TAKING DYADS SERIOUSLY

ABSTRACT. Much of international relations scholarship concerns dyads: dyadic hypotheses and especially dyadic data. Yet, contemporary research brushes aside the complications that come with analyzing interdependent relational data. Dyadic observations do not typically satisfy the conditional independence criterion required for many statistical approaches. As a result, many studies often produce results with biased coefficient estimates and poorly calibrated standard errors. These biases have profound consequences for evaluating parametric models. We present an alternative, regression-based, approach that accounts for the dependencies complicating this type of analysis. We first present a simulation exercise highlighting the model's ability to account for the dependencies that emerge in relational data. In addition, we replicate five recent studies in recent international relations scholarship, comparing the standard approach to our alternative. For each study, we find that conventional methods overstate the effect of key variables in the study of interstate conflict (and trade), underestimate the uncertainty in these effects, and in some cases lead researchers to faulty conclusions about the statistical significance and substantive importance of their variables. Further, we show that our approach dominates in terms of out-of-sample cross-validations, rendering it more useful in forecasting applications and in modeling the data generating process behind outcomes of interest.

1. Introduction

Aronow et al. (2015) estimate that during the period 2010 to 2015, over sixty articles were published in the *American Political Science Review*, *American Journal of Political Science*, and *International Organization* using dyadic data.[1] Most of these studies use a generalized linear model (GLM) to test hypothesized relationships. However, this approach to studying dyadic data increases the chance of faulty inferences by assuming data are conditionally independent and identically distributed (iid). Most standard approaches assume that the problems raised by having non-iid relational data can be addressed by recalculating the standard errors of estimated parameters, to reflect the potential clustering of cases. In practice, these palliatives rarely work because they fail to address the fundamental data generating process that remains a threat to inference. Namely, it is not just the diagonals of the variance-covariance matrix that are affected (Beck, 2012; Franzese and Hayes, 2007; King and Roberts, 2014).

In this article, we present the Additive and Multiplicative Effects (AME) model, a Bayesian approach for directly modeling relational data to better reflect the interdependencies underlying the data generating process of dyadic data structures (Hoff, 2008; Minhas et al., 2016). We focus on three types of interdependencies that can complicate dyadic analyses. First, dependencies may arise within a set of dyads if a particular actor is more likely to send or receive actions such as conflict.[2] Additionally, if the event of interest has a clear sender and receiver, we are likely to observe dependencies within a dyad; for example, if a rebel group initiates a conflict against a government, the government will likely reciprocate that behavior. We capture these effects, often referred to as first- and second-order dependencies, respectively, within the additive effects portion of the model. Third order dependencies capture relationships of transitivity, balance, and clusterability between different dyads–for example, we can only understand why Poland was involved in a dyadic conflict with Iraq in 2003 if we understand that the United States invaded Iraq in 2003 and that Poland often participates in US-led coalitions. The multiplicative effects capture these sorts of dependencies, especially those that result because the specified model has not accounted for

[1] In 2017, *International Studies Quarterly* published a special issue on Dyadic Research Designs along with an online symposium to discuss the papers.

[2] In the case of undirected data where there is no clear sender or receiver, it is still essential to take into account the variance in how active actors are in the system.

some latent set of shared attributes possessed by actors that affect their probability of interacting with one another.

We begin with a discussion of these dependencies and an introduction to the AME model. Next, we conduct a simulation study to show how the AME approach can recover unbiased and well-calibrated regression coefficients in the context of dyadic data. Last, to highlight the utility of this approach, we apply the AME model to five recent studies in the international relations (IR) literature. The comparison reveals that in accounting for observational dependence, AME produces more precise estimates and better-calibrated confidence intervals for key variables in the literature. Consequently, AME produces results that, at times, differ from those found in these studies in particular as well as the broader literature from which they are drawn. Moreover, we demonstrate the latent factor approach offers substantive insights that are often occluded by ignoring the interdependencies found in the relational data of IR studies. Finally, we show that for each replication our network-based approach provides substantively more accurate out-of-sample predictions than the models used in the original studies.

The framework that we present advances statistical analysis of dyadic data by accounting for observational dependence while enabling scholars to still focus on testing the substantive effect of variables of interest. Thus, the AME approach can be used by scholars in the field to continue to generate substantive insights, while having the benefit of being able to better account for the data generating process behind political events of interest. Most importantly, it concentrates on the relational aspect of the field of international relations through a statistical framework that is familiar to most scholars.

2. Dependencies in Dyadic Data

Scholars working with dyadic data typically begin by stacking observations associated with each dyad on top of one another. This makes sense if each observation is independent of the others. For example, a conflict initiated from the United States against Japan is assumed to be independent of any conflictual action that Japan may send to the United States. Additionally, every action sent by Japan to others in the system is considered independent even though each of those interactions involves a common sender, i.e, Japan. While most scholars begin with the assumption that each dyadic interaction is taking place in isolation of the others, we know this assumption to be false both in theory and in practice. Relational data comes with an explicit structure that generally leads to particular types of dependencies. The importance of accounting for the underlying structure of our data has been a lesson well understood, at least, when it comes to time-series cross-sectional data (TSCS) within political science (Beck and Katz, 1995; Beck et al., 1998). As a result, it is now standard practice to take explicit steps to account for the complex data structures that emerge in TSCS applications and the unobserved heterogeneity that they cause.

To uncover the underlying structure of relational data, it is helpful to restructure dyadic data in the form of a matrix—often referred to as an adjacency matrix—as shown in Figure 1. Rows designate the senders of an event and columns the receivers. The cross-sections in this matrix represent the actions that were sent by an actor in the row to those designated in the columns. Thus y_{ij} designates an action y, such as a conflictual event or trade flow, that is sent from actor i to actor j. In many applications, scholars are interested in studying undirected (i.e., symmetric) outcomes in which there is no clear sender or receiver, these type of outcomes still can, and we would argue should, be studied using the type of framework we discuss below.

Using the structure of an adjacency matrix, Figure 1 visualizes the types of first- and second-order dependencies that can complicate the analysis of relational data in traditional GLMs. The adjacency matrix on the top left highlights a particular row to illustrate that these values may be more similar to each other than other values because each has a common sender. Interactions involving a common sender also manifest heterogeneity in how active actors are across the network when compared to each other. In most relational datasets (e.g., trade flows, conflict, participation in international organizations, even networks derived from Twitter or Facebook), we often find that there are some actors that are much more active than others (Barabási and Réka, 1999). For example, in an analysis of international trade certain countries (e.g., China) export in much larger volumes than other countries for a variety of structural, contextual, and idiosyncratic reasons. Unless one is able to develop a model that can account for the variety of explanations that may play a role in determining why a particular actor is more active than others, parameter estimates from standard statistical models will be biased.[3]

[3]In an undirected setting instead of studying sender and receiver heterogeneity, we would just be concerned with actor heterogeneity in general.

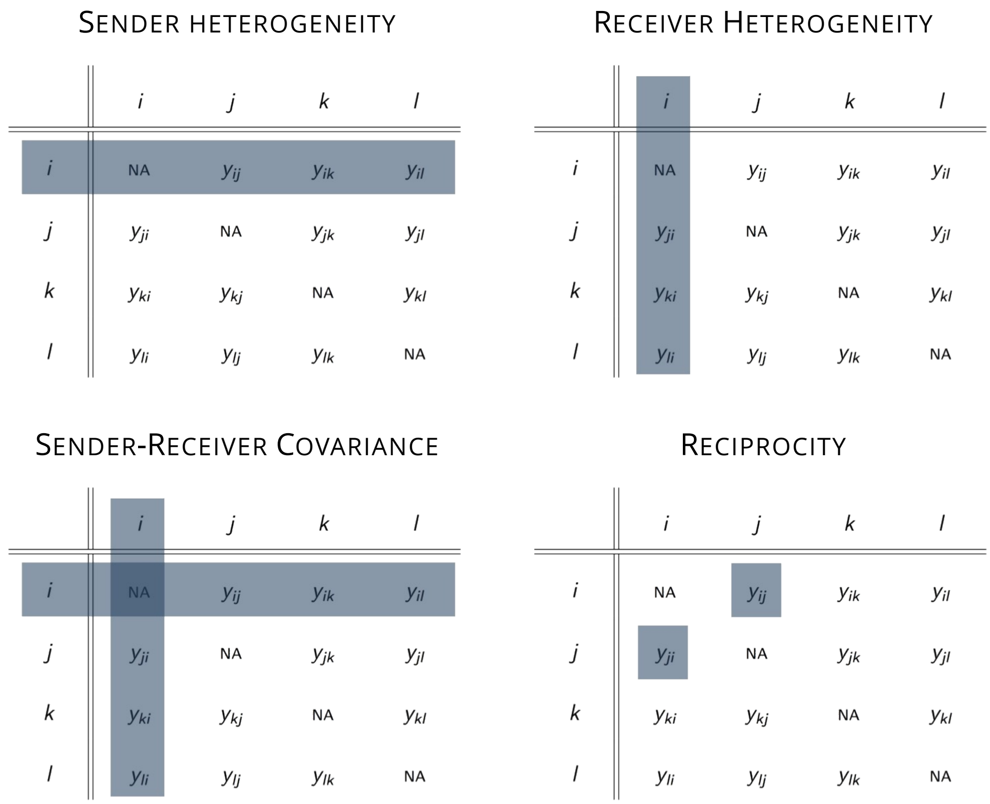


Figure 1. Nodal and dyadic dependencies in relational data.

For similar reasons one also needs to take into account the shared dependence between observations that share a common receiver. The bottom-left panel in Figure 1 illustrates that sender and receiver type dependencies can also blend together. Specifically, actors who are more likely to send ties in a network tend to also be more likely to receive them. As a result, the rows and columns in an adjacency matrix are often correlated. For example, consider trade flows both from and to many wealthy, developed countries. The bottom-right panel highlights a second-order dependence, specifically, reciprocity. This is a dependency occurring within dyads involving the same actors whereby values of y_{ij} and y_{ji} are correlated. The concept of reciprocity has deep roots in the study of relations between states (Richardson, 1960; Keohane, 1989).

For most relational data, however, dependencies do not simply manifest at the nodal or dyadic level. More often we find significant evidence of higher-order structures that result from dependencies between multiple groups of actors. These dependencies arise because there may be some set of latent attributes between actors that affects their probability of interacting with one another

(Zinnes, 1967; Wasserman and Faust, 1994). In Figure 2 we provide a visualization of a simulated relational dataset wherein the nodes designate actors and edges between the nodes indicate that an interaction between the two took place. To highlight third-order dependence patterns, nodes with similar latent attributes are colored similarly.

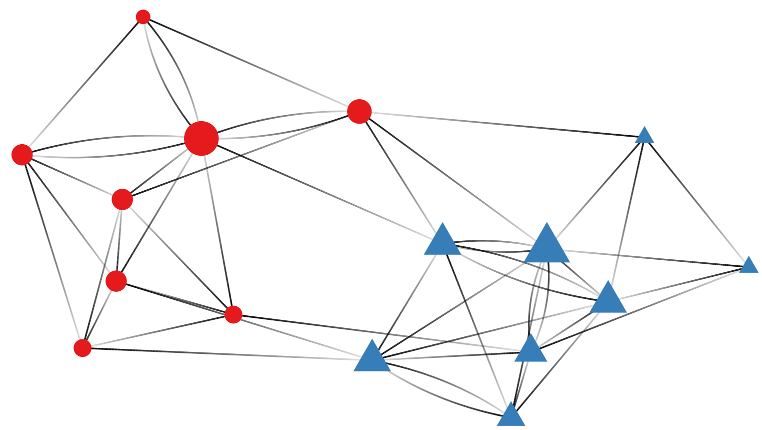


Figure 2. Visualization of a network with higher-order dependence patterns.

