Research Module in Macroeconomics and Public Economics: Household Heterogeneity and Inequality: Does Warm Glow Matters? Altruism and Saving Behavior

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1 Introduction

The importance of intergenerational links to explain the wealth inequality in real economy is first noticed by Kotlikoff and Summers (1981). They have the result that the majority of the U.S. capital stock can be attributed to the intergenerational transfers rather than to the accumulation of earnings, which is used to understand the wealth distribution in the most basic life-cycle model put forward by Atkinson (1971).

The basic life-cycle model has several problems to match the real wealth distribution. Hence some modifications are developed to improve the basic framework. One of these modifications is taking account of the uncertain life span. In this context the intergenerational transfers can be brought up as accidental bequests. Huggett (1996) developes a pure life-cycle model with accidental bequests and productivity shocks to match the wealth Gini coefficient of the U.S.. His model can explain the wealth inequality and the age-wealth distribution in the real economy. However, it fails to match the wealth distribution of different wealth groups. Specifically speaking, the Huggett's model generates a much thinner upper tail wealth distribution than that of the real economy: there are too few people in the top of wealth distribution. In addition, his model also cannot explains why there still exist bequests because there is nothing left after the agent passes away in his model.

In order to solve the problem in the Huggett's model (1996), our project extends the definition of intergenerational transfer to include the voluntary bequests motivation (adding a utility function named *warm glow*, first introduced by Andreoni (1990)). Our model follows closely the De Nardi's work (2004) but simplifies in several ways. In general, we use a general equilibrium, over-lapping generation model (OLG) with uncertain life span, intergenerational transfer links and productivity shocks. In this model, households will save to insure against the earnings shock, for retirement, and uncertain life span. And also save to get the altruistic utility through the *warm glow* preference. We use this model to explore how the motivation to leave voluntary bequests affects the wealth inequality and saving behaviors in the model economy and whether the affluent and less affluent have the same response to the variation of altruistic motivations

Our results are roughly presented as follows: First, our model can match some features of the actual data as the Huggett's model does. Second, with introducing the voluntary bequests the agents dissave slower after their retirement, and may leave bequests to the descendants. Third, affluent and less affluent agents react differently to the voluntary bequest motivation: the concern about leaving bequests mainly affects the affluent agents and the extent to which the bequest is treated as a luxury good has a larger effect on the less affluent agents.

This paper is organized as follows. Section 2 introduces the data our model tries to match. Section 3 reviews the most related paper and explains how we generate our own model based on it. Section 4 introduces our theoretical model in details. Section 5 introduces our calibration for the U.S. economy. Section 6 gives the results of our experiments and related discussion. Section 7 gives the final conclusion and deficiency of our model.

2 Wealth observations in the U.S.

Line 1 of table 1 presents some wealth observations in the U.S..[1]

[1] Statistics in table 1 are from De Nardi (2004). Moreover, throughout our paper, wealth observations in the U.S. are obtained from De Nardi (2004). All the statistics are estimated from the wealth data of the 1989 Survey of Finances (SCF) (for households who are at age 25 and older).

First, the capital-output ratio is the average capital-to-GDP ratio for the U.S. during the year 1959-1992 (Auerbach and Kotlikoff, 1995). It is about 3.

Second, the transfer-wealth ratio is estimated by Gale and Scholz (1994), which includes bequests and inter vivos transfers (adjusted for underreporting). This number also gives support to the Kotlikoff and Summers's result (1981) we have mentioned in the section 1: more than a half of wealth is from the wealth transfer and therefore the role of inherited wealth cannot be ignored.

Third, table 1 states two measurements of wealth concentration for the U.S.. The wealth Gini coefficient is 0.78. The top 1%, 5%, 20%, and 60% owns 29%, 53%, 80%, and 98% of total wealth. The wealth distribution in the U.S. has a fat upper tail.

Table 1: The U.S. wealth observations, results of Huggett's model and De Nardi's model

Capital-output	Transfer-wealth	Wealth	Percentage wealth in the top					Percentage with
ratio	ratio	Gini	1%	5%	20%	40%	60%	zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 - 15
(2) Huggett's model: Accidental bequests to all								
3.0	0.67	0.67	7	27	69	90	98	17
(3) De Nardi's model: With intergenerational links								
3.0	0.6	0.76	18	42	79	95	100	19

3 Closed related paper

Our project is closely related to De Nardi's study (2004).

De Nardi introduces productivity inheritance and intentional bequests as two intergenerational links in OLG model used by Huggett (1996). In De Nardi's model (2004), households save to insure against earning shocks, for retirement, to insure against having a long life span, and to leave voluntary bequests to their children. Including the two intergenerational links in the OLG model improves the performance of the model to measure wealth concentration. Line 2 and 3 of table 1 represents the results of Huggett's model (1996) and De Nardi's model (2004) which are taken from De Nardi (2004). First, comparing with Huggett's model (1996), De Nardi's model (2004) estimates larger fraction of assets held by the top wealthiest in the model economy. Second, De Nardi's model (2004) generates a lifetime wealth profile which is consistent with the data.

Our project simplifies De Nardi's study (2004) in several ways. First, we do not include human capital transmission in our model. In De Nardi's model (2004), children enters the economy at age 20. Parent's productivity shock at age 40 is transmitted to the children as their initial productivity shock. In our model, initial productivity shock of all agents is drawn from an exogenous initial distribution rather than inherited from the parents.

