LET'S SAY AMEN FOR LATENT SPACE MODELS

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Interest in networks

Popular approach has been to employ latent space models

Variants.

Response to Cranmer et al. (2016).

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Network analysis provides a way to represent and study "relational data", that is data, with characteristics extending beyond those of the individual, or in the parlance of International Relations (IR), characteristics beyond the monadic. Data structures that extend beyond the monadic level are quite simply the norm when it comes to the study of events such as trade, interstate conflict, or the formation of international agreements. The dominant paradigm in IR for dealing with data structures of this sort, however, is not a network approach but rather a dyadic design, in which an interaction between a pair of countries is considered independent of interactions between any other pair in the system.[1]

The implication of this assumption is that when, for example, Vietnam and the United States decide to form a trade agreement they make this decision independently of what they have done with other countries and what other countries in the international system have done amongst themselves.[2] An even harder assumption to maintain is that Japan declaring war against the United States is independent of the decision of the United States going to war against Japan.[3] A common refrain from those that continue to favor the dyadic approach is that many events are not multilateral (Diehl & Wright, 2016), thus alleviating the need for an approach that incorporates interdependencies between observations. The network perspective, however, is that even the bilateral events we study are taking place within a broader system, and what takes place in one part of that system may be dependent upon another.

The potential for interdependence between observations poses a challenge to statistical modeling as the assumption made by standard approaches used across the social

[1]To highlight the ubiquity of this approach the following represent just a sampling of the articles published from the 1980s to the present in the American Journal of Political Science (AJPS) and American Political Science Review (APSR) that assume dyadic independence: Dixon (1983); Mansfield et al. (2000); Lemke & Reed (2001); Mitchell (2002); Dafoe (2011); Fuhrmann & Sechser (2014); Carnegie (2014).

[2]There has been plenty of work done on treaty formation that would challenge this claim, e.g., see Manger et al. (2012); Kinne (2013).

[3]Maoz et al. (2006); Ward et al. (2007); Minhas et al. (2016) would each note the importance of taking into account network dynamics in the study of interstate conflict.

sciences is that observations are, at least, conditionally independent (Snijders, 2011). The consequence of ignoring this assumption have been frequently noted within the political science literature already.[4] More relevant is the fact that a wealth of research from other disciplines would argue that carrying the independence assumption into a study with relational data is misguided and likely to lead to biased inferences.[5]

Despite the hesitation among some in the discipline to adopt network analytic approaches, in recent years we have at least seen a greater level of interest in understanding these approaches. For instance, in the past year special issues focused on the application of a variety of network approaches have come out in the Journal of Peace Research and International Studies Quarterly. Particularly notable is a piece by Cranmer et al. (2016) that provides an overview and comparison of a handful of widely used network based approaches, specifically, they focus on the exponential random graph model (ERGM), the multiple regression quadratic assignment procedure (MRQAP), and a latent distance approach developed by Hoff et al. (2002). Their discussion around the differences in these approaches and their empirical comparison of them is extremely valuable and necessary, at the same time, they overlook a decade worth of developments that latent variable models have undergone. This is particularly relevant in the context of providing an overview for the field as by focusing on the results from an earlier attempt at a latent variable model, they end up overlooking much of the work that has actually been done using this type of approach in political science. The principal latent variable approach used in political science is the latent factor model, also known as the bilinear mixed-effects model, developed by Hoff (2005). Political science applications of the latent factor model include Hoff & Ward (2004); Ward et al. (2007); Metternich et al. (2015), we are not aware of any political science applications using

[4]For example, see Beck et al. (1998); Signorino (1999); Hoff & Ward (2004); Franzese & Hayes (2007); Cranmer & Desmarais (2011); Erikson et al. (2014).

[5]From Computer Science see: Bonabeau (2002); Brandes & Erlebach (2005). From Economics see: Goyal (2012); Jackson (2014). From Psychology see: Pattison & Wasserman (1999); Kenny et al. (2006). From Statistics see: Snijders (1996); Hoff et al. (2002).

the earlier latent distance approach.[6] As Hoff (2008) notes, the distinction between the latent distance and factor models is consequential when accounting for higher order interdependencies.

In this paper we introduce a more general form of the latent factor model and show that this approach provides a far superior goodness of fit to the application presented in Cranmer et al. (2016) than any of the models they discuss. This approach has been operationalized in the Additive and Multiplicative Effects Models for Networks and Relational Data, **amen**, package available on CRAN.[7] The **amen** package provides for the estimation of many different relational data structures (e.g., binomial, gaussian, and ordinal edges), and it can estimate models for both cross-sectional and longitudinal networks. The rest of this paper proceeds as follows, we introduce the modelling framework used in the **amen** package, compare it to previous implementations of latent variable approaches, and then end by showing how this approach fits the application presented in Cranmer et al. (2016). We believe that this modelling framework can provide a flexible and easy to use scheme through which scholars can study relational data. It addresses the issue of interdependence while still allowing scholars to test theories that may only be relevant in the monadic or dyadic level.[8] At the network level it provides estimates of degree related effects, reciprocity, and provides a descriptive visualization of higher order dependencies such as transitivity. We agree with Cranmer et al. (2016) that it does not allow for the explicit testing of parameter such as the number of two-paths in a network system, however, we would note that for the most part

[6] The software for the latent factor model used in these papers has been available since 2004 at the following address: `http://www.stat.washington.edu/people/pdhoff/Code/hoff_2005_jasa/`.

[7] This package was published on CRAN in mid-2015.

[8] We are aware of the emerging critique from Jones et al. (2016) that latent variable models do not reduce the possibility for inferential error. However, like Cranmer et al. (2016) they use an earlier version of the latent variable approach that, to our knowledge, no one in Political Science has actually applied. Further in our replication of the application presented in Cranmer et al. we show that the approach we present here produces a far better fit of the data than alternative network approaches, while also returning parameter estimates in line with the theoretical arguments described in the original paper.

those parameters are of little use in the types of theories that political scientists tend to develop.

1. Social Relations Model

Warner et al. (1979)

$$
\begin{aligned}
y_{i,j} &= \mu + e_{i,j},\ i \neq j \\
e_{i,j} &= a_i + b_j + \epsilon_{i,j}
\end{aligned} \tag{1}
$$

Decompose variance around μ into parts describing:

heterogeneity across row means (outdegrees) heterogeneity across column means (indegrees) correlation between row and column means correlation within dyads

Hoff (2005) added random sender and receiver effects to model inhomogeneity of the actors, similar to those in the p2 model (van Duijn et al., 2004), and described its generalized linear model formulation, applying it to non-binary data.

Li & Loken (2002) provide a random effects representation

$$
\begin{aligned}
y_{i,j} &= \mu + e_{i,j} \\
e_{i,j} &= a_i + b_j + \epsilon_{i,j} \\
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{a,b}) \\
\{(\epsilon_{i,j}, \epsilon_{j,i}) :\ i \neq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{\epsilon})
\end{aligned} \tag{2}
$$

Modelling non-normal data...probit regression

$$
\begin{aligned}
\epsilon_1, \ldots, \epsilon_n &\overset{\text{iid}}{\sim} N(0,1) \\
z_i &= \beta^T x_i + \epsilon_i \\
y_i &= 1(z_i > 0)
\end{aligned} \tag{3}
$$

Latent variable representation

$$
\begin{aligned}
Pr(Y_i = 1) = P(z_i > 0) &= \Phi(\beta^T x_i) \\
p(y|\beta, X) &= \prod_{i=1}^{n} \Phi(\beta^T x_i)^{y_i} [1 - \Phi(\beta^T x_i)]^{1-y_i}
\end{aligned} \tag{4}
$$

Threshold model linking latent z to observed y

$$
\begin{aligned}
y_{i,j} &= 1(z_{i,j} > 0) \\
z_{i,j} &= \beta^T x_{i,j} + e_{i,j}
\end{aligned} \tag{5}
$$

social relations model including network covariance

$$
\begin{aligned}
e_{i,j} &= a_i + b_j + \epsilon_{i,j} \\
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{a,b}) \\
\{(\epsilon_{i,j}, \epsilon_{j,i}) : \; i \neq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_\epsilon), \text{ where} \\
\Sigma_{a,b} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} & \quad \Sigma_\epsilon = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}
\end{aligned} \tag{6}
$$

ML estimation is problematic for non-Gaussian random effects models: the likelihood involves high-dimensional integrals; MLEs of variance components can be negative.

