LET'S SAY AMEN FOR LATENT SPACE MODELS

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Interest in networks

Popular approach has been to employ latent space models

Variants.

Response to Cranmer et al. (2016).

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Network analysis provides a way to represent and study "relational data", that is data, with characteristics extending beyond those of the individual, or in the parlance of International Relations (IR), characteristics beyond the monadic. Data structures that extend beyond the monadic level are quite simply the norm when it comes to the study of events such as trade, interstate conflict, or the formation of international agreements. The dominant paradigm in IR for dealing with data structures of this sort, however, is not a network approach but rather a dyadic design, in which an interaction between a pair of countries is considered independent of interactions between any other pair in the system.[1]

The implication of this assumption is that when, for example, Vietnam and the United States decide to form a trade agreement they make this decision independently of what they have done with other countries and what other countries in the international system have done amongst themselves.[2] An even harder assumption to maintain is that Japan declaring war against the United States is independent of the decision of the United States going to war against Japan.[3] A common refrain from those that continue to favor the dyadic approach is that many events are not multilateral (Diehl & Wright, 2016), thus alleviating the need for an approach that incorporates interdependencies between observations. The network perspective, however, is that even the bilateral events we study are taking place within a broader system, and what takes place in one part of that system may be dependent upon another.

The potential for interdependence between observations poses a challenge to statistical modeling as the assumption made by standard approaches used across the social

[1] To highlight the ubiquity of this approach the following represent just a sampling of the articles published from the 1980s to the present in the American Journal of Political Science (AJPS) and American Political Science Review (APSR) that assume dyadic independence: Dixon (1983); Mansfield et al. (2000); Lemke & Reed (2001); Mitchell (2002); Dafoe (2011); Fuhrmann & Sechser (2014); Carnegie (2014).

[2] There has been plenty of work done on treaty formation that would challenge this claim, e.g., see Manger et al. (2012); Kinne (2013).

[3] Maoz et al. (2006); Ward et al. (2007); Minhas et al. (2016) would each note the importance of taking into account network dynamics in the study of interstate conflict.

sciences is that observations are, at least, conditionally independent (Snijders, 2011). The consequence of ignoring this assumption have been frequently noted within the political science literature already.[4] More relevant is the fact that a wealth of research from other disciplines would argue that carrying the independence assumption into a study with relational data is misguided and likely to lead to biased inferences.[5]

Despite the hesitation among some in the discipline to adopt network analytic approaches, in recent years we have at least seen a greater level of interest in understanding these approaches. For instance, in the past year special issues focused on the application of a variety of network approaches have come out in the Journal of Peace Research and International Studies Quarterly. Particularly notable is a piece by Cranmer et al. (2016) that provides an overview and comparison of a handful of widely used network based approaches, specifically, they focus on the exponential random graph model (ERGM), the multiple regression quadratic assignment procedure (MRQAP), and a latent distance approach developed by Hoff et al. (2002). Their discussion around the differences in these approaches and their empirical comparison of them is extremely valuable and necessary, at the same time, they overlook a decade worth of developments that latent variable models have undergone. This is particularly relevant in the context of providing an overview for the field as by focusing on the results from an earlier attempt at a latent variable model, they end up overlooking much of the work that has actually been done using this type of approach in political science. The principal latent variable approach used in political science is the general bilinear mixed-effects (GBME) model, developed by Hoff (2005). Examples of political science applications of the GBME include Hoff & Ward (2004); Ward et al. (2007); Metternich et al. (2015), we

[4]For example, see Beck et al. (1998); Signorino (1999); Hoff & Ward (2004); Franzese & Hayes (2007); Cranmer & Desmarais (2011); Erikson et al. (2014).

[5]From Computer Science see: Bonabeau (2002); Brandes & Erlebach (2005). From Economics see: Goyal (2012); Jackson (2014). From Psychology see: Pattison & Wasserman (1999); Kenny et al. (2006). From Statistics see: Snijders (1996); Hoff et al. (2002).

are not aware of any political science applications using the earlier latent distance approach.[6] As Hoff (2008) notes, the distinction between the latent distance and factor models is consequential when accounting for higher order interdependencies.

In this paper we introduce a more general form of the GBME that we refer to as the additive and multiplicative effects network model (AMEN) and show that this approach provides a far superior goodness of fit to the application presented in Cranmer et al. (2016) than any of the models they discuss.[7] Through the AMEN approach we can estimate many different types of cross-sectional and longitudinal relational data structures (e.g., binomial, gaussian, and ordinal edges). The rest of this paper proceeds as follows, we briefly motivate the need for network oriented approaches, introduce the AMEN modeling framework, compare it to previous implementations of latent variable approaches, and then end by showing how this approach fits the application presented in Cranmer et al. (2016).

We believe that this modeling framework can provide a flexible and easy to use scheme through which scholars can study relational data. It addresses the issue of interdependence while still allowing scholars to test theories that may only be relevant in the monadic or dyadic level.[8] Further at the network level it provides estimates of degree related effects, reciprocity, and provides a descriptive visualization of higher order dependencies such as transitivity.

[6]The software for the latent factor model used in these papers has been available since 2004 at the following address: `http://www.stat.washington.edu/people/pdhoff/Code/hoff_2005_jasa/`.

[7]The AMEN approach has already been developed into a package named **amen** and is available on CRAN (Hoff et al., 2015).

[8]We are aware of the emerging critique from Jones et al. (2016) that latent variable models do not reduce the possibility for inferential error. However, like Cranmer et al. (2016) they use an earlier version of the latent variable approach that, to our knowledge, no one in political science has actually applied. Further in our replication of the application presented in Cranmer et al. we show that the approach we present here produces a far better fit of the data than alternative network approaches, while also returning parameter estimates in line with the theoretical arguments described in the original paper.

1. Addressing Dependencies in Dyadic Data

Relational, or dyadic, data structures provide measurements of how pairs of actors relate to one another. These structures encompass events of interest as diverse as the level of trade between i and j to the occurrence of an interstate conflict. The easiest way to organize such data is the directed dyadic design in which the unit of analysis is some set of n actors that have been paired together to form a dataset of z directed dyads. A tabular design such as this for a set of n actors, $\{i, j, k, l\}$ results in $n \times (n-1)$ observations, as shown in Table 1.

Sender	Receiver	Event
i	j	y_{ij}
$\vdots$	k	y_{ik}
	l	y_{il}
j	i	y_{ji}
$\vdots$	k	y_{jk}
	l	y_{jl}
k	i	y_{ki}
$\vdots$	j	y_{kj}
	l	y_{kl}
l	i	y_{li}
$\vdots$	j	y_{lj}
	k	y_{lk}

Table 1. Structure of datasets used in canonical design.

$\longrightarrow$

	i	j	k	l
i	NA	y_{ij}	y_{ik}	y_{il}
j	y_{ji}	NA	y_{jk}	y_{jl}
k	y_{ki}	y_{kj}	NA	y_{kl}
l	y_{li}	y_{lj}	y_{lk}	NA

Table 2. Adjacency matrix representation of data in Table 1. Senders are represented in the rows and receivers the columns.

1.1. Limitations of the Standard Framework.

When modeling these types of data structures, scholars typically employ a generalized linear model (GLM) estimated via maximum-likelihood. This type of model can be expressed via a stochastic and systematic component (Ward & Ahlquist, 2010). The stochastic component reflects our assumptions about the probability distribution from which the data is generated: $y_{ij} \overset{\text{iid}}{\sim} \mathcal{F}(\theta_{ij})$, where $\mathcal{F}$ represents a probability distribution or mass function such as normal or binomial, and $\overset{\text{iid}}{\sim}$ represents the assumption that each dyad in our sample is independently drawn from that particular distribution. The systematic component characterizes the model for the parameters of that distribution and describes how θ_{ij} varies as a function of a set of nodal and dyadic covariates, $\mathbf{X}_{ij}$: $\theta_{ij} = \beta^T \mathbf{X}_{ij}$. A fundamental assumption we make when applying this modeling technique is that given $\mathbf{X}_{ij}$ the parameters of our distribution each of the dyadic observations are conditionally independent.

The usage of this assumption becomes clearer if we go a bit further in the process of estimating a GLM via maximum likelihood. After having chosen a set of covariates and specifying a distribution we construct joint density function over all dyads.

$$\begin{aligned} Pr(y_{ij}, y_{ik}, \ldots, y_{lk} | \theta_{ij}, \theta_{ik}, \ldots, \theta_{lk}) = \mathcal{F}(\theta_{ij}) \times \mathcal{F}(\theta_{ik}) \times \ldots \times \mathcal{F}(\theta_{lk}) \\ Pr(\mathbf{Y} = (y_{ij}, y_{ik}, \ldots, y_{lk}) \mid \boldsymbol{\theta} = (\theta_{ij}, \theta_{ik}, \ldots, \theta_{lk})) = \prod_{a=1}^{z} \mathcal{F}(\theta_a) \end{aligned} \tag{1}$$

We next convert the joint probability into a likelihood by assuming the observations are fixed but the distributional parameters, $\boldsymbol{\theta}$, to be random:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\theta}|\mathbf{Y}) &= k(\mathbf{Y}) \times Pr(\mathbf{Y}|\boldsymbol{\theta}) \\ \mathcal{L}(\boldsymbol{\theta}|\mathbf{Y}) &= k(\mathbf{Y}) \times \prod_{a=1}^{z} \mathcal{F}(y_a|\theta_a) \\ \mathcal{L}(\boldsymbol{\theta}|\mathbf{Y}) &\propto \prod_{a=1}^{z} \mathcal{F}(y_a|\theta_a) \end{aligned} \tag{2}$$

Having constructed the likelihood we can proceed by solving through maximization or numerical analysis. However, the important point to note here is that the likelihood as defined above is only valid if we are able to make the assumption that, for example,

y_{ij} is independent of y_{ji} and y_{ik} given the set of covariates we specified.[9] Assuming y_{ij} is independent of y_{ji} asserts that there is no level of reciprocity in a dataset, an assumption that in many cases would seem quite untenable.[10] A harder problem to handle is the assumption that y_{ij} is independent of y_{ik}, the difficulty here follows from the possibility that i's relationship with k is dependent on how i relates to j and how j relates to k, or more simply put the "enemy of my enemy [may be] my friend".

