Package 'dualtrees'

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Title Decimated and Undecimated 2D complex dual-tree wavelet transform

Version 0.0.1

Description What the package does (one paragraph).

Depends R (>= 3.5.0)

License What license is it under?

Encoding UTF-8

LazyData true

RoxygenNote 6.1.1

R topics documented:

A *Bias correction matrices*

Description

Matrices supposedly needed in order to eliminate the effect of "spectral leakage" for the local dtcwt-spectra. Used by biascor(...).

Usage

```
A_b_bp

A_b
```

Format

A list with entries N512, N256, N128, N64, N32, each containing the bias correction matrix of appropriate size

Source

Calculated by hand via /user/s6sebusc/wavelets_verification/general_scripts/Amats_cdtwt.r

Examples

```
image( A_b_bp$N512 )
```

biascor *spectral bias correction, implemented in FORTRAN*

Description

Unfold scale and direction into a vector and multiply by A at each grid point

Usage

```
biascor(en, a)
```

Arguments

en	J x nx x ny x 6 array of squared wavelet coefficients
a	bias correction matrix

Value

an array of the same dimensions as en

Note

this step can introduce negative values into the spctrum which have no intuitive interpretation in terms of energy and need to be dealt with.

blossom — *Two meteorologists in front of cherry blossoms*

Description

A photograph of two meteorologists in front of the famous cherry trees in Bonn.

Usage

```
blossom
```

Format

A 256x256 matrix of gray-scale values

Source

real life

Examples

```
image(blossom, col=gray.colors(128,0,1))
```

boundaries — *Various boundary conditions for the 2D wavelet transform.*

Description

Various boundary conditions for the 2D wavelet transform.

Usage

```
pad(x, N, Ny = N, value = min(x, na.rm = TRUE))

put_in_mirror(x, N, Ny = N)

period_bc(x, N, Ny = N)
```

Arguments

x	a real matrix
N	the number of rows of the desired output
Ny	the number of columns of the desired output, defaults to N
value	the value with which the picture is padded by pad

Details

pad pads the fields with a constant value on all sides, be careful what you pick here. put_in_mirror reflects the input at all edges (with repeated end samples), period_bc simply repeats the input periodically. In any case, you can retrieve the initial area via bc$res[bc$px, bc$py].

Value

an object of class `bc_field`

Examples

```
bc <- put_in_mirror( blossom, N=300 )
plot( bc )
print( range( bc$res[ bc$px, bc$py ] - blossom ) )
```

`cen2uv` *centre to vector*

Description

transforms the angle and radius component of the spectral centre into vector components for visualization

Usage

```
cen2uv(cen)
```

Arguments

`cen`	an `nx x ny x 3` array, the output of `dt2cen`

Details

The resulting vector field represents the degree of anisotropy and dominant direction, averaged over all scales. In principle, large and small edges might compensate one another, but the result typically looks as one would intuitively expect.

Value

an `nx x ny x 2` array, the two matrices representing the u- and v-component of the vector field.

See Also

`dt2cen`, `uvplot`

Examples

```
dt <- fld2dt(blossom)
ce <- dt2cen(dt)
uv <- cen2uv(ce)
uvplot( uv, z=blossom )
```

cen_xy *xy <-> cen*

Description

Translate the centre of mass back and forth between polar and cartesian coordinates

Usage

```
cen2xy(cen)

xy2cen(xy)
```

Arguments

cen	the centre of mass of a wavelet spectrum (rho, phi, z), output of dt2cen
xy	the centre of mass in cartesian coordinates (x, y, z), output of cen2xy

Details

dt2cen represents the sepctrum's centre in cylinder coordinates because that is more intuitive than the x-y-z position within the hexagonal geometry. If you want to compare two spectra, it makes more sense to consider their distance in terms of x1-x2, y1-y2 since the difference in angle is only meaningful for reasonably large radii. These functions allow you to translate back and forth between the two coordinate systems.

Note

cen2xy is not the same thing as cen2uv !

See Also

dt2cen, cen2uv

---

dt2cen *centre of the DT-spectrum*

Description

calculate the centre of mass of the local spectra in hexagonal geometry

Usage

```
dt2cen(pyr, mask = NULL)
```

Arguments

pyr	a J x nx x ny x 6 array of spectral energies, the output of fld2dt
mask	a nx x ny array of logical values

Details

Each of the J x 6 spectral values is assigned a coordinate in 3D space with x(d,j)=cos(60*(d-1)), y(d,j)=sin(60*(d-1)),z(d,j)=j+1. Then the centre of mass in this space is calculated, the spectral values being the masses at each vertex. The x- and y-cooridnate are then transform into a radius rho=sqrt(x^2+y^2) and and angle phi=15+0.5*atan2(y,x). rho measures the degree of anisotropy at each pixel, phi the orientation of edges in the image, and the third coordinate, z, the central scale. If a mask is provided, values where mask==TRUE are set to NA.