Bayes estimation provides an alternative: high-dimensional integrals are replaced with MCMC approximations; parameter estimates are within the parameter space.

Z: Latent gaussian variable β: regression coefficients $\{(a_i, b_i),\ i = 1 \ldots n\}$: random effects $\Sigma_{a,b}$ $\Sigma_\epsilon(\sigma^2, \rho)$: network covariance

Gibbs sampler for Bayesian estimation

Iteratively simulate unknowns from their full conditional distributions

(1) simulate $Z \sim p(Z|Y, X, \beta, a, b, \Sigma_\epsilon)$
(2) simulate $\beta \sim p(\beta|X, Z, a, b, \Sigma_\epsilon)$
(3) simulate $a,\ b \sim p(a, b|X, Z, \beta, \Sigma_{a,b}, \Sigma_\epsilon)$
(4) simulate $\Sigma_\epsilon \sim p(\Sigma_\epsilon|X, Z, a, b)$
(5) simulate $\Sigma_{a,b} \sim p(\Sigma_{a,b}|a, b)$

ordinary regression models

(7) $$y_{i,j} \sim \beta^T x_{i,j} + e_{i,j}$$

simple latent variable might include additive node effects

(8) $$e_{i,j} = a_i + b_j + \epsilon_{i,j} \implies y_{i,j} \sim \beta^T x_{i,j} + a_i + b_j + \epsilon_{i,j}$$

$\{(a_1, b_1), \ldots, (a_n, b_n)\}$ represent nodal heterogeneity, additive on the regressor scale. But this model only captures heterogeneity of outdegree/indegree, and canâĂŹt represent more complicated structure, such as clustering, transitivity, etc. The a_i represent nodal sender features and b_j nodal receiver features. $(\epsilon_{i,j}, \epsilon_{j,i})$ represent heterogeneity among dyads.

Networks typically shows evidence AGAINST independence of $\{e_{i,j} : i \neq j\}$. Not accounting for independence can lead to biased effect estimation, uncalibrated confidence intervals, poor predictive performance, and inaccurate description of network phenomenon.

Now we still need to account for third-order dependence patterns.

Probit versions of three latent variable models all have the following form:

$$\begin{aligned}
y_{i,j} &= \begin{cases} 1 & if\ z_{i,j} > 0 \\ 0 & if\ z_{i,j} \leq 0 \end{cases} \\
z_{i,j} &= \mu + \alpha(\mu_i, \mu_j) + \epsilon_{i,j} \\
\{\epsilon_{i,j} : 1 \leq i \leq j \leq n\} &\overset{\text{iid}}{\sim} N(0,1) \\
\{\mu_1, \ldots, \mu_n\} &\overset{\text{iid}}{\sim} f(a|\psi)
\end{aligned} \tag{9}$$

Hoff et al. (2002)

Krivitsky & Handcock (2008) Krivitsky & Handcock (2015)

- Latent class model

$$\begin{aligned}
&\alpha(\mu_i, \mu_j) = \theta_{\mu_i,\mu_j} \\
&\mu_i \in \{1, \ldots, K\},\ i \in \{1, \ldots, n\} \\
&\Theta \text{ a } K \times K \text{ symmetric matrix}
\end{aligned} \tag{10}$$

- Latent distance model

$$\begin{aligned}
&\alpha(\mu_i, \mu_j) = -|\mu_i - \mu_j| \\
&\mu_i \in \Re^K,\ i \in \{1, \ldots, n\}
\end{aligned} \tag{11}$$

- Latent factor model

$$
\begin{aligned}
&\alpha(\mu_i, \mu_j) = \mu_i^T \wedge \mu_j \\
&\mu_i \in \Re^K,\ i \in \{1, \ldots, n\} \qquad (12)\\
&\wedge \text{ a } K \times K \text{ diagonal matrix}
\end{aligned}
$$

Latent factor models can represent row, column, and dyadic correlation, but not efficiently. It may be desirable to combine the latent factor and social relations model.

Hoff (2005) Hoff & Ward (2004)

$$
\begin{aligned}
z_{i,j} &= \beta^T x_{i,j} + u_i^T D v_j + a_i + b_j + \epsilon_{i,j} \\
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{a,b}) \qquad (13)\\
\{(\epsilon_{i,j}, \epsilon_{j,i}) :\ i \leq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_\epsilon)
\end{aligned}
$$

In sum, additive effects can capture network covariance. Multiplicative effects can capture higher-order dependence.

Hoff (2008)

homophily versus stochastic equivalence

euclidean

Most applications of this framework in political science have utilized the latent factor framework over the latent space approach (Hoff & Ward, 2004; Ward et al., 2007, 2012).

Hoff et al. (2015) develop the additive and multiplicative effects network modelling framework.

2. Comparison with Other Approaches

Ingold (2008)

Ingold & Fischer (2014)

Ingold & Leifeld (2014)

Figure 1 highlights how we can expect high levels of sender and receiver heterogeneity.

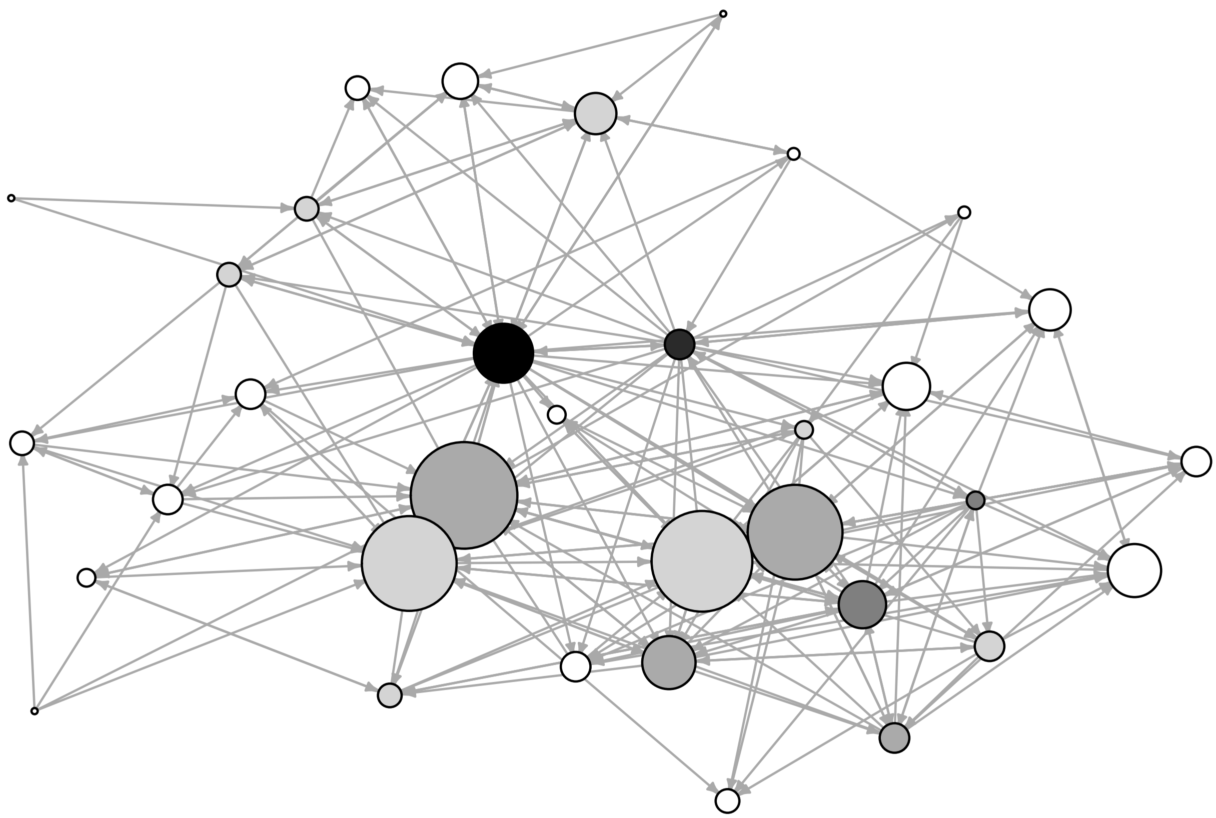

Figure 1. dv net

Krivitsky & Handcock (2015)