The presence of these types of interdependencies in relational data structures complicates the a priori assumption of observational independence, and without this assumption the joint density function cannot be written in the way described above and we cannot produce a valid likelihood.[11] Thus inferences drawn from models that ignore potential interdependencies between dyadic observations are likely to have a number of issues such as biased effect estimation, uncalibrated confidence intervals, and poor predictive performance. Just as important, however, is that by ignoring these interdependencies we ignore a potentially important part of the data generating process behind relational data structures, namely, network phenomena.

1.2. **Social Relations Model: Additive Part of AMEN.** The dependencies that tend to develop in relational data can be more easily understood when we move away from stacking dyads on top of one another and turn instead to adjacency matrices as shown in Table 2. Operationally, this type of data structure is represented as a $n \times n$ matrix, $\mathbf{Y}$, where the diagonals in the matrix are typically undefined.[12] The ij^{th} entry defines the relationship between i and j and can be continuous or discrete. If the matrix is undirected, the ji^{th} entry will equal the ij^{th} entry. In undirected data an event cannot be

[9]The difficulties of applying the GLM framework to data structures that have structural interdependencies between observations is a problem that has long been recognized. Beck & Katz (1995), for example, detail the issues with pooling observations in time-series cross-section datasets. Ward & Gleditsch (2008) have done the same in the case of spatial dependence.

[10]For example, see Ward et al. (2007); Cranmer et al. (2014).

[11]This problem has been noted in works such as Lai (1995); Manger et al. (2012); Kinne (2013).

[12]Most of the relational variables studied in political science do not involve events that countries can send to themselves.

attributed to a specific sender or receiver rather it is just an indication of a relationship between a pair of countries, an example of this that commonly arises in the IR literature involves models of alliance relationships. In directed matrices the off-diagonal values are not symmetric and there is a clear sender and receiver as in the case of exports or aid flows.

A common type of structural interdependency that arises in relational data structures is "preferential attachment" (Réka et al., 1999). This is typically categorized as a first-order, or nodal, dependency and represents the fact that we typically find significant heterogeneity in activity levels across nodes. The implication of this across-node heterogeneity is within-node heterogeneity of ties, meaning that values across a row, say $\{y_{ij}, y_{ik}, y_{il}\}$, will be more similar to each other than other values in the adjacency matrix because each of these values has a common sender i. This type of dependency would manifest in cases where sender i tends to be more active in the network than other senders. The emergence of this type of structure often occurs in relational datasets such as trade and conflict. In both these cases, there are a set of countries that tend to be more active than others. Similarly, while some actors may be more active in sending ties to others in the network, we might also observe that others are more popular targets, this would manifest in observations down a column, $\{y_{ji}, y_{ki}, y_{li}\}$, being more similar. Last, we might also find that actors who more likely to send ties in a network are also more likely to receive them, meaning that the row and column means of an adjacency matrix may be correlated. First-order dependencies are equally important to take into account in undirected relational structures, the only difference being that nodal heterogeneity will be equivalent across rows and columns. The presence of this type of heterogeneity in directed and undirected relational data structures leads to a violation of the conditional independence assumption underlying the models in our standard tool-kit.

Another ubiquitous type of structural interdependency is reciprocity. This is a second-order, or dyadic, dependency relevant only to directed datasets, and asserts that values of y_{ij} and y_{ji} may be statistically dependent. In studies of social and economic behavior, direct reciprocity – the notion that actors learn to “respond in kind" to one another – is argued to be an essential component of behavior.[13] More specifically, this concept actually has deep roots in political science theories of cooperation and the evolution of norms between states (Richardson, 1960; Choucri & North, 1972; Keohane, 1989). This concept has particular relevance in the conflict literature, as we would expect that if, for instance, Iran behaved aggressively towards Saudi Arabia that this would induce Saudi Arabia to behave aggressively in return. The prevalence of these types of potential interactions within directed dyadic data structures also complicates the basic assumption of observational independence.

The relevance of modeling first- and second-order dependencies has long been recognized within some social sciences such as psychology, and to do so Warner et al. (1979) developed the social relational model (SRM), a type of ANOVA decomposition technique.[14] The SRM is of particular note as it provides error structure for the additive effects component of the AMEN framework that we introduce here. The goal of the SRM is to decompose the variance of observations in an adjacency matrix in terms of heterogeneity across row means (out-degree), heterogeneity across column means (in-degree), correlation between row and column means, and correlations within dyads. Wong (1982) and Li & Loken (2002) provide a random effects representation of the SRM:

[13] For example, see Bolton et al. (1998); Cox et al. (2007).

[14] Dorff & Ward (2013) provide a detailed introduction to this model and Dorff & Minhas (2016) apply this approach to studying reciprocal behavior in economic sanctions.

$$
\begin{aligned}
y_{ij} &= \mu + e_{ij} \\
e_{ij} &= a_i + b_j + \epsilon_{ij} \\
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{a,b}) \\
\{(\epsilon_{ij}, \epsilon_{ji}) :\ i \neq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{\epsilon}), \text{ where} \\
\Sigma_{a,b} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} &\quad \Sigma_{\epsilon} = \sigma_{\epsilon}^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}
\end{aligned} \tag{3}
$$

The basic idea here is quite simple, μ provides a baseline measure of the density or sparsity of a network, and e_{ij} represents residual variation. We then decompose that residual variation into parts, namely, a row/sender effect (a_i), a column/receiver effect (b_j), and a within dyad effect (ϵ_{ij}). The row and column effects are modeled jointly to account for correlation in how active an actor is in sending and receiving ties. Heterogeneity in the row and column means is captured by σ_a^2 and σ_b^2, respectively, and σ_{ab} describes the linear relationship between these two effects (i.e., whether actors who send [receive] a lot of ties also receive [send] a lot of ties). Beyond these first-order dependencies, variation across second-order dependencies is described by σ_{ϵ}^2 and a within dyad correlation, or reciprocity, parameter ρ.

Hoff (2005) shows that the SRM covariance structure described in Equation 3 can be incorporated into the systematic component of the GLM framework we described earlier to produce a generalized linear mixed effects model: $\theta_{ij} = \beta^T \mathbf{X}_{ij} + a_i + b_j + \epsilon_{ij}$, where $\beta^T \mathbf{X}_{ij}$ accommodates the inclusion of dyadic, sender, and receiver covariates. Through this approach we can effectively incorporate row, column, and within-dyad dependence in a regression framework. Further this approach can be extended to a handle a diversity of outcome distributions (e.g., binomial, ordinal, etc.). In the case of binary data this can be done by utilizing a latent variable representation of a probit regression model. Specifically, we model a latent variable, z_{ij}, with a linear predictor and we model the error using the SRM from Equation 3: $z_{ij} = \beta^T \mathbf{X}_{ij} + e_{ij}$. Then we can simply utilize a threshold model linking z_{ij} to our observed values of y_{ij}: $y_{ij} = I(z_{ij} > 0)$. The result is actually a model that is very similar to the p1 and p2 ERGMs developed by Holland & Leinhardt (1981) and Duijn et al. (2004), respectively.

1.3. **Latent Space Models: Multiplicative Part of AMEN.** Missing from the framework provided by the SRM is an accounting of third order dependence patterns that can arise in relational data. The presence of third order effects in relational datasets can be seen in graphs that contain clumps of nodes all linked to each other. This "clumpage" of linked nodes indicates that there may be an unmodeled shared attribute(s) between those nodes, which make them more likely to interact with one another. For example, when modeling trade flows between a particular pair of countries it is often the case that neighboring countries are more likely to trade than others. This is an example of homophily, countries possessing some shared characteristic are more likely to interact.[15] Alternatively, we also find evidence of anti-homophily in networks, for example, countries that share a military alliance are obviously unlikely to engage in conflict with one another.

Both of these examples can be framed more generally, say that we have a binary network where actors tend to form ties to others based on some set of shared characteristics. This would inevitably lead to a network graph with a high number of "transitive triplets", that is cases in which we have sets of actors $\{i, j, k\}$ each being linked to one another. The left-most plot in figure 1 provides a representation of a network that exhibits a high degree of homophily. Structures such as this may develop when i interacts with j and k because i possesses some shared characteristic with those actors – in the case of anti-homophily i might not interact with those actors for the same reason.

[15]See Shalizi & Thomas (2011) for a more detailed discussion on the concept of homophily.

Higher-order dependencies result from the possibility that if i has some shared characteristics with j and k, j and k may also share that characteristic (i.e., they are likely also neighbors of one another or they are also a part of an alliance agreement). The implication of this is that the probability of j and k forming a tie is, first, not independent of the ties that already exist between those actors and i, and, second, higher than the probability that either of those actors might form a tie with another actor, l with whom they have no shared attributes.

