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1. (a) $f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(z-\mu)^2}$

(b) $P(-1 \le Z \le 1) = st.norm.cdf(1, 0, 1) - st.norm.cdf(-1, 0, 1)$
$= 0.6827$

(c) $x = st.norm.ppf(0.975, 0, 1) = 1.96$

(d) $f_Q(q) = \frac{1}{\sqrt{2\pi q}} \cdot e^{-\frac{q}{2}}$

$$f_Q(q) = \begin{cases} \frac{1}{\sqrt{2\pi q}} e^{-\frac{q}{2}} & , q > 0 \\ 0 & , q \le 0 \end{cases}$$

(e) $E[Q] = 1$

(f) $std[Q] = \sqrt{2}$

(g) $P(Q \le 1) = st.gamma.cdf(x=1, \alpha=0.5, scale=2) = 0.6827$

二. (a) $f_T(t) = \begin{cases} e^{-t} & , t > 0 \quad (\alpha=1, \beta=1) \\ 0 & , t \le 0 \end{cases}$

(b) $E[T] = \beta = 1$

(c) $std[T] = \sqrt{\beta^2} = \beta = 1$

(d) $P(T > 1) = st.gamma.sf(x=1, \alpha=1, scale=1) = 0.3679$

(e) $\alpha = 3, \beta = 1$

$f_{T_3}(t) = \frac{1}{1^3 \Gamma(3)} t^{3-1} e^{-t/1} = \frac{1}{2} t^2 e^{-t}$

$$f_{T_3}(t) = \begin{cases} \frac{1}{2} t^2 e^{-t} & , t > 0 \\ 0 & , t \ge 0 \end{cases}$$

(f) $E[T_3] = \alpha\beta = 3 \cdot 1 = 3$

(g) $std[T_3] = \sqrt{\alpha\beta^2} = \sqrt{3}$

(h) $P(T_3 > 3) = st.gamma.sf(x=3, a=3, scale=1) = 0.4232$

(i) $P(T_3 > 7) = st.gamma.sf(x=7, a=3, scale=1) = 0.0296$

假設該產品平均壽命為1年，即$\beta=1$在此條件算出使用3個產品超過7年以上的機率相當小，因此可接受此假設