	Logit	MRQAP	LSM	ERGM	AME
Intercept/Edges	-4.44*	-4.24*	0.94*	-12.17*	-3.39*
	(0.34)		[0.09; 1.82]	(1.40)	[-4.38; -2.50]
Conflicting policy preferences					
Business vs. NGO	-0.86	-0.87*	-1.37*	-1.11*	-1.37*
	(0.46)		[-2.42; -0.41]	(0.51)	[-2.44; -0.47]
Opposition/alliance	1.21*	1.14*	0.00	1.22*	1.08*
	(0.20)		[-0.40; 0.39]	(0.20)	[0.72; 1.47]
Preference dissimilarity	-0.07	-0.60	-1.76*	-0.44	-0.79*
	(0.37)		[-2.62; -0.90]	(0.39)	[-1.55; -0.08]
Transaction costs					
Joint forum participation	0.88*	0.75*	1.51*	0.90*	0.92*
	(0.27)		[0.86; 2.17]	(0.28)	[0.40; 1.47]
Influence					
Influence attribution	1.20*	1.29*	0.08	1.00*	1.09*
	(0.22)		[-0.40; 0.55]	(0.21)	[0.69; 1.53]
Alter's influence indegree	0.10*	0.11*	0.01	0.21*	0.11*
	(0.02)		[-0.03; 0.04]	(0.04)	[0.07; 0.15]
Influence absolute diff.	-0.03*	-0.06*	0.04	-0.05*	-0.07*
	(0.02)		[-0.01; 0.09]	(0.01)	[-0.11; -0.03]
Alter = Government actor	0.63*	0.68	-0.46	1.04*	0.55
	(0.25)		[-1.08; 0.14]	(0.34)	[-0.07; 1.15]
Functional requirements					
Ego = Environmental NGO	0.88*	0.99	-0.60	0.79*	0.67
	(0.26)		[-1.32; 0.09]	(0.17)	[-0.38; 1.71]
Same actor type	0.74*	1.12*	1.17*	0.99*	1.04*
	(0.22)		[0.63; 1.71]	(0.23)	[0.63; 1.50]
Endogenous dependencies					
Mutuality	1.22*	1.00*		0.81*	
	(0.21)			(0.25)	
Outdegree popularity				0.95*	
				(0.09)	
Twopaths				-0.04*	
				(0.02)	
GWIdegree (2.0)				3.42*	
				(1.47)	
GWESP (1.0)				0.58*	
				(0.16)	
GWOdegree (0.5)				8.42*	
				(2.11)	

Table 1. * $p < 0.05$ (or 0 outside the 95% confidence interval).

3. Capturing Network Attributes

To assess whether the model adequately captures the network parameters of the DV. Here we compare the observed with a set of simulated networks based on certain network statistics (Hunter et al., 2008).

See Morris et al. (2008) for details on each of these parameters.

- Dyad-wise shared partners - Number of dyads in the network with exactly i shared partners
- Edge-wise shared partners - Similar to above except this counts the number of dyads with the same number of edges
- Geodesic distances - The proportion of pairs of nodes whose shortest connecting path is of length k, for $k = 1, 2, \ldots$ Also, pairs of nodes that are not connected are classified as $k = \infty$.
- Incoming k-star - Propensities for individuals to have connections with multiple network partners
- Indegree - degree count is the number of nodes with the same value of the attribute as the ego node
- Outdegree - degree count is the number of nodes with the same value of the attribute as the ego node

Figure 3 give posterior predictive goodness of fit summaries for four network statistics: (1) the empirical standard deviation of the row means; (2) the empirical standard deviation of the column means; (3) the empirical within-dyad correlation; (4) a normalized measure of triadic dependence (Hoff et al., 2015).

Proportion of ties that are reciprocated.

$$t(Y) = \frac{\sum_{i \neq j} y_{i,j} y_{j,i}}{\sum_{i \neq j} y_{i,j}} \tag{14}$$

