Predicting Violence: Network Dynamics in Nigeria

Abstract

Canonical studies of civil war examine the conditions that favor conflict across the globe. While predicting the onset of intrastate war has seen great progress, analysis on the occurrence of battles between actors remains obscure. Despite growing interest in the logic of rebel group fighting, empirical assessment of these relationships is underdeveloped. We unmask the interdependent dynamics of civil conflict by conceptualizing actors and battles as nodes and ties in a network. We examine how these relationships change over time and how their evolution in one time period enables precise prediction of conflict in future periods. Our network approach yields theoretical implications for understanding how the entrance of particularly aggressive actors can decisively alter the trajectory of civil conflict.

Word Count: 9822

Introduction

Since 1946 more than half of the countries in the international system have experienced a civil war (Gleditsch et al., 2002), and although wars between countries have waned in recent years, there are a growing number of intrastate conflicts (Pettersson & Wallensteen, 2015). These types of conflicts have proven to be costly – civil wars in countries as diverse as the Ukraine, Syria, and Nigeria have been some of the most deadly conflicts of the post-Cold War period. The prevalence and costliness of such conflicts have resulted in the development of a rich literature on the causes and consequences of civil war.[1] In fact, Fearon's influential 2003 article exploring the conditions under which countries are most at risk for civil war has been cited over 6,000 times,[2] underscoring the value placed on understanding when, and why, conflicts begin. Such research agendas encompass the study of conflict onset and conflict prediction, and have been formative to scholarship on conflict processes more broadly, leading to investigation of not just how conflicts begin, but how they develop after initiation. Accordingly, researchers are now concerned with a wide variety of analyses on armed non-state actors' attributes, coordination, and relationships (Cunningham, 2006; Bakke et al., 2012; Findley & Rudloff, 2012; Akcinaroglu, 2012)

While these fine-grained approaches to the study of intrastate conflict signal a productive shift in the literature, most of these studies remain rooted in a dyadic research design. Importantly, this design has allowed for researchers to move away from state-centric analysis. Building on the work of Cunningham et al. (2009), Akcinaroglu (2012, p. 880) notes, "Existence of multiple rebel groups means we can no longer understand civil wars with a sole focus on state attributes. In fact, the government's strategies lead-

[1] For example, see Hegre et al. (2001); Collier et al. (2004); Salehyan (2008); Lacina (2014).

[2] According to Google Scholar at the time of this writing.

ing to victory, defeat, or continuation of war can only be understood in relation to the rebel group/groups it is fighting." Dyadic research designs have allowed for a closer examination of cases in which the government fights many different armed actors, rather than a single insurgent movement. Although such studies make significant theoretical and empirical contributions they are limited by the assumption of independence between dyads that is explicit in empirical dyadic analyses. In turn, current research has yet to provide full consideration of the interconnected multi-actor dynamics that characterize recent examples of intrastate conflicts; a notable restriction given that by 2003 over 30% of ongoing civil wars involved multiple dyads in conflict with one another (Harbom et al., 2008).

The contribution of this study is to bridge this disconnect and model civil conflict processes in a way that is reflective of their empirical form. Namely, we argue that intrastate conflicts are best understood as a single complex system composed of multiple actors in conflict with one another rather than as a set of independent, dyadic events. Accordingly, we conceptualize armed actors and battles as nodes and linkages in a network wherein the actors and their relationships change over time. This framework allows us to estimate the relationships that evolve between actors in a conflict network in order to predict the occurrence of battles between groups within the system. In doing so, our paper is the first to apply an empirical-based network methodology to the study of civil conflict evolution. Our network approach, which we employ to investigate civil conflict in Nigeria, significantly outperforms traditional dyad-group approaches at predicting the occurrence of battles between groups in an out-of-sample context. Finally, our findings yield theoretical implications for the study of civilian victimization and conflict occurrence, as well as insights about how the entrance of particularly aggressive actors can decisively alter the trajectory of conflict.

Network Patterns in an Intrastate Conflict Context

This study investigates how relationships between armed actors evolve over time, and how the structure of relationships in one time period influences the occurrence of violence between armed actors in a future period. To do this, we consider how interdependence across actors, changes in actor composition over time, and actor-level attributes predict violence in the network. In this section, our focus is two-fold. First we explain why intrastate conflict represents a network system and how network characteristics can reveal emergent patterns between armed actors over time. Understanding and taking these patterns into account can enhance our understanding of what violence might look like between actors in the future. Second, we discuss how the entrance of particularly aggressive actors can alter the trajectory of conflict in the network. We then turn to actor-level attributes in our modeling and data sections.

In the field of interstate conflict studies, network approaches are gaining more attention.[3] Yet, these approaches have seen little usage in the study of civil wars even though the dynamics underlying many modern day intrastate conflicts involve interdependent interactions between multiple groups. The reason for studying dyadic interactions using a network-based approach is to acknowledge the possibility that the actions of actors in a system are contingent on one another. A significant amount of work has shown that by moving away from the assumption that interactions within a system are independent, we can better account for the underlying process that generates relational data as well as make more accurate inferences.[4]

Below we show a stylized version of our dependent variable of interest: the oc-

[3]For example, see Kinne (2012); Metternich et al. (2015); Minhas et al. (2016b). Also see the 2016 special issues on network based approaches in International Studies Quarterly and the Journal of Peace Research.

[4]For example, see Snijders (1996); Dorff & Ward (2013); Erikson et al. (2014).

currence of conflict between actors in a network. Analyzing conflictual relations in a network context enables us to use information about dependencies between actors, rather than assuming these dependencies do not exist. A stylized representation of a dynamic network is shown below. In Figure 1, the nodes represent armed actors and the ties represent battles between armed actors. The graphic highlights several important features relevant to our study. Each panel represents a different phase, or new year, of the conflict. In each year, new armed actors might enter or exit the conflict (i.e., an armed group could stop fighting or dissolve). In the visualization, new entrants are shaded in grey. Second, new ties can form between actors in any period, but ties can also disappear over time. For example, moving from panel 1 to panel 2 we can see that some of the linkages in the first time period no longer appear in the second time period, which indicates that two actors that fought in the first time period are not fighting in the second time period. These features demonstrate the type of information that a network conceptualization of conflict development can provide.

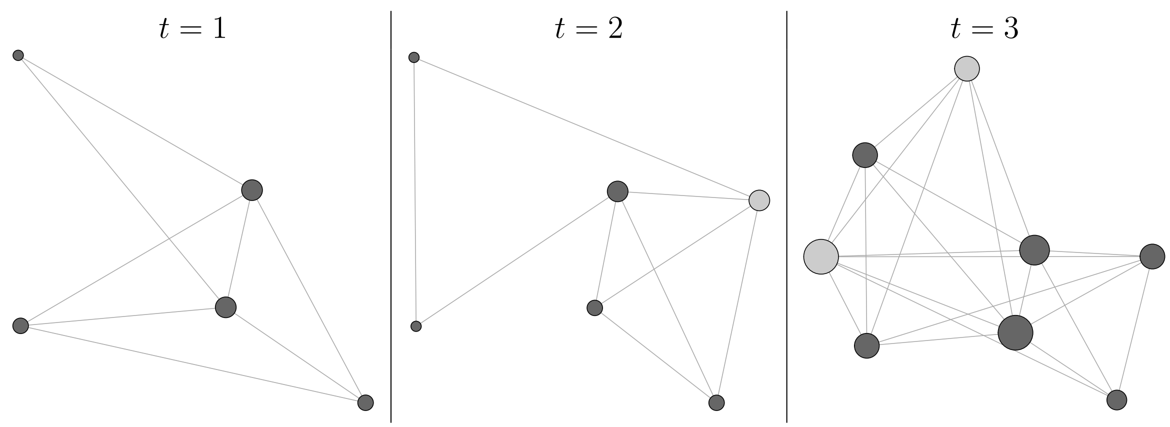

Table 1: Stylized network graphic depicting changing connections between actors overtime.

The second iteration of this visual example adds additional layers of information useful for understanding the development of conflict: directed linkages, reciprocity, and community clusters. By utilizing information about the direction of relationships, i.e., capturing which actors initiate battles with other actors, we can then reveal reciprocal relationships that evolve over time. The linked arrows in Figure 2 demonstrates these relational dependencies. Further, we also observe a cluster of actors, or a community of armed groups, in the bottom right corner of the network – these nodes are designated by squares instead of circles. This cluster's density increases over time and could suggest the existence of a conflict community wherein a shared latent attribute of each actor drives their propensity towards violence with one another.

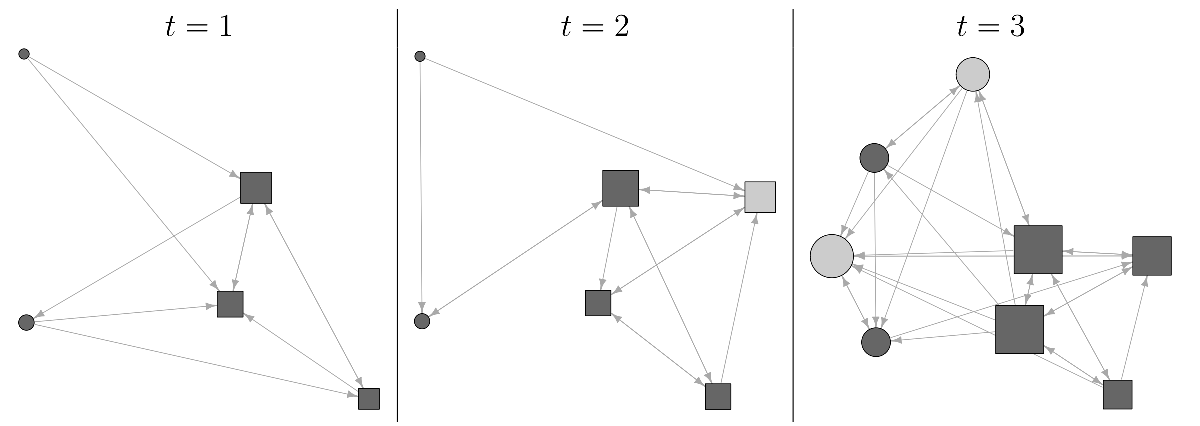

Table 2: Stylized network graphic depicting changing connections between actors overtime, directed linkages, and community clusters.

Drawing from the literature on rebel groups and non-state actors who operate in civil conflict environments, we focus on three forms of dependencies drive the occurrence of violence between actors: actor-level effects such as actor centrality, second-order effects such as reciprocity, and third-order effects that lead certain actors to form clusters of conflict.

Actor Centrality

We often observe heterogeneous actor centrality in social systems, revealing that some actors are more likely to initiate or receive ties (in this case, battles). Heterogeneity across actor centrality provides us with information on which set of events led to the creation of the observed network (Barabási & Réka, 1999). Heterogeneity *across* actors' activity in a network produces homogeneous ties *between* actors. For example, Weinstein (2007) argues that rebel groups with more ideologically motivated followers are thought to be less violent than groups whose members are motivated by greed. The motivation of group members, then, is a latent, unobserved attribute that accounts for why one rebel group is more prone to attack other groups. Thus, the relationships between the set of dyads $\{i \rightarrow j,\, i \rightarrow k,\, i \rightarrow l\}$ will be more similar to each other than they are to other interactions in the system, such as $m \rightarrow n$. This occurs because each of those interactions involve actor i as the sender of the event (Kenny & La Voie, 1984), and sender i possesses some latent attribute that makes the group more conflictual.

Another straightforward example of why this type of dependence emerges in intrastate conflict networks involves government actors. Government actors likely have interests in asserting their control across the country. Specifically, in a scenario where rebel groups $\{j, k, l\}$ operate in different parts of a country, we would expect government actor i to be more likely to initiate conflict with those rebel groups than an actor with non-competing interests. This logic naturally extends to non-government groups. For example, a rebel group may act upon ambitions of establishing control over broad swaths of the country, and, as a result, become a central initiator of conflicts in the network. For similar reasons we may observe heterogeneity in how likely actors are to receive conflict events, and, relatedly, find that actors initiating more battles are also likely to appear on the receiving end of violent attacks. Each of these processes creates dependence across observations that involve the same actor.

