TAKING DYADS SERIOUSLY

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ABSTRACT. Much international relations scholarship concerns dyads: dyadic hypotheses and especially dyadic data. Yet, contemporary research brushes aside the complications that come with analyzing interdependent relational data. Dyadic observations do not typically satisfy the conditional independence criterion required for many statistical approaches. As a result, many studies often produce results with biased coefficient estimates and poorly calibrated standard errors. These biases have profound consequences for evaluating parametric models. We present an alternative, regression-based, approach that accounts for the dependencies complicating this type of analysis. We first present a simulation exercise highlighting the model's ability to account for the dependencies that emerge in relational data. In addition, we replicate five prominent studies in recent international relations scholarship, comparing the standard approach to our alternative. For each study, we find that conventional methods overstate the effect of key variables in the study of interstate conflict (and trade), underestimate the uncertainty in these effects, and in some cases lead researchers to faulty conclusions about the statistical significance and substantive importance of their variables. Further, we show that our approach dominates in terms of out-of-sample cross-validations, rendering it more useful in forecasting applications and in modeling the data generating process behind outcomes of interest.

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1. Introduction

Aronow et al. (2015) estimate that during the period 2010 to 2015, over sixty articles were published in the *American Political Science Review*, *American Journal of Political Science*, and *International Organization* using dyadic data. Most of these studies use a generalized linear model (GLM) to estimate regression coefficients. However, this approach to studying dyadic data increases the chance of faulty inferences by assuming data are conditionally independent and identically distributed (iid). Most standard approaches assume that the problems raised by having non-iid relational data can be addressed by recalculating the standard errors of estimated parameters, so as to reflect the potential clustering of cases. In practice, these palliatives rarely work because they fail to address the fundamental data generating process that remains a threat to inference. Namely, it is not just the diagonals of the variance-covariance matrix that are affected (Beck, 2012; King and Roberts, 2014).

In this article, we discuss the Additive and Multiplicative Effects (AME) model, a Bayesian approach for directly modeling relational data to reflect the data generating process that yields interdependencies in dyadic data structures (Hoff, 2008; Minhas et al., 2016). We focus on three types of interdependencies that can complicate dyadic analyses. First, dependencies may arise within a set of dyads if a particular actor is more likely to send or receive actions such as conflict. Additionally, if the event of interest has a clear sender and receiver, we are likely to observe dependencies within a dyad; for example, if a rebel group initiates a conflict with a government, the government will likely reciprocate that behavior. We capture these effects, often referred to as first- and second-order dependencies, respectively, within the additive effects portion of the model. The multiplicative effects capture dependencies that result because the specified model has not accounted for some latent set of shared attributes possessed by actors that affect their probability of interacting with one another.

We begin with a brief review of these dependencies and the AME model. Next, we conduct a simulation study to show how the AME approach can recover unbiased and well-calibrated regression coefficients in the presence of the dependencies that arise in dyadic data. Then, we apply this approach to five prominent studies in the international relations (IR) literature and compare results from the current state-of-the-art approach (a GLM with robust standard errors) to those obtained using the AME framework. The comparison reveals that in ignoring observational dependence, the standard approach overestimates the effect of key variables in the literature and underestimates their uncertainty. Consequently, the latent factor approach (AME) produces results that, at times, contradict those found in these studies in particular, and the broader literatures from which they are drawn. Moreover, we demonstrate the latent factor approach offers substantive insights that are often occluded by ignoring the interdependencies in most relational data that the field of IR is concerned with. Finally, we show that for each replication our network-based approach provides substantively more accurate out-of-sample predictions than the models used in the original studies. Thus, the AME approach can be used by scholars in the field to not only generate substantive insights, but it also enables us to better model the data generating process behind events of interest. Most importantly, it facilitates concentration on the relations aspect of the field of international relations.

2. Dependencies in Dyadic Data

In working with relational data, scholars typically begin by structuring these data as a set of dyadic observations stacked on top of one another. This makes sense if each observation is independent of the others. For example, a conflict sent from the United States to Japan, is assumed to be independent of any action that Japan may send to the United States. Additionally, every action sent by Japan to others in the system is considered independent even though each of those interactions involves a common sender, i.e, Japan. While the assumption that most begin with is that each dyadic interaction is taking place in isolation of the others, we know this assumption to be false in theory and in practice. Relational data comes with an explicit structure that in general leads to particular types of dependencies. The importance of accounting for the underlying structure of our data has been a lesson well understood, at least, when it comes to time-series cross-sectional data (TSCS) within political science (Beck and Katz, 1995; Beck et al., 1998). As a result, it is now standard practice to take explicit steps to account for the complex data structures that emerge in TSCS applications and the unobserved heterogeneity that they cause. To uncover the underlying structure of relational data, it is helpful to shift towards restructuring dyadic data in the form of a matrix—often referred to as an adjacency matrix—as shown in Figure 1. Rows designate the senders of an event and columns the receivers. The cross-sections in this matrix represent the actions that were sent by an actor in the row to those designated in the columns. Thus y_{ij} designates an action y, such as a conflictual event or trade flow, that is sent from actor i to actor j.

Using the structure of an adjacency matrix, we visualize in Figure 1 the types of first- and second-order dependencies that can complicate the analysis of relational data in traditional GLMs. The adjacency matrix on the top left highlights a particular row to illustrate that these values may be more similar to each other than other values because each has a common sender. Interactions involving a common sender also manifest heterogeneity in how active actors are across the network when compared to each other. In most relational datasets (e.g., trade flows, conflict, participation in international organizations, even networks derived from Twitter or Facebook), we often find that there are some actors that are much more active than others (Barabási and Réka, 1999). Unless one is able to develop a model that can account for the variety of explanations that may play a role in determining why a particular actor is more active than others, parameter estimates from standard statistical models will be biased.

For similar reasons one also needs to take into account the shared dependence between observations that share a common receiver. The bottom-left panel, illustrates that sender and receiver type dependencies can also blend together. Specifically, actors who are more likely to send ties in a network tend to also be more likely to receive them. As a result, the rows and columns in an adjacency matrix are often correlated. For example, consider that trade flows both from and to many wealthy, developed countries. The bottom-right panel, highlights a second-order dependence, specifically, reciprocity. This is a dependency occurring within dyads involving the same actors whereby values of y_{ij} and y_{ji} are correlated. The concept of reciprocity has deep roots in the study of relations between states (Richardson, 1960; Keohane, 1989).

For most relational data, however, dependencies do not simply manifest at the nodal or dyadic level. More often we find significant evidence of higher-order structures that result from dependencies between multiple groups of actors. These dependencies arise because there may be a or some set of latent attributes between actors that affects their probability of interacting with one another (Zinnes, 1967; Wasserman and Faust, 1994). In Figure 2 we provide a visualization of a simulated relational dataset wherein the nodes designate actors and edges between the nodes

Sender heterogeneity

	i	j	k	l
i	NA	y_{ij}	y_{ik}	y_{il}
j	y_{ji}	NA	y_{jk}	y_{jl}
k	y_{ki}	y_{kj}	NA	y_{kl}
l	y_{li}	y_{lj}	y_{lk}	NA

Receiver Heterogeneity

	i	j	k	l
i	NA	y_{ij}	y_{ik}	y_{il}
j	y_{ji}	NA	y_{jk}	y_{jl}
k	y_{ki}	y_{kj}	NA	y_{kl}
l	y_{li}	y_{lj}	y_{lk}	NA

Sender-Receiver Covariance

	i	j	k	l
i	NA	y_{ij}	y_{ik}	y_{il}
j	y_{ji}	NA	y_{jk}	y_{jl}
k	y_{ki}	y_{kj}	NA	y_{kl}
l	y_{li}	y_{lj}	y_{lk}	NA

Reciprocity

	i	j	k	l
i	NA	y_{ij}	y_{ik}	y_{il}
j	y_{ji}	NA	y_{jk}	y_{jl}
k	y_{ki}	y_{kj}	NA	y_{kl}
l	y_{li}	y_{lj}	y_{lk}	NA

Figure 1. Nodal and dyadic dependencies in relational data.

indicate that an interaction between the two took place. To highlight third-order dependence patterns, nodes with similar latent attributes are colored similarly.

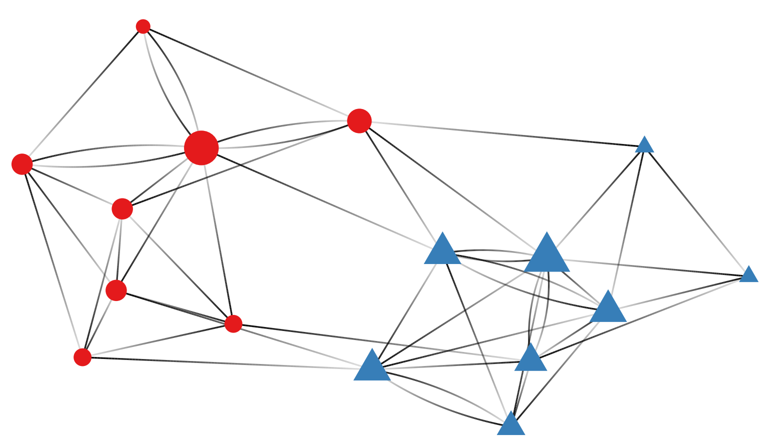


Figure 2. Visualization of network with meso-scopic features.

The visualization illustrates that the actors belonging to the same group have a higher likelihood of having an interaction with each other, whereas interactions across groups are rarer. A prominent example of a network with this type of structure was found by Adamic and Glance (2005), who visualized the ways in which right and left leaning political blogs linked to one another preceding the 2004 United States Presidential Election. Adamic and Glance find that the degree of interaction between right and left leaning blogs was minimal, and that most blogs linked to others that were politically similar. This showcases the types of higher-order dependencies that can emerge in relational data. First, the fact that interactions were determined by a shared attribute, in this case political ideology, is an example of *homophily*. Homophily explains the emergence of patterns such as transitivity ("a friend of a friend is a friend") and balance ("an enemy of a friend is an enemy"), which also have a long history in international relations. The other major type of meso-scopic feature that emerges in relational data is community structure (Mucha et al., 2010), which is often formalized through the concept of stochastic equivalence (Anderson et al., 1992). This concept simply refers to the idea that groups of nodes that act similarly in the network are stochastically equivalent. In the example we have laid out above each of the left leaning blogs would be considered stochastically equivalent to one another.

