Optimal climate and fiscal policy in an OLG economy

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April 25, 2018

Efficient policies price externalities through Pigouvian taxes, but these must be adjusted when there are other distortionary taxes in the economy. I develop a two-period overlapping generations climate-economy model to study integrated capital, labor, and carbon taxes. I derive four primary results. First, the optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon, but not always attains its Pigouvian level. Second, under weak separability in preferences over consumption and leisure, age-dependent labor income taxes allows the optimal carbon tax to achieve its first-best. Third, even if age-dependent taxes are available, non-separability in preferences and a decreasing labor supply over the life cycle leads to positive capital income taxes and an optimal price on carbon emissions that falls short of the Pigouvian tax. Fourth, an exogenous capital income tax rate implies an optimal carbon price that differs from both the market costs of carbon and its Pigouvian level.

JEL classification: E62; H21; H23; Q58.

Keywords: Fiscal policy, Optimal taxation, Externalities, Environmental policies .

*Department of Economics, Tilburg University. E-mail: R.V.JaimesBonilla@uvt.nl. I am indebted to Reyer Gerlagh and Sjak Smulders for their continuous guidance and support. I am very grateful to John Hassler, Ándres Erosa and Johanna Wallenius for helpful conversations. I also would like to thank the participants at the 11th RGS Doctoral Conference in Essen, and the ENTER Seminar in Stockholm for useful comments. All errors are my own. Link to most recent version: https://www.dropbox.com/s/3sbrfta6xt0rfxl/J17_dist_taxes.pdf?dl=0

1. Introduction

A fundamental observation in the literature about pricing externalities e.g., the setting of Pigouvian taxes, is that those prices should be adjusted in the presence of other distortions in the economy. This is particularly relevant in the context of environmental regulations, since recent economic research has suggested that optimal prices for pollution control respond differently to, for instance, non-observable private information (Jacobs and de Mooij, 2015; Kaplow, 2012; Tideman and Plassmann, 2010), financial frictions or productivity shocks (van den Bijgaart and Smulders, 2017; Hoffmann et al., 2017), the existence of positive capital taxes (Gerlagh and Liski, 2017; Barrage, 2016), time-inconsistency problems (Gerlagh and Liski, 2017; Schmitt, 2014), distributional issues (Jacobs and de Mooij, 2015; Chirole-Assouline and Fodha, 2011, 2014) or differences between private and social discounting (Barrage, 2017; Belfiori, 2017).

Using a two-period overlapping generations model with endogenous labor supply and a climate-module structure based on Howarth (1998), Iqbal and Turnovsky (2008) and Gerlagh and Liski (2017), this paper specifically studies the interaction between climate and fiscal policies when the government has no access to individualized lump-sum taxation and relies on the implementation of distortionary taxes to finance an exogenous stream of expenditures and inherited debt. I use this approach to inform about the conditions under which the market costs of carbon differs from the social costs of carbon (the Pigouvian tax).

First, I show that a non-zero optimal capital income tax drives a wedge between how the market values future output losses due to current carbon emissions and how the society values those losses, since such a tax does not affect the marginal productivity of capital.[1] Second, I find that under the assumption of weak separability in preferences over consumption and leisure, the availability of age-dependent labor income taxation allows the government to rely on a carbon price and labor income taxes to fulfill its spending requirements, and to avoid intertemporal distortions by imposing a non-zero capital income tax rate. Notice that although a labor tax affects intratemporal decisions, those do not impede that the carbon price attains its Pigouvian level.

Third, I show that even if age-dependent taxes are available, non-separability in preferences and a decreasing labor supply over the life-cycle leads to an optimal price on

[1] In the presence of no distortions in the economy, it has been shown that the marginal rate of substitution for consumption between two periods equals the opportunity cost of capital, so both valuations in equilibrium are the same.

carbon emissions that falls short of the Pigouvian tax. In this case, since a labor income tax also influence consumption-savings decisions due to the complementarity between consumption and leisure, the government finds optimal to tax capital income using a non-zero tax rate to offset the distortion on consumption optimal paths by changes in labor supply; importantly, the sign of this tax rate is determined by the optimal allocation of labor over the life-cycle and the intertemporal elasticity of substitution. It is worth noting that in the presence of no age-dependent taxation, regardless of assumptions about preferences, in general the optimal carbon price does not correspond to the Pigouvian tax. Fourth, when the government is constrained to set a non-optimal capital income tax rate, I find that the prescription which indicates that the optimal carbon price should be equal to the market costs of carbon does not hold anymore. The intuition behind this result relies on the fact that since the government cannot adjust optimally the capital income tax, this affects the optimal accumulation of capital, its net return, and therefore, how to discount future marginal damages from emissions.

This paper relates to distinct strands of literature. Firstly, one of most critical issues in climate change policy has to do with the decision about which discount rate a policy maker should use to calculate the net present value of future production damages due to the emission of one unit of CO_2 today, in order to determine a carbon price that accounts for those production losses. In this matter, for instance, Nordhaus (2008) and Stern (2007) provide somehow different recommendations. Nordhaus (2008) argues that current investments in climate change mitigation should earn the same return that other investments in the economy, e.g., the market interest rate. Stern (2007) suggests to follow an ethics-based approach and recommends to use a very 'low' rate of pure time preference. However, such assumption would imply higher savings rates than the ones observed in the data (Belfiori, 2017).

In the same vein, Goulder and Williams (2012), Weisbach and Sunstein (2009), and Dasgupta (2008) discuss the reasons behind these differences and point to the concepts attached to social discounting in each prescription as the cause of disagreement. I add to this literature by considering explicitly the difference between the market costs of carbon and the social costs of carbon as suggested in Goulder and Williams (2012). These concepts differ with respect to the discount rate used to evaluate future marginal damages to production by current pollutant activities. While the first one uses the market interest rate or the return on capital, the second one employs the consumption discount rate which is given by the marginal rate of substitution for consumption between two periods of successive generations. In this context, I consider thus the situations and the causes in which their valuations may be different.

Recent studies has also pointed to the importance of understanding the relationship between the existence of capital income taxes and the setting of climate policies, e.g. the carbon price, in dynamic climate-economy models. For instance, using a infinitely-lived agent model as in Golosov et al. (2014) and a climate structure as in Nordhaus (2008), Barrage (2016) shows that when climate change only has an impact in the production of the final good, a zero capital income tax does not distort the optimal carbon price and, therefore, it equals the social costs of carbon. Since a capital income tax distorts the consumption-savings decisions of households, it is well known that in infinitely-lived representative agent (ILA) frameworks, the government finds optimal to fully rely on labor income taxes, in absence of lump-sum taxation, because that fiscal policy is welfare improving; that is, the optimal capital income tax should be zero in the long run (Chamley, 1986; Judd, 1985).[2] However, when the government faces an exogenous constraint implying a positive capital income tax, Barrage (2016) finds that the optimal carbon prices should be set below its Pigouvian level. In the same line, Schmitt (2014) proposes a dynamic model based on Bovenberg and de Mooij (1994) without commitment technologies, characteristic that generates endogenously positive capital income tax rates, and finds out that governments set optimal carbon taxes below Pigouvian levels.

In contrast, a different strand of literature has suggested that there is space for positive capital income taxes in overlapping generations (OLG) models due to life-cycle characteristics no present in ILA models, such as differences in labor supply or productivity profiles, tax instruments available to the government, and preferences modeling (Garriga, 2017; Conesa et al., 2009; Iqbal and Turnovsky, 2008; Erosa and Gervais, 2002, 2001; Garriga, 2001). Since not so much effort has been done to analyze the setting of optimal carbon prices in OLG models with distortionary taxation,[3] I contribute to this literature by providing a set of additional results in terms of preferences modeling and

[2] Straub and Werning (2014) indicate, however, that this result is no longer valid whenever the elasticity of intertemporal substitution is below one. See also Albanesi and Armenter (2012) for an analysis of the effects of intertemporal distortions in ILA models.

