

A Network Approach to Measuring State Preferences

Abstract

State preferences play an important role in international politics. Unfortunately, actually observing and measuring these preferences is impossible. In general, scholars have tried to infer preferences using either UN voting or alliance behavior. The two most notable measures of state preferences that have flowed from this research area are ideal points (Bailey et al., 2017) and S-scores (Signorino & Ritter, 1999). The basis of both these models is a spatial weighting scheme that has proven useful but discounts higher-order effects that might be present in relational data structures such as UN voting and alliances. We begin by arguing that both alliances and UN voting are simply examples of the multiple layers upon which states interact with one another. To estimate a measure of state preferences, we utilize a tensor decomposition model that provides a reduced-rank approximation of the main patterns across the layers. Our new measure of preferences plausibly describes important state relations, and yields important insights on the relationship between preferences, democracy, and international conflict. Additionally, we show that a model of conflict using this measure of state preferences decisively outperforms models using extant measures when it comes to predicting conflict in an out-of-sample context.

Why we care about preferences

Understanding state interactions in the realm of international politics necessitates some knowledge of states' foreign policy preferences vis-à-vis each other. We can consider each state to have a preferred policy outcome on each possible issue that might arise in international relations. We can conceive of these preferred outcomes as an ideal point in multidimensional space. Given the fact that many issues in international politics are related, we can represent these preferences in far fewer dimensions than there are issues in international politics. These preferences will also change over time. When states have similar preferences on an issue, they will be more likely to collaborate to achieve their joint preference on that issue, and more generally states with similar foreign policy preferences will be cooperative in a larger proportion of their interactions. States with similar preferences are also unlikely to be involved in violent disputes with each other because the policy benefits of these disputes are unlikely to surpass the cost of fighting. Conversely, when states have highly dissimilar foreign policy preferences, cooperation will be difficult and violence will be more common. Unfortunately, while we have abundant data to measure the strength of states' economies, the volume of trade between states, or even their military power, it is much more difficult to measure states' preferences, because as with many social and political constructs they cannot be observed directly.

Effectively measuring state preferences would yield scholars a number of benefits. A number of formal theories of international relations require measures of preferences to be tested: the expected utility theory proffered by (Bueno de Mesquita, 1983) has similarity of preferences as an important input. Further attempts to expand studies of crisis bargaining to include mediation (Kydd, 2003), coalitional dynamics (Wolford, 2014), or the possibility of additional disputants (Gallop, 2017) require a measure of

state preferences in order to predict whether war will be the result of bargaining failure. Preferences have been used in empirical studies predicting bilateral trade, foreign aid, stability of international institutions and the incidence of conflict (DeRouen and Heo, 2004; Stone, 2004; Gartzke, 2007; Kastner, 2007; Braumoeller, 2008).

A substantive theoretical reason for why we need a good measure of preferences is to correctly understand the democratic peace. It is difficult to entangle whether democracies avoid war with other democracies because of the intrinsic nature of democracy, or simply because they appear to share similar ends. Farber and Gowa (1995) argue that democracies were only peaceful during the Cold War period because they had similar preferences and alliance structures. Similarly, Gartzke (1998) argues that dissimilar preferences are a necessary condition for conflict. Oneal and Russett (1999) respond by arguing that democracy has both a direct inhibiting effect on conflict, and an indirect one through influencing state preferences.¹ While there has been some impressive development with measures of preferences in recent years, a more accurate measure is essential to disentangle the extent to which peace is the product of shared preferences, and the extent to which institutions and norms are driving peace.

Much of the extant literature has focused on estimating state preferences by utilizing spatial weighting models on either alliance behavior or United Nations (UN) voting scores. These approaches have proven to be useful but there are two reasons to desire a different approach. First, alliances are rare and voting together in the UN is very common, so, by only focusing on the direct dyadic behavior, we risk mischaracterizing important relationships. Second, we would expect a better understanding of state preferences to help us predict state behavior, but as we show in Figure 1 adding mea-

¹Gartzke (2000) argued that even though democracies might have similar preferences, the residual of preferences from democracy explains conflict much better than the residual of democracy from preferences.

asures of state preferences to a traditional model of interstate disputes yields relatively scant increases in terms of predictive ability.

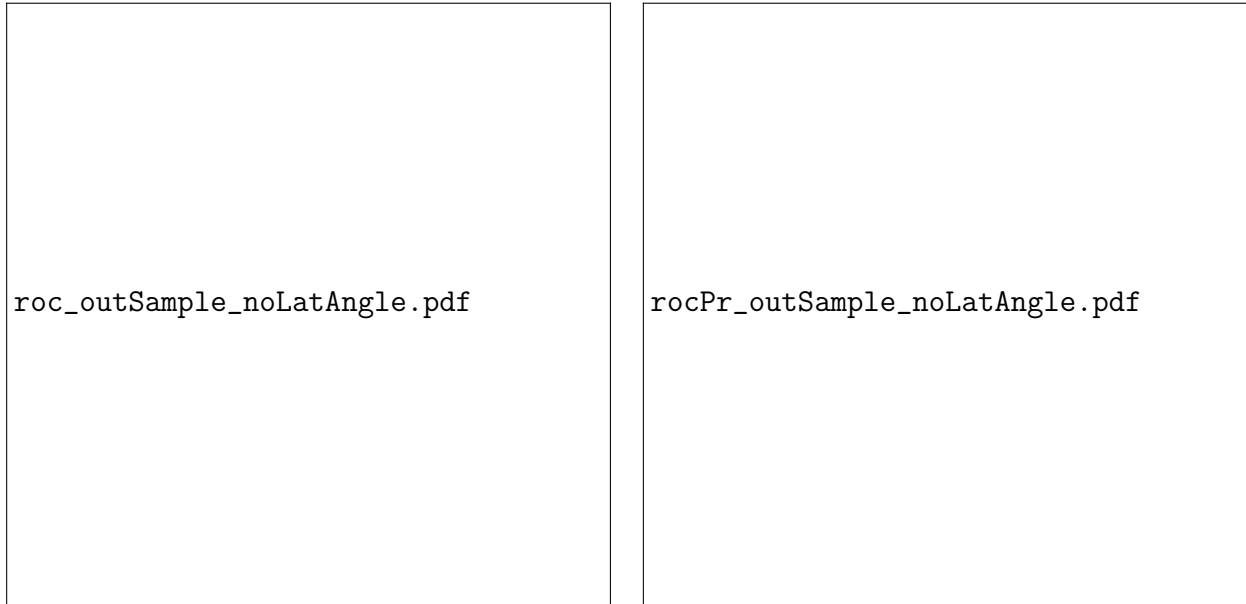


Figure 1: Assessments of out-of-sample predictive performance of Militarized Interstate Disputes using ROC curves and PR curves. AUC statistics are provided as well for both curves.

We can improve on these measures of preferences using the same raw material by acknowledging that both alliance membership and UN voting are layers of relationships that takes place simultaneously and in an interdependent context. Specifically, these relations between states constitute a multilayer network, in which the various layers correspond to different ways states are interacting with one another at a given time point. A bevy of research has shown that accounting for network structure necessitates an approach that can account for the indirect relations states share. As such, we make two contributions to the existing literature on state preferences. We utilize a multilinear tensor regression that enables us to measure how dependent actions of a particular dyad are across layers and time. We show that our revised approach of measuring state preferences both better characterizes relationships that have had counterintuitive results, and this measure greatly enhances our ability to predict instances

of conflict.

