(1) $\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ vektorer i $\mathbb{R}^2$

$\vec{c} = \begin{pmatrix} -1 \\ 2 \\ 3 \\ -4 \end{pmatrix}$, $\vec{d} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ —— " —— $\mathbb{R}^4$

a) Udregn $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$, $\vec{b}-\vec{a}$, $\vec{c}+\vec{d}$, $\vec{c}-\vec{d}$, $\vec{a}\cdot\vec{b}$, $\vec{c}\cdot\vec{d}$.

$$\vec{a}+\vec{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2+(-1) \\ 1+4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 5 \end{pmatrix}}}$$

$$\vec{a}-\vec{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 \\ -3 \end{pmatrix}}}$$

$$\vec{b}-\vec{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -3 \\ 3 \end{pmatrix}}}$$

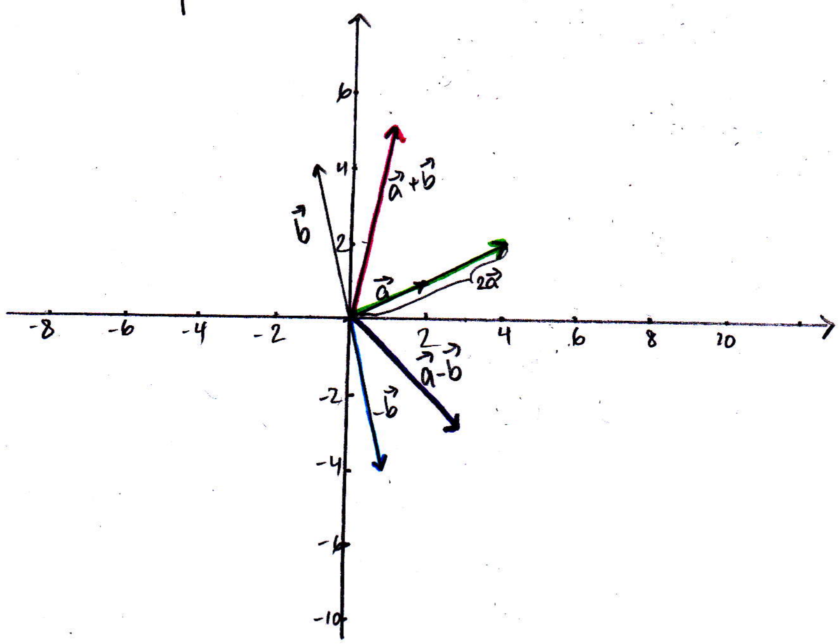
$$\vec{c}+\vec{d} = \begin{pmatrix} -1 \\ 2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ 3 \\ 4 \\ -3 \end{pmatrix}}}$$

$$\vec{c}-\vec{d} = \begin{pmatrix} -1 \\ 2 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ 1 \\ 2 \\ -5 \end{pmatrix}}}$$

$$\vec{a}\cdot\vec{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = 2\cdot(-1) + 1\cdot 4 = -2+4 = \underline{\underline{2}}$$

$$\vec{c}\cdot\vec{d} = \begin{pmatrix} -1 \\ 2 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = -1\cdot 1 + 2\cdot 1 + 3\cdot 1 + (-4)\cdot 1 = \underline{\underline{0}}$$

b) Geometrisk fortolkning af $2\vec{a}$, $-\vec{b}$, $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$ i $\mathbb{R}^2$



(1) c) Beregn længderne af $\vec{a}, \vec{b}, \vec{c}$ og $\sqrt{2}\vec{d}$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2} = \sqrt{2^2+1^2} = \underline{\underline{\sqrt{5}}} \approx 2,236$$

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2} = \sqrt{(-1)^2+4^2} = \underline{\underline{\sqrt{17}}} \approx 4,123$$

$$|\vec{c}| = \sqrt{c_1^2+c_2^2+c_3^2+c_4^2} = \sqrt{(-1)^2+2^2+3^2+(-4)^2} = \underline{\underline{\sqrt{30}}} \approx 5,477$$

$$|\sqrt{2}\vec{d}| = \sqrt{(\sqrt{2}d_1)^2+(\sqrt{2}d_2)^2+(\sqrt{2}d_3)^2+(\sqrt{2}d_4)^2} = \sqrt{2\cdot1^2+2\cdot1^2+2\cdot1^2+2\cdot1^2} = \sqrt{8}$$

$$= \sqrt{2\cdot4} = \underline{\underline{\sqrt{2}\cdot2}} \approx 2,828$$

d) Hvorfor giver det ikke mening at beregne længden af $\vec{a}\bullet\vec{b}$?