[3] Rausch and Abrell (2014) provide a characterization of capital-carbon tax interactions in an OLG framework. However, they do not discuss the consequences of distinct preferences specification and the role of age-dependent taxation in those interactions, as this paper does. Fried et al. (2016) study the introduction of a revenue-neutral carbon tax policy in a life-cycle model with distortionary taxes and quantify their distributional effects. They do not derive optimal carbon prices and do not consider the implications of existence of age-dependent taxation

tax instruments available to the government.[4]

Finally, this paper also relates to the literature that evaluate the role of age-dependent taxation in the setting of fiscal policy as in Bastani et al. (2013), Weinzierl (2011), and Blomquist and Micheletto (2008). I complement these studies by showing that the introduction of labor income taxes which can be conditioned by age, at least under the assumption of weak separability in preferences over consumption and leisure, leads to a zero capital income tax and to an optimal carbon price that attains its Pigouvian level. The intuition behind this result is straightforward. When the government has access to more fiscal instruments to finance spending, it is optimal to choose the ones that avoid or reduce intertemporal distortions.

The organization of this paper is as follows. Section 2 lays out the main characteristics of the model, provide some definitions about the costs of carbon and describe first-best allocations. Section 3 presents the Ramsey problem and derive optimal taxes using the primal approach. Section 4 studies the role of age-related income taxation and characterize optimal taxes. Section 5 provides a discussion about the implications of non-optimal constant capital income taxation in the setting of climate policies. Section 6 concludes.

2. The model

I consider a two-period overlapping generations model with endogenous labor supply based on Howarth (1998) and Iqbal and Turnovsky (2008), and add a climate-module structure as in Gerlagh and Liski (2017) to derive optimal environmental policies when the government has no access to individualized lump-sum taxes to finance an exogenous stream of government spending and inherited debt.

2.1. Household's problem

Each generation lives only two periods. Households supply labor in both periods and there is no population growth. I assume a constant population normalized to 1 and full capital depreciation. Each household is endowed with one unit of time. The time-separable utility function U_t is strictly increasing, strictly concave, twice continuosly differentiable and satisfies the usual Inada conditions. In period t, each household

[4]Notice that I do not consider other topics typical of OLG-climate-economy models such as demographic change (Gerlagh et al., 2017) and political economy features (Karp and Rezai, 2014).

solves the following problem taking as given the path for prices, fiscal policy and initial asset holdings:[5]

$$\max_{\{C_{1,t},C_{2,t+1},L_{1,t},L_{2,t+1},K_{t+1},B_{t+1}\}} W_t \equiv U(C_{1,t}, L_{1,t}) + \beta U(C_{2,t+1}, L_{2,t+1}) \tag{1}$$

subject to,

$$C_{1,t} + K_{t+1} + B^D_{t+1} = (1 - \tau^L_{1,t})\phi_1 w_t L_{1,t} \tag{2}$$

$$C_{2,t+1} = (1 - \tau^L_{2,t+1})\phi_2 w_{t+1} L_{2,t+1} + (1 - \tau^K_{t+1}) r_{t+1} K_{t+1} + R_{t+1} B^D_{t+1} \tag{3}$$

where $C_{1,t}$ and $C_{2,t+1}$ denote consumption at young and old age, respectively; $\beta \in (0, 1)$ is the subjective utility discount factor; $L_{1,t}$ and $L_{2,t+1}$ are the fractions of time allocated to work in each period; ϕ_1 and ϕ_2 identify labor productivities at each age; K_{t+1} represents savings; w_t and w_{t+1} describe wage payments; r_{t+1} is the return to capital investments; B^D_{t+1} is the demand for government bonds which have one-period maturities and a return R_{t+1}; households pay labor and capital income taxes $\{\tau^L_{1,t}, \tau^L_{2,t+1}, \tau^K_{t+1}\}$, accordingly. Solving the household's problem, the first-order conditions imply:

$$\frac{U_{C_{1,t}}}{U_{C_{2,t+1}}} = \beta(1 - \tau^K_{t+1}) r_{t+1} \tag{4}$$

$$-\frac{U_{L_{1,t}}}{U_{C_{1,t}}} = (1 - \tau^L_{1,t})\phi_1 w_t \tag{5}$$

$$-\frac{U_{L_{2,t+1}}}{U_{C_{2,t+1}}} = (1 - \tau^L_{2,t+1})\phi_2 w_{t+1} \tag{6}$$

$$R_{t+1} = (1 - \tau^K_{t+1}) r_{t+1} \tag{7}$$

where $U_{X_{i,t}}$ is the derivative of the utility function U_t with respect to $X_{i,t}$. Condition (4) is the usual Euler equation which relates marginal rates of substitution for consumption between two periods to the discounted after-tax returns on capital. Conditions (5-6) define intratemporal marginal rates of substitution over consumption and labor relatively to after-tax labor income weighted by age-specific productivities. The last equation, (7), corresponds to the no-arbitrage condition which establishes that in equilibrium government bonds and capital should earn the same net return.

[5] A general formulation of this household's problem, with generations living more than two periods, can be found in Erosa and Gervais (2002) and Garriga (2017).

2.2. Firms

Each period, under perfect competition a representative firm employs a technology that exhibits constant returns to scale to produce aggregate output Y_t, which depends on capital K_t, aggregate labor L_t, energy E_t, and a climate variable Z_t that affects output as in Gerlagh and Liski (2017). The production function F_t is strictly concave, twice continuously differentiable and satisfies the usual Inada conditions:

$$Y_t = F_t(K_t, L_t, E_t, Z_t) = \Omega(Z_t) K_t^{\alpha} [A_t(E_t, L_t)]^{1-\alpha} \tag{8}$$

where $\alpha \in (0, 1)$ and $A_t(E_t, L_t)$ has constant returns to scale. Likewise, let $\Omega(Z_t)$ and $Z_t = (E_1, E_2, \ldots, E_{t-2}, E_{t-1})$ be the output losses due to past carbon emissions and the history of emissions, respectively.[6] I consider a general formulation as follows:

$$\Omega(Z_t) = \exp(-\sum_{i=1}^{\infty} \theta_i E_{t-i}) \tag{9}$$

Taking $\Omega(z_t)$ as exogenous, the firm's problem is then as follows:

$$\max_{\{K_t, L_t, E_t\}} Y_t - w_t L_t - r_t K_t - \tau_t^E E_t \tag{10}$$

The first-order conditions are given by:

$$r_t = \alpha \frac{Y_t}{K_t} \tag{11}$$

$$w_t = (1-\alpha) Y_t \frac{A_{L,t}}{A_t} \tag{12}$$

$$\tau_t^E = (1-\alpha) Y_t \frac{A_{E,t}}{A_t} \tag{13}$$

As usual inputs are paid their marginal productivities. It is important to note that since the firm does not fully internalize the social cost of emitting one unit of carbon at period t, the emissions price τ_t^E (a carbon tax) has to be selected (optimally) by the government to correct this inefficiency, that is, without intervention $\tau_t^E = 0$. Finally, effective labor supply is:

$$L_t = \phi_1 L_{1,t} + \phi_2 L_{2,t} \tag{14}$$

[6] I assume implicitly that energy use maps one to one with emissions.

2.3. The government

To finance an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$ and inherited debt B_0, the government can issue one-year maturity bonds B_t^S, impose proportional taxes on labor and capital income, and set an excise tax on carbon emissions τ_t^E. For simplicity, I assume full commitment. The government's budget constraint is:

$$R_t B_t^S + G_t = B_{t+1}^S + \tau_{1,t}^L \phi_1 w_t L_{1,t} + \tau_{2,t}^L \phi_2 w_t L_{2,t} + \tau_t^K r_t K_t + \tau_t^E E_t \tag{15}$$

In this case, for $t \geq 0$, the intertemporal constraint can be written as follows:

$$B_t^S = \sum_{i=1}^{\infty} (T_{t+i} - G_{t+i}) / \prod_{i=1}^{\infty} R_{t+i} \tag{16}$$

where $T_t = \tau_{1,t}^L \phi_1 w_t L_{1,t} + \tau_{2,t}^L \phi_2 w_t L_{2,t} + \tau_t^K r_t K_t + \tau_t^E E_t$ defines government revenues.