Value

a nx x ny x 3 array where the third dimension denotes degree of anisotropy, angle and central scale, respectively.

Note

Since the centre of mass is not defined for negative mass, any values below zero are removed at this point.

Examples

```
dt <- fld2dt(blossom)
ce <- dt2cen(dt)
image( ce[,,3], col=gray.colors(32, 0, 1) )
```

---

dtcwt *The 2D forward dualtree complex wavelet transform*

Description

This function performs the dualtree complex wavelet analysis, either with or withour decimation

Usage

```
dtcwt(mat, fb1 = near_sym_b, fb2 = qshift_b, J = NULL, dec = TRUE,
  verbose = TRUE)
```

Arguments

mat	the real matrix we wish to transform
fb1	A list of analysis filter coefficients for the first level. Currently only near_sym_b and near_sym_b_bp are implemented
fb2	A list of analysis filter coefficients for all following levels. Currently only qshift_b and qshift_b_bp are implemented
J	number of levels for the decomposition. Defaults to log2(min(Nx,Ny)) in the decimated case and log2(min(Nx,Ny)) - 3 otherwise
dec	whether or not the decimated transform is desired
verbose	if TRUE, the function tells you which level it is working on

Details

This is the 2D complex dualtree wavelet transform as described by Selesnick et al 2005. It consists of four discerete wavelet transform trees, generated from two filter banks a and b by applying one set of filters to the rows and the same ot the other to the columns. In the decimated case (dec=TRUE), each convolution is followed by a downsampling, meaining that the size of the six coefficient fields is cut in half at each level.

Value

if dec=TRUE a list of complex coefficient fields, otherwise a complex J * Nx * Ny * 6 array.

Note

The inverse transform only works if the input is of size `2^N x 2^N`.

References

Selesnick, I.W., R.G. Baraniuk, and N.C. Kingsbury. "The Dual-Tree Complex Wavelet Transform." IEEE Signal Processing Magazine 22, no. 6 (November 2005): 123–51. https://doi.org/10.1109/MSP.2005.1550194.

See Also

idtcwt

Examples

```
dt <- dtcwt( blossom )
par( mfrow=c(2,3), mar=rep(2,4) )
for( j in 1:6 ){
    image( blossom, col=grey.colors(32,0,1) )
    contour( Mod( dt[[3]][ ,,j ] )**2, add=TRUE, col="green" )
}
```

---

dtmean *spatial mean spectrum*

Description

average the output of fld2dt or dtcwt over space

Usage

```
dtmean(x)
```

Arguments

x	either a J x nx x ny x 6 array of energies (output of dtcwt) or a list of complex wavelet coefficients (the output of `dtcwt(...,dec=FALSE)`)

Value

a J x 6 matrix of spatially averaged energies

Note

In the undecimated case, the coefficients are not averaged but summed up and then scaled by the area of the first level. This yields a comparable scale as the undecimated case.

`dt_analysis` *complete dual-tree analysis*

Description

Transform an input field and calculate a number of summary quantities like the histograms for the three central components, the mean spectrum and the centre of the mean spectrum.

Usage

```
dt_analysis(fld, neg.rm = TRUE, rbr = NULL, pbr = NULL, zbr = NULL,
  mask = NULL, return_fields = TRUE, ...)
```

Arguments

`fld`	a real matrix to be analysed
`rbr, pbr, zbr`	either the number of breaks to use for the histograms of rho, phi and z or vectors containing those breaks.
`mask`	a boolean matrix telling the function which pixels to remove from the analyis (see `dt2cen`)
`return_fields`	whether or not you want the function to return the 2D fields of rho, phi, z, u, v and fld itself.
`...`	further parameters passed to `fld2dt`, including wavelet selection and boundary conditions
`neg_rm`	whether or not negative values are immediately removed from the local spectra - recommended.

Details

The input is transformed via `fld2dt( fld, ... )` and the plugged into `dt2cen`. The results are summarized as histograms of radii, angles and central scales. By default, 33 equally spaced breaks between the theoretical extrema of these quantities are used. The default breaks for rho and phi are thus always the same, those for z range from 1 to the largest considered scale.