Number of transitive triplets

(15)
$$t(Y) = \sum_{i \neq j \neq k} y_{i,j} y_{i,k} y_{j,k}$$

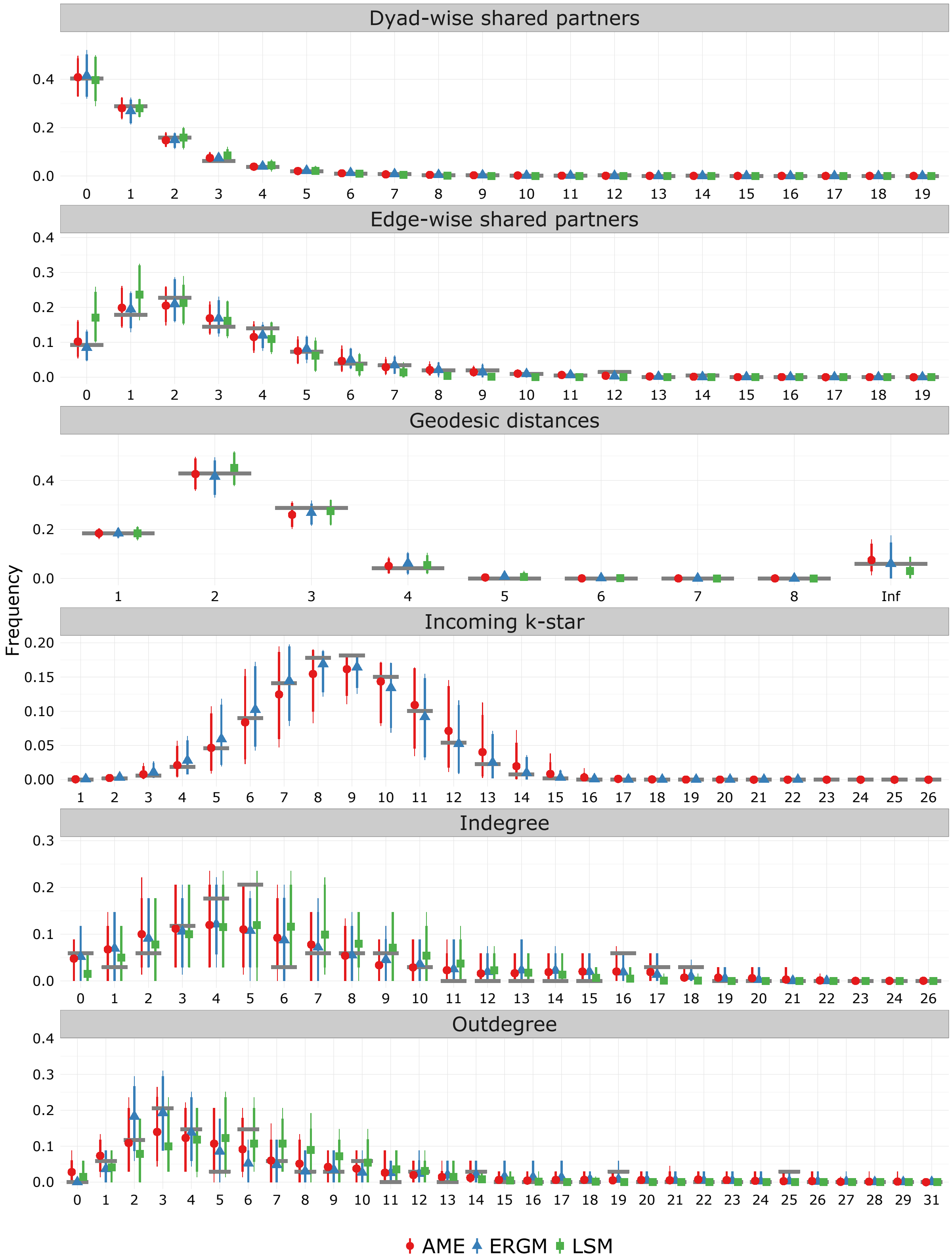

Figure 2. network stats

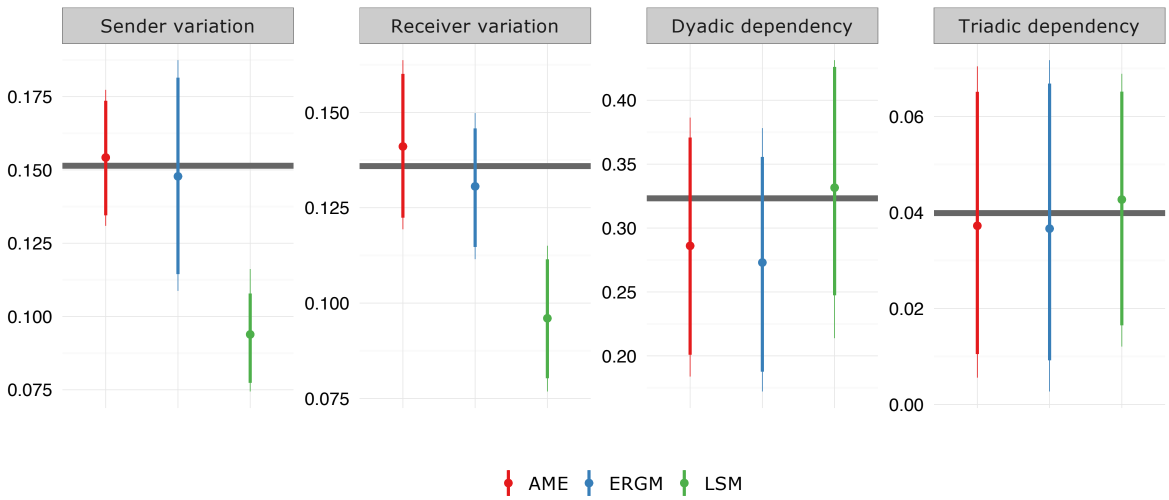

Figure 3. Posterior predictive goodness of fit summary

4. Tie Formation Prediction

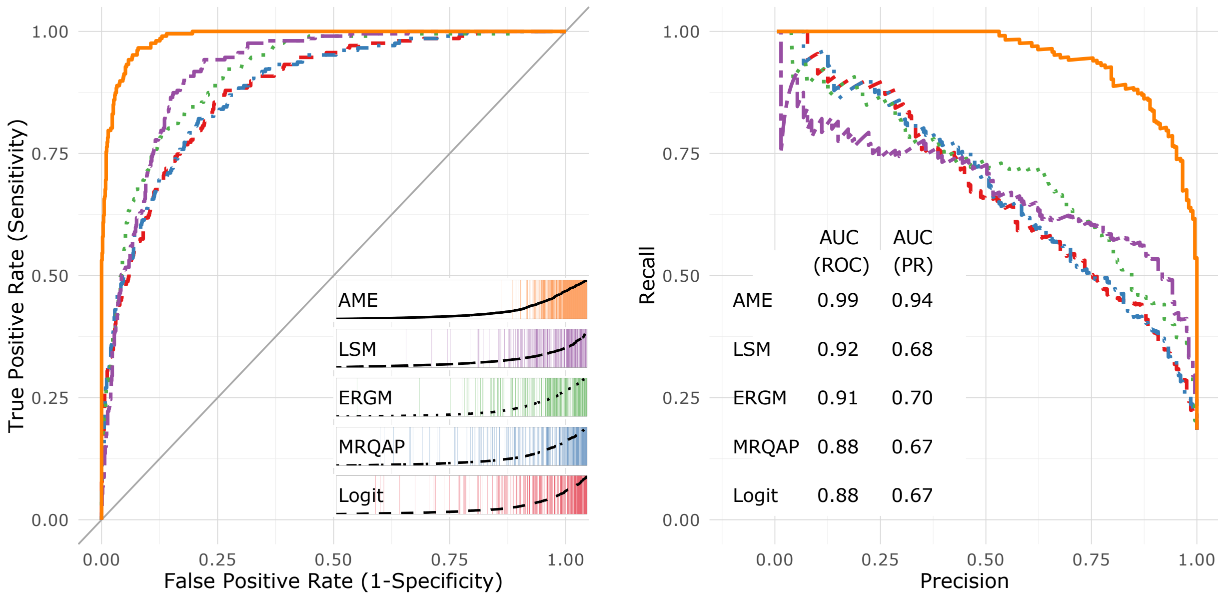

Figure 4. ROC and separation plots

5. Conclusion

6. Appendix

7. AMEN Model Convergence

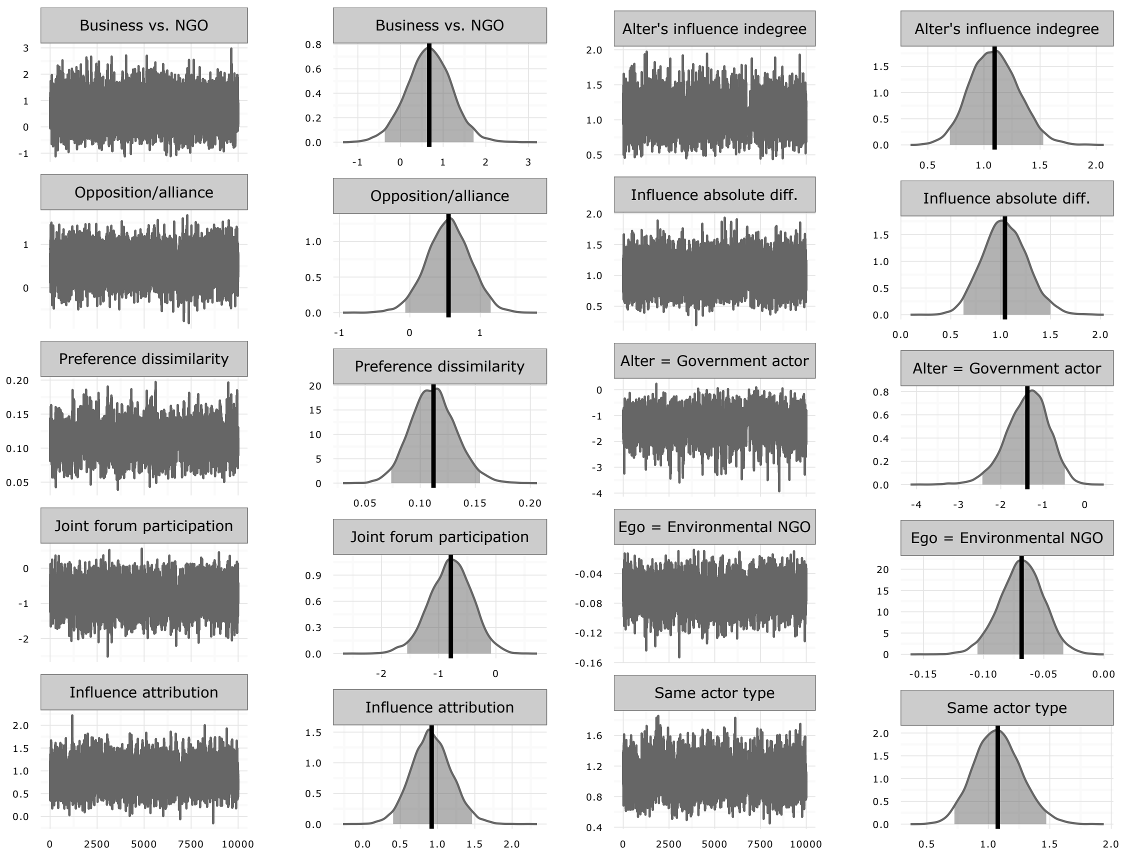

Figure 5. ame convergence k = 2

8. Comparison of amen & latentnet $\mathcal{R}$ Packages

	LSM	LSM (Bilinear)	LSM (SR)	LSM (Bilinear + SR)	AME
Intercept/Edges	0.94*	-2.66*	0.60	-2.50*	-3.39*
	[0.09; 1.82]	[-3.53; -1.87]	[-1.10; 2.37]	[-4.14; -0.88]	[-4.38; -2.50]
Conflicting policy preferences					
Business vs. NGO	-1.37*	-2.64*	-3.07*	-2.87*	-1.37*
	[-2.42; -0.41]	[-4.61; -0.96]	[-4.77; -1.56]	[-4.63; -1.29]	[-2.44; -0.47]
Opposition/alliance	0.00	0.04	0.31	0.24	1.08*
	[-0.40; 0.39]	[-0.44; 0.54]	[-0.24; 0.86]	[-0.36; 0.82]	[0.72; 1.47]
Preference dissimilarity	-1.76*	-2.00*	-1.88*	-2.20*	-0.79*