The presence of these dependencies in relational data points to the fact that there is a complex structure underlying the dyadic events that we observe, and that accounting for this structure is necessary if we are to adequately represent the data generating process. Of course, if one can specify each of the nodal, dyadic, and triadic set of attributes that influence interactions then one can be assured that the conditional independence assumption underlying standard approaches will be satisfied. However, it is rarely the case that we can do this even for TSCS data, thus we more than often make modeling decisions to account for that structure. Failing to account for the underlying structure in either TSCS or dyadic data leads to a number of well-known challenges: a) biased estimates of the effect of independent variables, b) uncalibrated confidence intervals, and c) poor predictive performance. Further by ignoring these potential interdependencies, we often ignore substantively interesting features of the problem. The study of international relations is founded on the relations among actors. Why ignore the interdependencies that led to the study of IR in the first place?

3. ADDITIVE AND MULTIPLICATIVE EFFECT MODELS FOR NETWORKS

To account for the dependencies that are prevalent in dyadic data, we turn to the AME model. The AME approach can be used to conduct inference on cross-sectional and longitudinal networks with binary, ordinal, or continuous linkages. It is flexible and easy to use for analyzing the kind of relational data often found in social science. It accounts for nodal and dyadic dependence patterns, as well as higher-order dependencies such as homophily and stochastic equivalence. Hoff (2015); Minhas et al. (2016) provide a detailed introduction to this framework, and the latter piece also details how it contrasts with alternative network-based approaches. Here we provide a brief review and then move to a simulation exercise.

The AME model combines the social relations regression model (SRRM) to account for nodal and dyadic dependencies and the latent factor model (LFM) for third-order dependencies. For details on the SRRM see Li and Loken (2002); Dorff and Minhas (2017). An earlier version of the LFM used in AME is presented as the general bilinear mixed effects (GBME) model in Hoff and Ward (2004). The GBME model is more limited in the types of dependence patterns that it can capture due to the formulation of the matrix decomposition procedure underlying the LFM. The AME model is specified as follows:

$$
\begin{aligned}
y_{ij} &= f(\theta_{ij}), \text{ where} \\
\theta_{ij} &= \boldsymbol{\beta}_d^{\top}\mathbf{X}_{ij} + \boldsymbol{\beta}_s^{\top}\mathbf{X}_i + \boldsymbol{\beta}_r^{\top}\mathbf{X}_j && \text{(Exogenous parameters)} \\
&\quad + a_i + b_j + \epsilon_{ij} && \text{(SRRM parameters)} \\
&\quad + \mathbf{u}_i^{\top}\mathbf{D}\mathbf{v}_j && \text{(LFM parameters)}
\end{aligned} \tag{1}
$$

where $y_{ij,t}$ captures the interaction between actor i (the sender) and j (the receiver). We use a Bayesian probit framework, in which we model a latent variable, θ_{ij}, using first a set of exogenous dyadic ($\boldsymbol{\beta}_d^{\top}\mathbf{X}_{ij}$), sender ($\boldsymbol{\beta}_s^{\top}\mathbf{X}_i$), and receiver covariates ($\boldsymbol{\beta}_r^{\top}\mathbf{X}_j$). Next, to account for the dependencies that emerge in dyadic data and that may complicate inference on the parameter associated with exogenous covariates, we add parameters from the SRRM and LFM. a_i and b_j in Equation 1 represent sender and receiver random effects incorporated from the SRRM framework:

$$
\begin{aligned}
\{(a_1, b_1), \ldots, (a_n, b_n)\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{ab}) \\
\{(\epsilon_{ij}, \epsilon_{ji}) : \ i \neq j\} &\overset{\text{iid}}{\sim} N(0, \Sigma_{\epsilon}), \text{ where} \\
\Sigma_{ab} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} &\quad \Sigma_{\epsilon} = \sigma_{\epsilon}^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}
\end{aligned} \tag{2}
$$

The sender and receiver random effects are modeled jointly from a multivariate normal distribution to account for correlation in how active an actor is in sending and receiving ties. Heterogeneity in the the sender and receiver effects is captured by σ_a^2 and σ_b^2, respectively, and σ_{ab} describes the linear relationship between these two effects (i.e., whether actors who send [receive] a lot of ties also receive [send] a lot of ties). Beyond these first-order dependencies, second-order dependencies are described by σ_{ϵ}^2 and a within dyad correlation, or reciprocity, parameter ρ.

The LFM contribution to the AME comes in the multiplicative term: $\mathbf{u}_i^{\top}\mathbf{D}\mathbf{v}_j = \sum_{k \in K} d_k u_{ik} v_{jk}$. K denotes the dimensions of the latent space. This model posits a latent vector of characteristics $\mathbf{u_i}$ and $\mathbf{v_j}$ for each sender i and receiver j. The similarity or dissimilarity of these vectors will then influence the likelihood of activity, and provides a representation of third-order interdependencies (Minhas et al., 2016). The LFM parameters are estimated by a process similar to computing the singular value decomposition (SVD) of the observed network. When taking the SVD we factorize our observed network into the product of three matrices: $\mathbf{U}$, $\mathbf{D}$, and , $\mathbf{V}$. This provides us with a low-dimensional representation of our original network.[1] Values in $\mathbf{U}$ provide a representation of how stochastically equivalent actors are as senders in a network or, for example, how similar actors are in terms of who they initiate conflict with. $\hat{\mathbf{u}}_i \approx \hat{\mathbf{u}}_j$ would indicate that actor i and j initiate battles with similar third actors. $\mathbf{V}$ provide a similar representation but from the perspective of how similar actors are as receivers. The values in $\mathbf{D}$, a diagonal matrix, represent levels of homophily in the network.[2]

By integrating the SRRM and LFM into a Bayesian probit framework, we can account for the underlying structure in dyadic data that if left unestimated would complicate any inferences we might

[1]The dimensions of $\mathbf{U}$ and $\mathbf{V}$ are $n \times K$ and $\mathbf{D}$ is a $K \times K$ diagonal matrix.
[2]Unlike traditional SVD, in the latent factor model, the singular values are not restricted to be positive, thus allowing us to account for both the presence and absence of homophily.

wish to draw for the exogenous parameters. Parameter estimation in the AME takes place within the context of a Gibbs sampler in which we iteratively sample from the posterior distribution of the full conditionals for each parameter. Specifically, given initial values of $\{\boldsymbol{\beta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2\}$, the algorithm proceeds as follows until convergence:

- sample $\boldsymbol{\theta} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
- sample $\boldsymbol{\beta} \mid \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
- sample $\mathbf{a}, \mathbf{b} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
- sample $\Sigma_{ab} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \rho, \text{ and } \sigma_\epsilon^2$ (Inverse-Wishart)
- update ρ using a Metropolis-Hastings step with proposal $p^*|p \sim$ truncated normal$_{[-1,1]}(\rho, \sigma_\epsilon^2)$
- sample $\sigma_\epsilon^2 \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \text{ and } \rho$ (Inverse-Gamma)
- For each $k \in K$:
  - Sample $\mathbf{U}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}_{[,-k]}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
  - Sample $\mathbf{V}_{[,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}_{[,-k]}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)
  - Sample $\mathbf{D}_{[k,k]} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{a}, \mathbf{b}, \mathbf{U}, \mathbf{V}, \Sigma_{ab}, \rho, \text{ and } \sigma_\epsilon^2$ (Normal)[3]

4. Simulation study

We utilize a simulation study to highlight the utility of AME as an inferential tool for dyadic analysis. The goal of the simulation is to assess how well AME can provide unbiased and well-calibrated estimates of coefficient parameters in the presence of unobserved dependencies. Specifically, we are concerned with conducting inference on regression parameters of a linear model for a network in the case where there is an omitted variable. Say that the true data-generating process for a particular Y is given by:

$$y_{i,j} \sim \mu + \beta x_{i,j} + \gamma w_{i,j} + \epsilon_{i,j} \tag{3}$$

[3]Subsequent to estimation, **D** matrix is absorbed into the calculation for **V** as we iterate through K.

where $Y = \{y_{i,j}\} \in \mathbb{R}^{n \times n}$ is an observed sociomatrix, $X = \{x_{i,j}\} \in \mathbb{R}^{n \times n}$ is a matrix of observed dyad-specific characteristics, and $W = \{w_{i,j}\} \in \mathbb{R}^{n \times n}$ is a matrix of unobserved dyad-specific characteristics. Y can be thought of as a dyadic dependent variable, X and W are both dyadic covariates that are a part of the data-generating process for Y, but W is not observed. We compare inference for μ and β—the latter parameter would be of primarily theoretical concern for applied scholars—using three models :

- the standard international relations approach assuming independent errors;
- the AME approach outlined in the previous section with a unidimensional latent factor space ($K = 1$);
- and an "oracle" regression model that assumes we have measured all sources of dependencies and thus includes both $x_{i,j}$ and $w_{i,j}$.

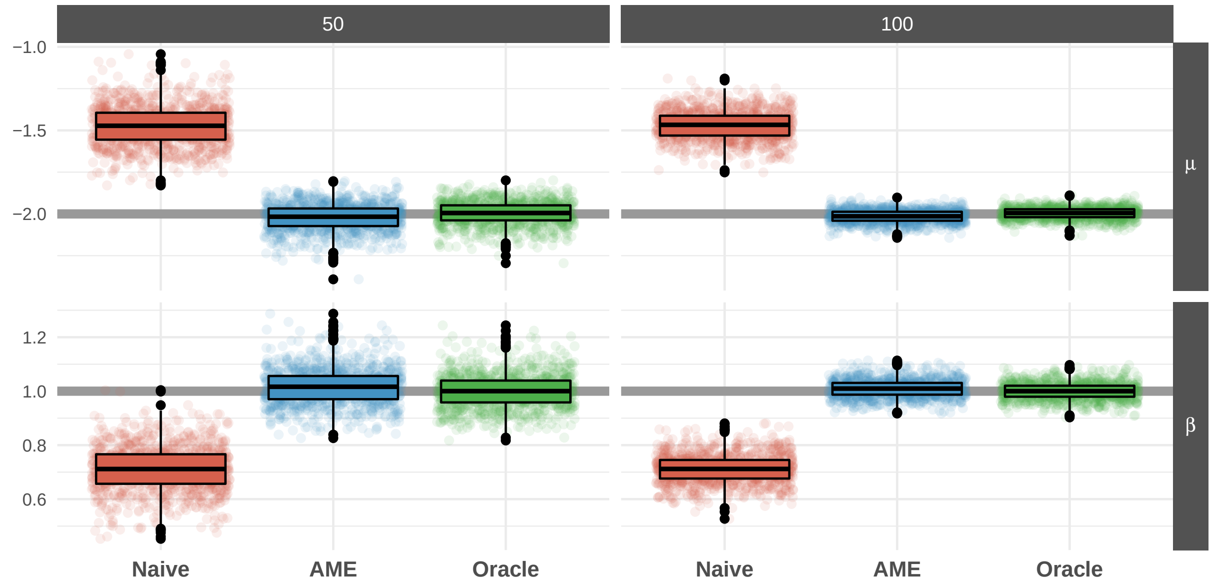
The first model corresponds to the "naive" approach in which little is explicitly done to account for latent dependencies in dyadic data. In the second model, we account for dependencies in dyadic data using the AME framework described in the previous section. For both the first and second models, we are simply estimating a linear model of X on Y, and assessing the extent to which inference on the regression parameters are complicated in the presence of unobserved dependencies, W. In the last model, we provide an illustration of the ideal case in which we have observed and measured W and include it in our specification for Y. The oracle case provides an important benchmark for the naive and AME approaches.

