

A.1. Appendix: Network Dynamics in Nigeria

A.1.1. Trace plots for parameter estimates

Figure A1 below shows trace plots for the parameters summarized in Figure 4 of the manuscript.

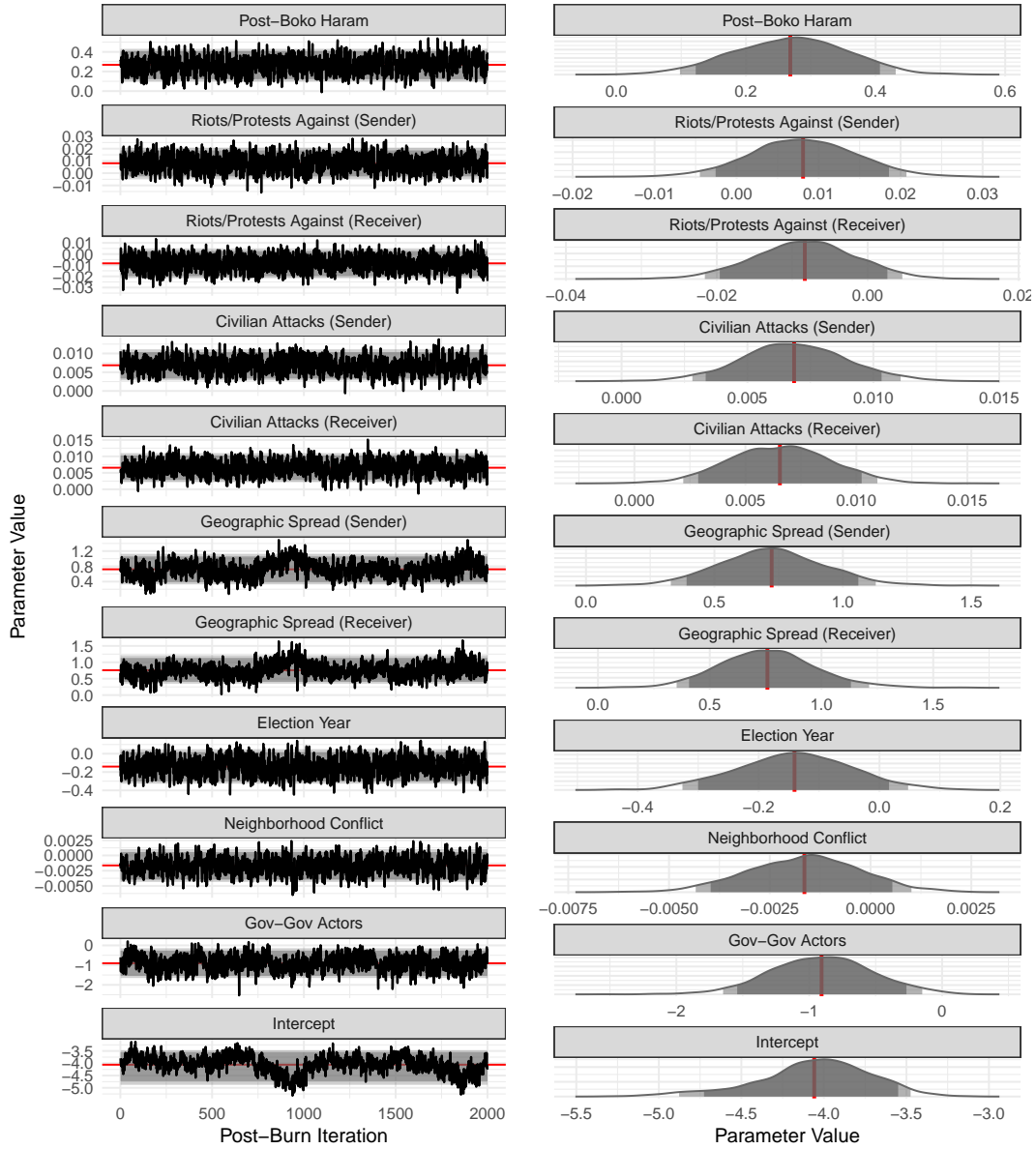


Figure A1: Trace plots for exogenous parameters.