The visualization illustrates that the actors belonging to the same group have a higher likelihood of interacting with each other, whereas interactions across groups are rarer. A prominent example of a network with this type of structure is discussed by Adamic and Glance (2005), who visualize linkages between right and left leaning political blogs preceding the 2004 United States Presidential Election. Adamic and Glance find that the degree of interaction between right and left leaning blogs was minimal, and that most blogs were linked to others that were politically similar. This showcases the types of higher-order dependencies that can emerge in relational data. First, the fact that interactions were determined by a shared attribute, in this case political ideology, is an example of *homophily*. Homophily explains the emergence of patterns such as transitivity ("a friend of a friend is a friend") and balance ("an enemy of a friend is an enemy"), which also have a long history in international relations. The other major type of meso-scopic features that emerge in relational data is community structure (Mucha et al., 2010), which is often formalized through the concept of stochastic equivalence (Anderson et al., 1992). Stochastic equivalence refers to a type of pattern in which actors can be divided into groups such that members of the same group have similar patterns of relationships. In the example we have laid out above, each of the left leaning blogs would be considered stochastically equivalent to one another because any given left-leaning blog is more likely to interact with a blog of a similar political position and less likely to interact with one with a divergent political position.

These types of patterns frequently emerge in IR contexts as well.[4] For example, a perennial finding in the interstate trade literature emphasizes the role that geography plays in determining trade flows. Geographic proximity in the network context is an example of homophily — a shared attribute between actors that corresponds to a greater likelihood of the event of interest taking place. Alternatively, in the interstate conflict literature, we may find that actors who are each a member of a particular (formal or informal) alliance are likely to act similarly in the conflict network. Specifically, they will tend to initiate conflictual events with actors that their fellow alliance members initiate conflict with, and they will be unlikely to initiate conflict with members of their alliance — an example of stochastic equivalence. In both these examples, we are able to explicitly parameterize the attribute that might explain the emergence of higher order dependence patterns, but it is likely that at times we will not able to account for the variety of reasons why higher order dependence patterns in networks may develop.

3. Additive and Multiplicative Effect Models for Networks

To account for the dependencies that are prevalent in dyadic data, we turn to the AME model. The AME approach can be used to conduct inference on cross-sectional and longitudinal networks with binary, ordinal, or continuous linkages. It is flexible and easy to use for analyzing the kind of relational data often found in social science. It accounts for nodal and dyadic dependence patterns, as well as higher-order dependencies such as homophily and stochastic equivalence.[5] The AME model combines the social relations regression model (SRRM) to account for nodal and dyadic dependencies and the latent factor model (LFM) for third-order dependencies. For details on the SRRM see Li and Loken (2002); Hoff (2005); Dorff and Minhas (2017).[6] The AME model is specified as follows:

[4] For example, see: Manger et al. (2012); Kinne (2013); Chyzh (2016).

[5] Minhas et al. (2016) detail how this framework contrasts with alternative network-based approaches.

[6] An earlier version of the LFM used in AME is presented as the general bilinear mixed effects (GBME) model in Hoff and Ward (2004). The GBME model is more limited in the types of dependence patterns that it can capture due to the formulation of the matrix decomposition procedure.

$$
\begin{aligned}
y_{ij} &= f(\theta_{ij}), \text{ where} \\
\theta_{ij} &= \boldsymbol{\beta}_d^\top \mathbf{X}_{ij} + \boldsymbol{\beta}_s^\top \mathbf{X}_i + \boldsymbol{\beta}_r^\top \mathbf{X}_j && \text{(Exogenous parameters)} \\
&\quad + a_i + b_j + \epsilon_{ij} && \text{(SRRM parameters)} \\
&\quad + \mathbf{u}_i^\top \mathbf{D} \mathbf{v}_j && \text{(LFM parameters)}
\end{aligned} \tag{1}
$$

where $y_{ij,t}$ captures the interaction between actor i (the sender) and j (the receiver). We use a Bayesian probit framework, in which we model a latent variable, θ_{ij}, using first a set of exogenous dyadic ($\boldsymbol{\beta}_d^\top \mathbf{X}_{ij}$), sender ($\boldsymbol{\beta}_s^\top \mathbf{X}_i$), and receiver covariates ($\boldsymbol{\beta}_r^\top \mathbf{X}_j$). Next, to account for the dependencies that emerge in dyadic data and that may complicate inference on the parameter associated with exogenous covariates, we add parameters from the SRRM and LFM. a_i and b_j in Equation 1 represent sender and receiver random effects incorporated from the SRRM framework:

$$
\begin{aligned}
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{ab}) \\
\{(\epsilon_{ij}, \epsilon_{ji}) : \; i \neq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_\epsilon), \text{ where} \\
\Sigma_{ab} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} &\quad \Sigma_\epsilon = \sigma_\epsilon^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}
\end{aligned} \tag{2}
$$

The sender and receiver random effects are modeled jointly from a multivariate normal distribution to account for correlation in how active an actor is in sending and receiving ties. Heterogeneity in the the sender and receiver effects is captured by σ_a^2 and σ_b^2, respectively, and σ_{ab} describes the linear relationship between these two effects (i.e., whether actors who send [receive] a lot of ties also receive [send] a lot of ties). Beyond these first-order dependencies, second-order dependencies are described by σ_ϵ^2 and a within dyad correlation, or reciprocity, parameter ρ.

The LFM contribution to the AME is in the multiplicative term: $\mathbf{u}_i^\top \mathbf{D} \mathbf{v}_j = \sum_{k \in K} d_k u_{ik} v_{jk}$. K denotes the dimensions of the latent space. The construction of the LFM here is actually quite similar to the recommender systems that companies like Amazon and Netflix have used to model customer behavior (Resnick and Varian, 1997; Bennett et al., 2007). This model posits a latent vector of characteristics $\mathbf{u_i}$ and $\mathbf{v_j}$ for each sender i and receiver j. The similarity or dissimilarity of these vectors will then influence the likelihood of activity, and provides a representation of third-order interdependencies. The LFM parameters are estimated by a process similar to computing the singular value decomposition (SVD) of the observed network. When computing the SVD we factorize our observed network into the product of three matrices: $\mathbf{U}, \mathbf{D},$ and $, \mathbf{V}$. This provides us with a low-dimensional representation of our original network.[7] Values in $\mathbf{U}$ provide a representation of how stochastically equivalent actors are as senders in a network or, for example, how similar actors are in terms of who they initiate conflict with. $\hat{\mathbf{u}}_i \approx \hat{\mathbf{u}}_j$ would indicate that actor i and j initiate battles with similar third actors. $\mathbf{V}$ provide a similar representation but from the perspective of how similar actors are as receivers. The values in $\mathbf{D}$, a diagonal matrix, represent levels of homophily in the network.[8]

Note that this model easily generalizes to the case, common in IR, where interactions are undirected (for example the presence of conflict or a bilateral investment treaty). In the case of the SRRM, ρ is constrained to be one and instead of separate sender and receiver random effects a single actor random effect is utilized. For the LFM, an eigen-decomposition scheme is used to capture higher order dependence patterns. In the application section, we show the applicability of the AME approach to both directed and undirected dyadic data.

By integrating the SRRM and LFM into a Bayesian probit framework, we can account for the underlying structure in dyadic data that, if left un-estimated, would complicate any inferences we might wish to draw for the exogenous parameters. Parameter estimation in the AME takes place within the context of a Gibbs sampler in which we iteratively sample from the posterior distribution of the full conditionals for each parameter. Specifically, given initial values of $\{\boldsymbol{\beta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho,$ and $\sigma_\epsilon^2\}$, the algorithm proceeds as follows until convergence:

- sample $\boldsymbol{\theta} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)
- sample $\boldsymbol{\beta} \mid \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)
- sample $\mathbf{a}, \mathbf{b} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)

[7]The dimensions of $\mathbf{U}$ and $\mathbf{V}$ are $n \times K$ and $\mathbf{D}$ is a $K \times K$ diagonal matrix.

[8]Unlike traditional SVD, in the latent factor model, the singular values are not restricted to be positive, thus allowing us to account for both positive and negative homophily.

- sample $\Sigma_{ab} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \rho,$ and σ_ϵ^2 (Inverse-Wishart)
- update ρ using a Metropolis-Hastings step with proposal $p^*|p \sim$ truncated normal$_{[-1,1]}(\rho, \sigma_\epsilon^2)$
- sample $\sigma_\epsilon^2 \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab},$ and ρ (Inverse-Gamma)
- For each $k \in K$:
  - Sample $\mathbf{U}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}_{[,-k]}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)
  - Sample $\mathbf{V}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}_{[,-k]}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)
  - Sample $\mathbf{D}_{[k,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho,$ and σ_ϵ^2 (Normal)[9]

The presence of these dependencies in relational data points to the fact that there is a complex structure underlying the dyadic events that we observe, and that accounting for this structure is necessary if we are to adequately represent the data generating process. Of course, if one can specify each of the nodal, dyadic, and triadic set of attributes that influence interactions then the conditional independence assumption underlying standard approaches will be satisfied. However, it is rarely the case that this is possible even for TSCS data and thus modeling decisions must account for underlying structure. Failing to account for the underlying structure in either TSCS or dyadic data leads to a number of well-known challenges: a) biased estimates of the effect of independent variables, b) uncalibrated confidence intervals, and c) poor predictive performance. Additionally, by ignoring these potential interdependencies, we often ignore substantively interesting features of the problem. The study of international relations is founded on the relations among actors. Why ignore the interdependencies that led to the study of IR in the first place?

4. Simulation study

We utilize a simulation study to highlight the utility of AME as an inferential tool for dyadic analysis.[10] Most scholars working with dyadic data are primarily concerned with understanding the effect of a particular independent variable on a dyadic dependent variable. The goal of the simulation is to assess how well AME can provide unbiased and well-calibrated estimates of coefficient

[9]Subsequent to estimation, **D** matrix is absorbed into the calculation for **V** as we iterate through K.
[10]Alternative network based approaches for dyadic data are exponential random graph models (ERGMs) and the related stochastic actor oriented model (SAOM). While both these models have led to numerous contributions to a variety of literatures, the applicability of these approaches may be limited to certain types of networks and individual level characteristics. Specifically, Block et al. (2017) note that these types of models may not be appropriate in situations where network and behavioral data depend on unobserved latent variables, which is explicitly the focus of our analysis here.