Reciprocity

Reciprocity is the notion that the interaction between $i \rightarrow j$ and $j \rightarrow i$ are dependent upon one another. The concept of reciprocity has deep roots in the study of relations between states (Richardson, 1960; Keohane, 1989), and it is no less relevant for studying relations between actors in an intrastate conflict context. Reciprocity captures the intuition that if one actor is, for example, particularly aggressive towards another actor in the network, the recipient of such aggression will "respond in kind" and exhibit aggressive behavior in return. In addition to this direct relationship, where attacks lead to retaliation, we might also see reciprocity when a common factor is both causing i to attack j and j to attack i. For example, literature on competition between armed groups suggests that when two groups are competing over scarce resources, (Fjelde & Nilsson, 2012), we should expect each group to attack the other and thus attacks by i on j are likely to be paired with attacks by j on i. We expect that in the context of battles, reciprocity is likely to play a role in motivating actors to respond to attacks by opponent groups.

Homophily & Stochastic Equivalence

Apart from these nodal and dyadic patterns that commonly arise in social systems, networks often exhibit patterns of dependence involving multiple actors. These types of patterns can broadly be summarized in terms of the concepts of stochastic equivalence and homophily (Wasserman & Faust, 1994). Stochastic equivalence may manifest in a network in which actors cluster into groups, and the group determines an actor's patterns of relations with other groups. More concretely, a pair of actors ij are stochastically equivalent if the probability of i relating to, and being related to, by every other actor is the same as the probability for j (Anderson et al., 1992). Consider a case in which actors nest in groups based on, for example, a shared but unobserved ideology: $\mathbf{a} = \{i, j\}$ and $\mathbf{b} = \{k, l\}$. Here we may likely find that the relations of i towards k and l is similar to how j behaves towards those actors, and this induces a dependence in the relations of actors within the system. Specifically, the behavior of actors within the conflict system may be a function of a latent community to which they belong. A large body of literature in conflict studies motivates this reasoning, since ethnicity and religion are often tied to the behavior of actors within their community context (Reynal-Querol, 2002; Denny & Walter, 2014). In the case of the conflict in Nigeria, we might see groups that share a common religion attacking a group whose followers follow a different religion, or groups with a given tribal identity coming into conflict with a group from a rival tribe.

Homophily is a type of dependence pattern involving multiple actors, that often leads to the emergence of cliques within social systems (Shalizi & Thomas, 2011).[5] In the intrastate setting, conflict cliques may emerge when multiple actors are competing for control over the same territory. Typically, conflict scholars attempt to account for this by explicitly identifying a territorial objective, but in some cases this may not be observed. Using a network based approach, we can examine the conflictual patterns of actors to parse out this underlying geographic structure. Thus a latent shared attribute that explains the occurrence of homophily in relational data may be geographical in nature.[6]

The principal idea underlying each of the dependence patterns discussed above is that the actions, or inactions, of any actor, are taking place within an interdependent system. The interdependencies result because of latent attributes possessed by a single rebel group or shared by many groups. Attempting to estimate the presence of these interdependencies allows us to better understand whether there is latent structure to an intrastate conflict system. Further by incorporating the estimation of these

[5]A clique in a network is a structure in which three actors $\{i, j, k\}$ each interact with one another to form a triangle.

[6]Interestingly, Wong et al. (2006) show that homophily in social networks is frequently determined by geographical space.

network effects into a modeling framework, we reduce the likelihood of making faulty inferences and improve our ability to forecast conflict.

Entrance of Key Players

An often undiscussed concept in both networks and conflicts relates to the ripple effects that the entrance of a key player can have on a system. The entry of a particularly active or violent group could lead to an increase in conflict through a number of mechanisms. As Kathman & Wood (2015, p. 168) have argued "conflict systems are fluid, and competition varies in response to the arrival or exit of violent combat groups...", thus the entrance of certain groups could increase levels of conflict. One possible mechanism through which violence might increase is through a process of rebel group outbidding, where in order for different groups to get support and resources from the population, they need to prove their conviction and capabilities through increasingly violent acts (Crenshaw, 1981; Bloom, 2004; Nemeth, 2014). The entrance of additional groups might also make ending conflict harder, as Cunningham et al. (2009) notes, because the entrance of an additional (dominant) group makes de-escalating conflict more difficult as it increases the number of veto players in peace negotiations.

We suggest that the entrance of key players affects violence because of their strategic relationship with the government. The arrival of a new challenger group that successfully challenges the government may shift perceptions of the government as weak or vulnerable, leading other groups to more willingly challenge the government, as shown in Walter (2006) or other armed groups, as studied by Fjelde & Nilsson (2012). This particular dynamic also gives the government incentives to react with violence, rather than negotiation, to deter the development of future challengers. Fjelde & Nilsson (2012, p. 613) find "states with weak coercive power create opportunities for non-state actors to engage in armed struggle against each other."

Boko Haram is an excellent example of this effect in Nigeria. Boko Haram gained such success that in mid-2014 they were able to effectively control swathes of territory in and around their home state of Borneo. The rise of Boko Haram and the challenges it posed to Nigeria's security apparatus provides incentives for other groups to increase their violent behavior, even in dyads that do not involve Boko Haram.

The Conflict[s] in Nigeria

Over the past decade and a half, the people of Nigeria have experienced violence through ethnic and religious group tensions, organized insurgency, and government repression. In this section, we describe Nigeria's conflict landscape and summarize the nature of warfare between multiple armed actors, government agents, and civilians. This overview highlights the complex nature of Nigeria's conflict(s) and the sustained toll on human life that results from such hostilities.

The year 2000 marks the beginning of a critically tense transition period in Nigeria. Following the 1999 presidential election of Olusegun Obasanjo, Nigeria's 15-year long military dictatorship ends. Meanwhile, a resurgent Islamic political movement consolidates influence in Nigeria's predominantly Muslim northern states. Sharia Law, a penal code in effect for hundreds of years in Nigeria but disbanded in 1960, is re-implemented following the 2000 elections. The implementation of Sharia law then leads to numerous riots and clashes, largely between Muslims and Christians (one clash, in Nigeria's second largest city of Kano, ends with over 100 dead in October of 2001).[7]

President Obasanjo faces many more violent episodes during his first term. As Christian-Muslim disputes continue in the Northern regions, a tribal war foments in the eastern-central state of Benue. Thousands of people are forced to flee the area,

[7] See `http://articles.latimes.com/2001/oct/16/news/mn-57819`.

and troops reportedly target unarmed civilians in retaliation for the abduction of nearly 20 soldiers. Continuing Christian-Muslim clashes in the north mark Obasanjo's second term (beginning in 2003) as well as deepening violence in the Niger Delta where militants vie for control of oil production centers. Fighting in the Okere district of Warri occurs between Itsekiris and Urhobos over contentious primary elections held for the Delta South district. Reports state that government forces attempt to suppress violence through execution-style killings. The Nigerian Red Cross reports that more than 6,000 people are displaced as result of the fighting.[8]

Mohammed Yusuf emerges in the early 2000s as the radical leader of the religious extremist group, the Boko Haram.[9] Yusuf spreads an anti-state, anti-elite ideology couched in Sunni Muslim teachings. Conflict in the northern region of Nigeria continues along religious lines, and by 2008 Boko Haram has grown increasingly well organized, operating as a mini-state with institutions, a welfare system, and religious police. Clashes between Boko Haram and state security forces gain momentum as the 2009 uprising begins in the state of Bauchi and spreads to Borno, Yobe, and Kano. These battles are the first sustained conflict between Boko Haram and the government, and culminates in the death of scores of police officers, more than 700 members of Boko Haram, and the capture of Mohammed Yusuf.[10]

Vice President Goodluck Jonathan succeeds to the presidency following the death of his predecessor in 2010. The death of religious leader Mohammed Yusuf, killed by police in a raid, leads to an intensification of tactics by Boko Haram, including greater

[8] See `https://www.hrw.org/reports/2003/nigeria1103/3.htm` for more details.

[9] There is evidence that the early leadership of the movement formed around 2000 or 2002, though in many reports the fully armed, organized, radical group forms closer to the year 2008 following a clash with government forces. See Walker (2016) for details.

[10] See `http://edition.cnn.com/2014/06/09/world/boko-haram-fast-facts/`.

violence against civilians. Demonstratively, in December of 2010, Boko Haram bombs kill an estimated 80 people in the central city of Jos. This event sparks a response from opposing Christians leading to roughly 200 more deaths.[11] Attacks on civilians continue and in April of 2014 Boko Haram chooses a target that sparks international condemnation: 276 Chibok schoolgirls. Though this is not the first time that civilians organize against the insurgency's violent tactics, high levels of protest and vigilantism ensue following the children's kidnapping.

[11] See http://www.aljazeera.com/news/africa/2013/09/201397155225146644.html.

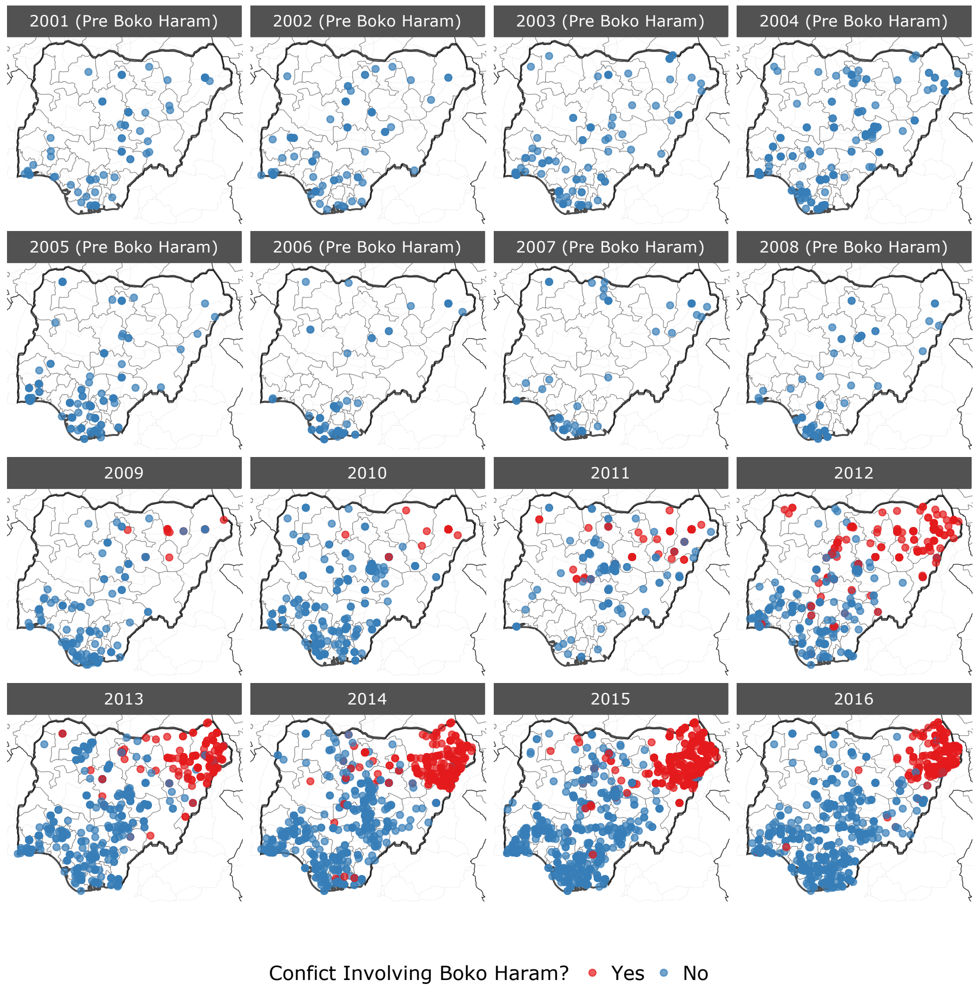

Figure 1: Spatial distribution of conflict in Nigeria from 2001 to 2016. Points colored in blue represent conflict events between sets of actors not including Boko Haram, and points in red events that do involve Boko Haram as either the initiator or target of conflict.

Violence against civilians, both on behalf of insurgent and government actors, continues to displace and disappear civilians. In March 2014, the Director-General of the Nigerian National Emergency Management Agency (NEMA) reported that more than 250,000 people were displaced as a result of the fighting in north-eastern Nigeria. Battles between Boko Haram, local militant groups, and security forces continue today. While Boko Haram has evolved as perhaps the most well known armed insurgent movement in Nigeria, there are major ethnic militias such as the Urhobo Ethnic Militia that engage in localized battles and groups such as MEND which battle with other armed actors and the government over natural resources. MEND is one of the largest militant groups in the Niger Delta region. The militant organization is expressly concerned with the public and private production of oil in the local area. Despite these important armed actors, Boko Haram's entrance into the conflict network in 2009 has had unique consequences on the stability of the entire region.