A.1.2. Comparison with GLM Estimates

Figure A2 compares parameter estimates returned from the AME and GLM frameworks.

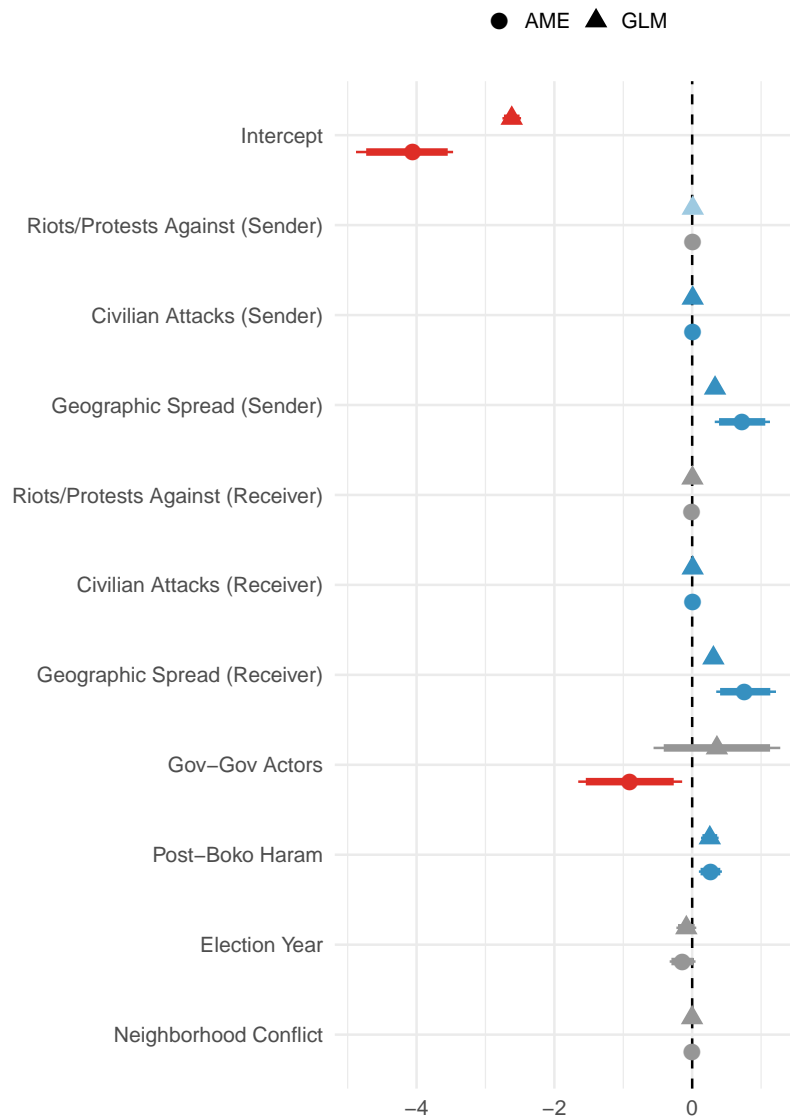


Figure A2: Comparison of AME and GLM estimates for base model.

A.1.3. Trace plots for parameter estimates when using fatality threshold

Figure A3 below shows trace plots for an alternative formulation of the model we presented in the paper. Specifically, here in defining the dependent variable from ACLED, we only set $y_{ij} = 1$ when the corresponding battle between i and j led to at least one fatality.