parameters in the presence of unobserved dependencies. Specifically, we are concerned with conducting inference on regression parameters of a linear model for a network in the case where there is an unaccounted for higher order dependence pattern. For instance, assume that the true data-generating process for a particular Y is given by:

$$y_{i,j} \sim \mu + \beta x_{i,j} + \gamma w_{i,j} + \epsilon_{i,j} \tag{3}$$

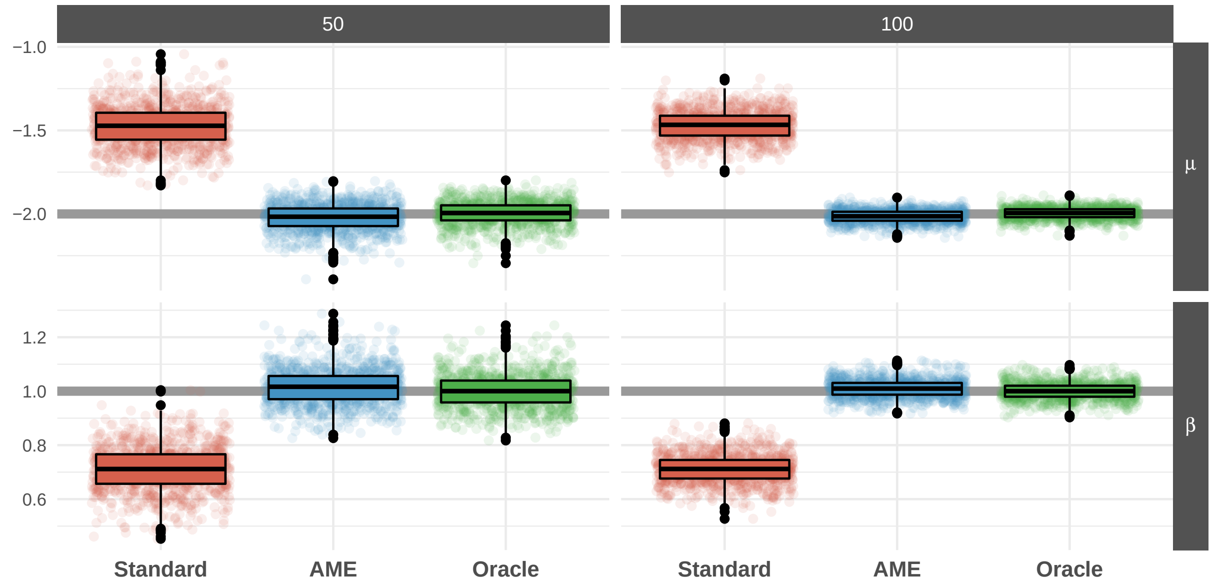
where $Y = \{y_{i,j}\} \in \mathbb{R}^{n \times n}$ is an observed sociomatrix, $X = \{x_{i,j}\} \in \mathbb{R}^{n \times n}$ is a matrix of observed dyad-specific characteristics, and $W = \{w_{i,j}\} \in \mathbb{R}^{n \times n}$ is a matrix of unobserved dyad-specific characteristics. Y can be thought of as a dyadic dependent variable, X and W are both dyadic covariates that are a part of the data-generating process for Y, but W is not observed. We compare inference for μ and β—the latter parameter would be of primarily theoretical concern for applied scholars—using three models:

- the "standard" international relations approach estimated through a typical generalized linear model;
- the AME approach outlined in the previous section with a unidimensional latent factor space ($K = 1$);[11]
- and an "oracle" regression model that assumes we have measured all sources of dependencies and thus includes both $x_{i,j}$ and $w_{i,j}$.

The first model corresponds to the "standard" approach in which little is explicitly done to account for dependencies in dyadic data. In the second model, we use the AME framework described in the previous section. For both the first and second models, we are simply estimating a linear model of X on Y, and assessing the extent to which inference on the regression parameters are complicated in the presence of unobserved dependencies, W. In the last model, we provide an illustration of the ideal case in which we have observed and measured W and include it in our specification for Y. The oracle case provides an important benchmark for the standard and AME approaches.

[11]Results with higher values of K are similar.

Figure 3. Regression parameter estimates for the standard, AME, and oracle models from 1,000 simulations. Summary statistics are presented through a traditional box plot, and the estimates from each simulation are visualized as well as points.



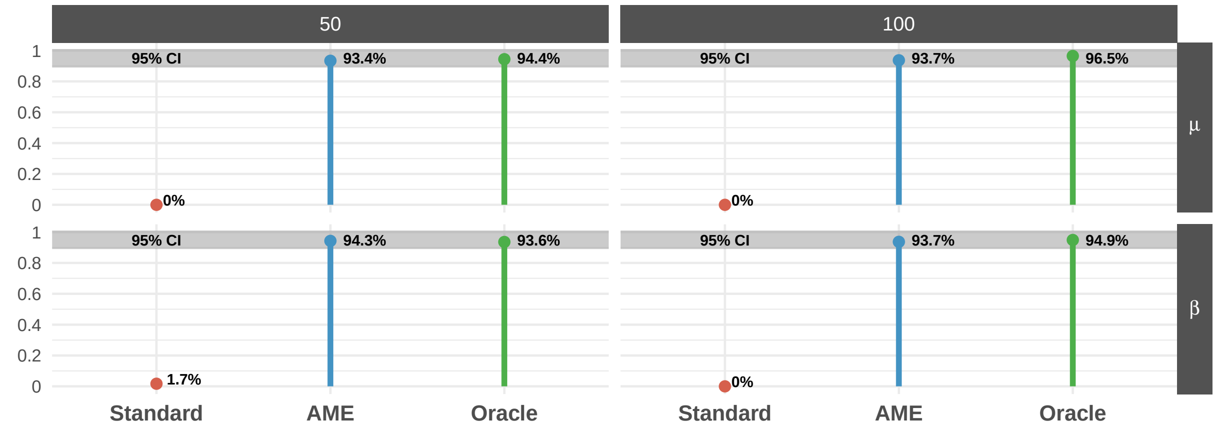
For the simulation we set the true value of μ (the intercept term) to -2 and β (the effect of X on Y) to 1.[12] We conduct two sets of simulations, one in which the number of actors in the network is set to 50 and the other at 100. In total, we ran 1,000 simulations where we begin by simulating Y from the specification given in Equation 3 and then for each simulated Y we estimated a standard, AME, and oracle model.

We compare the performance of the models first in terms of how well they estimate the true values of μ and β in Figure 3 by depicting the average μ and β estimates from the simulations for the three models. The panels in the left show the results for when the number of actors is set to 50 and those on the right for 100; and the top pair of panels represents the estimates for μ while the bottom pair do the same for β. In each case, we find that the estimates for μ and β produced by the standard approach are notably off from their true values. On the other hand, the AME model performs just as well as the oracle case in estimating the true parameter values.

Next, we estimate the 95% confidence interval for the three models in each of the simulations and estimate the proportion of times that the true value fell within those intervals. The results are

[12]The value of γ is also set to 1, which corresponds to a an example where the W character is associated with homophily.

Figure 4. Proportion of times the true value fell within the estimated 95% confidence interval for the standard, AME, and oracle models from 1,000 simulations.

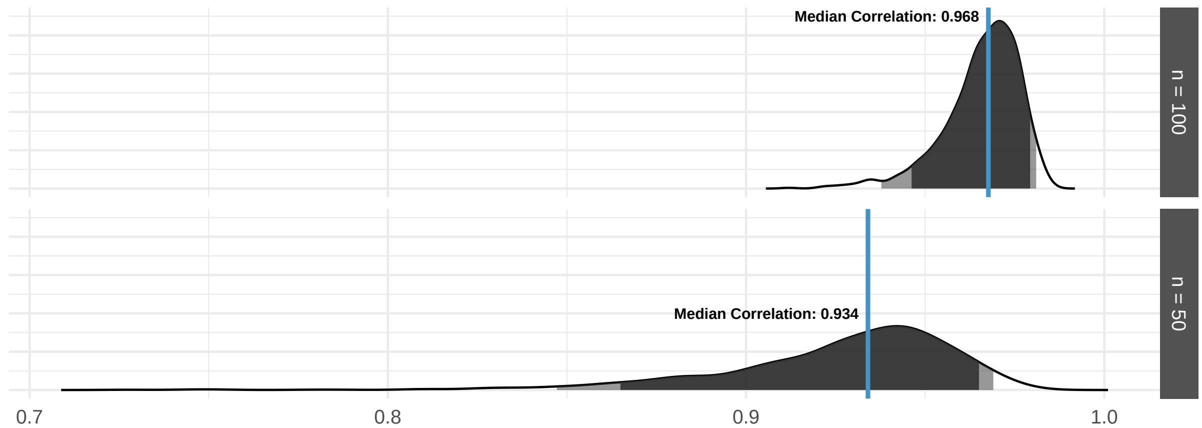


summarized in Figure 4, and again we see that the AME approach performs as well as the oracle, while the standard approach performs poorly by comparison. The implication of the results presented in Figures 3 and 4 is that standard approaches can often fail at estimating parameter values and conducting inferential tasks in the presence of unobserved dependencies. The AME approach by comparison can be used as a tool for scholars working with dyadic data to still estimate the true effects of their main variables of interest, while accounting for dependencies that do often emerge in dyadic data.

Moreover, the AME approach allows scholars to better understand what parameters their model may be missing. In the case of the simulation here, W is set as an unobserved dyadic covariate that had a homophilous effect on Y. Homophilous because W within this framework is simply an example of a dyadic attribute involving i and j that positively affects the degree to which they will interact with one another, i.e., y_{ij}. This type of unobserved dependency will be captured through the multiplicative effects portion of the model, $\mathbf{U}^\top\mathbf{D}\mathbf{V}$. To estimate how well the model performs in doing this we recover the multiplicative effects term for each simulation and calculate the correlation between it and the unobserved dependency, W.[13] We visualize the distribution of the correlations from each of the 1,000 simulations in Figure 5 for the case where the number of actors is set to 100 (top pair of panels) and 50. Additionally, we calculate the median across the

[13]Specifically, since both the multiplicative effects term and W are continuous dyadic variables, we calculate the Pearson correlation coefficient.

Figure 5. Distribution of correlation between missing variable and multiplicative random effect in AME across the 1,000 simulations. Vertical line through the distribution represents the median value across the simulations.



correlations and display the result using a vertical line. For both $n = 50$ and $n = 100$, we find that the multiplicative effects perform very well in capturing the unobserved dependency, which indicates that the structure provided by this framework is not simply capturing noise but can be used as a tool to estimate unobserved structure.

What this simulation has shown is that beyond obtaining less biased and better-calibrated parameter estimates, a key benefit of the AME framework is that we can also estimate unobserved dependencies through the random effects structure of the model. Scholars can use this framework in an iterative fashion where once they have estimated a model, they can empirically study the structure of the random effects portion of the model to assess whether there are unobserved covariates that they want to account for in their models. At the very least, what this simulation section shows is that the AME model can help in estimating the true effect and conducting inference on the independent variables that scholars have operationalized to test their theoretical propositions.

5. Applications with AME

5.1. **Design.** We apply AME to five recent IR studies: Reiter and Stam (2003); McDonald (2004); Rose (2004); Weeks (2012); Gibler (2017). Each of these studies are representative of broader trends in the field in that they use relational data of state interactions and propose both dyadic, monadic,

Table 1. Features of the Studies Re-estimated.

	Model	Date Range	N. Actors	N. Dyads	Dyads Type	Clustering $\sigma_{\hat{\beta}}$
Reiter & Stam (2003)	Logit	1945–1995	193	$753,456$	Directed	Robust
McDonald (2004)	Logit	1959–2002	198	$92,354$	Undirected	Robust
Rose (2004)	OLS	1948–1999	177	$234,597$	Directed	Robust
Weeks (2012)	Logit	1946–1999	197	$901,540$	Directed	Robust
Gibler (2017)	Logit	1816–2008	193	$650,557$	Undirected	None

and structural explanations for behavior of actors in the system. We choose to demonstrate the capabilities of AME with reference to existing studies in order to highlight several features of the AME approach. First, the results of AME estimation are interpretable alongside results using standard approaches, but, as shown in the simulation section, have the additional benefit of being able to take into account dependencies that may complicate inference. Second, through using this approach, we can also quantify the degree to which first, second, and third order dependencies are present in events of interest. Third, we show that by using the AME framework scholars can better model the data generating process behind their events of interest.

We obtained the data for each of these studies from their replication archives and replicated the main results of each article.[14] The five studies used below were selected based on how recent they were and whether they had more then 100 citations.[15] Each of these pieces, published in prominent journals well-known in their respective literatures, posited a theory in which interdependencies are consequential. Reflecting the dominant approach in the literature, each of the authors tested their hypothesis by employing some form of a general linearized model.[16]

Each of the five studies has a crucial finding that we hone in on to further draw into focus the analytical power of the AME estimation procedure. In Table 2, we present the overall results; the

[14]Without exception this was straightforward to accomplish, thanks to the authors' transparency and an increasing norm in the social sciences of open data sharing.

[15]Note that we chose papers with at least 100 citations as an indicator of influence within the discipline. Of course, by the criteria of influence many other papers could have been considered for the replication. However, as described in the following sections, the AME model has specific data requirements, which limited the scope of potential papers.