For the simulation we set the true value of μ (the intercept term) to -2 and β (the effect of X on Y) to 1.[4] We conduct two sets of simulations, one in which the number of actors in the network is set to 50 and the other at 100. In total, we ran 1,000 simulations where we begin by simulating Y from the specification given in Equation 3 and then for each simulated Y we estimated a naive, AME, and oracle model.

We compare the performance of the models first in terms of how well they estimate the true values of μ and β in Figure 3 by depicting the average μ and β estimates from the simulations for the three models. The panels in the left show the results for when the number of actors is set to 50 and those on the right for 100; and the top pair of panels represents the estimates for μ while

[4]The value of γ is also set to 1, which corresponds to a an example where the W character is associated with homophily.

Figure 3. Regression parameter estimates for the naive, AME, and oracle models from 1,000 simulations. Summary statistics are presented through a traditional box plot, and the estimates from each simulation are visualized as well as points.

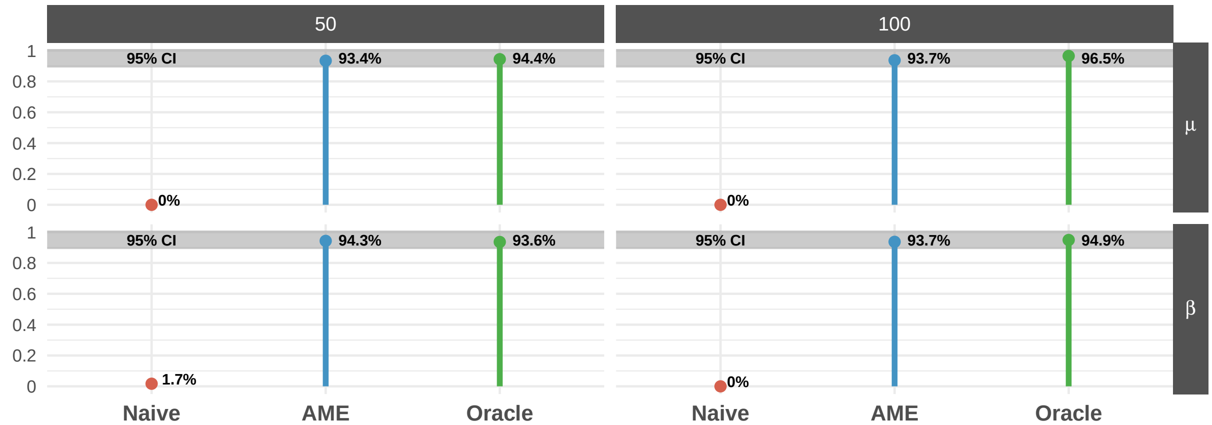


the bottom pair do the same for β. In each case, we find that the estimates for μ and β produced by the naive approach are notably off from their true values. On the other hand, the AME model performs just as well as the oracle case in estimating the true values.

Next, we estimate the 95% confidence interval for the three models in each of the simulations and estimate the proportion of times that the true value fell within those intervals. The results are summarized in Figure 4, and again we see that the AME approach performs as well as the oracle, while the naive approach performs poorly by comparison.

Last, beyond obtaining less biased and better-calibrated parameter estimates, a key benefit of the AME framework is that we can also estimate unobserved dependencies through the random effects structure of the model. In the case of the data generating process for Y, W is set as an unobserved dyadic covariate that had a homophilous effect on Y. Homophilous because W within this framework is simply an example of a dyadic attribute involving i and j that positively affects the degree to which they will interact with one another, i.e., y_{ij}. This type of unobserved dependency will be captured through the multiplicative effects portion of the model, $\mathbf{U}^{\top}\mathbf{D}\mathbf{V}$. To estimate how well the model performs in doing this we recover the multiplicative effects term for

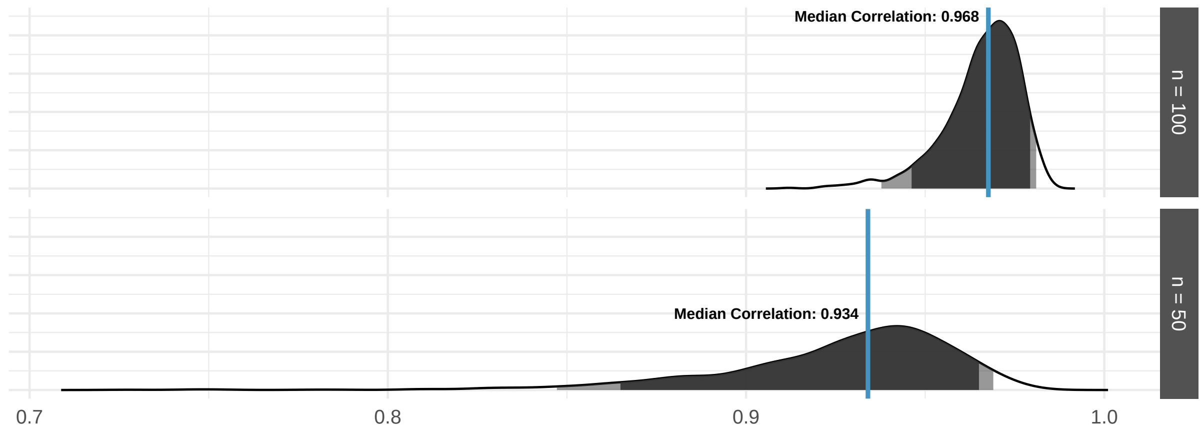
Figure 4. Proportion of times the true value fell within the estimated 95% confidence interval for the naive, AME, and oracle models from 1,000 simulations.



each simulation and calculate the correlation between it and the unobserved dependency, W.[5] We visualize the distribution of the correlations from each of the 1,000 simulations in Figure 5 for the case where the number of actors was set to 100 (top pair of panels) and 50. Additionally, we calculate the median across the correlations and display that result using a vertical line. For both $n = 50$ and $n = 100$, we find that the multiplicative effects perform very well in capturing the unobserved dependency, which indicates that the structure provided by this framework is not simply capturing noise but can be used as a tool to estimate unobserved structure.

[5]Specifically, since both the multiplicative effects term and W are continuous dyadic variables, we just calculate the Pearson correlation coefficient.

Figure 5. Distribution of correlation between missing variable and multiplicative random effect in AME across the 1,000 simulations. Vertical line through the distribution represents the median value across the simulations.



5. Re-estimation with AME

5.1. **Design.** We choose five prominent studies from the broad field of international relations and international political economy that utilize relational data (Reiter and Stam, 2003; McDonald, 2004; Rose, 2004; Weeks, 2012; Gibler, 2017). These studies are recent and have been cited over 100 times. Each of these pieces was published in a prominent journal and is well-known in the literature. Each used the standard approach in political science, which is to employ some form of a general linearized regression that ignores dyadic interdependencies, except as they may reveal themselves in included variables. In addition, most of these studies adjust the standard errors in an attempt to account for the clustering of observations. We follow the strategy of the original study in our re-estimation of the original models.

Table 1. Features of the Studies Re-estimated.

	Model	Date Range	N. Actors	N. Dyads	Dyads Type	Clustering $\sigma_{\hat{\beta}}$
Reiter & Stam (2003)	Logit	1945–1995	193	753,456	Directed	Robust
McDonald (2004)	Logit	1959–2002	198	92,354	Undirected	Robust
Rose (2004)	OLS	1948–1999	177	234,597	Directed	Robust
Weeks (2012)	Logit	1946–1999	197	901,540	Directed	Robust
Gibler (2017)	Logit	1816–2008	193	650,557	Undirected	none

We obtained the data for each of these studies from their replication archives and replicated the main results of each article.[6] The broader goal in doing this, beyond introducing the use of the AME framework in an applied setting, is to examine the extent to which interdependencies within typical dyadic data make much difference in what we have learned about international relations from recent empirical studies using a framework that a priori assumes conditional independence.

Typically, scholars have a small set of independent variable in a complicated empirical model that they look at for evaluating the validity of their empirically estimated models. For example, in the study by Reiter and Stam (2003) the goal is to determine whether in mixed dyads—consisting of a democratic country and country ruled by a personalist dictatorship—it is the democratic country or the personalist country that is associated with a greater likelihood of the dyad being in a dispute. In a regression including fourteen covariates, they focus on whether the democratic nature of the initiating country is a statistically significant covariate.[7] In our re-examination of Reiter and Stam, we focus on whether the model replicates when re-estimated within the AME framework. Each of the other four studies also has a crucial finding that we hone in on to further draw into focus the potential differences brought about by utilizing an AME estimation procedure. In Table 2, we present the overall results; the term *does not replicate* indicates only that the sign and/or significance of the putatively crucial finding in the original study is not found to hold in the AME estimation.[8]

Beyond just comparing parameter estimates, we examine how well each approach can represent the data generating process using an out-of-sample cross validation strategy. Specifically, for each study, we randomly divide the data into $k = 30$ sets, letting $s_{ij,t}$ be the set to which pair ij, t is assigned. Then for each $s \in \{1, \ldots, k\}$, we:

(1) estimate model parameters with $\{y_{ij,t} : s_{ij,t} \neq s\}$, the data not in set s,

(2) and predict $\{\hat{y}_{ij,t} : s_{ij,t} = s\}$ from these estimated parameters.