Sources of Preference Measures: Alliance Portfolios and UN Voting

Given that we cannot directly observe state preferences, scholars have attempted to estimate preferences using two main behavioral indicators: who states choose to ally with and how states vote at the UN. The idea behind alliance portfolio measures is that we can infer a state's foreign policy by looking at the states they choose to align with. In the extreme case, if two states have all of the same allies, it is likely that their foreign policy goals are quite similar. Conversely, if all allies of one state are not allied to another, and vice versa, our best guess is that these states would have different aims and desires in foreign policy. Bueno De Mesquita and Lalman (2008) encapsulate the logic when they note that "alliance commitments reflect a nation's position on major international issues". Measures of alliance behavior do, however, suffer from the fact that these measures are largely static and sparsely occurring. Formal alliances are relatively constant over time, whereas in many cases state preferences will be more fluid, and therefore these scores will be at best a lagging indicator of preferences. Furthermore, as Häge (2011) points out, the fact that links are so rare creates an artificial similarity of alliance portfolios.

We also have a relatively large corpus of behavioral information in UN Voting Records. The cost of voting in the UN is low, and so, scholars have argued that measures of affinity based on UN voting are relatively representative of the underlying distribution of preferences (Gartzke, 1998). This is especially fortuitous because the methodology of inferring preferences from voting in a legislature is advanced. A few issues with these measures are that the potential benefit of winning UN votes is low, and so states might have incentives to vote against their preference as they are not costly signals, and the distribution of UN voting is prone to large supermajorities of the type rarely seen in

“ordinary” legislatures.

Current measures of preferences: S-Scores

The initial measure used to measure preference similarity based on alliance portfolios was Kendall’s τ_B (Bueno De Mesquita and Lalman, 2008). This measure is:

$$\tau_B = \frac{n_c - n_d}{\sqrt{(n_0 - n_1)(n_0 - n_2)}} \quad (1)$$

where n_c is the number of pairs where both actor i and j have the same rank ordering (for example both the United Kingdom and the United States are more closely allied to Israel than to Iran), n_d is the number of pairs where they have discordant rankings (the United States is more closely allied to Saudi Arabia than to Russia, Syria is more closely allied to Russia than to Saudi Arabia). The denominator attempts to adjust the total number of pairs with the number of ties: n_0 is the total number of pairs ($n(n - 1)/2$), n_1, n_2 are measures for ties in both i and j ’s rankings respectively.

Signorino and Ritter (1999) convincingly pointed to flaws in this measure, notably its focus on rank-ordering as applied to a context where we instead care mostly about the presence or absence of an alliance. In addition, if we add additional strategically irrelevant states, we will create artificially high τ_B statistics. Thus, Signorino and Ritter introduce the S-score, which has since been the most widely used alliance similarity measure.² The equation for the S-score is:

$$S(P^i, P^j, W, L) = 1 - 2w_k \frac{d(P^i, P^j, W, L)}{d^{\max}(W, L)} \quad (2)$$

here P^i and P^j are vectors of length N , showing the relation that states i and k have

²Bennett and Rupert (2003) also find a stronger relationship between theoretical predictions and results when using S-scores than when using τ_B .

to each of the N states (including themselves). W is a vector of weights (normalized to sum to 1) allowing a scholar to put higher (lower) values of more (less) important states – allowing relations with China to play a larger role in two states' similarity score than relations with San Marino $d(P^i, P^j, W, L)$ is the distance between portfolio i (P^i) and portfolio j (P^j) based on the vector of weights W and the scoring rule L . If using an absolute distance scoring rule, then $d(P^i, P^j, W, L) = \sum_{k=1}^N \frac{w_k}{\Delta_k^{\max}} |p_k^i - p_k^j|$. In other words, we would calculate the distance between the two portfolios by looking at the absolute distance between each item in the portfolio (p_k^i for state i 's relationship with state k) adjusted by the weight given to this relationship (w_k) and then normalized by the highest possible difference between p_k^i and p_k^j , Δ_k^{\max} . $d^{\max}(W, L)$ is the largest distance between portfolios observed in the policy space, so this means that states at the maximum observed distance will have an S-score of -1 , and those with identical portfolios will have an S-score of 1 . For the weight matrix, generally, analysts have used S-scores calculated with a weight matrix of ones—giving each potential ally equal weight—though the other plausible choice would be to weight states by importance, for example using their share of world military capability, as calculated by Singer and Small (1995). Gartzke (1998) attempted to apply a similar S-score methodology to UN voting data and created the “Affinity of Nations” index. An important point for these scores is that they are purely dyadic. One can look at the S-score between two states, but one cannot look at a state's preferences in comparison to a larger cluster, or note the movement in a states preferences over time. In monadic analysis, these score measures are not even available, and once we are dealing with situations involving more than two states, the number of S-scores necessary to fully characterize the preferences balloons quickly (it is the number of actors choose two).

In recent years, many scholars have pointed out that forming and maintaining alliances are not independent dyadic phenomena, but are rather part of an interdepen-

dent network (or series of networks). Warren (2010) pointed out how extra-dyadic factors mattered for alliance formations, in particular alliances with allies of your allies (and enemies of your allies) were more likely. Further investigation uncovered more network effects were important such as popularity and 2-stars (Cranmer et al., 2012b). The strategic nature of states when they form alliance ties leads to interdependence in the alliance network, as states seek to ensure their strength and security as efficiently as possible, privileging ties that complete triangles, and paying attention to the decreasing returns of additional alliances (Cranmer et al., 2012a). Later, Warren (2016) demonstrated not only were network effects (like preferential attachment, density, and transitivity) important in the generation of the alliance network, but the alliance network and the conflict network were mutually constitutive. Similarly, Kinne and Bunte (2018) uncovered links between the network of defense cooperation, and the network of bilateral loans, as states strategically loans to exert policy influence on other states, and strategically formed communities of defense cooperation in order to enhance the power and independence of its members.

An advantage of utilizing UN General Assembly Voting, is that it allowed the field to take advantage of methodological advances that have been made in the study of legislatures (Poole and Rosenthal, 1985). Bailey et al. (2017) do so by using an Item Response Theory model on UNGA voting. This model seeks to place states on a unidimensional latent preference space using their voting behavior. The assumption of this model is that states' votes on a resolution are a function of states' ideal points, characteristics of the vote, and random error. In particular, for each bill v , a state's vote will be based on the latent variable Z_{itv} , representing the preference of state i , on vote v , at time t , defined such that:

$$Z_{itv} = \beta_v \theta_{it} + \epsilon_{iv} \tag{3}$$

Where β_v is the discrimination factor of bill v , determining whether states with high ideal points will vote yes (if β_v is negative) or no (positive). θ_{it} is state i 's ideal point at time t , and ϵ_{itv} is actor and bill specific error. Of course, we do not observe the latent variable, and only observe the outcome Y_{itv} , which will either be yes, no, or abstention. The model creates a pair of cutpoints γ_{1v} and γ_{2v} in the preference space, such that if $Z_{itv} < \gamma_{1v}$, $Y_{itv} = \text{Yes}$, if $Z_{itv} > \gamma_{2v}$, $Y_{itv} = \text{No}$ and otherwise abstain. This allows the model to use observed vote outcomes in order to determine each states' ideal point in each year.

The authors specifically fix the parameters γ_{1v} and γ_{2v} such that the same bill will have the same value in different years, and they standardize and normalize θ . They also use θ_{it-1} as a prior on θ_{it} . With these constraints, they solve for the ideal points using a Metropolis Hastings Markov Chain Monte Carlo (MCMC).