Second, in our model, both of the voluntary bequests and accidental bequests are equally distributed to all living agents. In De Nardi's model (2004), all voluntary bequests and accidental bequests of the parents are inherited by their children. Our model adopts the approach used in Hugeett's model (1996): all the bequests are fully taxed by the government and redistributed to all living agents in equal amounts each period. In addition, we do not include estate taxes in our model.[2]

Third, all agents in our model start to receive transfer at age 20. In contrast, in De Nardi's model (2004), the children can receive the bequests only when their parents are dead. Since all people do not die before age 65 and the parents are older than their children by 25 years, the children can only inherit the bequests from their parents since they reach 45 years old. Difference in the timing of receiving the bequests has a great impact on the measure of per capita aggregate transfer.

Our model also has several settings which are different from De Nardi's model (2004). First, our model period is 1 year long. There are total 67 periods representing agent's age from 20 to 86. Second, our model has 18 productivity shocks. Grid points and transition matrix are estimated by Tauchen's (1986) method for approximating AR(1) process. Third, our algorithm for computing stationary equilibrium iterates over the per capita capital stocks and transfers.

4 Theoretical Model

Our project investigates an overlapping generation economy. Agents differ in their productivity levels as well as their labor income. During the working period, all agents face stochastic productivity shocks. When agents retire, they may die with a positive probability in each retirement period. All the assets left by the agents who die prematurely become accidental bequests. Without perfect annuity markets in the economy, agents save for precautionary, retirement, and lifetime uncertainty. An additional savings motive is warm glow motive. Agents choose to save in order to derive utility from leaving voluntary bequests.

4.1 Demographics

We define one model period as 1 year long. During each period, a continuum of agents are born. Agents face a one-year ahead survival probability s_t and die for certain in the terminal period N. Population grows at a constant rate n. Based on these stable demographic patterns, age t agents constitute a constant fraction μ_t of the population in every point in time.[3]

Agents enter the model at age 20 ($t = 1$) with zero wealth holdings. Agents pay labor and capital income taxes when they are working. At age 65 ($t = 46$), agents retired and start to receive social security benefits (p) from the government. During the retirement period, agents faces a positive death probability and die for sure at age 86 ($t = 67$). For simplicity, we assume that all agents do not die before age 65.

4.2 Preferences and technology

At all age, we assume agents have identical constant relative risk aversion (CRRA) preferences as $u(c_t) = c_t^{1-\sigma}/1 - \sigma$ and constant discount factor β.

In our model, agents also derive utility from leaving voluntary bequests. The utility function is given by $\phi(a_t) = \phi_1(1 + \frac{a_t}{\phi_2})^{1-\sigma}$ where ϕ_1 reflects agent's overall concern about bequests and ϕ_2 reflects the the extent that agents treat bequests as a luxury good.

[2] Introducing the estate tax significantly reduces the labor earnings tax rate in our model economy. Particularly, the labor tax can drop by almost 85% when the capital/output ratio is controlled. The reason lies in the assumption that government expenditure is constant in the government budget constraint.

[3] We normalize the weights μ_t to sum to 1 and the weight is measured as: $\mu_t = (s_{t-1}/(1+n))\mu_{t-1}$.

During the working period, agents face an exogenous age-efficiency profile ϵ_t. As well, agents face stochastic productivity shocks y at each period. y is in the finite set Y and follows a Markov process. Agents face the same Markov process with the same transition matrix Q. Agent's labor endowment at time t is determined by the product of her stochastic productivity state at age t and the age t efficiency profile ($\epsilon_t y_t$). At the initial period ($t = 1$), agent's productivity shock is drawn from the invariant distribution of productivity shock.

In each period, agents can choose to consume or invest in physical capital a with net return r and deprecation rate δ. Agents cannot own negative capital stocks at any time.

Our economy has an aggregate production function $F(K, L) = AK^{\alpha}L^{1-\alpha}$ which is assumed to be constant return to scale.

4.3 Government

The government set labor and capital income tax to finance the social security benefits to the retired agents and an exogenous public expenditure.

At each period, all retired agents can receive an exogenous lump-sum social security benefits p from the government until they die. A constant fraction g of aggregate income of the economy is considered as the public expenditure.

The capital income tax rate τ_a is constant and exogenous. The labor income tax rate τ_l is flat and is chosen to balance the government budget constraint.

In addition, government collects all accidental and voluntary bequests in the economy and redistributes the bequests to all living agents in equal amounts. Government does not levy any tax on bequests. In each period, all living agents receive a lump-sum transfer T from the government.

4.4 Household's recursive problem

We consider an arrangement where in each period a t-years old agent chooses consumption c in the current period and investment in risk-free asset holdings a' for the next period. Agent's state at a point of time is denoted as $x = (a, y)$ where a represents the capital holdings brought from the last period and y represents the productivity shocks at the current period. Optimal decision rules are consumption in current period $c(x, t)$ and the capital holdings for the next period $a'(x, t)$ which solve the dynamic programming problem faced by agents.