Figure 1 shows the spatial distribution of violence in Nigeria from 2001 to 2016, points in red represent Armed Conflict Location and Event Data Project (ACLED) battle events in which Boko Haram is involved (either as the initiator or target) while points in blue designate events that do not involve Boko Haram. What becomes apparent is that Boko Haram's entrance into the system corresponds to an increase in both the level and spread of conflict. Additionally, the increase in conflict is not attributable to just the violent actions of Boko Haram alone. Boko Haram's entrance to the system corresponds to other actors engaging in more conflictual events. Figure 2 further highlights the complexity of Nigeria's conflict and the relevance of relationships between actors: the Fulani Militia is central to the network, as is (understandably) the Nigerian police and military forces. Reciprocal conflictual relations are also visible, such as between the Boko Haram and the Military.

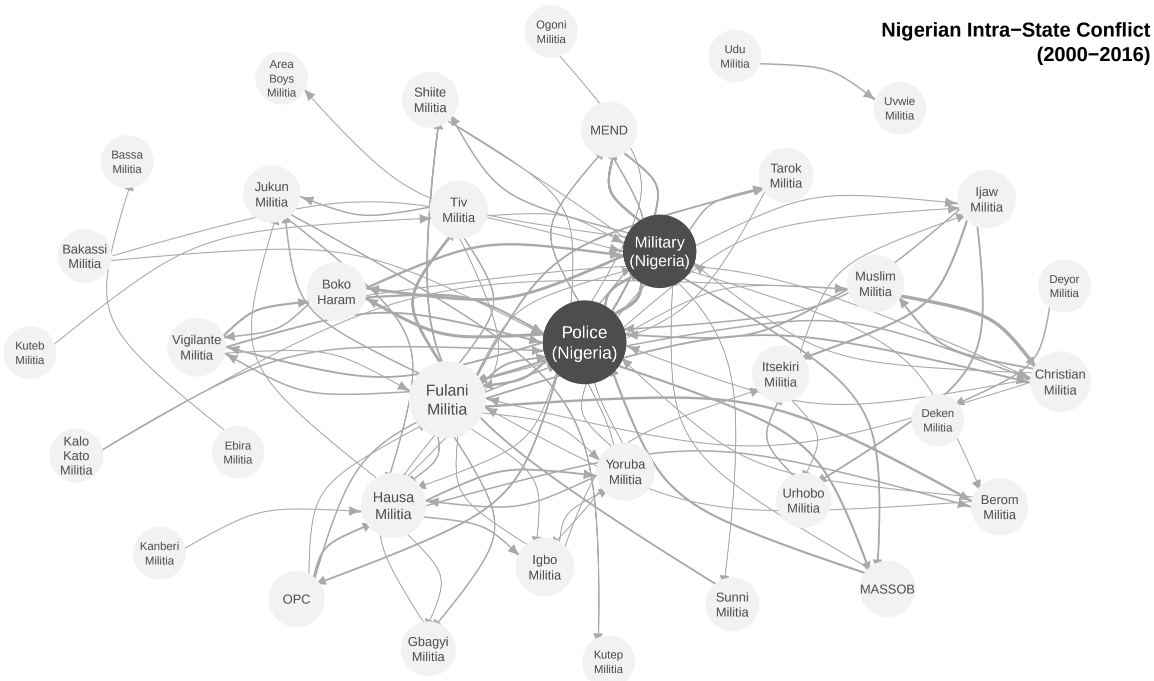

Figure 2: Network graph depicting battles between actors for the entire 2000-2016 time period. Pairs of actors experiencing more conflict have thicker links. All ties are directed, as indicated by arrows.

Data

To study intrastate conflict patterns in Nigeria we utilize the ACLED data initiated by Raleigh et al. (2010). This dataset records armed conflict and protest events in over 60 developing countries. ACLED's *battles* data is used to generate our measure of conflict where $y_{ij,t} = 1$ indicates that a conflict occurred when actor i attacked actor j at time t ($y_{ij,t} = 0$ if no conflict occurred).[12,13] We focus only on armed groups that are engaged

[12]In the appendix, we show results when only including battles from ACLED that resulted in at least one battle death. The results when using a fatality threshold and not are similar, thus we focus on the latter for the remainder of this paper.

[13]Some scholars have noted that the ACLED conflict data should be understood as defining symmetric events, since who started a particular battle can at times be difficult to entangle. In the appendix, we show that the results presented in the paper are robust to utilizing a directed or undirected formulation.

in battles for at least 5 years during the 2000-2016 period, which results in a total of 37 armed groups.

The groups included in our battle-data analysis are politically violent actors as defined by the ACLED codebook. Such actors include rebels, militias, and ethnic groups.[14] Additionally, we use secondary resources to confirm the existence of each actor in the data. Overall, the data capture several key types of actors: government armed forces (military and police), insurgent groups (Boko Haram), and ethnic militias (including the Yoruba and Fulani Militias).

The ACLED data also allows us to explore civilians' role in shaping conflict within the Nigerian system. Building on the research agenda motivated by macro-level studies (Gurr, 1970; Opp, 1988; Tarrow, 1994; Tucker, 2007; Chenoweth & Stephan, 2011), we investigate the link between civilian mobilization and violence between armed actors at the local level by creating a count of the number of protests/riots led by civilians against a given actor at time $t-1$. While there is a robust debate over what causes civilian victimization,[15] discussion of the consequences of civilian victimization, particularly as they relate to the potential for future conflict has been more limited (Hultman, 2007; Raleigh, 2012). Berman & Matanock (2015) argue that when rebels groups [governments] kill civilians, other civilians are more [less] likely to share information with the government: information allows the government to carry out attacks against insurgents, and the lack of information makes insurgent attacks less likely. Condra & Shapiro (2012) find support for this relationship in Iraq.[16] To account for both these dynamics

[14] For detailed definitions of these actors, see the on-line ACLED Africa data codebook maintained at `www.acleddata.com`.

[15] For example, see Valentino et al. (2004); Humphreys & Weinstein (2006); Downes (2006); Kalyvas (2006); Idean Salehyan & Wood (2015); Prorok & Appel (2014).

[16] Lyall (2009), however, finds a contrary effect and demonstrates that indiscriminate violence against civilians in Chechnya is associated with a lower risk of rebel attacks, possibly because these attacks

we create a count of the number of violent actions that an actor committed against civilians at time $t-1$ and nodal covariates to explain both the probability of an actor sending and receiving a conflictual tie.

Additionally, we attempt to account for spatial effects in modeling the likelihood of conflict between a particular pair of actors. First, we add a measure of how dispersed a rebel group's activity is across the country. Armed groups whose actions are more dispersed across Nigeria are more likely to come into conflictual contact with others. We include a "Geographic Spread" variable as both a sender and receiver covariate.[17] Second, a notable literature has developed explicating the mechanisms through which civil conflict can diffuse between countries.[18] To account for this effect, we use the ACLED dataset to count the number of conflict events occurring within Nigeria's contiguous neighbors.

We add three other covariates to the model. First, we create a control for whether both actors are a part of the government. There are only two actors within the 37 that we study in the Nigerian system that would fall under this classification, namely, the Police and Military. We include this control to account for the fact that the probability of an interaction between these two is minimal relative to the other potential dyads in this system.[19] Second, we include a binary variable that takes on the value of one if the following year is an election year and zero otherwise. This is to account for the fact that elections in Nigeria are rarely waged peacefully in the ballot box, but instead are typ-

deprive insurgents of resources.

[17]For example, to model the spread of group i in period t, we calculate this by taking all the events involving group i in the 3 years prior to t and estimate the variance.

[18]For example, see Starr & Most (1983); Salehyan (2009); Salehyan & Gleditsch (2007); Metternich et al. (2015); Braithwaite (2010); Beardsley (2011).

[19]An alternative approach would be to include sender and receiver level government indicator variables, our results are similar when using that approach.

ically followed or preceded by episodes of conflict. Most importantly, to examine the affect that Boko Haram's entrance has had on the level of conflict in this system we include a binary variable that takes on the value of one after the Boko Haram insurgency has begun (in 2009) and zero beforehand.[20]

Modeling Approach

To estimate conflict in this system in a way that explicitly models interdependencies between actors, we rely on a network based approach that combines the social relations regression model (SRRM) – for details on this model see Li & Loken (2002); Dorff & Minhas (2017) – and the latent factor model (LFM). Together the SRRM provides a set of additive effects that we can use to capture first and second order dependencies, and the latent factor model provides a set of multiplicative effects that we use to model third order dependencies (Hoff, 2008; Hoff et al., 2013; Minhas et al., 2016a). This estimator is referred to as the additive and multiplicative effects (AME) model:

$$
\begin{aligned}
&y_{ij,t} = g(\theta_{ij,t}) \\
&\quad \theta_{ij,t} = \boldsymbol{\beta}_d^\top \mathbf{X}_{ij,t} + \boldsymbol{\beta}_s^\top \mathbf{X}_{i,t} + \boldsymbol{\beta}_r^\top \mathbf{X}_{j,t} + e_{ij,t} \\
&\quad e_{ij,t} = a_i + b_j + \epsilon_{ij} + \alpha(\mathbf{u}_i, \mathbf{v}_j) \text{ , where} \\
&\qquad \alpha(\mathbf{u}_i, \mathbf{v}_j) = \mathbf{u}_i^\top \mathbf{D} \mathbf{v}_j = \sum_{k \in K} d_k u_{ik} v_{jk}
\end{aligned}
\tag{1}
$$

where $y_{ij,t}$ represents whether or not conflict occurred between actor i (the sender)

[20] We choose this date both because 2009 is considered the beginning of the Boko Haram insurgency in most media reports, despite their founding in 2002, because 2009 features the first battle including Boko Haram in the ACLED Dataset.

and actor j (the receiver) at time t. To model our binary dependent variable we employ a latent variable representation of a probit regression framework, in which we model a latent variable, θ_{ij}, using a set of time varying dyadic ($\boldsymbol{\beta}_d^\top \mathbf{X}_{ij,t}$), sender ($\boldsymbol{\beta}_s^\top \mathbf{X}_{i,t}$), and receiver covariates ($\boldsymbol{\beta}_r^\top \mathbf{X}_{j,t}$).

To be able to assess the effect that the entrance of actors such as Boko Haram have on the Nigerian conflict network, we extend the AME framework to handle networks in which the composition of actors change over time.[21] Typically, when employing a latent variable framework to longitudinal networks one assumes that the composition of actors is uniform across time (Sarkar & Moore, 2005; Ward et al., 2012; Sewell & Chen, 2015), and if the composition is not truly uniform the authors arbitrarily make it so by choosing a select group of actors to study. This is obviously problematic in general and especially in the case of intrastate conflict as rebel groups often emerge and dissolve – meaning that in time t we may have actors $\{i, j, k\}$ in the network, and in time $t+1$ actors $\{i, j, k, l\}$ or just a different set of actors $\{i, j, l\}$. A change in composition can occur in two ways: 1) an actor dissolves after some time, 2) an actor enters the network after some time.[22]

To account for these types of changes to a network, we adjust the estimation procedure of the AME model such that actor observations only contribute to the likelihood of the model once they enter into the network. This procedure is analogous to that adopted by Huisman & Snijders (2003) for the stochastic actor oriented model (SAOM).

[21]This extension is one reason why we choose not to use the Exponential Random Graph Model (ERGM). The ERGM framework requires that the set of actors remain constant over time, and so using that framework would require us to either ignore important actors in the network, or treat them as if they were present and non-violent for long periods of time.

[22]Theoretically, an actor can also dissolve and then reappear in the network, but this does not occur in the particular context of the Nigerian conflict system between 2000 and 2016.

They limit the set of actors who can change their outgoing relations (or have incoming relations changed with) to only those that are part of an active set at time t. We implement a process for the AME framework in which composition changes are modeled as exogenous events such that actors are allowed to enter and leave the network at fixed time points.[23] The utility of this approach is that the baseline probability of a tie shifts in accordance with the number of active actors in the network at time t and the time-varying covariates already included in the model.