[16]It is important to note that the AME framework is applicable only where a full set of dyads for an outcome of interest is observable—for example it is unsuitable for studying onsets, where dyad-years representing conflict continuations are removed from the sample. The AME is also not applicable to analyses of case-level data, for example, studies that examine the decision to go to war by particular states.

term *Unconfirmed* indicates only that the sign and/or significance of the putatively crucial finding in the original study is not found to hold in the AME estimation.[17]

Table 2. Here we provide a brief summary of the key variable in each of the five replications and a note about whether or not the highlighted finding remains when using our network-based approach.

Study	Central Finding	Confirmed accounting for dependencies?
Reiter & Stam (2003)	Personalist Regimes Attack Democracies, Not Vice Versa	Partially Confirmed
McDonald (2004)	Lower Trade Barriers and Higher Trade Lead to Peace	Unconfirmed
Rose (2004)	WTO Membership Does not Affect Trade	Partially Confirmed
Weeks (2012)	Bosses, Juntas, and Strongmen are more Aggressive, Machines are Not	Unconfirmed
Gibler (2017)	Power Parity at Time of Entry to International System Inceases Conflict	Unconfirmed

An important takeaway here is that many scholars are forced to make knowledge claims based on the statistical significance of a small set of covariates, or the differences between these covariates. These differences may change dramatically when interdependencies are taken into account directly. This outcome follows from AME's ability to better account for the dependencies discussed in the previous section, whereas GLM approaches explicitly assume observational independence conditional on the specified covariates. As this is a widely-known limitation of GLM approaches, scholars often attempt to account for clustering of observations by including additional variables and adjusting the standard errors of the resulting estimates. At best, this method introduces noise and imprecision into results, and at worst can produce misleading outcomes.

Beyond just comparing parameter estimates, we examine how well each approach can represent the data generating process using an out-of-sample cross validation strategy. Specifically, for each study, we randomly divide the data into $k = 30$ sets, letting $s_{ij,t}$ be the set to which pair ij, t is assigned.

Then for each $s \in \{1, \ldots, k\}$, we:

[17]Full tabular results for each of the original and reestimated models are presented in the Appendix.

(1) estimate model parameters with $\{y_{ij,t} : s_{ij,t} \neq s\}$, the data not in set s,

(2) and predict $\{\hat{y}_{ij,t} : s_{ij,t} = s\}$ from these estimated parameters.

The result of this procedure is a set of sociomatrices $\hat{Y}$, in which each entry $\hat{y}_{ij,t}$ is a predicted value obtained from using a subset of the data that does not include $y_{ij,t}$. We summarize the performance of the various models in Table 3 below. For the binary models we provide the area under the Receiver Operator Characteristic (ROC) and Precision Recall (PR) curves. Only one of the studies here had a continuous dependent variable and for this we provide the root mean squared error (RMSE) and root median squared error (RMDSE).[18] For each of the replications, we find that the AME approach substantially outperforms the original models in terms of out-of-sample predictive performance. This is important as it indicates that switching to the AME framework—even when using the exact same specification as the original studies—enables scholars to better represent the data generating process of their events of interest. The fact that this analysis is done in an out-of-sample context ensures that the AME framework is not simply overfitting with more parameters, rather the dependence parameters we include are capturing underlying structure previously missed by the exogenous covariates in the models.

[18] More details on the performance of each of these models can be found in the Appendix.

		GLM	AME
Reiter & Stam (2003)	Area Under ROC Curve, AUC-ROC	0.92	0.96
	Area Under PR Curve, AUC-PR	0.08	0.15
McDonald (2004)	AUC-ROC	0.92	0.99
	AUC-PR	0.13	0.28
Rose (2004)	RMSE	3.23	1.99
	RMDSE	2.01	1.06
Weeks (2012)	AUC-ROC	0.64	0.97
	AUC-PR	0.00	0.15
Gibler (2017)	AUC-ROC	0.52	0.91
	AUC-PR	0.00	0.08

Table 3. Here we provide a summary of the out-of-sample performance based on our cross-validation strategy for each of the five replications when using the standard dyadic approach and our network-based approach. Four of the five studies involved a binary dependent variable; for those measures, area under the curve (AUC) statistics are reported. The Rose study involved a Gaussian dependent variable and for that we use the root mean squared error (RMSE) and root median squared error (RMDSE).

Next we discuss each of the replications in more detail and highlight the substantive insights that can be drawn from the AME framework.

5.2. **Re-estimation of Reiter & Stam (2003).** Reiter and Stam (2003) examine the relationship between democracy, dictatorship and the initiation of militarized disputes. Their work contests prior scholarship that had claimed interstate dyads containing democracies and personalist dictatorships were particularly prone to conflict because of aggression on the part of the democratic state. Using directed dyads, they find evidence against this hypothesis: dictators are in fact more likely to challenge democracies, but not the other way around. In addition, military regimes and single-party regimes are more prone to initiate disputes with democracies, but the opposite is not true. Independent variables focus on various encodings of regime types, contiguity, alliance, and capability measures. As is prevalent in this literature, Reiter & Stam employ a logistic regression that includes an indicator of the time since the last dispute as well as three cubic splines. The database for this study is constructed using `EUGene` (Bennett and Stam, 2000) and comprises approximately three-quarters of a million stacked dyads. Based on their statistical analysis, they conclude that institutional constraints affect the propensity of democratic and non-democratic leaders to engage in military conflict.

In the original model, the variable “Pers/Democ Directed Dyad" (which represents a Personalist $\rightarrow$ Democractic directed dyad) has a positive effect while the variable “Democ/Personalist Directed Dyad” is too imprecisely measured to indicate a direction. In our re-estimation using the AME framework, we also find that Pers-Democ directed dyad has a positive effect while Democ-Pers directed dyad is still too imprecisely measured to indicate a direction. Using this model, however, we can no longer conclusively say that the Pers/Democratic coefficient is larger than the Democ/Personalist one.[19] Our re-estimation using the AME approach therefore cast some doubt on Reiter & Stam’s key claim that MIDs initiated by personalist dictatorships against democracies are more likely than MIDS initiated by democracies, though many of the major effects uncovered by the authors remains robust when accounting for dyadic interdependencies.

5.3. **Re-estimation of McDonald (2004).** McDonald (2004) studies whether trade promotes peace between nations. McDonald (2004, p. 547) introduced the argument that free trade between states “makes conflict less likely because of its efficiency over conquest in acquiring resources. . . ”. Accordingly, he provided evidence challenging the generalized linkage between peace and trade and refined the measurement of the standard “trade” variable, arguing that *free* trade, rather than trade alone, reduces the likelihood of conflict between states. His key hypothesis is that greater levels of protection increase the probability of interstate conflict, an argument that builds on the work of classic liberalism and connects free trade to the power of domestic audiences. McDonald (2004) measured free trade in two ways. The first, the “protection” variable measures the proportion of customs revenue divided by total imports in the state that possesses the greater such ratio in each dyad. reflects the notion that larger protected sectors generate greater societal pressures resulting in pockets of support for war. This measure thus captures the score of the state in the dyad that possesses higher barriers to trade. McDonald (2004, p. 560) also includes a measure of economic integration calculated as “the lower proportion of total dyadic trade (imports plus exports) divided by state i’s GDP or total dyadic trade divided by state j’s GDP”. The (binary) dependent variable is the onset of a new militarized interstate dispute within a given dyad, and

[19] Comparing

McDonald (2004) includes splines to correct for temporal dependence with robust standard errors clustered on each dyad.

Our re-estimation with AME reveals that trade relations are highly interdependent and exhibit important patterns of transitivity. Or, in other words, if countries i and j are highly dependent and countries j and k are also highly dependent, then we are likely to observe high dependency between countries i and k. This indicates that conflict is less likely among members of a trade community. Once we control for these dependencies, we can more clearly interpret the positive link between trade and conflict. The most striking thing is that AME finds a positive conditional association between trade dependence and conflict ($\hat{\beta} = 18.4, \quad \sigma_{\hat{\beta}} = 28.6$), while the comparable numbers for the logistic regression found in the original articles are negative ($\hat{\beta} = -22.2, \quad \sigma_{\hat{\beta}} = 15.2$). At the same time, the AME has ROC and PR curves (shown in the Appendix) that dominate the results found in McDonald (2004).

5.4. **Re-estimation of Rose (2004).** In 2004, Andrew Rose published a study in the *American Economic Review* that proved to be quite controversial in terms of macroeconomic trade theory and in terms of trade policy in a variety of nations. It also provoked a number of responses in the international political economy literature (Tomz et al., 2007). Rose's basic argument is that despite longstanding arguments, made by trade theorists and the World Trade Organization, that WTO membership fosters greater cooperation and thereby more trade among its members, the empirics do not bear out such claims. He uses a standard gravity model with dyadic data on bilateral merchandise trade (not services) for 175 countries over a period of five decades. Estimating this model using OLS within many differing contexts, his conclusion is that: "An extensive search reveals little evidence that countries joining or belonging to the GATT/WTO have different trade patters from outsiders . . . (2004, page 98, abstract)." The data for this study have been widely used in replications by many searching for the missing effects of the WTO—as well as preferential trade agreements, bilateral investment theories, and other aspects of modern trade theory.

When we compare the results of Rose's original OLS model to the AME model accounting for network dependencies, the results are similar. The main result of the model—the null effect of membership in the WTO, as represented by the "One-In" and "Both-In" variables—does not prevail in the AME model. However, in a sense the AME result provides even stronger evidence against

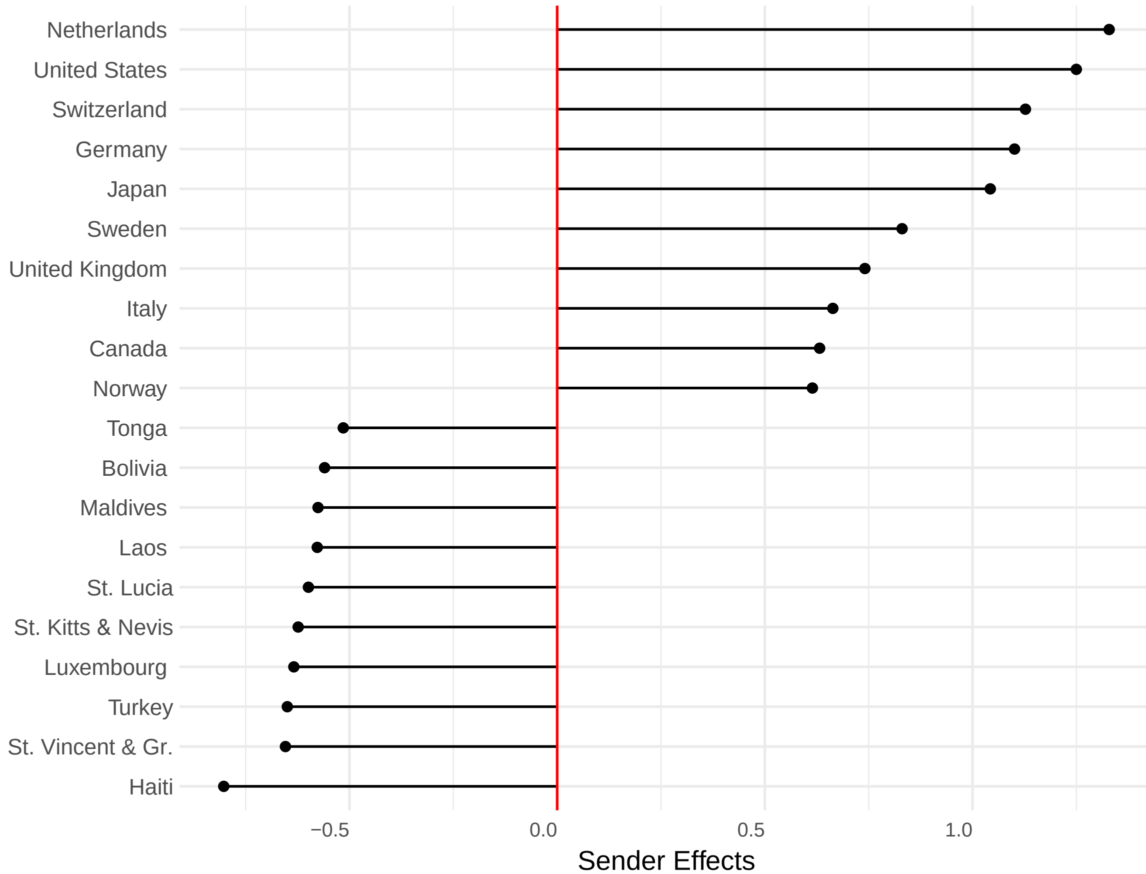


Figure 6. Nodal Random Effects for AME estimation of Rose (2004), for the countries with the highest and lowest Random Effects.

a positive effect of WTO membership: in the original model, this negative linkage is not significant, but in the AME estimation this negative linkage is more precisely estimated. The fact that this variable attains conventional statistical significance under an AME estimation shows an additional value of this approach to applied researchers – while accounting for interdependencies and removing that source of bias will remove false positive findings, it can also avoid false negative findings where the bias prevented a researcher from uncovering a consistent relationship.[20] In both cases we find a rejection of the conventional wisdom, that WTO membership increases trade. Of course, the controversial result of the paper, strengthened in the AME analysis might be an artifact of measurement error. (Tomz et al., 2007). While in the original model there was a clear positive relationship between Real GDP and Trade, most of this effect vanishes in the AME model.

