

**MANUSCRIPT ID PA-2018-043 ENTITLED “INFERENTIAL
APPROACHES FOR NETWORK ANALYSIS: AMEN FOR LATENT
FACTOR MODELS”**

Dear Professor Gill,

Thank you for the opportunity to revise and resubmit our manuscript. We believe the manuscript has benefited from the Reviewers’ helpful and thoughtful comments. We have revised the manuscript, taking seriously each individual point raised by the Reviewers. Our comments and responses are shown in *BLUE* below each point raised by the reviewers.

We hope you agree that the manuscript has improved through this process and we are looking forward to your response.

Sincerely,

Shahryar Minhas

1. REVIEWER 1

1.1. Major Comments.

- (1) The dataset suffers from two notable issues with regard to a fair comparison between ERGM, LSM, and the proposed AME approach ... To address both issues, the authors could either describe in detail the merits and demerits of this empirical application for comparing AME to existing frameworks, or could draw up a simple simulation to show the sensitivity of the model comparison results to the factors identified above.
- The first is that each node’s latent position is likely to be correlated with the model terms of interest – a point raised by Cranmer et al. as well (pp. 248-9). In this case, key government players have lower Euclidean distances because they tend to be positioned in the center of the latent space. If this were not the case, would we see such a large divergence between the LSM and AME results? For example, would the AME perform much “better” than the LSM if the network data exhibited a less hierarchical/more egalitarian structure?
 - *We thank the reviewer for his/her comments. To answer this question we conduct a simulation exercise – included in the appendix. Specifically, we set up six simulation scenarios representing varying degrees of “egalitarianism” in terms of network structure. For each scenario, we simulate 50 networks with 100 nodes each and then evaluate the performance of LFM and LSM to predict this network structure. We focus on just comparing the LFM and LSM to make the test more fair. If we were to include the full AME, then degree heterogeneity would be captured within the additive effects portion of the model. The results are shown in Figure 8 of the appendix and included below in Figure 1 as well. Each panel here represents one scenario in which we vary the extent to which the degree distribution is egalitarian. The left most panel represents the situation in which the structure of the network is most egalitarian. Next, we run a LFM and LSM on each of the simulated networks from each scenario and compare the predictive performance based on AUC (ROC) and AUC (PR). We set $K = 2$ for both the LFM and LSM. The results of this analysis indicate that under each scenario LFM performs better than LSM. However, the performance of both models tends to decline as the structure of the simulated networks become less egalitarian (i.e., the extent of tie formation among just a few nodes becomes much higher than the typical node in the network). If covariate information was provided to the model about which nodes were more likely to form ties, then the predictive performance of both models would obviously improve. Additionally, if we were to estimate the full AME model (SRM + LFM) then the additive effects would be able to capture the degree heterogeneity. Typically, in most applied scenarios one would always to include both the additive and multiplicative effects portions when using AME.¹*
 - The second is that the data exhibit high reciprocity such that the directed network is effectively a symmetric undirected network. How would the results differ

¹When using the **amen** package, the SRM portion of the model will be included by default.