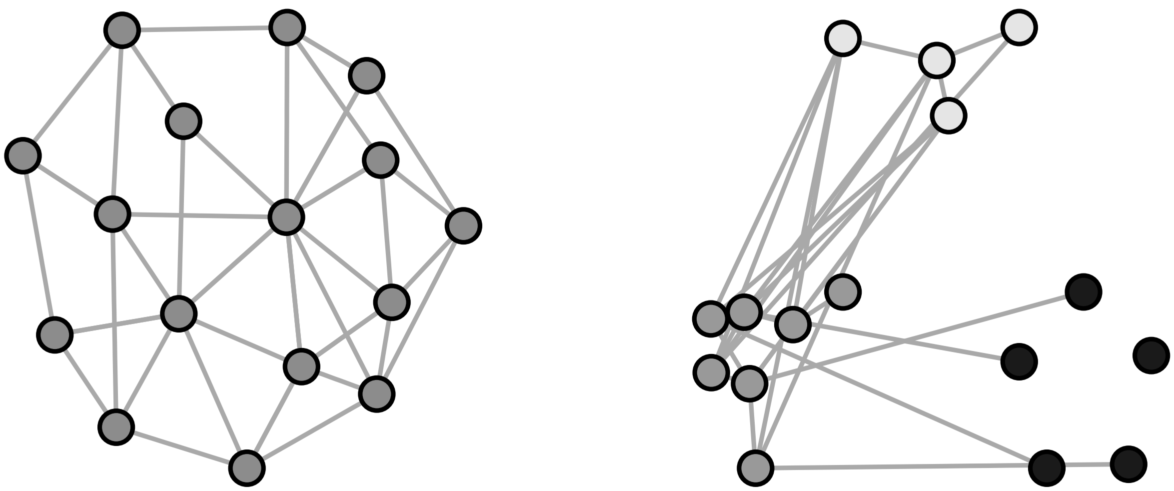

Figure 1. Graph on the left is a representation of an undirected network that exhibits a high degree of homophily, while on the left we show an undirected network that exhibits stochastic equivalence.

Another dependence pattern that cannot be accounted for in the framework discussed above is stochastic equivalence. A pair of actors ij are stochastically equivalent if the probability of i relating to, and being related to, by every other actor is the same as the probability for j (Anderson et al., 1992). More simply put this refers to the idea that there will be groups of nodes in a network with similar relational patterns. The occurrence of a dependence pattern such as this is not uncommon in an IR context. Manger et al. (2012) theorize and estimate a stochastic equivalence structure to explain the formation of preferential trade agreements (PTAs). Specifically, they begin by dividing up countries into high, middle, and low income groups. They find that PTA formation occurs with greater probability in the following order high-middle, high-high, and middle-middle income groups, and that low income countries are rather unlikely to form PTAs with any partner. Such a structure is represented in the right-most panel of Figure 1, here the lightly shaded group of nodes at the top can represent high-income countries, nodes on the bottom-left middle-income, and the darkest shade of nodes low-income countries. The point here is just that the behavior of actors in a network can at times be governed by group level dynamics rather than attributes specific to an actor, and failing to account for patterns such as this could lead to discounting an important part of the data generating process.

If we are able to account for the variety of shared attributes that might cause third order dependence patterns to develop then the additive effects described above is likely enough to justify our assumption of conditional independence and we can proceed to interpret our results. The **amen** package even provides for the estimation of that type of model using a Bayesian framework.[16] In the context of most observational research, however, this assumption is untenable. The implausibility of an assumption such as this is, in spirit, the same reason why we no longer model time-series cross-sectional data without including, for example, country level fixed or random effects.

As Cranmer et al. (2016) note a number of solutions have been suggested to this problem. They provide brief descriptions of the MRQAP and ERGM that we will not rehash here. The only point that we would emphasize from their discussion is that ERGMs should only be used when researchers are not only interested in modeling the specific set of third order patterns that may have given rise to a certain network, but also able to provide a theoretical justification for why those endogenous dependencies were specified over others. Failure to properly account for higher order dependence structures through an appropriate specification can at best lead to model degeneracy,

[16] The main function in the **amen** package is titled "ame" and by default it runs a model assuming that no multiplicative effects are necessary. There are also a set of utilities one can use to determine whether the inclusion of multiplicative effects is necessary, we will review these in the following section.

which provides an obvious indication that the specification needs to be altered, and at worst a result that converges but does not appropriately capture the interdependencies in the network (Handcock, 2003; Hunter et al., 2012).[17] The consequence of the latter case is a set of inferences that will continue to be biased as a result of unmeasured heterogeneity, thus defeating what we see to be a major motivation for pursuing an inferential network model in the first place.

Of more relevance to us in this paper is the review that Cranmer et al. (2016) provide of latent space approaches. Of more relevance to us in this paper is the review that Cranmer et al. (2016) provide of latent space approaches. Generally, the utilization of latent variable models for network analysis is a popular approach for modeling relational data in fields as diverse as computer science to the social sciences. One obvious reason for their continued usage is that they enable us to capture and visualize third order dependencies in a way that other approaches are not able to replicate. Additionally, the conditional independence assumption that these models are able to provide implies that model degeneracy is not an issue, facilitating the testing of a variety of nodal and dyadic level theories, and providing a range of computational advantages (Hunter et al., 2012).

They focus primarily on an approach that addresses third order dependency structures through embedding a graph onto a latent k-dimensional Euclidean space where the probability of a tie between nodes is decreasing in their distance in that space (we refer to this as the latent distance model). This approach was developed by Hoff et al. (2002), but, to our knowledge, the dominant approach in political science has been the

[17]For a related discussion on the questionable tractability and consistency of modeling relational data through ERGMs see Bhamidi et al. (2008); Chatterjee & Diaconis (2013); Chandrasekhar & Jackson (2014).

latent factor approach, also known as the GBME, developed by Hoff (2005) and introduced to political science by Hoff & Ward (2004).[18] The distinction between these approaches is consequential and also informs the choices that were made in the creation of the alternative approach, AMEN, that we are introducing here.[19]

In the latent distance model, each node i has some unknown latent position in k dimensional space, $u_i \in R^k$, and the probability of a tie between a pair ij is a function of the negative Euclidean distance between them: $-|u_i$ and $u_j|$. The basis

The closer two actors are in this latent Euclidean space, the more likely they are to form an edge. This low-dimensional space can be thought of as a characteristic space where the distance between actors represents how similar they are (Hoff, Raftery, and Handcock 2002), or as a social space where the distance between two actors corresponds to the strength of the relationship between the two.

Because latent distances for a triple of actors must obey the triangle inequality, this formulation models the tendencies toward transitivity commonly found in social networks. A latent cluster model (Handcock, Raftery, and Tantrum 2007) is a variation on (10), specifying that latent positions for individual actors are mixtures of patterns associated with two or more latent categorical groups of actors. The LatentNet package in R (Handcock et al. 2007) uses Bayesian methods to fit such models.

Nodes nearby one another are more likely to have a tie and will likely have similar ties to others. One of the characteristics of this model is that if nodes are nearby one another they are highly likely to have a tie. But also if the nodes are closer together

[18]The latent factor approach is what has been used in: Ward et al. (2007); Cao & Ward (2014); Metternich et al. (2015).

[19]There is one other major latent variable approach that we will leave undiscussed here referred to as the latent class model, or stochastic block model. In this model, nodes are assumed to belong to an unobserved latent class and a probability distribution is used to describe the relationships between classes (Nowicki & Snijders, 2001). The probability of a tie between a pair of actors is a function of the classes to which they have been assigned. This type of model is useful as a community detection tool and is well suited for networks that exhibit high degrees of stochastic equivalence.

then they are going to be close to the same other nodes. So if you have two nodes that interact they are also similar form the perspective of other nodes.

In sum, additive effects can capture network covariance. Multiplicative effects can capture higher-order dependence.

$$
\begin{aligned}
y_{ij} &= g(\theta_{ij}) \\
z_{ij} &= \beta^T \mathbf{X}_{ij} + e_{ij} \\
e_{ij} &= a_i + b_j + \epsilon_{ij} \\
\epsilon_{ij} &= \gamma(u_i, u_j) + \mathcal{E}_{ij}
\end{aligned} \tag{4}
$$

2. Comparison with Other Approaches

Ingold (2008)

Ingold & Fischer (2014)

Ingold & Leifeld (2014)

Figure 2 highlights how we can expect high levels of sender and receiver heterogeneity.

Krivitsky & Handcock (2015)

2.1. Parameter Estimates.

2.2. Capturing Network Attributes.

To assess whether the model adequately captures the network parameters of the DV. Here we compare the observed with a set of simulated networks based on certain network statistics (Hunter et al., 2008).

See Morris et al. (2008) for details on each of these parameters.

- Dyad-wise shared partners - Number of dyads in the network with exactly i shared partners

