Graphical Analysis of Calhoun’s Formula

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Abstract

The findings of this paper were to establish an algorithm that was created to find a polynomial function that relates voltage readings to temperature in °C. The purpose of this algorithm is to create a program for an integrated circuit to regulate the procedure of the ignition process in diesel automobiles with the lowest spatial complexity possible for the calculation of temperature. To achieve this, we graphed the data in *LibreOffice Calc* and created a polynomial trendline calculating the coefficient of determination and the subsequent theoretical percent error to determine a function with a third degree polynomial allowing the spatial complexity of the temperature calculation to be improved from a hash table $O(n)$ to an explicit algorithm $O(1)$. This was done with a coefficient of determination with a %99.9 line of best fit and an average %error of %3.

Objective

The objective of the analysis was to find a mathematical formula that would allow us to find the temperature readings of an onboard peripheral using the voltage measured through the analog spectrum as an input parameter. The formulas that we will be using in this analysis would be coefficient of determination, and percentage error.

$$R^2 = \frac{\sum_i (\hat{y}_i - \bar{y})^2}{\sum_i (y_i - \bar{y})^2} = \frac{SSR}{SST} \text{ (coefficient of determination)}$$

SSR: Sum of squared regression

SST: Sum of squared total

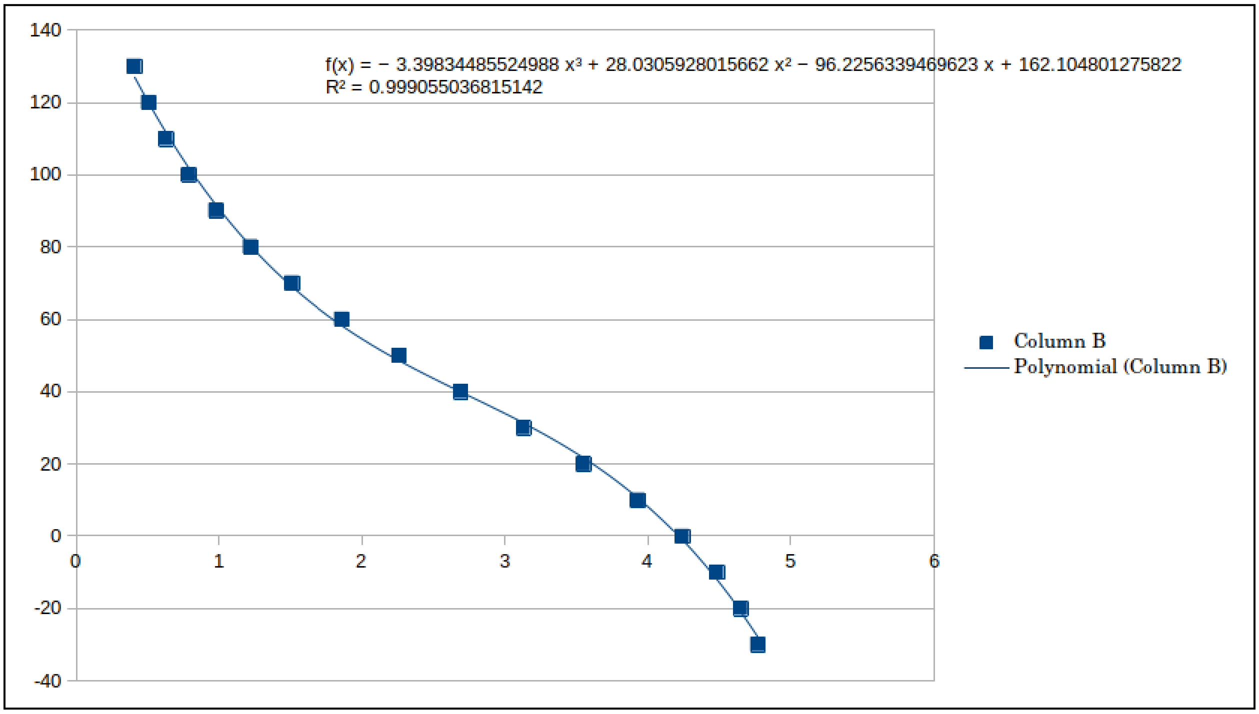
y_i: Y value for observation

$\bar{y}$: Mean Y value

$\hat{y}_i$: Predicted value of Y for observation i

$$\%error = \left| \frac{v_{theoretical} - v_{exact}}{v_{theoretical}} \right| * 100$$

Data



Voltage	Temp °C	Theo. V	%error
0.41	130	127.130016681742	2.2575182424798
0.51	120	119.869691307165	0.108708624685809
0.63	110	111.758248236157	1.57326037577232
0.79	100	101.904926876092	1.86931774006172
0.98	90	91.5257663434208	1.66703476450102
1.22	80	80.2593844776634	0.323182739752426
1.51	70	69.0163138450326	1.42529512250706
1.86	60	58.2319012088967	3.03630613872714
2.26	50	48.5762315761228	2.9309980986195
2.69	40	39.9408655028785	0.148055121933379
3.13	30	31.3236256896896	4.22564649061446
3.55	20	21.7212208599264	7.9241441861212

3.93	10	10.5934735576139	5.60225646843986
4.24	0	-1.00801966156018	N/A
4.48	-10	-11.964338852991	16.4182816712848
4.65	-20	-20.938212817466	4.48086389055801
4.77	-30	-27.941094830394	7.36873476899806

Error Analysis

After finding the correlation between Calhoun's formula as a third degree polynomial and the recorded values, using the formula for the average value over a set $\frac{\sum_{v}^{s}\left|\frac{v_t - v_e}{v_t}\right|(100)}{s}$ and the coefficient of determination $R^2 = \frac{\sum_i (\hat{y}_i - \bar{y})^2}{\sum_i (y_i - \bar{y})^2}$ we can conclude that the percentage error averages to %3 and the line of best fit has an accuracy of %99.9 with respect to the model.

