Presentation: Does Warm Glow Matters? Saving Behavior with Bequest Motivation

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Motivations for the Project

- Based a most popular assumption in Economics, we are totally selfish.
- However, it's not always the case in the real world.
- For example, the existence of leaving a bequest.

Motivations for the Project

- Saving behaviors cannot be fully explained by ordinary over-lapping generational model.
- Also the agent will leave nothing after death.

Motivations for the Project

Median Net Worth by age and Net Worth in US,1984 (in thousands of dollars)

	Net Worth Quintile				
Age	20%	40%	60%	80%	100%
Under 25	-1.3	0.2	2.2	5.6	18.1
25-34	-0.6	1.7	8.1	23.1	65.6
35-44	0	11.3	35.5	66.6	152.1
45-54	0.5	23.3	56.4	97.8	205.3
55-64	2.4	35.3	72.4	118.9	245.4
65 and over	0.8	26.7	59.5	99.3	200.1
65-74	0.8	29.0	62.0	103.8	209.6
65-69	1.1	32.6	65.6	108.7	219.7
70-74	0.5	24.4	59.4	96.5	197.6
75 and over	0.7	24.0	54.6	92.5	181.1
All ages	0	7.5	32.5	71.7	166.9

Motivations for the Project

Thus we try to use an OLG model with altruistic motivation to explain this:

- Agents still save during retirement and leave bequest.
- Different scale of bequest across different wealth quantiles.

Most Closest Research Paper

One closest research paper related to our project:

De Nardi, M. (2004). Wealth inequality and intergenerational links.

Most Closest Research Paper
De Nardi's Study

Quick overview of De Nardi's research:

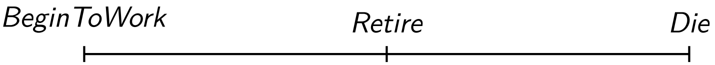
- Over-lapping generation model(OLG) with two intergenerational links:
    - productivity inheritance
    - intentional bequests
- Including these two links improves the performance of the model:
    - wealth inequality more consistent with data
    - save for intentional bequests

Most Closest Research Paper
Simplification

We simplfied De Nardi's study in several ways:

- We exclude human capital transmission in our model: exogeneous initial distribution.
- We distributed voluntary bequests and accidental bequests equally to all living agents.
- All agents in our model receive transfer in all periods.

Theoretical Model
Model Overview



Two periods

- Working period – work with stochastic productivity
- After retirement – stop working and starts facing a positive probability of dying

Theoretical Model

Model Overview

More ingredients

- Agents save for precautionary purpose and retirement
- All the assets left by the agents who die prematurely become accidental bequests
- Furthermore, we add *warm glow* saving motive: utility from leaving bequests

Theoretical Model

Demographics

	$t \quad (n_{age}^{generation})$						
Generations	1	2	3	4	...	T	...
j	n_1^j	n_2^j	n_3^j	n_4^j	...	n_T^j	0
$j+1$		n_1^{j+1}	n_2^{j+1}	n_3^{j+1}	...	n_{T-1}^{j+1}	n_T^{j+1}
$j+2$			n_1^{j+2}	n_2^{j+2}	...	n_{T-2}^{j+2}	n_{T-1}^{j+1}
$j+3$				n_1^{j+3}	...	n_{T-3}^{j+3}	n_{T-2}^{j+2}

Theoretical Model
Demographics

- One model period 1 year long
- During each period a continuum of agents are born
- T: terminal period
- n: constant population growth rate
- s_t: exogeneous survial rate for $t+1$ period
  note: for the jth generation $n^j_{t+1} = s_t * n^j_t$
- μ_t: constant fraction of the jth generation in the total population

Theoretical Model
Demographics

In our model, we let agents:

- Enter the model at $t = 1$, when we assume he is of the age of 20
- Pay labor and capital taxes when they are in working period
- At $t = 46$ (age 65) retire, start to receive pensions and face a nonzero death probability
- (For simplify) Not die before $t = 46$ (age 65), but die for sure at $t = 67$ (age 86)

Theoretical Model
Preferences and Technology

Two preferences:

