(1) a) Udregn euklidisk afstand mellem $P(1,1)$, $Q(2,4)$ i $\mathbb{R}^2$

$$\text{Dist}(P,Q) = \sqrt{(Q_1 - P_1)^2 + (Q_2 - P_2)^2} = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{10} \approx \underline{\underline{3{,}16}}$$

b) Udregn euklidisk afstand mel $P(1,2,3)$ og $Q(-2,4,0)$ i $\mathbb{R}^3$

$$\text{Dist}(P,Q) = \sqrt{(Q_1-P_1)^2 + (Q_2-P_2)^2 + (Q_3-P_3)^2} = \sqrt{(-2-1)^2 + (4-2)^2 + (0-3)^2}$$

$$= \sqrt{9+4+9} = \sqrt{22} \approx \underline{\underline{4{,}69}}$$

c) Bestem kuglen m. radius $\frac{1}{2}$ omkring pkt'et $(1,2,3) \in \mathbb{R}^3$

Centrum: $(1,2,3)$, radius: $\frac{1}{2}$

Kuglens ligning:

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

$$\Updownarrow$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\Updownarrow$$

$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = \frac{1}{4}$$

$$\Updownarrow$$

$$x^2 + y^2 + z^2 - 2(x + 2y + 3z) = \frac{1}{4} - \frac{4}{4} - \frac{16}{4} - \frac{36}{4} = -\frac{55}{4}$$

Dvs: $B_{\frac{1}{2},(1,2,3)} = \{(x,y,z) \mid x^2 + y^2 + z^2 - 2(x+2y+3z) \leq -\frac{55}{4}\}$

(2) 12.2.4+6: Evaluer grænsen eller argumenter hvorfor den ikke findes.

12.2.4: $\lim_{(x,y)\to(0,0)} \frac{x}{x^2+y^2}$

Lad $f(x,y) = \frac{x}{x^2+y^2}$:

Da vil $|f(x,0)| = \left|\frac{x}{x^2+0^2}\right| = \left|\frac{1}{x}\right| \to \infty$ for $x \to 0$

$|f(0,y)| = \left|\frac{0}{0^2+y^2}\right| = 0 \to 0$ for $y \to 0$

Da de to grænser ikke er ens eksisterer grænsen ikke.

Dvs. fkt'en er ikke kontinuert i $(0,0)$ (Def 3, s.682) og vi kan ikke tilføje en værdi så den bliver.

12.2.6 $\lim_{(x,y)\to(0,1)} \frac{x^2(y-1)^2}{x^2+(y-1)^2}$

Lad $f(x,y) = \frac{x^2(y-1)^2}{x^2+(y-1)^2}$

Da vil

$$0 \leq |f(x,y)| = \left|\frac{x^2(y-1)^2}{x^2+(y-1)^2}\right|$$

$$= \left|\frac{x^2(y-1)^2+x^4-x^4}{x^2+(y-1)^2}\right| = \left|\frac{x^2((y-1)^2+x^2)}{x^2+(y-1)^2} - \underbrace{\frac{x^4}{x^2+(y-1)^2}}_{\geq 0}\right|$$

$$\leq \left|\frac{x^2((y-1)^2+x^2)}{(y-1)^2+x^2}\right| = x^2$$

Dvs: $0 \leq \left|\frac{x^2(y-1)^2}{x^2+(y-1)^2}\right| \leq x^2$.

$\downarrow 0$ for $(x,y) \to (0,1)$

$\Rightarrow \lim_{(x,y)\to(0,1)} f(x,y) = 0$

Let $f(0,1) = 0$ then f is cont.

(3) Find alle første partielle afledte og evaluer dem i det givne pkt.

regnsler

12.3.2: $f(x,y) = xy + x^2, \quad (2,0)$

$f_1(x,y) = \frac{\partial}{\partial x} f(x,y) = \underline{y + 2x} \;; \; f_1(2,0) = 0 + 2\cdot 2 = \underline{\underline{4}}$

$f_2(x,y) = \frac{\partial}{\partial y} f(x,y) = \underline{\underline{x}} \qquad f_2(2,0) = \underline{\underline{2}}$

12.3.4: $g(x,y,z) = \frac{xz}{y+z}, \quad (1,1,1)$

$g_1(x,y,z) = \frac{\partial}{\partial x} g(x,y,z) = \underline{\underline{\frac{z}{y+z}}} : g_1(1,1,1) = \underline{\underline{\frac{1}{2}}}$