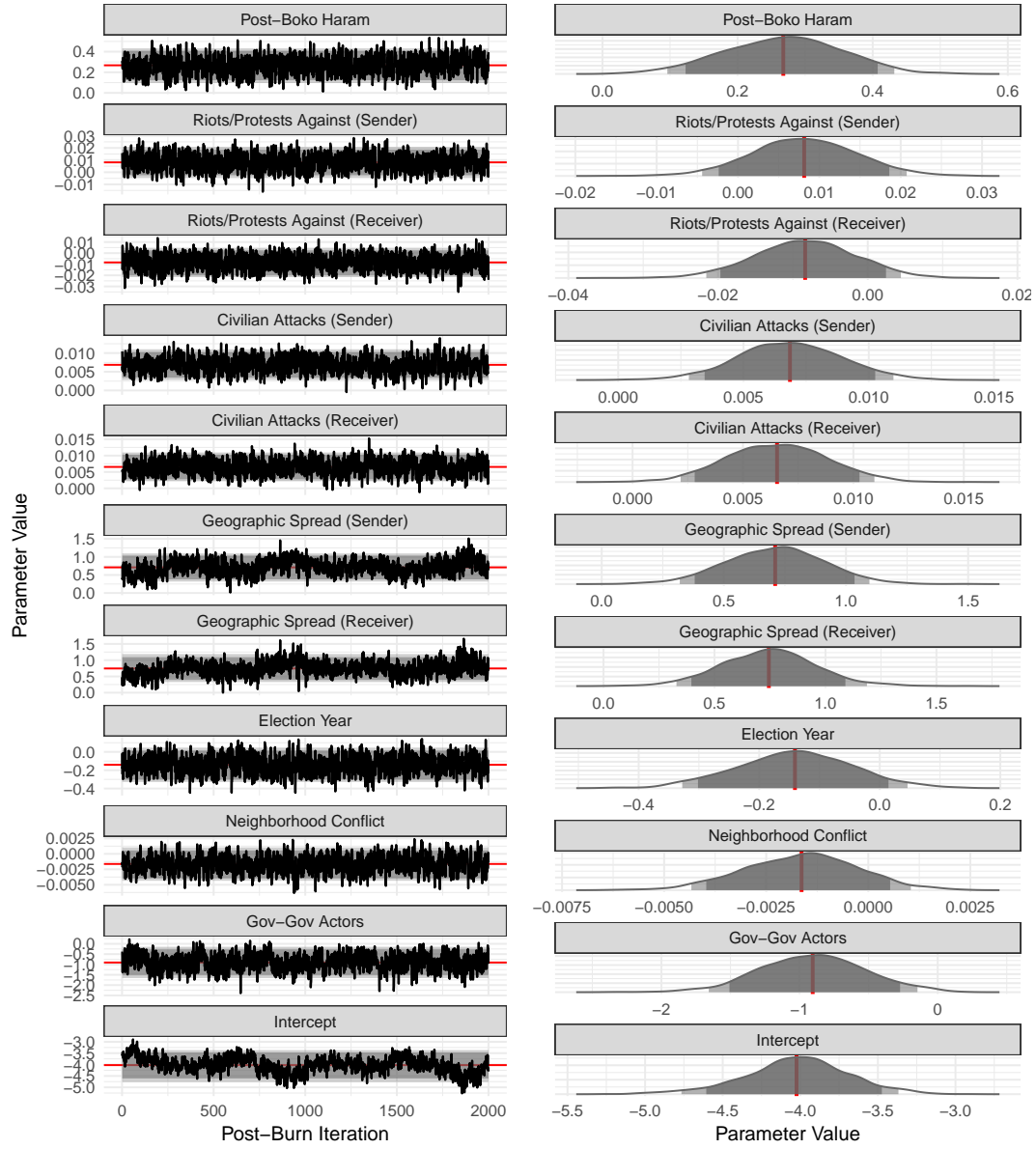


Figure A3: Trace plots for exogenous parameters.

A.1.4. Trace plots for parameter estimates when setting battles to be symmetric

Figure A4 below shows trace plots for an alternative formulation of the model we presented in the paper. Specifically, here in defining the dependent variable from ACLED, when $y_{ij} = 1$ we set $y_{ji} = 1$ as well.