Value

and object of class `"dtana"`, containing the following:

`ms`	the mean spectrum
`ce`	the centre of mass of `ms` (rho, phi, z)
`rhist`	the histogram of radii
`phist`	the histogram of angles
`zhist`	the histogram of central scales
`rmi`	the bin centres for `rhist`
`pmi`	the bin centres for `phist`
`zmi`	the bin centres for `zhist`

if you set `return_fields=TRUE` (the default), the resulf also contains

`cen`	the output of `dt2cen`
`dt`	the output of `fld2dt`
`uv`	the centre converted into cathesian coordinates
`fld`	the input field

See Also

`fld2dt`, `dt2cen`

Examples

```
dta <- dt_analysis( blossom )
plot( dta )
dtb <- dt_analysis( blossom, return_fields=FALSE )
X11()
plot( dtb )
```

`filterbanks` *filterbanks for the dtcwt*

Description

Some of the filters implemented in the python package dtcwt.

Usage

```
qshift_b

qshift_b_bp

near_sym_b

near_sym_b_bp
```

Format

A list of high- and low-pass filters for analysis and synthesis

Details

The near-sym filterbanks are biorthogonal wavelets used for the first level, they have 13 and 19 taps. The qshift filterbanks, each with 14 taps, are suitable for all higher levels of the dtcwt, as the a- and b-filters for an approximate Hilbert-pair. The naming convention follows the python-package:

h:	analysis
g:	synthesis
0:	low-pass
1:	high-pass
a,b:	shifted filters

The b_bp-versions of the filterbanks contain a second high-pass for the diagonal directions, denoted by 2.

Source

dtcwt python package

`fld2dt` *transform a field into an array of spectral energies*

Description

Handles the transformation itself, boundary conditions and bias correction and returns the unbiased local wavelet spectrum at each grid-point.

Usage

```
fld2dt(fld, Nx = NULL, Ny = NULL, J = NULL, correct = NULL,
  rsm = 0, verbose = FALSE, boundaries = "pad",
  fb1 = near_sym_b_bp, fb2 = qshift_b_bp)
```

Arguments

`fld`	a real matrix
`Nx`	size to which the field is padded in x-direction
`Ny`	size to which the field is padded in y-direction
`J`	number of levels for the decomposition
`correct`	how to correct the bias, either "b" or "b_bp" - any other value results in no correction
`rsm`	number of pixels to be linearly smoothed along each edge before applying the boundary conditions (see `smooth_borders`).
`verbose`	whether or not you want the transform to talk to you
`boundaries`	how to handle the boundary conditions, either "pad", "mirror" or "periodic"
`fb1`	filter bank for level 1
`fb2`	filter bank for all further levels

Details

The input is blown up to `Nx x Ny` and the thrown into `dtcwt`. Then the original domain is cut out, the coefficients are squared and the bias is corrected.

Value

an array of size `J x nx x ny x 6` where `dim(fld)=c(nx,ny)`

See Also

`biascor`

Examples

```
dt <- fld2dt( blossom )
par( mfrow=c(2,2), mar=rep(2,4) )
for( j in 1:4 ){
    image( blossom, col=gray.colors(128, 0,1), xaxt="n", yaxt="n" )
    for(d in  1:6) contour( dt[j,,,d], levels=quantile(dt[,,,], .995),
                           col=d+1, add=TRUE, lwd=2, drawlabels=FALSE )
    title( main=paste0("j=",j) )
}
x0  <- seq( .1,.5,,6 )
y0  <- rep( 0.01,6 )
a   <- .075
phi <- seq( 15,,30,6 )*pi/180
x1  <- x0 + a*cos( phi )
y1  <- y0 + a*sin( phi )
rect( min(x0,x1)-.05, min(y0,y1)-.05,
      max(x0,x1)+.05, max(y0,y1), col="black", border=NA )
arrows( x0, y0, x1, y1, length=.05, col=2:7, lwd=2, code=3 )
```

---

get_en *get energy from the dualtree transform*

Description

square the wavelet coefficients and apply some form of correction

Usage

```
get_en(pyr, correct = "none", N = ncol(pyr))
```

Arguments

pyr	array of `J x nx x ny x 6` complex wavelet coefficients, output of `dtcwt(..., dec=FALSE)`
correct	type of correction, either `"b"` or `"b_bp"`, any other value results in no correction at all.

Details

The bias correction matrix should correspond to the filter bank used in the transform. It is computed by brute force for the so-called auto-correlation wavelets, for more details, see Nelson et al 2017 and Eckley et al 2010.

Value

an array of the same dimensions as `pyr`

References

Eckley, Idris A., Guy P. Nason, and Robert L. Treloar. "Locally Stationary Wavelet Fields with Application to the Modelling and Analysis of Image Texture: Modelling and Analysis of Image Texture." Journal of the Royal Statistical Society: Series C (Applied Statistics), June 21, 2010, no-no. https://doi.org/10.1111/j.1467-9876.2009.00721.x.