- CRRA: $u(c) = c^{1-\sigma}/(1-\sigma)$
- *warm glow*: $\phi(a) = \phi_1(1 + \frac{a}{\phi_2})^{1-\sigma}$

ϕ_1: agent's overall concern of leaving intentional bequest

ϕ_2: the extent that agent treats intentional bequests as a luxury good

Theoretical Model
Preferences and Technology

- CRRA: $u(c) = c^{1-\sigma}/1 - \sigma$
- *warm glow*: $\phi(a) = \phi_1(1 + \frac{a}{\phi_2})^{1-\sigma}$

ϕ_1: agent's overall concern of leaving intentional bequest
ϕ_2: the extent that agent treats intentional bequests as a luxury good

We let $\phi_1 < 0$, $\phi_2 > 0$. Why?

Theoretical Model
Preferences and Technology

- $\phi(a) = \phi_1(1 + \frac{a}{\phi_2})^{1-\sigma}$

  ϕ_1: agent's overall concern of leaving intentional bequest

  ϕ_2: the extent that agent treats intentional bequests as a luxury good

According to our intuition, we expect:
$D_{\phi_1}\phi = (1 + a/\phi_2)^{1-\sigma} > 0$
$D_{\phi_2}\phi = -(1 - \sigma)\phi_1(1 + a/\phi_2)^{-\sigma}(a/\phi_2^2) < 0$

But in our later setting, $\sigma > 1$!
Why $\phi_2 > 0$? $D_a\phi = \phi_1(1 - \sigma)(1 + a/\phi_2)^{-\sigma}(1/\phi_2) > 0$

Theoretical Model
Preferences and Technology

Agents labor endowments:

- $\{\epsilon_t\}$: exogeneous age-efficiency profile
- Z: finite shock space, with shock $z \in Z$
- Q: transition matrix, derived from a Markov procecss $\{z_t\}$
  In our model the agents face the same Markov process and transition matrix
- $\epsilon_t z_t$: Agent's labor endowment at time t
- w: endogeneous agent's wage

Theoretical Model
Preferences and Technology

Investments and Aggregation Production Function

- Agent can only invest in physical capital, a
- r: endogeneous net return
- δ: exogeneous depreciation rate
  $r + \delta$ is gross-of-depreciation rate
- Agents cannot own negative capital stocks at any time.
- aggregation production function: $F(K, L) = AK^{\alpha}L^{1-\alpha}$

Theoretical Model
Government

Government collects tax:

- τ_a: exogeneous constant capital income tax
- τ_l: endogeneous labor income tax to balance the government budget constraint

Government uses tax to pay:

- p: pensions to retirees until the agents die

(We imagine) Government helps to allocate the accedental bequests equally as transfer:

- T: lump-sum transfer, all agents can receive at each period.

Furthermore, we assume that the total government expenditure is 18% of the total output, that is:

- $g \equiv 0.18 * (r * K + w * L)$

Theoretical Model
Household's Recursive Problem

- In each period a t-years old agent chooses consumption c in the current period and investment in risk-free asset holdings a' for the next period.
- A individual state at a point in time is denoted by $x = (a, z)$
- Optimal decision rules: $c(x, t)$ and $a(x, t)$ solve the dynamic programming problem faced by agents.

Theoretical Model
Household's Recursive Problem

From $t = 1$ to $t = 44$ (Age 20 to 63)

$$V(t, a, z) = max_{c,a'}\{u(c) + \beta E_t V(t + 1, a', z')\}$$

subject to

$$a' + c = (1 - \tau_l)w\epsilon z + (1 + (r - \tau_a))a + T$$

$$a' \geq 0, c \geq 0$$

Theoretical Model

Household's Recursive Problem

$t = 45$ **(Age 64)**

$$V(t,a,z) = max_{c,a'}\{u(c) + s_t \beta E_t W(t+1, a', z') + (1-s_t)\phi(a'))\}$$

subject to

$$a' + c = (1-\tau_l)w\epsilon z + (1 + (r - \tau_a))a + T$$

$$a' \geq 0, c \geq 0$$

Theoretical Model

Household's Recursive Problem

From $t = 46$ to $t = 66$ (Age 65 to 85)

$$W(t, a, z) = max_{c,a'}\{u(c) + s_t \beta E_t W(t+1, a', z') + (1 - s_t)\phi(a')\}$$

subject to

$$a' + c = (1 + (r - \tau_a))a + T + p$$

$$a' \geq 0, c \geq 0$$

Theoretical Model

Household's Recursive Problem

Terminal period $T = 67$ **(Age 86)**

$$W(T, a, z) = \phi(a)$$

Stationary Equilibrium

Distributions of Individual States Across Agents

For convenience, we use some concepts in probability theory:

