

NETWORKS MATTER IN INTERNATIONAL RELATIONS: A LATENT NETWORK APPROACH TO ANALYSIS

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ABSTRACT. International relations scholarship is filled with dyads. This includes dyadic hypotheses and propositions, but especially data. These relational data contain information about the interdependencies of various phenomena, including countries, dyads, and even triads. However, most empirical studies of such data do not take into account these dependencies. As a result, the studies are often contradictory and produce results that are not compelling. One reason is that the independence required by the typical methods employed does not exist in the data being studied. We present a different, regression based method, which constructs a latent network which incorporates 1st, 2nd, and 3rd-order dependencies. We replicate five prominent studies in recent IR scholarship and compare the standard approach to the latent factor approach. The additive and multiplicative latent factor approach is shown to produce more precise estimates of covariate effects, and it also dominates standard approaches in terms of out-of-sample cross-validations.

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1. INTRODUCTION

Aronow et al. (2015) estimate that over the period from 2010 to 2015, over sixty articles utilizing dyadic data were published in the *American Political Science Review*, *American Journal of Political Science*, and *International Organization*. Most of these studies use a generalized linear model (GLM) to estimate regression coefficients. However, extant approaches to studying dyadic data increase the chance of faulty inferences by treating data as independent and identically distributed (iid) when observations may be highly dependent. Most standard approaches assume that the problems raised by having non-iid relational data can be addressed by recalculating the standard errors of estimated parameters in the link function, so as to reflect the potential clustering of cases. This may work in limited situations, but is not generally effective because these palliatives do not address the fundamental data generating process that remains a threat to inference because of the interdependence of observations or measurements. Namely, it is not just the diagonals of the variance-covariance matrix this affects.

In this article, we discuss a Bayesian approach, the Additive and Multiplicative Effects (AME) model, for directly modeling relational data to reflect the data generating process that yields interdependencies in these types of data structures (Hoff, 2008; Minhas et al., 2016). We focus on three types of interdependencies that can complicate dyadic analyses. First, dependencies may arise within a set of dyads as there may be a particular actor that is more likely to send or receive events such as conflict. Additionally, if the event of interest has a clear sender and receiver, we are likely to observe dependencies within a dyad; specifically, if a rebel group initiates a conflict with a government, the government will likely reciprocate that conflictual behavior. We capture these two dependencies, often referred to as first- and second-order dependencies, respectively, within the additive effects portion of the model. The multiplicative effects capture dependencies that results from groups of actors clustering together or organizing into communities due to *meso-scopic* features of networks, such as homophily and stochastic equivalence. These type of meso-scopic features often arise in relational data because actors possess some latent set of shared attributes that affect their probability of interacting with one another.

We begin by providing a brief review of these dependencies and the AME model. Next, we move to conducting a simulation study to show how this AME approach can recover unbiased and

well-calibrated regression coefficients in the presence of network dependencies. Then, we apply this approach to five prominent studies in the international relations (IR) literature and compare results from the current state-of-the-art approach (a GLM with robust standard errors) to those obtained with the AME framework. The latent factor approach (AME) is able to better capture first, second, and third-order interdependencies than the standard approach. It also produces results that are more precise and at times at odds with those found in these studies in particular, and the broader literatures from which they are drawn. As such, this approach offers substantive insights which are often occluded by ignoring the interdependent nature of the relational data that characterize many studies in the field of international relations. Finally, we show that for each replication our network based approach provides substantively more accurate out-of-sample predictions than the models used in the original studies. Thus, the AME approach is one that can be used by scholars in the field to not only generate substantive insights, but it also enables us to better model the data generating process behind events of interest in international relations. It facilitates the concentration on international relations in the field of international relations.

2. DEPENDENCIES IN DYADIC DATA

In working with relational data, scholars in the field begin by structuring it as a set of dyadic observations stacked on top of one another. This makes sense if each observation is independent of the others. Thus, for example, a conflict sent from the United States to Japan, is assumed to be independent of any action that Japan may send to the United States. Additionally, every action sent by Japan to others in the system is considered independent even though each of those interactions involves a common sender, i.e, Japan. As a result, the assumption that most begin with is that each dyadic interaction is taking place in isolation of the others. We know this assumption to be false in theory and in practice because relational data comes with an explicit structure that in general leads to particular types of dependencies. The importance of accounting for underlying structure of our data has been a lesson well understood at least when it comes to time-series cross-sectional data (TSCS) within political science (Beck and Katz, 1995; Beck et al., 1998). As a result, it is now standard practice to take explicit steps to account for the complex data structures that emerge in TSCS applications and the unobserved heterogeneity that they cause.

To uncover the underlying structure that relational data comes with it is helpful to shift towards restructuring dyadic data in the form of a matrix – often referred to as an adjacency matrix – as shown in Figure 1. Rows designate the senders of an event and columns the receivers. The cross-sections in this matrix represent the actions that were sent by an actor in the row to those designated in the columns. Thus y_{ij} designates an action y , such as a conflictual event or trade flows, that is sent from actor i to actor j .

Using the structure of an adjacency matrix, we visualize in Figure 1 the types of first- and second-order dependencies that can complicate the analysis of relational data in traditional GLMs. The adjacency matrix on the top left highlights a particular row of an adjacency matrix, to illustrate that these values may be more similar to each other than other values in the adjacency matrix because each has a common sender. Homogeneity in interactions involving a common sender also manifest heterogeneity in how active actors are across the network when compared to each other. Thus in most relational datasets (e.g., trade flows, conflict, participation in international organizations, even networks derived from Twitter or Facebook) we often find that there are some actors which are much more active than others (Barabási and Réka, 1999). Unless one is able to develop a model that can account for the variety of explanations that may play a role in determining why a particular actor may be more active than others, parameter estimates from standard statistical models will be biased.

For similar reasons one also needs to take into account that there is a shared dependence between observations that share a common receiver. The bottom-left panel, illustrates that sender and receiver type dependencies can also blend together. Specifically, actors who are more likely to send ties in a network tend to also be more likely to receive them. As a result, the rows and columns in an adjacency matrix are often correlated. For example, consider that trade flows both from and to many wealthy, developed countries. The bottom-right panel, highlights a second order dependence, specifically, reciprocity. This is a dependency occurring within dyads involving the same actors whereby values of y_{ij} and y_{ji} are correlated. The concept of reciprocity has deep roots in the study of relations between states (Richardson, 1960; Keohane, 1989).

Sender heterogeneity					Receiver Heterogeneity				
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	NA	y_{ij}	y_{ik}	y_{il}	<i>i</i>	NA	y_{ij}	y_{ik}	y_{il}
<i>j</i>	y_{ji}	NA	y_{jk}	y_{jl}	<i>j</i>	y_{ji}	NA	y_{jk}	y_{jl}
<i>k</i>	y_{ki}	y_{kj}	NA	y_{kl}	<i>k</i>	y_{ki}	y_{kj}	NA	y_{kl}
<i>l</i>	y_{li}	y_{lj}	y_{lk}	NA	<i>l</i>	y_{li}	y_{lj}	y_{lk}	NA

Sender-Receiver Covariance					Reciprocity				
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>i</i>	NA	y_{ij}	y_{ik}	y_{il}	<i>i</i>	NA	y_{ij}	y_{ik}	y_{il}
<i>j</i>	y_{ji}	NA	y_{jk}	y_{jl}	<i>j</i>	y_{ji}	NA	y_{jk}	y_{jl}
<i>k</i>	y_{ki}	y_{kj}	NA	y_{kl}	<i>k</i>	y_{ki}	y_{kj}	NA	y_{kl}
<i>l</i>	y_{li}	y_{lj}	y_{lk}	NA	<i>l</i>	y_{li}	y_{lj}	y_{lk}	NA

Figure 1. Nodal and dyadic dependencies in relational data.

For most relational data, however, dependencies do not simply manifest at the nodal or dyadic level. More often we find significant evidence of higher order structures that result from dependencies between multiple groups of actors (Poast, 2010). These dependencies arise because there may be a or some set of latent attributes between actors that affects their probability of interacting with one another (Zinnes, 1967; Wasserman and Faust, 1994). In Figure 2 we provide a visualization of a hypothetical relational dataset wherein the nodes designate actors and edges between the nodes indicate that an interaction between the two took place. To highlight third order dependence patterns, nodes with similar latent attributes are colored similarly.