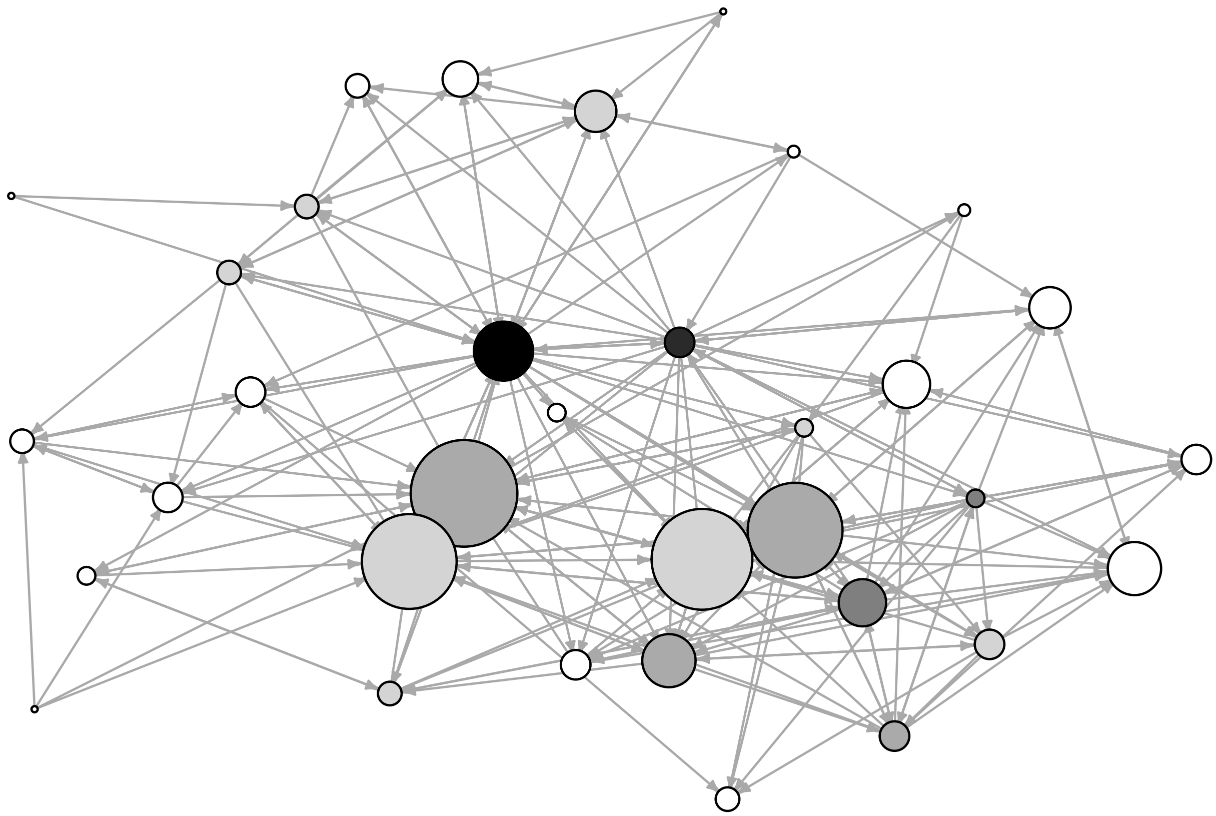

Figure 2. dv net

- Edge-wise shared partners - Similar to above except this counts the number of dyads with the same number of edges
- Geodesic distances - The proportion of pairs of nodes whose shortest connecting path is of length k, for $k = 1, 2, \ldots$ Also, pairs of nodes that are not connected are classified as $k = \infty$.
- Incoming k-star - Propensities for individuals to have connections with multiple network partners
- Indegree - degree count is the number of nodes with the same value of the attribute as the ego node
- Outdegree - degree count is the number of nodes with the same value of the attribute as the ego node

Figure 4 give posterior predictive goodness of fit summaries for four network statistics: (1) the empirical standard deviation of the row means; (2) the empirical standard deviation of the column means (heterogeneity of nodes with incoming activity); (3) the empirical within-dyad correlation; (4) a normalized measure of triadic dependence (Hoff et al., 2015).

For a given summary statistic g() we first simulate $\mathbf{Y}_{sim} \approx p(\mathbf{Y}_{sim}|\mathbf{Y}_{obs}) = \int p(\mathbf{Y}_{sim}|\theta)p(d\theta|\mathbf{Y}_{obs})$ and then we compare $g(\mathbf{Y}_{sim})$ to $g(\mathbf{Y}_{obs})$. Histograms represent predicted value of statistics under the model and red dash line represents the observed value.

Proportion of ties that are reciprocated.

(5) $$t(Y) = \frac{\sum_{i \neq j} y_{i,j} y_{j,i}}{\sum_{i \neq j} y_{i,j}}$$

Number of transitive triplets, number of triangles in network, number of times ijk are all connected.

(6) $$t(Y) = \sum_{i \neq j \neq k} y_{i,j} y_{i,k} y_{j,k}$$

2.3. **Tie Formation Prediction.** The results are displayed in Figure 5 using separation plots and Receiver Operating Characteristic (ROC) curves.

We compare the sensitivity and specificity trade-off for each model using ROC curves. Models that have a better fit according to this test should have curves that follow the left-hand border and then the top border of the ROC space. Here again it is apparent that accounting for the interstate relations and the endogenous network effects leads to noticeable improvements in performance. Last, by calculating the area under the ROC curve (AUC) we can assess the accuracy of each model.

Separation plots provide a visual interpretation of model fit by plotting all observations, in this case country pairs, in the data set according to their predicted value from left (low values) to right (high values). Models with a good fit should should have all actual (dark blue) observations towards the right of the separation plot (Greenhill et al., 2011).

Beger (2015)

In addition, we also highlight the difference in performance through the utilization of a precision-recall curve. Precision is a measure of result relevancy, while recall is a measure of how many truly relevant results are returned. A high area under the curve represents both high recall and high precision, where high precision relates to a low false positive rate, and high recall relates to a low false negative rate. High scores for both show that the classifier is returning accurate results (high precision), as well as returning a majority of all positive results (high recall).

Precision is defined as the number of true positives over the number of true positives plus the number of false positives.

Recall is defined as the number of true positives over the number of true positives plus the number of false negatives.

	Logit	MRQAP	LSM	ERGM	AME
Intercept/Edges	-4.44*	-4.24*	0.94*	-12.17*	-3.39*
	(0.34)		[0.09; 1.82]	(1.40)	[-4.38; -2.50]
Conflicting policy preferences					
Business vs. NGO	-0.86	-0.87*	-1.37*	-1.11*	-1.37*
	(0.46)		[-2.42; -0.41]	(0.51)	[-2.44; -0.47]
Opposition/alliance	1.21*	1.14*	0.00	1.22*	1.08*
	(0.20)		[-0.40; 0.39]	(0.20)	[0.72; 1.47]
Preference dissimilarity	-0.07	-0.60	-1.76*	-0.44	-0.79*
	(0.37)		[-2.62; -0.90]	(0.39)	[-1.55; -0.08]
Transaction costs					
Joint forum participation	0.88*	0.75*	1.51*	0.90*	0.92*
	(0.27)		[0.86; 2.17]	(0.28)	[0.40; 1.47]
Influence					
Influence attribution	1.20*	1.29*	0.08	1.00*	1.09*
	(0.22)		[-0.40; 0.55]	(0.21)	[0.69; 1.53]
Alter's influence indegree	0.10*	0.11*	0.01	0.21*	0.11*
	(0.02)		[-0.03; 0.04]	(0.04)	[0.07; 0.15]
Influence absolute diff.	-0.03*	-0.06*	0.04	-0.05*	-0.07*
	(0.02)		[-0.01; 0.09]	(0.01)	[-0.11; -0.03]
Alter = Government actor	0.63*	0.68	-0.46	1.04*	0.55
	(0.25)		[-1.08; 0.14]	(0.34)	[-0.07; 1.15]
Functional requirements					
Ego = Environmental NGO	0.88*	0.99	-0.60	0.79*	0.67
	(0.26)		[-1.32; 0.09]	(0.17)	[-0.38; 1.71]
Same actor type	0.74*	1.12*	1.17*	0.99*	1.04*
	(0.22)		[0.63; 1.71]	(0.23)	[0.63; 1.50]
Endogenous dependencies					
Mutuality	1.22*	1.00*		0.81*	
	(0.21)			(0.25)	
Outdegree popularity				0.95*	
				(0.09)	
Twopaths				-0.04*	
				(0.02)	
GWIdegree (2.0)				3.42*	
				(1.47)	
GWESP (1.0)				0.58*	
				(0.16)	
GWOdegree (0.5)				8.42*	
				(2.11)	

Table 3. * $p < 0.05$ (or 0 outside the 95% confidence interval).