2.4. Competitive equilibrium

A competitive equilibrium for this economy can be defined as follows:

Definition 1. *Given a set of policies $\{\tau_{1,t}^L, \tau_{2,t+1}^L, \tau_t^K, \tau_t^E, B_{t+1}^S\}_{t=0}^{\infty}$, initial debt B_0 and an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy consists of relative prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, allocations for the households $\{C_{1,t}, C_{2,t+1}, L_{1,t}, L_{2,t+1}, K_{t+1}, B_{t+1}^D\}_{t=0}^{\infty}$ and the firm $\{K_t, L_t, E_t\}_{t=0}^{\infty}$ such that:*

1. *The allocations for the households solve (4-7),*
2. *The allocations for the firm solve (11-13),*
3. *The intertemporal budget constraint for the government (16) is satisfied, subject to the transversality condition:* $\lim_{i \to \infty} \frac{B_i^S}{\prod_{i=1}^{\infty} R_{t+i}} = 0$.
4. *Market clearing conditions are satisfied:*

$$\begin{aligned} C_{1,t} + C_{2,t} + K_{t+1} + G_t &= exp(-\sum_{i=1}^{\infty} \theta_i E_{t-i}) K_t^{\alpha} [A_t(E_t, L_t)]^{1-\alpha} && (17) \\ L_t &= \phi_1 L_{1,t} + \phi_2 L_{2,t} && (18) \\ B_{t+1}^D &= B_{t+1}^S && (19) \end{aligned}$$

2.5. First-best allocations

If there were not other distortions in the economy, except the climate externality, a benevolent government who has access to individualized lump-sum taxes for financing government spending would seek to maximize a social welfare function, taking K_0 and B_0 as given, to solve the following problem:

$$\max_{\{C_{1,t},C_{2,t+1},L_{1,t},L_{2,t+1},K_{t+1},E_t\}_{t=0}^{\infty}} \gamma^{-1}U_0 + \sum\nolimits_{t=0}^{\infty} \gamma^t W_t \tag{20}$$

subject to the set of technological and resource constraints described above. W_t is the utility function of generation t and $1 > \gamma > 0$ is the intergenerational discount factor.[7] Let $\gamma^t \mu_t$ denote the Lagrange multiplier associated to the resource constraint (17). This problem is similar to the one described in Howarth (1998). Here, I extend his framework by considering endogenous labor supply and a distinct climate-module structure. The first-best allocations can be then derived from the optimality conditions which are given by:

$$U_{C_{1,t}} = \mu_t \tag{21}$$

$$\frac{\beta}{\gamma} U_{C_{2,t+1}} = \mu_{t+1} \tag{22}$$

$$U_{L_{1,t}} = -\mu_t \phi_1 F_{L_t} \tag{23}$$

$$\frac{\beta}{\gamma} U_{L_{2,t+1}} = -\mu_{t+1} \phi_2 F_{L_{t+1}} \tag{24}$$

$$\frac{1}{F_{K_{t+1}}} = \gamma \frac{\mu_{t+1}}{\mu_t} \tag{25}$$

$$\sum\nolimits_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \underbrace{\frac{\partial Y_{t+i}}{\partial \Omega_{t+i}} \frac{-\partial \Omega_{t+i}}{\partial E_t}}_{\theta_i Y_{t+i}} = F_{E_t} \tag{26}$$

where F_{X_t} is the derivative of the production function F_t with respect to X_t. The conditions (21-25) characterize consumption and labor paths at young and old age when there are not distortionary taxes in the economy. Equation (26) relates the marginal benefits of emitting one unit of CO_2 at period t (right-hand side) to the discounted marginal future damages (left-hand side).

[7] For the case when welfare weights are chosen such that the government does not redistribute income between generations see Gerlagh et al. (2017).

2.6. Carbon policies

Before describing and discussing the results from the social planner's problem under a distortionary fiscal policy scheme, I provide two key definitions:

Definition 2. *The Pigouvian tax in this economy denotes the net present value of marginal output losses due to one unit of energy consumption at period t evaluated at the optimal allocation and valued at the successive generations' marginal rates of substitution for consumption:*

$$\tau_t^{PIGOU} = \sum\nolimits_{i=1}^{\infty} \beta^i \prod_{j=1}^{i} \frac{U_{C_{2,t+j}}}{U_{C_{1,t+j-1}}} \theta_i Y_{t+i} \tag{27}$$

Definition 3. *The market costs of carbon emissions in this economy is defined as the net present value of future marginal damages evaluated at the market interest rate:*

$$MCC_t = \sum\nolimits_{i=1}^{\infty} \frac{1}{\prod_{j=1}^{i} r_{t+i}} \theta_i Y_{t+i} \tag{28}$$

Notice that while the Pigouvian tax (27) values the net present value of marginal climate damages using the marginal rates of substitution between consumption today and tomorrow of successive generations that live only two periods, the market costs of carbon discount future damages using the market interest rate. It is well known that in a economy with no distortions, optimality implies that the marginal rate of substitution for consumption between two periods equals the real interest rate, thus:

Proposition 1. *At the optimal allocation, in absence of any other distortion in the economy, it follows that:*

$$\tau_t^{PIGOU} = MCC_t \tag{29}$$

Proof. Using the first-order conditions (21-22) and (25-26), and according to the previous definitions, we get the result. ■

The result pointed out in Proposition 1 resembles the analysis in Howarth (1998) which indicates that the social costs of carbon (The Pigouvian tax) corresponds one to one to the market costs of carbon. The next section however describes under which conditions, in terms of tax interaction effects, the optimal carbon price may differ from its Pigouvian level.

3. Optimal taxation

To determine the path for optimal taxes I follow the primal approach as in Iqbal and Turnovsky (2008).[8] That is, instead of solving for tax rates directly, I characterize optimal allocations which are compatible with a competitive equilibrium and then derive prices and taxes that implement such allocations. The following lemma allows me to apply this approach:

Lemma 1. *Any competitive equilibrium which is a set of allocations for the firm* $\{K_t, L_t, E_t\}_{t=0}^{\infty}$ *and the household* $\{C_{1,t}, C_{2,t+1}, L_{1,t}, L_{2,t+1}, K_{t+1}, B_{t+1}^D\}_{t=0}^{\infty}$*, supported by a particular set of policies* $\{\tau_{1,t}^L, \tau_{2,t+1}^L, \tau_t^K, \tau_t^E, B_{t+1}^S\}_{t=0}^{\infty}$*, an exogenous stream of expenditures* $\{G_t\}_{t=0}^{\infty}$ *and initial debt* B_0*, satisfy:*

$$C_{1,t} + C_{2,t} + K_{t+1} + G_t = exp(-\sum_{i=1}^{\infty} \theta_i E_{t-i}) K_t^{\alpha} [A_t(E_t, L_t)]^{1-\alpha} \tag{30}$$

$$L_t = \phi_1 L_{1,t} + \phi_2 L_{2,t} \tag{31}$$

$$U_{C_{1,t}} C_{1,t} + \beta U_{C_{2,t+1}} C_{2,t+1} + U_{L_{1,t}} L_{1,t} + \beta U_{L_{2,t+1}} L_{2,t+1} = 0 \tag{32}$$

$$U_{C_{2,0}} C_{2,0} + U_{L_{2,0}} L_{2,0} = U_{C_{2,0}} \left[(1 - \tau_0^K) F_{K_0} K_0 + R_0 B_0 \right] \tag{33}$$

Any allocation that satisfies (30)-(33), can be decentralized as a competitive equilibrium for a particular set of policies, prices, and asset holdings.