Both methods relying on UN data, and those relying on alliances have difficulties distinguishing within 'o's and 'i's. For example, if we know two states are allies, we have reason to believe they have similar preferences, but if we know they are not allies, it is not clear whether they are enemies or they are indifferent – the United States is “not-allies” with both Bhutan and North Korea for example. As of 2012, using the Correlates of War projects alliance data, only about 1/8th of all dyads were between countries with any sort of alliance. Similarly, with UN voting, so many UN votes contain super-majorities and states vote together a huge proportion of the time. If we only look at yes and no votes in the UN general assembly, the median pair of states has voted together about 96% of the time. If we include abstentions, they have voted together 86% of the time. So when two states vote together it is hard to distinguish between states voting together because of similar preferences, or just preferences that are not radically dissimilar. Both S-scores and Item Response theory succeed in adding granularity and nuance to these rough measures, but they are both limited by focusing only on a

relationship between two states.

We can see the potential issues of focusing only on direct relations when we view how these two measures of preference treat relations over the Korean peninsula. If we take China, North Korea, South Korea, and the United States, we would expect the United States and South Korea to have preferences that are similar to each other and dissimilar from China and North Korea (and vice versa). Yet, if we look at extant measures of preferences (as of 2012), as depicted in Figure 2, they do not seem to effectively characterize this relationship. S-scores based on alliance portfolios posit that China and the two Koreas are closely clustered, with the United States distant from all three, while ideal point distances put China's relationship with South Korea on par with its relationship with North Korea. Now it could be that these measures are producing a novel, counterintuitive, result, but given the failures of extant preference measures to add much to predictive models of conflict, one might be skeptical.

The sources of these surprising results become more clear when looking at the data on which they are based. In 2012, according to the Correlates of War, North Korea only had 3 alliances – a non-aggression pact with the South, and alliances with Russia and China. Similarly, South Korea's only alliances were that non-aggression pact, and an alliance with the United States. Given the United States' many other allies, there was much more divergence between their alliance portfolio and South Korea's, than there was between the two Koreas' portfolios (Gibler and Sarkees, 2004). Similarly, when looking at voting patterns at the UN, South Korea voted 50% of the time with the US, 63% of the time with North Korea, and 70% of the time with China.

Even using this data, we can better approximate state preferences when we treat alliances and UN voting as relational data. When it comes to alliance behavior, the fact that North Korea was allies with China and Russia, and South Korea with the United States could give us additional information, because the alliance behaviors of countries

such as the United States and China are so divergent. When looking at voting patterns at the UN, the need for a relational approach is even more apparent. While South Korea only voted with the US 50% of the time at the UN, this was in the top 15% of all countries in terms of voting with the US, whereas South Korea was in the bottom 20% in terms of the proportion of time voting with China.

We thus argue that two changes can substantially improve measures of state preferences. First, both alliances and UN voting contain information about state foreign policy preferences, and given the limits of this information, we should find a way to use both. Second, by using network techniques and treating this data as relational data, we can wring more information from the stone, and get both a more nuanced, and more accurate view of state affinity and state preferences.

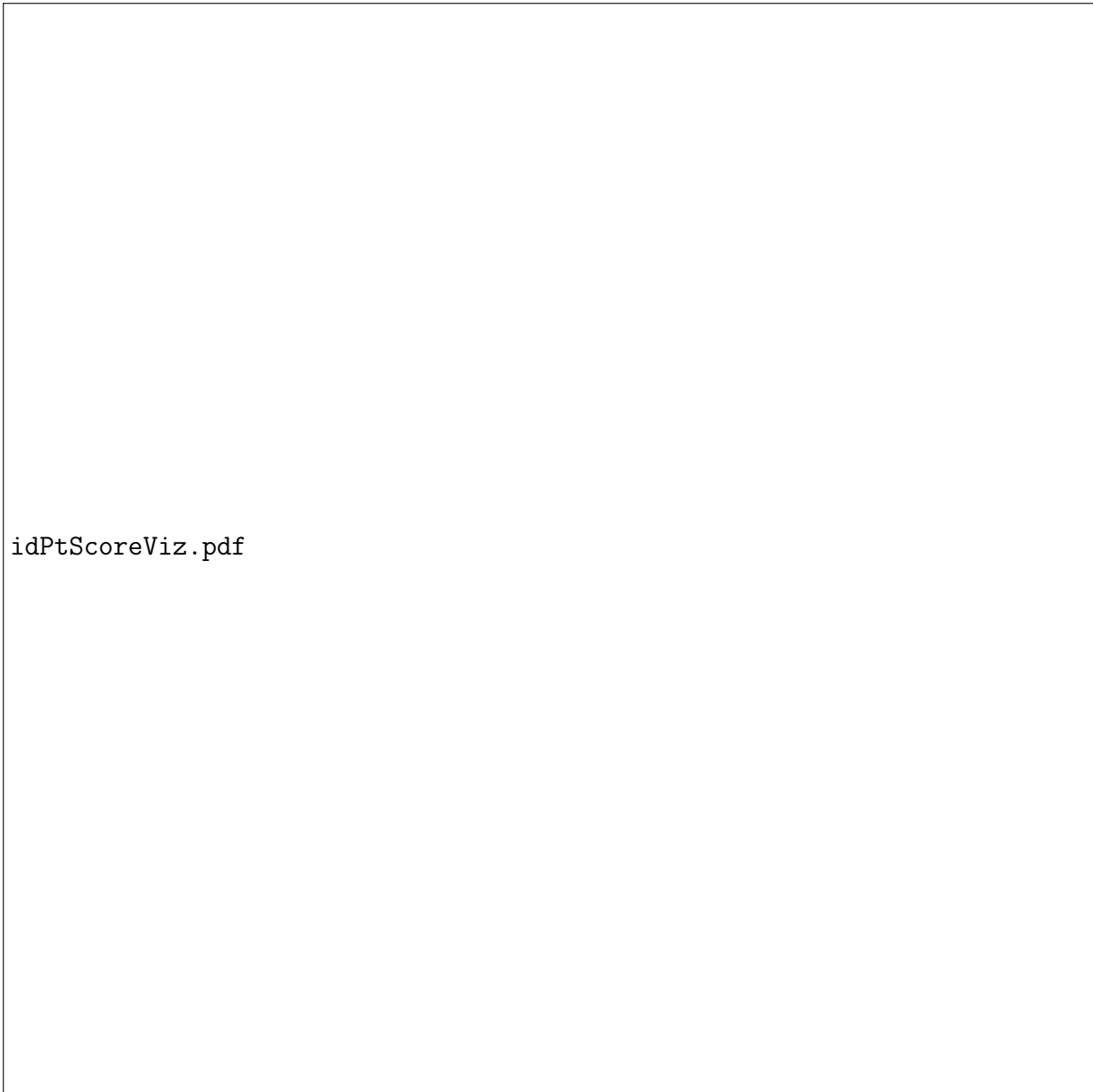


Figure 2: Visualization of ideal point distance and S-score relationships between China (CHN), North Korea (DPRK), South Korea (ROK), and the United States (USA) in 2012. Within each panel a higher score denotes that the countries have a more positive relationship.

Synthesizing Measures of State Preference

The approach that we introduce here to measure state preferences starts by assuming that both UN voting and alliance relationships are sources of information on how states relate to one another on the international stage. By accounting for the multiple layers upon which states interact with one another we can synthesize a better measure of state preferences than if we relied on any one measure alone.³ The downside of this extant approach, however, is that it does not account for structural patterns that we often see in relational data.