1. From age $t = 1$ to age $t = 44$ (20 years old to 63 years old), the agent works and receives labor income $(1 - \tau_l)w_t\epsilon_t y_t$, capital income $[1 + r(1 - \tau_a)]a_t$, and lump-sum transfer T. The agent will survive with probability one for the next period. The productivity shocks in the next period y' is determined by the transition matrix Q_t.

$$V(t, a, y) = max_{c,a'}\{u(c) + \beta E_t V(t + 1, a', y')\}$$

   subject to

$$a' + c = (1 - \tau_l)w\epsilon y + [1 + r(1 - \tau_a)]a + T$$

$$a' \geq 0, c \geq 0$$

2. At age $t = 45$ (64 years old), the agent will retire in the next period. The agent starts to face positive death probability and derive utility from leaving bequests.

$$V(t, a, y) = max_{c,a'}\{u(c) + s_t\beta E_t W(t + 1, a') + (1 - s_t)\phi(a'))\}$$

subject to

$$a' + c = (1 - \tau_l)w\epsilon y + [1 + r(1 - \tau_a)]a + T$$

$$a' \geq 0, c \geq 0$$

3. From age $t = 46$ to age $t = 66$ (65 years old to 85 years old), the agent retires and does not receive labor income any more. The resources are derived from capital a_t, lump-sum transfer T, and social security benefits p.

$$W(t, a) = max_{c,a'}\{u(c) + s_t\beta E_t W(t + 1, a') + (1 - s_t)\phi(a')\}$$

subject to

$$a' + c = [1 + r(1 - \tau_a)]a + T + p$$

$$a' \geq 0, c \geq 0$$

4. At the age $t = N = 67$ (86 years old), the agent dies with certainty. Optimal consumption in current period and the capital holdings for the next period are both zero.

$$W(N, a) = \phi(a)$$

4.5 Definition of stationary equilibrium

The stationary equilibrium is $\{c(x,t), a'(x,t), w, r, L, K, T, G, \tau_l, \tau_a, p\}$ and the distributions $(\psi_1, \psi_2, ..., \psi_N)$ such that:

1. Given the price, taxation, government transfers and state variables x, $c(x, t)$ and $a'(x, t)$ solve the agent's optimization problem.
2. Input market clears. Factor prices are equal to marginal products:

$$w = F_L(L, K)$$

and

$$r = F_K(L, K) - \delta$$

3. Distribution $\psi_t(B)$ is defined in the probability state space $(X, \mathscr{B}(X), \psi_t)$, where $X = [0, \inf) \times Y$ is the state space, $\mathscr{B}(X)$ is the borel σ-algebra on X, and $B \in \mathscr{B}(X)$. Such that, with an exogenous initial distribution, distributions $\psi_2, \psi_3, \cdots, \psi_{N-1}, \psi_N$ of individual states for agent's age at $t = 2, 3, ..., N - 1, N$ are consistent with agent's optimal decision rules:

$$\psi_t(B) = \int_X P(x, t - 1, B) d\psi_{t-1}$$

where $P(x, t, B)$ is exogenous transition probabilities on labor productivity shock.
4. All markets clear.
   (a) Capital market clears such that capital stock per capita is given by:

$$\sum_{t=1}^{N} \mu_t \int_X a d\psi_t = (1 + n)K$$

(b) Labor market clears such that labor per capita is summation of the labor input over the population:

$$\sum_{t=1}^{N} \mu_t \int_X (\epsilon y) d\psi_t = L$$

5. Government sets τ_l to balance the budget constraint:

$$G + p(\sum_{t=1}^{N} \mu_t) = (\tau_l L) + (r\tau_k K)$$

where G is the exogenous public expenditure.[4]

6. Lump-sum transfer equals to the sum of accidental and voluntary bequests. Government taxes all bequests and redistributes to all living agents equally each period.

$$T = \left[\sum_{t=1}^{N} \mu_t (1 - s_t) \int_X a(1 + r(1 - \tau_k)) d\psi_t \right] \Big/ (1 - n)$$

and per capita aggregate transfer is the sum of the transfer of each living agent received in each period:

$$T_{agg} = \sum_t \mu_t \sum_{j=0}^{t-1} T(1 + r(1 - \tau_a))^j$$

5 Calibration

Our calibration for the U.S. economy follows the calibration in De Nardi's study (2004).

Table 2 presents the full set of parameters used in this paper. α is capital's share of output and we fix at 0.36 (Prescott, 1986). A is technological level which is normalized such that wage w and interest rate on capital r are normalized as 1 and 0.06 when the capital-output ratio is 3 and the annual depreciation rate δ is 0.06.

n and g are population growth rate and government's share of output (Council of Economic Advisors (1998)). Capital income tax τ_a is computed by Kotlikoff *et al.* (1999).

s_t is the vector of conditional survival probabilities of people who born in 1965 which is estimated by using the mortality probabilities provided by Bell, Wade and Gross (1992). ϵ_t is the age-efficiency profile vector which is provided by Hansen (1993).

Based on the estimates by Attanasio *et al.* (1999) and Gourinchas and Parker (2002), relative risk aversion σ is set at 1.5.

The pensions p which is set at 0.4 to match with the implied replacement rate for the U.S..

We assume that the logarithm of the productivity process follows an AR(1) process. The persistence p_y and the variance σ_{er}^2 are provided by Huggett (1996). Besides, σ_{y1}^2 is the variance of the productivity state in the initial period (Huggett, 1996).

In addition, we follow Huggett's approach to define the productivity process. The productivity process is a Markov process with 18 states. The states are equally spaced and ranged from $-4\sigma_{y1}^2$ to $4\sigma_{y1}^2$ with an extreme state $6\sigma_{y1}^2$. Agents with the highest productivity state can receive earnings which are 40 times earnings of the median agents

[4] $G \equiv 0.18 * (r * K + w * L)$, from De Nardi (2004).