Proof. In appendix A. ■

The main advantage of the primal approach has to do with the fact that allows me to reduce the number of variables and equations needed to solve for optimal allocations, and then decentralize them in a transparent manner. For example, notice that by replacing out prices and taxes in the budget constraint for the households using their first-order conditions, we can get the implementability conditions (32).[9] This step assures that if condition (32) is satisfied, the same allocations also solve (4-7). Likewise, equations (30) and (31) are equivalent to the first two constraints that come from the market clearing conditions in definition 1. Finally, using the first-order conditions for both the household and the firm I can solve for prices and taxes.

[8] Similar results are derived in Erosa and Gervais (2001), Garriga (2001), Erosa and Gervais (2002) and Conesa et al. (2009).

[9] The same argument applies for the implementability condition for the initial old, (33).

According to the Lemma (1), the government thus maximizes social welfare, taking K_0 and B_0 as given, to solve again:

$$\max_{\{C_{1,t},C_{2,t+1},L_{1,t},L_{2,t+1},K_{t+1},E_t\}_{t=0}^{\infty}} \gamma^{-1}u_0 + \sum\nolimits_{t=0}^{\infty} \gamma^t W_t \tag{34}$$

Let $\gamma^t\mu_t$, and $\gamma^t\lambda_t$ denote the Lagrange multipliers associated to the following constraints: (i) the resource constraint (30), and (ii) the implementability condition (32). Notice that I substitute constraint (31) into (30), and that condition (33) is not needed to solve the problem for generation t. The optimality conditions are:

$$\begin{aligned}
U_{C_{1,t}} \cdot \Lambda^{C_{1,t}} &= \mu_t && (35)\\
\frac{\beta}{\gamma} U_{C_{2,t+1}} \cdot \Lambda^{C_{2,t+1}} &= \mu_{t+1} && (36)\\
U_{L_{1,t}} \cdot \Lambda^{L_{1,t}} &= -\mu_t \phi_1 F_{L_t} && (37)\\
\frac{\beta}{\gamma} U_{L_{2,t+1}} \cdot \Lambda^{L_{2,t+1}} &= -\mu_{t+1} \phi_2 F_{L_{t+1}} && (38)\\
\frac{1}{F_{K_{t+1}}} &= \gamma \frac{\mu_{t+1}}{\mu_t} && (39)\\
\sum\nolimits_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} &= F_{E_t} && (40)
\end{aligned}$$

where

$$\begin{aligned}
\Lambda^{C_{1,t}} &= 1 + \lambda_t \Gamma^{C_{1,t}} && (41)\\
\Lambda^{C_{2,t+1}} &= 1 + \lambda_t \Gamma^{C_{2,t+1}} && (42)\\
\Lambda^{L_{1,t}} &= 1 + \lambda_t \Gamma^{L_{1,t}} && (43)\\
\Lambda^{L_{2,t+1}} &= 1 + \lambda_t \Gamma^{L_{2,t+1}} && (44)
\end{aligned}$$

with,

$$\begin{aligned}
\Gamma^{X_{i,t}} &= 1 + \Theta^{X_{i,t}} && (45)\\
\Theta^{C_{i,t}} &= \frac{C_{i,t} U_{C_{i,t}C_{i,t}} + L_{i,t} U_{L_{i,t}C_{i,t}}}{U_{C_{i,t}}} && (46)\\
\Theta^{L_{i,t}} &= \frac{L_{i,t} U_{L_{i,t}L_{i,t}} + C_{i,t} U_{C_{i,t}L_{i,t}}}{U_{L_{i,t}}} && (47)\\
& && (48)
\end{aligned}$$

for $X = \{C, L\}$ and $i = \{1, 2\}$. Let $\Lambda^{X_{1,t}}$ and $\Lambda^{X_{2,t+1}}$ denote the marginal cost of public funds (MCF), as defined in Barrage (2016); Jacobs and de Mooij (2015), which can be used to measure the costs of using distortionary taxation, that is, the costs associated of transferring a marginal unit of private consumption at each age to the government. Let $\Theta^{C_{i,t}}$, $\Theta^{L_{i,t}}$ be general equilibrium elasticities in the spirit of Atkeson et al. (1999), which account for the interactions between consumption-leisure marginal utilities. From equation (40), we know thus that the optimal carbon tax in period $t > 0$ that decentralizes the optimal allocation under distortionary taxation is implicitly defined as follows:

$$\tau_t^{E^*} = \sum\nolimits_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} \tag{49}$$

Notice that, however, from equations (35)-(39), once the government has to rely on distortionary taxation, this creates a wedge between the marginal rate of substitution for consumption and the marginal rate of transformation i.e., the return on physical capital investment.

Lemma 2. *The optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon:*

$$\tau_t^{E^*} = MCC_t \tag{50}$$

but not attains its Pigouvian level

$$\tau_t^{E^*} = \sum\nolimits_{i=1}^{\infty} \beta^i \prod_{j=1}^{i} \frac{U_{C_{2,t+j}}}{U_{C_{1,t+j-1}}} \Xi_{t+j-1}^C \theta_i Y_{t+i} \neq \tau_t^{PIGOU} \tag{51}$$

as long as $\Xi_t^C \neq 1$, *where* the consumption-savings wedge, Ξ_t^C, *is given by:*

$$\Xi_t^C = \frac{\Lambda^{C_{2,t+1}}}{\Lambda^{C_{1,t}}} \tag{52}$$

Proof. To get the first result, equation (50), replace the first-order condition for capital from the social planner's problem, (39), into (40). The second result, equation (51), follows from conditions (35-36) and (40). ∎

Lemma 2 provides the characterization for the setting of optimal carbon prices. Note that the consumption-savings wedge, Ξ_t^C, reflects the ratio of inter-temporal welfare costs, from using distortionary taxes, represented by the marginal cost of public funds in period t and $t+1$. In general, the optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon, but not always attains its Pigouvian level unless I provide certain conditions for the consumption-savings wedge to be one. For instance, the requirements for the marginal cost of funds to be constant over time. In addition, under this fiscal structure, optimal income taxes can be derived as:

Lemma 3. *The optimal capital and labor income taxes in an economy with distortionary fiscal policy are given by:*

$$\tau_{t+1}^{K} = 1 - \Xi_t^C \tag{53}$$

$$\frac{1-\tau_{t+1}^{L}}{1-\tau_t^{L}} = \frac{\Xi_t^C}{\Xi_t^L} \tag{54}$$

where Ξ_t^L, the labor supply wedge, *is given by:*

$$\Xi_t^L = \frac{\Lambda^{L_{2,t+1}}}{\Lambda^{L_{1,t}}} \tag{55}$$

Proof. Using the primal approach, see Lemma 1, by combining the FOC's for the social planner (35)-(36) and for the households (4), it yields the optimal capital income tax (53). In addition, from (35-38) and (5-6), we can get the path for labor income taxes, (54). ■

Taking together Lemma 2 and Lemma 3, it turns out that if $\Xi_t^C = 1$, then from (53) the optimal capital income tax is zero, and using (51) the optimal carbon tax attains its Pigouvian level, without imposing any restrictions on the path of labor income taxes. The following proposition points out the conditions, in terms of wedges, under which optimal carbon prices would differ from the Pigouvian level and the setting of other taxes in the economy.