The a_i and b_j in Equation 1 represent sender and receiver random effects that we incorporate from the SRRM framework:

$$
\begin{aligned}
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{ab}) \\
\{(\epsilon_{ij}, \epsilon_{ji}) : \; i \neq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{\epsilon}), \text{ where} \\
\Sigma_{ab} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} & \qquad \Sigma_{\epsilon} = \sigma_{\epsilon}^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}
\end{aligned}
\tag{2}
$$

The interpretation of these parameters is straightforward. The sender and receiver random effects are modeled jointly from a multivariate normal distribution to account for correlation in how active an actor is in sending and receiving ties. Heterogeneity in the the sender and receiver effects is captured by σ_a^2 and σ_b^2, respectively, and σ_{ab} de-

[23]This is straightforward to do in the context of AME mostly because this is a fully Bayesian model. Since the AME is estimated in a fully Bayesian way, we are able to incorporate new nodes in the estimation process by essentially setting all their edges at the time point in which they were not active as missing. By doing so we implicitly make an assumption that those observations are missing at random and thus we can integrate over any contribution they have to the likelihood, which means that edges involving actors not active at a particular time point will not contribute to the inference on any of the parameters.

scribes the linear relationship between these two effects (i.e., whether actors who send [receive] a lot of ties also receive [send] a lot of ties). Beyond these first-order dependencies, second-order dependencies are described by σ_ϵ^2 and a within dyad correlation, or reciprocity, parameter ρ.[24]

While the additive effects from the SRRM can deal with first (differing levels of activity across actors) and second order interdependencies (reciprocity), the multiplicative effects are used to deal with third order dependencies. Specifically, this multiplicative effect allows us to model homophily – the tendency of actors with similar characteristics to form strong relationships than those with differing characteristics – and stochastic equivalence, the possibility that two actors i and j will have similar relationships with every other actor in the network. An AME model accounts for these third order effects using the multiplicative term: $\alpha(\mathbf{u}_i, \mathbf{v}_j) = \mathbf{u}_i^\top \mathbf{D} \mathbf{v}_j$. This model posits a latent vector of characteristics $\mathbf{u_i}$ and $\mathbf{v_j}$ for each sender i and receiver j. The similarity or dissimilarity of these vectors will then influence the likelihood of activity, and provides a representation of third order interdependencies (Minhas et al., 2016a).

The representation of third order interdepedencies is accomplished by a process similar to computing the singular value decomposition (SVD) of the observed network. When taking the SVD we factorize our observed conflict network into the product of three matrices: $\mathbf{U}, \mathbf{D},$ and $, \mathbf{V}$.[25] This provides us with a low-dimensional representation of our original conflict network in terms of third order dependence patterns arising in

[24]Note that the latent parameters being estimated are not indexed by t, which is to indicate that they represent the average value over time for a particular actor. The interpretation of a_i, for instance, is the average sender effect across the sample timeframe after accounting for the exogenous covariates.

[25]The benefit of using an SVD to factorize our observed network is that it is guaranteed to provide the optimal matrix approximation, given K, of the observed network.

the network.[26] Specifically, values in **U** provide a representation of how stochastically equivalent actors are as senders in a network, or more simply put how similar actors are in terms of who they are initiating battles with. For instance, $\hat{\mathbf{u}}_i \approx \hat{\mathbf{u}}_j$ would indicate that actor i and j initiate battles with similar third parties. **V** provide a similar representation but from the perspective of how similar actors are as receivers. The values in **D**, a diagonal matrix, represent levels of homophily in the network.[27] Parameter estimation in the AME takes place by sampling from the posterior distribution of the full conditionals – further details on the estimation procedure can be found in the Appendix.

Results

Parameter Estimates

We report the results for the sender, receiver, and dyadic covariates included in the AME model in figure 3.[28] First, we find little evidence to support the argument that conflict in neighboring countries drives violence between actors in the Nigerian conflict system. We also do not find that actors become particularly more violent during election years. We do, however, find significant evidence for the argument that actors operating across a larger region of Nigeria are likely to come into more conflict with other actors.

[26]The dimensions of **U** and **V** are $n \times K$ and **D** is a $K \times K$ diagonal matrix.

[27]Unlike traditional SVD, in the latent factor model, the singular values are not restricted to be positive, thus allowing us to account for both the presence and absence of homophily.

[28]Excluded from these results is the dyadic variable for "both are government actors." This is included and has a negative effect in both models, which simply indicates that government actors are unlikely to fight one another. Trace plots for each of these parameter estimates can be found in figure A1 of the Appendix.

We see a non-trivial relationship between violence against civilians and participation in a greater number of battles. Targeting civilians is linked both to a higher likelihood of engaging in violence against other military groups, and a higher likelihood of being targeted by other military groups.[29] Moving from the 2.5%ile of violence against civilians to the 97.5%ile (moving from no violence to 17 attacks), while all other variables are held at their measure of central tendency[30], increases the risk of starting a battle by the perpetrator by a factor of about 1.58, and the risk of being targeted for a battle by about 1.55. We cannot causally say that these are the result of violence against civilians, but the fact that the relationship holds even in a mixed-effects model (and one that accounts for network dependencies) is suggestive. Further, anecdotal evidence maintains a similar story. For example, the Óodua People's Congress (OPC), an organization active in the southwest of Nigeria organized to protect the Yoruba ethnic group, is known to engage in two primary modes of conflict: large-scale ethnic battles that directly result in mass casualties to both rivals and civilians, and smaller-scale episodes wherein the OPC targets the civilian base of rival groups.

[29]This effect holds even if we leave government actors out of the network or add in binary sender and receiver controls for government actors. See the appendix for details.

[30]We held whether the dyad was in the Post-Boko Haram period, the presence of neighboring conflicts, whether it was a governmental dyad, and whether it was an election year at their median, all other variables, as well as the random and multiplicative effects at their mean level.

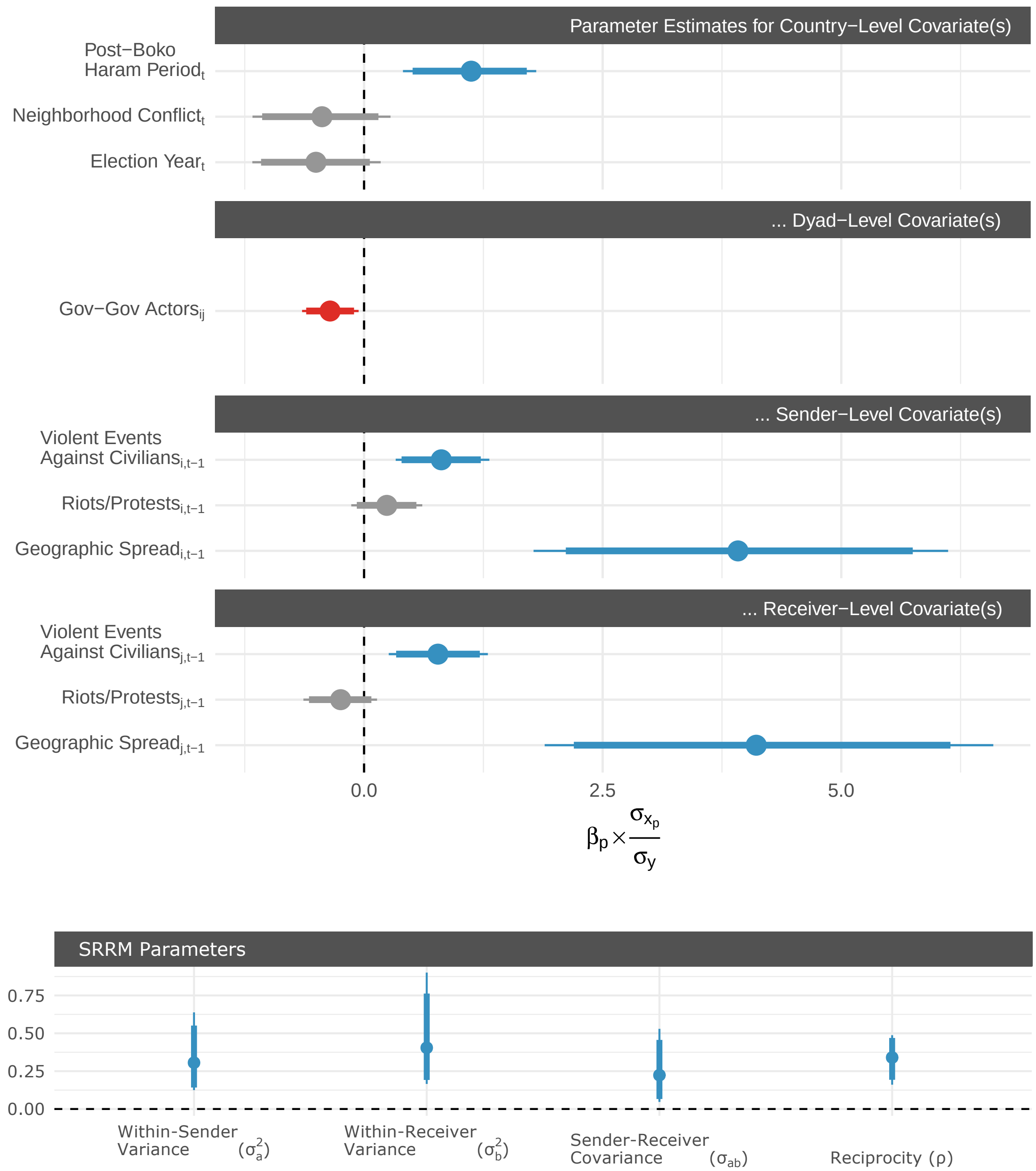

Figure 3: Standardized parameter estimates from AME model. Points represent average value of parameters, thicker line represents the 90% credible interval, and thinner line the 95%.

In the case of this Nigerian conflict network, there is no evidence of a link between civilian protests and the occurrence of a conflict event. There are a number of possible reasons for this null finding: the protest data we use might contain uncertainty about the targets of the protest, some protests might agitate towards greater military involvement–such as the “Bring Back Our Girls" protests pushing for the government to confront Boko Haram–while other protests might be against violence, or different actors might respond to protest in divergent ways.[31]

Finally, we examine the effect of the Boko Haram insurgency on the level of conflict in the system and find a clear increase in violence in the years that follow Boko Haram’s entry. With all other variables held at their measure of central tendency, a battle between any of the dyads in the system is about 2.82 times as likely in the period after the entrance of Boko Haram as it was before their entrance. Again this is not just the effect of Boko Haram’s propensity to target and be targeted in battles, as those first order effects are accounted for in the AME model. The Boko Haram Insurgency is associated with an increase in conflicts even in the dyads that *do not contain Boko Haram*.[32]

We plot variance parameters from the SRRM in the bottom of figure 3. By looking at the sender and receiver heterogeneity estimates (σ_a^2, σ_b^2), we see significant first order effects–different actors not only have different baseline levels of conflict, but the variance changes too. We also see a moderate value for σ_{ab} indicating that actors which

[31]We see no consistent effect of protest, whether we focus only on anti-government protest, only on protests against non-government groups, or when we look at all protests.

[32]One concern about a temporal variable such as this one is that, when relying on event data from ACLED, we might see more stories from Nigeria with Boko Haram’s entrance even if there was not an increase in violence. However, when we limit our dependent variable to only those battles causing at least 1 casualty, which are less likely to be subject to this inflation, we see quite similar results, and the greater violence in the post-Boko Haram period remains.

initiate more conflictual links also receive more conflictual links in return. Relatedly, the positive value for ρ indicates that actors whom receive a conflict from a particular sender reciprocate that conflictual behavior. The fact that each of these variance parameters is positive and significantly greater than zero indicates that the assumption of observational independence relied on by standard generalized linear models (GLM) are violated in this dyadic conflict dataset.

Boko Haram's Entrance

In figure 4 we show that there is both more intense and more widespread conflict with the beginning of Boko Haram's uprising. In this visualization, we shade dyadic relationships that experience more conflict after the beginning of Boko Haram's insurgency in blue, those that saw less in red, and those that stay the same in green. Interestingly, much of this conflict does not actually involve Boko Haram. Their conflict is predominantly against the Nigerian Police and Military forces, and yet after their entrance we also see increases in conflict between other groups, and in regions of the country where Boko Haram is not present.

The uptick in conflict after 2009 is most consistent with a logic of multiple rebel groups strategically interacting with the government. Other rebel groups are able to witness Boko Haram's success against the government, and this increases their beliefs that they can challenge the government successfully (or, as Fjelde & Nilsson (2012) argue, fight eachother due to governmental weakness). At the same time, if the government is unsuccessful against Boko Haram, they are incentivized to prove their continued effectiveness against other groups who try to deter future challenges. After Boko Haram's entry in 2009 we see increased attacks between rebel groups, and increased attacks by those groups against the government, but we also see increased attacks by the government–consistent with this logic.