The visualization illustrates that the actors belonging to the same group have a higher likelihood of having an interaction with each other, whereas interactions across groups are more rare. A prominent example of a network with this type of structure was found in American Politics by Adamic and Glance (2005), who visualized the ways in which right and left leaning political blogs linked to one another in the 2004 United States Election. Adamic and Glance find that the degree

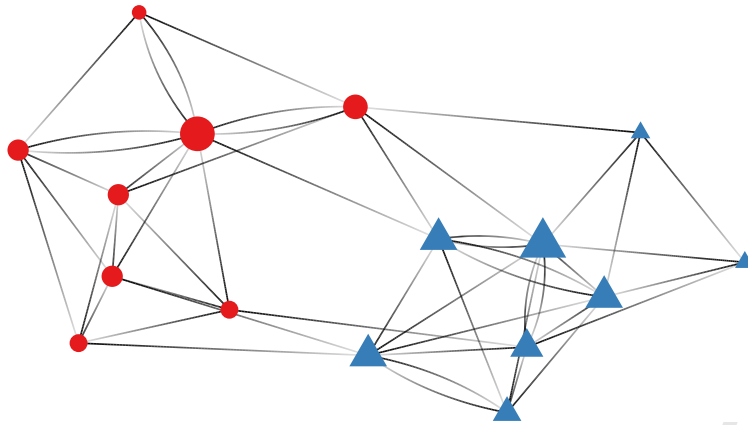


Figure 2. Visualization of network with meso-scope features.

of interaction between right and left leaning blogs was minimal, and that most blogs just linked to others that were political similar. This showcases the types of higher-order dependencies that can emerge in relational data. First, the fact that interactions were determined by a shared attribute, in this case political ideology, is an example of what is known as *homophily*. Homophily can be used to explain the emergence of patterns such as transitivity (“a friend of a friend is a friend”) and balance (“an enemy of a friend is an enemy”), which also have a long history in international relations. The other major type of meso-scope feature that emerges in relational data is community structure (Mucha et al., 2010), which is often formalized through the concept of stochastic equivalence (Anderson et al., 1992). This concept simply refers to the idea that groups of nodes that act similarly in the network are stochastically equivalent. In the example we have laid out above each of the left leaning blogs would be considered stochastically equivalent to one another.

The presence of these dependencies in relational data points to the fact that there is a complex structure underlying the dyadic events that we observe, and that accounting for this structure is necessary if we are to adequately represent the data generating process. Of course, if one is able to specify each of the nodal, dyadic, and triadic set of attributes that influence interactions then one can be assured that the conditional independence assumption underlying standard approaches will be satisfied. However, it is rarely the case that we can do this even for TSCS data, thus we more than often make certain modeling decisions to account for that structure. Failing to account for the underlying structure in either TSCS or dyadic data leads to a number of well-known challenges: a) biased estimates of the effect of independent variables, b) uncalibrated confidence

intervals, and c) poor predictive performance. Further by ignoring these potential interdependencies, we often ignore substantively interesting features of the problem under study. The study of international relations is founded on the relations among actors. Why ignore the interdependencies that led to the study of IR in the first place?

3. ADDITIVE AND MULTIPLICATIVE EFFECT MODELS FOR NETWORKS

To account for the dependencies that are prevalent in dyadic data, we turn to the AME model. The AME approach can be used to conduct inference on cross-sectional and longitudinal networks with binary, ordinal, or continuous linkages. It is flexible and easy to use for analyzing the kind of relational data often found in social science. It accounts for nodal and dyadic dependence patterns, as well as higher-order dependencies such as homophily and stochastic equivalence. Hoff (2015); Minhas et al. (2016) provide a detailed introduction to this framework, and the latter piece details how it contrasts with alternative network based approaches. Here we just provide a brief review and then move onto presenting a simulation exercise.

The AME model combines the social relations regression model (SRRM) to account for nodal and dyadic dependencies and the latent factor model (LFM) for third order dependencies.¹ The AME model is specified as follows:

$$\begin{aligned}
 y_{ij} &= f(\theta_{ij}), \text{ where} \\
 \theta_{ij} &= \beta_d^\top \mathbf{X}_{ij} + \beta_s^\top \mathbf{X}_i + \beta_r^\top \mathbf{X}_j + && \text{(Exogenous parameters)} \\
 (1) \quad &a_i + b_j + \epsilon_{ij} + && \text{(SRRM parameters)} \\
 &\mathbf{u}_i^\top \mathbf{D} \mathbf{v}_j && \text{(LFM parameters)}
 \end{aligned}$$

where $y_{ij,t}$ captures the interaction between actor i (the sender) and j (the receiver). We use a Bayesian probit framework, in which we model a latent variable, θ_{ij} , using first a set of exogenous dyadic ($\beta_d^\top \mathbf{X}_{ij}$), sender ($\beta_s^\top \mathbf{X}_i$), and receiver covariates ($\beta_r^\top \mathbf{X}_j$). Next, to account for the

¹For details on the SRRM see: for details on this model see Li and Loken (2002); Dorff and Minhas (2016). An earlier version of the LFM used in AME is presented as the general bilinear mixed effects (GBME) model in Hoff and Ward (2004). The GBME model is more limited in the types of dependence patterns that it can capture due to the formulation of the matrix decomposition procedure.

dependencies that emerge in dyadic data and that may complicate inference on the parameter associated with exogenous covariates we add parameters from the SRRM and LFM. a_i and b_j in Equation 1 represent sender and receiver random effects incorporated from the SRRM framework:

$$(2) \quad \begin{aligned} \{(a_1, b_1), \dots, (a_n, b_n)\} &\stackrel{\text{iid}}{\sim} N(0, \Sigma_{ab}) \\ \{(\epsilon_{ij}, \epsilon_{ji}) : i \neq j\} &\stackrel{\text{iid}}{\sim} N(0, \Sigma_\epsilon), \text{ where} \\ \Sigma_{ab} &= \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} \quad \Sigma_\epsilon = \sigma_\epsilon^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \end{aligned}$$

The sender and receiver random effects are modeled jointly from a multivariate normal distribution to account for correlation in how active an actor is in sending and receiving ties. Heterogeneity in the the sender and receiver effects is captured by σ_a^2 and σ_b^2 , respectively, and σ_{ab} describes the linear relationship between these two effects (i.e., whether actors who send [receive] a lot of ties also receive [send] a lot of ties). Beyond these first-order dependencies, second-order dependencies are described by σ_ϵ^2 and a within dyad correlation, or reciprocity, parameter ρ .

The LFM contribution to the AME comes in the multiplicative term: $\mathbf{u}_i^\top \mathbf{D} \mathbf{v}_j = \sum_{k \in K} d_k u_{ik} v_{jk}$. K denotes the dimensions of the latent space. This model posits a latent vector of characteristics \mathbf{u}_i and \mathbf{v}_j for each sender i and receiver j . The similarity or dissimilarity of these vectors will then influence the likelihood of activity, and provides a representation of third order interdependencies (Minhas et al., 2016). The LFM parameters are estimated by a process similar to computing the singular value decomposition (SVD) of the observed network. When taking the SVD we factorize our observed network into the product of three matrices: \mathbf{U} , \mathbf{D} , and \mathbf{V} . This provides us with a low-dimensional representation of our original network.² Values in \mathbf{U} provide a representation of how stochastically equivalent actors are as senders in a network, or more simply put how similar actors are in terms of who they are initiating, for example, conflict with. $\hat{\mathbf{u}}_i \approx \hat{\mathbf{u}}_j$ would indicate that actor i and j initiate battles with similar third actors. \mathbf{V} provide a similar representation but

²The dimensions of \mathbf{U} and \mathbf{V} are $n \times K$ and \mathbf{D} is a $K \times K$ diagonal matrix.

from the perspective of how similar actors are as receivers. The values in \mathbf{D} , a diagonal matrix, represent levels of homophily in the network.³

By integrating the SRRM and LFM into a Bayesian probit framework, we are able to account for the underlying structure in dyadic data that if left unestimated would complicate any inferences we draw

Parameter estimation in the AME takes place within the context of a Gibbs sampler in which we iteratively sample from the posterior distribution of the full conditionals for each parameter. Specifically, given initial values of $\{\beta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2\}$, the algorithm proceeds as follows until convergence:

- sample $\theta \mid \beta, \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
- sample $\beta \mid \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
- sample $\mathbf{a}, \mathbf{b} \mid \beta, \mathbf{X}, \theta, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
- sample $\Sigma_{ab} \mid \beta, \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \rho, \text{ and } \sigma_\epsilon^2$ (Inverse-Wishart)
- update ρ using a Metropolis-Hastings step with proposal $p^* \mid p \sim \text{truncated normal}_{[-1,1]}(\rho, \sigma_\epsilon^2)$
- sample $\sigma_\epsilon^2 \mid \beta, \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \text{ and } \rho$ (Inverse-Gamma)
- For each $k \in K$:
 - Sample $\mathbf{U}_{[k]} \mid \beta, \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}_{[-k]}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
 - Sample $\mathbf{V}_{[k]} \mid \beta, \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}_{[-k]}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
 - Sample $\mathbf{D}_{[k,k]} \mid \beta, \mathbf{X}, \theta, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)⁴

4. SIMULATION STUDY

We utilize a simulation study to highlight the utility of AME as an inferential tool for dyadic analysis. The goal of the simulation is to assess how well AME can provide unbiased and well calibrated estimates of coefficient parameters in the presence of unobserved dependencies. Specifically, we are concerned with conducting inference on a regression parameter β of a linear model for a network in the case where there is an omitted variable. Say that the true data-generating process for a particular Y is given by:

³Unlike traditional SVD, in the latent factor model, the singular values are not restricted to be positive, thus allowing us to account for both the presence and absence of homophily.

⁴Subsequent to estimation, \mathbf{D} matrix is absorbed into the calculation for \mathbf{V} as we iterate through K .

$$y_{i,j} \sim \mu + \beta x_{i,j} + \gamma w_{i,j} + \epsilon_{i,j}$$

where $Y = \{y_{i,j}\} \in \mathbb{R}^{n \times n}$ is an observed sociomatrix, $x = \{x_{i,j}\} \in \mathbb{R}^{n \times n}$ is a matrix of observed dyad-specific characteristics, and $w = \{w_{i,j}\} \in \mathbb{R}^{n \times n}$ is a matrix of unobserved dyad-specific characteristics. We compare inference for β using three models:

- the standard IR approach assuming independent errors;
- the AME approach outlined in the previous section with a unidimensional latent factor space ($K = 1$);
- and an “oracle” regression model that includes both $x_{i,j}$ and $w_{i,j}$.