Proposition 2. *In a second-best fiscal policy:*

1. *The optimal carbon tax always equals the market costs of carbon, (50); however, it is below (above) its Pigouvian level, (51), if the consumption-savings wedge, (52), is below (above) one.*

2. *The optimal capital tax, (53), is positive (negative) if the consumption-savings wedge, (52), is below (above) one.*

3. *The labor income taxes, (54), decrease over time,* $\tau_t^L > \tau_{t+1}^L$, *if the consumption-savings wedge, (52), is greater than the labor wedge, (55).*

Proof. The results follow directly from Lemma 2 and Lemma 3. ■

From proposition 2, a noteworthy implication has to do with the fact that the government always finds optimal to discount future marginal damages using the market interest rate and tax carbon emissions below (or above) its Pigouvian rate relatively to the consumption-savings wedge, Ξ_t^C. The results presented in numerals 2 and 3 are not novel, though. For instance, Conesa et al. (2009) and Erosa and Gervais (2002) find out that in absence of age-dependent labor income taxation, the capital income tax rate is different from zero, which in terms of this paper, would imply a consumption-savings wedge distinct of one.

It is important to note that so far I have assumed that the government has access to a full set of income taxes i.e., capital and age-dependent taxes and no constraints on households' preferences. That is, the second-best problem is not restricted. In particular, I show below that if we extend or restrict the set of available tax instruments to the government, different capital income tax policies could be optimal, conditional to the assumptions on separability in preferences over consumption and leisure. In this sense, to put more structure on the model, in the next section I also proceed to use separable and non-separable preferences as in Conesa et al. (2009) to draw some implications in terms of tax instruments and preferences modeling for the setting of carbon policies.

4. Age-dependent labor income taxes

Previous literature has pointed out that the existence of individual-specific taxation could generate welfare gains in the implementation of fiscal policies in the presence of consumption externalities (Jacobs and de Mooij, 2015; Kaplow, 2012). As mentioned in the introduction, recent research has also indicated that age-dependent labor income taxation in economies with heterogeneous agents, e.g, in terms of abilities, could reduce the costs associated to distortionary fiscal policy (Bastani et al., 2013; Gervais, 2012; Weinzierl, 2011; Blomquist and Micheletto, 2008). Thus, in order to understand the role of age-dependent labor income taxes in the setting of optimal climate and fiscal policy when there are production externalities, as special cases I first consider a policy with age-dependent taxation under two different assumptions about separability and non-separability in preferences over consumption and leisure. Then, I provide additional general results for how the set of optimal tax rates changes when I assume a constrained government who cannot enact differential labor income taxes, that is, the third-best fiscal policy.

4.1. Age-dependent taxes

In this subsection, I describe under which conditions the optimal carbon price can attain its Pigouvian level using two different preference specifications. It has been shown that the assumption about complementarity between consumption and leisure has important implications for the setting of optimal income taxes, since those interactions constrain how the government can reduce the distortions in the economy when individualized lump-sum taxation is not possible (see e.g., Conesa et al. (2009), Erosa and Gervais (2002)).

4.1.1. Separable preferences

One of the main implications of using separable preferences over consumption and labor is that there are not complementary effects, $U_{C_{1,t},L_{1,t}} = U_{C_{2,t+1},L_{2,t+1}} = 0$. For instance, suppose that the households' preferences can be represented by the following utility function as in Conesa et al. (2009):

$$U(C_t, L_t) = \frac{C_t^{1-\sigma_1} - 1}{1-\sigma_1} + \chi \frac{(1-L_t)^{1-\sigma_2}}{1-\sigma_2} \tag{56}$$

where σ_1 and σ_2 denote consumption and labor supply elasticities, respectively; and χ measures the distaste for work with respect to consumption. Under this assumption, as a special case for Proposition 2, it follows:

Proposition 3. *If the government has access to age-dependent labor income taxes, and the households have preferences over consumption and labor which can be represented by an utility function defined as in (56), then:*

1. *The optimal carbon tax equals the market costs of carbon, (50), and attains its Pigouvian level, (51).*

2. *The optimal capital tax, (53), is zero.*

3. *If $L_{1,t} > L_{2,t+1}$, then $\tau^L_{1,t} > \tau^L_{2,t+1}$.*

Proof. In appendix A. ■

By using a utility function which is separable in consumption and labor, the tax rate on capital income is zero and the carbon tax fully internalizes climate damages from carbon emissions that affect output. This result is equivalent to the one in Barrage (2016), in an infinitely-lived agent model, when climate change only affects production.[10] The intuition for these findings is straightforward. Non-complementarity between consumption and labor reduces the costs of implementing the second-best fiscal policy given that in this case the consumption-savings wedge, Ξ_t^C, is constant over time. Besides, since the government has access to a full set of age-dependent labor income taxes, it is optimal to avoid the distortions in inter-temporal consumption-savings decisions. Thus, considering that a zero optimal capital income tax rate does not affect the relative price between consumption at period t and consumption at period $t+1$, the marginal rate of substitution equals the marginal rate of transformation, and as a consequence the optimal carbon tax is set at its first-best (The Pigouvian level).

4.1.2. Non-separable preferences

Here, I assume that preferences are represented by the following Cobb-Douglas utility function which is not separable in consumption and labor as in Conesa et al. (2009):

$$U(C_t, L_t) = \frac{\left(C_t^{\xi}(1-L_t)^{1-\xi}\right)^{1-\sigma}}{1-\sigma} \tag{57}$$

where ξ measures the degree of substitutability between consumption and leisure and σ denotes the coefficient of risk aversion. Notice that in this case, the previous non-complementarity vanishes, $U_{C_{1,t},L_{1,t}}, U_{C_{2,t+1},L_{2,t+1}} \neq 0$, and therefore, labor income taxes affect both labor supply and consumption decisions. Under this specification, as a special case for Proposition 2, I get the following:

Proposition 4. *If the government has access to age-dependent labor income taxes, and the households have preferences over consumption and labor which can be represented by the usual Cobb-Douglas utility function (57), then:*

[10] Barrage (2016) also shows that this result holds using non-separable preferences. In an OLG framework, however, it is not valid since young and old households have different consumption and labor supply profiles, and that complementarity creates a motive for the government to set both labor and capital income taxes at the same time to smooth optimal paths for consumption and leisure (see e.g., Erosa and Gervais (2001) and Erosa and Gervais (2002)).

1. *The optimal carbon tax equals the market costs of carbon, (50), however, it is below (above) its Pigouvian level, (51), as long as the labor supply is decreasing (increasing) over the life-cycle*

2. *The optimal capital tax, (53), is positive (negative) as long as the labor supply is decreasing (increasing) over the life-cycle.*

3. $\tau^L_{1,t} > \tau^L_{2,t+1}$*, as long as* $L_{1,t} > L_{2,t+1}$ *and the intertemporal elasticity of substitution,* $1/\sigma$*, is above one.*

Proof. In appendix A. ■

The non-separability of consumption and leisure creates new interactions between the optimal allocation of consumption and labor supply over time and, therefore,it also affects the allocation of leisure. In this case, since labor income taxes distort both consumption and labor optimal paths, the government finds optimal to set a non-zero capital income tax to offset the changes in demand for leisure and consumption. In particular, the allocation of labor over the life-cycle will determine the sign of the capital income tax.

It is important to note that in contrast to Barrage (2016), since in infinitely-lived agent models it is optimal to set a zero capital income tax, here I can derive the implications for the optimal price on carbon emissions when non-zero capital income taxes are optimal. In this case, due to non-separability in preferences and heterogeneity in the age of households, it is possible to get positive (or negative) tax rates on capital returns as an optimal fiscal policy. Thus, I find that again, as in Proposition 2, the optimal carbon tax equals the market costs of carbon, although it differs from the Pigouvian level. The reason is straightforward. Even with a non-zero capital income tax, the marginal productivity of capital does not change, and it only affects the households' decisions with respect to consumption and leisure.