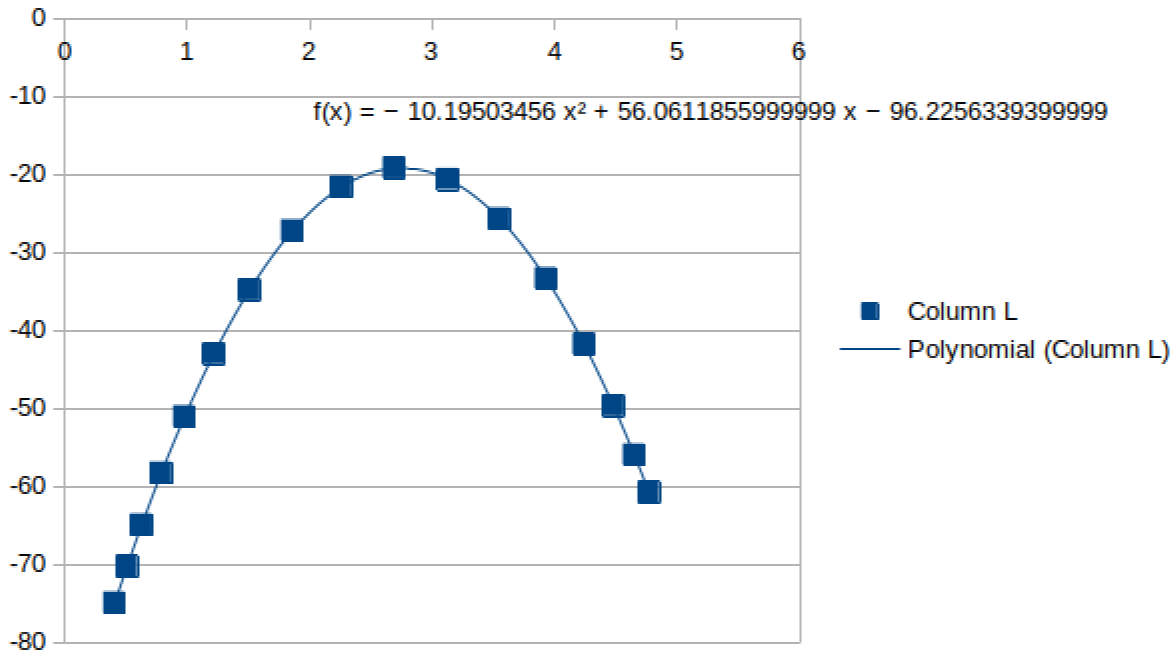
Avg %error
3.83497527781604

Further reading

Furthermore, we can calculate the derivative of Calhoun's formula to find the instantaneous rate of change for any voltage input. This is unimportant for our immediate application of the formula but I included the derivative because it can be used to find variations in temperature from different points of the program without recursion which may be important later.

Below is the data, graph, and math involved with the derivation:

Voltage	Derivative
0.41	-74.954333153536
0.51	-70.286157773056
0.63	-64.953496228864
0.79	-58.300018384896
0.98	-51.076983243424
1.22	-43.005276947104
1.51	-34.818941984256
1.86	-27.222570287776
2.26	-21.599513002656
2.69	-19.193334255616
3.13	-20.633857092864
3.55	-25.6913481024
3.93	-33.366463807744
4.24	-41.808460301856
4.48	-49.689944085024
4.65	-55.9832556736
4.77	-60.780380468224



$\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}[f(x)] = \frac{d}{dx}[-\frac{17}{5}x^3 + 28x^2 - 96x + 162]$

$\frac{d}{dx}[-\frac{17}{5}x^3 + 28x^2 - 96x + 162] =- \frac{51x^2}{5} + 56x - 96$

$\frac{d}{dx}[f(x)] = f'(x) =- \frac{51x^2}{5} + 56x - 96 \therefore$

Checking Work:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \frac{d}{dx}[f(x)] = \int - \frac{51x^2}{5} + 56x - 96\, dx = f(x)$ In theory*

$\int - \frac{51x^2}{5} + 56x - 96\, dx =- \frac{17}{5}x^3 + 28x^2 - 96x + 162$

$f(x) =- \frac{17}{5}x^3 + 28x^2 - 96x + 162 \therefore$

Conclusion

The conclusion of the analysis allows us to generate the formula $f(x) = -\frac{17}{5}x^3 + 28x^2 - 96x + 162$.

The subsequent calculation for converting the voltage reading to the temperature in °C can now be done in any programming language with a spatial and time complexity of $O(1)$ making this conversion algorithm the fastest and most accurate algorithm possible from my current understanding of mathematics.