Relational data is composed of observations between pairs of actors, or dyads. For both alliance relationships and UN voting, we are able to observe how the actors in the international system interact with one another across time. This system of interactions taken in its totality defines a network, and within these types of structures a bevy of research has shown that we need methods that go beyond assuming that interactions are taking place between just two actors in a vacuum (Wasserman and Faust, 1994; Snijders and Nowicki, 1997). As such we reformulate the problem of determining state preferences in terms of a network analysis. The goal of our approach is summarized in Figure 3. In the top row, we represent UN voting and alliance patterns at time t as a pair of adjacency matrices that form an evolving multiplex network.⁴ Our goal is to extract a lower dimensional representation of this system, such that the output is a series of $n \times n$ matrices, where n represents the number of actors and in which the cross-sections denote our estimates of the preference similarities between countries.

We generate these estimates through a multilinear tensor regression model that re-

³The idea of using multiple metrics to get a better handle on preferences is not new, in fact, Signorino and Ritter suggested it in introducing S-scores, which were designed to allow for aggregation of similarity on multiple dimensions (such as alliances and UN voting).

⁴The approach that we describe here can be generalized to a multiplex with more than two dimensions.

lies on a high to estimate a Gaussian score for each country pair. We will show that by combining different measures of state preferences, and better accounting for network dependencies through our approach, we are able to generate a measure for preference that maintains the insights of both UN voting scores and S-scores and which can yield new insights, in particular when it comes to predicting and explaining interstate conflict.

Multilayer Tensor Decomposition

In essence, our goal here is to measure an unobservable concept. This is a problem that is very familiar in political science and a number of techniques based on latent variable models have been developed to study political ideology (Martin and Quinn, 2002; König et al., 2013), human rights abuses (Fariss, 2014), and judicial independence (Linzer and Staton, 2015). Obviously, the ideal points measure developed by Bailey et al. (2017) also follows in this growing practice of extracting unobserved information via latent variable modeling. Here we employ this general framework to develop a latent measurement of how a state relates to other states in a network context. The factor analysis we employ seeks to take as an input the interactions that actors have with others across a variety of dimensions and project this onto a low-dimensional space. In many ways this goal is no different than how others have sought to find simpler representations of legislators and bills (Poole and Rosenthal, 1985; Clinton et al., 2004).

However, in the network case there are specific types of patterns that often occur and that should be captured when reducing the dimensionality of a network. One such pattern is stochastic equivalence. Stochastic equivalence refers to the idea that there are communities of nodes in a network, and actors within a community act similarly towards those in other communities. Thus the community membership of an actor provides us with information on how that actor will act towards others in the network.

Put more concretely, a pair of actors i and j are stochastically equivalent if the probability of i relating to, and being related to, by every other actor is the same as the probability for j (Anderson et al., 1992). This concept simply speaks to the assertion that we can learn something about how an actor will interact with an entire network based on, for example, the existing set of alliances that they are enmeshed in.

An additional dependence pattern that often manifests in networks is homophily – the tendency of actors to form links they share some latent attribute. In many cases, homophily will lead to greater levels of transitivity in a network because, if i and j share an attribute and thus are more likely to be linked, and j and k share the same attribute and are linked, then we should expect to see a link between i and k . In the context of clustering in alliance relationships, we are likely to find that states like the United States, United Kingdom, and Germany may cluster together because they share some latent state level attribute. We would ignore salient information if we did not use, for example, the United Kingdom's behavior towards third parties, when trying to understand the United States' preference similarity with those parties. Doing so is likely to paint an incomplete picture of the preferences that states share with one another.

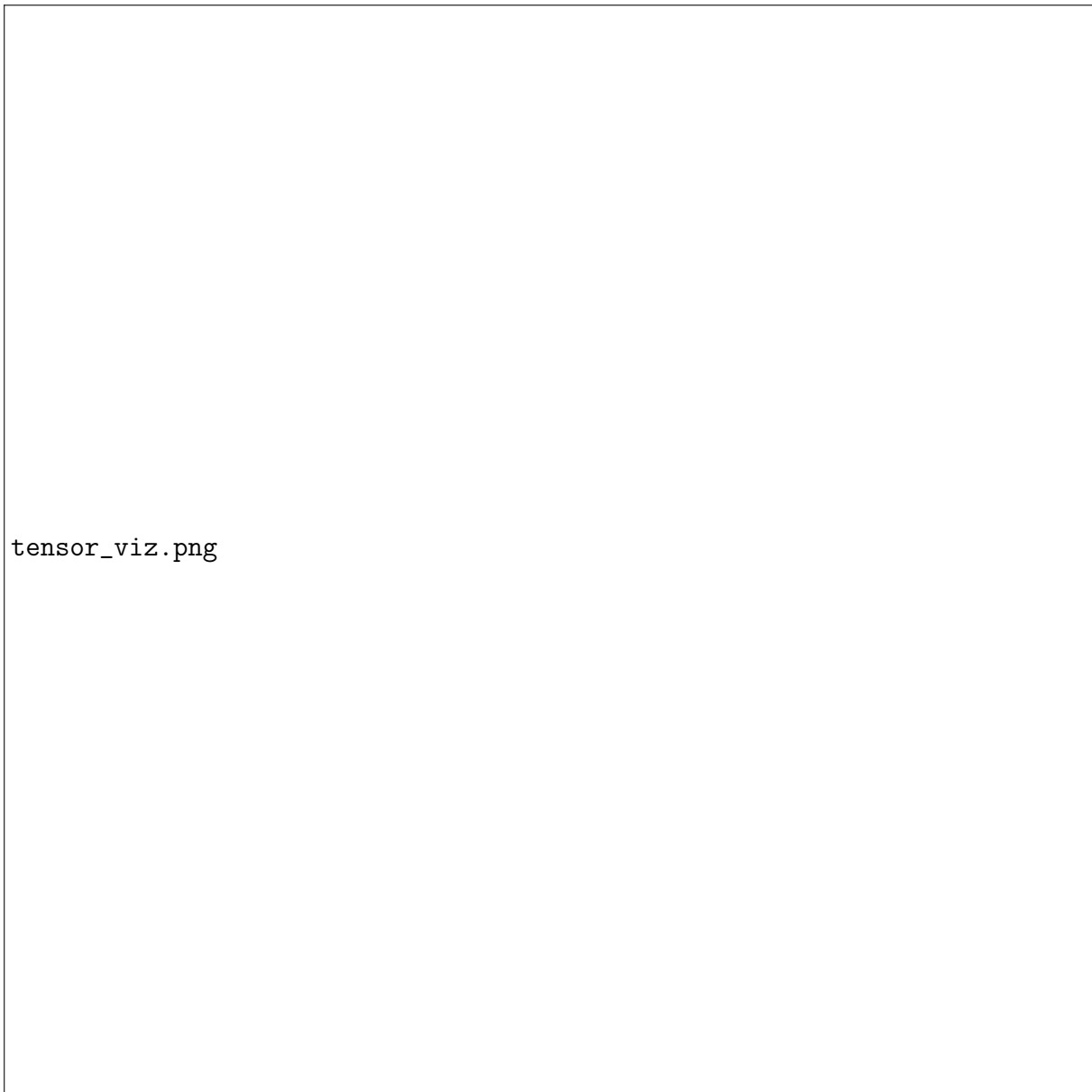


Figure 3: The green and blue colors represent different relational measures and darker shading indicates later time periods. Our goal is to reduce the patterns found across those layers of relationships into a single measure.

Modeling Approach

In our model, we go a step further by estimating the relationship between trade and conflict in a multilinear tensor regression. Our estimation approach here utilizes the framework established in Hoff (2015) and Minhas et al. (2016). We review the approach briefly here.