4.2. Age-independent taxes

In the previous apart, I consider special cases in terms of preferences modeling that complement and change the results pointed out in Proposition 2. Here, I assume that the government has no access to age-dependent labor income taxes, that is, $\tau^L_{1,t} = \tau^L_{2,t} = \tau^L_t$. In this case, notice that the government losses one degree of freedom in the set of policy instruments it can use to finance its stream of expenditures and inherited debt.

Thus, bearing in mind that, an additional constraint has to be added to the government's problem described using the primal approach (see Lemma 1).

$$\phi_2 U_{L_{1,t}} U_{C_{2,t}} - \phi_1 U_{L_{2,t}} U_{C_{1,t}} = 0 \tag{58}$$

As shown in Erosa and Gervais (2002) and Conesa et al. (2009), this restriction generates a role for capital income taxes as they can help the government to tax individuals at different rates conditional on age. To see why it is the case, notice that from Lemma 3, it follows:

$$\frac{1-\tau^L_{t+1}}{1-\tau^L_t} = \frac{1-\tau^K_{t+1}}{\Xi^L_t} \tag{59}$$

Given that under age-independent labor income taxes, $\tau^L_{1,t} = \tau^L_{2,t} = \tau^L_t$, using the previous equation I can obtain the following:

$$\tau^K_{t+1} = 1 - \Xi^L_t \tag{60}$$

Since labor supply in an OLG framework is in general not flat, given the heterogeneity with respect to age and productivities, the labor wedge, Ξ^L_t, is not equal to one and it depends on the allocation of labor over the life-cycle. Thus, capital taxes can be used to generate the same wedge as in the situation with age-dependent taxation (see e.g., Conesa et al. (2009)).

Let $\gamma^t \psi_t$ denote the Lagrange multiplier associated to the labor-income taxation constraint (58). The optimality conditions become:[11]

$$\begin{aligned}
U_{C_{1,t}} \cdot \underline{\Lambda}^{C_{1,t}} &= \mu_t && (61)\\
\frac{\beta}{\gamma} U_{C_{2,t+1}} \cdot \underline{\Lambda}^{C_{2,t+1}} &= \mu_{t+1} && (62)\\
U_{L_{1,t}} \cdot \underline{\Lambda}^{L_{1,t}} &= -\mu_t \phi_1 F_{L_t} && (63)\\
\frac{\beta}{\gamma} U_{L_{2,t+1}} \cdot \underline{\Lambda}^{L_{2,t+1}} &= -\mu_{t+1} \phi_2 F_{L_{t+1}} && (64)\\
\frac{1}{F_{K_{t+1}}} &= \gamma \frac{\mu_{t+1}}{\mu_t} && (65)\\
\sum_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} &= F_{E_t} && (66)
\end{aligned}$$

[11] Notice that I multiply the constraint, (58), by $\frac{\beta}{\gamma}$ for the ease of calculations.

where,

$$\underline{\Lambda}^{C_{1,t}} = 1 + \lambda_t \Gamma^{C_{1,t}} + \psi_t \Upsilon^{C_{1,t}} \tag{67}$$

$$\underline{\Lambda}^{C_{2,t+1}} = 1 + \lambda_t \Gamma^{C_{2,t+1}} + \psi_{t+1} \Upsilon^{C_{2,t+1}} \tag{68}$$

$$\underline{\Lambda}^{L_{1,t}} = 1 + \lambda_t \Gamma^{L_{1,t}} + \psi_t \Upsilon^{L_{1,t}} \tag{69}$$

$$\underline{\Lambda}^{L_{2,t+1}} = 1 + \lambda_t \Gamma^{L_{2,t+1}} + \psi_{t+1} \Upsilon^{L_{2,t+1}} \tag{70}$$

with,

$$\Gamma^{X_{i,t}} = 1 + \Theta^{X_{i,t}} \tag{71}$$

$$\Theta^{C_{i,t}} = \frac{C_{i,t} U_{C_{i,t} C_{i,t}} + L_{i,t} U_{L_{i,t} C_{i,t}}}{U_{C_{i,t}}} \tag{72}$$

$$\Theta^{L_{i,t}} = \frac{L_{i,t} U_{L_{i,t} L_{i,t}} + C_{i,t} U_{C_{i,t} L_{i,t}}}{U_{L_{i,t}}} \tag{73}$$

$$\Upsilon^{X_{1,t}} = \frac{\phi_2 U_{L_{1,t} X_{1,t}} U_{C_{2,t}} - \phi_1 U_{L_{2,t}} U_{C_{1,t} X_{1,t}}}{U_{X_{1,t}}} \tag{74}$$

$$\Upsilon^{X_{2,t+1}} = \frac{\phi_2 U_{L_{1,t+1}} U_{C_{2,t+1} X_{2,t+1}} - \phi_1 U_{L_{2,t+1} X_{2,t+1}} U_{C_{1,t+1}}}{U_{X_{2,t+1}}} \tag{75}$$

for $X = \{C, L\}$ and $i = \{1, 2\}$. Here, $\underline{\Lambda}^{X_{i,t}}$ is the modified marginal cost of public funds, and $\Upsilon^{X_{i,t}}$ corresponds to the welfare costs involved with the no availability of age-dependent labor income taxation, constraint (58). Notice that now the non-availability of individualized labor income taxes implies a marginal cost of public funds in the implementation of a distortionary fiscal policy that now depends on two terms, and not in only one as described in Proposition 2: (i) the implementability condition, and (ii) the age-independent labor income tax constraint.

Proposition 5. *In a third-best fiscal policy:*

1. *The optimal carbon tax always equals the market costs of carbon, (50), however, it is below (above) its Pigouvian level, (51), if the modified consumption-savings wedge, $\Xi_t^C = \underline{\Lambda}^{C_{2,t+1}}/\underline{\Lambda}^{C_{1,t}}$, is below (above) one.*

2. *The optimal capital tax, (53), is positive (negative) if the modified consumption-savings wedge, Ξ_t^C, is below (above) one.*

3. *The labor income taxes decrease over time, $\tau_t^L > \tau_{t+1}^L$, if the modified consumption-savings wedge, Ξ_t^C, is greater than the modified labor wedge, $\Xi_t^L = \underline{\Lambda}^{L_{2,t+1}}/\underline{\Lambda}^{L_{1,t}}$.*

Proof. The results follow directly from Lemma 2 and Lemma 3, but now taking into account the modified version for both the consumption-savings and labor wedges. ■

Proposition 5 implies that the main results derived in Proposition 2 still hold, however, now they are more general in the sense that allow us to determine how the reduction in the set of policy instruments alters the second-best optimal fiscal policy. In the context of optimal environmental policies, it is also easy to check that again the optimal carbon price equals the market costs of carbon, since the production side of the economy is not affected, that is, the marginal productivity of capital does not change under an age-independent taxation fiscal structure.

5. Exogenous capital income taxes

In the previous section I derived the implications for optimal climate and fiscal policy of facing constraints with respect to the use of age-related labor income taxation. Here, I describe why those prescriptions are no longer valid when the government is constrained to implement an exogenous capital income tax rate in the spirit of Barrage (2016). Following the same procedure as before, an additional constraint has to added to the government's problem illustrated in Lemma 1:

$$\frac{U_{C_{1,t}}}{U_{C_{2,t+1}}} - \beta(1-\overline{\tau}^K)F_{K_{t+1}} = 0 \tag{76}$$