- $X = [0, \infty) \times Z$: sample space, or state space
- ψ_t: a probability measure
- $\mathscr{B}(X)$: Borel σ-algebra of state space X
  note: $\mathscr{B}(X)$ can be understood as a particular collection of subsets of X, similar as power set
- $\psi_t(B)$: can be seen as the fraction of age t agents whose individual state lies in $B \in \mathscr{B}(X)$
- $(X, \mathscr{B}(X), \psi_t)$: the probability space we are interested in

Stationary Equilibrium
Distributions of Individual States Across Agents

Define the distribution of indivual states across agents, recrusively:

- Initial distribution ψ_1 is taken as exogeneous.
- For $t = 2, 3, ..., T$:

$$\psi_t(B) = \int_X P(x, t-1, B) d\psi_{t-1} \qquad \forall B \in \mathscr{B}(X)$$

- $P(x, t-1, B)$ is the transition function which gives the probability that an age $t-1$ agent transits to the set B next period given the agent's current state x.
- $P(x, t-1, B)$ is determined by the optimal decision rule we derived from dynamic programming (see our example next slide).

Stationary Equilibrium

Distributions of Individual States Across Agents

In our Matlab implementation:

- $X = A \times Z$, where A is capital space and Z is labor productivity shock space. Both discrete.
- $\mathscr{B}(X) = \mathscr{P}(A) \times \mathscr{P}(Z)$
- $$P(x, t-1, B) = \mathbb{1}_{\{a_t \in B_A\}} \sum_{z_t \in B_Z} \pi(z_{t-1}, z_t)$$

  where
  a_t: optimal saving policy
  $B_A = \{a \in A | (a(x,t), z') \in B\}$
  $B_Z = \{z' \in Z | (a(x,t), z') \in B\}$
  $\pi(z_{t-1}, z_t)$: transition probability
  $((t-1, t)$ element in transition matrix Q)

Stationary Equilibrium

Definition of the Stationary Equilibrium

$\{c(x,t), a(x,t), w, r, L, K, T, g, \tau_l, \tau_a, b\}$ and $(\psi_1, \psi_2, ..., \psi_T)$

- $c(x,t)$ and $a(x,t)$ are optimal decision rules.
- Factor prices are equal to marginal products: $w = F_L(L,K)$, $r = F_K(L,K) - \delta$.

Stationary Equilibrium

Definition of the Stationary Equilibrium

(cont.)

- All markets clear:
  - $\sum_{t=1}^{T} \mu_t \int\limits_{X} a(x,t) d\psi_t = K$
  - $\sum_{t=1}^{T} \mu_t \int\limits_{X} (\epsilon_t z_t) d\psi_t = L$
- $\psi_t(B) = \int\limits_{X} P(x, t-1, B) d\psi_{t-1} \qquad \forall B \in \mathscr{B}(X)$
- $g + p(\sum_{t=46}^{T} \mu_t) = \tau_l L + \tau_k r K$

  where $g \equiv 0.18 * (r * K + w * L)$

Stationary Equilibrium

Definition of the Stationary Equilibrium

(cont.)