An intuitive interpretation suggests that Boko Haram's conflict with the government weakened the Nigerian state's ability to monopolize violence and deter other violent actors. This suggests that the government is so busy fighting Boko Haram that other militias are able to run amok. Yet if this were the case, we would expect to see the Police and Military involved in fewer conflicts with other groups in the later period. We do not observe this: not only are the government groups involved in *more* battles on average in the second period, but there level of conflict with nearly every group has increased.

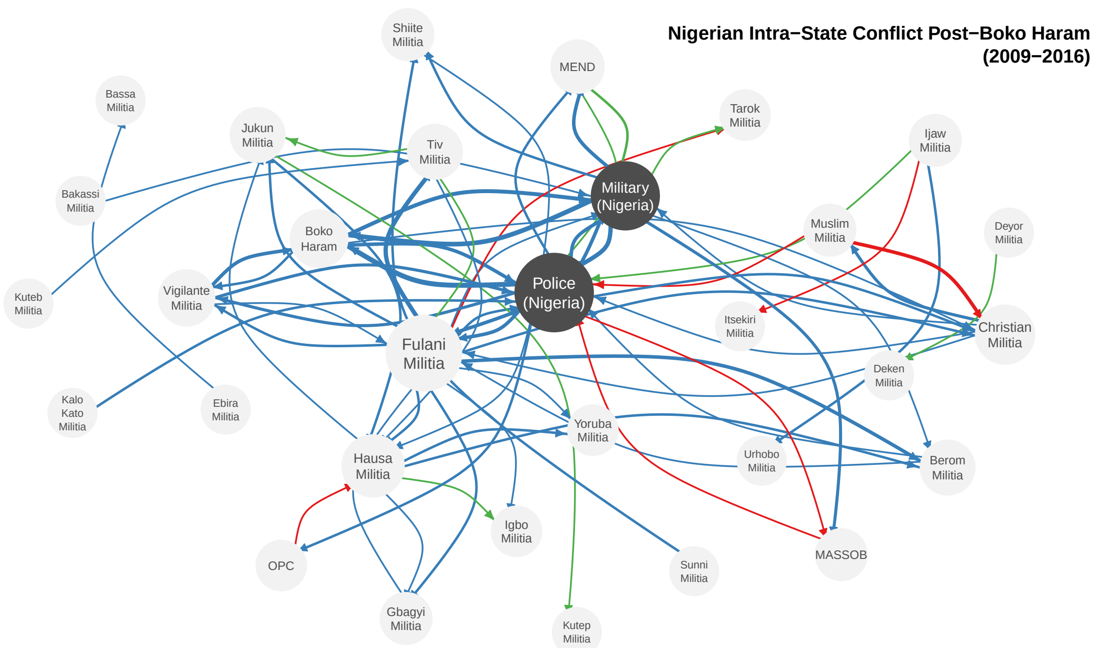

Figure 4: Conflict network 2009-2016. Pairs with more conflict have larger links. Government groups are in black, all other groups are in grey. Blue links indicate more interactions in the 2009-2016 period than in the 2000-2008 period, red indicates less, green no change.

The effect of Boko Haram's entrance underscores the need for a networked approach to interstate conflict. A standard approach to interstate conflict, which focuses on the government-rebel dyads, or in particularly ambitious cases, also looks at rebel-rebel dyads, would capture Boko Haram's direct effect on conflict, but it would miss this indirect effect.

Network Dependencies

We use a network model to understand the Nigeria conflict system not just because it can give more precise parameter estimates, but also because it aids in understanding the interdependencies among the actors in the network.[33] Such an approach allows us to infer actor relationships that are typically unobservable. We depict the sender ($\hat{a}_i$) and receiver ($\hat{b}_j$) random effects in figure 5.

[33]A comparison of our network model to the GLM approach can be found in the appendix.

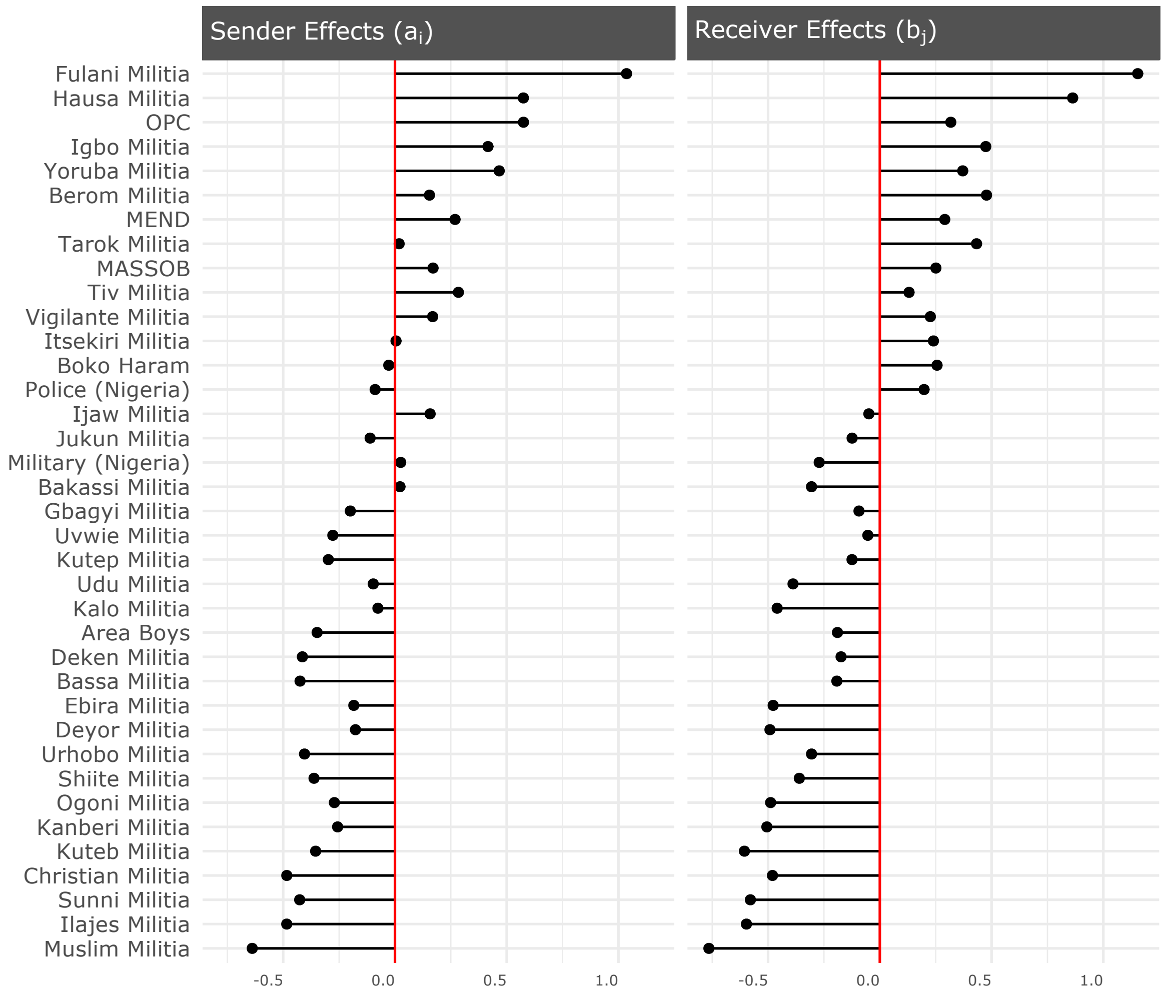

Figure 5: Sender and receiver random effect estimates.

This figure shows which actors are more (and less) violent than would be predicted by just accounting for the exogenous covariates we reviewed in the previous section. We can see, for example, that Boko Haram actually has a random sender effect that is approximately zero, implying that once we account for their high tendency to target civilians and the secular increase in violence that followed their entry into the conflict the model is able to accurately predict the group's tendency towards initiating conflict. On the other hand, actors like the Fulani and Hausa militias have notably positive sender and receiver effects (and the Christian and Muslim Militias have notably negative effects). This gives us cause to believe that our model has done a somewhat worse job capturing the behavior of these groups; it indicates that other unobserved factors drive their tendency to send and receive conflict.

Figure 6 shows the directions of actors' latent factors for both sending, left panel, and receiving violence, right panel. The directions of $\hat{u}_i$'s (for sending conflict) and $\hat{v}_i$'s (for receiving conflict) are noted in blue and red, respectively. The size of the label assigned to the points is a function of the magnitude of the vectors.[34] The purpose of this figure is to discern groups of actors that are more similar to each other in terms of whom they send conflict to (left panel) or receive conflict from (right). Actor similarity here results from third order dependence patterns – specifically, homophily and stochastic equivalence – that remain after accounting for the other parameters estimated by the model.

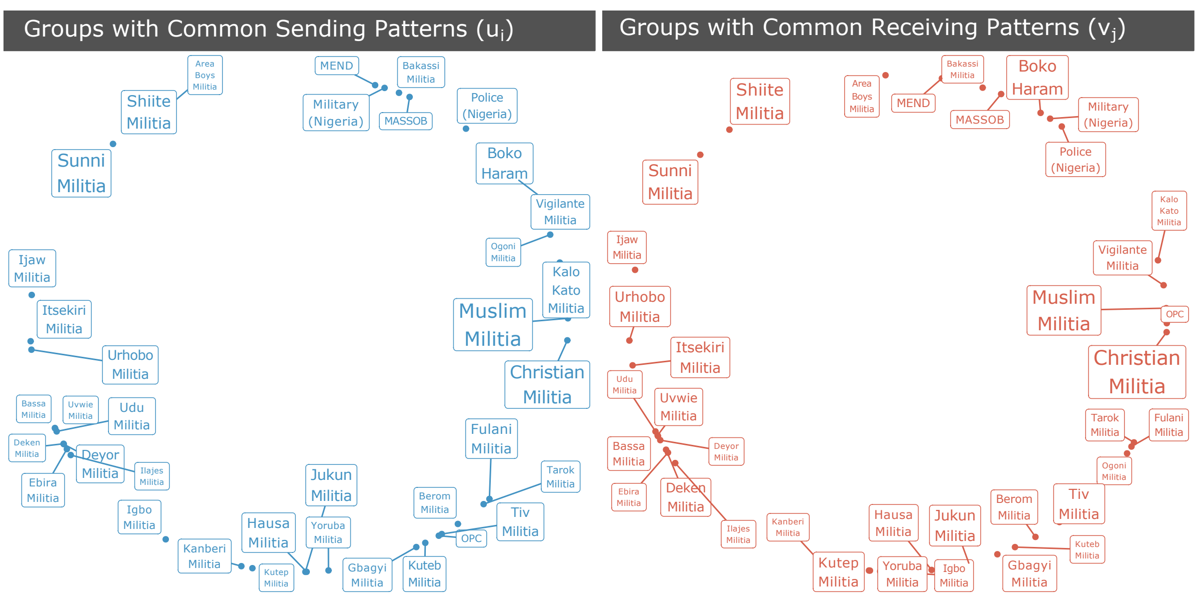

Figure 6: Visualization of multiplicative effects.

[34]Figure A6 in the appendix summarizes how well we capture first, second, and third order dependencies using our model.

By using an AME approach, we are able to better identify previously unobservable patterns of conflict in Nigeria. For example the cluster involving Boko Haram, MASSOB, and MEND is an example of stochastic equivalence: in particular these parties, though spatially heterogeneous, are each predominantly engaged in fighting with the Nigerian military and police for control of territory (the Niger Delta for MEND, the state of Biafra for MASSOB, and the North West for Boko Haram). We also find certain groups of actors who are not just more likely to fight common third parties, but they are also more likely to fight *each other*: for prominent examples of this we can see the proximity between the Sunni and Shiite militias, or the Christian and Muslim militias. Additionally, groups in the bottom of figure 6 such as the Fulani, Tiv, Berom, and Kuteb militias each tend to predominantly operate in the Middle Belt of Nigeria, and form geographically concentrated conflict clusters in which each group initiates or receives conflict with one another – this is an example of a set of homophilous relations.

Out-of-Sample Performance Analysis

By accounting for exogenous and network dependent patterns that give rise to conflict systems we are able to better account for the data generating process underlying relational data structures. To show that this is the case, we examine whether our approach achieves better predictive performance in an *out-of-sample context* than traditional dyadic models. Given that our model is more complex than a GLM, we would expect it to do better *in-sample*, but inclusion of more parameters will only improve performance *out-of-sample* if these parameters are helping us to better capture the underlying data generating process. To evaluate our model, we randomly divide the $\binom{n}{2} \times T$ data values into $k = 30$ sets, letting $s_{ij,t}$ be the set to which pair ij, t is assigned. Then for each $s \in \{1, \ldots, k\}$, we:

1. estimate model parameters with $\{y_{ij,t} : s_{ij,t} \neq s\}$, the data not in set s,

2. and predict $\{\hat{y}_{ij,t} : s_{ij,t} = s\}$ from these estimated parameters.