[6] Without exception this was straightforward to accomplish, thanks to an increasing norm in the social sciences of open data sharing.

[7] In particular, the variables `Pers/Democ` and `Democ/Pers` in Model 3, in Table 1, page 335.

[8] Full tabular results for each of the original and reestimated models are presented in the Appendix.

Table 2. Here we provide a brief summary of the key variable in each of the five replications and a note about whether or not the finding is replicated when using our network-based approach.

Study	Central Finding	Replicates in a Network Model?
Reiter & Stam (2003)	Personalist Regimes Attack Democracies, Not Vice Versa	Does Not Replicate
McDonald (2004)	Lower Trade Barriers and Higher Trade Lead to Peace	Does Not Replicate
Rose (2004)	WTO Membership Does not Affect Trade	Partially Replicates
Weeks (2012)	Bosses, Juntas, and Strongmen are more Aggressive, Machines are Not	Does Not Replicate
Gibler (2017)	Power Parity at Time of Entry to International System Inceases Conflict	Does Not Replicate

The result of this procedure is a set of sociomatrices $\hat{Y}$, in which each entry $\hat{y}_{ij,t}$ is a predicted value obtained from using a subset of the data that does not include $y_{ij,t}$.[9] We summarize the performance of the various models in Table 3 below. For the binary models we just provide the area under the Receiver Operator Characteristic (ROC) and Precision Recall (PR) curves. Only one of the studies here had a continuous dependent variable and for this we provide the root mean squared error (RMSE) and root median squared error (RMDSE).[10] For each of the replications, we find that the AME approach substantially outperforms the original models in terms of out-of-sample predictive performance.

[9]For more details on this type of cross-validation strategy see Minhas et al. (2016).
[10]More details on the performance of each of these models can be found in the Appendix.

		GLM	AME
Reiter & Stam (2003)	Area Under ROC Curve, AUC-ROC	0.92	0.96
	Area Under PR Curve, AUC-PR	0.08	0.15
McDonald (2004)	AUC-ROC	0.92	0.99
	AUC-PR	0.13	0.28
Rose (2004)	RMSE	3.23	1.99
	RMDSE	2.01	1.06
Weeks (2012)	AUC-ROC	0.64	0.97
	AUC-PR	0.00	0.15
Gibler (2017)	AUC-ROC	0.52	0.91
	AUC-PR	0.00	0.08

Table 3. Here we provide a summary of the out-of-sample performance based on our cross-validation strategy for each of the five replications when using the standard dyadic approach and our network-based approach. Four of the five studies involved a binary dependent variable; for those measures, area under the curve (AUC) statistics are reported. The Rose study involved a Gaussian dependent variable and for that we use the root mean squared error (RMSE) and root median squared error (RMDSE).

Next we discuss, each of the replications in more detail and highlight the substantive insights that can be drawn from the AME framework.

5.2. **Re-estimation of Reiter & Stam (2003).** Reiter and Stam (2003) examine the relationship between democracy, dictatorship and the initiation of militarized disputes. They use directed dyads and find that dyads involving a democratic leader on the one hand and a personalist dictator on the other tend to be violent. They also discover that dictators are likely to challenge democracies, but that the converse is not true. In addition, military regimes and single-party regimes are more prone to initiate disputes with democracies, than the other way around. Independent variables are largely taken from an earlier study and focus on various encodings of regime types, contiguity, alliance, and capability measures. As is prevalent in these kinds of studies, Reiter & Stam employ a logistic regression that includes an indicator of the time since the last dispute as well as three cubic splines. The database for this study is constructed using `EUGene` (Bennett and Stam, 2000) and comprises approximately three-quarters of a million stacked dyads. Based on their statistical analysis, they conclude that institutional constraints affect the propensity of democratic and non-democratic leaders to engage in military conflict.

In the original model, the variable “Pers/Democ Directed Dyad" (which represents a Personalist → Democractic directed dyad) is clearly positive while the variable “Democ/Personalist Directed Dyad” is zero and the difference between the two coefficients is clearly distinct from zero. In our re-estimation using the AME framework, we also find that Pers-Democ directed dyad has a positive effect with zero excluded from the 95% confidence interval while Democ-Pers directed dyad is indistinguishable from zero. Using this model, however, we can no longer conclusively say that the Pers/Democratic coefficient is larger than the Democ/Personalist one. Our re-estimation using the AME approach therefore cast some doubt on Reiter & Stam’s key claim that MIDs initiated by personalist dictatorships against democracies are more likely than MIDS initiated by democracies. Further, the effect of most of the covariates in the literature thought to predict interstate MIDs are much closer to zero when using the AME framework. An important takeaway here is that many studies make knowledge claims based on the statistical significance of a small set of covariates, or the differences between these covariates. These differences may change dramatically when interdependencies are taken into account directly, as they do in the case of this study.

5.3. **Re-estimation of McDonald (2004).** McDonald (2004) studies whether trade promotes peace between nations. The link between conflict and trade is perplexing, with many persistent, yet competing explanations. McDonald (2004, p. 547) includes the argument that interdependence between states “makes conflict less likely because of its efficiency over conquest in acquiring resources. . . ”. Accordingly, his primary contribution is to provide evidence challenging the generalized linkage between peace and trade and to offer a new measurement of the key independent variable, trade. McDonald (2004) refines the trade variable, arguing that *free* trade, rather than trade alone, reduces the likelihood of conflict between states. His key hypothesis is that greater levels of protection increase the probability of interstate conflict, an argument that builds on the work of classic liberalism and connects free trade to the power of domestic audiences. McDonald (2004) measures free trade in two ways. The first captures the idea that larger protected sectors generate greater societal pressures resulting in pockets of support for war. This protection variable measures the proportion of customs revenue divided by total imports in the state that possesses the greater such ratio in each dyad. This measure captures the score of the state in the dyad that possesses higher barriers to trade. McDonald (2004, p. 560) also includes a measure of economic integration calculated as "the lower proportion of total dyadic trade (imports plus exports) divided by state *i*'s GDP or total dyadic trade divided by state *j*'s GDP". The binary, dependent variable is the onset of a new militarized interstate dispute within a given dyad. McDonald (2004) employs logistic regression to examine the putative statistical significance of these variables. The models include splines to correct for temporal dependence, and robust standard errors clustered on each dyad.

Our re-estimation with AME reveals that trade relations are highly interdependent and exhibit important patterns of transitivity. Or, in other words, if countries i and j are highly dependent and countries j and k are also highly dependent, then we are likely to observe high dependency between countries i and k. This indicates that conflict is less likely among members of a trade community. Once we control for these dependencies, we can more clearly interpret the positive link between trade and conflict. The most striking thing is that AME finds a positive conditional association between trade dependence and conflict ($\hat{\beta} = 18.4, \quad \sigma_{\hat{\beta}} = 28.6$), while the comparable numbers for the logistic regression found in the original articles are negative ($\hat{\beta} = -22.2, \quad \sigma_{\hat{\beta}} = 15.2$). At the same time, the AME has ROC and PR curves (shown in the Appendix) that dominate the results found in McDonald (2004).

5.4. **Re-estimation of Rose (2004).** In 2004, Andrew Rose published a study in the *American Economic Review* that proved to be quite controversial in terms of macroeconomic trade theory and in terms of trade policy in a variety of nations. It also provoked a number of responses in the international political economy literature (Tomz et al., 2007; Ward et al., 2013). Rose's basic argument is that despite longstanding arguments, made by trade theorists and the World Trade Organization, that WTO membership fosters greater cooperation and thereby more trade among its members, the empirics do not bear out such claims. He uses a standard gravity model with dyadic data on bilateral merchandise trade (not services) for 175 countries over a period of five decades. Estimating this model using OLS within many differing contexts, his conclusion is that: "An extensive search reveals little evidence that countries joining or belonging to the GATT/WTO have different trade patters from outsiders . . . (2004, page 98, abstract)." The data for this study have been widely used in replications by many searching for the missing effects of the WTO—as well as preferential trade agreements, bilateral investment theories, and other aspects of modern trade theory.

When we compare the results of Rose's original OLS model to the AME model accounting for network dependencies, the results are similar. The main result of the model—the null effect of membership in the WTO, as represented by the "One-In" and "Both-In" variables—does not prevail in the AME model. In the original, this negative linkage is not significant, but in the AME estimation this negative linkage is more precisely estimated. However, while in the original model there was a clear positive relationship between Real GDP and Trade, most of this effect vanishes in the AME model. The random effects shown in Figure 6 reveal the cause of much of this divergence. Here, the states with the most positive random effects are also states with high GDP, though not necessarily high GDP/capita.[11] Thus, the effect of GDP in the original model was, in part, an artifact of first-order dependencies. Most of the other results of the model are constant across each model, though some geographic features, such as islands and landlocked states, have a more clear effect on trade once we account for these network dependencies. When we account for network interdependencies, we observe a markedly lower Root Mean Squared Error out-of-sample — 3.23 for the OLS model and 1.77 for the AME model. While the AME model replicate the main result, there are substantial and substantive improvements gleaned from the AME results.

[11]Note: Qatar exhibits strongly negative random effects.