Table 2: Model parameters

Parameter	Value
α	0.36
A	0.895
β	0.95 - 0.97
r	endogenous = 0.06
w	endogenous = 1
δ	0.06
n	0.012
g	0.18
τ_a	0.2
σ	1.5
p	0.4
s_t	Vector
ϵ_t	Vector
ρ_y	0.96
σ^2_{er}	0.045
σ^2_{y1}	0.38
ϕ_1	-9.6
ϕ_2	11.6

in the same cohort. The transition probabilities between the states are estimated by using the method proposed by Tauchen (1986).

ϕ_1 represents the agent's general concern about bequests and ϕ_2 is the degree to which bequests are a luxury good. In experiment 1, we use the values provided by De Nardi (2004).[5] In experiment 2, we set different values of ϕ_1 and ϕ_2 to see the mechanism of the bequest motive for savings.

6 Results and Disccussion

We conduct two experiments in this paper. Firstly, we compare the performance of Huggett's model with the model which includes bequest motive. Secondly, we manipulate the values of ϕ_1 and ϕ_2 to investigate how bequest motive influences wealth concentration and the saving behaviors of the agents in the model economy.

6.1 Experiment 1

Table 3 summarizes the results for the U.S. economy.

6.1.1 Huggett's model

Line 2 of table 3 represents the result of Huggett's model. When facing non-zero death probabilities, the retired agents save for retirement and to insure against uncertain life span. They leave accidental bequests if they die prematurely. Huggett's model can generate various features of wealth distribution in the U.S. economy. The top 20% agents

[5] De Nardi (2004) sets $\phi_1 = -9.6$ and $\phi_2 = 11.6\%$ to match the U.S. transfer wealth ratio of 0.6.

Table 3: Experiment 1: results for the U.S. calibration

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 - 15
(2) Huggett's model: Accidental bequests to all								
3.0	0.73	0.71	9	33	72	92	99	17
(3) With voluntary bequests to all ($\phi_1 = -9.5$, $\phi_2 = 11.6$)								
3.0	0.9	0.72	9	33	73	93	99	18

hold 72% of total wealth. The top 40% and top 60% agents hold 92% and 99% of total wealth respectively. Also the wealth Gini coefficient is 0.71 in the model economy. These statistics can match with the U.S. data. However, Huggett's model underestimates the wealth concentration of the upper tail of wealth distribution. This model fails to generate the total wealth hold by the top 1% and top 5% in the U.S. economy. The top 1% agents only hold 9% of total wealth which is one third of fraction of total wealth hold by the top 1% shown in the data (i.e. 29%). Also Huggett's model cannot generate 53% of total wealth hold by the top 5% agents in the U.S. economy. Top 5% agents in the model economy only hold 33% of total wealth. Only introducing uncertain life span to the OLG model cannot generate all of the features of wealth distribution in the U.S. economy, especially a substantial part of wealth concentration in the upper tail of wealth distribution is underestimated.

There is a high transfer-wealth ratio in the model economy. A high transfer-wealth ratio is mainly due to the timing and the way the accidental bequests are redistributed across agents. In Huggett's model, accidental bequests are collected by the government and are redistributed to all living agents in equal amounts. All agents start to receive transfer each period at age 20. On one hand, all agents can receive transfer at least for 45 years until they start to face positive death probability at age 65. Possibly, agents can receive transfers each period until they die at age 86 (i.e. at the terminal period). On the other hand, once agents receive transfer at age 20, interest on transfer also start to accumulate. Since the estimate of per capita aggregate transfer includes both of the total amounts of transfer received starting at age 20 and the interest on the transfer, Huggett's model generates higher transfer-wealth ratio than the ratio in the data.

Moreover, Huggett's model generates higher fraction of population with zero wealth holdings than in the data. This population is mainly constituted by the young agents in the economy. In Huggett's model, all agents start with zero wealth holdings and have expectation of higher future income. Therefore, consumption smoothing tendency leads to most of the young agents save little. When agents become older, they have a higher earnings and thus accumulate assets to insure against earning shocks.

Figure 1 displays the age-asset profiles for various quantiles of wealth distribution generated by Huggett's model. All agents start from zero wealth holdings and they save against earning uncertainty during their working period. At age 65, all agents keep positive wealth holdings. Agents save for retirement and uncertain life-time during the retirement period. While rich agents have a higher saving at retirement age, all agents dissave rapidly during their advanced age and do not leave any positive wealth holdings at the terminal period (86 years old). Without bequest motive, the bequests left by agents are only the accidental bequests.

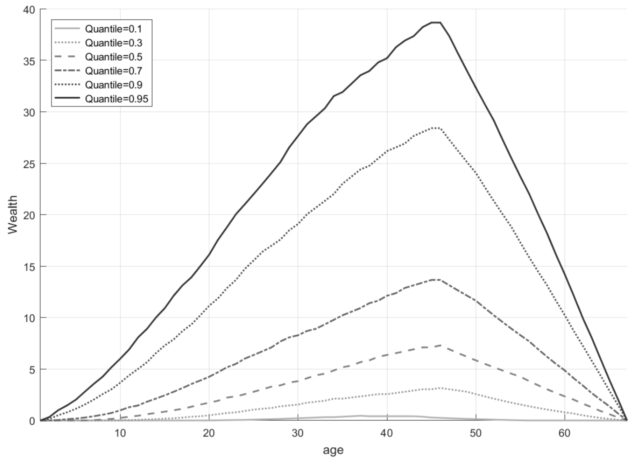

Fig. 1: Wealth profiles 0.1, 0.3, 0.5, 0.7, 0.9, 0.95 quantiles. Huggett's model.