[20]This benefit comes up less often in these replications because most published studies are positive, rather than null findings.

The random effects shown in Figure 6 reveal the cause of much of this divergence. Here, the states with the most positive random effects are also states with high GDP, though not necessarily high GDP/capita.[21] Most of the other results of the model are constant across each model, though some geographic features, such as islands and landlocked states, have a more clear effect on trade once we account for these network dependencies. When we account for network interdependencies, we observe a markedly lower Root Mean Squared Error out-of-sample — 3.23 for the OLS model and 1.77 for the AME model. While the AME model replicates the main (null) result, there are substantial and substantive improvements gained from moving to an AME estimation.

5.5. **Re-estimation of Weeks (2012).** Weeks (2012) examines the influence of domestic institutions on the initiation of military conflicts by autocratic leaders. She argues that in some circumstances autocrats are held accountable for their foreign policy decisions. She adds the nuance that autocratic audiences are not homogeneous. When the autocratic regime is nonmilitary, domestic audiences do not favor military actions, but in military autocracies this is not the case. Further she argues that in personalistic regimes without a military or civilian domestic audience, the leaders tend to be more likely to employ military force in their foreign policy. To study this question, she uses a dyadic design in which the dependent variable is "whether country A in a directed dyad initiated military conflict against country B during year t" (page 337). One major innovation in her study resides in the nuanced way in which she conceptualizes and codes regimes into four types: a) Machine, b) Junta, c) Boss, and d) Strongmen. She also includes a variety of putative control variables focusing on capabilities for both sides of the dyad, alliances, geography, trade dependence, regime instability, and the regime type of "side B." She uses a logistic regression, but follows Beck et al. (1998) and includes splines to capture temporal covariation in the dependent variable along with fixed, unit effects. The analysis is done for dyads, but is considered to be from the perspective of the actor that initiated the dispute. The basic finding is that a) juntas, boss type, and strongmen type regimes are more likely to initiate conflict than machine-type regimes (and maybe democracies) and that b) machine-type regimes are no more belligerent than democracies. These insights are mainly emphasized in the paper by the parameter estimates depicted in Tables 1 and 2 (pages

[21]Note: Qatar exhibits strongly negative random effects.

339–340) from the paper. She argued that ignoring important nuances between different types of autocracies hinders our understanding of the initiation of military conflict by autocracies.

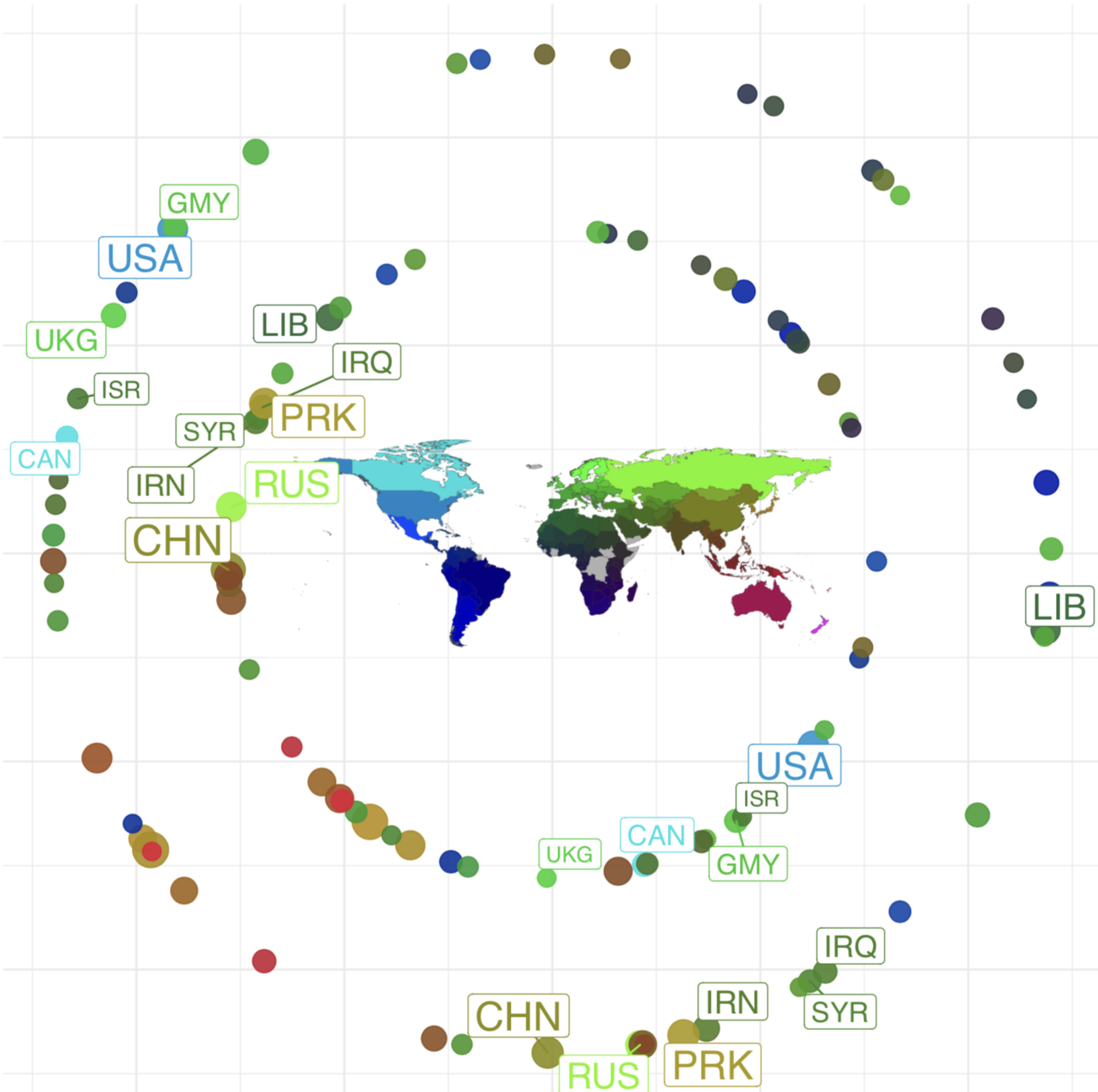


Figure 7. Visualization of multiplicative effects for Weeks (2012). Each circle designates a country and the color corresponds to the legend at the center of the visualization. Countries that cluster together in the outer ring are those that were found by the model to have similar sending patterns, meaning that they tend to send conflict to similar sets of countries. The inner ring clusters countries by the similarity of who they receive conflict from.

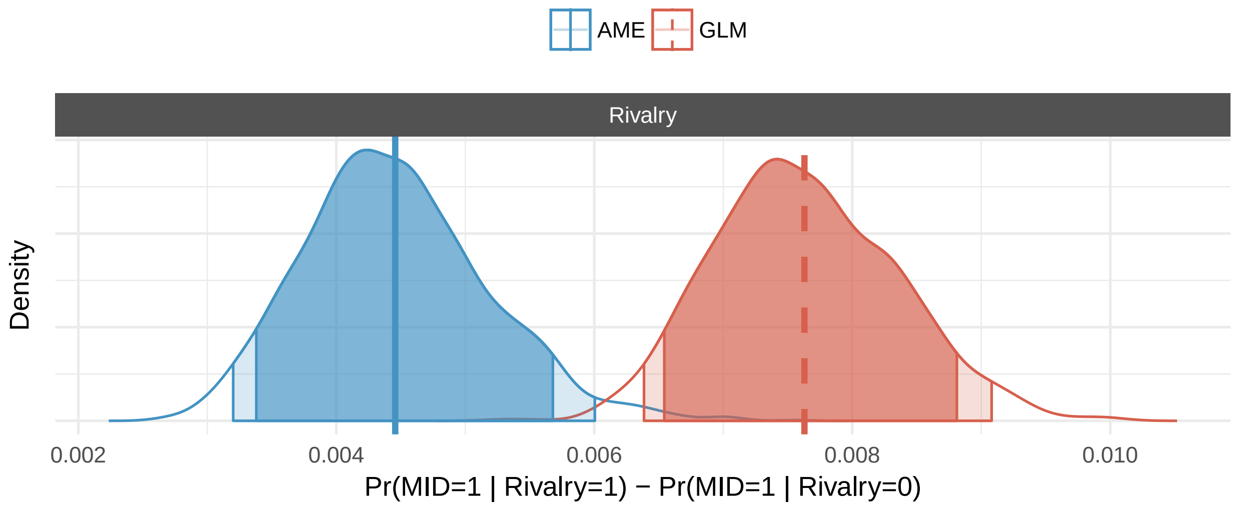
The re-estimation of Weeks (2012) has the sharpest divergence between the GLM results and those of the AME Model. In Weeks' initial models, she finds that machines are less prone to initiate conflict than the reference category, whereas Juntas, Bosses, and Strong-men are more conflict-prone. When we examine the AME results, we find that none of these values are distinguishable from zero. Similarly, we find less pronounced effects for military capabilities. One explanation for this divergence is the AME model's ability to account for third-order effects. Inspection of the multiplicative effects in Figure 7 reveals a number of clusters of states which exhibit structural equivalence—in the top left corner we see the US, the UK, Germany, Canada, and Israel. These states cluster together in the outer ring of this visualization because they tend to send conflicts to similar targets. Conversely, in the bottom right of the outer ring, we see a cluster of authoritarian countries: Iraq, Russia, Syria, North Korea, and China. In the inner ring, the proximity of countries is determined by the degree to which they receive conflict from the same countries. In general, the clusters found on both the inner and outer rings have similar governmental types (Iraq, Syria, Libya, and North Korea all fell under the "boss" category). They tend to initiative and be targets of conflict by similar actors but they are not more likely to initiate

Thus she is kinda right but wrong in that they are more likely to intiate conflict in general ... but they do have a similar pattern of relationshsips

In the GLM, which ignores these third-order dependencies, many of these results might have been attributed to regime type. Structural equivalence is present even when accounting for nodal characteristics like regime type. The AME model, on the other hand, shows that this can be specified in terms of the interdependencies captured by the multiplicative effects.

5.6. **Re-estimation of Gibler (2017).** A more recent example is Gibler (2017) which examines the onset of militarized disputes using capabilities, joint democracy, alliances, and power parity in a undirected dyadic study using logistic regression and dyad clustered standard errors. In addition, Gibler shows that the long-standing relationship between the relative parity of capabilities and initiation of international conflict is almost completely mediated by the initial conditions for the members of the dyad when they joined the international system as sovereign members. This finding calls into question many IR theories about the role of balance in terms of generating international conflict (Organski, 1958).

