Matematiske metoder (MM529)

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Syddansk Universitet, Odense

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Sets

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Representations on sets

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Real numbers visualized as points on the real line.

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Important Cartesian product:
$\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R} = \{(a_1, a_2, a_3, \ldots, a_n) \mid a_i \in \mathbb{R} \text{ for all } i \in$
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Range of f: $\mathcal{R}(f) = f(A) = \{y \in B \mid f(x) = y \text{ for an } x \in A\}$.

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b is the image (or function value) of a under f
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Range of f: $\mathcal{R}(f) = f(A) = \{y \in B \mid f(x) = y \text{ for an } x \in A\}$.
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Composition of functions

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Not commutative in general!

Powers

kth power of x

Function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^k = \underbrace{x \cdot x \cdots x}_{k \text{ times}}$.

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x: basis. k: exponent.

$x \in \mathbb{R}$, $x > 0$:

Power rules

$$
\begin{aligned}
x^n \cdot x^m &= x^{n+m}. \\
x^n : x^m &= x^{n-m}. \\
x^{-n} &= \frac{1}{x^n}. \\
x^{\frac{n}{m}} &= \sqrt[m]{x^n} = (\sqrt[m]{x})^n, \text{ if } m > 0. \\
x^0 &= 1.
\end{aligned}
$$

Multiplication/division of powers with the same basis: addition/subtraction of the exponents.

Polynomial

A (real) polynomial is a function $p : \mathbb{R} \to \mathbb{R}$

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Two polynomials:

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Example:

$p(x) = x^4 - 4x^3 + \frac{3}{2}x + 1,\ q(x) = 2x^2 + x + 4,$
$r(x) = -2x^2 - x + 1.$