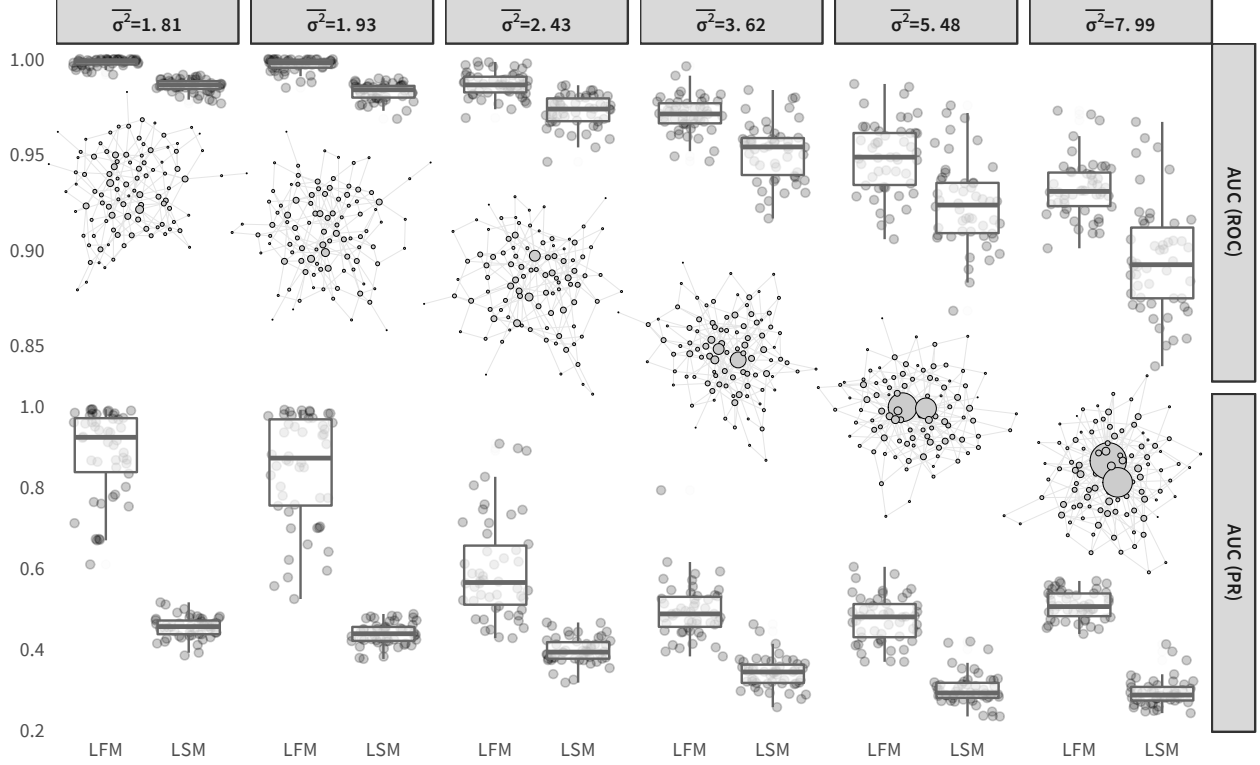


FIGURE 1. Predictive performance of LFM vs LSM for networks under five scenarios (the panels) that vary the extent to which the distribution of ties are egalitarian. We use a box plot to represent the performance of LFM and LSM across fifty simulations for each scenario. The set of network visualizations across the diagonal of the plot illustrate a representative network from one simulation under that scenario, and the size of nodes corresponds to their number of ties. The labels at the top of each panel indicate the standard deviation of the number of ties, which are averaged across the fifty simulations for that scenario.

for a more evenly-balanced directed network (i.e. one where the probability of a reciprocal tie is 50%)?

- *To answer this question we again conduct a simulation exercise – included in the appendix. Here the simulation scenarios vary the level of reciprocity. For each scenario, we again simulate 50 binary, directed networks. The results are included in Figure 9 of the appendix and shown in Figure 2. Each panel here represents represents one scenario with a certain degree of reciprocity. The left-most panel highlights the case where there is little to no reciprocity in the network and the right-most where the level of reciprocity is quite high. The average level of reciprocity across the fifty simulated networks is given at the top of each panel. To compare LFM and LSM, we again utilize AUC (ROC) and AUC (PR) statistics. $K = 2$ for both models and no covariate information is provided. Here again we find that the LFM consistently outperforms the LSM, though at higher levels of reciprocity the performance difference between the two approaches does*

shorten. If we were to estimate the full AME model, then we would be better able to capture reciprocity in the network, as dyadic reciprocity is estimated within the additive effects portion of AME.