Tensor Model

We represent relational data that is longitudinal be represented as a series of matrices $\{\mathbf{Y}_t : t = 1, \dots, n\}$, with each \mathbf{Y}_t representing the actions or flows between the pairs of actors, in our case countries, at a given month t . A cross-sectional entry such as $y_{i,j,v,t}$ from \mathbf{Y}_t denotes an interaction that occurred between actors i and j at time t across variable v . These series of matrices can be assembled in a tensor, \mathbf{Y} , which we use to represents actors interacting over time across multiple measures. Specifically, \mathbf{Y} will have dimensions $m \times m \times v \times n$, where m corresponds to the number of countries, v to the number of variables (i.e., relational measures), and n to the number of time periods. Figure 3 provides an example of this for the simple case of four countries, three time points, and two relational measures.

Now we can also rewrite the information contained in this set of matrices as a vector, where (\mathbf{y}_t) represents a time series of vectors representing the dyadic interactions across multiple variables for m countries through the period $t = 1, \dots, n$. Transforming the data in this way enables us to think of what we are modeling in an explicitly time series context, where the evolution of multiple series is a function of their past behavior. Modeling such processes can often be accomplished using a vector autoregression (VAR) framework. The benefit of this approach is that it models a set of time series as resulting from a dynamic and endogenous process, rather than imposing a priori assumptions of exogeneity between the various explanatory variables, and we can posit

the autoregressive relation as:

$$\mathbf{y}_t = \Theta \mathbf{y}_{t-1} + \mathbf{e}_t \quad (4)$$

$$E[\mathbf{e}_t] = 0 \quad (5)$$

$$E[\mathbf{e}_t \mathbf{e}_s^T] = \begin{cases} \Sigma & \text{if } t = s, \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where \mathbf{y}_t has been demeaned to eliminate intercepts such that $\sum_t y_{i,j,t}/n \equiv 0$. This is easy to estimate, but has extreme data requirements since Θ has m^4 entries. One basic solution to this is to use a bilinear model such that the regression matrix is given by $\Theta = \mathbf{B} \otimes \mathbf{A}$, where \otimes is the Kronecker product. As a result of this specification we obtain a basic specification that somewhat mimics alternatives approaches to modeling cross-sectional networks such as the latent factor model (see Minhas et al. (2019)): $\mathbf{Y}_t = \mathbf{A} \mathbf{Y}_{t-1} \mathbf{B}^T$. The key difference here, however, is that in latent factor models we are projecting the relations between actors onto lower a dimensional space, where similarity in orientation indicates that actors are more likely to have similar dependence patterns. In the context of this model, \mathbf{A} and \mathbf{B} are $m \times m$ matrices of regressions coefficients, where each cross-sectional entry measures the level of dependence between the sending relationships of a particular actor and the B the receiving relationships.

$a_{i,i'}$ capture how previous actions of i' affect i and $b_{j,j'}$ shows how actions that target j are influenced by prior actions toward j' . If $a_{i,i'}$ is positive, this gives us a measure of how likely i is to send an event to a third party k given that i' has already sent an event to k . More concretely, if we imagine that the event is verbal cooperation, then a positive $a_{i,i'}$ would indicate that i and i' are more likely to send cooperative events to the same third country. If $b_{j,j'}$ is positive on the other hand, this gives us a measure of

how likely j is to receive an event from a third party k given that j' has already received an event from k . Another way would be to think of this is $b_{j,j'} > 0$ gives us a warning that if j' has been attacked by k , then k 's j should expect to be attacked soon as well.

Given these parameters, the model for a single country-country-variable observation is shown in equation 7, where i represents a sender country, j a target, k a particular relational variable, and t a time point:

$$y_{ijvt} = \sum_i \sum_j \sum_k a_{1ii'} b_{2jj'} b_{3kk'} x_{i'j'k't} \quad (7)$$

The regression problem can also be written somewhat more simply in tensor form, where it is clearer that we are essentially regressing the relational tensor \mathbf{Y}_t from time t on the tensor $\mathbf{X}_t = \mathbf{Y}_{t-1}$ from time $t - 1$:

$$\mathbf{Y} = \mathbf{X} \times \{\mathbf{A}, \mathbf{B}, \beta\} + \mathbf{E}, \quad (8)$$

where “ \times ” is a multilinear operator known as the “Tucker product”.⁵ Here \mathbf{Y} and \mathbf{X} are $m \times m \times v \times n$ tensors and each entry for \mathbf{X} is lagged by one period.

Thus \mathbf{A} and \mathbf{B} represent how previous actions along any of the v parameters affects future interactions. For example, the ij coefficient in \mathbf{A} measures the effect of an interaction from actor j to actor k along any of the v parameters on the likelihood of an interaction from i to k , in any of the parameters, in the next time period.⁶

⁵The Tucker product is a multilinear operator that is used for higher-order singular value decomposition (SVD), the same way that matrix multiplication is used for matrix SVD (Kolda & Bader 2009). To illustrate how the Tucker product is calculated, say that we want to get the following expression: $\mathbf{Y} = \mathbf{X} \times \{\mathbf{A}, \mathbf{B}, \mathbf{B}_3\}$, where \mathbf{Y} has dimensions $n_1 \times n_2 \times n_3$ and \mathbf{X} has dimensions $m_1 \times m_2 \times m_3$. The first step involves reshaping \mathbf{X} so that it is a matrix with dimensions $m_1 \times (m_2 \times m_3)$, then we multiply on the left by \mathbf{B} , next we reshape the result to an $n_1 \times m_2 \times m_3$ matrix. This procedure would be applied iteratively among the remaining dimensions. Further details on this operator can be found in: Kolda (2006).

⁶For example, for each county i , the coefficient b_{1iUSA} describes how predictive the actions of the USA are of the future actions of country i . Heterogeneity of these coefficients across countries i de-

Within this framework, we can also include nodal and dyadic covariates. Relations between this set of covariates and \mathbf{Y} are captured in β , which enables us to model the relations between each of the p covariates we pass to the model and the v dependent variables. The first set of variables that we include in the model are lagged versions of each of the dependent variables. We also create additional terms for to capture reciprocal and transitive effects.⁷ In the next section, we will discuss how we operationalize conflict, trade, and a set of control variables into the model.

The key output from the LFM for our purpose here is $U\Lambda U^T$. Actor positions in the latent factor space are characterized by the concepts of homophily and stochastic equivalence discussed above. Interpretation of this space needs to be done with care. Distances between actors cannot be interpreted using Euclidean metrics, as actor's latent positions are actually embedded within a k -dimensional hypersphere. This means that an actor's vector direction within the k -dimensional hypersphere indicates which latent preferences an actor i has and does not have. Comparing the similarity of preferences between two states, $\{i, j\}$, can be accomplished by comparing the direction to which their respective factor vectors point. A commonly used metric for this sort of problem in the recommender system literature from computer science is the cosine of the angle formed by the latent vectors of both actors.⁸ We refer to this distance metric as latent angle distance.⁹ Thus, if the estimated latent vectors of two states are in the same direction, they are apt to have both alliances and UN co-voting to similar partners. The way we measure this similarity in dimension is by looking at the absolute distance of the angles created by each states position and the center of the latent space.

scribes heterogeneity of the nodes in terms of their dependence on the actions of USA.

⁷For further details on the construction of these effects see Minhas et al. (2016).

⁸For a review of this literature see (Xavier Amatriain and Pujol, 2015).

⁹This measure is defined as: Latent angle distance $_{ij} = \frac{u_i \cdot u_j}{||u_i|| \cdot ||u_j||}$.

As an example of how our tensor dependence measure is effected by changes in alliance relations or UN voting similarity consider the United States and Israel, in 1981. With the signing of the "Joint Memorandum of Understanding," the two states went from having no alliance relationship to having one, the states were voting together at the UN 80% of the time (up 3% from 1980, below the median yearly change for this dyad). These factors, coupled with changes in the third order effects for the dyad, led the tensor dependency to increase from about 2.72 to about 3.08.