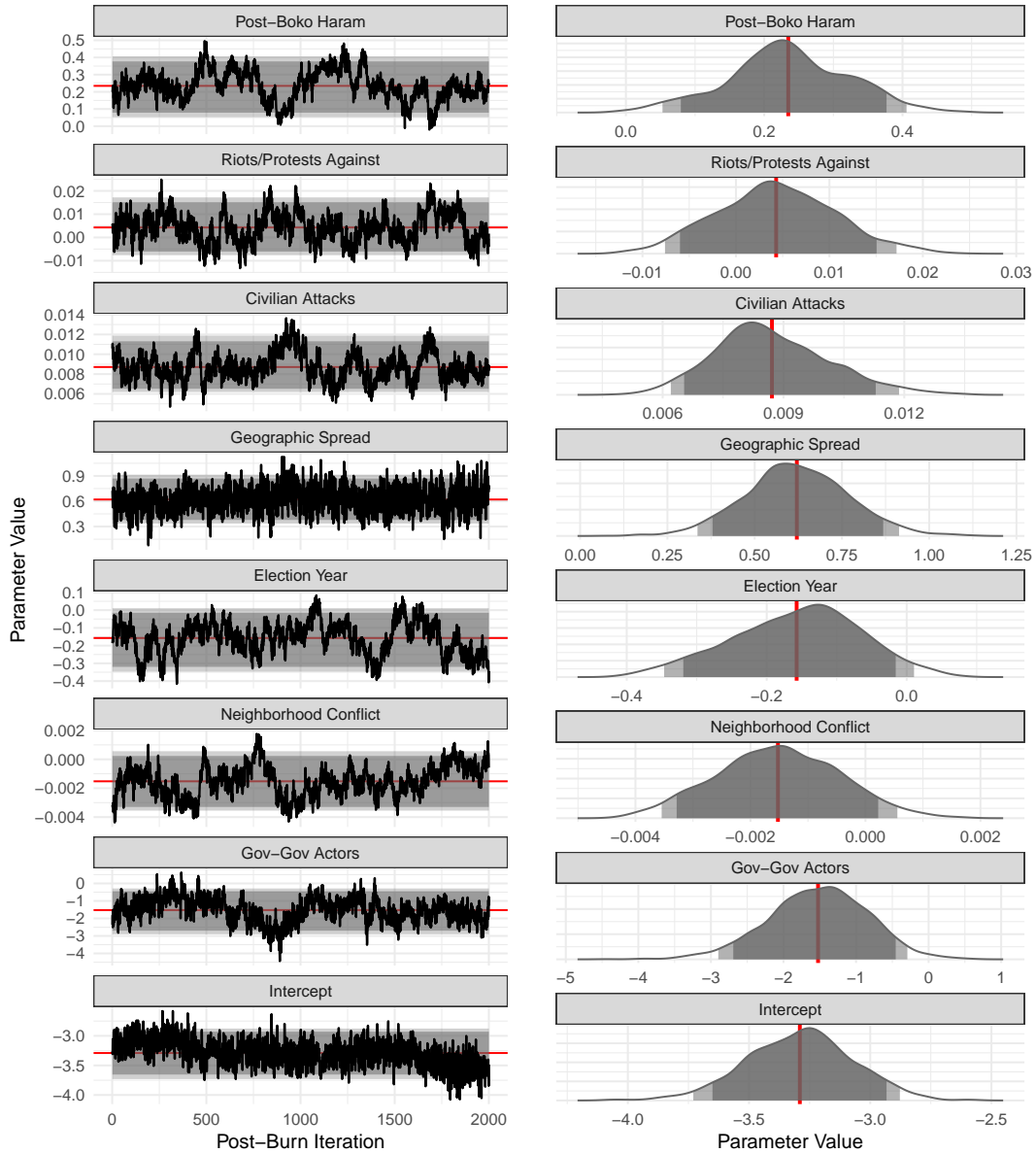


Figure A4: Trace plots for exogenous parameters.

A.1.5. Trace plots for parameter estimates when excluding Government actors

Figure A5 below shows trace plots for an alternative formulation of the model we presented in the paper. Specifically, here we exclude government actors from the analysis.