6.1.2 With voluntary bequests to all

Line 3 of table 3 shows the results of model economy with voluntary bequests. With bequest motive, agents now care about leaving voluntary bequests. Since bequests are a luxury good due to the nonhomothetic bequest motive, the richest agents in the economy choose to leave a large stock of wealth as bequests. The wealth distribution of the model economy coincides with the data in several aspects. As compared to 80%, 93%, and 98% of total wealth which is held by the top 20%, 40%, and 60% agents in the data, the top 20%, 40%, and 60% agents hold 73%, 93%, and 99% of total wealth in the model economy respectively. The wealth Gini coefficient is 0.72.

However, wealth distribution generated by model with bequest motive has a too thin upper tail which does not match with the data. The top 1% and top 5% agents only hold 9% and 33% of total wealth in the model economy but they hold 29% and 53% of total wealth in the data. Even though the top wealthiest agents do maintain a large stock of assets at the advanced age and leave a substantial amount of bequests due to the nonhomothetic bequest motive, all bequests are redistributed to all living agents equally via lump-sum transfer from the government each period. Without intergenerational links in our model, the top wealthiest agents cannot transfer their assets via voluntary bequests only to their children. Therefore, our model cannot account for the emergence of large asset holdings accumulated by agents from different generations.

With voluntary bequests, the model generates a transfer-wealth ratio which is 0.9. This estimates is larger than that in the Huggett's model. The intuition is that now agents have an additional motive to save. A fraction of them choose to leave voluntary bequests at their advanced age. In our setting, all accidental bequests and voluntary bequests would be equally redistributed to all living agents. Each living agents can receive larger amounts of transfer each period than they would in the Huggett's model. With the accumulated interest on transfer, the estimate of per capita aggregate transfer becomes larger than that in the Huggett's model. Therefore, the model with voluntary bequest generates an even higher transfer-wealth ratio.

There is 18% of population with zero wealth holdings in the model economy which is higher than the ratio shown in the data and in the Huggett's model. Although all agents start with zero wealth holdings in the model economy as in the Huggett's model, they now can receive a large amount of transfer each period due to the introduction of bequest motive. Therefore, there is a larger fraction of young agents saving little for precautionary during the early phase of the working period.

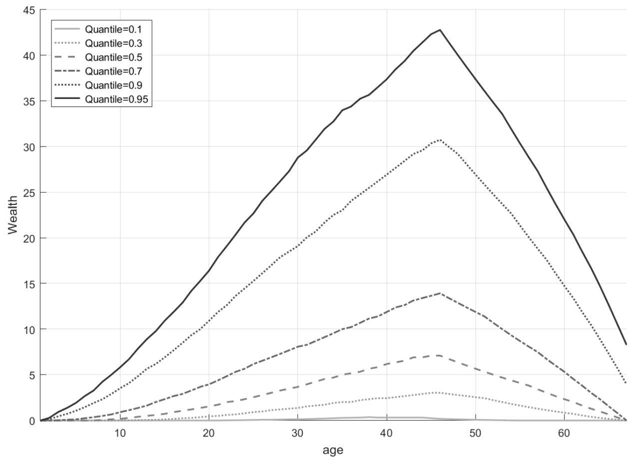

Fig. 2: Wealth profiles 0.1, 0.3, 0.5, 0.7, 0.9, 0.95 quantiles. With voluntary bequests to all.

Figure 2 displays wealth profiles for various quantiles of the wealth distribution which are generated by the model with voluntary bequests. All agents have a positive amount of saving to insure against the earning shocks during their working period and accumulate the highest level of wealth at age 65. Agents in different quantiles response differently to the additional bequest motive. Except the agents in the top 5% and top 10% , agents dissave rapidly when they are retired and consume all the wealth holdings until the terminal period (age 86). Bequest motive does not affect their saving behaviours. They save only for retirement as well as the lifetime uncertainty. Hence, they leave zero voluntary bequests.

In contrast, agents in the top 5% and top 10% leave voluntary bequests at their advanced age. During the retirement period, they maintain a substantial amount of saving. Even at age 86, when both types of agents are dead certainly, they still leave the bequests which are around 18% and 11% of their wealth holdings when they are at age 65. These results match with Dynan *et al.*'s (1996) findings. The wealthiest retires have different saving motives from those retires who are less well off. While the less affluent agents save mainly for retirement, the richest agents also save to leave bequests.

6.2 Experiment 2

Bequest motive involves two key parameters. In the previous sections, we follow De Nardi (2004) and set $\phi_1 = -9.5$ and $\phi_2 = 11.6$. In this section, we manipulate value of ϕ_1 and ϕ_2 to examine how bequest motive influences wealth concentration and saving behaviors of agents in the economy.

6.2.1 Manipulate ϕ_1

ϕ_1 is the agent's concern about leaving bequests. ϕ_1 is a multiplier which multiplies agent's utility received from leaving bequests.

The first partial derivative of utility of leaving bequests (ϕ) with respect to ϕ_1 is positive and constant.[6]

[6] This condition holds if $\phi_2 > 0$.