Let $\gamma^t\varphi_t$ denote the Lagrange multiplier associated to the capital income tax rate constraint. The optimality conditions are now given by:

$$\begin{aligned}
U_{C_{1,t}} \cdot \overline{\Lambda}^{C_{1,t}} &= \mu_t && (77)\\
\frac{\beta}{\gamma}U_{C_{2,t+1}} \cdot \overline{\Lambda}^{C_{2,t+1}} &= \mu_{t+1} && (78)\\
U_{L_{1,t}} \cdot \overline{\Lambda}^{L_{1,t}} &= -\mu_t\phi_1 F_{L_t} && (79)\\
\frac{\beta}{\gamma}U_{L_{2,t+1}} \cdot \overline{\Lambda}^{L_{2,t+1}} &= -\mu_{t+1}\phi_2 F_{L_{t+1}} && (80)\\
\frac{1}{F_{K_{t+1}}} + \frac{\varphi_t}{\mu_t}(1-\overline{\tau}^K)\frac{F_{K_{t+1}K_{t+1}}}{F_{K_{t+1}}} &= \gamma\frac{\mu_{t+1}}{\mu_t} && (81)\\
\sum_{i=1}^{\infty}\gamma^i\frac{\mu_{t+i}}{\mu_t}\theta_i Y_{t+i} + \alpha\beta(1-\overline{\tau}^K)\sum_{i=1}^{\infty}\theta_i\frac{\gamma^{i-1}\varphi_{t+i-1}}{\mu_t}\frac{Y_{t+i}}{K_{t+i}} &= F_{E_t} && (82)
\end{aligned}$$

where,

$$\overline{\Lambda}^{C_{1,t}} = 1 + \lambda_t \Gamma^{C_{1,t}} + \varphi_t \Pi^{C_{1,t}} \tag{83}$$
$$\overline{\Lambda}^{C_{2,t+1}} = 1 + \lambda_t \Gamma^{C_{2,t+1}} - \varphi_t \Pi^{C_{2,t+1}} \tag{84}$$
$$\overline{\Lambda}^{L_{1,t}} = 1 + \lambda_t \Gamma^{L_{1,t}} + \varphi_t \Pi^{L_{1,t}} \tag{85}$$
$$\overline{\Lambda}^{L_{2,t+1}} = 1 + \lambda_t \Gamma^{L_{2,t+1}} - \varphi_t \Pi^{L_{2,t+1}} \tag{86}$$

with,

$$\Pi^{C_{1,t}} = \frac{1}{U_{C_{2,t+1}}} \frac{U_{C_{1,t}C_{1,t}}}{U_{C_{1,t}}} \tag{87}$$
$$\Pi^{C_{2,t+1}} = \frac{1}{\beta U^2_{C_{2,t+1}}} \frac{U_{C_{2,t+1}C_{2,t+1}}}{U_{C_{2,t+1}}} \tag{88}$$
$$\Pi^{L_{1,t}} = \frac{1}{U_{C_{2,t+1}}} \frac{U_{C_{1,t}L_{1,t}}}{U_{L_{1,t}}} \tag{89}$$
$$\Pi^{L_{2,t+1}} = \frac{1}{\beta U^2_{C_{2,t+1}}} \frac{U_{C_{2,t+1}L_{2,t+1}}}{U_{L_{2,t+1}}} \tag{90}$$
$$\Gamma^{X_{i,t}} = 1 + \Theta^{X_{i,t}} \tag{91}$$
$$\Theta^{C_{i,t}} = \frac{C_{i,t}U_{C_{i,t}C_{i,t}} + L_{i,t}U_{L_{i,t}C_{i,t}}}{U_{C_{i,t}}} \tag{92}$$
$$\Theta^{L_{i,t}} = \frac{L_{i,t}U_{L_{i,t}L_{i,t}} + C_{i,t}U_{C_{i,t}L_{i,t}}}{U_{L_{i,t}}} \tag{93}$$
$$\tag{94}$$

for $X = \{C, L\}$ and $i = \{1, 2\}$. Under this specification, $\Pi^{X_{i,t}}$ corresponds to the welfare costs involved with the non-availability of a flexible capital income tax rate, constraint (76).

Proposition 6. *Under no flexibility in the setting of optimal capital income taxes, it follows that:*

1. *The optimal carbon tax differs from both the market costs of carbon, (50), and its Pigouvian level, (51).*

Proof. Using the first-order conditions (77-82), and according to our definitions, we get the result. ■

Notice that now the introduction of a constrained capital income tax rate, since now the government has lost again a policy instrument, creates an inefficiency in the accumulation of capital, see equation (81) second term of the left-hand side, and one more direct interaction between future damages and capital returns, see equation (82) second term of the left-hand side, and the government has to take this into account when it wants to calculate the present value of future marginal damages of current marginal carbon emissions. This result is equivalent to the one in Barrage (2016) where a constrained set of policy instruments generate efficiency losses in the second-best fiscal policy. However, and connecting with the previous apart, my results are more general in the sense that an OLG economy allows me to study additionally the interaction of positive optimal capital income taxes and optimal prices for carbon emissions. Thus, in my framework, exogenous tax rates on capital returns impedes to the government to use the market interest rate to discount future marginal damages from carbon emissions, and therefore, the optimal carbon tax does not equals the market costs of carbon, , (50), a result that is robust to assumptions on preferences modeling and age-dependent taxation as shown above.

6. Discussion

This paper has studied the optimal climate and fiscal policy in an OLG economy. I show that in general the optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon, but not always attains its Pigouvian level. That is, future marginal damages to current carbon emissions are discounted using the market rate of interest which resembles the opportunity cost of capital investment, supporting the Nordhaus (2008) recommendation that claims that climate change mitigation investments should earn the same net return than other investments in the economy.

Moreover, I addressed the implications of separability in the utility function and different tax instruments available to the government on the setting of carbon taxes. In particular, I show that with a full set of tax instruments and separable preferences over consumption and labor supply, the optimal carbon tax attains its Pigouvian level. The intuition behind this outcome relies on the fact that separability implies a constant general equilibrium elasticity in consumption, result that avoids the distortions due to the introduction of proportional labor and capital income taxes. Thus, it turns out that since a zero capital income tax is optimal, the way households value current and future consumption is not affected and therefore the carbon tax equals both its Pigouvian level and the market costs of carbon.

However, once the government is constrained in the set of tax instruments, for instance, no age-dependent labor income taxation, there is space for an endogenous non-zero tax rate on capital income and the optimal carbon tax does not attain its Pigouvian level; that is, either a positive or a negative tax rate on capital income creates a wedge between the marginal rate of substitution for consumption and the marginal rate of transformation. When preferences are not separable over consumption and labor, even if I allow for the existence of age-dependent labor income taxes, the optimal carbon price differs from the Pigouvian tax. In general, I find out that the conditions which optimally generate a capital income tax rate different from zero, would also imply a distortion in the setting optimal environmental policies.

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A. Proofs

Lemma 1

Proof. The procedure follows closely the one in Iqbal and Turnovsky (2008) and Garriga (2001). I begin by showing that a competitive equilibrium must satisfy equations (30)-(33). Notice that equations (30) and (31) are equivalent to the first two constraints that come from the market clearing conditions in definition 1. To derive (32), I proceed to use the intertemporal budget constraint for the households which is given by:

$$C_{1,t} + \frac{C_{2,t+1}}{(1-\tau^K_{t+1})r_{t+1}} = (1-\tau^L_{1,t})\phi_1 w_t L_{1,t} + \frac{(1-\tau^L_{2,t+1})\phi_2 w_{t+1} L_{2,t+1}}{(1-\tau^K_{t+1})r_{t+1}} + B^D_{t+1}\left[\frac{R_{t+1}}{(1-\tau^K_{t+1})r_{t+1}} - 1\right] \tag{A.1}$$

Then, using the optimality conditions (4)-(7) from the household's problem, it follows that:

$$C_{1,t} + \frac{\beta U_{C_{2,t+1}}}{U_{C_{1,t}}} C_{2,t+1} = -\frac{U_{l_{1,t}}}{U_{C_{1,t}}} L_{1,t} - \frac{U_{L_{2,t+1}}}{U_{C_{2,t+1}}} L_{2,t+1} \frac{\beta U_{C_{2,t+1}}}{U_{C_{1,t}}} \tag{A.2}$$

which yields (32). The same procedure can be applied to derive (33) by considering that the budget constraint for the initial old is:

$$C_{2,0} = (1-\tau^L_{2,0}) w_0 \phi_2 L_{2,0} + \left[(1-\tau^K_0) r_0\right] K_0 + R_0 B_0 \tag{A.3}$$