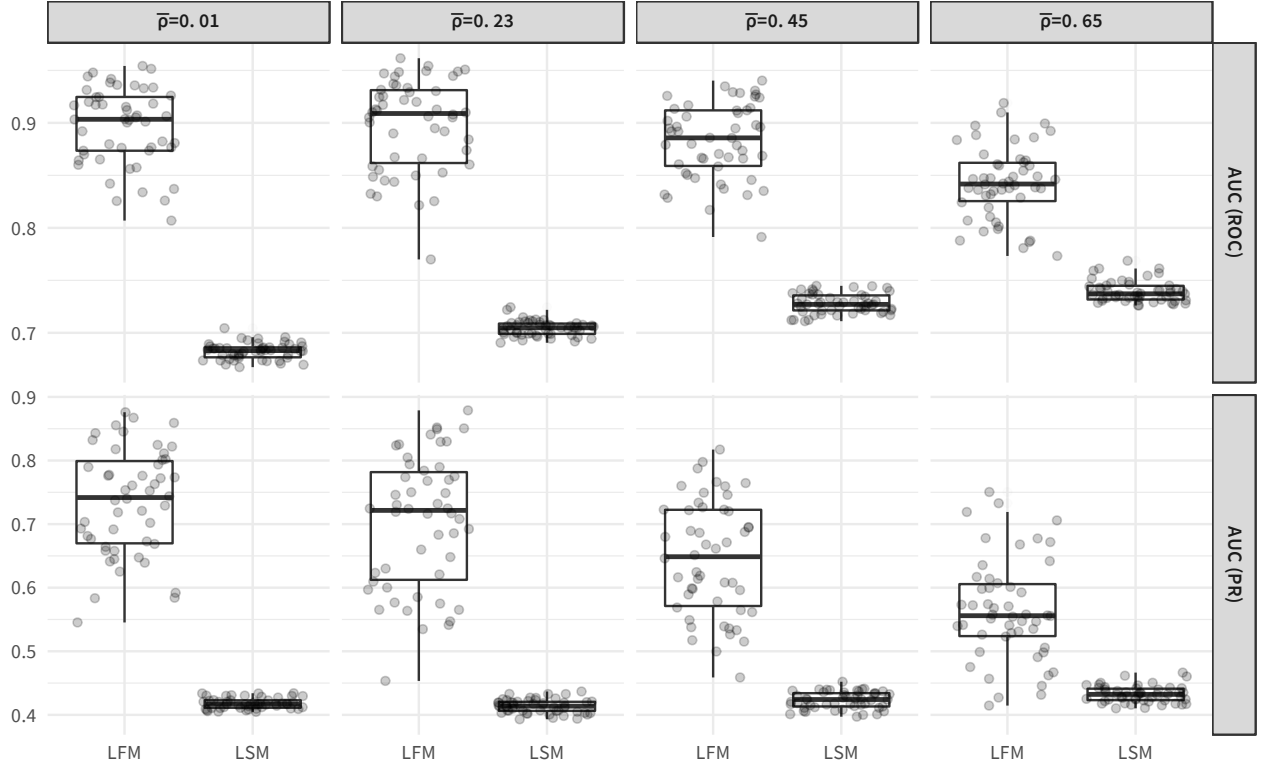


FIGURE 2. Predictive performance of LFM vs LSM for networks with varying levels of reciprocity. We use a box plot to represent the performance of LFM and LSM across fifty simulations for each scenario. The labels at the top of each panel indicate the average level of reciprocity across the fifty simulated in that scenario.

1.2. Minor Comments.

- (1) It would be helpful for the reader to compare AME to non-network models as well, so the authors could include a column in Table 3 for results from a logit model (or the like).
 - *We have included a Logit model for comparison. Figure 2 in the paper shows that the Logit model performs notably worse in terms of out-of-sample predictive performance than the AME.*
- (2) Typo on page 13, first sentence should read: $y = B'x - |u_i u_j|$ instead of $y = B'x + |u_i - u_j|$.
 - *Thank you for pointing this out, we have fixed this typo.*

2. REVIEWER 2

2.1. Major Comments.

- (1) Would it be appropriate to incorporate time dependence in your model given the empirical setup.
 - *We thank the reviewer for his/her comments. Time dependence can be incorporated into the model via time-dependent regression terms such as lags of the dependent variable or simply time-varying regression parameters. We have clarified this point in the conclusion.*
- (2) Talking about inference and using non-experimental data as you do for example on country dyads may be confusing to those who may expect a treatment effects estimator.
 - *The focus of our paper here is not directly related to the growing literature on treatment effect estimators in political science. However, for the reasons outlined in the section entitled “Addressing Dependencies in Dyadic Data” we do believe that this method can be used to more accurately assess the effect of a particular treatment on a dyadic outcome when network dependencies are present.*
- (3) It could be useful to compare your model to the current work going on in modelling network interdependence in statistics, engineering and epidemiology, That may make your model more relevant to the broader field.
 - *Since our primary audience are political scientists we wanted to center our discussion around the types of approaches that the field is currently familiar with and AME. However, we very much agree that a significant amount of relevant work is being done in each of these fields, so we have added a footnote to the conclusion pointing to current work and how it relates to the AME.*
- (4) I also recommend including the estimation of the basic logit model as that is still the starting point for many approaching this field.
 - *We have included a Logit model for comparison. Figure 2 in the paper shows that the Logit model performs notable worse in terms of out-of-sample predictive performance than the AME.*