Can you put in the explicit math that leads to this SM?

Of course, another pair of countries might sign an alliance (or withdraw from an alliance) and have a markedly different effect on this measure of tensor dependency. For example, in 1977, the United Kingdom and the United States went from being in 5 alliances together to being in 4, and this led to a drop in their tensor dependency from 3.13 to 2.28, despite a decrease in their UN voting similarity of less than 1%. This is because the alliance that ended in 1977 was the Southeast Asian Treaty Organization, which had 4 other members, and so the third order effects were concomittantly more severe. To capture the aggregate effects of dyadic variables on our measure of tensor dependency, we estimated a simple linear model, depicted in table ??, and these dyadic factors account for about 78% of the variance, with the rest coming from third order factors.

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.0773	0.001	74.26	0.000
Tensor Dependence _{t-1}	0.888	0.0004	2094.81	0.000
Change in Alliance Relationship	0.131	0.009	15.30	0.000
Change in UN Voting Similarity	4.401	0.014	313.14	0.000

Table 1: Regression of tensor dependence on dyadic components.

Data Sources, Modeling choices

Another important question is the amount of temporal aggregation used. In our baseline model, we treat each year as separate and gain a unique observation of each states ideal point in each year. However, this raises a real risk of temporal inconsistency in the values. An alternative approach would be to have a rolling average for the measures of similarity over a number of years. This would allow us to infer a country's relative position not just by their behavior in a given year, but also their behavior in the past few years. The risk if we use too much temporal aggregation is that we are including data which is no longer relevant to a country's relative preferences. For instance, Turkey and Russia's relationship looks a lot more positive when we look at 2013 and 2014 then when we look at 2015. To that end, in addition to our baseline model where years are seen as independent, we also evaluate models where ...

\mathbf{B}_1 and \mathbf{B}_2 contain $m \times m$ regression coefficients. The ij coefficient in \mathbf{B}_1 provides a measure of how likely i is to send, for example, a verbal or material cooperative event to a third party, k , given that j has already sent a cooperative event to k . Similarly, the ij coefficient in \mathbf{B}_2 provides a measure of how likely i is to receive a cooperative event from a third party, k , given that j has already received a cooperative event from k .

Face Plausibility

An important question for these different measures of preferences, are whether they give results that "make sense." In particular, we would hope that the measure both provides plausible levels to relationships – sorting states into friends and foes effectively – and that when these measures change, they do so in ways that correspond to changes in the world. One of our principle critiques of both alliance portfolio S-scores and UN voting based ideal points is that they describe certain important relationships in a way that some may find implausible. Using our measure we return to the exam-

ple of relationships in East Asia in 2012. We find that our tensor dependence measure, depicted in figure 4, corresponds much better to conventional understanding of the relations between these states.

Figure 4: Visualization of tensor dependence, S-score, and ideal point similarity relationships between China (CHN), North Korea (DPRK), South Korea (ROK), and the United States (USA) in 2012. Within each panel a higher score denotes that the countries have a more positive relationship.

To generalize this test of face plausibility, we present all three measures' accounts of six dyadic relationships over time. Additionally, we rescale each of these measures to be between 0 and 1 for comparability, where a 1 indicates that states are farther apart and a 0 that states are closer together.

We first look at three close relationships where we would expect to see similar preferences. Figure 5 depicts the relationships between France and Germany, the US and Israel, and North Korea and China. In all three cases our measure using tensor dependence distance has consistently low values – nearly 0 in the case of France and Germany, and China and North Korea, and low but less stable values for the US/Israel relationship. Alliance S-Scores correctly classifies Franco-German and Sino-North Korean friendship, while the US/Israel relationship is characterized as being relatively neutral. The measure based on UN voting is most out of step in terms of characterizing these relationships, with notable divergence in the preferences of many of these pairs.

In figure 6 we look at the United States' relationship with three countries that are characterized by change and major events. S-scores are the only measure that does

Figure 5: Dyadic relationships between France-Germany, United States-Israel, and China-North Korea according to the different preference measures. Yearly tensor dependence scores between dyads are denoted by red circles, ideal point similarity based on UN voting in blue triangles, and S-scores by green squares. Within each panel a higher score denotes that the countries have a more positive relationship.

Figure 6: Dyadic relationships between France-Germany, United States-Israel, and China-North Korea according to the different preference measures. Yearly tensor dependence scores between dyads are denoted by red circles, ideal point similarity based on UN voting in blue triangles, and S-scores by green squares. Within each panel a higher score denotes that the countries have a more positive relationship.

not detect a marked improvement in the US/Russian relationship at the end of the Cold War. Both the UN ideal points and tensor dependence space find significant improvements followed by a drift toward enmity, whereas S-scores have a constant (though slightly improving) neutral relationship. For the US and Iraq no measure depicts more similar preferences following the US occupation, though only tensor dependence distance finds an increase in enmity in the run-up to the United States' invasion. Finally, for the US and China, the S-score has a consistent neutral relationship, while both UN ideal points and tensor dependences both have more negative relationships, and more variance – with the tensor dependences metric finding particularly large spikes at the beginning of the Bush and Obama administrations.

As can be seen from these relationships, the measure of preferences based on tensor dependence distance is in many cases as plausible or more than incumbent measures of preferences. While the measure has more temporal instability than S-scores or UN ideal points, in these six cases it often does better, and rarely worse at conforming to our expectations of the relationships. Of course, this could just be a case of us picking particularly propitious cases. Next, we conduct a the large-N analysis of conflict and compare the added utility of these various measures of state preferences.

Model Competition

To evaluate the different measures of state preferences, we assess their utility in predicting the occurrence of interstate disputes. Here we look at four non-nested models: a model using no measures of state preferences, one using an S-Score based on

similarity in alliance portfolio (Signorino and Ritter, 1999), one using the ideal points determined by UN voting (Bailey et al., 2017), a model using both UN ideal points and alliance S-scores, and finally, a model using our tensor dependence approach that combines the raw data from UN voting and alliances. We evaluate the models on two criteria: whether state preferences have a consistent effect in the predicted direction, and how well each model does at predicting disputes in an out-of-sample context.

In each of these models, we utilize a logistic regression of Militarized Interstate Dispute (MID) participation on measures of state preferences and a vector of control variables. These control variables overlap with the standard framework used in O’Neal and Russett’s canonical work on the democratic peace (Oneal and Russett, 1997).¹⁰ In particular, we include a binary measure of joint democracy (whether both states have Polity IV scores ≥ 7), whether the states are contiguous, and the ratio of state capabilities as measured by the Correlates of War Project’s Composite Index of National Capabilities (CINC). We also account for temporal interdependence using a peace year spline (Carter and Signorino, 2010).

In-sample explanation

As detailed in figure 7, two of the measures of state preferences perform as we would expect. Our measure of state preference using tensor dependence difference is highly significant and positive: states with more dissimilar preferences will have greater difference in their tensor dependences, and this is highly associated with a greater risk of conflict. Similarly, UN ideal point similarity passes this test. The measure using UN voting ideal points is positive and clearly distinct from 0, indicating that states with more distant ideal points, and thus more dissimilar preferences, are more likely to

¹⁰The exception is that our models ignore trade interdependency, as including that data drastically decreases the number of observations.

find themselves in conflict. On the other hand, higher alliance S-scores are consistently associated with higher probabilities of conflict – so states with more similar preferences as measured using alliance portfolios are more likely to quarrel. This is, to say the least, surprising. These results hold when those measures of preference are used in isolation, or in tandem.