The result of this procedure is a set of sociomatrices $\hat{Y}$, in which each entry $\hat{y}_{ij,t}$ is a predicted value obtained from using a subset of the data that does not include $y_{ij,t}$.

We set a number of benchmarks for comparison. First we compare the AME model to a GLM model using the same covariates to show the effect of accounting for network dependencies on predicting conflict. We supplement this with an alternative GLM that includes not just these covariates, but also a lagged dependent variable and a lagged reciprocity term. The lagged dependent variable is the equivalent of saying that conflict and peace are relatively likely to persist between dyads, while the inclusion of a lagged reciprocity term in a GLM framework is a simple way to account for retaliatory strikes.

We utilize three performance criterions to compare the models: Receiver Operator Characteristic (ROC) curves, Precision Recall (PR) curves, and separation plots. ROC curves look at the trade-off between true positive rates and false positive rates at different thresholds of classification.ROC curves can be problematic for analyzing conflict because conflict is relatively rare at the dyadic level: in most years only 3% of possible dyads are in conflict with one another in our data. If peace is common, even a poor model will have a very low False Positive Rate.

To better assess which models predict the presence of conflict, not just its absence, we look at PR Curves. These examine the trade-offs between the percentage of conflicts a model predicts, and the percentage of predicted conflicts which occur. Lastly, we examine separation plots (Greenhill et al., 2011). These provide an intuitive visualization of the accuracy of our predictions by juxtaposing a line showing the predicted probability of conflict with whether conflict actually occurs for all cases (where the cases are sorted by the predicted probability and then colored to indicate the outcome). Here a perfect model would have all cases where conflict actually exists on the right with a predicted probability of 1, and would predict 0 in all other cases. All of the models' performance out-of-sample by these metrics are displayed in figure 7. The AME model with covariates is the best performing model out-of-sample in all cases. This model outperforms each of the GLM variants by a notable margin.

Typically, conflict scholars are also interested in generating forecasts from their models, and to assess the performance of their model instead of taking a cross-validation approach they often attempt to forecast the last period of data. We perform such an exercise as well by dividing up our sample into a training and test set, where the test set corresponds to the last period in the dataset that we have available. Results for this analysis are shown in Figure 8 and here again we find that our network based approach has better out-of-sample predictive performance than the more traditional logistic alternatives.

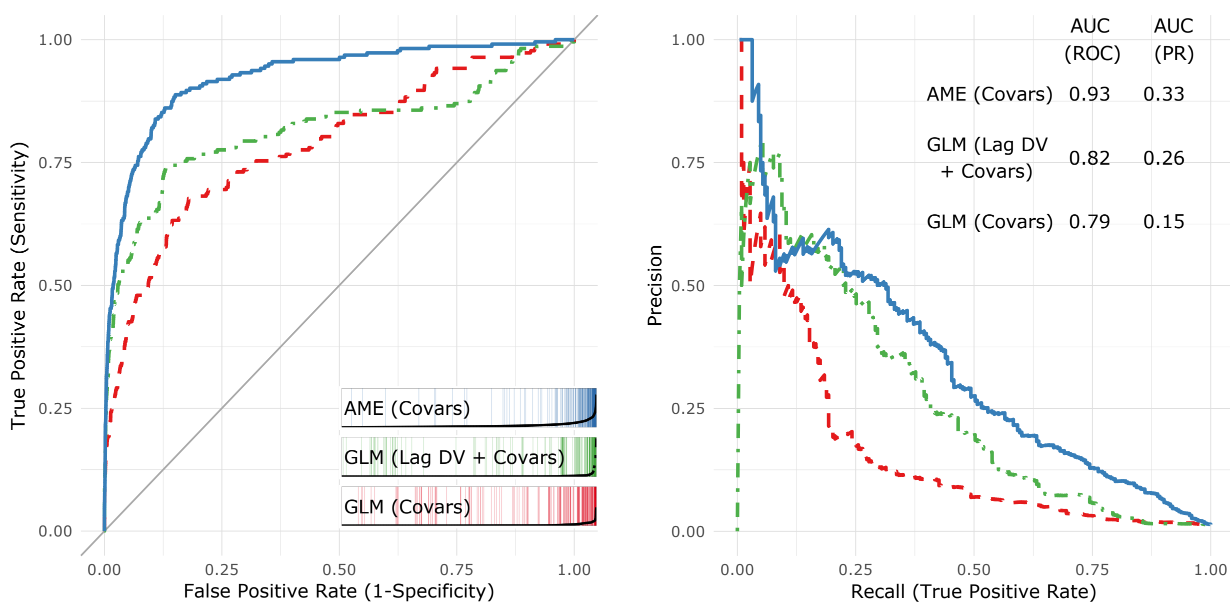

Figure 7: Assessments of out-of-sample predictive performance using ROC curves, separation plots, and PR curves. AUC statistics are provided as well for both curves. In each curve, the AME model is in blue, the GLM with lagged DV and covariates is in green, the GLM with only covariates is in red.

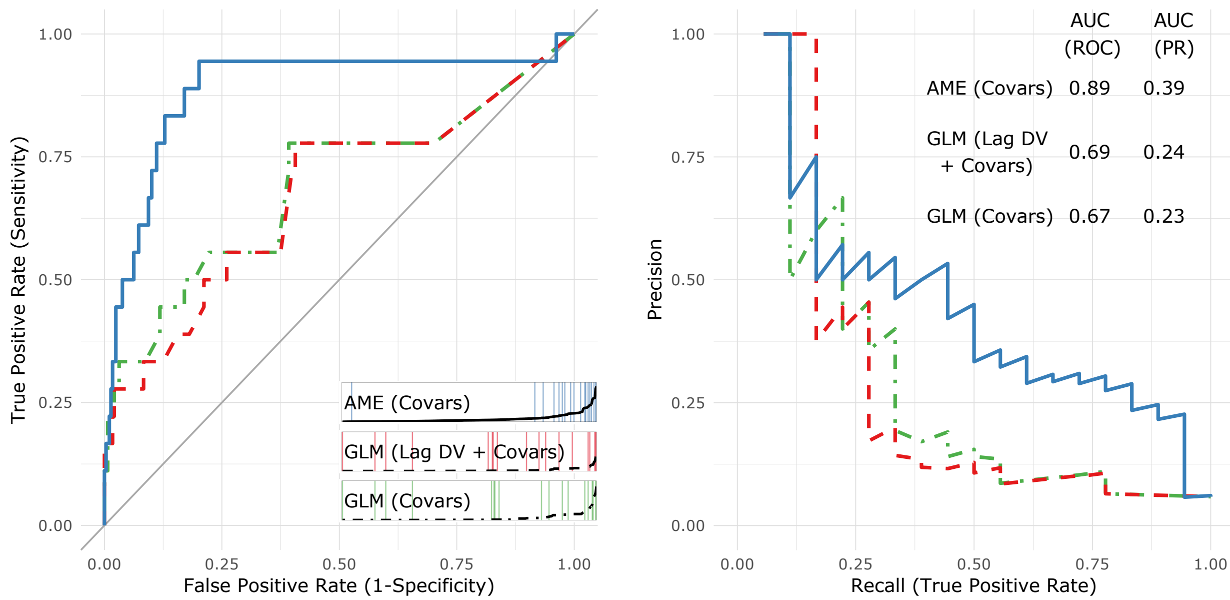

Figure 8: Assessments of out-of-sample predictive performance when forecasting out the last period using ROC curves, separation plots, and PR curves. AUC statistics are provided as well for both curves. In each curve, the AME model is in blue, the GLM with lagged DV and covariates is in green, the GLM with only covariates is in red.

Discussion

Intrastate Conflict as a Dynamic Network

Intrastate conflicts are often more complex than Manichaean struggles between a government and a unified opposition. More and more, they involve a number of actors operating in an environment where interactions can be interdependent. The improved predictive performance of our network approach is a result of the fact that there is significant network based structure underlying the Nigerian conflict system. Apart from pointing to the utility of taking network patterns into account, our study also highlights how we can use the AME model to uncover the types of first, second, and third order dependence patterns that emerge.

The implication of first order dependence patterns is that there are latent actor attributes not accounted for by our model specification that could be used to explain why some actors are more conflictual than others. Interestingly, one of the more striking findings is that the Fulani militia is particularly more active in engaging in battles than the exogenous sender, receiver, and dyadic variables in our model would predict. This highlights the opportunity for further research into understanding what latent attributes may drive such inflated conflictual behavior. Our findings with regards to second order dependencies demonstrate the strong role of reciprocity in this network. This reflects a common intuition from the interstate conflict literature and our study of Nigeria highlights that this type of pattern also persists in intrastate conflicts. Last, our visualization of the multiplicative effects captures third order dependencies and shows strong evidence that actors in this system form various communities of conflict. Each of these dependencies points to the fact that there is an underlying structure to the set of dyadic interactions that we observe in this network even after accounting for the effects of exogenous covariates. Since these patterns play a role in the data-generating process of intrastate conflict, our approach out-performs more traditional frameworks that ignore such dependencies.

Implications for Future Research

By utilizing information about dependencies between actors to accurately consider how conflicts develop over time, we are also able to more accurately test exogenous actor and dyadic level explanations for conflict. First, because we are able to estimate a network model in which actor composition changes over time, we can assess the impact that particular actors such as Boko Haram have on the system. We find that the entrance of this single actor profoundly changed interactions across the system. Boko Haram's entrance does not simply increase conflict *directly*, it also is associated with a marked rise in violence in the dyads that *do not include Boko Haram.* This raises an important question for future research which is whether or not we can formally study why some actors have stronger effects on shaping interactions across a network than others.

Our present study is necessarily limited in that we retroactively observe the influence of Boko Haram on the conflict network. However, our results dovetail with current findings about Boko Haram's rise and expansion and provides insights for defining key actors in other conflict settings. First, we have shown that Boko Haram is particularly violent and targets both the government and civilians. Weeraratne (2017) argues that a key determinant of Boko Haram's rise is their rapid increase in the use of violence against societal actors, government armed groups, religious communities, and civilians. Osumah (2013) details how the targeting strategies of Boko Haram has weakened the government's security apparatus and heightened tensions across the country. Second, there is evidence to suggest that Boko Haram was particularly well-organized and well-resourced even in the first few years of its operation (Osumah, 2013). Group-level attribute data would provide a way to test whether groups that are highly organized, well-resourced, and target a wide range of actor types are likely to have a broad affect on sub-national level violence, similar to the Boko Haram's influence in Nigeria.

Additionally, our results highlight the role that civilian victimization can play in further perpetuating violence within a network context. Even after controlling for network related dependencies within this system, we find that actors who target civilians are more likely to receive and send conflict themselves. Relatedly, we find little evidence that civilian led protest against a particular actor changes that actor's likelihood in initiating conflict. To some extent, these findings showcase the limitations of civilian populations to change the behavior of actors in environments of extreme violence. Future research should compare and contrast these findings beyond the Nigerian case to better illuminate the networked nature of civilian mobilization in conflict environments. The findings with regards to the role of civilian victimization and civilian riots/protests might not generalize beyond the Nigerian case. Nigeria may be unique for several reasons, its 150 million people are divided almost equally between Muslims and Christians. Further, Nigerians split into even finer divisions based on tribe, of which there are over 200 in the country.

Conclusion

Our study has embraced the complexity of multi-actor conflicts, rather than discounted it. We have shown that network dynamics help explain the occurrence of violence between groups over time, even as actors enter and exit the conflict. We have demonstrated how key, violent actors can increase violence within a network system. Additionally, our approach achieves a longstanding goal of conflict studies: to accurately predict violence over time. Our approach offers important insights into how conflict studies can utilize methodological innovations to expand theoretical inquiry.

These contributions are not without their limitations. While our study affirms the utility of ACLED data efforts, more actor-level characteristics are needed to fully engage the rich literature on civil war dynamics. Such data would enable even more precise investigation of covariates of interest for determining changes in rebel group behavior over time. Second, though our study has revealed meaningful findings with respect to key covariates such as civilian victimization, more research is needed to explore the generalizability of such results. We hope this study motivates future work that more fully considers the conflictual behavior of armed actors in complex settings.