- Lump-sum transfer equals to accidental bequests T equals

$$\left[\sum_{t=1}^{T}\mu_t(1-s_{t+1})\int_X a(x,t)(1+r(1-\tau_k))d\psi_t\right]\Big/(1-n)$$

note: $1-n=1+\frac{N_t-N_{t+1}}{N_t}$ *denotes all living agents*

Stationary Equilibrium

Aggregation Transfer Wealth

More about transfer wealth:

- Like K and L, we want to compute the transfer aggregation in this model economy.
- Aggregate wealth = life-cycle wealth + transfer
- How? Just rewrite the budget constraint recursively.

$$a_{t+1} = a_t(1 + r(1 - \tau_a)) + (1 - \tau_l)\epsilon_t z_t w + p_t - c_t + T$$

$$= \sum_{j=0}^{t-1} \{(1 - \tau_l)\epsilon_{t-j} z_{t-j} w + p_{t-j} - c_{t-j}\}(1 + r(1 - \tau_a))^j$$

$$+ \sum_{j=0}^{t-1} T(1 + r(1 - \tau_a))^j$$

- Aggregate transfer wealth $= \sum_t \mu_t \sum_{j=0}^{t-1} T(1 + r(1 - \tau_a))^j$

U.S. Calibration

Parameter	Value	Description
A	0.895	Technological level
β	0.95 - 0.97	Discount factor
r	endogenous $=$ 0.06	Interest rate of capital
w	endogenous $=$ 1	Wage rate
n	0.012	Annual population growth rate
g	0.18	Government expenditure to GDP
τ_a	0.2	Capital income tax
α	0.36	Capital share
δ	0.06	Depreciation rate
σ	1.5	Risk aversion
p	0.4	Pensions
s_t	Vector	Survival probabilities of people born in 1965 (Source: Bell, Wade and Goss (1992))
ϵ_t	Vector	Age-efficiency profile (Source: Hansen (1993))
ρ_y	0.96	AR(1) persistence of logarithm of the productivity process
σ^2_{er}	0.045	AR(1) variance of logarithm of the productivity process
σ^2_{y1}	0.38	Variance of log earnings of age 1 agents
ϕ_1	-9.6	Concern about bequests
$\phi 2$	11.6	Degree to which bequest is considered as a luxury good

Experiment 1

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 - 15
(2) Only productivity shock								
3.0	-	0.70	9	32	72	92	99	14
(3) Accidental bequests to all ($\phi_1) = 0$)								
3.0	0.73	0.70	9	32	71	92	99	14
(4) Both accidental and voluntary bequests to all ($\phi_1 = -9.5$, $\phi_2 = 11.6$)								
3.0	0.9	0.71	9	33	73	93	99	16

Experiment 1

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
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(4) Both accidental and voluntary bequests to all ($\phi_1 = -9.5$, $\phi_2 = 11.6$)								
3.0	0.9	0.71	9	33	73	93	99	16

- Including accidental bequests cannot generate a better match with data.
    1. Upper tail is too thin and lower tail is too fat.
    2. High fraction of population with zero wealth.
    3. High transfer-wealth ratio.

Experiment 1

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
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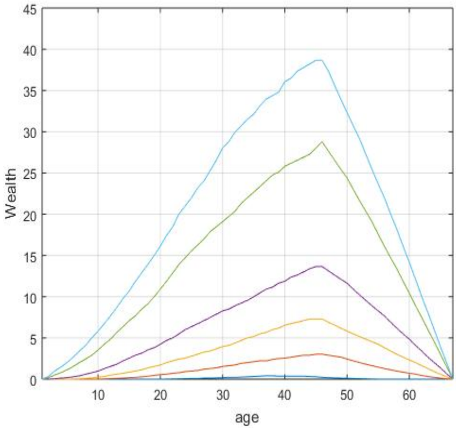
(4) Both accidental and voluntary bequests to all ($\phi_1 = -9.5$, $\phi_2 = 11.6$)								
$\boxed{3.0}$	0.9	0.71	9	33	73	93	99	16

- Including voluntary bequests does not improve much...
  - Contradict with the results of De Nardi.
  - Equal redistribution of bequests to all living agents matters!

Accidental bequests to all.

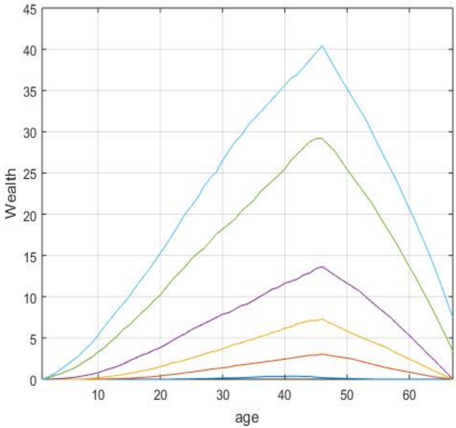
U.S. wealth 0.1 , 0.3 , 0.5 , 0.7 , 0.9 , 0.95 quantiles, by age.