To run the simulation we vary the size of the network under study at 50 and 100.

One way to compare inferences across models is with bias and variance of the parameter estimates. Both of these quantities are combined to get the MSE.

Alternatively, bias and precision can be summarized by confidence interval coverage and width. Coverage should ideally be at the nominal level. If two methods have the same actual coverage rate, the one with the narrower intervals is preferred.

Figure 3. Bias in parameter when homophily is ignored.

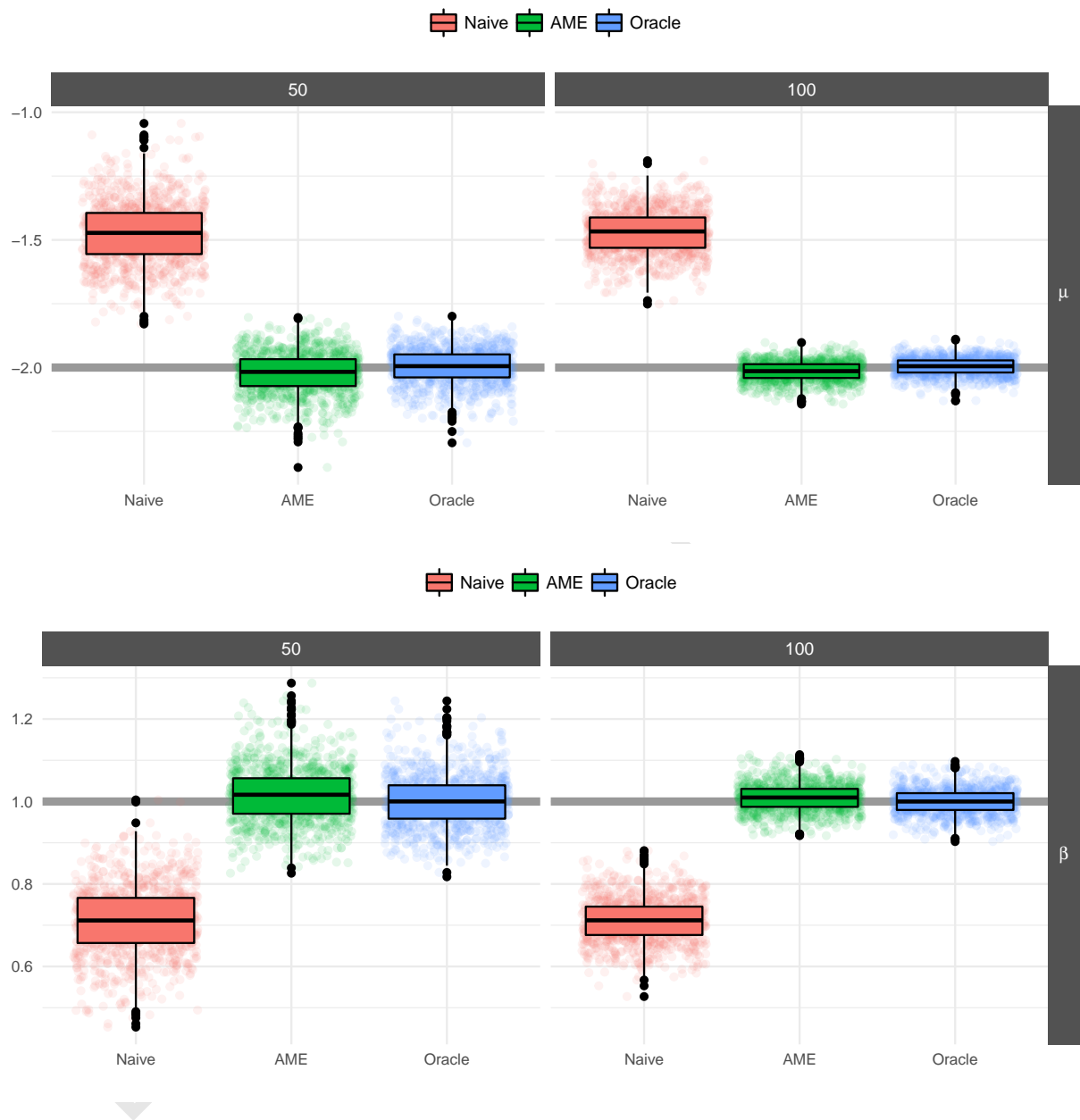
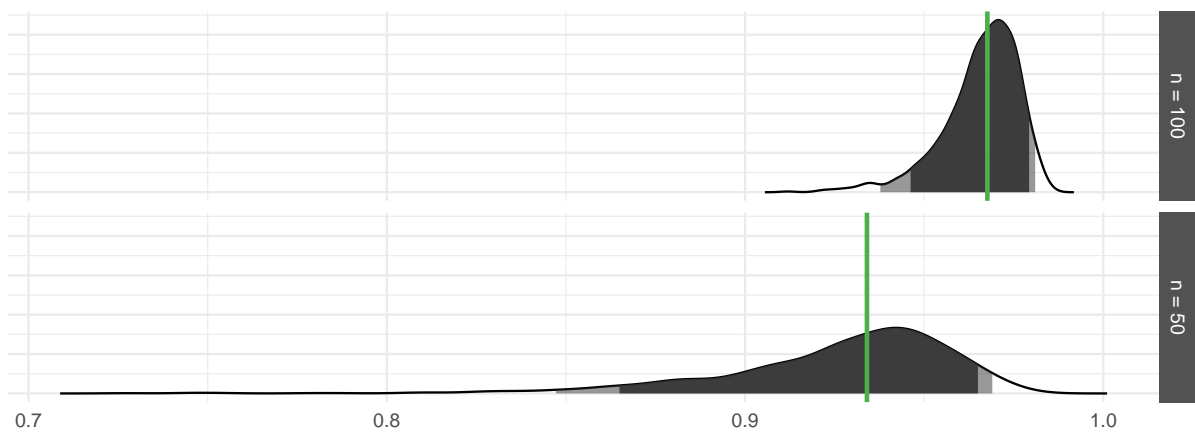


Figure 4. Coverage in parameter when homophily is ignored.



Figure 5. Correlation between missing variable and multiplicative random effect in AME.



5. RE-ESTIMATION WITH AME

5.1. **Design.** We choose five prominent studies from the broad field of international relations and international political economy that utilize relational data (Reiter and Stam, 2003; McDonald, 2004; Rose, 2004; Weeks, 2012; Gibler, 2017). These studies are recent and have been cited over 100 times. Each of these pieces was published in a prominent journal and is well-known in the literature. Each used the standard approach in political science, which is to employ some form of a general linearized regression that ignores dyadic interdependencies, except as they may reveal themselves in included variables. Also part of the gold standard is a post estimation cleanup thought to be a palliative, and each of these studies adjust the posterior standard errors in an attempt to account for the clustering of observations.

We obtained the data for each of these studies from their replication archives and replicated the main results of each of the articles.⁵ We also examine each of the models using the AME framework described above. Our goal is to ascertain whether the ignored interdependencies – the non-iid structure of the relational data – would result in different model estimates when they were addressed in an AME framework, and more importantly to see if there were substantive opportunities that were presented with the dynamic factor approach. Finally, in each study we assess whether there is any new substantive finding that emerges or indeed if any disappear once the interdependencies in the data are modeled.

The broader goal, beyond introducing the use of the AME framework in an applied setting, is to examine the extent to which interdependencies within typical dyadic data make much difference in what we have learned about international relations from recent empirical studies that use a

⁵Without exception this was straightforward to accomplish, thanks to an increasing norm in the social sciences of open data sharing.

Table 1. Features of the Studies Re-estimated.

	Model	Date Range	N. Actors	N. Dyads	Dyads Type	Clustering $\sigma_{\hat{\beta}}$
Reiter & Stam (2003)	Logit	1945–1995	193	753, 456	Directed	Robust
McDonald (2004)	Logit	1959–2002	198	92, 354	Undirected	Robust
Rose (2004)	OLS	1948–1999	177	234, 597	Directed	Robust
Weeks (2012)	Logit	1946–1999	197	901, 540	Directed	Robust
Gibler (2017)	Logit	1816–2008	193	650, 557	UnDirected	none

framework that ignores dependencies. Based in part on our results as well as the statistical and mathematical characteristics of the AME framework, we believe that it does. The dynamic latent factor model provides a step forward in the modeling of international relations.

Given current practices, most scholars have a single variable in a complicated empirical model that they look at for evaluating the validity of their empirically estimated models. Whether putative control variables, for example, are precisely estimated is less important. Generally, interpretations focus on a small set of independent variables. In the study by Reiter and Stam (2003) the main question is to determine in mixed dyads – consisting of a democratic country and country ruled by a personalist dictatorship – whether it is the democratic country or the personalist country that is associated with a greater likelihood that the dyad is in a dispute. In a regression including fourteen coefficients, they focus on whether the variables that capture whether the initiator is a democratic country are statistically significant.⁶ In our evaluation we focus on the same question in the re-examination of the Reiter and Stam study to determine whether the model replicates when re-estimated within the AME framework. Each of the other four studies also has a crucial finding that we hone in on to further draw into focus the potential differences brought about by utilizing an AME estimation procedure. In Table 2, we present the overall results; the term *does not replicate* indicates only that the sign and/or significance of the putatively crucial finding in the original study is not also found in the AME estimation.