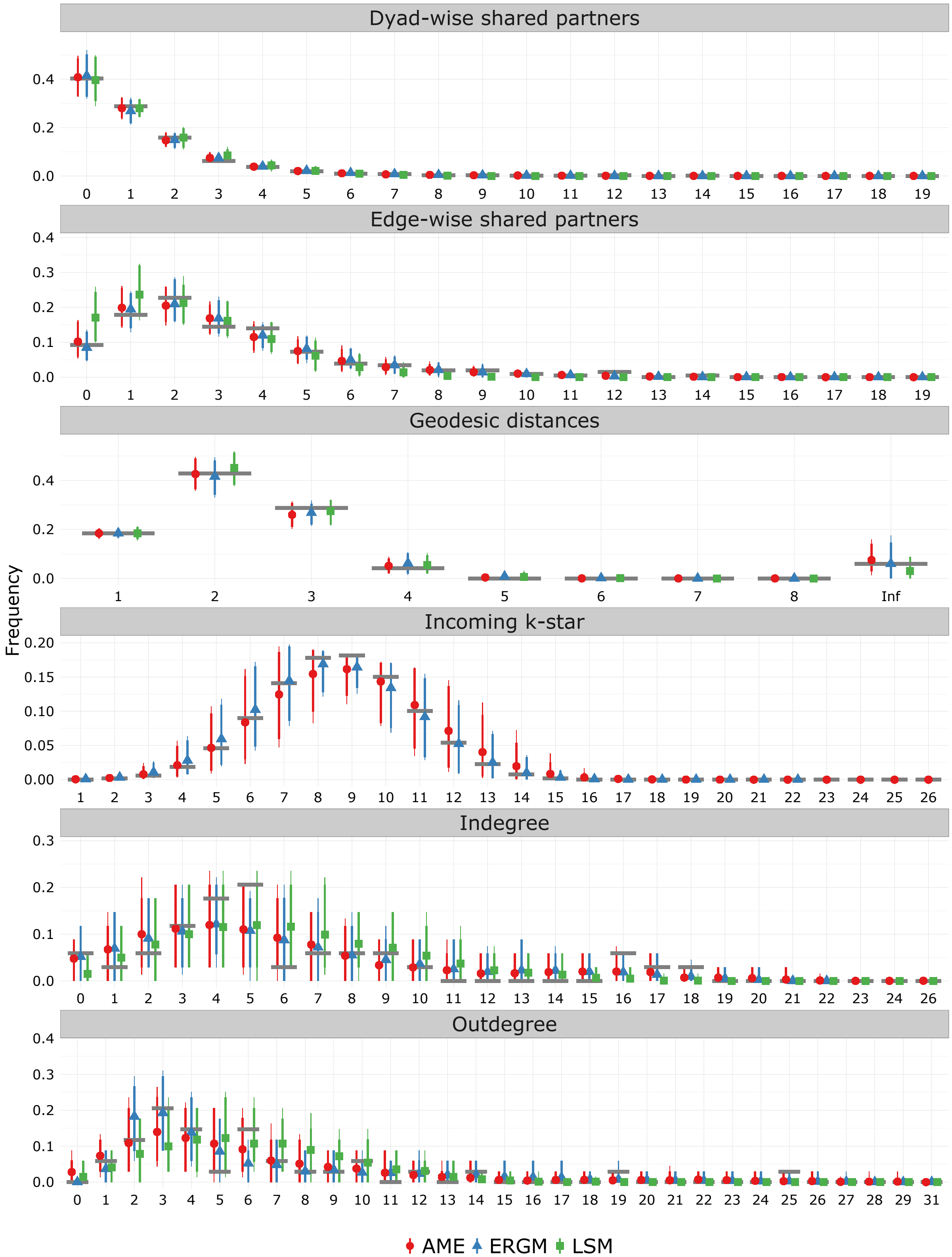

Figure 3. network stats

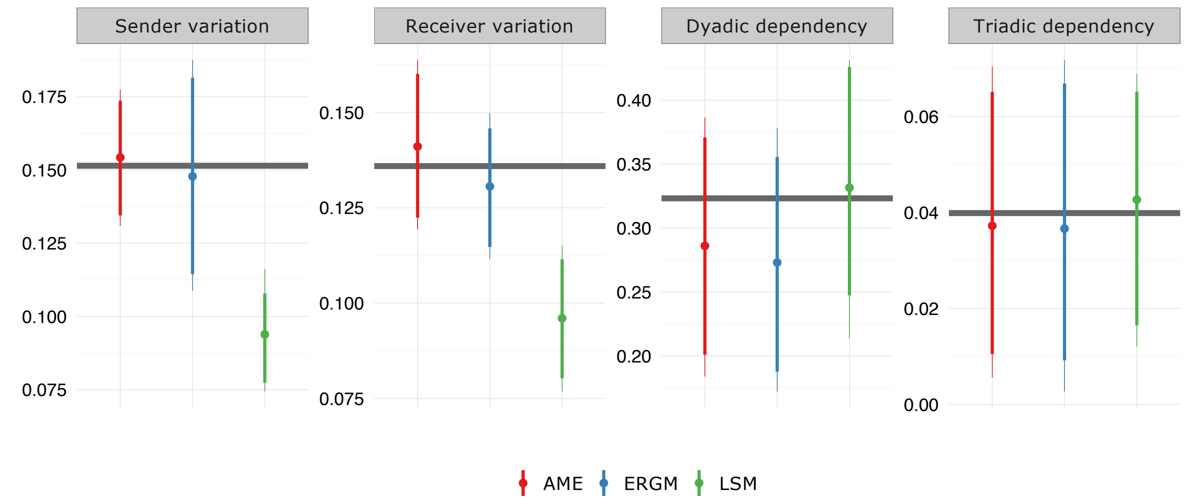

Figure 4. Posterior predictive goodness of fit summary

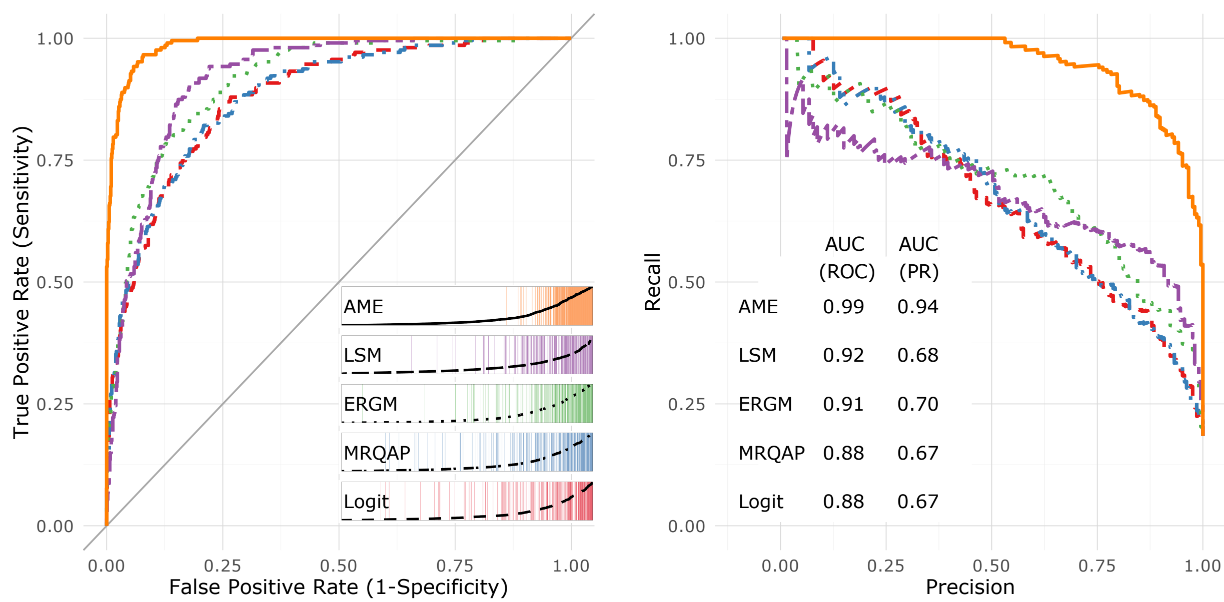

Figure 5. ROC and separation plots

3. Conclusion

4. Appendix

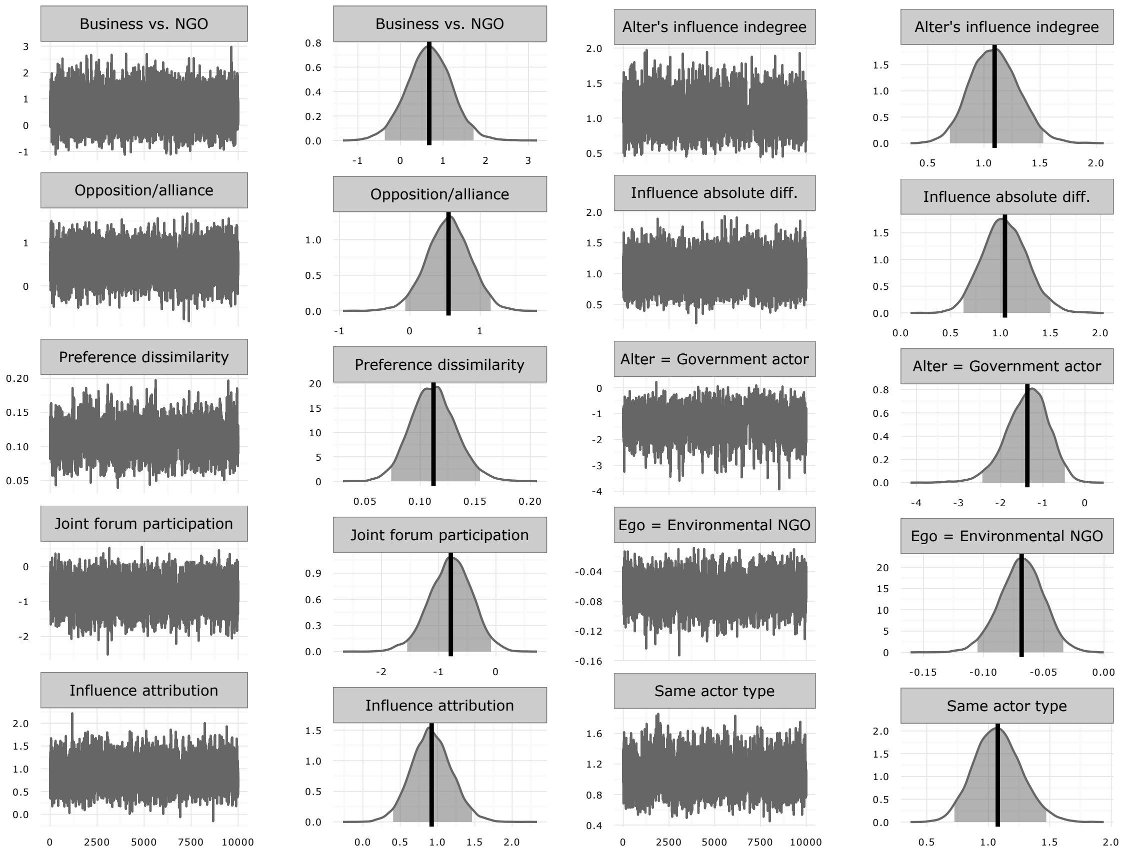

Figure 6. ame convergence k = 2

4.1. AMEN Model Convergence.

	LSM	LSM (Bilinear)	LSM (SR)	LSM (Bilinear + SR)	AME
Intercept/Edges	0.94*	-2.66*	0.60	-2.50*	-3.39*
	[0.09; 1.82]	[-3.53; -1.87]	[-1.10; 2.37]	[-4.14; -0.88]	[-4.38; -2.50]
Conflicting policy preferences					
Business vs. NGO	-1.37*	-2.64*	-3.07*	-2.87*	-1.37*
	[-2.42; -0.41]	[-4.61; -0.96]	[-4.77; -1.56]	[-4.63; -1.29]	[-2.44; -0.47]
Opposition/alliance	0.00	0.04	0.31	0.24	1.08*
	[-0.40; 0.39]	[-0.44; 0.54]	[-0.24; 0.86]	[-0.36; 0.82]	[0.72; 1.47]
Preference dissimilarity	-1.76*	-2.00*	-1.88*	-2.20*	-0.79*
	[-2.62; -0.90]	[-3.01; -1.03]	[-3.07; -0.68]	[-3.46; -0.96]	[-1.55; -0.08]
Transaction costs					
Joint forum participation	1.51*	1.24*	1.56*	1.62*	0.92*
	[0.86; 2.17]	[0.53; 1.93]	[0.69; 2.41]	[0.70; 2.52]	[0.40; 1.47]
Influence					
Influence attribution	0.08	-0.08	0.30	0.28	1.09*
	[-0.40; 0.55]	[-0.62; 0.46]	[-0.37; 0.96]	[-0.42; 0.97]	[0.69; 1.53]
Alter's influence indegree	0.01	-0.05*	0.06	0.05	0.11*
	[-0.03; 0.04]	[-0.09; -0.01]	[-0.03; 0.14]	[-0.04; 0.13]	[0.07; 0.15]
Influence absolute diff.	0.04	0.02	-0.08*	-0.08*	-0.07*
	[-0.01; 0.09]	[-0.03; 0.07]	[-0.14; -0.02]	[-0.14; -0.02]	[-0.11; -0.03]
Alter = Government actor	-0.46	-0.80	-0.11	-0.20	0.55
	[-1.08; 0.14]	[-1.67; 0.04]	[-1.91; 1.76]	[-2.14; 1.74]	[-0.07; 1.15]
Functional requirements					
Ego = Environmental NGO	-0.60	-1.90*	-1.69	-1.84	0.67
	[-1.32; 0.09]	[-3.10; -0.86]	[-3.74; 0.23]	[-4.02; 0.11]	[-0.38; 1.71]
Same actor type	1.17*	1.40*	1.82*	1.90*	1.04*
	[0.63; 1.71]	[0.85; 1.95]	[1.10; 2.54]	[1.19; 2.62]	[0.63; 1.50]

Table 4. * $p < 0.05$ (or 0 outside the 95% confidence interval).