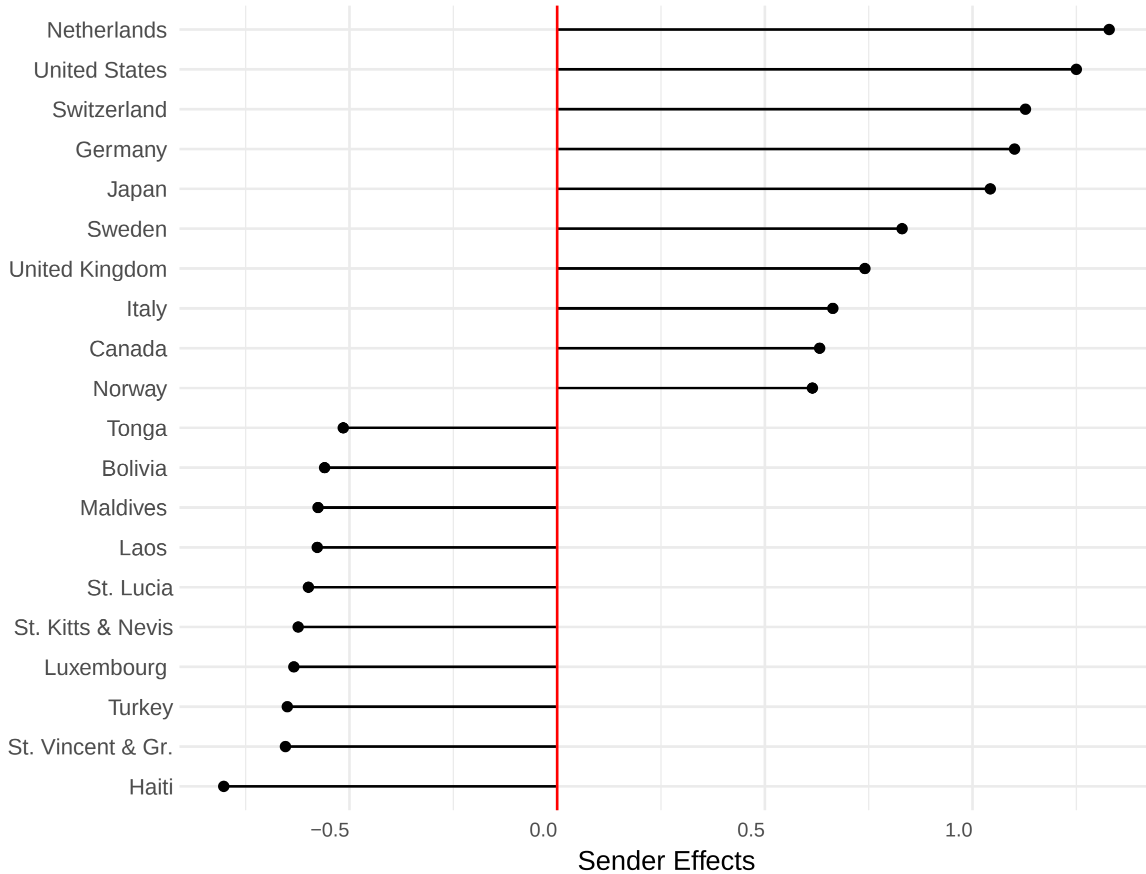


Figure 6. Nodal Random Effects for AME estimation of Rose (2004), for the countries with the highest and lowest Random Effects.

5.5. **Re-estimation of Weeks (2012).** Weeks (2012) examines the influence of domestic institutions on the initiation of military conflicts by autocratic leaders. She argues that in some circumstances autocrats are held accountable for their foreign policy decisions. She adds the nuance that autocratic audiences are not homogeneous. When the autocratic regime is nonmilitary, the domestic audience do not favor military actions, but in military autocracies this is not the case. Further she argues that in personalistic regimes without a military or civilian domestic audience, the leaders tend to be more likely to employ military force in their foreign policy. To study this question, she uses a dyadic design in which the dependent variable is "whether country A in a directed dyad initiated military conflict against country B during year t" (page 337). One major innovation in her study resides in the nuanced way in which she conceptualized and coded regime type into four types: a) Machine, b) Junta, c) Boss, and d) Strongmen. She also includes a variety of putative control variables focusing on capabilities for both sides of the dyad, alliances, geography, trade dependence, regime instability, and the regime type of "side B." She uses a logistic regression, but follows Beck et al. (1998) and includes splines to capture temporal covariation in the dependent variable along with fixed, unit effects. The analysis is done for dyads, but is considered to be from the perspective of the actor that initiated the dispute. The basic finding is that a) juntas, bosses, and strongmen are more likely to initiate conflict than machines (and maybe democracies) and that b) machines are no more belligerent than democracies. These insights are mainly emphasized in the paper by the parameter estimates depicted in Tables 1 and 2 (pages 339–340) from the paper. She makes the argument that ignoring important nuances between different types of autocracies hinders our understanding of the initiation of military conflict by autocracies.

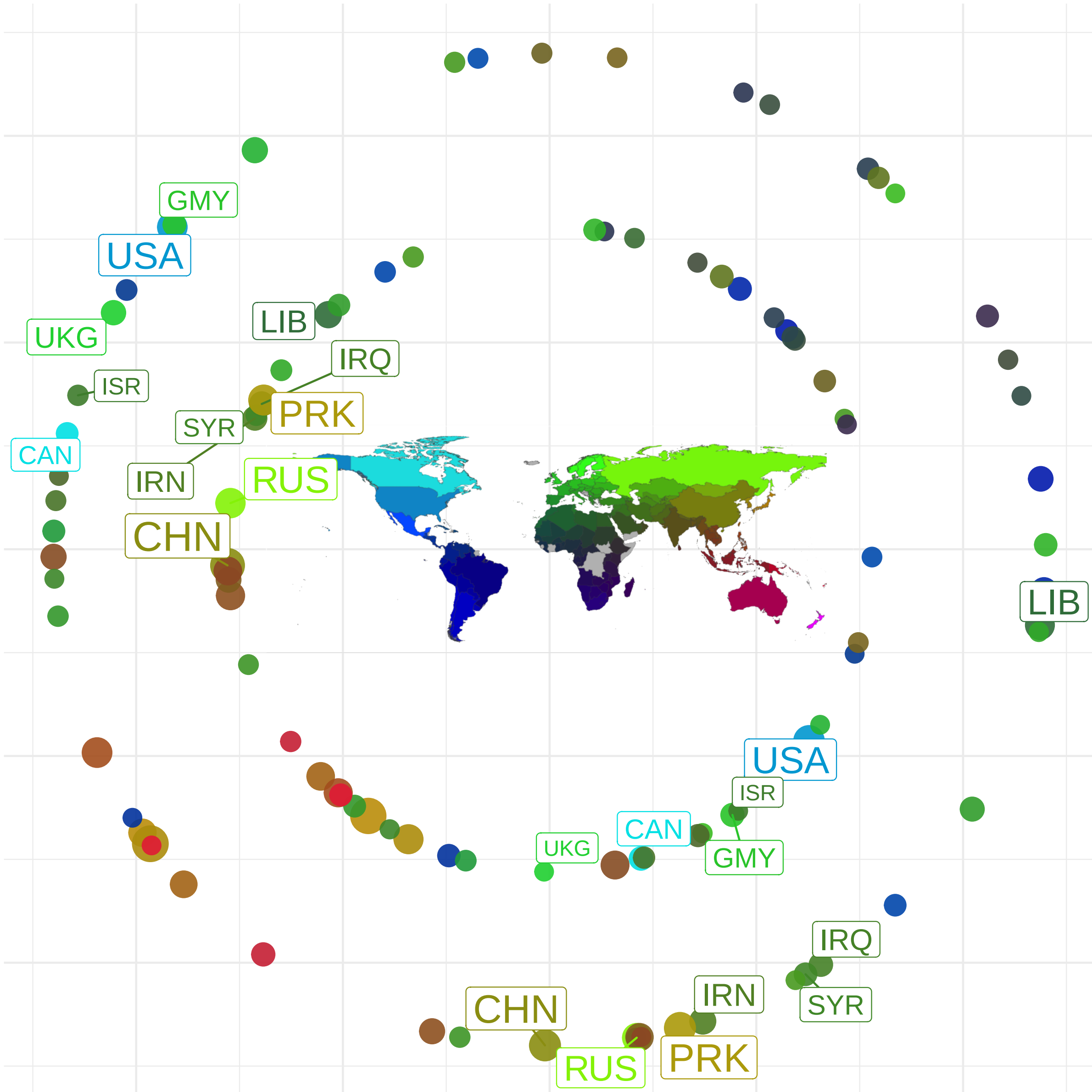


Figure 7. Visualization of multiplicative effects for Weeks (2012). Each circle designates a country and the color corresponds to the legend at the center of the visualization. Countries that cluster together in the outer ring are those that were found by the model to have similar sending patterns, meaning that they tend to send conflict to similar sets of countries. The inner ring clusters countries by the similarity of who they receive conflict from.

The re-estimation of Weeks (2012) has the sharpest divergence between the GLM results and those of the AME Model. In Weeks' initial models, she finds that machines are less prone to initiate conflict than the reference category, whereas Juntas, Bosses, and Strong-men are more conflict-prone. When we examine the AME results, we find that none of these values are distinguishable from zero. Similarly, we find less pronounced effects for military capabilities. One explanation for this divergence is the AME model's ability to account for third-order effects. Inspection of the multiplicative effects in Figure 7 reveals a number of clusters of states which exhibit structural equivalence—in the top left corner we see the US, the UK, Germany, Canada, and Israel. These states cluster together in the outer ring of this visualization because they tend to send conflicts to similar targets.[12] Conversely, in the bottom right of the outer ring, we see a cluster of authoritarian countries: Iraq, Russia, Syria, North Korea, and China. In the inner ring, the proximity of countries is determined by the degree to which they receive conflict from the same countries. In general, the clusters found on both the inner and outer rings have similar governmental types (Iraq, Syria, Libya, and North Korea all fell under the "boss" category). In the GLM, which ignores these third-order dependencies, many of these results might have been attributed to regime type. Structural equivalence is present even when accounting for nodal characteristics like regime type. The AME model, on the other hand, shows that this can be specified in terms of the interdependencies captured by the multiplicative effects.

5.6. **Re-estimation of Gibler (2017).** The replications we have undertaken are all from articles over the past fifteen years. A more recent example is Gibler (2017) which examines the onset of militarized disputes using capabilities, joint democracy, alliances, and power parity in a undirected dyadic study using logistic regression and dyad clustered standard errors. In addition, Gibler shows that the long-standing relationship between the relative parity of capabilities and initiation of international conflict is almost completely mediated by the initial conditions for the members of the dyad when they joined the international system as sovereign members. This finding calls into question many IR theories about the role of balance in terms of generating international conflict (Organski, 1958).

We re-estimated model 6 from Table 6 (2017, 34). The results are presented in the Appendix. The results obtained with AME stand in stark contrast to those found with a logistic regression (with dyad clustered, robust standard errors). Most importantly, the primary variable from the

[12]For details on how to interpret this type of visualization see Minhas et al. (2016).