- Households in all quantiles dis-save rapidly during the retirement period.
- They do not leave any voluntary bequests at the terminal period.
- They save totally for retirement.

Accidental and voluntary bequests to all.

U.S. wealth 0.1 , 0.3 , 0.5 , 0.7 , 0.9 , 0.95 quantiles, by age.



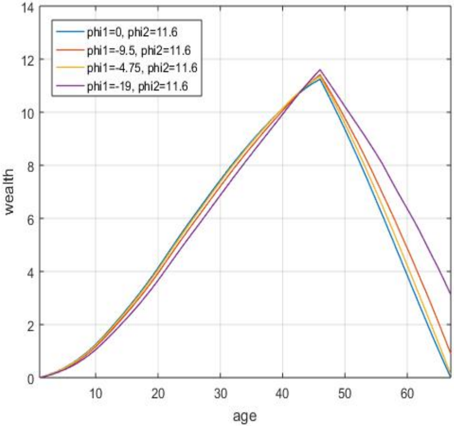
- Households in bottom quantiles still save totally for retirement.
- Households in top 10 % and top 5 % are also driven by bequests motive.
    - They maintain a substantial amount of savings to leave bequests.
    - At the terminal period, they leave around 10% and 20% of pre-retirement savings as bequests.

Experiment 2 – Manipulate ϕ_1

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 – 15
(2) $\phi_1 = 0$: Accidental bequests to all								
3.0	0.73	0.70	9	32	71	82	99	14
(3) $\phi_1 = -9.5$, $\phi_2 = 11.6$								
3.0	0.9	0.71	9	33	73	93	99	16
(4) $\phi_1 = -4.75$, $\phi_2 = 11.6$								
3.0	1.17	0.70	9	33	72	92	99	15
(5) $\phi_1 = -19$, $\phi_2 = 11.6$								
3.0	1.19	0.72	9	34	75	94	99	18

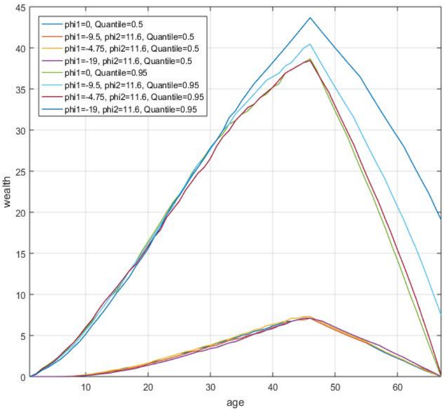
- Line(3) to line(5) show that change in values of ϕ_1 do not help our model to match better with the data ...

Manipulate ϕ_1: Weighted average wealth profiles.



- An increase in $|\phi_1|$ shifts the profiles upward.
- Doubling the value of $|\phi_1|$, the mean households leave the bequests at the terminal period by more than double.
- Decreasing $|\phi_1|$ value by a half, profiles of mean households become indifferent to the profiles as in the case of no bequests motive.

Manipulate ϕ_1: Wealth profiles of 0.5 and 0.95 quantiles.



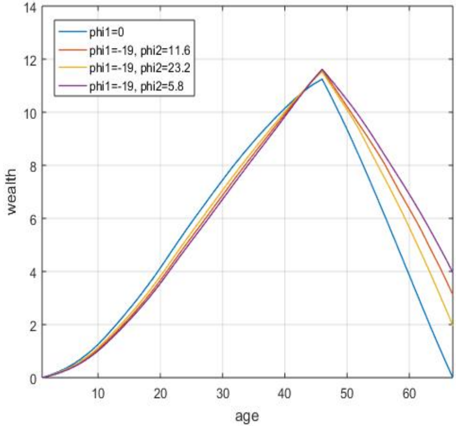
- Change the value of ϕ_1 does not have significant impact on profiles of median households.
  - No changes in the luxury good nature of bequests ...
  - They save totally for retirement.
- The top richest households' are greatly influenced by ϕ_1.
  - They are greatly driven by the bequests motive.
  - At the terminal period, they leave near a half of their pre-retirement assets as bequests.