$\vec{a}\bullet\vec{b}$ er et tal - ikke en vektor $\Rightarrow$ ingen længde.

e) Bestem cosinus af vinklen ml $\vec{a}$ og $\vec{b}$ og ml $\vec{c}$ og $\vec{d}$

Generelt:

$$\cos(\vec{u},\vec{v}) = \cos(\text{vinkel ml } \vec{u},\vec{v}) = \frac{u\bullet v}{|u|\cdot|v|}$$

$$\cos(\vec{a},\vec{b}) = \frac{\vec{a}\bullet\vec{b}}{|\vec{a}||\vec{b}|} = \frac{2}{\sqrt{5}\sqrt{17}} = \frac{2}{\sqrt{5\cdot17}} = \underline{\underline{\frac{2}{\sqrt{85}}}} \approx 0,2169$$

$\Updownarrow$ $v \approx 77,47°$

el $v = 1,35$ rad.

$$\cos(\vec{c},\vec{d}) = \frac{\vec{c}\bullet\vec{d}}{|\vec{c}||\vec{d}|} = \frac{0}{|\vec{c}||\vec{d}|} = \underline{\underline{0}}$$

$\Updownarrow$ $v = 90°$

$v = \frac{1}{2}\pi$

(2) Find: a) Gradienten af fkt'en i pkt'et.
b) En lign. for tangentplanet for fkt'en i pkt'et

<u>12.7.2</u>: $f(x,y) = \frac{x-y}{x+y}$, $(1,1)$

a) Def 6 p. 716: $\vec{\nabla} f(x,y) = f_1(x,y)\vec{i} + f_2(x,y)\vec{j}$

$f_1(x,y) = \frac{2y}{(x+y)^2}$ (12.3.11)

$f_2(x,y) = \frac{-2x}{(x+y)^2}$

$\nabla f(1,1) = f_1(1,1)\vec{i} + f_2(1,1)\vec{j} = \frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} = \underline{\underline{\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}}}$

b) Tangentplan:

$z = f(a,b) + \vec{\nabla} f(a,b) \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix}$

$= 0 + \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}\begin{pmatrix} x-1 \\ y-1 \end{pmatrix} = \frac{1}{2}(x-1) + \frac{1}{2}(y-1) = \underline{\underline{\frac{1}{2}x - \frac{1}{2}y}}$

<u>12.7.4</u> $f(x,y) = e^{xy}$, $(2,0)$

a) $\vec{\nabla} f(x,y) = ye^{xy}\vec{i} + xe^{xy}\vec{j}$

$\vec{\nabla} f(2,0) = 0e^{2\cdot 0}\vec{i} + 2e^{2\cdot 0}\vec{j} = 2\vec{j} = \underline{\underline{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}}$

b) Tangentplan:

$z = f(2,0) + \vec{\nabla} f(2,0) \cdot \begin{pmatrix} x-2 \\ y-0 \end{pmatrix} = 1 + \begin{pmatrix} 0 \\ 2 \end{pmatrix}\begin{pmatrix} x-2 \\ y-0 \end{pmatrix} = \underline{\underline{1+2y}}$

(3) Linear approx.

Brug (passende) lin approx til at estimere fktværdien i pkt'et.

<u>12.6.4</u> $f(x,y) = \frac{24}{x^2+xy+y^2}$, $(2{,}1; 1{,}8)$

$$f(x,y) \approx L(x,y) = f(a,b) + Df(a,b) \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix}$$

Vælg $a=2$, $b=2$

$$f(2,2) = \frac{24}{2^2+2\cdot2+2^2} = \frac{24}{12} = 2$$

$$f_1(x,y) = \left(24\cdot(x^2+xy+y^2)^{-1}\right)'$$

$$= -24\,(x^2+xy+y^2)^{-2}\cdot(2x+y)$$

$$= \frac{-24\,(2x+y)}{(x^2+xy+y^2)^2}$$

$$f_1(2,2) = \frac{-24\,(2\cdot2+2)}{12^2} = \frac{-24\cdot6}{12\cdot12} = \frac{-2\cdot6}{12} = -1$$

$$f_2(x,y) = \frac{-24\,(x+2y)}{(x^2+xy+y^2)^2} \Rightarrow f_2(2,2) = -1$$

$$f(2{,}1;1{,}8) \approx L(2{,}1;1{,}8) = 2 + \begin{pmatrix} -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2{,}1-2 \\ 1{,}8-2 \end{pmatrix}$$