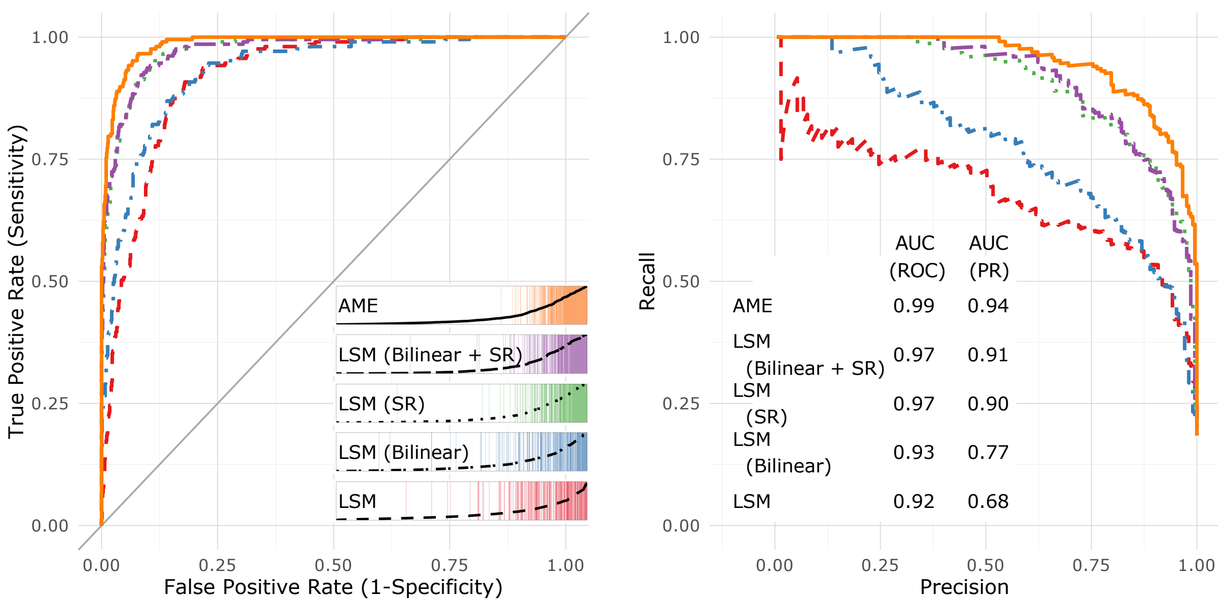

Figure 7. ROC and separation plots

4.2. Comparison of amen & latentnet $\mathcal{R}$ Packages.

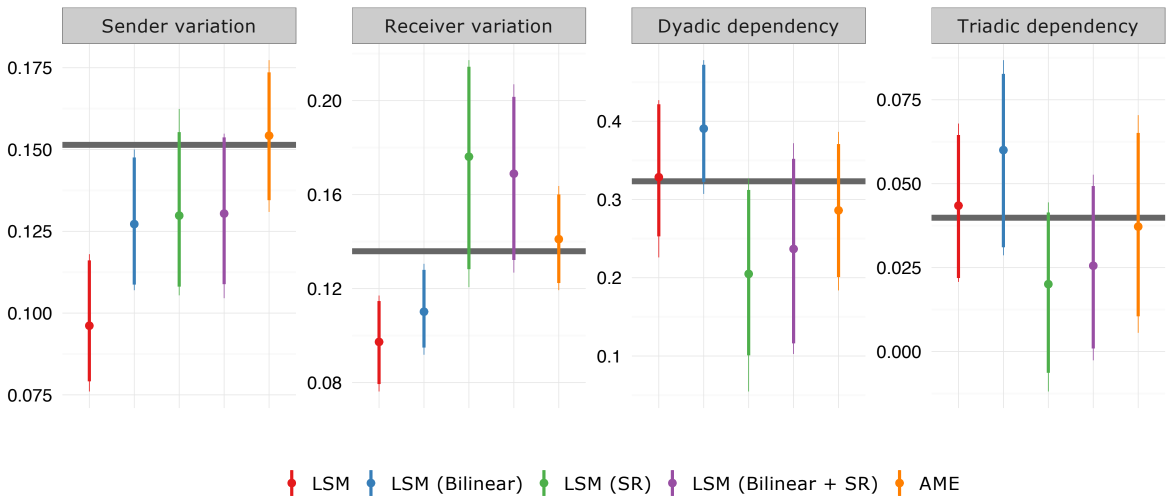

Figure 8. Posterior predictive goodness of fit summary

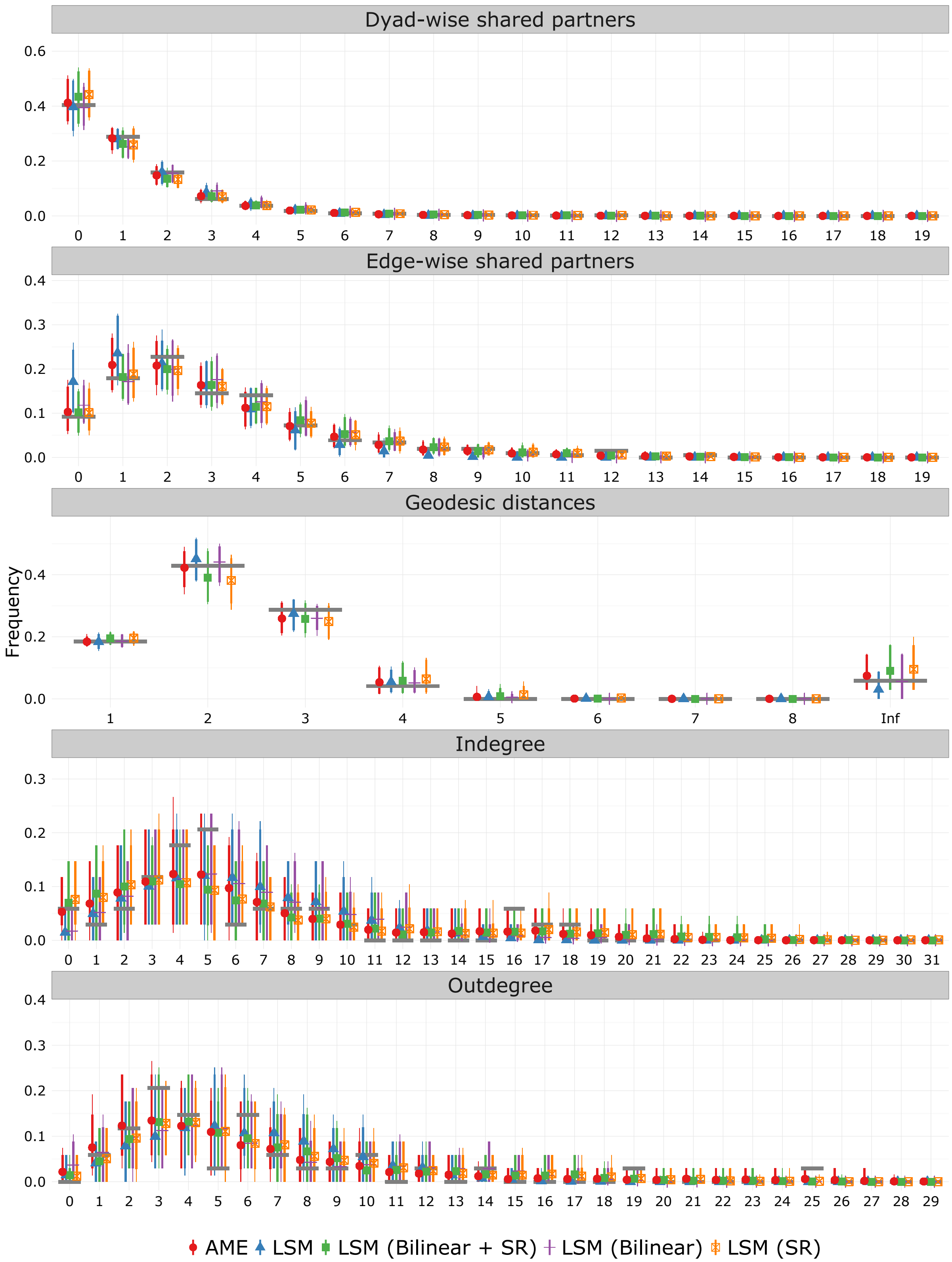

Figure 9. network stats

	AME (k=0)	AME (k=1)	AME (k=2)	AME (k=3)	AME (k=4)
Intercept/Edges	-2.75*	-3.08*	-3.39*	-3.72*	-3.93*
	[-3.43; -2.09]	[-3.91; -2.30]	[-4.38; -2.50]	[-4.84; -2.73]	[-5.12; -2.87]
Conflicting policy preferences					
Business vs. NGO	-1.08*	-1.28*	-1.37*	-1.48*	-1.51*
	[-1.82; -0.41]	[-2.20; -0.47]	[-2.44; -0.47]	[-2.63; -0.49]	[-2.69; -0.47]
Opposition/alliance	0.83*	0.95*	1.08*	1.19*	1.28*
	[0.57; 1.10]	[0.64; 1.27]	[0.72; 1.47]	[0.80; 1.64]	[0.86; 1.77]
Preference dissimilarity	-0.49	-0.65*	-0.79*	-0.89*	-0.95*
	[-1.06; 0.06]	[-1.30; -0.03]	[-1.55; -0.08]	[-1.71; -0.12]	[-1.80; -0.14]
Transaction costs					
Joint forum participation	0.73*	0.84*	0.92*	1.01*	1.06*
	[0.34; 1.12]	[0.38; 1.31]	[0.40; 1.47]	[0.44; 1.62]	[0.43; 1.72]
Influence					
Influence attribution	0.88*	1.00*	1.09*	1.21*	1.28*
	[0.57; 1.19]	[0.63; 1.39]	[0.69; 1.53]	[0.75; 1.71]	[0.80; 1.84]
Alter's influence indegree	0.09*	0.10*	0.11*	0.12*	0.13*
	[0.06; 0.12]	[0.07; 0.14]	[0.07; 0.15]	[0.08; 0.17]	[0.09; 0.18]
Influence absolute diff.	-0.06*	-0.06*	-0.07*	-0.07*	-0.08*
	[-0.08; -0.03]	[-0.10; -0.03]	[-0.11; -0.03]	[-0.12; -0.04]	[-0.12; -0.04]
Alter = Government actor	0.49	0.52	0.55	0.60	0.64
	[-0.01; 0.99]	[-0.04; 1.07]	[-0.07; 1.15]	[-0.07; 1.27]	[-0.07; 1.35]
Functional requirements					
Ego = Environmental NGO	0.54	0.61	0.67	0.76	0.80
	[-0.28; 1.36]	[-0.31; 1.56]	[-0.38; 1.71]	[-0.38; 1.90]	[-0.40; 2.04]
Same actor type	0.88*	0.97*	1.04*	1.11*	1.17*
	[0.55; 1.21]	[0.60; 1.35]	[0.63; 1.50]	[0.64; 1.59]	[0.68; 1.68]

Table 5. * $p < 0.05$ (or 0 outside the 95% confidence interval).