	[-2.62; -0.90]	[-3.01; -1.03]	[-3.07; -0.68]	[-3.46; -0.96]	[-1.55; -0.08]
Transaction costs					
Joint forum participation	1.51*	1.24*	1.56*	1.62*	0.92*
	[0.86; 2.17]	[0.53; 1.93]	[0.69; 2.41]	[0.70; 2.52]	[0.40; 1.47]
Influence					
Influence attribution	0.08	-0.08	0.30	0.28	1.09*
	[-0.40; 0.55]	[-0.62; 0.46]	[-0.37; 0.96]	[-0.42; 0.97]	[0.69; 1.53]
Alter's influence indegree	0.01	-0.05*	0.06	0.05	0.11*
	[-0.03; 0.04]	[-0.09; -0.01]	[-0.03; 0.14]	[-0.04; 0.13]	[0.07; 0.15]
Influence absolute diff.	0.04	0.02	-0.08*	-0.08*	-0.07*
	[-0.01; 0.09]	[-0.03; 0.07]	[-0.14; -0.02]	[-0.14; -0.02]	[-0.11; -0.03]
Alter = Government actor	-0.46	-0.80	-0.11	-0.20	0.55
	[-1.08; 0.14]	[-1.67; 0.04]	[-1.91; 1.76]	[-2.14; 1.74]	[-0.07; 1.15]
Functional requirements					
Ego = Environmental NGO	-0.60	-1.90*	-1.69	-1.84	0.67
	[-1.32; 0.09]	[-3.10; -0.86]	[-3.74; 0.23]	[-4.02; 0.11]	[-0.38; 1.71]
Same actor type	1.17*	1.40*	1.82*	1.90*	1.04*
	[0.63; 1.71]	[0.85; 1.95]	[1.10; 2.54]	[1.19; 2.62]	[0.63; 1.50]

Table 2. * $p < 0.05$ (or 0 outside the 95% confidence interval).