A.1. Appendix

A.1.1. Trace plots for parameter estimates

Figure A1 below shows trace plots for the parameters summarized in Figure 3.

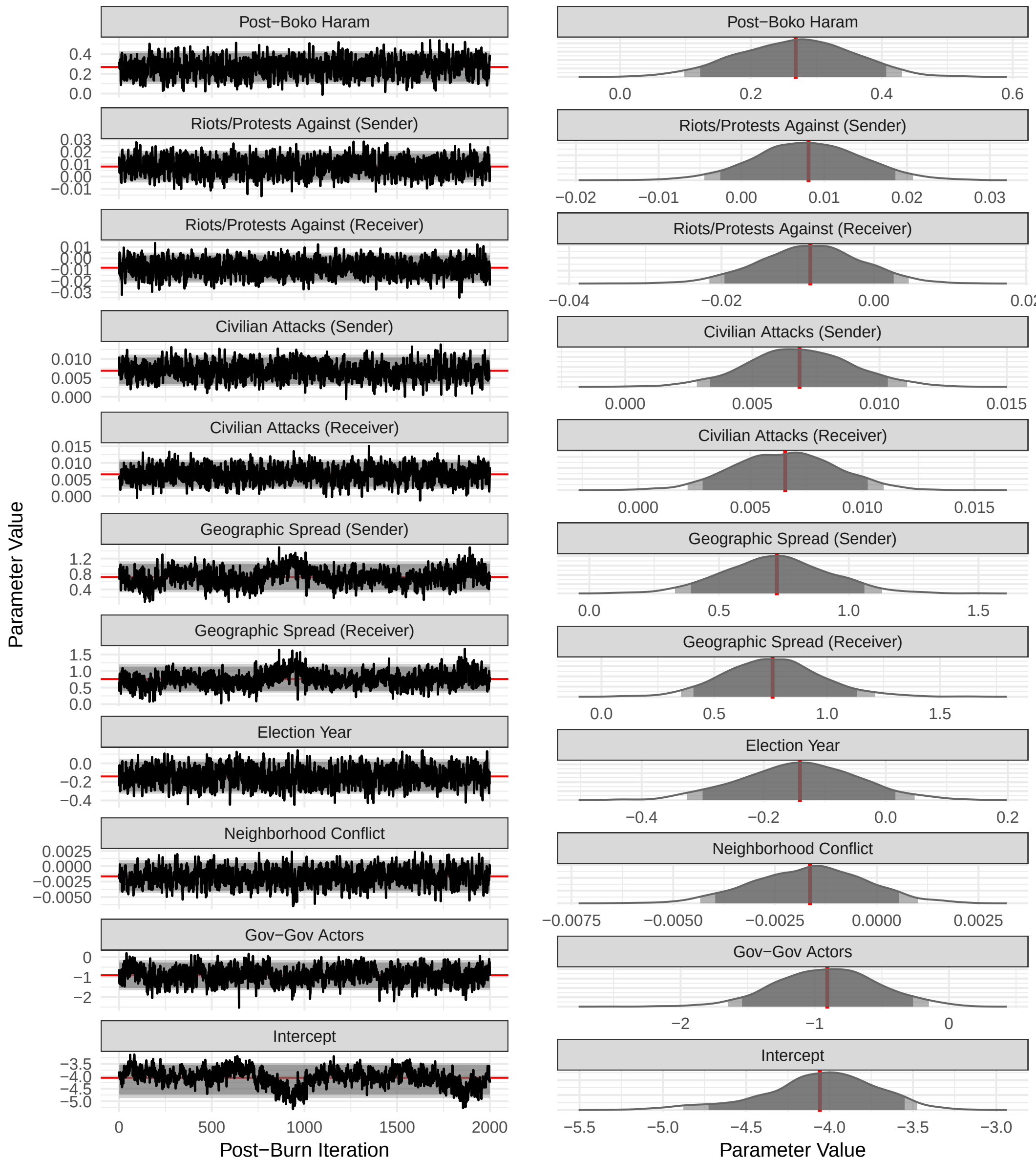

Figure A1: Trace plots for exogenous parameters.

A.1.2. Comparison with GLM Estimates

Figure A2 compares parameter estimates returned from the AME and GLM frameworks.

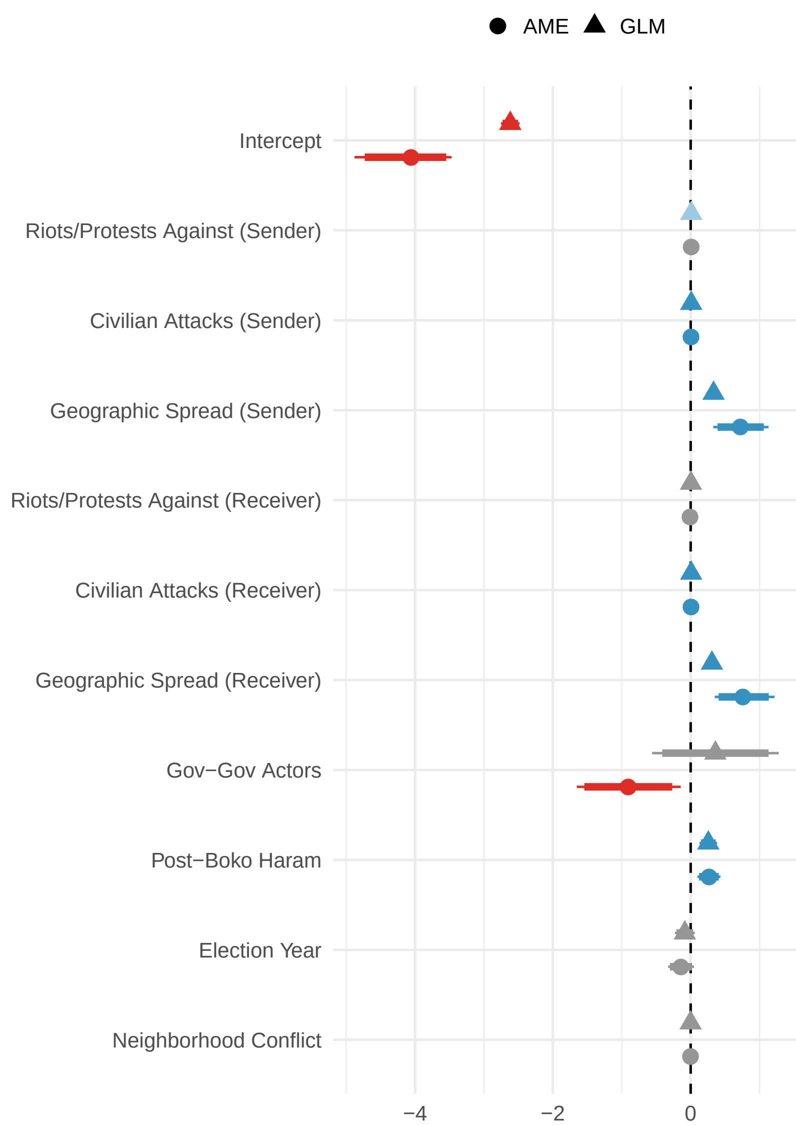

Figure A2: Comparison of AME and GLM estimates for base model.

A.1.3. Trace plots for parameter estimates when using fatality threshold

Figure A3 below shows trace plots for an alternative formulation of the model we presented in the paper. Specifically, here in defining the dependent variable from ACLED, we only set $y_{ij} = 1$ when the corresponding battle between i and j led to at least one fatality.

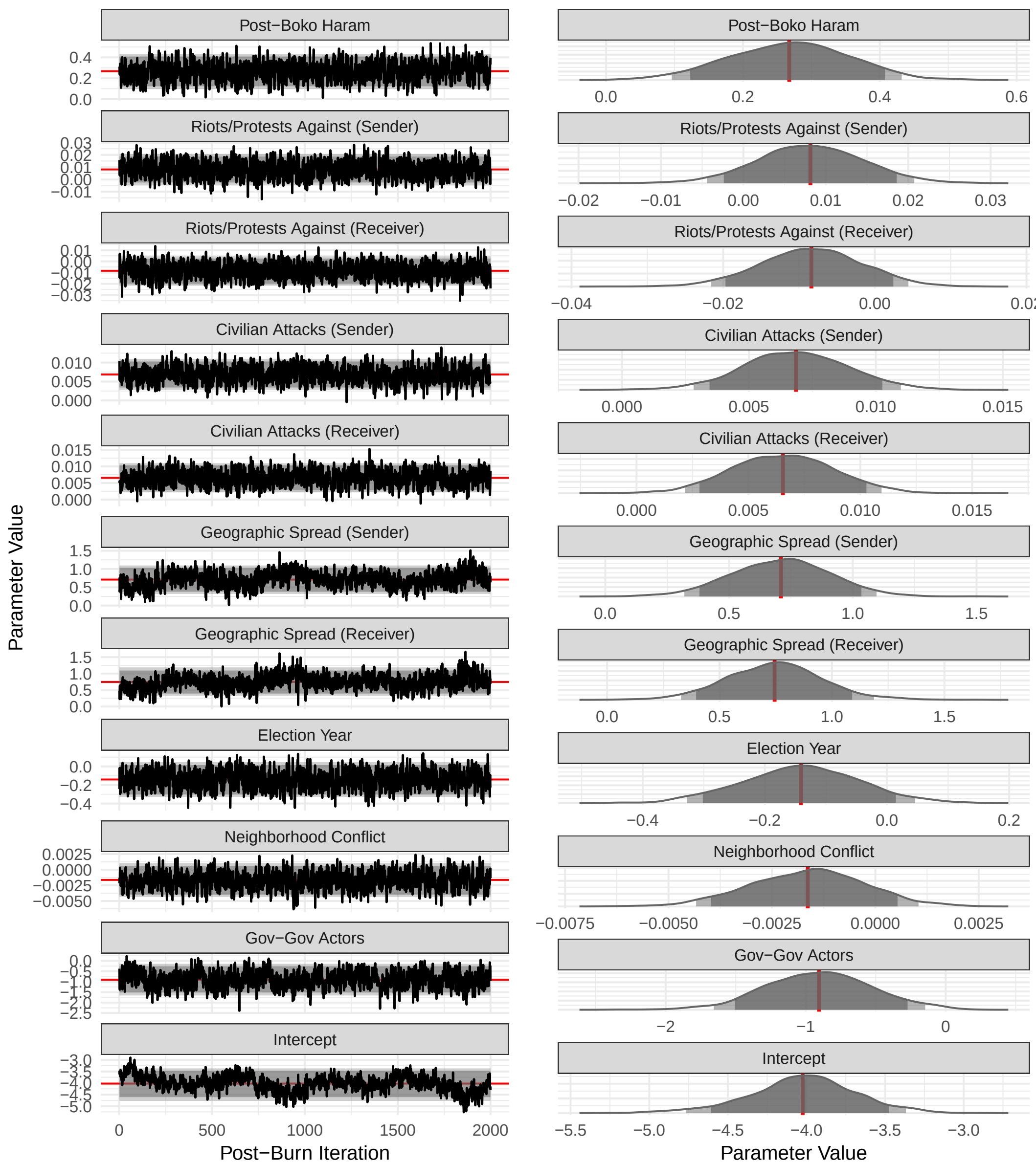

Figure A3: Trace plots for exogenous parameters.

A.1.4. Trace plots for parameter estimates when setting battles to be symmetric

Figure A4 below shows trace plots for an alternative formulation of the model we presented in the paper. Specifically, here in defining the dependent variable from ACLED, when $y_{ij} = 1$ we set $y_{ji} = 1$ as well.