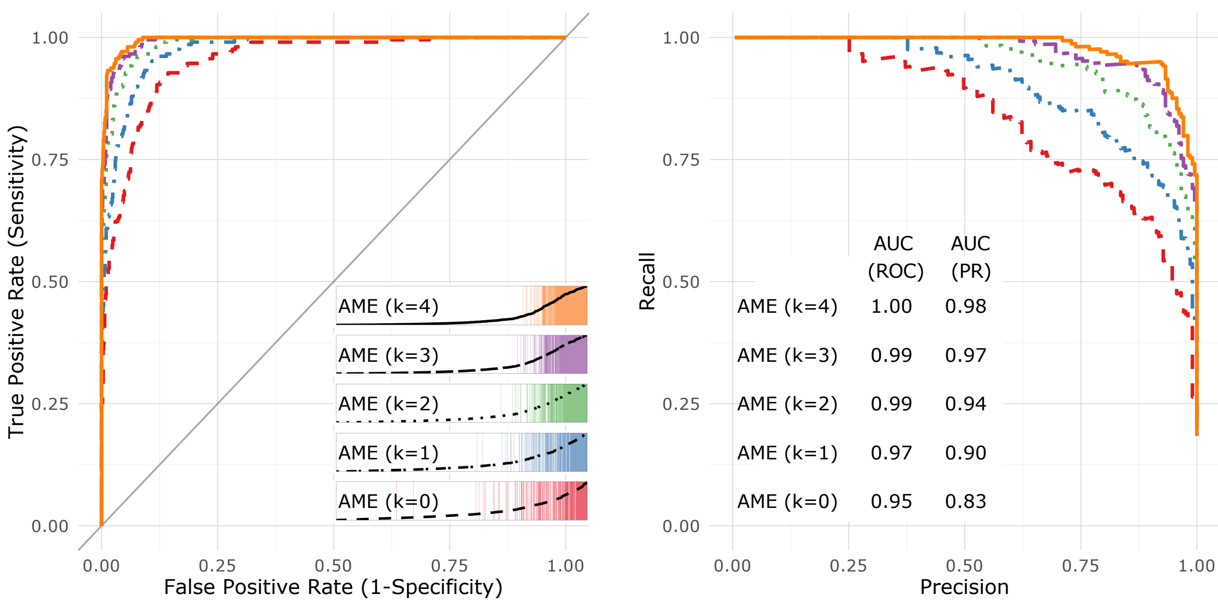

Figure 10. ROC and separation plots

4.3. Comparison with other AME Parameterizations.

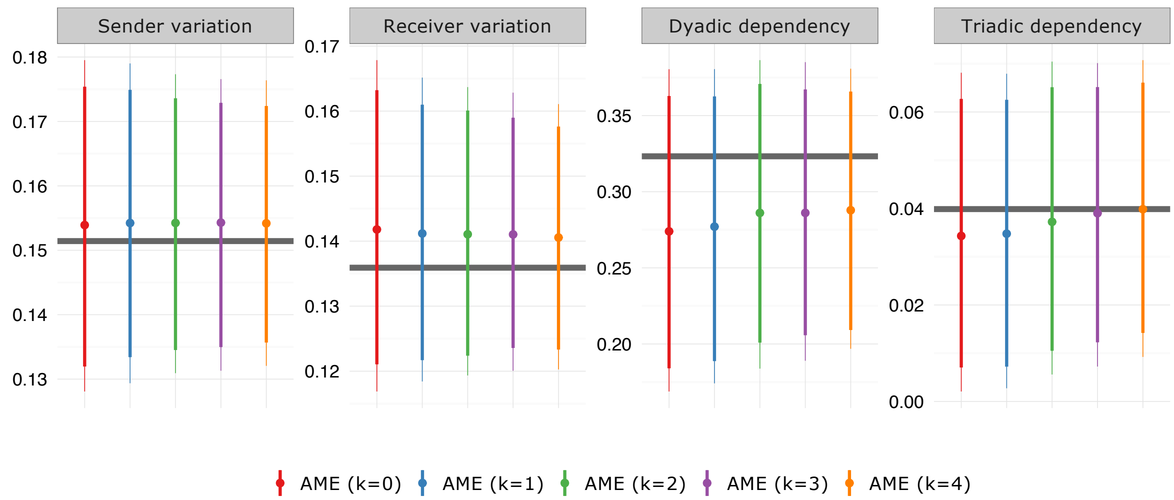

Figure 11. Posterior predictive goodness of fit summary

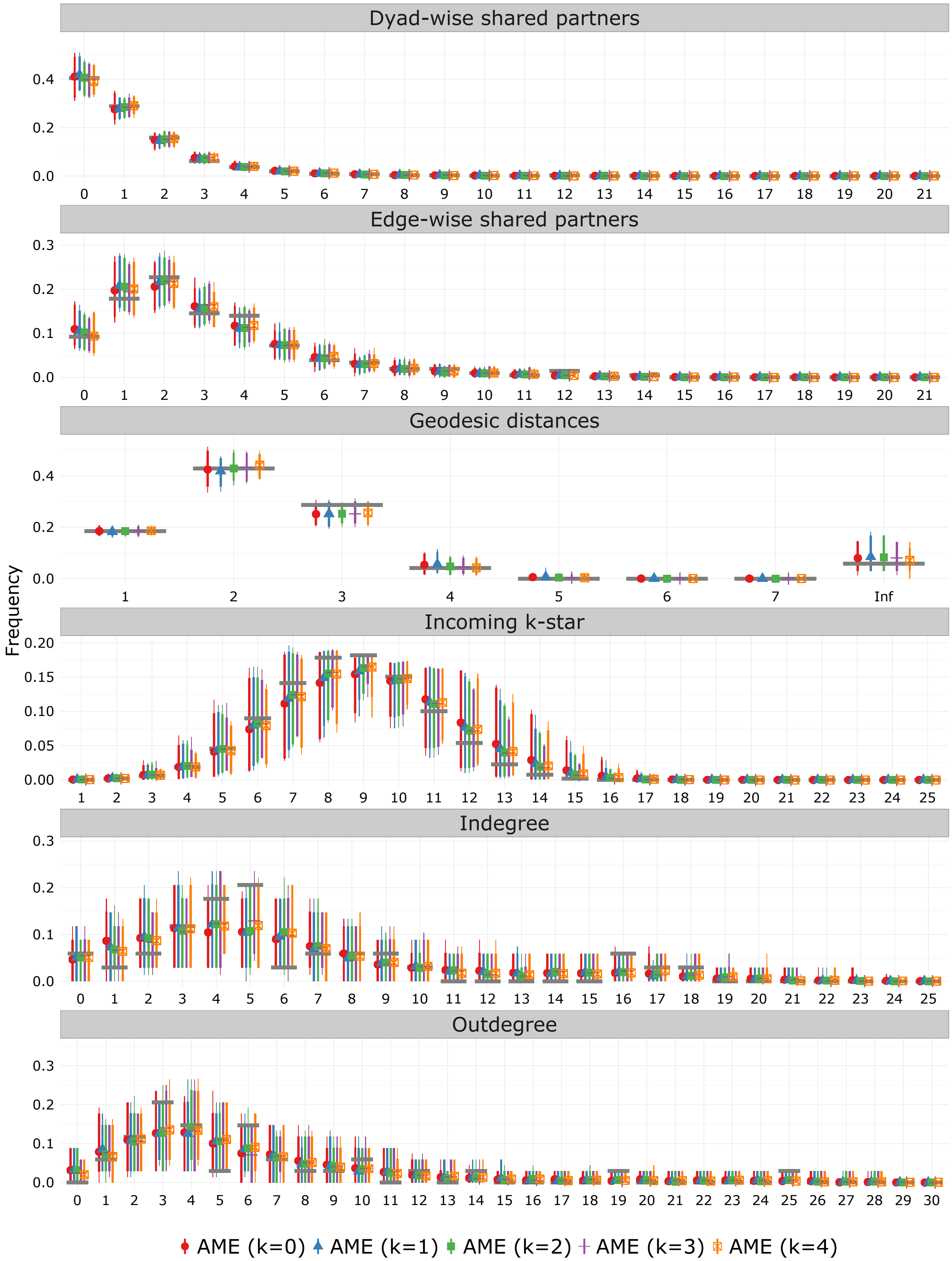

Figure 12. network stats

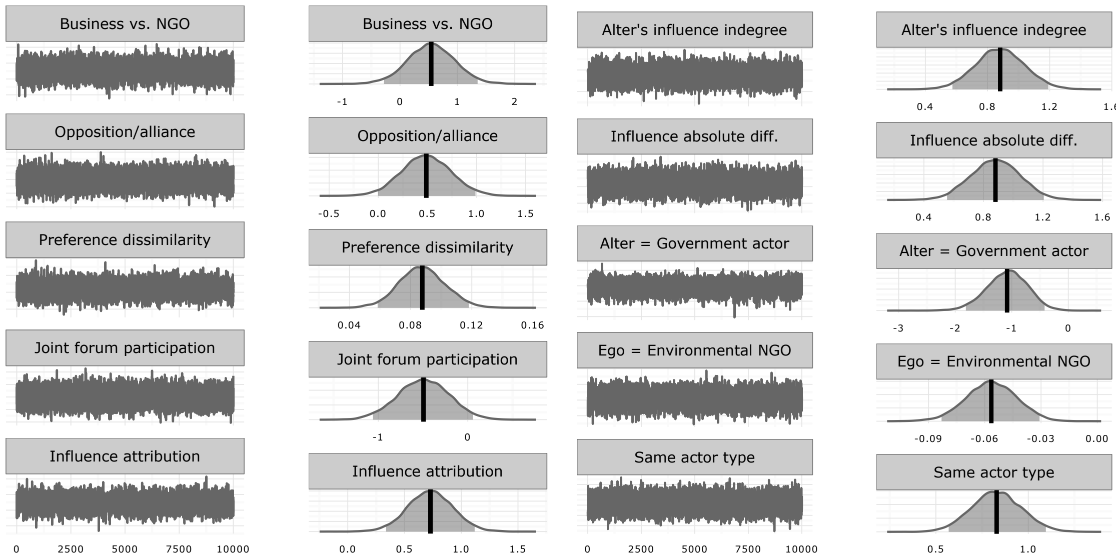