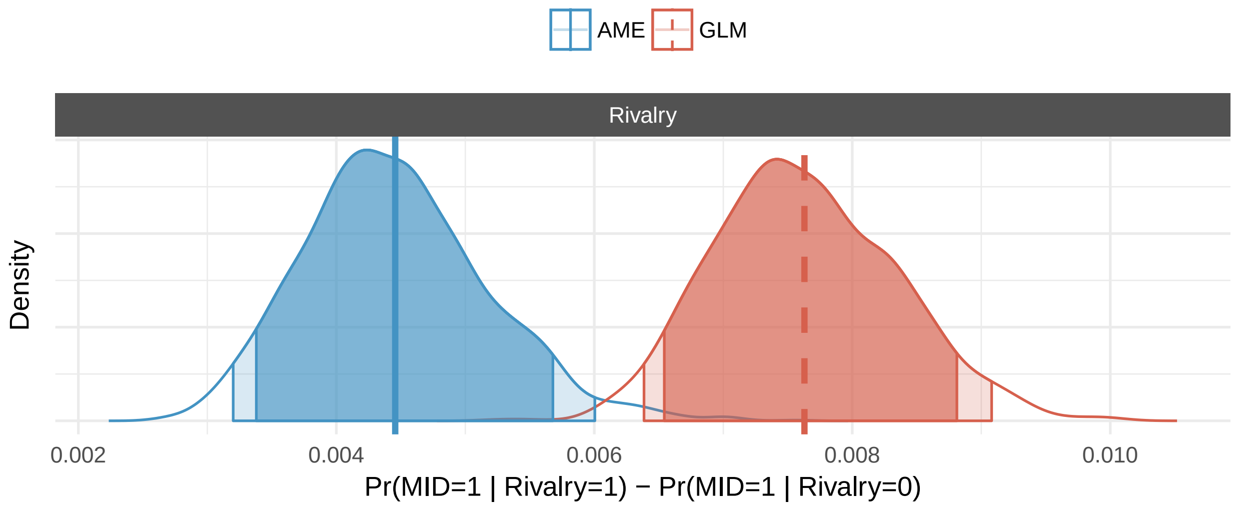
Gibler study, parity of the members of a dyad at the year in which they entered the international system, is shown to be unimportant in the AME results. Not only is the value of this parameter small, but it has a very large relative standard error—over a magnitude larger than the parameter itself ($z = 0.038$). In addition, the variable indicating whether both members of the dyad were coded as democracies (joint democracy) follows the same pattern: important and strong in the logistic results, but this disappears once interdependencies are modeled. As might be expected, the strong geographic clustering in the original study is about one-quarter as strong in the AME estimations. Similarly, rivalry coefficients are about one-third the size in the AME formulation, but a great deal more precisely measured ($z = 18.116$).

We utilized the original and the AME results from Gibler's model 6 in Table 6 to calculate the expected values for one scenario focusing on the variables measuring rivalry. We employed mean or modal values for all independent variables, except we changed the rivalry variable to indicate that there was a rivalry when the actual data suggest there is none. The expected values of this scenario are essentially a first difference plot comparing results with the model when estimated in two different ways: Gibler's GLM estimation and our AME approach. As this Figure 8 illustrates, the AME results differ notably. First, the expected value of the dependent variable—the probability of the onset of a militarized interstate dispute, is considerably lower when taking interdependencies into account with the AME model. These are rare events, so the probabilities are low, but the difference is a factor of almost 2. Thus, you get quite substantially different expected values from these two models.

5.7. **Lessons Learned from Re-estimating Five Prominent Studies.** First and foremost, many findings that emerge from models that do not take interdependencies into account lose their statistical significance when network effects are estimated via AME.[13] Not only are coefficients biased in the GLM approaches to the analysis of dyadic data, but they are often imprecisely measured, with poorly calibrated standard errors. This means that significance testing (for better or worse) is compromised when network effects are ignored.

[13]This should not be news—since the finding has long been in the theoretical literature, but given the state of current literature in international relations is still pertinent.

Figure 8. Marginal effects of a change in the Rivalry variable for both the AME and the Gibler estimation.



Second, even when the results from the AME estimation conform with those found in an OLS or logistic regression, new insights emerge from the additional information derived. In particular, there is actual information about the dependencies so that clusters can be identified, and the extent of reciprocity at the dyad level, as well as among senders and receivers. This kind of information is absent in standard approaches and adds to our ability to explain specific as well as general results.

Third, it is evident that the actual results—not the estimated coefficients and their covariances—which are generated by the models differ greatly in expectations. This implies that policy experimentations with the models, as well as scenario-based simulations and forecasting of GLM models are likely to often give misleading results compared to the AME approach.

Fourth, it is clear that the AME approach dominates the GLM approaches in terms of performance. Not only it is better at correctly identifying cases in which the dependent variable takes a value of 0 (via the ROC curves and associated statistics), but it also dominates at correctly identifying occurrences of the dependent variable in the data (seen via the PR curves and associated statistics). In the case of studies with continuous dependent variables, the AME approach has average error statistics that are about one-half that found in the OLS model. It is rare for studies in this area to provide performance statistics, but at the same time at least one of the studies is unable to identify a single case in spite of having almost a million observations.

6. Conclusion

International relations is generally about the interactions and dependencies among a set of countries or other important actors such as international governmental organizations (IGOs). This is particularly true of those scholars who work in the tradition of the Correlates of War Project, but is by no means limited to them.[14] Many scholars have debated the use and abuse of dyadic data.[15] It is clear from a survey of the literature and from work in this area published as recently as 2017 that many find dyadic data to be an important touchstone in the study of international relations (Erikson et al., 2014; Aronow et al., 2015).

At the same time, we know that research designs focusing on the statistical analysis of dyadic data quickly go astray if the dyadic data are assumed to be iid. Virtually all of the standard statistical models—ordinary least squares and logistic regressions, to name a few—fail if the data are not conditionally independent. This fact has been accepted when it comes to temporal dependencies, but adoption of methods to account for network dependencies have seen less progress. By definition dyadic data are not iid and thus the standard approaches can not be used cavalierly to analyze these data. Signorino (1999) showed why this is true of models of strategic interaction, but it is more broadly true of models that employ dyadic data. We show that the AME framework can be employed to account for the statistical issues that arise when studying dyadic data.

To explore this approach in the context of international relations we have presented two broad analyses. The first is a simulation where the characteristics of the network are known. This shows that, when there are unobserved dependencies, the AME approach is less biased in terms of parameter estimation compared with standard approach employed in international relations to study dyadic data (i.e., GLM models). The second is a replication of five prominent studies that have been published recently using a broad range of dyadic data to draw inferences about international relations. These five studies have been replicated with the original research designs, each of which used a statistical method that assumes the dyadic data are all independent from

[14]See Singer (1972) for an early description of the project and also see the project's Web site for an history and more recent efforts http://www.correlatesofwar.org/.

[15]One recent on-line symposium can be found at http://bit.ly/2wB2hab.

one another. We then re-analyzed each study using the AME model. In every case, we found that the AME approach provided a) increased precision of estimation, b) better out-of-sample fits, c) evidence of 1st-, 2nd-, and 3rd-order dependencies that were overlooked in the original studies.[16] In several cases, the new approach overturns the basic findings of the original research. This leads us to speculate that many of the findings in the international relations literature may be fragile in the sense that they can only be obtained under stringent assumptions that cannot possibly be valid. In turn, this leads to a certain arbitrariness in some research findings, which might lead to puzzles that are more apparent than real (Zinnes, 1980).

It is no longer necessary to assume that the interesting, innate interdependencies in relational data about international relations can be ignored. Nor do they have to be approximated with *ad hoc*, incomplete solutions that purport to control for dependencies (such as modifying the post-estimation standard errors of the estimated coefficients (King and Roberts, 2014)). Instead, the interdependencies may be addressed directly with additive and multiplicative effects in the context of a generalized linear model that provides more reliable inferences, better out-of-sample predictive performance, and new substantive insights.

[16] The Appendix contains performance data on all of these replications, as well as sample code illustrating how to undertake AME analysis using `amen`.

Appendix

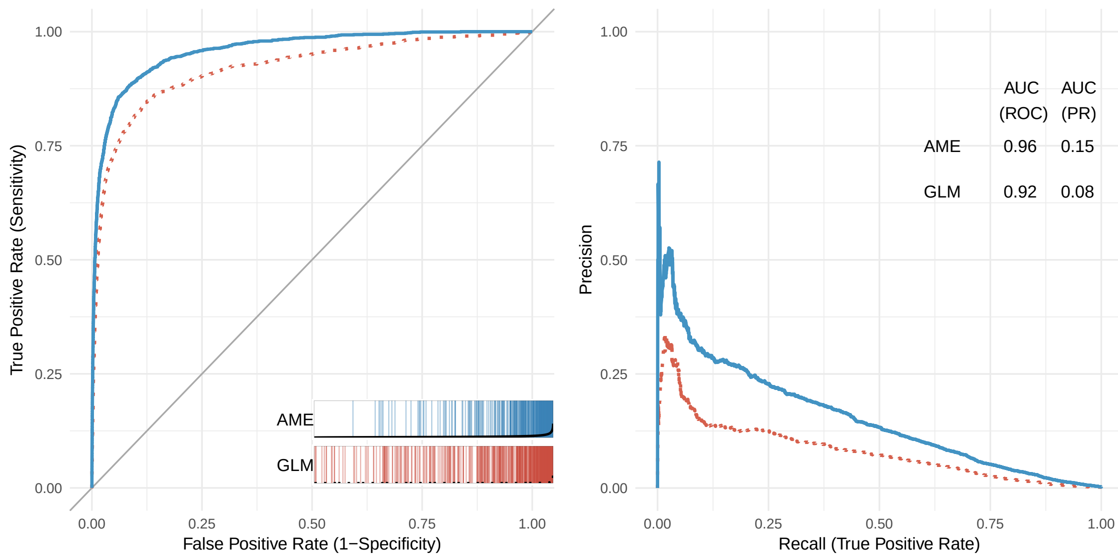
Additional Replication Information. For each of the replications involving a binary dependent variable we provide a table of coefficient estimates that includes the original GLM estimation with a logit link, a GLM estimation with a probit link, and the AME model. The GLM estimation with a probit link function is provided so as to ease comparison between the AME model, which is also based on the probit link. For Rose (2004) we just provide a table of coefficient estimates that includes the original OLS estimation and the AME model.

Additionally, for each replication we provide a more detailed visualization illustrating the results of our out-of-sample performance analysis.