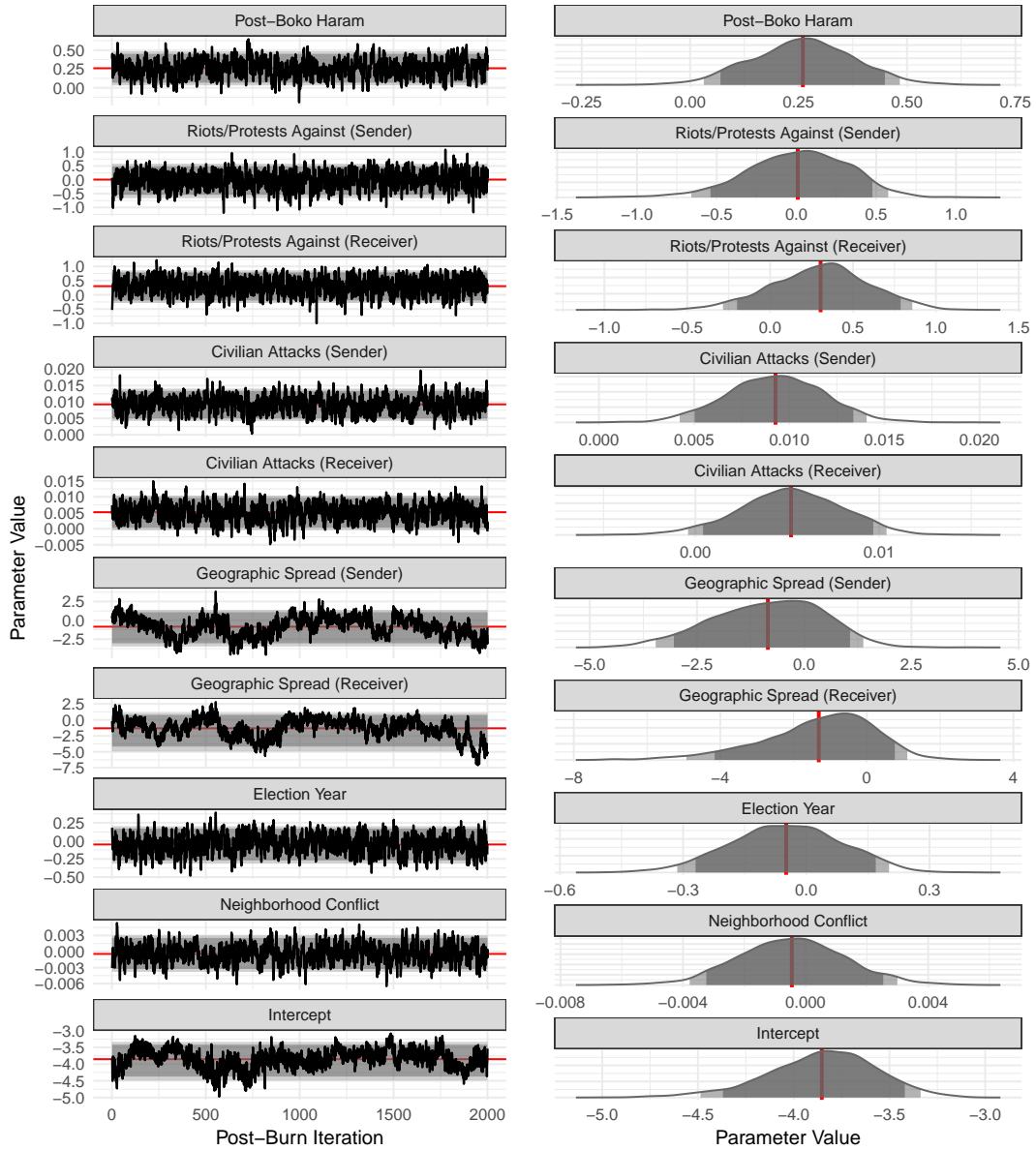


Figure A5: Trace plots for exogenous parameters.

A.1.6. Network Goodness of Fit Assessment

Figure A6 presents an assessment of how well our model captures the network attributes of the conflict system in Nigeria on a variety of dimensions. For details on interpreting this diagnostic see Minhas et al. (2016).