Figure 13. ame convergence k = 0

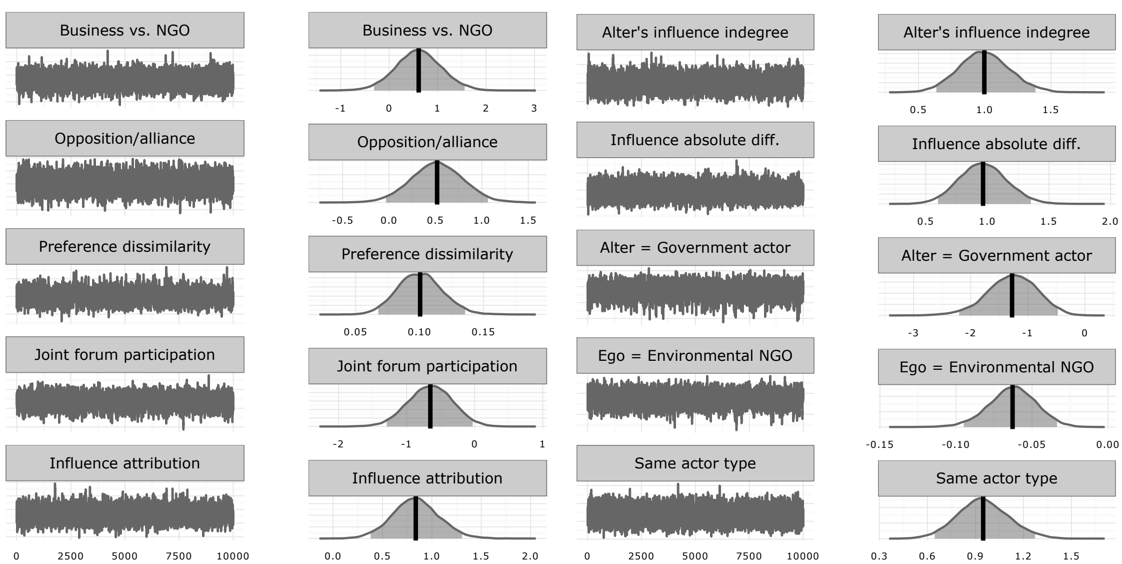

Figure 14. ame convergence k=1

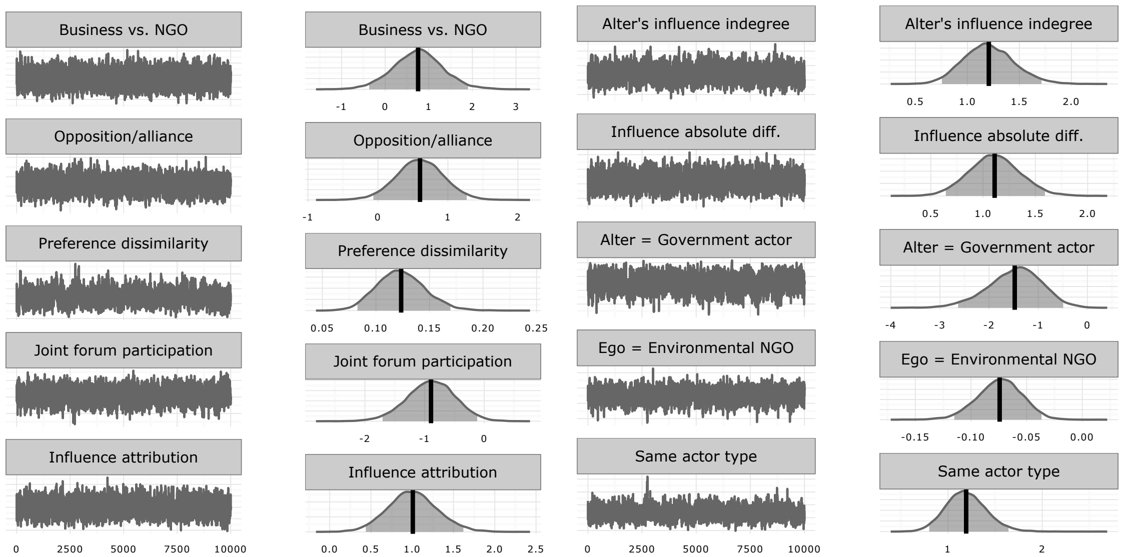

Figure 15. ame convergence k=3

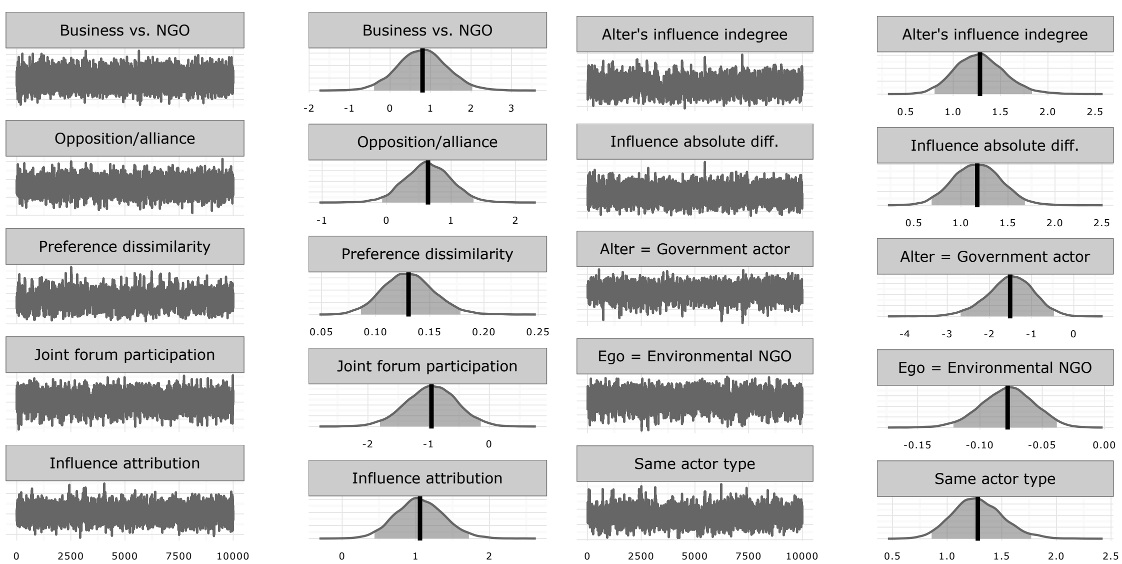

Figure 16. ame convergence k = 4

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