Experiment 2 – Manipulate ϕ_2

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 – 15
(2) $\phi_1 = 0$: Accidental bequests to all								
3.0	0.73	0.70	9	32	71	82	99	14
(3) $\phi_1 = -19$, $\phi_2 = 11.6$								
3.0	1.19	0.72	9	34	75	94	99	18
(4) $\phi_1 = -19$, $\phi_2 = 23.2$								
3.0	1.03	0.72	9	34	74	93	99	17
(5) $\phi_1 = -19$, $\phi_2 = 5.8$								
3.0	1.30	0.72	9	33	74	94	99	19

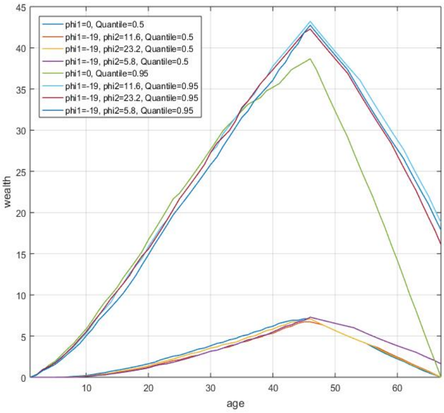
- Note: In order to amplifier the effect of ϕ_2, the default value of ϕ_1 set at -19.
- As usual, line(3) to line(5) show that change in values of ϕ_2 do not help our model to match better with the data ...

Manipulate ϕ_2: Weighted average wealth profiles.



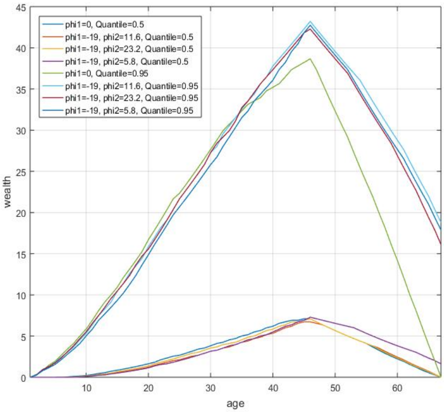
- Decrease in value of ϕ_2 shifts the entire wealth profiles upward.
- Marginal effect of change in ϕ_2 is decreasing in the value of ϕ_2.
  - Decreases ϕ_2 from 23.2 to 11.6, the mean households increase the bequests at the terminal period by 1.1 units.
  - However, decreases ϕ_2 from 11.6 to 5.8, they only increase the bequests by 0.9 units.

Manipulate ϕ_2: Wealth profiles of 0.5 and 0.95 quantiles.



- For the top affluent households, change in ϕ_2 does not affect their saving behaviors.
  - Even we double the value of ϕ_2, the wealth profiles of the top 5 % households do not change.
  - They are rich and have already saved a lot. Nature of bequest is not a matter.

Manipulate ϕ_2: Wealth profiles of 0.5 and 0.95 quantiles. (cont.)



- The median households are very sensitive to the value of ϕ_2.
    - They will not be driven by bequest motive if they consider bequest as a luxury good (i.e. high value of ϕ_2).
    - If they treat bequest as necessity, they change their saving behaviors.
    - They leave about one-fourth of their pre-retirement savings at the terminal period.

Conclusion

- Including bequest motive can influence household's saving behaviors.
  - $\uparrow |\phi_1| \ / \downarrow \phi_2 \rightarrow \uparrow$ Savings
  - $\downarrow |\phi_1| \ / \uparrow \phi_2 \rightarrow \downarrow$ Savings
- Change in ϕ_1 and change in ϕ_2 have different implications.
  - Change in ϕ_1 has greater impact on the richest.
  - Change in ϕ_2 has greater impact on the less affluent.
- Introducing voluntary bequests is not necessarily improve wealth inequality measurements.
  - How to redistribute bequests matters!

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