$g_2(x,y,z) = \frac{\partial}{\partial y} g(x,y,z) = xz \frac{\partial}{\partial y} \frac{1}{y+z}$

$= \underline{\underline{-\frac{xz}{(y+z)^2}}} \quad , g_2(1,1,1) = -\frac{1}{(1+1)^2} = \underline{\underline{-\frac{1}{4}}}$

$g_3(x,y,z) = \frac{\partial}{\partial z} g(x,y,z)$

$= x \frac{\partial}{\partial z} \frac{z}{y+z}$

$= x \left(\frac{(y+z)\cdot 1 - z \cdot 1}{(y+z)^2} \right) = \frac{xy}{(y+z)^2}$

$g_3(1,1,1) = \frac{1\cdot 1}{(1+1)^2} = \underline{\underline{\frac{1}{4}}}$

regnsler

12.3.8 $f(x,y) = \frac{1}{\sqrt{x^2+y^2}} \quad , (-3,4)$

$= (x^2+y^2)^{-\frac{1}{2}}$

$f_1(x,y) = \frac{\partial}{\partial x} f(x,y) = -\frac{1}{2} \cdot \frac{1}{(x^2+y^2)^{3/2}} \cdot 2x = \underline{\underline{\frac{-x}{(x^2+y^2)^{3/2}}}}$

$f_1(-3,4) = \frac{3}{(9+16)^{3/2}} = \frac{3}{(25)^{3/2}} = \frac{3}{5^3} = \underline{\underline{\frac{3}{125}}}$

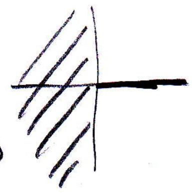
$f_2(x,y) = \frac{\partial}{\partial y} f(x,y) = \underline{\underline{\frac{-y}{(x^2+y^2)^{3/2}}}} \Rightarrow f_2(-3,4) = \underline{\underline{\frac{-4}{125}}}$

(4) a) Argumenter hvorfor $f(x,y) = \begin{cases} \frac{2xy^2}{x^2+y^4} & x>0 \\ 0 & x \le 0 \end{cases}$

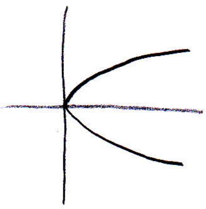
ikke er kont i $(x,y)=(0,0)$.

Niveau-kurver: $f(x,y)=k$

$$f(x,y)=k=0 \Leftrightarrow \begin{cases} \frac{2xy^2}{x^2+y^4} = 0 \quad (x>0) \Leftrightarrow y=0 \\ 0 = 0 \quad (x \le 0) \Leftrightarrow x \le 0 \end{cases}$$



$$f(x,y)=1 \Leftrightarrow \frac{2xy^2}{x^2+y^4} = 1 \Leftrightarrow 2xy^2 = x^2+y^4$$
$$\Leftrightarrow -y^4+2xy^2-x^2=0$$
$$\overset{t=y^2}{\Leftrightarrow} -t^2+2xt-x^2=0$$
$$\Leftrightarrow t = \frac{-2x \pm \sqrt{4x^2-4\cdot(-1)\cdot(-x^2)}}{2\cdot(-1)}$$
$$= \frac{-2x}{-2} = x$$
$$\Leftrightarrow y = \pm\sqrt{t} = \pm\sqrt{x}$$



$\Rightarrow$ limits not the same from every direction.....

$\Rightarrow$ not con't.

b) Eksisterer de partielle afledte af f i (0,0)? Hvis ja - hvad er deres værdi?

De partielle afledte $f_x(x,y)$ og $f_y(x,y)$, eksisterer ikke i 0 for fra venstre på x-aksen er de 0 men fra højre er de ikke defineret (dividere m. nul).
Da den afledte skal være ens fra alle sider for at eksistere, eksisterer de altså ikke.

(5) Find ligning for tangentplanen for grafen for flet'en i det givne plet.