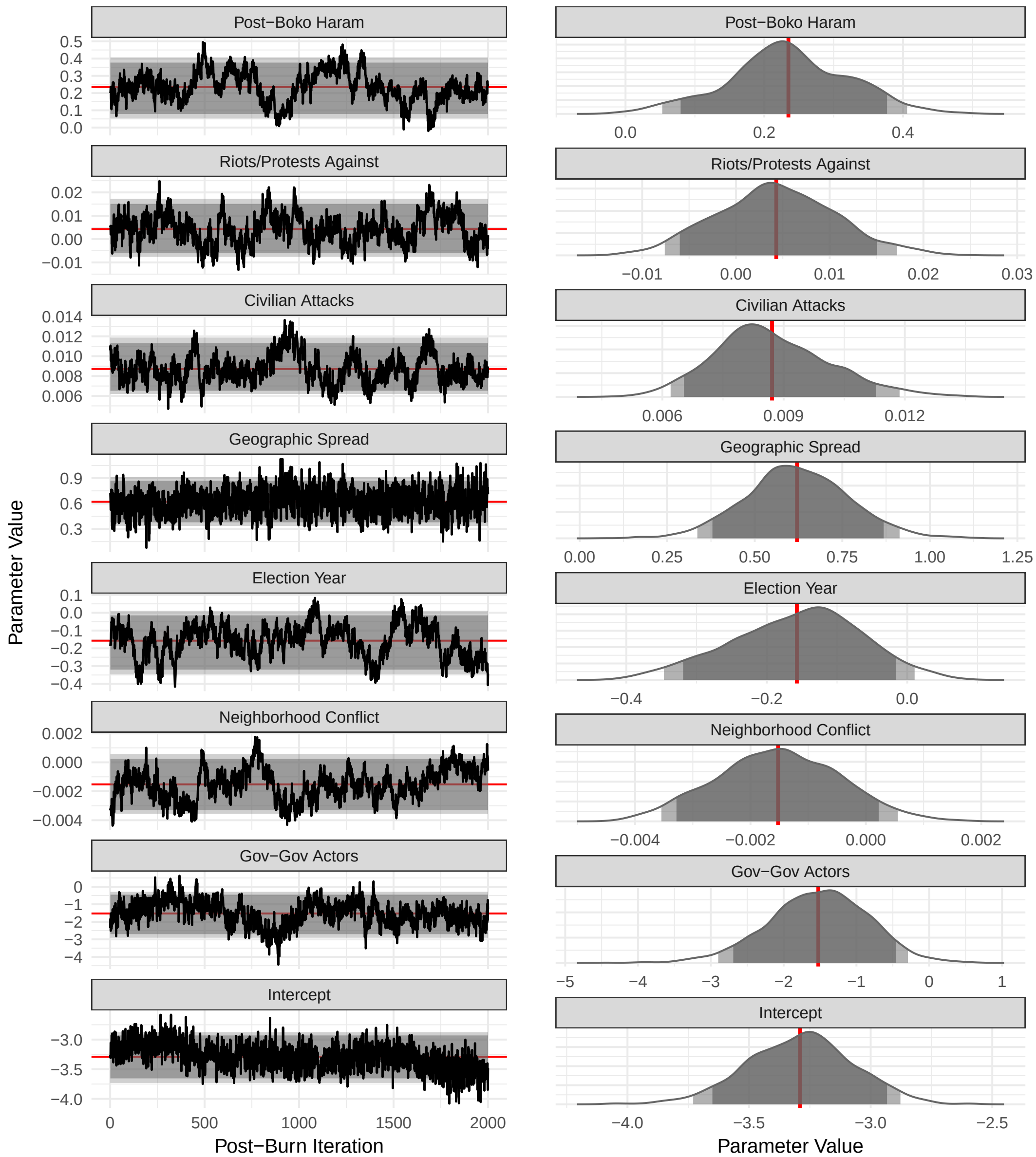

Figure A4: Trace plots for exogenous parameters.

A.1.5. *Trace plots for parameter estimates when excluding Government actors*

Figure A5 below shows trace plots for an alternative formulation of the model we presented in the paper. Specifically, here we exclude government actors from the analysis.

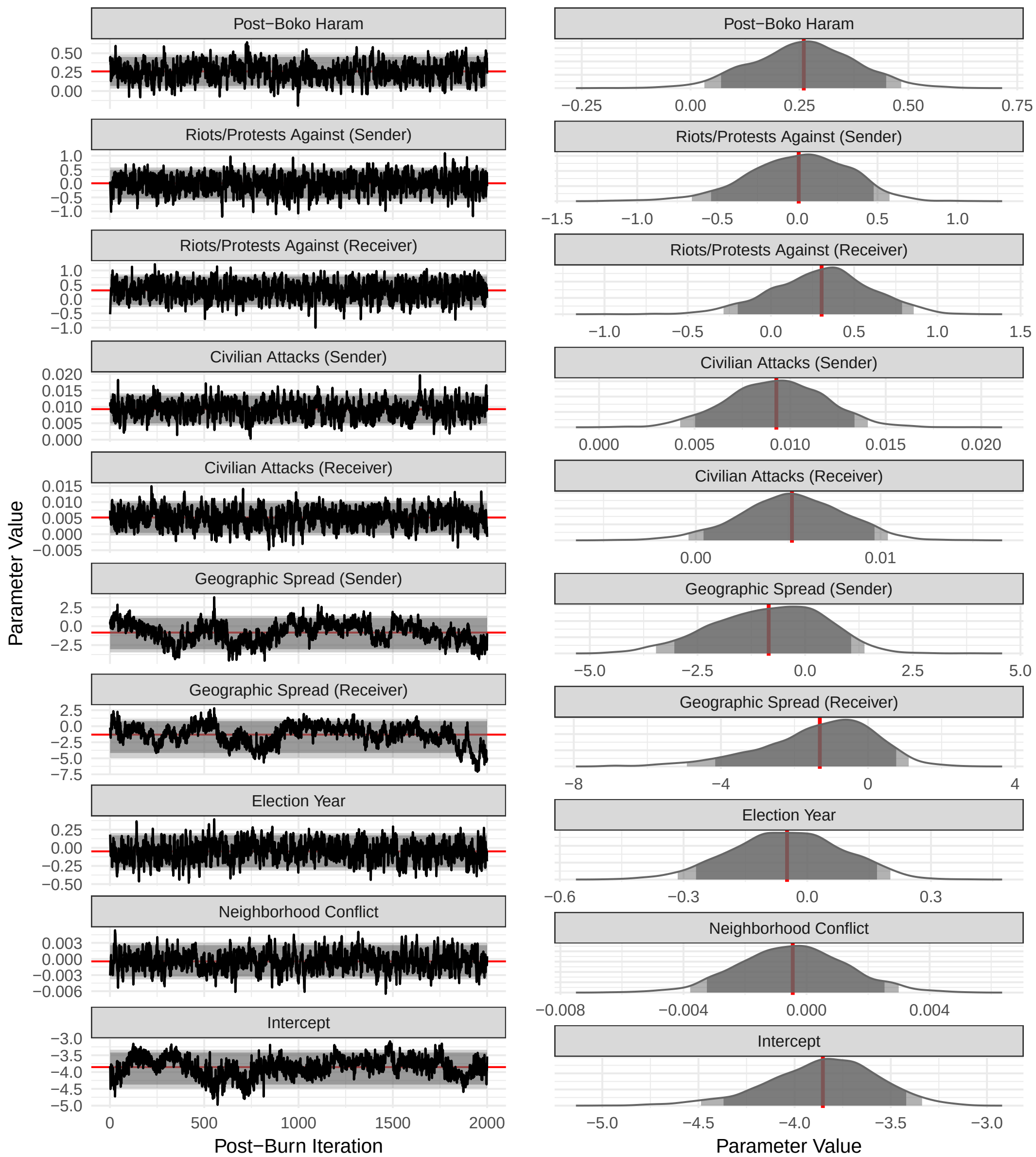

Figure A5: Trace plots for exogenous parameters.

A.1.6. Network Goodness of Fit Assessment

Figure A6 presents an assessment of how well our model captures the network attributes of the conflict system in Nigeria on a variety of dimensions. For details on interpreting this diagnostic see Minhas et al. (2016a).

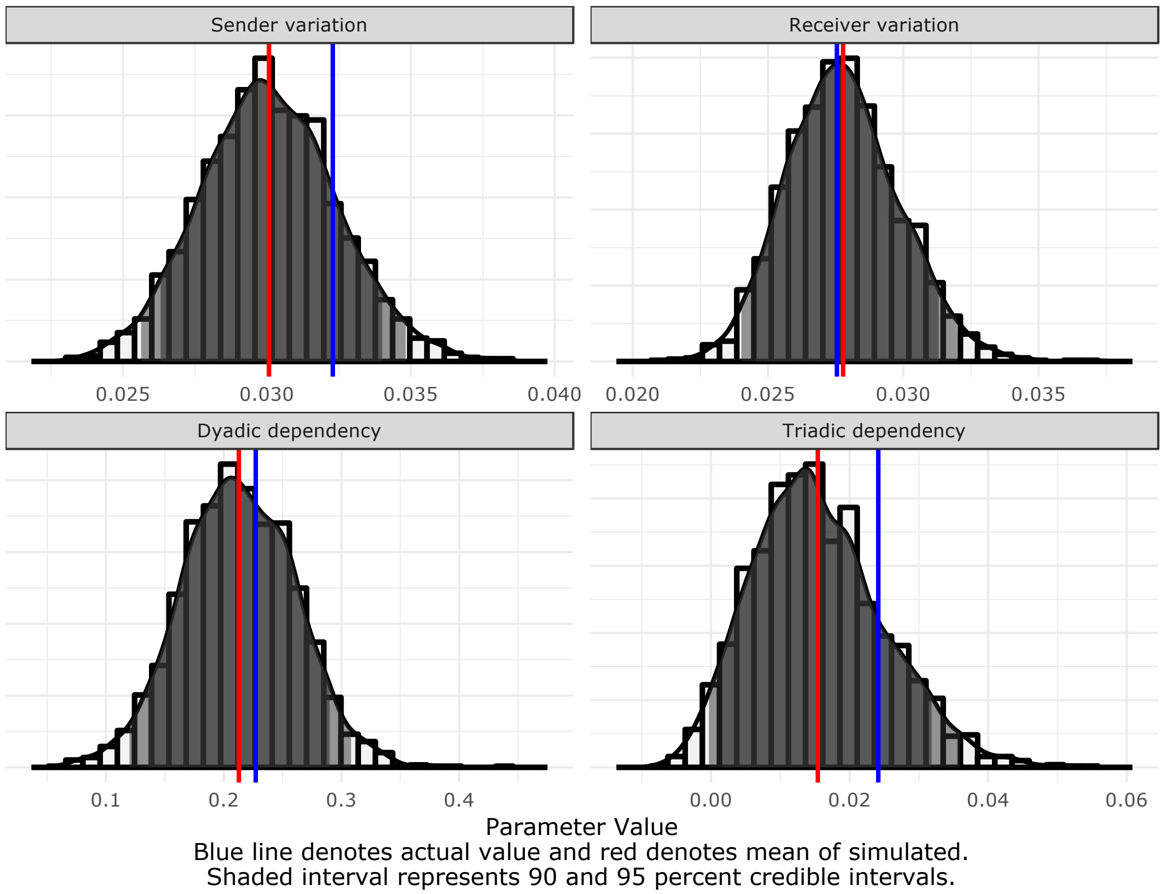

Figure A6: Network goodness of fit summary.

A.1.7. Additive and Multiplicative Effects Gibbs Sampler

To estimate, the effects of our exogenous variables and latent attributes we utilize a Bayesian probit model in which we sample from the posterior distribution of the full conditionals until convergence. Specifically, given observed data **Y** and **X** – where **X** is a design array that includes our sender, receiver, and dyadic covariates – we estimate our network of binary ties using a probit framework where: $y_{ij,t} = 1(\theta_{ij,t} > 0)$ and $\theta_{ij,t} = \boldsymbol{\beta}^\top \mathbf{X}_{ij,t} + a_i + b_j + \mathbf{u}_i^\top \mathbf{D}\mathbf{v}_j + \epsilon_{ij}$. The derivation of the full conditionals is described in detail in Hoff (2005) and Hoff (2008), thus here we only outline the Markov chain Monte Carlo (MCMC) algorithm for the AME model that we utilize in this paper.

- Given initial values of $\{\boldsymbol{\beta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2\}$, the algorithm proceeds as follows:
  - sample $\boldsymbol{\theta} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
  - sample $\boldsymbol{\beta} \mid \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
  - sample $\mathbf{a}, \mathbf{b} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
  - sample $\Sigma_{ab} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \rho, \text{ and } \sigma_\epsilon^2$ (Inverse-Wishart)
  - update ρ using a Metropolis-Hastings step with proposal $p^*|p \sim$ truncated normal$_{[-1,1]}(\rho, \sigma_\epsilon^2)$
  - sample $\sigma_\epsilon^2 \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \text{ and } \rho$ (Inverse-Gamma)
  - For each $k \in K$:
    * Sample $\mathbf{U}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}_{[,-k]}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
    * Sample $\mathbf{V}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}_{[,-k]}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
    * Sample $\mathbf{D}_{[k,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)[35]

[35]Subsequent to estimation, **D** matrix is absorbed into the calculation for **V** as we iterate through K.

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