

Beispiel (1).

$$\begin{array}{rrrrrcl} x_1 & + & 2x_2 & - & 3x_3 & - & 2x_4 & = & -4 \\ -9x_1 & + & 10x_2 & & & & - & 4x_4 & = & 0 \\ 3x_1 & + & 6x_2 & - & x_3 & + & 5x_4 & = & 2 \end{array}$$

$$\begin{array}{rrrrrcl} x_1 & + & 2x_2 & - & 3x_3 & - & 2x_4 & = & -4 \\ & + & 28x_2 & - & 27x_3 & - & 22x_4 & = & -36 \\ & & & + & 8x_3 & + & 11x_4 & = & 14 \end{array}$$

In verkürzter Form:

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ -9 & 10 & 0 & -4 & 0 \\ 3 & 6 & -1 & 5 & 2 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 11 & 14 \end{array} \right)$$

Unendliche viele Lösungen (1 Parameter):

$$x_4 = t, x_3 = \frac{11t - 14}{8}, x_2 = \frac{-121t + 90}{224}, x_1 = \frac{-117t + 50}{112}, \quad t \in \mathbb{R}$$

Beispiel (2).

$$\begin{array}{rrrrrcl} x_1 & + & 2x_2 & - & 3x_3 & - & 2x_4 & = & -4 \\ 3x_1 & + & 6x_2 & - & x_3 & + & 5x_4 & = & 2 \\ -9x_1 & + & 10x_2 & & & & - & 4x_4 & = & 0 \\ 2x_1 & + & 4x_2 & + & 2x_3 & + & 7x_4 & = & 6 \end{array}$$

In verkürzter Form:

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 3 & 6 & -1 & 5 & 2 \\ -9 & 10 & 0 & -4 & 0 \\ 2 & 4 & 2 & 7 & 6 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 0 & 8 & 11 & 14 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 11 & 14 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 11 & 14 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Unendliche viele Lösungen (1 Parameter):

$$x_4 = t, x_3 = \frac{11t - 14}{8}, x_2 = \frac{-121t + 90}{224}, x_1 = \frac{-117t + 50}{112}, \quad t \in \mathbb{R}$$

Beispiel (3).

$$\begin{array}{rrrrrcl} x_1 & + & 2x_2 & - & 3x_3 & - & 2x_4 & = & -4 \\ -9x_1 & + & 10x_2 & & & - & 4x_4 & = & 0 \\ 4x_1 & + & 8x_2 & + & 4x_3 & + & 14x_4 & = & 2 \\ 3x_1 & + & 6x_2 & - & x_3 & + & 5x_4 & = & 2 \end{array}$$

In verkürzter Form:

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ -9 & 10 & 0 & -4 & 0 \\ 4 & 8 & 4 & 14 & 2 \\ 3 & 6 & -1 & 5 & 2 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 16 & 22 & 18 \\ 0 & 0 & 8 & 11 & 14 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 11 & 9 \\ 0 & 0 & 8 & 11 & 14 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 11 & 9 \\ 0 & 0 & 0 & 0 & 5 \end{array} \right)$$

Keine Lösung!

Beispiel (4).

$$\begin{array}{rrrrrcl} x_1 & + & 2x_2 & - & 3x_3 & - & 2x_4 & = & -4 \\ -9x_1 & + & 10x_2 & & & - & 4x_4 & = & 0 \\ 4x_1 & + & 8x_2 & + & 4x_3 & + & 12x_4 & = & 2 \\ 3x_1 & + & 6x_2 & - & x_3 & + & 5x_4 & = & 2 \end{array}$$

In verkürzter Form:

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ -9 & 10 & 0 & -4 & 0 \\ 4 & 8 & 4 & 12 & 2 \\ 3 & 6 & -1 & 5 & 2 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 16 & 20 & 18 \\ 0 & 0 & 8 & 11 & 14 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 10 & 9 \\ 0 & 0 & 8 & 11 & 14 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -2 & -4 \\ 0 & 28 & -27 & -22 & -36 \\ 0 & 0 & 8 & 10 & 9 \\ 0 & 0 & 0 & 1 & 5 \end{array} \right)$$

Eindeutige Lösung:

$$x_4 = 5, x_3 = -\frac{41}{8}, x_2 = -\frac{515}{224}, x_1 = -\frac{535}{112}.$$