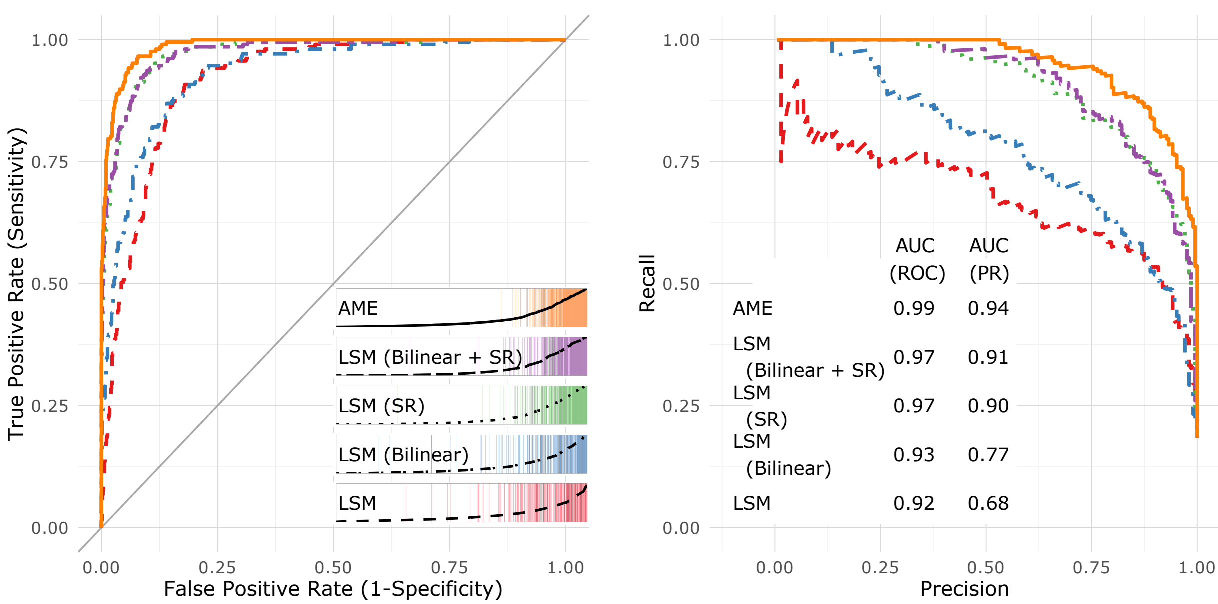

Figure 6. ROC and separation plots

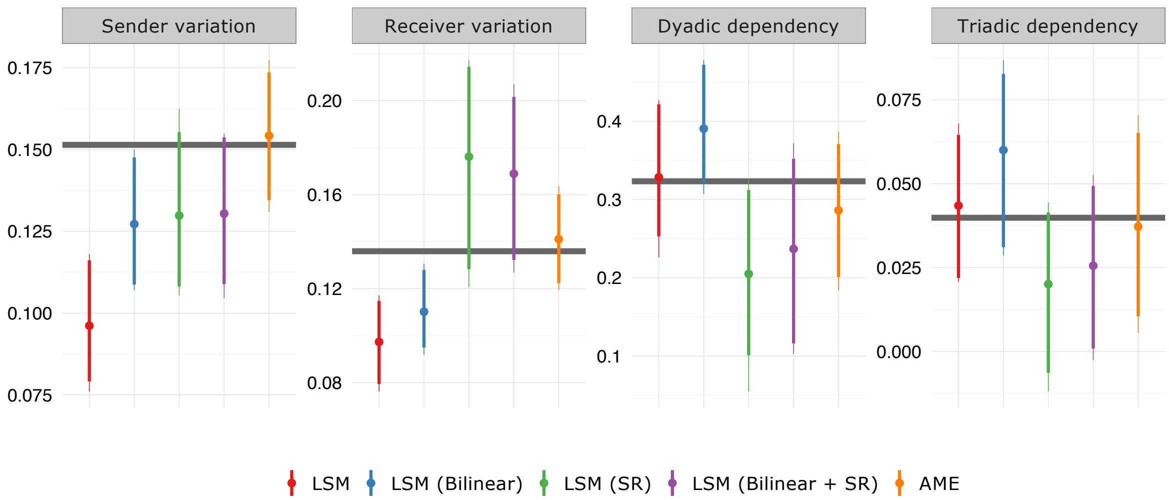

Figure 7. Posterior predictive goodness of fit summary

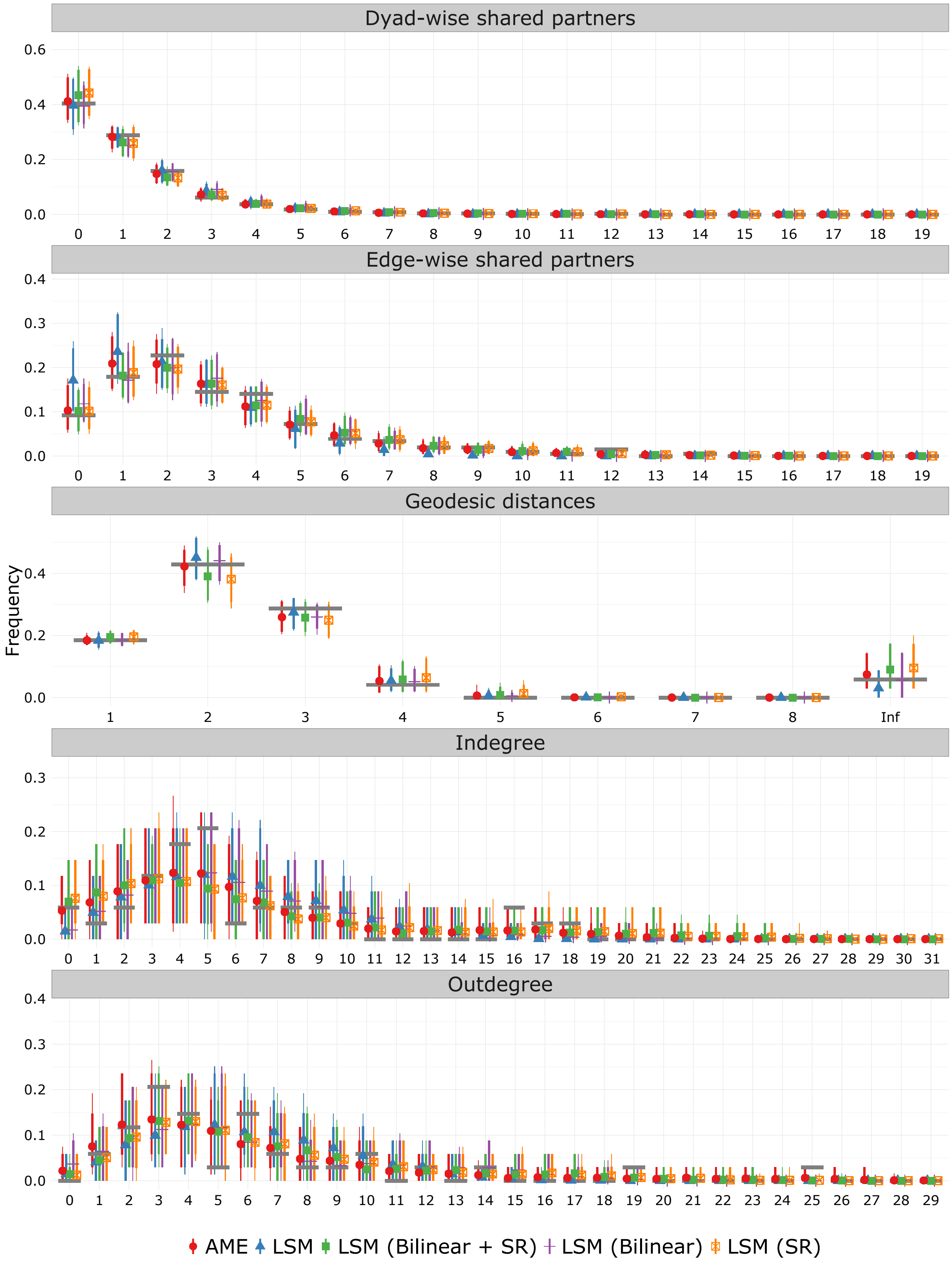

Figure 8. network stats

9. Comparison with other AME Parameterizations

	AME (k=0)	AME (k=1)	AME (k=2)	AME (k=3)	AME (k=4)
Intercept/Edges	-2.75*	-3.08*	-3.39*	-3.72*	-3.93*
	[-3.43; -2.09]	[-3.91; -2.30]	[-4.38; -2.50]	[-4.84; -2.73]	[-5.12; -2.87]
Conflicting policy preferences					
Business vs. NGO	-1.08*	-1.28*	-1.37*	-1.48*	-1.51*
	[-1.82; -0.41]	[-2.20; -0.47]	[-2.44; -0.47]	[-2.63; -0.49]	[-2.69; -0.47]
Opposition/alliance	0.83*	0.95*	1.08*	1.19*	1.28*
	[0.57; 1.10]	[0.64; 1.27]	[0.72; 1.47]	[0.80; 1.64]	[0.86; 1.77]
Preference dissimilarity	-0.49	-0.65*	-0.79*	-0.89*	-0.95*
	[-1.06; 0.06]	[-1.30; -0.03]	[-1.55; -0.08]	[-1.71; -0.12]	[-1.80; -0.14]
Transaction costs					
Joint forum participation	0.73*	0.84*	0.92*	1.01*	1.06*
	[0.34; 1.12]	[0.38; 1.31]	[0.40; 1.47]	[0.44; 1.62]	[0.43; 1.72]
Influence					
Influence attribution	0.88*	1.00*	1.09*	1.21*	1.28*
	[0.57; 1.19]	[0.63; 1.39]	[0.69; 1.53]	[0.75; 1.71]	[0.80; 1.84]
Alter's influence indegree	0.09*	0.10*	0.11*	0.12*	0.13*
	[0.06; 0.12]	[0.07; 0.14]	[0.07; 0.15]	[0.08; 0.17]	[0.09; 0.18]
Influence absolute diff.	-0.06*	-0.06*	-0.07*	-0.07*	-0.08*
	[-0.08; -0.03]	[-0.10; -0.03]	[-0.11; -0.03]	[-0.12; -0.04]	[-0.12; -0.04]
Alter = Government actor	0.49	0.52	0.55	0.60	0.64
	[-0.01; 0.99]	[-0.04; 1.07]	[-0.07; 1.15]	[-0.07; 1.27]	[-0.07; 1.35]
Functional requirements					
Ego = Environmental NGO	0.54	0.61	0.67	0.76	0.80
	[-0.28; 1.36]	[-0.31; 1.56]	[-0.38; 1.71]	[-0.38; 1.90]	[-0.40; 2.04]
Same actor type	0.88*	0.97*	1.04*	1.11*	1.17*
	[0.55; 1.21]	[0.60; 1.35]	[0.63; 1.50]	[0.64; 1.59]	[0.68; 1.68]

Table 3. * $p < 0.05$ (or 0 outside the 95% confidence interval).