Table 4: Experiment 2: results for the U.S. calibration with different values of ϕ_1

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 - 15
(2) Huggett's model: Accidental bequests to all								
3.0	0.73	0.71	9	33	72	92	99	17
(3) $\phi_1 = -9.5$, $\phi_2 = 11.6$								
3.0	0.9	0.72	9	33	73	93	99	18
(4) $\phi_1 = -4.75$, $\phi_2 = 11.6$								
3.0	0.77	0.71	9	33	72	93	99	17
(5) $\phi_1 = -19$, $\phi_2 = 11.6$								
3.0	1.19	0.73	9	34	75	94	99	20

$$\frac{\partial \phi}{\partial \phi_1} = (1 + \frac{a}{\phi_2})^{1-\sigma} > 0$$

The intuition is the following: when agents concern more about leaving bequests, same level of bequests can generate even higher utility to the agents and hence agents would choose to leave more bequests.[7] However, change in the value of ϕ_1 would not alter the nature of bequests as a luxury good.

Table 4 shows the results of the model economy with different values of ϕ_1.

Wealth distributions generated by the model economy with various values of ϕ_1 have the similar features of wealth distribution generated by Huggett's model. The top 20% and top 40% hold $72 - 75\%$ and $93 - 94\%$ of total wealth which are consistent with the data. Wealth Gini coefficient is about $0.71 - 0.73$. Nevertheless, the top 1% and 5% agents only hold 9% and $33 - 34\%$ of total wealth which do not coincide with the data.

Transfer-wealth ratio is higher if the value of ϕ_1 is lower. The ratio is 0.9 when ϕ_1 equals to -4.5. If the value of ϕ_1 equals to -9.5 (-19), the ratio goes up to 0.9 (1.19). Now, when the agents start to concern more about the bequests, they save more at their advanced age. As a result, all living agents receive a larger amount of transfer each period and hence there is a larger aggregate per capita transfer. Besides, the ratio of population with zero wealth holdings ranges from 17% to 20% and is decreasing with respect to the value of ϕ_1. Higher concern about bequests leads to larger amounts of transfer received by all living agents which in turn decreases the saving of the agents at the early phase of their working period.

Figure 3 displays the mean age-asset profiles for different values of ϕ_1. Firstly, during the working period, the agents tend to have a lower saving level if there is a higher concern about leaving bequests. The agents can receive a larger amount of transfer each period when ϕ_1 is lower. Consumption smoothing tendency causes the agents to save less for earnings uncertainty when they are working. Secondly, when agents now have higher concern about leaving bequests, agents keep a larger wealth holdings during their retirement age. They save more for bequests and leave a positive amount of wealth at the terminal period. Thirdly, the marginal effect of change in the value of ϕ_1 is decreasing to the original value of ϕ_1. The marginal effect of doubling the value of ϕ_1 from -9.5 to

[7] With our definition of nonhomothetic bequest motive, the value of ϕ and ϕ_1 are both negative. Therefore, when agents have a higher concern about leaving bequests, there is a lower value of ϕ_1.

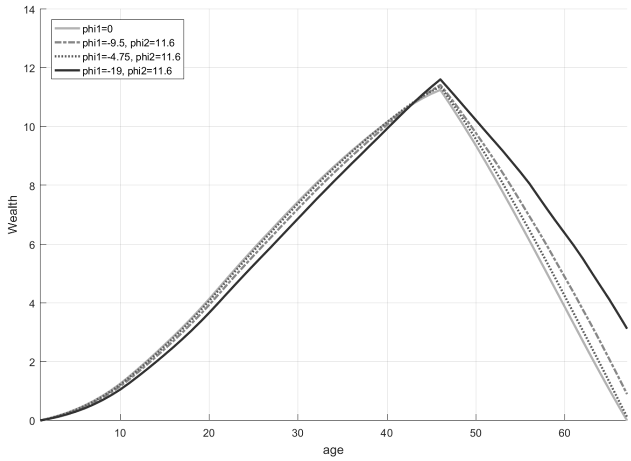

Fig. 3: Manipulate ϕ_1 : Weighted average wealth profiles.

-19 on wealth profile is larger than the effect of doubling the value of ϕ_1 from -4.75 to -9.5. Fourthly, agents just behave as no bequest motive if they only have smaller concern about leaving bequests. When $\phi_1 = -4.75$, agents keep smaller amounts of assets after they retired and leave around zero voluntary bequests at the terminal period. The age-asset profile is virtually the same as the age-asset profile generated by Huggett's model.

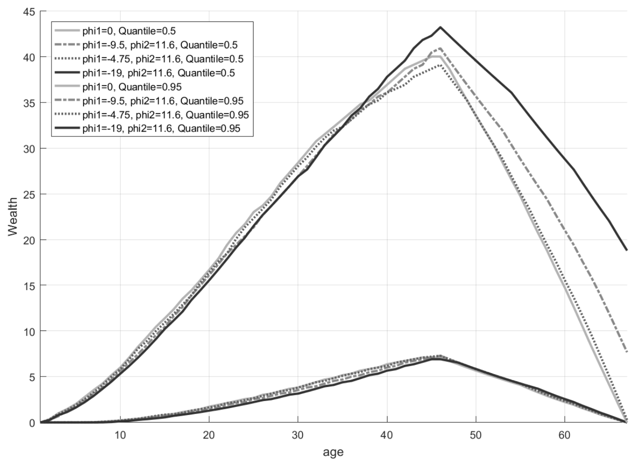

Fig. 4: Manipulate ϕ_1 : Wealth profiles for 0.5 and 0.95 quantiles.

Figure 4 shows the age-asset profiles for 0.5 and 0.95 quantiles. The median agents and the top 5% richest agents response differently to the change in the value of ϕ_1. Change in the concern about leaving bequests only have insignificant effect on median age-asset profiles. The median agents do not save for bequest motive regardless of the value of ϕ_1. They dissave rapidly after retirement and consume all of their assets at the terminal period. Change in the concern about bequests do not alter the nature of bequests as a luxury good. Therefore, the median agents only save for retirement and lifetime uncertainty at the advanced age.