<u>12.3.14</u> $f(x,y) = \frac{x-y}{x+y}$, $(1,1)$

Tangentligning: $z = f(a,b) + f_1(a,b)(x-a) + f_2(a,b)(y-b)$

$$f_1(x,y) = \frac{(x+y)\cdot 1 - (x-y)\cdot 1}{(x+y)^2} = \frac{x+y-x+y}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$f_1(1,1) = \frac{2\cdot 1}{(1+1)^2} = \underline{1/2}$$

$$f_2(x,y) = \frac{(x+y)(-1) - (x-y)\cdot 1}{(x+y)^2} = \frac{-x-y-x+y}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

$$f_2(1,1) = \frac{-2\cdot 1}{(1+1)^2} = \underline{\underline{-\frac{1}{2}}}$$

$$f(1,1) = \frac{1-1}{1+1} = \underline{0}$$

Dvs:

$$z = 0 + \frac{1}{2}(x-1) - \frac{1}{2}(y-1) = \frac{1}{2}x - \frac{1}{2} - \frac{1}{2}y + \frac{1}{2} = \underline{\underline{\frac{1}{2}x - \frac{1}{2}y}}$$

regn selv <u>12.3.16</u> $f(x,y) = e^{xy}$, $(2,0)$

$$f_1(x,y) = ye^{xy} \Rightarrow f_1(2,0) = 0\cdot e^{2\cdot 0} = 0$$

$$f_2(x,y) = xe^{xy} \Rightarrow f_2(2,0) = 2\cdot e^{2\cdot 0} = 2$$

$$f(2,0) = e^{2\cdot 0} = 1$$

$$z = 1 + 0(x-2) + 2(y-0) = \underline{\underline{1+2y}}$$

(6) Find alle 2. ordenspartielle afledte.

12.4.2: $f(x,y) = x^2+y^2$

$f_1(x,y) = 2x$, $f_{11}(x,y) = \underline{\underline{2}}$

$f_2(x,y) = 2y$, $f_{22}(x,y) = \underline{\underline{2}}$

$f_{12}(x,y) = \underline{\underline{0}} = f_{21}(x,y)$

12.4.4: $z = \sqrt{3x^2+y^2} = f(x,y)$

$$f_1(x,y) \underset{\text{kæderegl.}}{=} \frac{6x}{2\sqrt{3x^2+y^2}} = \frac{3x}{\sqrt{3x^2+y^2}}$$

$$f_{11}(x,y) \underset{\text{kæde + brøk}}{=} \frac{\sqrt{3x^2+y^2}\cdot 3 - \frac{3x}{\sqrt{3x^2+y^2}}\cdot 3x}{3x^2+y^2}$$

$$= \frac{\frac{(3x^2+y^2)\cdot 3}{\sqrt{3x^2+y^2}} - \frac{9x^2}{\sqrt{3x^2+y^2}}}{3x^2+y^2}$$

$$= \frac{9x^2+3y^2-9x^2}{(3x^2+y^2)^{3/2}}$$

$$= \underline{\underline{\frac{3y^2}{(3x^2+y^2)^{3/2}}}}$$

(6) fortsat.

$$f_{12}(x,y) = \frac{\partial}{\partial y}\left(3x \cdot \left(\sqrt{3x^2+y^2}\right)^{-1}\right)$$

$$= 3x \cdot (-1)\left(\sqrt{3x^2+y^2}\right)^{-2} \cdot \frac{1}{2 \cdot \sqrt{3x^2+y^2}} \cdot 2y$$

kædereglen to gange

$$= \underline{\underline{\frac{-3xy}{(3x^2+y^2)^{3/2}}}}$$

$$f_2(x,y) = \frac{2y}{2\sqrt{3x^2+y^2}} = \frac{y}{\sqrt{3x^2+y^2}}$$

$$f_{22}(x,y) = \frac{\sqrt{3x^2+y^2} \cdot 1 - \frac{y}{\sqrt{3x^2+y^2}} \cdot y}{3x^2+y^2}$$

$$= \frac{\frac{3x^2+y^2}{\sqrt{3x^2+y^2}} - \frac{y^2}{\sqrt{3x^2+y^2}}}{3x^2+y^2}$$

$$= \underline{\underline{\frac{3x^2}{(3x^2+y^2)^{3/2}}}}$$

$$f_{21}(x,y) = \frac{\partial}{\partial x}\left(y\left(\sqrt{3x^2+y^2}\right)^{-1}\right)$$

$$= y \cdot (-1) \cdot (3x^2+y^2)^{-2} \cdot \frac{1}{2\sqrt{3x^2+y^2}} \cdot 6x$$

$$= \underline{\underline{\frac{-3xy}{(3x^2+y^2)^{3/2}}}}$$