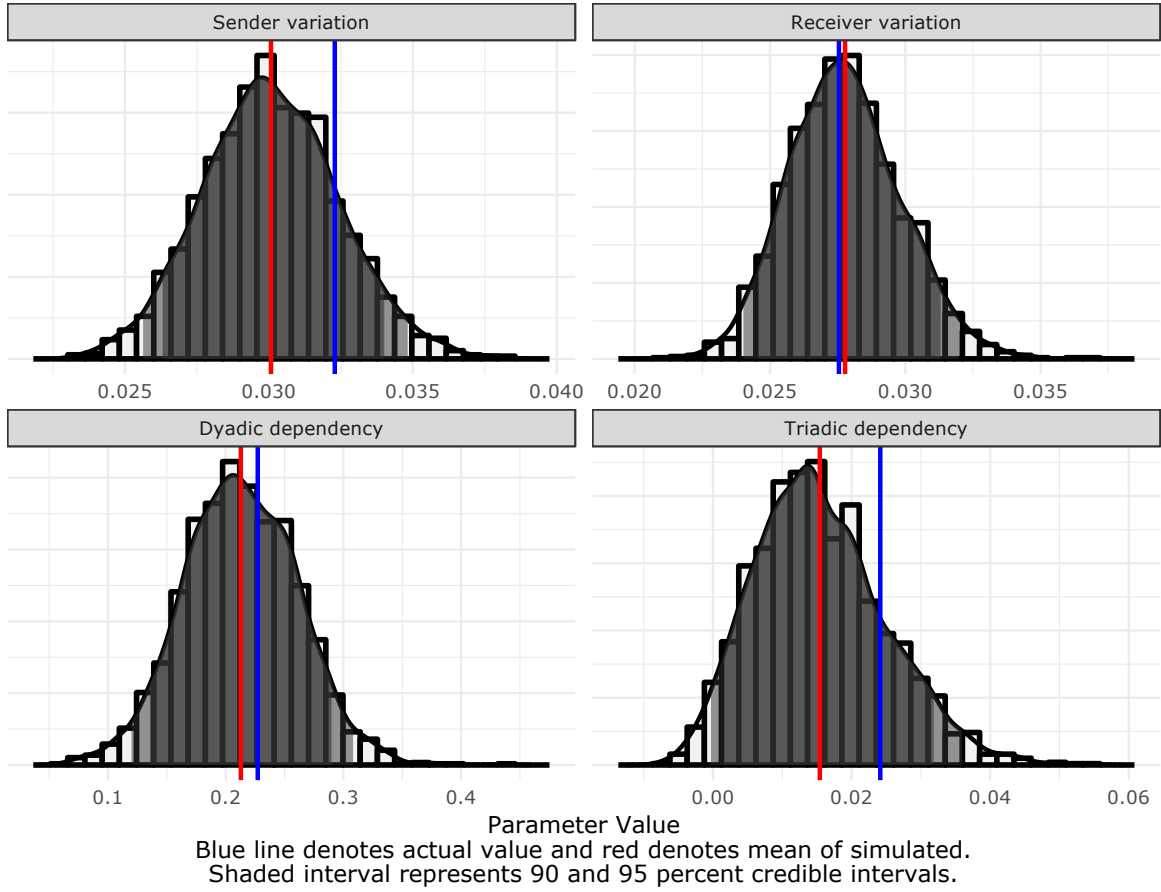


Figure A6: Network goodness of fit summary.

A.1.7. Temporal Forecasting Performance

Scholars of violence are often concerned with models abilities to generated forecasts of future events. To do this, we compare the AME and GLM models' abilities to predict battles in the final year of our data with those in the rest of the data.