On the contrary, for the top 5% wealthiest agents, the value of ϕ_1 matters. When there is an increase in the concern about the bequests, the age-asset profile of the top 5% agents shifts upward. During the latter stage of working period, they start to save more and accumulate a larger peak level of assets at the age 65. At the advanced age, they dissave at a slower pace and leave a larger amount of bequests at the age 85. Particularly, if the value of ϕ_1 is doubled, the bequests left at the terminal period increase by more than a double and account for more than half of the peak level savings of the agents. In contrast, when the value of ϕ_1 is halved, the top 5% wealthiest agents do not consider leaving bequests. Their age-asset profile is nearly identical to the wealth profile they have in the Huggett's model.

6.2.2 Manipulate ϕ_2

Table 5: Experiment 2: results for the U.S. calibration with different values of ϕ_2

Capital-output ratio	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with zero wealth
(1) U.S. data								
3.0	0.6	0.78	29	53	80	93	98	5.8 - 15
(2) Huggett's model: Accidental bequests to all								
3.0	0.73	0.71	9	33	72	92	99	17
(3) $\phi_1 = -19$, $\phi_2 = 11.6$								
3.0	1.19	0.73	9	34	75	94	99	20
(4) $\phi_1 = -19$, $\phi_2 = 23.2$								
3.0	1.03	0.73	9	35	75	94	99	19
(5) $\phi_1 = -19$, $\phi_2 = 5.8$								
3.0	1.30	0.73	9	34	75	94	99	21

ϕ_2 is the degree to which bequests are considered as a luxury good. The first partial derivative of utility of leaving bequests (ϕ) with respect to ϕ_2 is non-positive.[8]

$$\frac{\partial \phi}{\partial \phi_2} = -(1-\sigma)a\frac{\phi_1}{(\phi_2)^2}(1+\frac{a}{\phi_2})^{-\sigma} \leq 0$$

The intuition is that agents receive higher utility from bequests and tend to save more if they consider bequests are more of a necessity good.

Line $3-5$ of table 5 represents the performance of model with different value of ϕ_2.[9] The top 20% and the top 40% hold 75% and 94% of total wealth in the model economies which are consistent with the data. Nonetheless, the upper tail of wealth distribution is too thin for all of the model economies. The top 1% and top 5% only hold 9% and $34-35$% of total wealth while they hold 29% and 53% of total wealth in the data.

Moreover, with higher value of ϕ_2, the model generates lower transfer-wealth ratio and smaller fraction of population with zero wealth holdings. When agents consider bequests as more of a luxury good, they save less for leaving bequests and hence all the living agents receive smaller amounts of lump-sum transfer in every periods. As a result, per capita aggregate transfer drops and the young agents save more.

The wealth profiles of the mean of wealth distribution are displayed in figure 5. When the value of ϕ_2 is low, agents tend to save less when they are working. The intuition is that

[8] This condition holds if $\sigma \leq 0$, $\phi_1 \leq 0$, and $\phi_2 > 0$.

[9] We fix $\phi_1 = -19$ in this subsection in order to amplifier the effect of change in the value of ϕ_2.

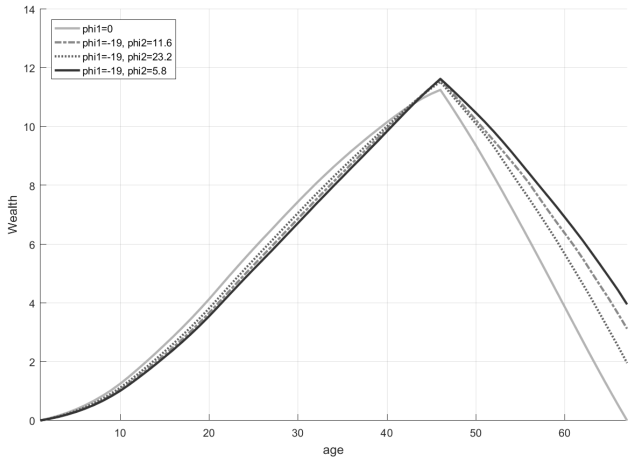

Fig. 5: Manipulate ϕ_2 : Weighted average wealth profiles.

all living agents receive larger amounts of transfer each period when bequests are more of a necessity good and hence the consumption smoothing tendency leads to lower saving of the agents during the working period. At age 65, agents have virtually the same level of wealth holdings with different value of ϕ_2. During the retirement period, with a lower value of ϕ_2, agents maintain a larger wealth holdings. In other words, when bequests become more of a necessity good, agents at the advanced age save more for leaving voluntary bequests.

Figure 6 displays the wealth profiles for 0.5 and 0.95 quantiles of wealth distribution. Change in the value of ϕ_2 only has insignificant impact on the saving behaviors of the top 5% wealthiest agents. No matter bequests are treated as more of a luxury good or a necessity good, the top wealthiest agents still save for leaving voluntary bequests. With the same value of ϕ_1, the wealth profiles of top 5% richest agents are nearly identical regardless the value of ϕ_2.