Reiter & Stam (2003). Additional information for the Reiter & Stam (2003) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
Intercept	-4.784^{**}	-2.339^{**}	-3.144^{**}
	(0.097)	(0.034)	(0.06)
Pers/Democ Directed Dyad	1.026^{**}	0.378^{**}	0.255^{**}
	(0.14)	(0.051)	(0.068)
Democ/Pers Directed Dyad	0.083	0.033	0.112
	(0.191)	(0.066)	(0.079)
Personal	0.281	0.15	0.211^{*}
	(0.265)	(0.099)	(0.11)
Military	-0.323	-0.105	-0.025
	(0.574)	(0.204)	(0.249)
Single	-0.677^{**}	-0.261^{**}	-0.07
	(0.144)	(0.062)	(0.073)
Democracy	-1.073^{**}	-0.428^{**}	-0.254^{**}
	(0.194)	(0.07)	(0.063)
Contiguous	2.912^{**}	1.147^{**}	1.296^{**}
	(0.09)	(0.031)	(0.033)
Major Power	2.174^{**}	0.919^{**}	0.906^{**}
	(0.101)	(0.037)	(0.093)
Ally	0.078	-0.003	0.136^{**}
	(0.086)	(0.035)	(0.037)
Higher/Lower Power Ratio	-0.316^{**}	-0.122^{**}	-0.111^{**}
	(0.027)	(0.01)	(0.011)
Economically Advanced	-0.175	-0.054	0.053
	(0.131)	(0.051)	(0.05)
Years Since Last Dispute	-0.381^{**}	-0.149^{**}	-0.129^{**}
	(0.023)	(0.009)	(0.008)
Cubic Spline 1	-0.004^{**}	-0.001^{**}	-0.001^{**}
	(0.000)	(0.000)	(0.000)
Cubic Spline 2	0.002^{**}	0.001^{**}	0.001^{**}
	(0.000)	(0.000)	(0.000)
Cubic Spline 3	-0.001^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)

Table A.1. Parameter comparison for Reiter & Stam (2003). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



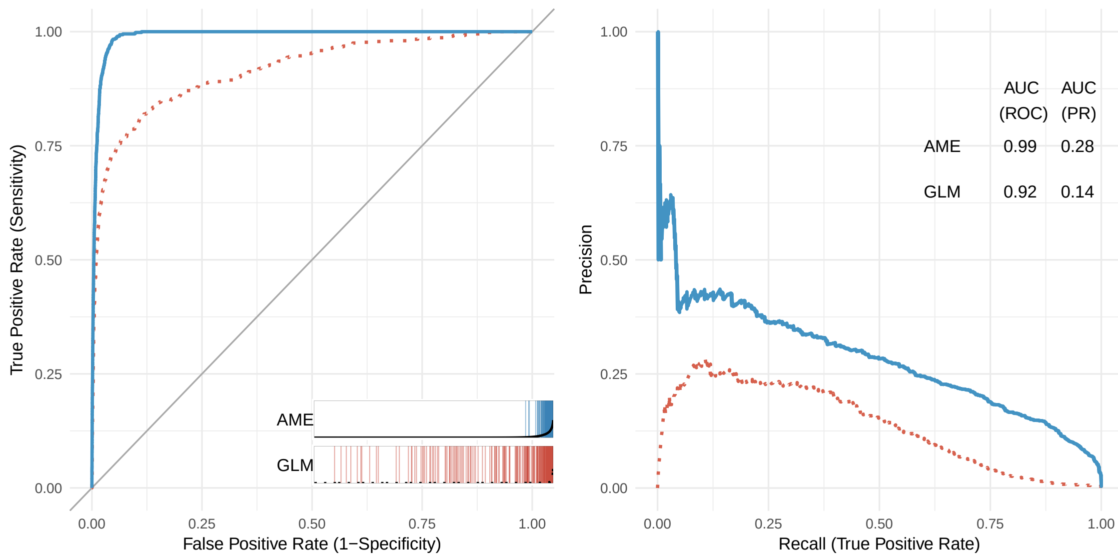
(a) AUC (b) Precision and Recall

Figure A1. Assessments of out-of-sample predictive performance for Reiter & Stam (2003) using ROC curves, PR curves, and separation plots.

McDonald (2004). Additional information for the McDonald (2004) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
(Intercept)	0.054	0.085	-1.171^{**}
	(1.179)	(0.409)	(0.096)
Spline0	-0.438^{**}	-0.222^{**}	-0.145^{**}
	(0.061)	(0.026)	(0.019)
Spline1	-0.003^{**}	-0.002^{**}	-0.001^{**}
	(0.001)	(0.000)	(0.000)
Spline2	0.001	0.001^{**}	0.000^{*}
	(0.001)	(0.000)	(0.000)
Spline3	0.000	0.000	0.000^{**}
	(0.000)	(0.000)	(0.000)
Shared Alliance	0.483^{**}	0.155	0.342^{**}
	(0.233)	(0.095)	(0.069)
Contiguous	2.011^{**}	0.789^{**}	0.988^{**}
	(0.343)	(0.118)	(0.066)
Log Capabilities Ratio	-0.146^{**}	-0.054^{**}	0.029^{**}
	(0.072)	(0.026)	(0.013)
Trade Dependence	-22.244	-7.051	-13.134^{**}
	(15.184)	(5.536)	(4.938)
Preconflict GDP Change	-6.79^{**}	-3.155^{**}	-2.651^{**}
	(2.033)	(0.788)	(0.574)
Lowest Dyadic Polity Score	-0.036^{**}	-0.014^{**}	-0.026^{**}
	(0.015)	(0.006)	(0.002)
Capabilities	-0.995^{**}	-0.349^{**}	0.022
	(0.377)	(0.14)	(0.079)
Logged GDP	0.000^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Logged Cap. Distance	-0.425^{**}	-0.224^{**}	-0.275^{**}
	(0.14)	(0.047)	(0.012)
Major Power In Dyad	0.769^{**}	0.312^{**}	0.212^{**}
	(0.322)	(0.122)	(0.098)
Highest Barrier To Trade	0.024^{**}	0.011^{**}	0.004^{**}
	(0.008)	(0.003)	(0.001)

Table A.2. Parameter comparison for McDonald (2004). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



(a) AUC (b) Precision and Recall

Figure A2. Assessments of out-of-sample predictive performance for McDonald (2004) using ROC curves, PR curves, and separation plots.

Rose (2004). Additional information for the Rose (2004) re-estimation.

Variable	LM	AME
Intercept	-24.96^{**}	-22.532^{**}
	(0.409)	(0.103)
Both in GATT/WTO	-0.042	-0.56^{**}
	(0.053)	(0.013)
One in GATT/WTO	-0.058	-0.317^{**}
	(0.049)	(0.012)
GSP	0.859^{**}	0.399^{**}
	(0.032)	(0.009)
Log Distance	-1.119^{**}	-1.097^{**}
	(0.022)	(0.005)
Log Product Real GDP	0.916^{**}	0.798^{**}
	(0.01)	(0.002)
Log Product Real GDPpc	0.321^{**}	0.244^{**}
	(0.014)	(0.004)
Regional FTA	1.199^{**}	0.826^{**}
	(0.106)	(0.027)
Currency Union	1.118^{**}	1.144^{**}
	(0.122)	(0.029)
Common language	0.313^{**}	0.345^{**}
	(0.04)	(0.009)
Land Border	0.526^{**}	0.483^{**}
	(0.111)	(0.02)
Number Landlocked	-0.271^{**}	-0.42^{**}
	(0.031)	(0.009)
Number Islands	0.042	0.058^{**}
	(0.036)	(0.009)
Log Product Land Area	-0.097^{**}	-0.024^{**}
	(0.008)	(0.002)
Common Colonizer	0.585^{**}	0.418^{**}
	(0.067)	(0.013)
Currently Colonized	1.075^{**}	1.762^{**}
	(0.235)	(0.081)
Ever Colony	1.164^{**}	1.335^{**}
	(0.117)	(0.024)
Common Country	-0.016	-0.672^{**}
	(1.097)	(0.19)

Table A.3. Parameter comparison for Rose (2004). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.

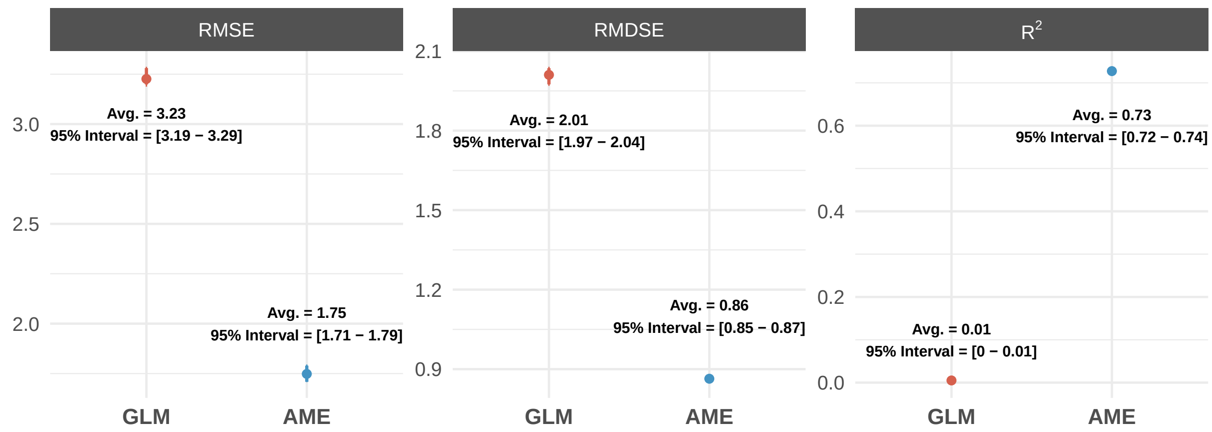


Figure A3. Assessments of out-of-sample predictive performance for Rose (2004) using root mean squared error (RMSE), root median squared error (RMDSE), and R^2.