The models have one major difference in terms of the controls: in the model using tensor dependence distance, joint democracy is indistinguishable from 0. This is particularly interesting as one of the major criticisms of democratic peace theory is that democracies have similar preferences. Some argue that it is similarity in preferences that leads to peace among democracies. Despite this dispute, most attempts to include preferences in the standard democratic peace regressions still find a consistent pacifying effect of democracy. (as do those models with UN voting and S-scores presented here).

Figure 7: Parameter estimates from models with different measures of state preference. Point represents average estimate, line through the point represents the 95% confidence interval.

Out-of-sample prediction

Given these results, we can say that two of the measures of preferences behave as we would expect, while S-scores do not. To adjudicate between the two measures of preference similarity that pass this test, and help determine what we should make of the differing effect of democracy, we turn to out-of-sample prediction. We undertake a cross-validation procedure in which we partition our data into 30 different folds. This process works by taking each fold, k , and running a logistic model excluding data from that fold. Once we have parameter estimates from a model that excluded fold k , we predict the probability of a MID in fold k using only the parameter estimates and the covariates from fold k .

We then compare the performance of these models using two metrics: area under the Receiver Operator Characteristic Curve (AUC-ROC) and the Precision Recall Curve (AUC-PR). The ROC curve examines the tradeoff between true positives and false positives, and the PR examines the tradeoff between making only correct predictions and predicting all the disputes that occurred. In general, the ROC will disproportionately reward those models that predict 0 well – and we can interpret the ROC as the likelihood a prediction is correct. The numeric value for the PR has less of a clear interpretation, but models with a higher PR do a better job of predicting when events actually occur.

As shown in figure 8, the model using tensor dependence distance decisively outperforms all the other models. While the AUC-ROC is somewhat higher with the tensor dependence model, the real difference between the measures shines through in the AUC-PR, where the model using this measure performs twice as well as the base model. In contrast, models using other measures of state preferences yield only minimal improvements in prediction over the base model. This is especially relevant because AUC-PR specifically focuses on the difficult task of predicting conflict, compared to the relatively easier task of predicting non-conflict that is rewarded by the AUC-ROC measure. Thus we have reason for confidence in both the usefulness of this measure of state preferences and renewed reason for skepticism in the effect of joint democracy once we control for state preferences.

Figure 8: Assessments of out-of-sample predictive performance using ROC curves and PR curves. AUC statistics are provided as well for both curves.

Conclusion

We use a network methodology to create a new measure of state preferences using both UN Voting data and alliance data. This measure of state preferences demon-

strates face plausibility comparable or superior to existing measures when it comes to capturing the dynamics of a number of notable dyadic relationships. We then attempt to use this measure of state preferences in a predictive model of interstate disputes. In doing so, we find that the measure of preferences has the expected effect (states with similar preferences are less likely to be involved in disputes) and inclusion of our measure leads to the measure of joint democracy becoming indistinguishable from 0. Most importantly, a model of interstate conflict that includes our measure of preferences decisively outperforms models that include both of the most prominent existing measures of preferences.

While this measure of state preferences has yielded leverage in predicting conflict, it should also be useful in answering a number of other questions. One possible use would be to look at the measure of state preferences as an outcome variable, rather than a predictor. We could here look at how preferences change in tandem with leadership change – can we find evidence, for example, that the election of Donald Trump moved the United States’ foreign policy preferences away from the major Western European states and towards Russia’s? We could also see how well this measure of state preferences predicts more collaborative behavior – treaty membership or trade for example.

Finally, this particular method allows us to control for confounding fixed effects. A closer examination of the relationship between democracy, preferences, and conflict could create a measure of the component of state preferences not explained by democracy.¹¹ Similarly, the underlying methodology could be usefully applied to other questions. For example, it could be used to estimate the alignments of elites in non-democratic states or even as a way of combining a number of related variables on

¹¹We look at the entire measure of state preferences because our primary goal here is not to examine this relationship, but to create the most accurate measure of preferences possible.

development or democracy.

Ours is not the first paper to use network analysis to attempt to measure a theoretically important but difficult to observe phenomenon. For example, Moody and White (2003) measured social cohesion by looking at the minimum number of paths connecting each actor in a group (and applied to high school friendship networks and among American businesses. Beardsley et al. (2018) generates a measure of hierarchy by looking at a network of arms transfers and first finding the clusters in each group, then how central an actor is within a given cluster, they go on to use this measure of hierarchy in a downstream model to explain interstate conflict, and similarly Cranmer et al. (2012b) use a measure of multiplex modularity in the network of alliances, trade, and international organization membership to measure “Kantian fractionalization”—the extent to which the international system is fragmented between liberal and illiberal states—which predicts the aggregate level of violence. In general, there has been a dichotomy between latent variable and network measurement models; for example ? criticized the dominant latent variable approach to measuring comorbidity, and argued that a network approach was a superior way to measure the underlying process. Where we differ from this work, however, is that while previous attempts at measurement using networks have relied on descriptive statistics of observed networks, we use a Latent Factor approach which assumes that the network we study is observed with error from the true data-generating process. In cases where the network is observed accurately, and measurement directly flows from this network, our approach is unnecessary and selection of appropriate descriptive statistics should suffice for measurement. However, in many cases, especially in the social sciences, we either observe the true network with error, or we select a network as a proxy for the true process we want to measure.

In addition to helping us to understand state preferences, this sort of technique

could help in measuring phenomenon like social cohesion and hierarchy, or spatial contagion. If one wanted to use a latent network approach to measure social cohesion, one could potentially do this with a Latent Class Model like in ?. Using some measure of social interaction, this model would give the probability that each member is in a particular clique or subgroup in society. We could then look at how many members of society have probability of membership in multiple groups above a certain threshold to see how many individuals bridge different groups, contributing to social cohesion in the way that Moody and White (2003) measured it. In understanding how phenomenon – violence, diseases, new technology to name a few examples – diffuse, many scholars rely on spatial lag models, which require the scholar to specify a spatial weights matrix that indicates how likely a phenomenon is to move from one area to another. Of course, we can never know the true adjacency matrix, and so scholars use a proxy, most often some form of geographic distance. We can (and in the case of political violence have) improved the predictive performance of spatial models by inferring a weights matrix using a latent network approach based on how the phenomenon has diffused in the past.

If a latent network approach is needed, the specific type used should depend on the problem.¹²

¹²For more information on different types of latent networks, see Minhas et al. (2018) or Goldenberg et al. (2010). If your beliefs about the true data-generating process you wish to measure are that it is about membership in disjoint groups, and you have a strong prior on the number of extant groups, a Latent Class Model (see Airoldi et al. (2008)) will clearly measure the probability of membership in these groups, and as Goldenberg et al. (2010) note, it is the easiest model to interpret. However, many phenomena cannot meet the assumption of the Latent Class Model. The Latent Space Model popularized by Handcock et al. (2008) is another model where results are easy to interpret, and requires less assumptions and priors about the data-generating process, but if you wish to measure something related to homophily or stochastic equivalence, this model will not capture those. As Goldenberg et al. (2010)[p.179] note, the latent factor model (which we use) “can capture more connectivity patterns than the latent class and the latent distance models for a given degree of model complexity...There is a price to pay, however. The eigenmodel is the least amenable to interpretation among the three models.” Regardless of the model used, if you are trying to use a latent network model for measurement, and especially if you are using the measure in a downstream model, you need to be aware of and account

for uncertainty. To do this, we suggest running the downstream model repeatedly with different draws from the distribution for the latent space, then combining the models using Rubin (1976)'s rules to obtain an estimate of the effect of your measure that accounts for uncertainty.