Nelson, J. D. B., A. J. Gibberd, C. Nafornita, and N. Kingsbury. "The Locally Stationary Dual-Tree Complex Wavelet Model." Statistics and Computing 28, no. 6 (November 2018): 1139–54. https://doi.org/10.1007/s11222-017-9784-0.

See Also

A

`idtcwt` *The 2D inverse dualtree complex wavelet transform*

Description

Reconstructs an image from the pyramid of complex directional wavelet coefficients.

Usage

```
idtcwt(pyr, fb1 = near_sym_b, fb2 = qshift_b, verbose = TRUE)
```

Arguments

`pyr`	a list containing arrays of complex coefficients for each level of the decomposition, produced by `dtcwt( ..., dec=TRUE )`.
`fb1`	the filter bank for the first level
`fb2`	the filter bank for all following levels
`verbose`	if true, the function will say a few words while doing its thing.

Details

This function re-arranges the six complex daughter coefficients back into the four trees, convolves them with the synthesis wavelets and adds everything up to recover an image. For the `near_sym_b` and `qshift_b` filter banks, this reconstrcution should be basically perfect. In the case of the the `b_bp` filters, non-negligible artifacts appear near +-45° edges.

Value

a real array of size `2N x 2M` where `dim( pyr[[1]] ) = (M,N,6)` .

Note

At present, the inverse transform only works if the input image had dimensions `2^N x 2^N`. Sorry.

References

Selesnick, I.W., R.G. Baraniuk, and N.C. Kingsbury. "The Dual-Tree Complex Wavelet Transform." IEEE Signal Processing Magazine 22, no. 6 (November 2005): 123–51. https://doi.org/10.1109/MSP.2005.1550194.

See Also

dtcwt

Examples

```
py <- dtcwt( blossom )
blossom_i <- idtcwt( py )
image( blossom - blossom_i )
```

---

my_conv *Column-convolutions*

Description

This function convolves the columns of a matrix mat with a filter fil.

Usage

```
my_conv(mat, fil, dec = TRUE, odd = FALSE, inverse = FALSE)
```

Arguments

mat	a matrix
fil	the filter to convolve the columns with
dec	if TRUE, every second row is discarded after the convolution
odd	if TRUE, the first row is discarded, otherwise the second row is.
inverse	set TRUE for the inverse transformation

Details

This functions does all of the actual computations inside the wavelet transform via FFT. If dec=TRUE or inverse=TRUE, the filters may be longer than the data. In these cases, the data is periodically extended to exceed the filter size.

Value

a matrix with as many columns and either the same (dec=FALSE) or half (dec=TRUE) the number of rows as mat

Examples

```
dboysdy <- my_conv( blossom, c(-1,1), dec=FALSE )
dboysdx <- t( my_conv( t(blossom), c(-1,1), dec=FALSE ) )
par( mfrow=c(1,2) )
image( dboysdx, col=gray.colors(32) )
image( dboysdy, col=gray.colors(32) )
```

smooth_borders *smoother borders*

Description

let a field decrease linearly towards its edges

Usage

```
smooth_borders(x, r)
```

Arguments

x	a real matrix
r	a positive integer

Details

Values within the field are linearly reduced from their original value to the field minimum, starting r pixels away from the edge. This enforces truely periodic boundaries. It may be a good idea to apply this to you input data before doing the wavelet analysis. Consider removing the altered border afterwards ...

Value

a matrix of the same dimensions as x

Note

r must not be larger than `min( dim(x) )/2`.

Examples

```
image( smooth_borders(blossom, r=64), col=gray.colors(128,0,1) )
```

uvplot *plot centre as vectors*

Description

display the radial and angular component of the spectrum's centre as arrows.

Usage

```
uvplot(uv, z = NULL, x = NULL, y = NULL, col = "green",
  zcol = gray.colors(32, 0, 1), n = 42, f = 1, code = 0,
  length = 0.05, ...)
```

Arguments

uv	either an array of dimension nx x ny x 2 , containing the u- and v-component, result of cen2uv or an object of class "dtana"
z	image to show in the background, defaults to the absolute velocity
col	color of the arrows
zcol	color scale for the image
n	number of arrows in one direction
f	factor by which to enlarge the arrows
code	determines the type of arrow, passed to arrows()
length	length of the arrowhead in inches, does nothing if code=0
...	further arguments passed to image

Details

The pivot of the arrows is at the location to which the u- and v-component belong. By default, no arrowhead is displayed (code=0) since the egdges detcted by the cdtwt have an orientation but no sign (SW and NE are equivalent). The default size of the arrows is such that a 'velocity' of 1 corresponds to 5

See Also

cen2uv

Examples

```
uv <- cen2uv( dt2cen( fld2dt( blossom ) ) )
uvplot( uv, z=blossom )
```

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