In addition, we also examine the predictions made with each approach using an out-of-sample cross validation strategy. By accounting for exogenous and network dependent patterns that give rise to conflict systems we are able to better account for the data generating process underlying relational data structures. To show that this is the case, we examine whether our approach achieves better predictive performance in an out-of-sample context than traditional dyadic models. To evaluate our model, we randomly divide the $\binom{n}{2} \times T$ data values into $k = 30$ sets, letting $s_{ij,t}$ be the set to which pair ij, t is assigned. Then for each $s \in \{1, \dots, k\}$, we:

- (1) estimate model parameters with $\{y_{ij,t} : s_{ij,t} \neq s\}$, the data not in set s ,
- (2) and predict $\{\hat{y}_{ij,t} : s_{ij,t} = s\}$ from these estimated parameters.

⁶In particular, the variables Pers/Democ and Democ/Pers in Model 3, in Table 1, page 335.

Table 2. Here we provide a brief summary of the key variable in each of the five replications and a note about whether or not the finding is replicated when using our network based approach. Cases in which the substantive finding is not replicated are highlighted in bold.

Study	Central Finding	Replicates in a Network Model?
Reiter & Stam (2003)	Personalist Regimes Attack Democracies, Not Vice Versa	Replicates
McDonald (2004)	Lower Trade Barriers and Higher Trade Lead to Peace	Does Not Replicate
Rose (2004)	WTO Membership Does not Effect Trade	Replicates
Weeks (2012)	Bosses, Juntas and Strongmen are more Aggressive, Machines are Not	Does Not Replicate
Gibler (2017)	Power Parity at Time of Entry to International System Inceases Conflict	Does Not Replicate

The result of this procedure is a set of sociomatrices \hat{Y} , in which each entry $\hat{y}_{ij,t}$ is a predicted value obtained from using a subset of the data that does not include $y_{ij,t}$.

We set a number of benchmarks for comparison. First, we compare the AME model to a GLM model (that used in the original study) using the same covariates to show the effect of accounting for network dependencies on predicting conflict. We supplement this with an alternative GLM that includes not just these covariates, but also a lagged dependent variable and a lagged reciprocity term. The lagged dependent variable is the equivalent of saying that conflict and peace are relatively likely to persist between dyads, while the inclusion of a lagged reciprocity term in a GLM framework is a simple way to account for retaliatory strikes.

We utilize three performance criteria to compare the models: Receiver Operator Characteristic (ROC) curves, Precision Recall (PR) curves, and separation plots. ROC curves look at the trade-off between true positive rates and false positive rates at different thresholds of classification. An issue with an ROC Curve when looking at conflict, is that it is relatively rare at the dyadic level: in most years only 3% of possible dyads are in conflict with one another. If peace is common, even a poor model will have a very low False Positive Rate. The ROC curves are best for balanced samples with as many events as non-events. To better assess which models predict the presence

of conflict, not just its absence, we look at PR Curves. These examine the trade-offs between the percentage of conflicts a model predicts, and the percentage of predicted conflicts which occur. Lastly, we examine separation plots (Greenhill et al., 2011). These provide an intuitive visualization of the accuracy of our predictions by juxtaposing a line showing the predicted probability of conflict with whether conflict actually occurs for all cases (where the cases are sorted by the predicted probability and then colored to indicate the outcome). Here a perfect model would have all cases where conflict actually exists on the right with a predicted probability of 1, and would predict 0 in all other cases. All of the models' performance out of sample by these metrics are displayed in figure ???. The AME model with covariates is the best performing model out of sample in all cases. This model outperforms each of the GLM variants by a notable margin.

		GLM	AME
Reiter & Stam (2003)	Area Under ROC Curve, AUC-ROC	0.92	0.96
	Area Under PR Curve, AUC-PR	0.08	0.15
McDonald (2004)	AUC-ROC	0.92	0.99
	AUC-PR	0.13	0.28
Rose (2004)	Root Mean Squared Error RMSE	3.23	1.99
	Root Median Squared Error RMDSE	2.01	1.06
Weeks (2012)	AUC-ROC	0.64	0.97
	AUC-PR	0.00	0.15
Gibler (2017)	AUC-ROC	0.52	0.91
	AUC-PR	0.00	0.08

Table 3. Here we provide a summary of the out-of-sample performance based on our cross-validation strategy for each of the five replications when using the standard dyadic approach and our network based approach. Four of the five studies involved a binary dependent variable, so for those measures we provide area under the curve (AUC) statistics. The Rose study involved a Gaussian dependent variable and for that we use the root mean squared error (RMSE) and root median squared error (RMDSE). Cases in which our network based approach outperformed the standard approach are highlighted in bold.

5.2. Replication of Reiter & Stam (2003). Reiter and Stam (2003) examine the relationship between democracy, dictatorship and the initiation of militarized disputes. They use directed dyads and find that dyads involving a democratic leader on the one hand and a personalist dictator on the other tend to be violent. They also discover that dictators are likely to challenge democracies,

but that this is not reciprocal. In addition, military regimes and single-party regimes are more prone to initiate disputes with democracies, than the other way around. They use the MID data, but note that “We code a state as having initiated a dispute if it is on ‘side A’ of a MID, the conventional approach to coding initiation. This means that the state was on the side that took the first action in the dispute, whether that action was the threat, display, or use of force. We code joiners as initiators or targets, though the results do not change if we do not code joiners as initiators or targets. ... Though coding initiation will always be difficult, the ‘side A’ variable has been widely used in past conflict scholarship (page 334).” Independent variables are largely taken from an earlier study and focus on various encodings of regime types, contiguity, alliance, and capability measures. As is prevalent in these kinds of studies, Reiter & Stam employ a logistic regression that includes an indicator of the time since the last dispute as well as three cubic splines. The database for this study is constructed using EUGene (Bennett and Stam, 2000) and comprises approximately three-quarters of a million stacked dyads. Based on their statistical analysis, they conclude that institutional constraints affect the propensity of democratic and non-democratic leaders to engage in military conflict.

In the original model, the variable “Pers/Democ Directed Dyad” (which represents a Personalist → Democratic directed dyad) is clearly positive while the variable “Democ/Personalist Directed Dyad” is zero and the difference between the two coefficients is clearly distinct from zero. In our re-estimation using the AME framework, we also find that Pers-Democ directed dyad has a positive effect with zero excluded from the 95% confidence interval while Democ-Pers directed dyad is indistinguishable from zero. Using this model, however, we can no longer conclusively say that the Pers/Democratic coefficient is larger than the Democ/Personalist one. Our re-estimation using the AME approach therefore cast some doubt on Reiter & Stam’s key claim that MIDs initiated by personalist dictatorships against democracies are more likely than MIDS initiated by democracies. Further, the effect of most of the covariates in the literature thought to predict interstate MIDs are much closer to zero when using the AME framework. Finally, our modeling approach outperforms the original model by better, and more accurately, predicting MIDs out-of-sample (Figure 6(a)).

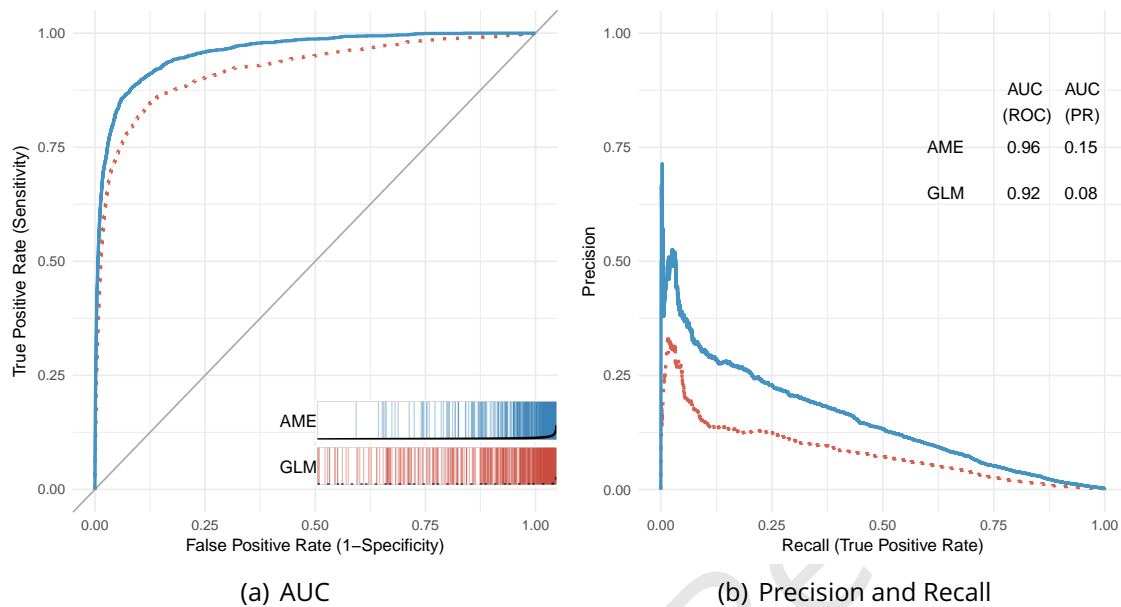


Figure 6. Assessments of out-of-sample predictive performance for Reiter & Stam (2003) using ROC curves and PR curves.