Figure 8. Marginal effects of a change in the Rivalry variable for both the AME and the Gibler estimation.



We re-estimated model 6 from Table 6 (2017, 34). The results are presented in the Appendix. The results obtained with AME stand in stark contrast to those found with a logistic regression (with dyad clustered, robust standard errors). Most importantly, the primary variable from the Gibler study, parity of the members of a dyad at the year in which they entered the international system, is shown to be unimportant in the AME results. Not only is the value of this parameter small, but it has a very large relative standard error—over a magnitude larger than the parameter itself ($z = 0.038$). In addition, the variable indicating whether both members of the dyad were coded as democracies (joint democracy) follows the same pattern: important and strong in the logistic results, but this disappears once interdependencies are modeled. As might be expected, the strong geographic clustering in the original study is about one-quarter as strong in the AME estimations. Similarly, rivalry coefficients are about one-third the size in the AME formulation, but a great deal more precisely measured ($z = 18.116$).

We utilized the original and the AME results from Gibler's model 6 in Table 6 to calculate the expected values for one scenario focusing on the variables measuring rivalry. We employed mean or modal values for all independent variables, except we changed the rivalry variable to indicate that there was a rivalry when the actual data suggest there is none. The expected values of this scenario are essentially a first difference plot comparing results with the model when estimated in two different ways: Gibler's GLM estimation and our AME approach. As this Figure 8 illustrates, the AME results differ notably. First, the expected value of the dependent variable—the probability of the onset of a militarized interstate dispute, is considerably lower when taking interdependencies into account with the AME model. These are rare events, so the probabilities are low, but the difference is a factor of almost 2. Thus, you get quite substantially different expected values from these two models.

5.7. **Lessons Learned from Re-estimating Five Prominent Studies.** First and foremost, many findings that emerge from models that do not take interdependencies into account lose their statistical significance when network effects are estimated via AME. Not only are coefficients biased in the GLM approaches to the analysis of dyadic data, but they are often imprecisely measured, with poorly calibrated standard errors. This means that significance testing (for better or worse) is compromised when network effects are ignored.

Second, even when the results from the AME estimation conform with those found in an OLS or logistic regression, new insights emerge from the additional information derived. In particular, there is actual information about the dependencies so that clusters can be identified, and the extent of reciprocity at the dyad level, as well as among senders and receivers. This kind of information is absent in standard approaches and adds to our ability to explain specific as well as general results.

Third, it is evident that the actual results—not the estimated coefficients and their covariances—which are generated by the models differ greatly in expectations. This implies that policy experimentations with the models, as well as scenario-based simulations and forecasting of GLM models are likely to often give misleading results compared to the AME approach.

Fourth, it is clear that the AME approach dominates the GLM approaches in terms of performance. Not only it is better at correctly identifying cases in which the dependent variable takes a value of 0 (via the ROC curves and associated statistics), but it also dominates at correctly identifying occurrences of the dependent variable in the data (seen via the PR curves and associated statistics). In the case of the study with a continuous dependent variable, the AME approach has average error statistics that are about one-half that found in the original model.

6. Conclusion

International relations is generally about the interactions and dependencies among a set of countries or other important actors such as international governmental organizations (IGOs). This is particularly true of scholarship in the tradition of the Correlates of War Project, but it is by no means limited to it.[22] Many scholars have debated the use and abuse of dyadic data.[23] It is clear from a survey of the literature and from work in this area published as recently as 2017 that many find dyadic data to be an important touchstone in the study of international relations (Erikson et al., 2014; Aronow et al., 2015).

At the same time, we know that research designs focusing on the statistical analysis of dyadic data quickly go astray if the dyadic data are assumed to be iid. Virtually all of the standard statistical models—ordinary least squares and logistic regressions, to name a few—fail if the data are not conditionally independent. This fact has been accepted when it comes to temporal dependencies, but adoption of methods to account for network dependencies have seen less progress. By definition dyadic data are not iid and thus the standard approaches can not be used cavalierly to analyze these data. Signorino (1999) showed why this is true of models of strategic interaction, but it is more broadly true of models that employ dyadic data. We show that the AME framework can be employed to account for the statistical issues that arise when studying dyadic data.

To explore this approach in the context of international relations we have presented two broad analyses. The first is a simulation where the characteristics of the network are known. This shows that when there are unobserved dependencies, the AME approach is less biased in terms of parameter estimation compared with standard approach employed in international relations to study dyadic data (i.e., GLM models). The second analysis is a replication of five prominent studies that have been published recently using a broad range of dyadic data to draw inferences about international relations. These five studies have been replicated with the original research designs, each of which used a statistical method that assumes the dyadic data are all independent from one another. We then re-analyzed each study using the AME model. In every case, we found that the AME approach provided a) increased precision of estimation, b) better out-of-sample fit, and c)

[22] See Singer (1972) for an early description of the project and also see the project's Web site for an history and more recent efforts http://www.correlatesofwar.org/.

[23] One recent on-line symposium can be found at http://bit.ly/2wB2hab.

evidence of 1st-, 2nd-, and 3rd-order dependencies that were overlooked in the original studies.[24] In several cases, the new approach overturns the basic findings of the original research. This leads us to speculate that many of the findings in the international relations literature may be fragile in the sense that they can only be obtained under stringent assumptions that are not valid.

It is no longer necessary to assume that the interesting, innate interdependencies in relational data about international relations can be ignored. Nor do they have to be approximated with *ad hoc*, incomplete solutions that purport to control for dependencies (such as modifying the post-estimation standard errors of the estimated coefficients (King and Roberts, 2014)). Instead, the interdependencies may be addressed directly with additive and multiplicative effects in the context of a generalized linear model that provides more reliable inferences, better out-of-sample predictive performance, and new substantive insights.

[24]The Appendix contains performance data on all of these replications, as well as sample code illustrating how to undertake AME analysis using `amen`.

APPENDIX

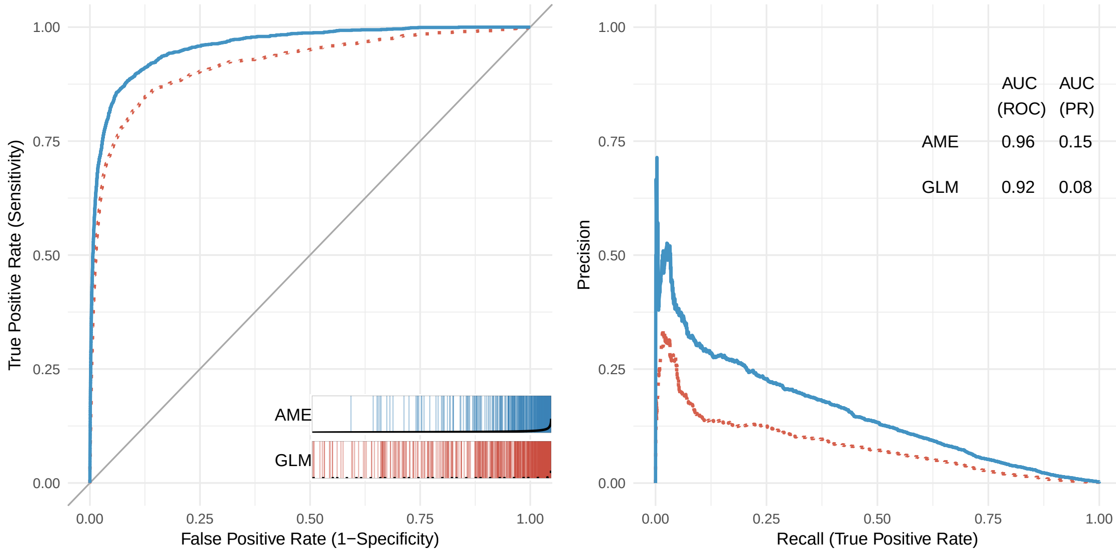
Additional Replication Information. For each of the replications involving a binary dependent variable we provide a table of coefficient estimates that includes the original GLM estimation with a logit link, a GLM estimation with a probit link, and the AME model. The GLM estimation with a probit link function is provided so as to ease comparison between the AME model, which is also based on the probit link. For Rose (2004) we just provide a table of coefficient estimates that includes the original OLS estimation and the AME model.

Additionally, for each replication we provide a more detailed visualization illustrating the results of our out-of-sample performance analysis.

Reiter & Stam (2003). Additional information for the Reiter & Stam (2003) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
Intercept	-4.784^{**}	-2.339^{**}	-3.144^{**}
	(0.097)	(0.034)	(0.06)
Pers/Democ Directed Dyad	1.026^{**}	0.378^{**}	0.255^{**}
	(0.14)	(0.051)	(0.068)
Democ/Pers Directed Dyad	0.083	0.033	0.112
	(0.191)	(0.066)	(0.079)
Personal	0.281	0.15	0.211^{*}
	(0.265)	(0.099)	(0.11)
Military	-0.323	-0.105	-0.025
	(0.574)	(0.204)	(0.249)
Single	-0.677^{**}	-0.261^{**}	-0.07
	(0.144)	(0.062)	(0.073)
Democracy	-1.073^{**}	-0.428^{**}	-0.254^{**}
	(0.194)	(0.07)	(0.063)
Contiguous	2.912^{**}	1.147^{**}	1.296^{**}
	(0.09)	(0.031)	(0.033)
Major Power	2.174^{**}	0.919^{**}	0.906^{**}
	(0.101)	(0.037)	(0.093)
Ally	0.078	-0.003	0.136^{**}
	(0.086)	(0.035)	(0.037)
Higher/Lower Power Ratio	-0.316^{**}	-0.122^{**}	-0.111^{**}
	(0.027)	(0.01)	(0.011)
Economically Advanced	-0.175	-0.054	0.053
	(0.131)	(0.051)	(0.05)
Years Since Last Dispute	-0.381^{**}	-0.149^{**}	-0.129^{**}
	(0.023)	(0.009)	(0.008)
Cubic Spline 1	-0.004^{**}	-0.001^{**}	-0.001^{**}
	(0.000)	(0.000)	(0.000)
Cubic Spline 2	0.002^{**}	0.001^{**}	0.001^{**}
	(0.000)	(0.000)	(0.000)
Cubic Spline 3	-0.001^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)

Table A.1. Parameter comparison for Reiter & Stam (2003). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



(a) AUC

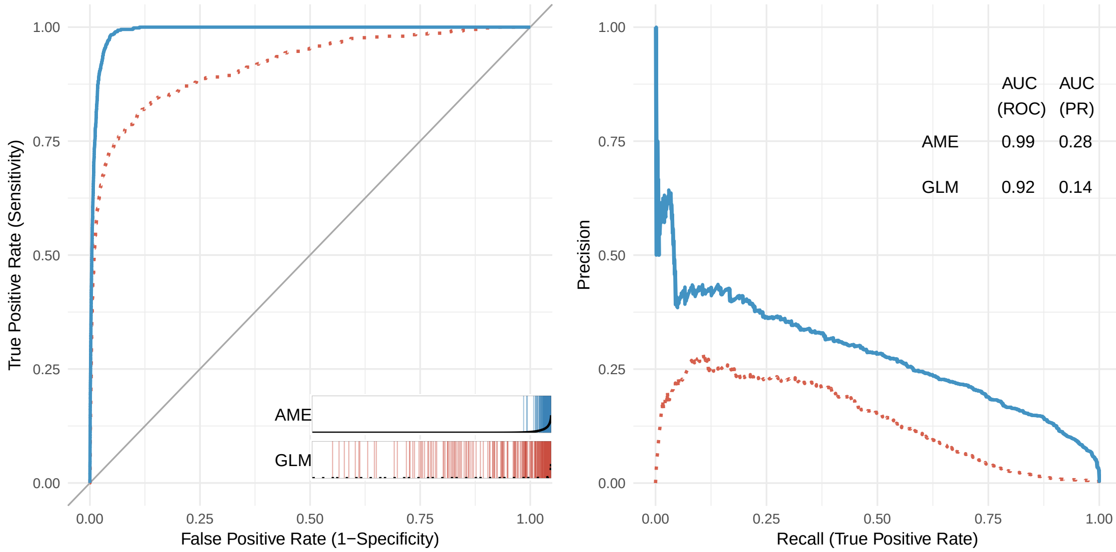
(b) Precision and Recall

Figure A1. Assessments of out-of-sample predictive performance for Reiter & Stam (2003) using ROC curves, PR curves, and separation plots.