Weeks (2012). Additional information for the Weeks (2012) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
(Intercept)	−3.784**	−1.797**	−2.409**
	(0.423)	(0.159)	(0.132)
Machine	−0.459**	−0.162**	-0.006
	(0.174)	(0.062)	(0.04)
Junta	0.515**	0.194**	0.034
	(0.169)	(0.062)	(0.046)
Boss	0.649**	0.281**	-0.044
	(0.153)	(0.05)	(0.044)
Strongman	0.832**	0.295**	0.032
	(0.132)	(0.048)	(0.044)
Other Type	0.147	0.051	-0.01
	(0.132)	(0.046)	(0.034)
New/Unstable Regime	−0.312**	−0.123**	-0.043
	(0.092)	(0.033)	(0.031)
Democracy Target	0.185	0.052	0.024
	(0.115)	(0.04)	(0.026)
Military Capabilities Initiator	5.234**	2.136**	0.071
	(1.69)	(0.554)	(0.412)
Military Capabilities Target	6.34**	2.865**	−0.969**
	(1.675)	(0.573)	(0.48)
Low Trade Dependence	−24.794*	-8.197	-4.733
	(12.866)	(5.582)	(3.017)
Both Major Powers	1.136**	0.687**	1.122**
	(0.547)	(0.183)	(0.241)
Minor/Major	0.772**	0.292**	0.496**
	(0.239)	(0.086)	(0.118)
Major/Minor	0.711**	0.332**	0.778**
	(0.225)	(0.075)	(0.16)
Contiguous	2.172**	0.738**	0.705**
	(0.32)	(0.125)	(0.06)
Log Dist. Between Capitals	−0.209**	−0.095**	−0.129**
	(0.038)	(0.015)	(0.01)
Alliance Similarity Dyad	−0.999**	−0.386**	-0.073
	(0.144)	(0.05)	(0.065)
Alliance Similarity With System Leader Initiator	0.11	0.011	0.068
	(0.24)	(0.082)	(0.057)
Alliance Similarity Leader Target	0.203	0.032	0.08
	(0.244)	(0.081)	(0.056)
Time Since Last Conflict	−0.229**	−0.089**	−0.067**
	(0.018)	(0.007)	(0.007)
Spline1	−0.001**	0.000**	0.000**
	(0.000)	(0.000)	(0.000)
Spline2	0.000**	0.000**	0.000**
	(0.000)	(0.000)	(0.000)
Spline3	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)

Table A.4. Parameter comparison for Weeks (2012). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.

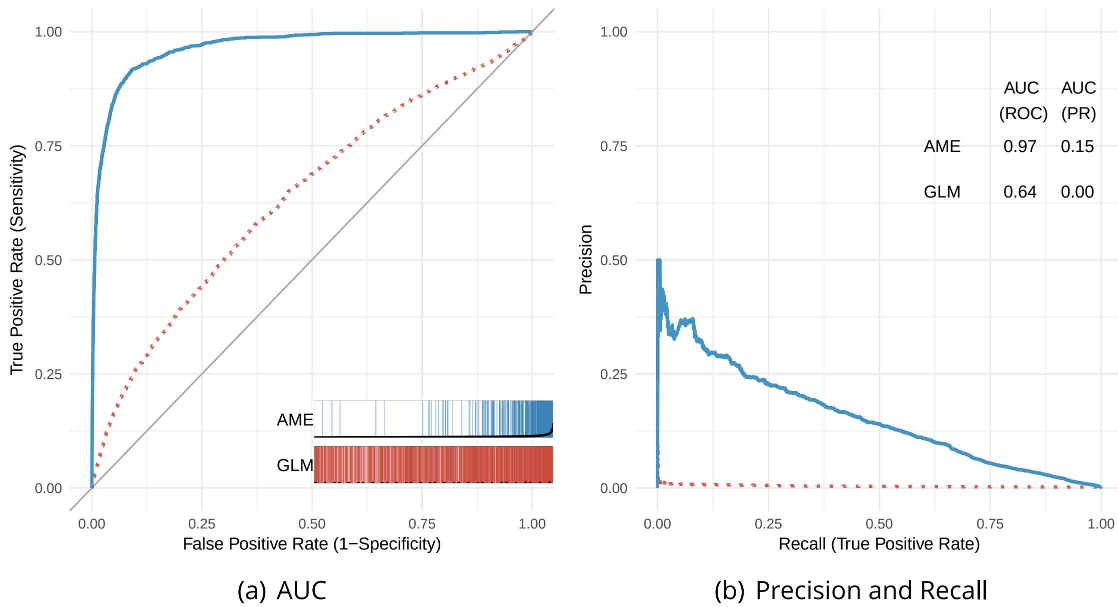
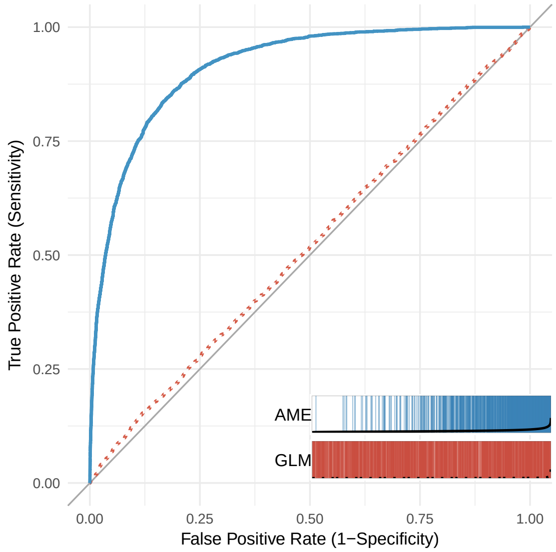


Figure A4. Assessments of out-of-sample predictive performance for Weeks (2012) using ROC curves and PR curves

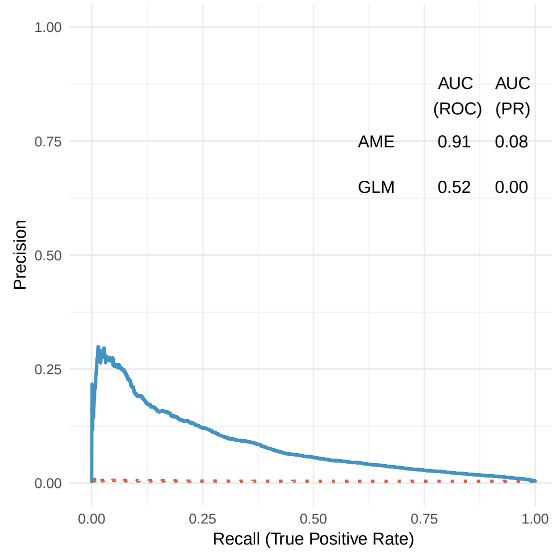
Gibler (2017). Additional information for the Gibler (2017) re-estimation.

Variable	GLM (Logit)	GLM (Probit)	AME
(Intercept)	-5.826^{**}	-2.793^{**}	-2.758^{**}
	(0.366)	(0.366)	(0.045)
Allied	0.133	0.067	0.078^{**}
	(0.102)	(0.102)	(0.021)
Joint Democracy	-0.527^{**}	-0.186^{*}	0.005
	(0.099)	(0.099)	(0.022)
Peace Years	-0.261^{**}	-0.099^{**}	-0.058^{**}
	(0.016)	(0.016)	(0.004)
Spline 1	-0.001^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Spline 2	0.000^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)
Spline 3	0.000	0.000	0.000^{**}
	(0.000)	(0.000)	(0.000)
Contiguity	2.427^{**}	0.95^{**}	0.66^{**}
	(0.196)	(0.196)	(0.023)
Parity	-0.77	-0.228	-0.067
	(0.551)	(0.551)	(0.057)
Parity at Entry Year	2.034^{**}	0.739	-0.05
	(0.617)	(0.617)	(0.065)
Rivalry	2.034^{**}	1.035^{**}	0.655^{**}
	(0.213)	(0.213)	(0.028)

Table A.5. Parameter comparison for Gibler (2017). Standard errors in parentheses. ** and * indicate significance at $p < 0.05$ and $p < 0.10$, respectively.



(a) AUC



(b) Precision and Recall

Figure A5. Assessments of out-of-sample predictive performance for Gibler (2017) using ROC curves, PR curves, and separation plots.

AME Tutorial. Using the AMEN function requires formatting data into a particular structure. The primary distinction in data formatting is whether the outcome of interest represents a directed or undirected network.

If undirected, the AMEN function has three main inputs:

- Y: a T length **list** of $n \times n$ adjacency matrices, where T = number of years in the dataset and n = number of nodes in the network.
- Xdyad: a T length **list** of $n \times n \times p$ arrays, where p = number of dyadic covariates in dataset.
- Xrow: a T length **list** of $n \times p$ matrices, where p = number of monadic (nodal) covariates in dataset.

If directed, AMEN further requires:

- Xrow: a T length list of $n \times p$ matrices, where p = number of sender (nodal) covariates in dataset.
- Xcol: a T length list of $n \times p$ matrices, where p = number of receiver (nodal) covariates in dataset.

Beyond the data inputs, the AMEN function requires additional specification:

- model: how to model the outcome variable, e.g., 'logit'
- symmetric: whether the input network is symmetric
- intercept: whether to estimate an intercept
- nscan: number of iterations of the Markov chain
- burn: burn-in period
- odens: thinning interval
- R: dimension of the multiplicative effect (referred to as K in the paper)
- gof: whether to calculate goodness of fit statistics

There is often little theoretical reason to choose a particular value of R (above). One strategy is to estimate models at different values of R and compare goodness of fit statistics across models.

Given the computational intensity needed for parameter estimates to converge, parallelization strategies are recommended to speed up analysis. In addition, providing AMEN function with starting values, either dictated by theory, previous research, or previous runs can also help speed up convergence time.

The code below presents an example of an AME model running in parallel across 4 different levels of R. Note also that the model is using starting values from a previous run, defined in *startVals0*.

```
# running in parallel varying k
imps = 10000 ; brn = 25000 ; ods = 10 ; latDims = 0:3

# Run amen in parallel
library(doParallel) ; library(foreach) ; cl=makeCluster(4) ; registerDoParallel(cl)
foreach(ii=1:length(latDims), .packages=c("amen")) %dopar% {

  # load previous model run
  load(prevModelFiles[ii])
  # extract start vals
  startVals0 = ameFit$'startVals'
  # dump rest
  rm(ameFit)

  ameFit = ame_repL(
    Y=yList,Xdyad=xDyadList,Xrow=NULL,Xcol=NULL,
    model="bin",symmetric=FALSE,intercept=TRUE,R=latDims[ii],
    nscan=imps, seed=1, burn=brn, odens=ods,
    plot=FALSE, print=FALSE, gof=TRUE, startVals=startVals0,
    periodicSave=TRUE )
  save(ameFit, file=paste0('model_k', latDims[ii],'_v2.rda') )
}
stopCluster(cl)
```

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