5.3. Replication of McDonald (2004). McDonald (2004) studies whether trade promotes peace between nations. The link between conflict and trade is perplexing, with many persistent, yet competing explanations. McDonald (2004) includes the argument that interdependence between states “makes conflict less likely because of its efficiency over conquest in acquiring resources... (547)”. Accordingly, his primary contribution is to provide evidence challenging the generalized linkage between peace and trade and to offer a new measurement of the key independent variable, trade. McDonald (2004) refines the trade variable, arguing that *free* trade, rather than trade alone, reduces the likelihood of conflict between states. His key hypothesis that greater levels of protection increase the probability of interstate conflict, an argument that builds on the work of classic liberalism and connects free trade to the power of domestic audiences. McDonald (2004) measures free trade in two ways. The first captures the idea that larger protected sectors generate greater societal pressures resulting in pockets of support for war. This protection variable measures the proportion of customs revenue divided by total imports in the state that possesses the greater such ratio in each dyad. This measure captures the score of the state in the dyad that possesses higher barriers to trade (560). McDonald (2004) also includes a measure of economic integration calculated as “the lower proportion of total dyadic trade (imports plus exports) divided by state

i 's GDP or total dyadic trade divided by state j 's GDP" (560). The binary, dependent variable is the onset of a new militarized interstate dispute within a given dyad. McDonald (2004) employs logistic regression to examine the putative statistical significance of these variables. The models include splines to correct for temporal dependence, and robust standard errors clustered on each dyad.

Our re-estimation with AME reveals that trade relations are highly interdependent and exhibit important patterns of transitivity. Or, in other words, if countries i and j are highly dependent and countries j and k are also highly dependent, then we are likely to observe high dependency between countries i and k . Our findings support those of Traag and Lupu (2013) which argue that indirect trade relations reduce the probability of conflict. This indicates that conflict is less likely between members of a trade community. Once we control for these dependencies, we can more clearly interpret the positive link between trade and conflict. The most striking thing is that AME finds a positive conditional association between trade dependence and conflict ($\hat{\beta} = 18.4$, $\sigma_{\hat{\beta}} = 28.6$), while the comparable numbers for the logistic regression found in the original articles are negative ($\hat{\beta} = -22.2$, $\sigma_{\hat{\beta}} = 15.2$). At the same time, the AME has ROC and PR curves (shown in the Appendix) that dominate the results found in McDonald (2004).

5.4. Replication of Rose (2004). In 2004, Andrew Rose published a study in the *American Economic Review* that proved to be quite controversial in terms of macroeconomic trade theory and in terms of trade policy in a variety of nations. It also provoked a number of responses in the international political economy literature Tomz et al. (2007); Ward et al. (2013). Rose's basic argument is that despite longstanding arguments made by trade theorists and the World Trade Organization that WTO membership fosters greater cooperation and thereby more trade among its members, the empirics do not bear out such claims. He uses a standard gravity model with dyadic data on bilateral merchandise trade (not services) for 175 countries over a period of five decades. Estimating this model using OLS within many differing contexts, his conclusion was that: "An extensive search reveals little evidence that countries joining or belonging to the GATT/WTO have different trade patters from outsiders . . . (2004, page 98, abstract)." The data for this study have been widely used in replications by many searching for the missing effects of the WTO – as well as preferential trade agreements, bilateral investment theories, and other aspects of modern trade theory.

When we compare the results of Rose's original OLS model to the AME model accounting for network dependencies, the results are generally similar. The main result of the model – the null effect of membership in the WTO, as represented by the "One-In" and "Both-In" variables – persists in the AME model. However, while in the original model there was a clear positive relationship between Real GDP and Trade, most of this effect vanishes in the AME model. The random effects shown in Figure 8 reveal the cause of much of this divergence. Here, the states with the most positive random effects are also states with high GDP, though not necessarily high GDP/capita.⁷ Thus, the effect of GDP in the original model was, in part, an artifact of first-order dependencies. Most of the other results of the model are constant across each model, though some geographic features, such as islands and landlocked states, have a more clear effect on trade once we account for these network dependencies. When we account for network interdependencies, we observe a markedly lower Root Mean Squared Error out-of-sample – 3.23 for the OLS model and 1.77 for the AME model. While the basic model replicates with the basic result, there are substantial and substantive improvements gleaned from the AME results.

⁷Note: Qatar exhibits strongly negative random effects.

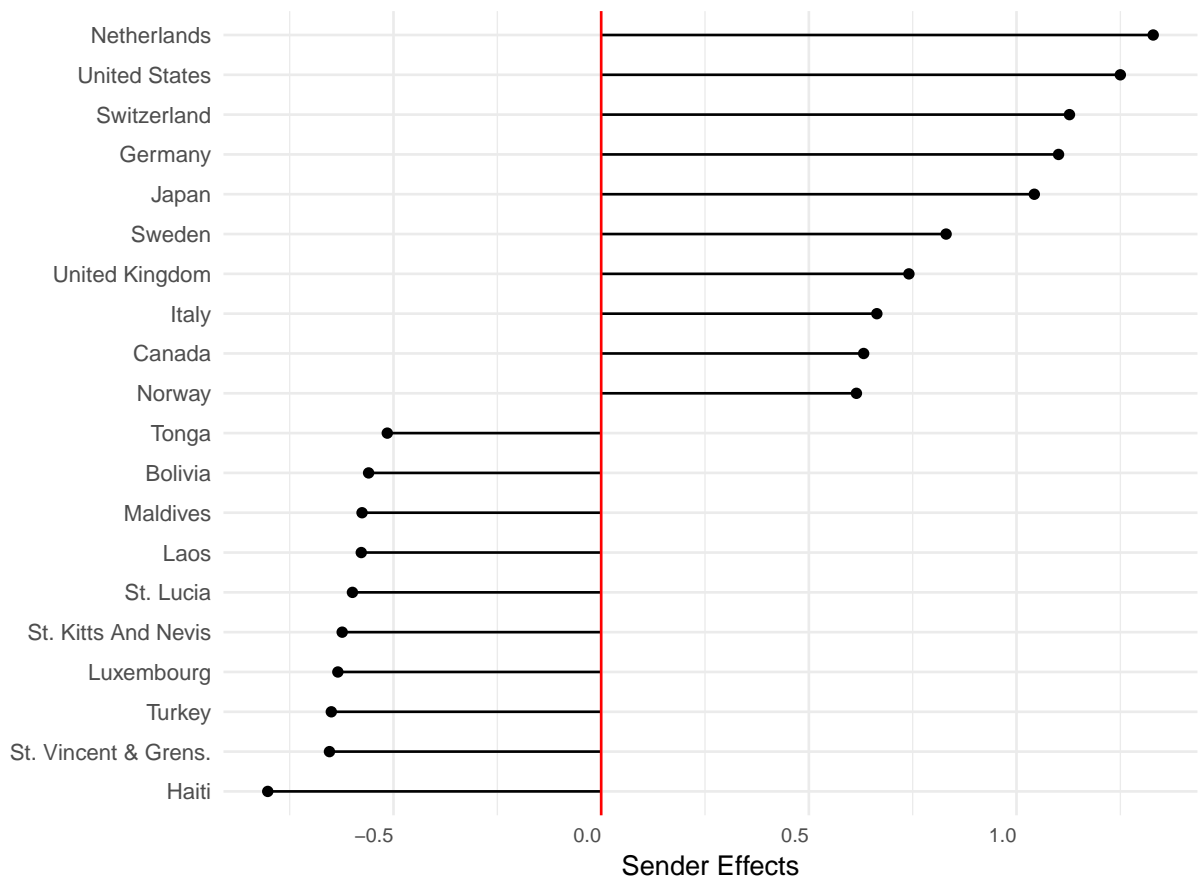


Figure 7. Nodal Random Effects for AME estimation of Rose (2004)

5.5. Replication of Weeks (2012). Weeks (2012) examines the influence of domestic institutions on the initiation of military conflicts by autocratic leaders. She argues that in some circumstances autocrats are held accountable for their foreign policy decisions. She adds the nuance that autocratic audiences are not homogeneous. When the autocratic regime is nonmilitary, the domestic audience do not favor military actions, but in military autocracies this is not the case. Further she argues that in personalistic regimes without a military or civilian domestic audience, the leaders tend to be more likely to employ military force in their foreign policy. To study this question, she uses a dyadic design in which the dependent variable is “whether country A in a directed dyad initiated military conflict against country B during year t ” (page 337). These data come from the Militarized Interstate Disputes database (Maoz, n.d.). One major innovation in her study resides in the nuanced way in which she conceptualized and coded regime type into four types: a) Machine,

b) Junta, c) Boss, and d) Strongmen. She also includes a variety of putative control variables focusing on capabilities for both sides of the dyad, alliances, geography, trade dependence, regime instability, and the regime type of "side B." She uses a logistic regression, but follows Beck et al. (1998) and includes splines to capture temporal covariation in the dependent variable along with fixed, unit effects. The analysis is done for dyads, but is considered to be from the perspective of the actor that initiated the dispute. Her time frame is from 1946-1999. The basic finding is that a) juntas, bosses, and strongmen are more likely to initiate conflict than machines (and maybe democracies) and that b) machines are no more belligerent than democracies. These insights are mainly determined by looking at coefficients and their putative significance in Tables 1 and 2 (pages 339-340). She makes the argument that ignoring important nuances between different types of autocracies hinders our understanding of the initiation of military conflict by autocracies.

