrm{eh}}$ reverses sign with reversal of A47, $\frac{\delta}{\delta \mathrm{eh}}$ reverses sign with reversal of A48, and $\frac{\delta}{\delta \mathrm{ulf}}$ reverses sign with reversal of A49.