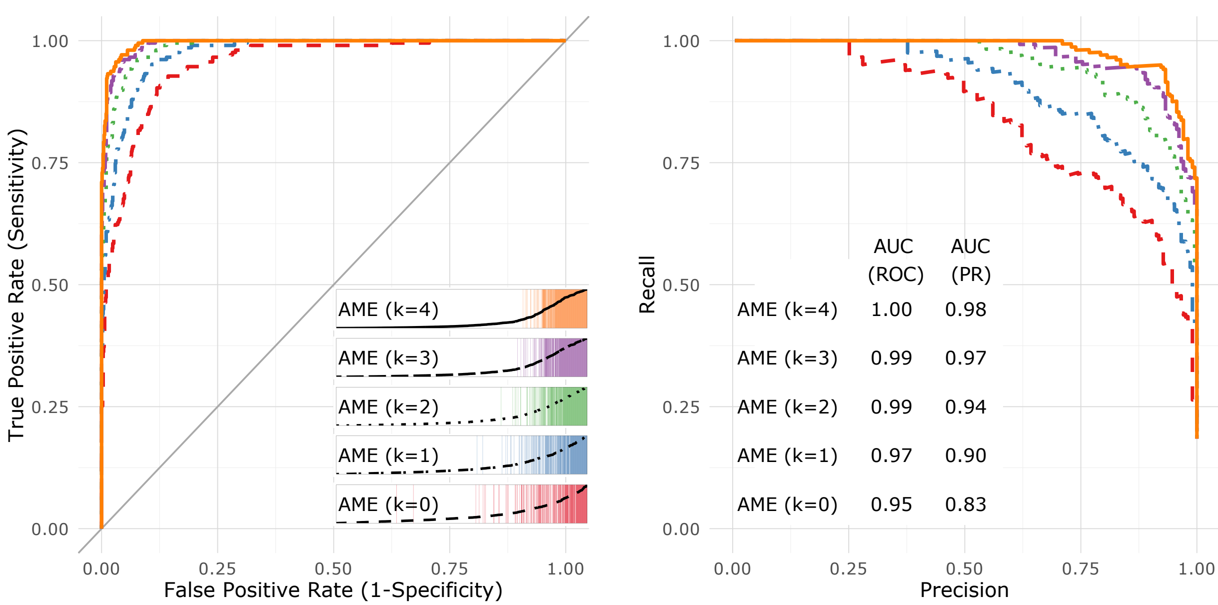

Figure 9. ROC and separation plots

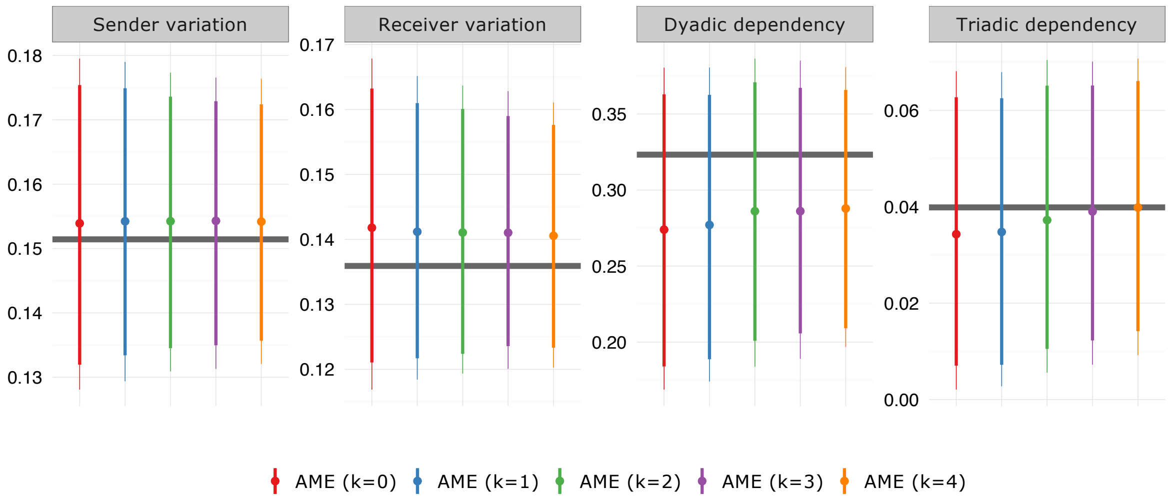

Figure 10. Posterior predictive goodness of fit summary

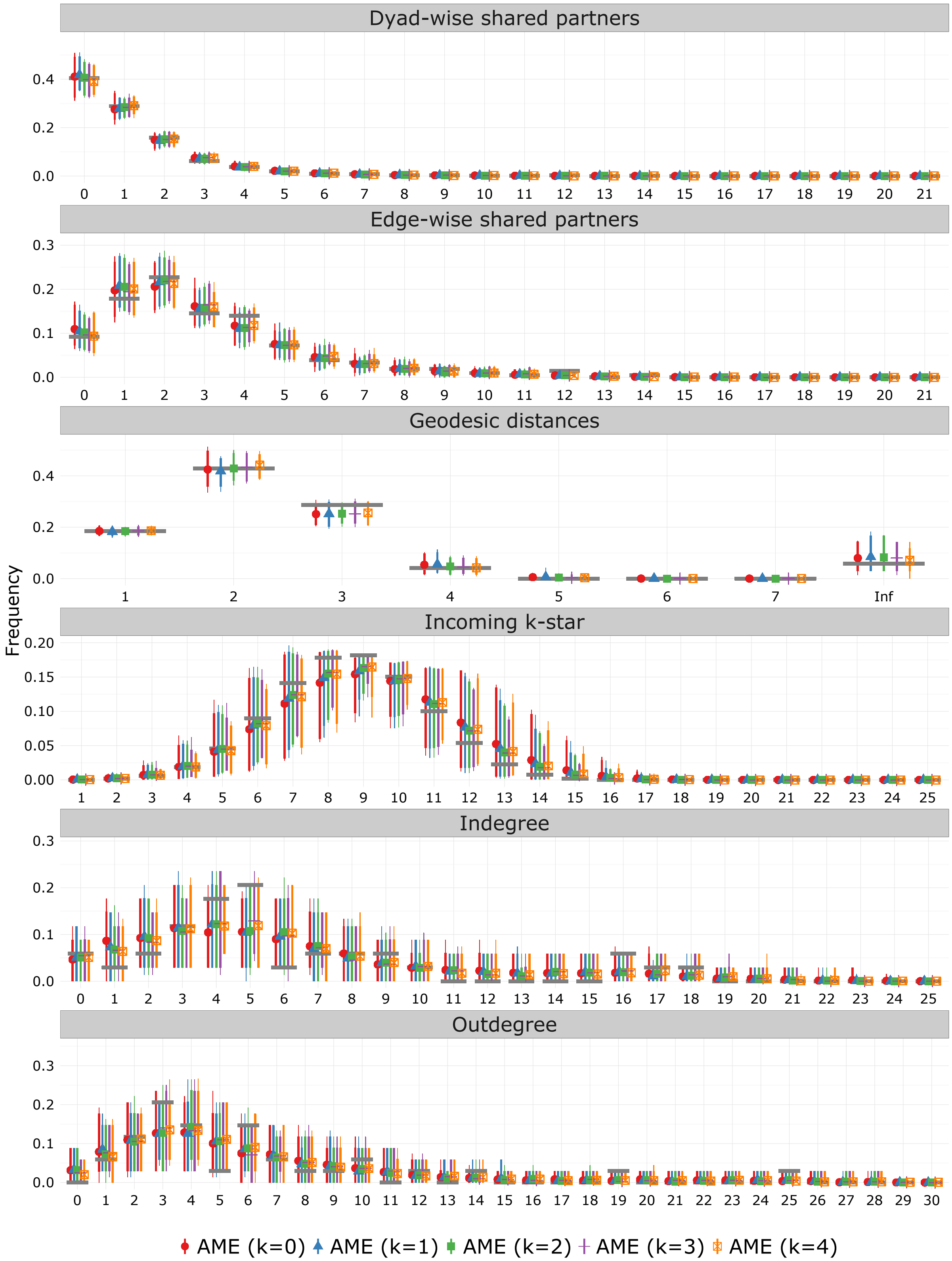

Figure 11. network stats

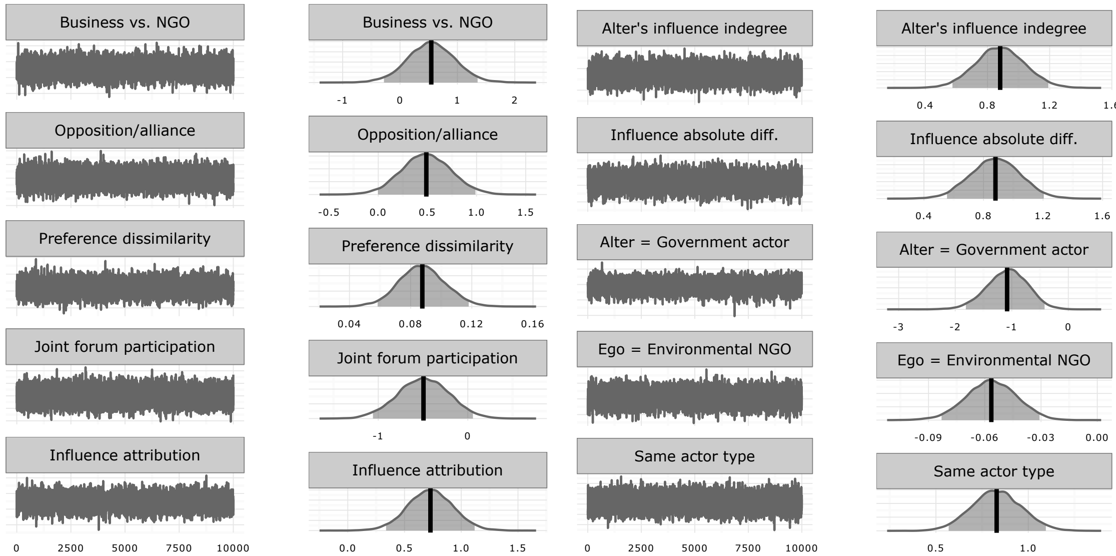