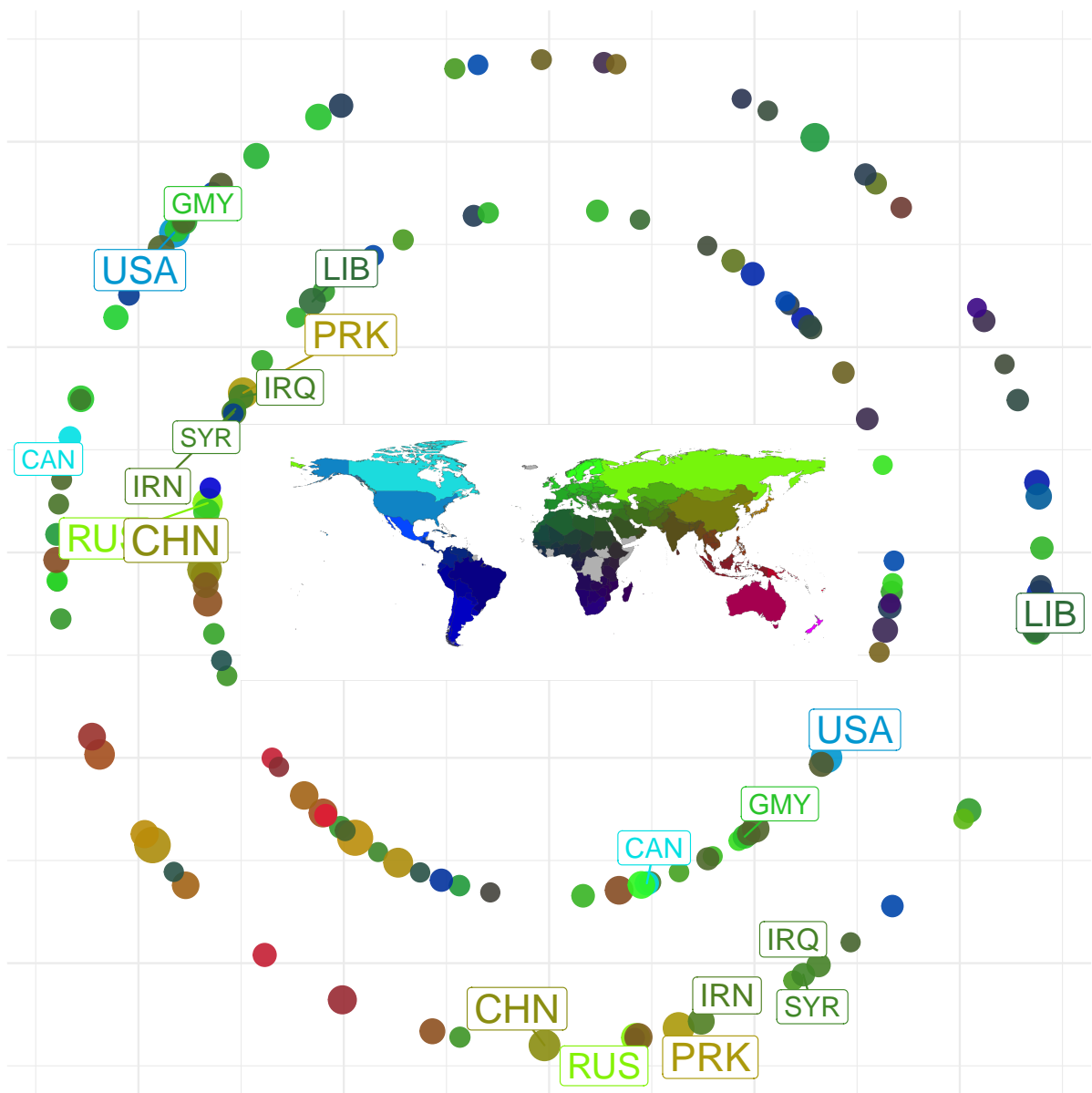


Figure 8. Visualization of multiplicative effects for Weeks (2012). Blue represents groups with common sending patterns and red represents groups with common receiving patterns.

The re-estimation of Weeks (2012) likely has the sharpest divergence between the General Linear Model results and those of the AME Model. In Weeks's initial models, she finds that machines are less prone to initiate conflict than the reference category, whereas Juntas, Bosses and Strong-men are more conflict-prone, in line with her theoretical expectations. When we look at the posterior

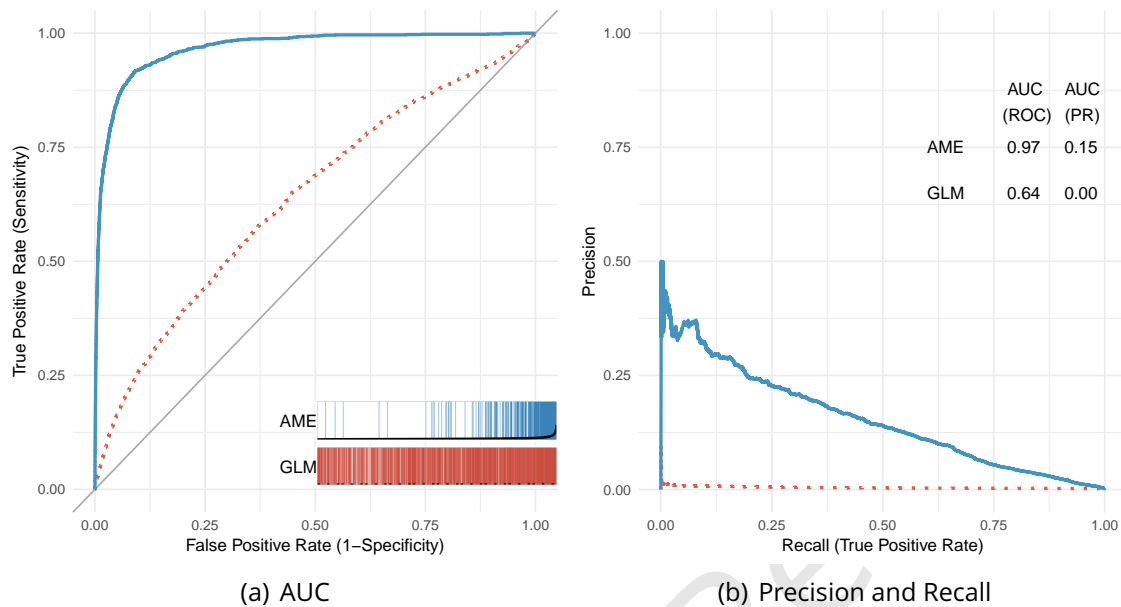


Figure 9. Assessments of out-of-sample predictive performance for Weeks (2012) using ROC curves and PR curves

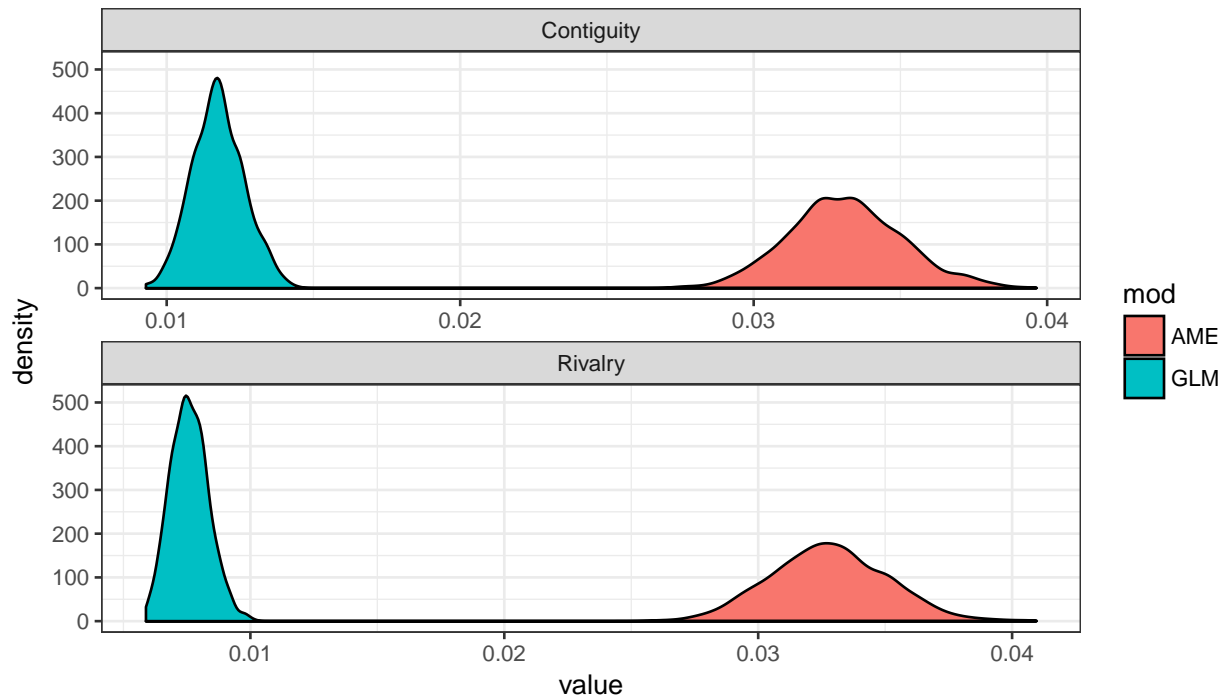
distribution of these coefficients in the AME results, we find that none of these values are distinguishable from zero. We similarly find less pronounced effects for military capabilities. One explanation for this divergence is the AME model's ability to account for third order effects. Inspection of the multiplicative effects in Figure 9 reveals a number of clusters of states which exhibit structural equivalence – in the top right corner we see states like the US, the UK, and Israel who often send conflict to similar targets. In particular, we observe a cluster of “rogue states,” who are receivers of conflict, in the top right (Iran, Iraq, Syria, Libya, North Korea). In the bottom corner we see a cluster of authoritarian senders including Iraq, Russia, Syria, North Korea and China. In general, these clusters have similar governmental types (Iraq, Syria, Libya, and North Korea all fell under the “boss” category), and similar conflict behavior. So, in the GLM, which ignores these third order dependencies, much of these results might have been attributed to regime type. The AME model, on the other hand, finds that it is more efficient to attribute this behavior to the multiplicative effects. In terms of out-of-sample performance, shown in Figure 10(a), the AME model performs markedly better than the GLM out of sample, lending credence to the possibility that these third order dependencies cause spurious effects for regime type.

