Monday, 23 June 2025 17:19

$\theta = \sqrt{\frac{m^2 k \|A\|^2}{n^2 k \|A\|^2 + 1}}$

$Fix := \{ c(x) = D(x) \}$

$A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$ $b = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$ $Ax = b$

$Fix: \ span\left\{ \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$ $\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \xrightarrow{2D-X} \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{C} \begin{pmatrix} 0 \\ 0 \\ 8 \\ 8 \end{pmatrix}$ $\begin{pmatrix} a \\ b \\ a \\ b \end{pmatrix}$

$L = span\{ C \cup D \} = span\left\{ \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}^{\perp}$

$d_1 - d_2 = \ D(x) \overset{-X}{=} C(x) - X$

$\rightsquigarrow L$ closed under C, D and DR

$A^+ A x = A^T b$

$C(D(x)) - d_2 = X$

$D(x) - X = d_2 \Rightarrow d_1 = 0$ $\quad \sum_i \frac{-(a_i^T x_i - b_i)}{\|a_i\|^2} a_i = 0$

$D \perp D(x) - X = C(x) - X \perp C$

$(2D(x) - X) + X - D(x) = C(D(x)) - d_2$

$x = (x - \bar{x}) + \bar{x}$

$C(2D(x) - X) = C(D(x)) + C(D(x) - X)$

$d_1 + d_2$

In L: $\theta < 1 \rightsquigarrow$ Project to L and perform D.R.

DR

For $Ax \leq b$: $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$Fix: \left\{ \begin{pmatrix} a & +b \\ -a & +b \\ a & -b \\ -a & -b \end{pmatrix} \mid a \in \mathbb{R}, b \geq 0 \right\}$

$\begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \in D$

$L = span(C \cup D) = \mathbb{R}^4$

$X = X'' + X^{\perp}$ $\quad X'' = \begin{pmatrix} b \\ b \\ -b \\ -b \end{pmatrix}$ $\quad \langle X^{\perp}, \begin{pmatrix} b \\ b \\ -b \\ -b \end{pmatrix} \rangle = 0$

$DR(X) = DR(X'') + DR(X^{\perp})$?