McDonald (2004). Additional information for the McDonald (2004) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
(Intercept)	0.054	0.085	−1.171**
	(1.179)	(0.409)	(0.096)
Spline0	−0.438**	−0.222**	−0.145**
	(0.061)	(0.026)	(0.019)
Spline1	−0.003**	−0.002**	−0.001**
	(0.001)	(0.000)	(0.000)
Spline2	0.001	0.001**	0.000*
	(0.001)	(0.000)	(0.000)
Spline3	0.000	0.000	0.000**
	(0.000)	(0.000)	(0.000)
Shared Alliance	0.483**	0.155	0.342**
	(0.233)	(0.095)	(0.069)
Contiguous	2.011**	0.789**	0.988**
	(0.343)	(0.118)	(0.066)
Log Capabilities Ratio	−0.146**	−0.054**	0.029**
	(0.072)	(0.026)	(0.013)
Trade Dependence	-22.244	-7.051	−13.134**
	(15.184)	(5.536)	(4.938)
Preconflict GDP Change	−6.79**	−3.155**	−2.651**
	(2.033)	(0.788)	(0.574)
Lowest Dyadic Polity Score	−0.036**	−0.014**	−0.026**
	(0.015)	(0.006)	(0.002)
Capabilities	−0.995**	−0.349**	0.022
	(0.377)	(0.14)	(0.079)
Logged GDP	0.000**	0.000**	0.000**
	(0.000)	(0.000)	(0.000)
Logged Cap. Distance	−0.425**	−0.224**	−0.275**
	(0.14)	(0.047)	(0.012)
Major Power In Dyad	0.769**	0.312**	0.212**
	(0.322)	(0.122)	(0.098)
Highest Barrier To Trade	0.024**	0.011**	0.004**
	(0.008)	(0.003)	(0.001)

Table A.2. Parameter comparison for McDonald (2004). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



(a) AUC

(b) Precision and Recall

Figure A2. Assessments of out-of-sample predictive performance for McDonald (2004) using ROC curves, PR curves, and separation plots.

Rose (2004). Additional information for the Rose (2004) re-estimation.

Variable	LM	AME
Intercept	-24.96^{**}	-22.532^{**}
	(0.409)	(0.103)
Both in GATT/WTO	-0.042	-0.56^{**}
	(0.053)	(0.013)
One in GATT/WTO	-0.058	-0.317^{**}
	(0.049)	(0.012)
GSP	0.859^{**}	0.399^{**}
	(0.032)	(0.009)
Log Distance	-1.119^{**}	-1.097^{**}
	(0.022)	(0.005)
Log Product Real GDP	0.916^{**}	0.798^{**}
	(0.01)	(0.002)
Log Product Real GDPpc	0.321^{**}	0.244^{**}
	(0.014)	(0.004)
Regional FTA	1.199^{**}	0.826^{**}
	(0.106)	(0.027)
Currency Union	1.118^{**}	1.144^{**}
	(0.122)	(0.029)
Common language	0.313^{**}	0.345^{**}
	(0.04)	(0.009)
Land Border	0.526^{**}	0.483^{**}
	(0.111)	(0.02)
Number Landlocked	-0.271^{**}	-0.42^{**}
	(0.031)	(0.009)
Number Islands	0.042	0.058^{**}
	(0.036)	(0.009)
Log Product Land Area	-0.097^{**}	-0.024^{**}
	(0.008)	(0.002)
Common Colonizer	0.585^{**}	0.418^{**}
	(0.067)	(0.013)
Currently Colonized	1.075^{**}	1.762^{**}
	(0.235)	(0.081)
Ever Colony	1.164^{**}	1.335^{**}
	(0.117)	(0.024)
Common Country	-0.016	-0.672^{**}
	(1.097)	(0.19)

Table A.3. Parameter comparison for Rose (2004). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.

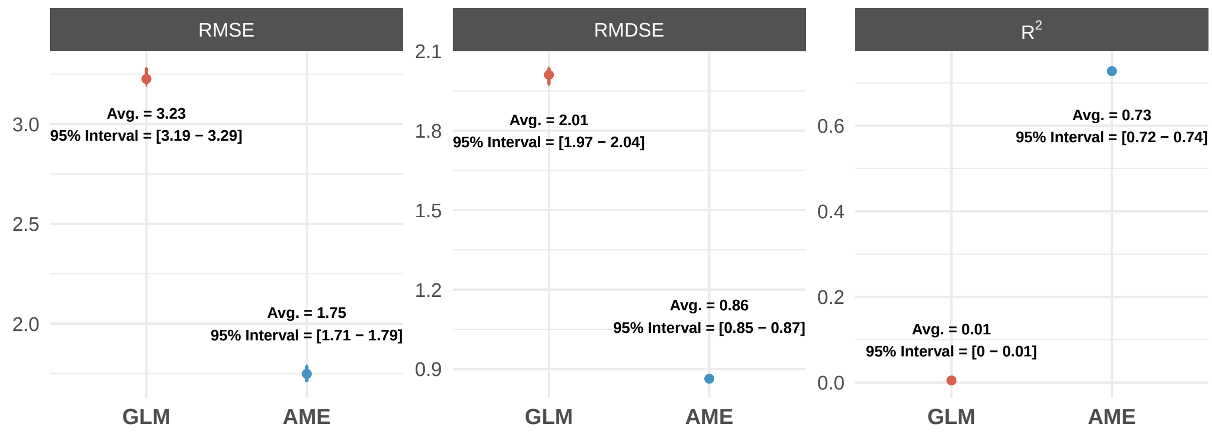
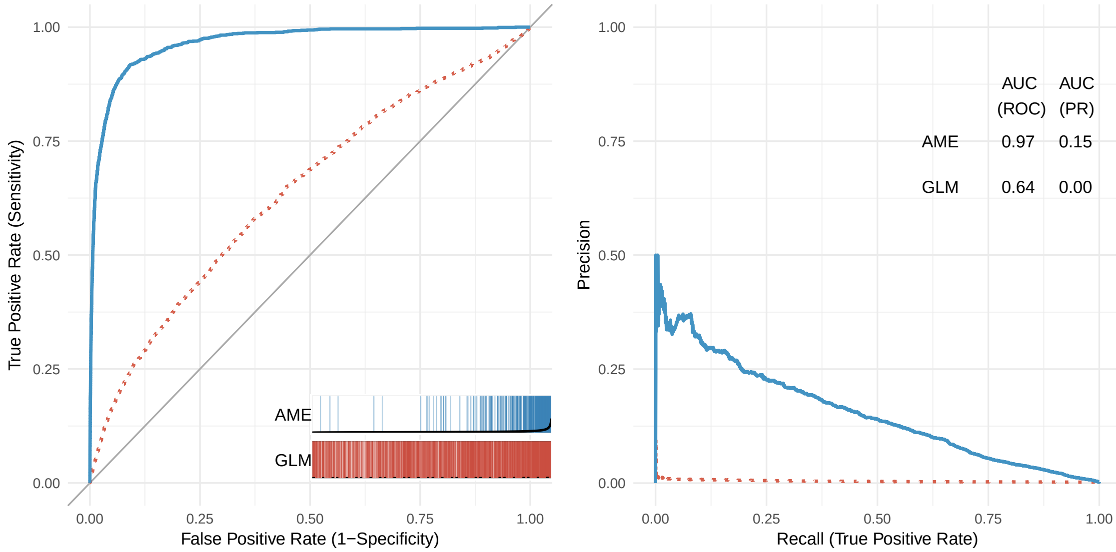


Figure A3. Assessments of out-of-sample predictive performance for Rose (2004) using root mean squared error (RMSE), root median squared error (RMDSE), and R^2.

Weeks (2012). Additional information for the Weeks (2012) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
(Intercept)	-3.784^{**}	-1.797^{**}	-2.409^{**}
	(0.423)	(0.159)	(0.132)
Machine	-0.459^{**}	-0.162^{**}	-0.006
	(0.174)	(0.062)	(0.04)
Junta	0.515^{**}	0.194^{**}	0.034
	(0.169)	(0.062)	(0.046)
Boss	0.649^{**}	0.281^{**}	-0.044
	(0.153)	(0.05)	(0.044)
Strongman	0.832^{**}	0.295^{**}	0.032
	(0.132)	(0.048)	(0.044)
Other Type	0.147	0.051	-0.01
	(0.132)	(0.046)	(0.034)
New/Unstable Regime	-0.312^{**}	-0.123^{**}	-0.043
	(0.092)	(0.033)	(0.031)
Democracy Target	0.185	0.052	0.024
	(0.115)	(0.04)	(0.026)
Military Capabilities Initiator	5.234^{**}	2.136^{**}	0.071
	(1.69)	(0.554)	(0.412)
Military Capabilities Target	6.34^{**}	2.865^{**}	-0.969^{**}
	(1.675)	(0.573)	(0.48)
Low Trade Dependence	-24.794^{*}	-8.197	-4.733
	(12.866)	(5.582)	(3.017)
Both Major Powers	1.136^{**}	0.687^{**}	1.122^{**}
	(0.547)	(0.183)	(0.241)
Minor/Major	0.772^{**}	0.292^{**}	0.496^{**}
	(0.239)	(0.086)	(0.118)
Major/Minor	0.711^{**}	0.332^{**}	0.778^{**}
	(0.225)	(0.075)	(0.16)
Contiguous	2.172^{**}	0.738^{**}	0.705^{**}
	(0.32)	(0.125)	(0.06)
Log Dist. Between Capitals	-0.209^{**}	-0.095^{**}	-0.129^{**}
	(0.038)	(0.015)	(0.01)
Alliance Similarity Dyad	-0.999^{**}	-0.386^{**}	-0.073
	(0.144)	(0.05)	(0.065)
Alliance Similarity With System Leader Initiator	0.11	0.011	0.068
	(0.24)	(0.082)	(0.057)
Alliance Similarity Leader Target	0.203	0.032	0.08
	(0.244)	(0.081)	(0.056)
Time Since Last Conflict	-0.229^{**}	-0.089^{**}	-0.067^{**}
	(0.018)	(0.007)	(0.007)
Spline1	-0.001^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Spline2	0.000^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Spline3	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)

Table A.4. Parameter comparison for Weeks (2012). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