5.6. **Replication of Gibler (2017).** The replications we have undertaken are all from articles over the past fifteen years. A more recent example is Gibler (2017) which examines the onset of militarized disputes using capabilities, joint democracy, alliances, and power parity in a undirected dyadic study using logistic regression and dyad clustered standard errors. In addition to this, Gibler shows that the long-standing relationship between the relative parity of capabilities and initiation of international conflict is almost completely mediated by the initial conditions for the members of the dyad when they joined the international system as sovereign members. This finding calls into question many IR theories about the role of balance in terms of generating international conflict (Organski, 1958).

Insert coefplot here, without splines...

We re-estimated model 6 from Table 6 (2017, 34). The results are presented in Figure 5.6 in order to facilitate explicit comparison. The results obtained with `ame` stand in stark contrast to those found with a logistic regression (with dyad clustered, robust standard errors). Most importantly, the primary variable from the Gibler study, parity of the members of a dyad at the year in which they entered the international system, is shown to be unimportant in the AME results. Not only is the value of this parameter small, but it has a very large relative standard error – over a magnitude larger than the parameter itself ($z = 0.038$). In addition, the variable indicating whether both members of the dyad were coded as democracies (joint democracy) follows the same pattern: important and strong in the logistic results, but this disappears once interdependencies are modeled. As might be expected, the strong geographic clustering in the original study is about one-quarter as strong in the AME estimations. Similarly, rivalry coefficients are about one-third the size in the AME formulation, but a great deal more precisely measured ($z = 18.116$).

We utilized the original and the AME results from Gibler's model 6 in Table 6 to calculate the expected values for one scenario. We focused on the variables measuring rivalry. We used mean or modal values for all independent variables, except we changed the rivalry variable to indicate that there was a rivalry when the actual data suggest there is none. The expected values of this scenario are essentially a first difference plot comparing results with the model when estimated in two different ways: Gibler's GLM estimation and our AME approach. As this Figure 11 illustrates,

Figure 10. THIS NEEDS TO BE FIXED.

the AME results differ from the GLM in two distinct ways. First, the expected value of the dependent variable – the probability of the onset of a militarized interstate dispute, is considerably higher when taking interdependencies into account with the AME model. These are rare events, so the probabilities are low, but the difference is a factor of $3x$. Thus, you get quite substantially different expected values from these two models. Second, the GLM model has a standard errors which underestimate the true variability, in part because some of them are assumed to be zero. This shows up in the narrow density of predicted outcomes for the GLM model, compared to the density seen with the AME. The latter reflects a wider range of potential outcomes, reflecting the incorporation of uncertainty in the AME model.

Beyond more informative fixed-effects coefficients, the AME approach also provides information about the interdependencies that were modeled. The assumption of independence among the dyadic data in this study can be strongly rejected. Perhaps most importantly, the main substantive conclusions of the 2017 Gibler study do not seem warranted from the perspective of the

results obtained with the `amen` estimation that explicitly models 1st-, 2nd-, and 3rd-order interdependencies. Not only do the estimated coefficients tell a different story, but the assumptions under which the original results would hold are shown to be violated by the data.

5.7. Lessons Learned from Re-estimating Five Prominent Studies. First and foremost, many findings which emerge from models that do not take interdependencies into account lose their statistical significance when network effects are estimated via AME. This should not be news – since the finding has long been in the theoretical literature, but given the state of current literature in international relations is still pertinent. Not only are coefficients biased in the OLS and logit approaches to the analysis of dyadic data, but they are often imprecisely measured, with inflated standard errors. This means that significant testing (for better or worse) is compromised when network effects are ignored.

Second, even when the results from the AME estimation conform with those found in an OLS or logistic regression, new insights emerge from the additional information derived. In particular, there is actual information about the dependencies so that clusters can be identified, and the extent of reciprocity at the dyad level, as well as among senders and receivers. This kind of information is absent in standard approaches and add to our ability to explain specific as well as general results.

Third, it is evident that the actual results – not the estimated coefficients and their covariances – which are generated by the models differ greatly in expectations. This implies that policy experimentations with the models, as well as scenario-based simulations and forecasting of GLM models are likely to often give misleading results compared to the AME approach.

Fourth, it is clear that the AME approach dominates the OLS and Logit-based approaches in terms of performance. Not only it is better at correctly identifying cases in which the dependent variable takes a value of 0 (via the ROC curves and associated statistics), but it also dominates at correctly identifying occurrences of the dependent variable in the data (seen via the PR curves and associated statistics). In the case of studies with continuous dependent variables, the AME approach has average error statistics that are about one-half that found in the OLS model. It is rare for studies in this area to provide performance statistics, but at the same time that at least one of the studies is unable to identify a single case in spite of having almost a million observations.

Fifth, it is useful to suggest that a lot of what is *known* about international relations based on studies of dyadic data should be taken with a grain of salt, and awaits a reassessment using methods that do not rely on an assumption that all the dyads are independent and identically distributed.

6. CONCLUSION

International relations is generally about the interactions and dependences among a set of countries or other important actors such as international governmental organizations (IGOs, such as the WTO) and non-governmental organizations (NGOs, such as the Gates Foundation). Some approaches focus on only looking at a small number of these actors, but many scholars examine a large number of actors at a time. This is particularly true of those scholars who work in the tradition of the Correlates of War Project, but is by no means limited to them.⁸ Many scholars have debated the use and abuse of dyadic data. One recent on-line symposium can be found at <http://bit.ly/2wB2hab>. It is clear from a survey of the literature and from work in this area published as recently as 2017 that many find dyadic data to be an important touchstone in the study of international relations Erikson et al. (2014); Aronow et al. (2015).

At the same time, we know that research designs which focus on the statistical analysis of dyadic data quickly go astray if the dyadic data are assumed to be independent and identically distributed (iid). Virtually all of the standard statistical models—ordinary least squares, logistic and probit regressions, to name a few—fail if the data are not iid. By definition dyadic data are not iid and thus the standard approaches can not be used cavalierly to analyze these data. Signorino (1999) showed why this is true of models of strategic interaction, but it is more broadly true of models that employ dyadic data. We show that latent networks can be employed to defeat this vulnerability of dyadic data in the realm of international relations. These approaches have been developed for a while, but are not yet widely used in international relations scholarship. The statistical model of the latent network captures first-order (example or restatement), second-order (example or restatement), and third-order (example or restatement) dependencies in dyadic data using a familiar regression framework that has been adapted for relational data—such as dyadic data—which

⁸See Singer (1972) for an early description of the project and also see the project's Web site for an history and more recent efforts <http://www.correlatesofwar.org/>.

are not independent nor identically distributed. There is an available, open source computer packages that implements this approach.

To explore this approach in the context of international relations we conduct two broad analyses. The first is a simulation of XCLKFJDSLFL. The second is a replication of five prominent studies that have been published recently using a broad range of dyadic data to draw inferences about international relations. These five studies have been replicated with the original research designs, each of which used a statistical method that assumes the dyadic data are all independent from one another. We then reanalyzed each study using the latent network approach which captures that additive and multiplicative aspects of interdependencies among the dyadic data. In every case, we found that the AME approach provided a) increased precision of estimation, b) better out-of-sample fits, c) evidence of 1st-, 2nd-, and 3rd-order dependencies that were overlooked in the original studies. In several cases, the new approach overturns the basic findings of the original research. This leads us to speculate that many of the findings in the international relations literature may be fragile in the sense that they only obtain under stringent assumptions that can not possibly be valid. This in turn leads to a certain arbitrariness in some research findings, which might lead to puzzles that are more apparent than real (Zinnes, 1980). At the same time, the latent factor model we present and demonstrate herein provides a way to easily examine and if necessary defeat these assumptions.

It is no longer necessary to assume that the interesting, innate interdependencies in relational data about international relations can be ignored. Nor do they have to be approximated with *ad hoc*, incomplete solutions that purport to control for dependencies (such as modifying the post-estimation standard errors of the estimated coefficients (King and Roberts, 2014)). Instead, the interdependencies may be addressed directly with additive and multiplicative effects in the context of a generalized linear model that provides more reliable inferences and better out-of-sample predictive performance, along with new substantive insights.