$$= 2 - 0{,}1 + 0{,}2 = \underline{\underline{2{,}1}}$$

$$f(2{,}1;1{,}8) = 2{,}0997$$

(3) <u>12.6.6</u> $f(x,y) = xe^{y+x^2}$, $(2.05; -3.92)$

Valg $a=2$, $b=-4$

$f(2,-4) = 2 \cdot e^{-4+2^2} = 2 \cdot e^0 = 2$

$f_1(x,y) = 1e^{y+x^2} + x \cdot 2x e^{y+x^2} = (1+2x^2)e^{y+x^2}$

$f_1(2,-4) = (1+2\cdot 2^2)e^{-4+2^2} = 1+8 = 9$

$f_2(x,y) = xe^{y+x^2} \Rightarrow f_2(2,-4) = 2$

$$f(2{,}05; -3{,}92) \approx L(2{,}05; -3{,}92)$$
$$= f(2,-4) + \vec{\nabla} f(2,-4) \cdot \begin{pmatrix} 2{,}05 - 2 \\ -3{,}92 - (-4) \end{pmatrix}$$
$$= 2 + \begin{pmatrix} 9 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0{,}05 \\ 0{,}08 \end{pmatrix}$$
$$= 2 + 9 \cdot 0{,}05 + 2 \cdot 0{,}08$$
$$= 2 + 0{,}45 + 0{,}16$$
$$= \underline{\underline{2{,}61}}$$

$f(2{,}05; -3{,}92) = 2{,}719$

(4) Retningsafledte.

a) For $f(x,y) = 2x + xy^2$: • Find retn.afl. i retning $\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ i pkt $\vec{a} = (1,1)$.

• Find enhedsvektor v_0 i alle retninger hvor den ret.afl. i a er -1.

• Retningsafledt:

Thm 7 p 718:

Hvis f diff i (a,b) og $\vec{u} = u\vec{i} + v\vec{j}$ er enhedsvektor så er den retn.afl. af f i (a,b) m. retn. $\vec{u}$ givet ved

$$D_{\vec{u}} f(a,b) = \vec{u} \bullet \nabla f(a,b)$$

Er $\vec{v}$ enh.vektor: $|v| = \sqrt{3^2+4^2} = \sqrt{25} = 5 \neq 1$

Brug $\vec{u} = \frac{\vec{v}}{|v|} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$: $|u| = \sqrt{(3/5)^2 + (4/5)^2} = \sqrt{\frac{25}{25}} = 1$

$\vec{\nabla} f(x,y) = (f_1(x,y), f_2(x,y)) = \begin{pmatrix} 2+y^2 \\ 2xy \end{pmatrix}$

$\vec{\nabla} f(1,1) = \begin{pmatrix} 2+1 \\ 2\cdot 1\cdot 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$D_{\vec{u}} f(1,1) = \vec{u} \bullet \vec{\nabla} f(1,1) = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{9}{5} + \frac{8}{5} = \underline{\underline{\frac{17}{5}}}$$

• Søger v_0 hvor $D_{v_0} f(1,1) = v_0 \bullet \vec{\nabla} f(1,1) = -1$

$\Leftrightarrow \begin{pmatrix} v_{01} \\ v_{02} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 1 \Leftrightarrow 3v_{01} + 2v_{02} = -1$

$\Leftrightarrow v_{01} = \frac{-1-2v_{02}}{3}$

og hvor $|v_0|^2 = v_{01}^2 + v_{02}^2 = 1$

$\Leftrightarrow \left(\frac{-1-2v_{02}}{3}\right)^2 + v_{02}^2 = 1$

$\Leftrightarrow 1 + 4v_{02}^2 + 4v_{02} + 9v_{02}^2 = 9$

$\Leftrightarrow 13v_{02}^2 + 4v_{02} - 8 = 0$

$\Leftrightarrow v_{02} = \frac{-4 \pm \sqrt{16 - 4\cdot 13\cdot(-8)}}{2\cdot 13} = \begin{cases} 0{,}6455 \\ -0{,}9533 \end{cases}$

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(4) a)

$$v_{01} = \frac{-1-2v_{02}}{3} = \begin{cases} \frac{-1-2\cdot(0,6455)}{3} = -0,7637 \\ \frac{-1-2\cdot(-0,9533)}{3} = +0,3022 \end{cases}$$

Dvs: $v_0 = \begin{pmatrix} -0,7637 \\ 0,6455 \end{pmatrix} \vee v_0 = \begin{pmatrix} +0,3022 \\ -0,9533 \end{pmatrix}$

Tjek: $|v_0| = 0,9999 \qquad |v_0| = 1,0001$

$v_0 \cdot \vec{\nabla} f(1,1) = -1,0001 \qquad v_0 \cdot \vec{\nabla} f(1,1) = -1,000$

b) For $f(x,y,z) = x^2+y^2+z^2$: Find retn.afl: $v = (1,1,1)$, $a = (0,1,2)$ (a, b, c)

$|v| = \sqrt{3} \neq 1$ Vælg $u = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$

$$\vec{\nabla} f(x,y,z) = \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \\ f_3(x,y) \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\vec{\nabla} f(0,1,2) = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$$

$$D_{\vec{u}} f(a,b,c) = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \frac{2}{\sqrt{3}} + \frac{4}{\sqrt{3}} = \frac{6}{\sqrt{3}} = \frac{2\sqrt{3}^2}{\sqrt{3}} = \underline{\underline{2\sqrt{3}}}$$