Using the first-order conditions from the household's problem, (4-6), we arrive to conditions (58-76). To prove the last part of the proposition, the prices can be derived using the first-order conditions (11)-(13) from the firm's problem, in addition to a carbon tax, to make them consistent with a competitive equilibrium, that is:

$$\tau_t^E = F_{E_t} \tag{A.4}$$

$$w_t = F_{L_t} \tag{A.5}$$

$$r_t = F_{K_t} \tag{A.6}$$

Likewise, the set of policies for labor and capital income can be constructed by replacing allocations and equilibrium prices into the first order conditions from the household's problem such that tax rates satisfy those conditions, Therefore, using equations (4)-(6), the following conditions characterize labor and capital income taxes:

$$\tau_{t+1}^K = 1 - \frac{U_{C_{1,t}}}{\beta r_{t+1} U_{C_{2,t+1}}} \tag{A.7}$$

$$\frac{1 - \tau_{2,t+1}^L}{1 - \tau_{1,t}^L} = \frac{\phi_1 w_t U_{L_{2,t+1}} U_{C_{1,t}}}{\phi_2 w_{t+1} U_{L_{1,t}} U_{C_{2,t+1}}} \tag{A.8}$$

Finally, the return on debt holdings can be defined using the no arbitrage condition (7). Notice that the household's budget constraint also holds under those allocations and prices. To see that, replace the first order conditions from the household's problem into equations (32) and (33) to get the intertemporal budget constraint for the households. Since the feasibility constraint and the intertemporal budget constraint for the households are satisfied, by Walras' Law, the government budget constraint holds as well and government debt B_{t+1} is set accordingly.

■

Proposition 3

Proof. Using the fact that the constraint, (58), is not binding, and that consumption-labor separability implies $U_{CL} = U_{LC} = 0$, it follows:

$$\begin{aligned}\Theta^{C_{i,t}} &= -\sigma_1 &\text{(A.9)}\\ \Theta^{L_{i,t}} &= \frac{\sigma_2 L_{i,t}}{1-L_{i,t}} &\text{(A.10)}\end{aligned}$$

with,

$$\begin{aligned}U_{C_{i,t}} &= C_{i,t}^{-\sigma_1}\\ U_{C_{i,t}C_{i,t}} &= -\sigma_1 C_{i,t}^{-\sigma_1-1}\\ U_{L_{i,t}} &= -\chi(1-L_{i,t})^{-\sigma_2}\\ U_{L_{i,t}L_{i,t}} &= -\chi\sigma_2(1-L_{i,t})^{-\sigma_2-1}\end{aligned}$$

Thus, if $\Theta^{C_{1,t}} = \Theta^{C_{2,t+1}} = -\sigma_1$, then $\Lambda^{C_{1,t}} = \Lambda^{C_{2,t+1}}$, and using (52), it yields $\Xi_t^C = 1$. To check that the first two numerals of the proposition hold under this condition, replace $\Xi_t^C = 1$ in equations (53) and (51), to get the zero optimal capital income tax and the optimal carbon tax at the Pigouvian level, respectively. Finally, to derive optimal labor income taxes, we use (54) to obtain:

$$\frac{1-\tau_{2,t+1}^L}{1-\tau_{1,t}^L} = \frac{1}{\Xi_t^L} \tag{A.11}$$

From the previous equation, $\tau_{2,t+1}^L < \tau_{1,t}^L$, as long as $\Xi_t^L < 1$. This latter condition requires that $\Lambda^{L_{1,t}} > \Lambda^{L_{2,t+1}}$, which is only possible if $L_{2,t+1} < L_{1,t}$. That is, a declining labor supply requires labor income taxes that decrease with age, then $\Xi_t^L < 1$, and we obtain the third result.

■

Proposition 4

Proof. Without separability, the general equilibrium elasticities imply the following:

$$\Theta^{C_{i,t}} = -1 + (1-\sigma)\left[\xi - \frac{(1-\xi)L_{i,t}}{1-L_{i,t}}\right] \tag{A.12}$$

$$\Theta^{L_{i,t}} = \frac{[(1-\sigma)\xi + \sigma]L_{i,t}}{1-L_{i,t}} + \xi(1-\sigma) \tag{A.13}$$

with,

$$\begin{aligned}
U_{C_{i,t}} &= \frac{\xi(1-\sigma)U_{i,t}}{C_{i,t}} \\
U_{L_{i,t}} &= -\frac{(1-\xi)(1-\sigma)U_{i,t}}{1-L_{i,t}} \\
U_{C_{i,t}C_{i,t}} &= \frac{\left[(1-\sigma)\xi^2-\xi\right](1-\sigma)U_{i,t}}{C_{i,t}^2} \\
U_{C_{i,t}L_{i,t}} = U_{L_{i,t}C_{i,t}} &= -\frac{(1-\xi)\xi(1-\sigma)^2U_{i,t}}{C_{i,t}(1-L_{i,t})} \\
U_{L_{i,t}L_{i,t}} &= -\frac{(1-\xi)(1-\sigma)U_{i,t}}{(1-L_{i,t})^2}\left[(1-\xi)\sigma+\xi\right]
\end{aligned}$$

Using the previous results, it follows that:

$$\Xi_t^C = \frac{1+\lambda_t(1-\sigma)\left[\xi-\frac{(1-\xi)L_{2,t+1}}{1-L_{2,t+1}}\right]}{1+\lambda_t(1-\sigma)\left[\xi-\frac{(1-\xi)L_{1,t}}{1-L_{1,t}}\right]} \tag{A.14}$$

In this case, notice that $\Xi_t^C = 1$ if and only if households feature a flat labor supply. So, if the labor supply is decreasing (increasing) over the life-cycle, it is optimal to set a positive (negative) capital income tax, and the optimal carbon tax would be lower (higher) than the Pigouvian level.

Likewise, with respect to labor income taxes, I obtain that:

$$\Xi_t^L = \frac{1+\lambda_t(1-\sigma)\left[\xi-\frac{(1-\xi)L_{2,t+1}}{1-L_{2,t+1}}\right]+\frac{\lambda_t}{1-L_{2,t+1}}}{1+\lambda_t(1-\sigma)\left[\xi-\frac{(1-\xi)L_{1,t}}{1-L_{1,t}}\right]+\frac{\lambda_t}{1-L_{1,t}}} \tag{A.15}$$

We know thus that:

$$\frac{1-\tau_{2,t+1}^L}{1-\tau_{1,t}^L} = \frac{\Xi_t^C}{\Xi_t^L} > 1 \tag{A.16}$$

as long as $L_{2,t+1} < L_{1,t}$ and $1+\lambda_t(1-\sigma) > 0$. To see this, define $m_{i,t}$ and $n_{i,t}$ as:

$$m_{i,t} = 1+\lambda_t(1-\sigma)\left[\xi-\frac{(1-\xi)L_{i,t}}{1-L_{i,t}}\right]$$

$$n_{i,t} = \frac{\lambda_t}{1-L_{i,t}}$$

such that,

$$\frac{\Xi_t^C}{\Xi_t^L} = \frac{\frac{m_{2,t+1}}{m_{1,t}}}{\frac{m_{2,t+1}+n_{2,t+1}}{m_{1,t}+n_{1,t}}} \tag{A.17}$$

After some algebra, and by replacing our auxiliary variables, it yields the condition for $\Xi_t^C/\Xi_t^L > 1$:

$$\{L_{1,t} - L_{2,t+1}\}\{1 + \lambda_t(1-\sigma)\} > 0 \tag{A.18}$$

Note that this expression is particularly true whenever that the intertemporal elasticity of substitution, $1/\sigma$, is above one.

■