Figure 12. ame convergence k = 0

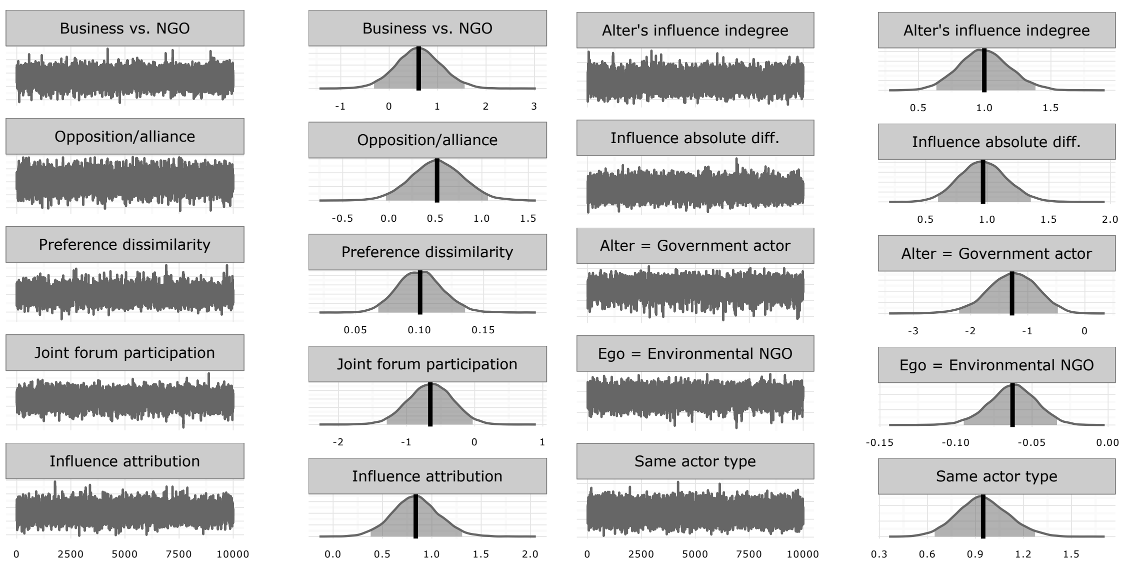

Figure 13. ame convergence k=1

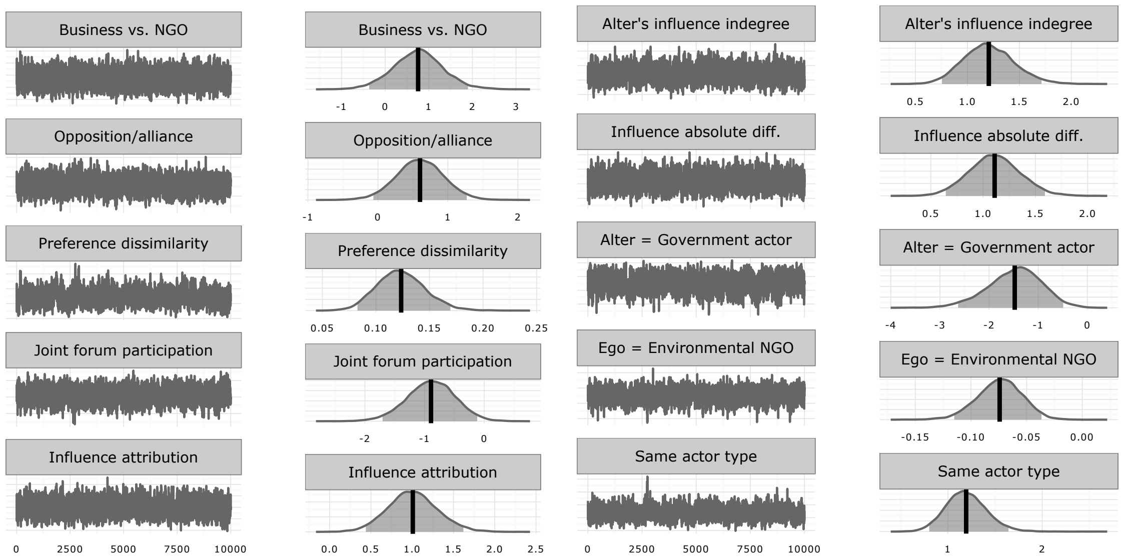

Figure 14. ame convergence k=3

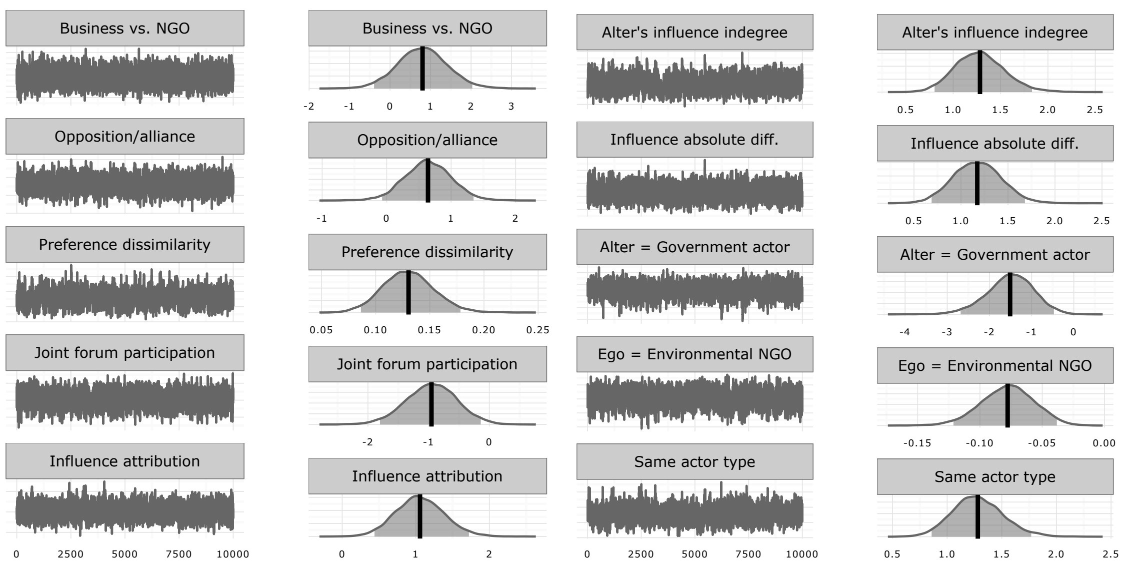

Figure 15. ame convergence k = 4

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