APPENDIX

Performance Analysis.

Gibler (2017).

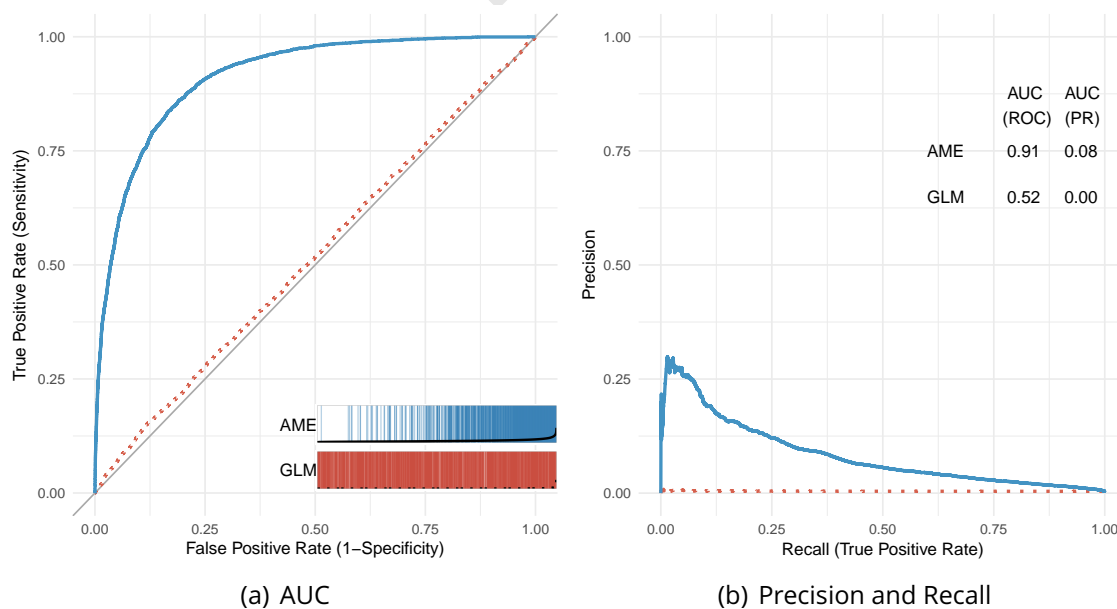


Figure A1. Assessments of out-of-sample predictive performance for Gibler (2017) using ROC curves and PR curves.

McDonald (2004).

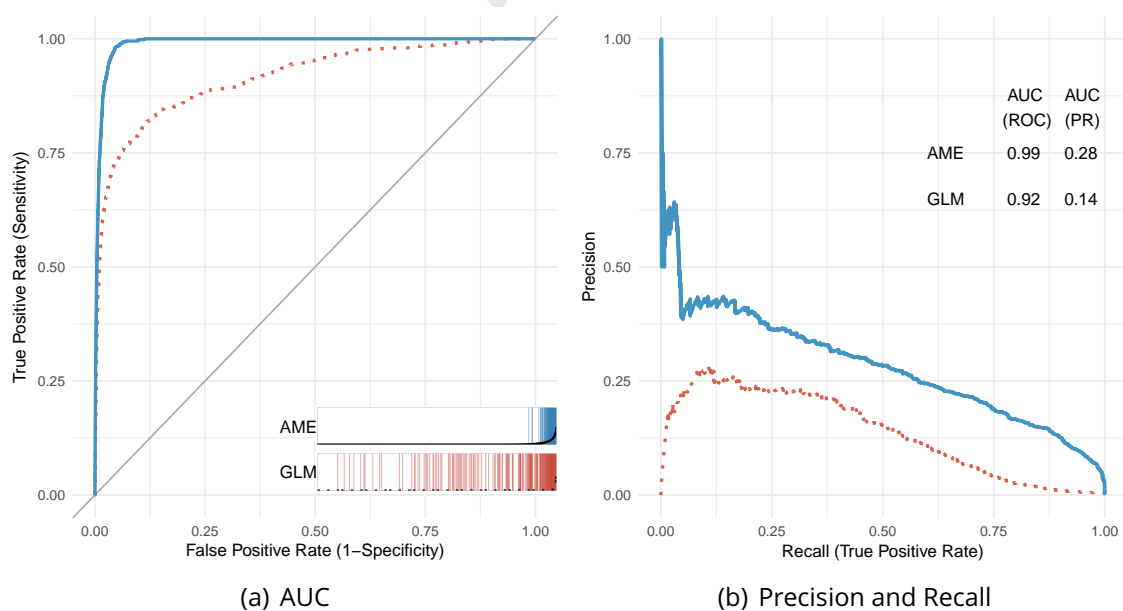


Figure A2. Assessments of out-of-sample predictive performance for McDonald (2004) using ROC curves and PR curves.

Reiter & Stam (2003).

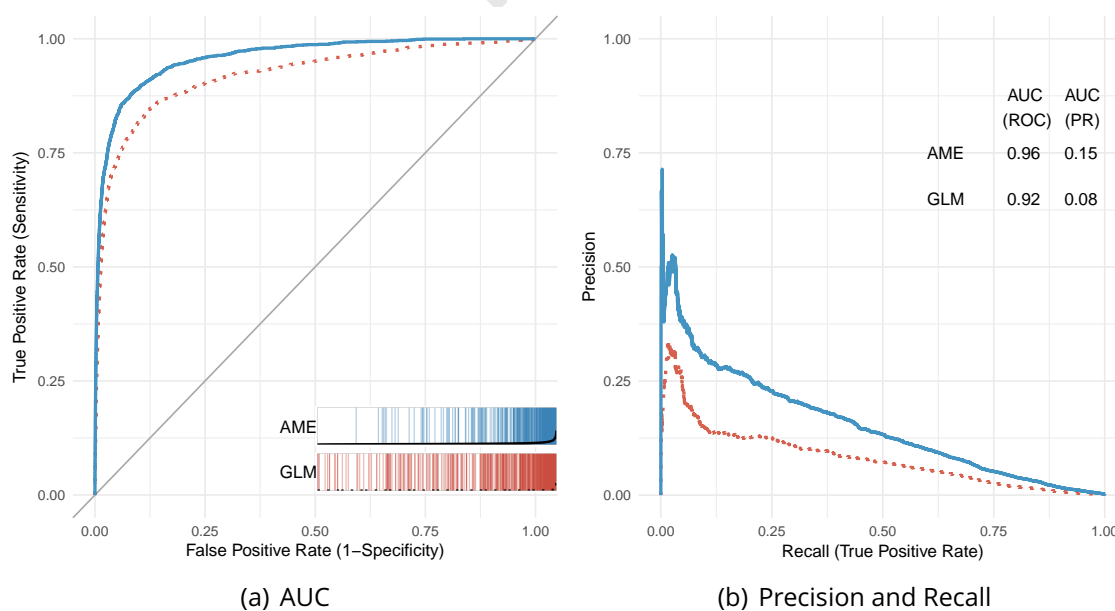


Figure A3. Assessments of out-of-sample predictive performance for Reiter & Stam (2003) using ROC curves and PR curves.

Rose (2004).

Weeks (2012).

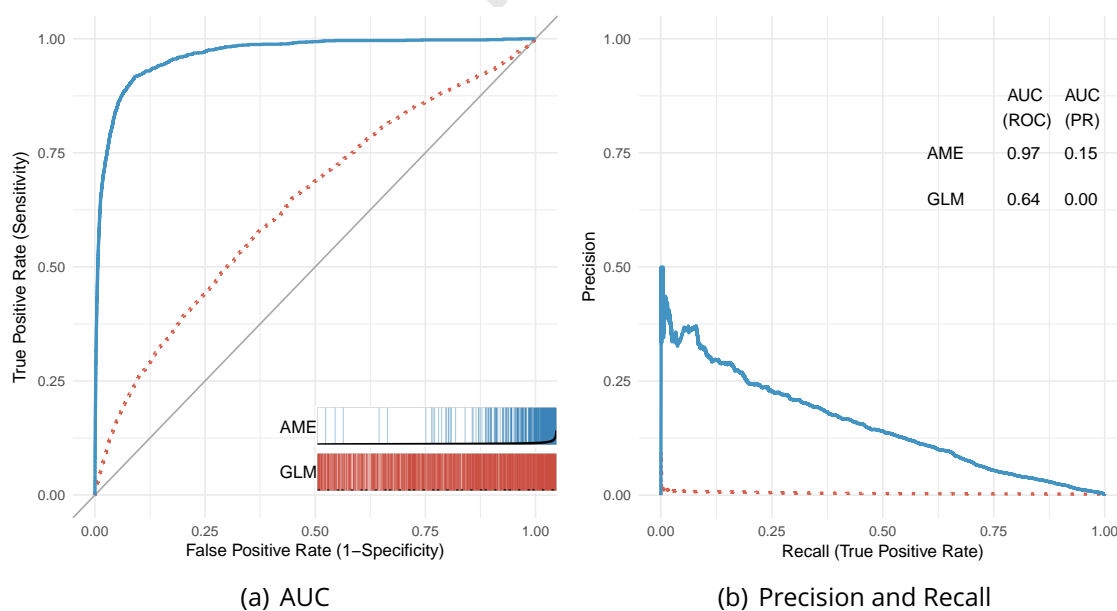


Figure A4. Assessments of out-of-sample predictive performance for Weeks (2012) using ROC curves and PR curves.

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