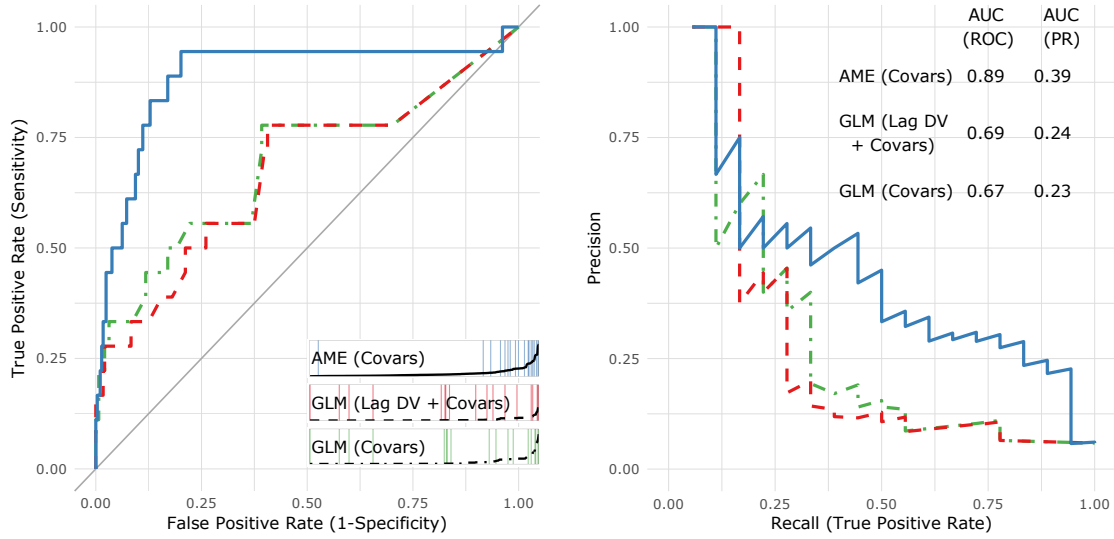


Figure A7: Assessments of out-of-sample predictive performance when forecasting out the last period using ROC curves, separation plots, and PR curves. AUC statistics are provided as well for both curves. In each curve, the AME model is in blue, the GLM with lagged DV and covariates is in green, the GLM with only covariates is in red.

A.1.8. Additive and Multiplicative Effects Gibbs Sampler

To estimate, the effects of our exogenous variables and latent attributes we utilize a Bayesian probit model in which we sample from the posterior distribution of the full conditionals until convergence. Specifically, given observed data \mathbf{Y} and \mathbf{X} – where \mathbf{X} is a design array that includes our sender, receiver, and dyadic covariates – we estimate our network of binary ties using a probit framework where: $y_{ij,t} = 1(\theta_{ij,t} > 0)$ and $\theta_{ij,t} = \boldsymbol{\beta}^\top \mathbf{X}_{ij,t} + a_i + b_j + \mathbf{u}_i^\top \mathbf{D}\mathbf{v}_j + \epsilon_{ij}$. The derivation of the full conditionals is described in detail in Hoff (2005) and Hoff (2008), thus here we only outline the Markov chain Monte Carlo (MCMC) algorithm for the AME model that we utilize in this paper.

- Given initial values of $\{\boldsymbol{\beta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2\}$, the algorithm proceeds as follows:
 - sample $\boldsymbol{\theta} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
 - sample $\boldsymbol{\beta} \mid \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
 - sample $\mathbf{a}, \mathbf{b} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
 - sample $\Sigma_{ab} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \rho, \text{ and } \sigma_\epsilon^2$ (Inverse-Wishart)
 - update ρ using a Metropolis-Hastings step with proposal $p^* \mid p \sim \text{truncated normal}_{[-1,1]}(\rho, \sigma_\epsilon^2)$
 - sample $\sigma_\epsilon^2 \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \text{ and } \rho$ (Inverse-Gamma)

– For each $k \in K$:

* Sample $\mathbf{U}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}_{[:, -k]}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)

* Sample $\mathbf{V}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}_{[:, -k]}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)

* Sample $\mathbf{D}_{[k,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)¹

¹Subsequent to estimation, \mathbf{D} matrix is absorbed into the calculation for \mathbf{V} as we iterate through K .

A.2. References

- Hoff, Peter D (2005) Bilinear mixed-effects models for dyadic data. *Journal of the American Statistical Association* 100(4690): 286–295.
- Hoff, Peter D (2008). Modeling homophily and stochastic equivalence in symmetric relational data. In: Platt, J. C, Koller, D, Singer, Y & Roweis, S. T (eds.) *Advances in Neural Information Processing Systems 20* *Processing Systems 21* , 657–664. Cambridge, MA, USA. MIT Press.
- Minhas, Shahryar; Peter D Hoff & Michael D Ward (2016) Let’s say amen for latent factor models. Working paper.
- URL:** <https://arxiv.org/abs/1611.00460>