CS3243 : Introduction to Artificial Intelligence

Tutorial 3

NUS School of Computing

July 7, 2022

Admin

- Consultations
- Extra material for practice

Review

- Heuristics, more importantly on how do we come up with them
- Dominance and how it helps
- Relaxed problems, the recipe for obtaining heuristics

Review

Admissibility	h is admissible iff $\forall n : h(n) \leq h*(n)$ An admissible heuristic will never overestimate the cost to reach the goal
Consistency	h is consistent, iff for every node n and every successor n' of n generated by action a $h(n) \leq c(n, a, n') + h(n')$
Dominance	$h_1(n)$ dominates $h_2(n)$ iff $\forall n : h_1(n) \geq h_2(n)$

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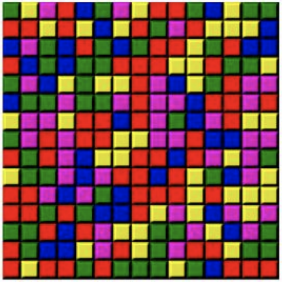
- Relaxed Version 2 : A tile can move from square X to square Y if X is adjacent to Y ~~and Y is blank~~

Tutorial Question 1

- Design Admissible Heuristics for SameGame Puzzle
- Given information :
  - Initial state : Grid with $n \times m$ tiles with c different colours
  - Neighbors : Directly adjacent tiles (Internal tiles : 4 neighbors, Edge tiles : 3 neighbors, Corner tiles : 2 neighbors)
  - Group : Set of ≥ 2 neighboring tiles of same color
  - Singleton : Tile not belonging to any group
  - Action/Move : Deleting a group (not singleton). Thereafter, vertical gravity and column shifting (right-to-left) applies (in transition model)
  - Goal state : Empty grid (assume solvable)
  - Transition cost : 1 (or ∞ if no groups exist)

Tutorial Question 1

- An example of the initial state :



- Top-left corner is $(1,1)$, $1 \leq ROW \leq n$, and $1 \leq COL \leq m$

Tutorial Question 1

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- We have two approaches
- Approach 1 : By reasoning/inference (ie What information could I use at each state to get a estimate of the number of moves needed to reach the goal? Important : Must be an underestimate!)
- Approach 2 (better approach) : Relaxation of the game rules (ie. I now need less moves than what I originally need to reach the goal because the game has become "easier" $=$ admissible. Important : We should be able to easily obtain the number of moves needed with the relaxed rules)

Tutorial Question 1

- Proof of admissibility : $h(s) =$ Number of colours remaining.
- Each group contains exactly 1 colour
- Each remaining color has 1 or more groups
- Each move may reduce groups by more than 1 (example when groups of same color combine), but will never reduce remaining groups of same color below 1 (unless that color is removed by the move)
- Hence, each legal move will leave the number of colors remaining unchanged or reduce it by at most 1 if the color is removed by the move, ie. $h(s) \leq h^*(s)$

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- Note : You may also prove admissibility by showing that heuristic was derived from a relaxed version of the game (Please refer to the forum post on LumiNUS for details)

Tutorial Question 3

- Given information :
  - State : Position of Pac-Man in the grid at any point in time and the positions of the remaining (i.e., uneaten) pellets.
  - Initial state: Grid entirely filled with pellets except at Pac-Man's starting position.
  - Action: Move up/down/left/right.
  - Transition model: Updating the position of Pac-Man given current grid and action, and eating (ie. removing) the pellet at this new position if a pellet exists. Returns updated grid.
  - Goal state: No (uneaten) pellets left in the grid.
  - Transition/Step cost: 1 for each action taken.

Tutorial Question 3

- Let's say admissible heuristics : h_1, h_3, h_4
- On to comparing the dominance of each on another

Tutorial Question 3

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- h_1 *vs* h_4
- What do you think?
- Let's look at the previous counter-examples

Tutorial Question 3

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Tutorial Question 3

- h_3 *vs* h_4
- What do you think?
- Average $\leq$ Maximum, and Euclidean distance $\leq$ Manhattan distance; hence Average Euclidean distance (h_4) $\leq$ Maximum Manhattan distance (h_3) for all nodes ie h_3 dominates h_4

Tutorial Question 4

- Traveling Salesman Problem (TSP)
- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once, and returns to the origin city?

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- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once, and returns to the origin city?
- Now, this is a hard problem. We haven't been able to find a polynomial time algorithm for it. But, we have heuristics to the rescue.

Tutorial Question 4(a)

- How can MST heuristic be derived from a relaxed version of TSP?

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- How can MST heuristic be derived from a relaxed version of TSP?
- The TSP problem is to find a minimal length path through the cities, which forms a closed loop.
- MST is a relaxed version of the TSP problem because it only seeks for a minimal length graph that need not be a closed loop – it can be any fully-connected graph.

Tutorial Question 4(b)

- Is the MST heuristic admissible?

Tutorial Question 4(b)

- Is the MST heuristic admissible?
- Yes, because it is always shorter than or equal to the length of a closed loop through the cities.

Tutorial Question 4(c)

- Hill Climbing Approach to solve TSP?

Tutorial Question 4(c)

- Hill Climbing Approach to solve TSP?
- Algorithm:
  - Choose a random path that connects all the cities
  - Along that path, pick two cities at random
  - Partition the whole path across these two cities, which leaves us with 3 paths
  - Try all 6 possible permutations of arranging the partitions to obtain the least cost loop
  - Upon getting the best cost, repeat the steps to further improve
  - Halt when done with a certain k number of iterations

Thank you!

If you have any questions, please don't hesitate. Feel free to ask!
We are here to learn together!