References

- Airoldi, E. M., Blei, D. M., Fienberg, S. E., Xing, E. P., 2008. Mixed membership stochastic blockmodels. *Journal of Machine Learning Research* 9 (September), 1981–2014.
- Anderson, C. J., Wasserman, S., Faust, K., 1992. Building stochastic blockmodels. *Social Networks* 14 (1), 137–161.
- Bailey, M. A., Strezhnev, A., Voeten, E., 2017. Estimating dynamic state preferences from United Nations voting data. *Journal of Conflict Resolution* 61 (2), 430–456.
- Beardsley, K., Liu, H., Mucha, P. J., Siegel, D. A., Tellez, J., 2018. Hierarchy and the provision of order in international politics. Tech. rep., Working paper, Duke University.
- Bennett, D. S., Rupert, M. C., June 2003. Comparing measures of political similarity: An empirical comparison of S versus τ_b in the study of international conflict. *Journal of Conflict Resolution* 47 (3), 367–393.
- Braumoeller, B. F., 2008. Systemic politics and the origins of great power conflict. *American Political Science Review* 102 (1), 77–93.
- Bueno de Mesquita, B., 1983. *The War Trap*. Yale University Press, New Haven, CT.
- Bueno De Mesquita, B., Lalman, D., 2008. *War and reason: Domestic and international imperatives*. Yale University Press.
- Carter, D. B., Signorino, C. S., 2010. Back to the future: Modeling time dependence in binary data. *Political Analysis* 18 (3), 271–292.
- Clinton, J., Jackman, S., Rivers, D., May 2004. The statistical analysis of roll call data. *American Political Science Review* 98 (2), 355–370.

- Cranmer, S. J., Desmarais, B. A., Kirkland, J. H., 2012a. Toward a network theory of alliance formation. *International Interactions* 38 (3), 295–324.
- Cranmer, S. J., Desmarais, B. A., Menninga, E. J., 2012b. Complex dependencies in the alliance network. *Conflict Management and Peace Science* 29 (3), 279–313.
- DeRouen, K., Heo, U., 2004. Reward, punishment or inducement? US economic and military aid, 1946–1996. *Defence and Peace Economics* 15 (5), 453–470.
- Farber, H., Gowa, J., 1995. Politics and peace. *International Security* 20 (2), 123–146.
- Fariss, C. J., 2014. Respect for human rights has improved over time: Modeling the changing standard of accountability. *American Political Science Review* 108 (2), 297–318.
- Gallop, M., 2017. More dangerous than dyads: How a third party enables rationalist explanations for war. *Journal of Peace Research*.
- Gartzke, E., 1998. Kant we all just get along? Opportunity, willingness, and the origins of the democratic peace. *American Journal of Political Science* 42 (1), 1–27.
- Gartzke, E., 2000. Preferences and the democratic peace. *International Studies Quarterly* 44, 191–212.
- Gartzke, E., 2007. The capitalist peace. *American Journal of Political Science* 51 (1), 166–191.
- Gibler, D. M., Sarkees, M., March 2004. Measuring alliances: The correlates of war formal interstate alliance data set, 1816–2000. *Journal of Peace Research* 41 (2), 211–222.
- Goldenberg, A., Zheng, A. X., Fienberg, S. E., Airolidi, E. M., et al., 2010. A survey of statistical network models. *Foundations and Trends® in Machine Learning* 2 (2), 129–233.

- Häge, F. M., 2011. Choice or circumstance? Adjusting measures of foreign policy similarity for chance agreement. *Political Analysis*, 287–305.
- Handcock, M. S., Hunter, D. R., Butts, C. T., Goodreau, S. M., Morris, M., 2008. *statnet*: Software tools for the representation, visualization, analysis and simulation of network data. *Journal of Statistical Software* 24 (1), 1548.
- Hoff, P. D., 2005. Bilinear mixed-effects models for dyadic data. *Journal of the American Statistical Association* 100 (4690), 286–295.
- Hoff, P. D., Fosdick, B., Volfovsky, A., Stovel, K., 2013. Likelihoods for fixed rank nomination networks. *Network Science* 1 (3), 253–277.
- Kastner, S. L., 2007. When do conflicting political relations affect international trade? *Journal of Conflict Resolution* 51 (4), 664–688.
- Kinne, B. J., Bunte, J. B., 2018. Guns or money? defense co-operation and bilateral lending as coevolving networks. *British Journal of Political Science*, 1–22.
- König, T., Marbach, M., Osnabrügge, M., 2013. Estimating party positions across countries and time—a dynamic latent variable model for manifesto data. *Political Analysis* 21 (4), 468–491.
- Kydd, A., 2003. Which side are you on? Bias, Credibility, and Mediation. *American Journal of Political Science* 47 (4), 597–611.
- Linzer, D. A., Staton, J. K., 2015. A global measure of judicial independence, 1948–2012. *Journal of Law and Courts* 3 (2), 223–256.
- Martin, A. D., Quinn, K. M., 2002. Dynamic ideal point estimation via markov chain, monte carlo for the U.S. Supreme Court, 1953–1999. *Political Analysis* 10 (2), 134–153.

- Minhas, S., Hoff, P. D., Ward, M. D., 2018. Inferential approaches for network analysis: AMEN for latent factor models. *Political Analysis*.
- Moody, J., White, D. R., 2003. Structural cohesion and embeddedness: A hierarchical concept of social groups. *American sociological review*, 103–127.
- Oneal, J., Russett, B. M., 1999. Is the liberal peace just an artifact of cold war interests? Assessing recent critiques. *International Interactions* 25 (3), 213–241.
- Oneal, J. R., Russett, B. M., 1997. The classical liberals were right: Democracy, interdependence, and conflict, 1950-1985. *International Studies Quarterly* 41 (2), 267–293.
- Poole, K. T., Rosenthal, H., 1985. A spatial model for legislative roll call analysis. *American Journal of Political Science* 29 (2), 357–384.
- Rubin, D. B., 1976. Inference and missing data. *Biometrika* 63 (3), 581–592.
- Signorino, C., Ritter, J., 1999. Tau-b or not tau-b. *International Studies Quarterly* 43 (1), 115–144.
- Singer, J. D., Small, M., 1995. National Military Capabilities Data. *Correlates of War Project*, Ann Arbor, MI.
- Snijders, T. A., Nowicki, K., 1997. Estimation and prediction for stochastic blockmodels for graphs with latent block structure. *Journal of Classification* 14 (1), 75–100.
- Stone, R. W., 2004. The political economy of IMF lending in Africa. *American Political Science Review* 98 (4), 577–591.
- Voeten, E., 2013. Data and analyses of voting in the United Nations. Routledge, Ch. 4.
- Warren, T. C., 2010. The geometry of security: Modeling interstate alliances as evolving networks. *Journal of Peace Research* 47 (6), 697–709.

- Warren, T. C., 2016. Modeling the co-evolution of international and domestic institutions: Alliances, democracy, and the complex path to peace. *Journal of Peace Research* 53, tbd.
- Wasserman, S., Faust, K., 1994. *Social Network Analysis: Methods and Applications*. Cambridge University Press, Cambridge.
- Wolford, S., 2014. Showing restraint, signaling resolve: Coalitions, cooperation, and crisis bargaining. *American Journal of Political Science* 58 (1), 144–156.
- Xavier Amatriain, Alejandro Jaimes, N. O., Pujo, J. M., 2015. Data mining methods for recommender systems. In: *Recommender Systems Handbook*. Springer, pp. 227–262.