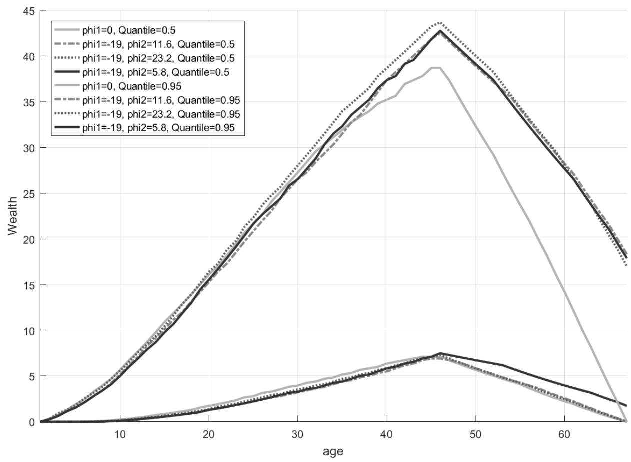

Fig. 6: Manipulate ϕ_2 : Wealth profiles for 0.1 and 0.95 quantiles.

In contrast, the median agents have a larger response on different value of ϕ_2. With a higher value of ϕ_2, the median agents save only for retirement and lifetime uncertainty after retirement. They do not consider leaving voluntary bequests. As a result, when bequests are considered as more of a luxury good, the wealth profile of the median agents is nearly the same with the wealth profile generated by Huggett's model. However, the median agents start to save for voluntary bequests when they consider bequests are more of a necessity good. With a lower value of ϕ_2, the median agents tend to dissave slowly and maintain larger stock of assets after they retired. They also leave one-fourth of their peak level of savings as bequests at the terminal period.

7 Conclusion

This paper introduces a bequest motive to a life-cycle model with earning and lifetime uncertainty. The main findings are as below. Firstly, the model with voluntary bequests can generate a wealth distribution which can match several features of the actual distribution shown in the data. Secondly, with an additional motive for savings, agents now save more and keep larger amounts of assets after retirement. Affluent agents in the economy tends to have a larger response to the introduction of nonhomothetic bequest motive because bequests are considered as a luxury good. Thirdly, change in the value of ϕ_1 and ϕ_2 have different implications. With an increase of the concern about leaving the bequests, affluent agents response greatly to this change and save more for leaving bequests. However, without altering the luxury good nature of bequests, median agents in the economy does not response to this change and only save for retirement and lifetime uncertainty. In contrast, when bequests become more of a necessity good, the median agents now keep a larger stock of assets at the advanced age and even leave a positive amount of assets at the terminal period.

However, our model does not generate all of the wealth concentration in the upper tail of wealth distribution. Particularly, the top 1% and 5% wealthiest agents in our model only hold 9% and 33% of total wealth in the model economy while they hold 29% and 53% in the U.S. data. Even though our model includes nonhomothetic bequest motive, the bequests are all redistributed to all living agents and hence our model cannot greatly improve the estimate of wealth concentration while comparing with the Huggett's model.

In the future, it would be interesting to investigate two extensions to our model. Firstly, we would introduce intergenerational links to our model. De Nardi (2004) introduces two links to Huggett's model which greatly improve the wealth concentration generated by the model. Agents in De Nardi's model (2004) can inherent both the bequests and human capital from their parents. Especially, the parent's bequest motive is the most important link. This link helps De Nardi's model in generating the upper tail of wealth distribution which is better consistent with the data. Moreover, including intergenerational links make more economic sense. These links can help to explain the emergence of human capital and the large estates which are accumulated by family members from different generations.

Secondly, it would be interesting to investigate heterogeneity of bequest motive. In our model, all agents have identical bequest motive with same value of key parameters. On one hand, agents may have different preference about leaving bequests. Some agents may behave more altruistically and are more willingness to leave bequests. Even there is an identical bequest motive, agents may have different values of key parameters which reflect their heterogeneous preference of leaving bequests. On the other hand, even with intergenerational links, some agents may also interested in leaving bequests not to their children but to all agents. The bequest motive used in our model and De Nardi's model (2004) cannot account for this preference.

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Appendix

In this part we want to say something about the algorithm we use to calculate the equilibria and how we implement the wealth distribution in programming.

We follow the equilibrium conditions described in the section 4.5 closely. But for the evolving part, we first have some guessed values of aggregate capital K and transfer T; second, we calculate the labor income tax rate τ_l, aggregate labor L, interest rate r and wage rate w using those initial values and condition 2; third, we use the optimal decision rules to calculate K and T; finally, we calculate the gaps between the calculated values and initial guessed values. If they are sufficiently small (in our setting, less than $1e-3$), then we find the equilibria; if not, we adjust the guessed values with the mean of these two values, and repeat the second step. The largest number of iterations is set to 20.

The wealth distribution is discretized as following: The state space is organized as $X = A \times Z$, where A is capital space and Z is labor productivity shock space. They are both discrete: A is divided into 50 grids with decreasing distances and Z has 18 states derived from Tauchen's method (1986). The Borel σ-algebra is simply the Cartesian product of two power sets:

$$\mathscr{B}(X) = \mathscr{P}(A) \times \mathscr{P}(Z)$$

The transition function in our MATLAB implementation is given by:

$$P(x, t-1, B) = \mathbb{1}_{\{a_t \in B_A\}} \sum_{z_t \in B_Z} \pi(z_{t-1}, z_t)$$

where
a_t: optimal saving policy
$B_A = \{a \in A | (a(x,t), z') \in B\}$
$B_Z = \{z' \in Z | (a(x,t), z') \in B\}$
$\pi(z_{t-1}, z_t)$: transition probability ($(t-1, t)$ element in transition matrix)