(a) AUC

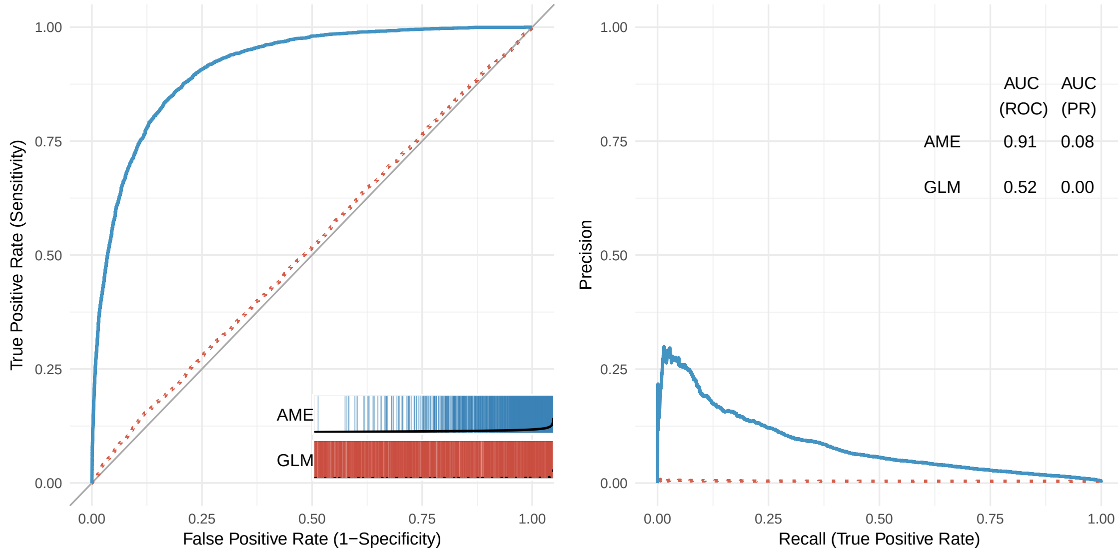
(b) Precision and Recall

Figure A4. Assessments of out-of-sample predictive performance for Weeks (2012) using ROC curves and PR curves

Gibler (2017). Additional information for the Gibler (2017) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
(Intercept)	-5.826^{**}	-2.793^{**}	-2.758^{**}
	(0.366)	(0.366)	(0.045)
Allied	0.133	0.067	0.078^{**}
	(0.102)	(0.102)	(0.021)
Joint Democracy	-0.527^{**}	-0.186^{*}	0.005
	(0.099)	(0.099)	(0.022)
Peace Years	-0.261^{**}	-0.099^{**}	-0.058^{**}
	(0.016)	(0.016)	(0.004)
Spline 1	-0.001^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Spline 2	0.000^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Spline 3	0.000	0.000	0.000^{**}
	(0.000)	(0.000)	(0.000)
Contiguity	2.427^{**}	0.95^{**}	0.66^{**}
	(0.196)	(0.196)	(0.023)
Parity	-0.77	-0.228	-0.067
	(0.551)	(0.551)	(0.057)
Parity at Entry Year	2.034^{**}	0.739	-0.05
	(0.617)	(0.617)	(0.065)
Rivalry	2.034^{**}	1.035^{**}	0.655^{**}
	(0.213)	(0.213)	(0.028)

Table A.5. Parameter comparison for Gibler (2017). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



(a) AUC

(b) Precision and Recall

Figure A5. Assessments of out-of-sample predictive performance for Gibler (2017) using ROC curves, PR curves, and separation plots.

AME Tutorial. Using the AMEN function requires formatting data into a particular structure. The primary distinction in data formatting is whether the outcome of interest represents a directed or undirected network.

If undirected, the AMEN function has three main inputs:

- Y: a T length **list** of $n \times n$ adjacency matrices, where T = number of years in the dataset and n = number of nodes in the network.
- Xdyad: a T length **list** of $n \times n \times p$ arrays, where p = number of dyadic covariates in dataset.
- Xrow: a T length **list** of $n \times p$ matrices, where p = number of monadic (nodal) covariates in dataset.

If directed, AMEN further requires:

- Xrow: a T length list of $n \times p$ matrices, where p = number of sender (nodal) covariates in dataset.
- Xcol: a T length list of $n \times p$ matrices, where p = number of receiver (nodal) covariates in dataset.

Beyond the data inputs, the AMEN function requires additional specification:

- model: how to model the outcome variable, e.g., 'logit'
- symmetric: whether the input network is symmetric
- intercept: whether to estimate an intercept
- nscan: number of iterations of the Markov chain
- burn: burn-in period
- odens: thinning interval
- R: dimension of the multiplicative effect (referred to as K in the paper)
- gof: whether to calculate goodness of fit statistics

There is often little theoretical reason to choose a particular value of R (above). One strategy is to estimate models at different values of R and compare goodness of fit statistics across models.

Given the computational intensity needed for parameter estimates to converge, parallelization strategies are recommended to speed up analysis. In addition, providing AMEN function with starting values, either dictated by theory, previous research, or previous runs can also help speed up convergence time.

The code below presents an example of an AME model running in parallel across 4 different levels of *R*. Note also that the model is using starting values from a previous run, defined in *startVals0*.

```
# running in parallel varying k
imps = 10000 ; brn = 25000 ; ods = 10 ; latDims = 0:3

# Run amen in parallel
library(doParallel) ; library(foreach) ; cl=makeCluster(4) ; registerDoParallel(cl)
foreach(ii=1:length(latDims), .packages=c("amen")) %dopar% {

  # load previous model run
  load(prevModelFiles[ii])
  # extract start vals
  startVals0 = ameFit$'startVals'
  # dump rest
  rm(ameFit)

  ameFit = ame_repL(
    Y=yList,Xdyad=xDyadList,Xrow=NULL,Xcol=NULL,
    model="bin",symmetric=FALSE,intercept=TRUE,R=latDims[ii],
    nscan=imps, seed=1, burn=brn, odens=ods,
    plot=FALSE, print=FALSE, gof=TRUE, startVals=startVals0,
    periodicSave=TRUE )
  save(ameFit, file=paste0('model_k', latDims[ii],'_v2.rda') )
}
stopCluster(cl)
```

References

Adamic, Lada A., and Glance, Natalie. 2005. The Political Blogosphere and the 2004 US Election: Divided they Bloglog. Pages 36–43 of: *Proceedings of the 3rd International Workshop on Link Discovery*. New York, N.Y.: ACM, for ACM.

Anderson, Carolyn J., Wasserman, Stanley, and Faust, Katherine. 1992. Building Stochastic Blockmodels. *Social Networks*, **14**(1), 137–161.

Aronow, Peter M., Samii, Cyrus, and Assenova, Valentina A. 2015. Cluster-Robust Variance Estimation for Dyadic Data. *Political Analysis*, **23**(4), 564–577.

Barabási, Albert-László, and Réka, Albert. 1999. Emergence of Scaling in Random Networks. *Science*, **286**(October 15), 509–510.

Beck, Nathaniel. 2012. Sweeping Fewer Things under the Rug: Tis Often (Usually?) Better to Model than be Robust. *The Society for Political Methodology POLMETH XXIX*.

Beck, Nathaniel, and Katz, Jonathan N. 1995. What To Do (and Not To Do) with Pooled Time-Series Cross-Section Data. *American Political Science Review*, **89**(3), 634–647.

Beck, Nathaniel, Katz, Jonathan N., and Tucker, Richard. 1998. Taking Time Seriously: Time-Series-Cross-Section Analysis with a Binary Dependent Variable. *American Journal of Political Science*, **42**(2), 1260–1288.

Bennett, D. Scott, and Stam, Allan C. 2000. Research Design and Estimator Choices in the Analysis of Interstate Dyads: When Decisions Matter. *Journal of Conflict Resolution*, **44**(5), 653–685.

Bennett, James, Lanning, Stan, et al. 2007. The Netflix Prize. Page 35 of: *Proceedings of KDD cup and workshop*, vol. 2007.

Block, Per, Stadtfeld, Christoph, and Snijders, Tom A.B. 2017. Forms of Dependence: Comparing SAOMs and ERGMs from Basic Principles. *Sociological Methods & Research*.

Chyzh, Olga. 2016. Dangerous Liaisons: An Endogenous Model of International Trade and Human Rights. *Journal of Peace Research*, **53**, tbd.

Dorff, Cassy, and Minhas, Shahryar. 2017. When Do States Say Uncle? Network Dependence and Sanction Compliance. *International Interactions*, **43**(4), 563–588.

Erikson, Robert S., Pinto, Pablo M., and Rader, Kelly T. 2014. Dyadic Analysis in International Relations: A Cautionary Tale. *Political Analysis*, **22**(4), 457–463.

Franzese, Robert, and Hayes, Jude C. 2007. Spatial Econometric Models for the Analysis of TSCS Data in Political Science. *Political Analysis*, **15**(15), 2.

Gibler, Douglas M. 2017. State Development, Parity, and International Conflict. *American Political Science Review*, **111**(1), 21–38.

Hoff, Peter D. 2005. Bilinear Mixed-Effects Models for Dyadic Data. *Journal of the American Statistical Association*, **100**(4690), 286–295.

Hoff, Peter D. 2008. Modeling Homophily and Stochastic Equivalence in Symmetric Relational Data. Pages 657–664 of: Platt, John C., Koller, Daphne, Singer, Yoram, and Roweis, Sam T. (eds), *Advances in Neural Information Processing Systems 20*. Processing Systems 21. Cambridge, MA, USA: MIT Press.

Hoff, Peter D., and Ward, Michael D. 2004. Modeling Dependencies in International Relations Networks. *Political Analysis*, **12**(2), 160–175.

Keohane, Robert O. 1989. Reciprocity in International Relations. *International Organization*, **40**(1).

King, Gary, and Roberts, Margaret E. 2014. How Robust Standard Errors Expose Methodological Problems They Do Not Fix, and What to Do about It. *Political Analysis*, **23**(2), 159–179.

Kinne, Brandon J. 2013. Network Dynamics and the Evolution of International Cooperation. *American Political Science Review*, **107**(04), 766–785.

Li, Heng, and Loken, Eric. 2002. A Unified Theory of Statistical Analysis and Inference for Variance Component Models For Dyadic Data. *Statistica Sinica*, **12**(2), 519–535.

Manger, Mark S., Pickup, Mark A., and Snijders, Tom A.B. 2012. A Hierarchy of Preferences: A Longitudinal Network Analysis Approach to PTA Formation. *Journal of Conflict Resolution*, **56**(5), 852–877.

McDonald, Patrick J. 2004. Peace through Trade or Free Trade? *Journal of Conflict Resolution*, **48**(4), 547–572.

Minhas, Shahryar, Hoff, Peter D., and Ward, Michael D. 2016 (October). *Let's Say Amen for Latent Factor Models*. Working paper.

Mucha, Peter J., Richardson, Thomas, Macon, K., Porter, Mason A., and Onnela, J. P. 2010. Community Structure in Time-Dependent, Multiscale, and Multiplex Networks. *Science*, **328**(5980), 876ff.

Organski, A.F.K. 1958. *World Politics: The Stages of Political Development*. New York: Alfred A. Knopf.

Reiter, Dan, and Stam, Allan C. 2003. Identifying the Culprit: Democracy, Dictatorship, and Dispute Initiation. *American Political Science Review*, **97**(2), 333–337.

Resnick, Paul, and Varian, Hal R. 1997. Recommender Systems. *Communications of the ACM*, **40**(3), 56–58.

Richardson, Lewis F. 1960. *Arms and Insecurity*. Chicago and Pittsburgh, PA: Quadrangle/Boxwood.

Rose, Andrew K. 2004. Do We Really Know That the WTO Increases Trade? *American Economic Review*, **94**(1), 98–114.

Signorino, Curtis. 1999. Strategic Interaction and the Statistical Analysis of International Conflict. *American Political Science Review*, **92**(2), 279–298.

Singer, J. David. 1972. The Correlates of War Project: Continuity, Diversity, and Convergence. Chap. 2 of: Singer, J. David (ed), *Quantitative International Politics: An Appraisal*. Special Studies in International Politics and Government, vol. VI. Praeger.

Tomz, Michael, Goldstein, Judith, and Rivers, Douglas. 2007. Do We Really Know That the WTO Increases Trade? *American Economic Review*, **97**(5), 2005–2018.

Wasserman, Stanley, and Faust, Katherine. 1994. *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

Weeks, Jessica L. 2012. Strongmen and Straw Men: Authoritarian Regimes and the Initiation of International Conflict. *American Political Science Review*, **106**(2), 326–347.

Zinnes, Dina A. 1967. An Analytical Study of the Balance of Power Theories. *Journal of Peace Research*, **3**, 270–288.