Global Value Numbering with SSA
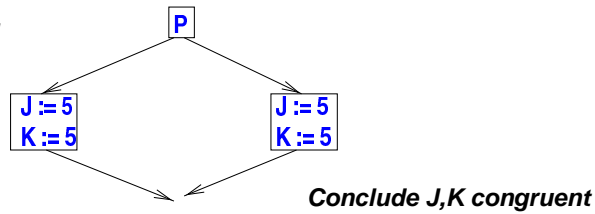
X, Y are **dynamically equivalent** at P if they have the same values whenever control reaches P on execution

Undecidable ⟶ Develop static notion   **Congruence**

X congruent to Y ⟶ **X dynamically equivant to Y**

Go beyond Basic Block

Ex.

```
        ┌───┐
        │ P │
        └───┘
       ╱      ╲
┌──────────┐   ┌──────────┐
│ J := 5   │   │ J := 5   │
│ K := 5   │   │ K := 5   │
└──────────┘   └──────────┘
       ╲      ╱
        ╲    ╱
```
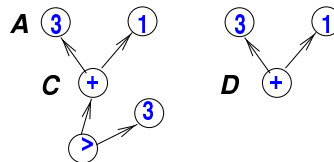   **Conclude J,K congruent**

Value Graph for Basic Block

```
A := 3
B := 3
C := A+1       ⟶
D := B+1
if (C>3)...
```
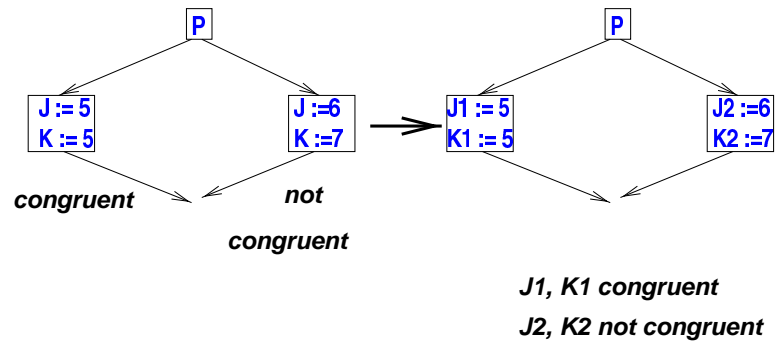

C and D are **congruent :**

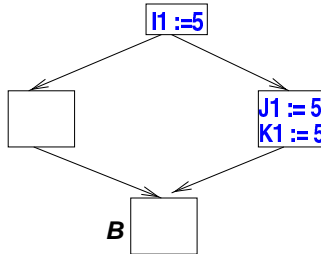   **have identical operators, and like operands are congruent**

Like value-numbering

Why SSA?

```
                P                              P
           ╱         ╲                    ╱         ╲
     J := 5           J :=6         J1 := 5          J2 :=6
     K := 5           K :=7    ⟶    K1 := 5          K2 :=7
           ╲         ╱                    ╲         ╱
  *congruent*      *not*
              *congruent*
```

J1, K1 congruent

J2, K2 not congruent

What about control flow?

```
                 I1 :=5
            ╱            ╲
      ┌────┐          J1 := 5
      │    │          K1 := 5
      └────┘            ╱
            ╲          ╱
          B  ┌────┐
             │    │
             └────┘
```

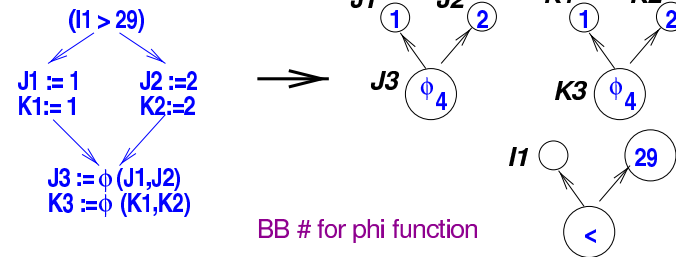I1, J1, K1 congruent at B if assignments dominate B

Value Graph for SSA

Nodes   *Constants, operators, phi-functions*

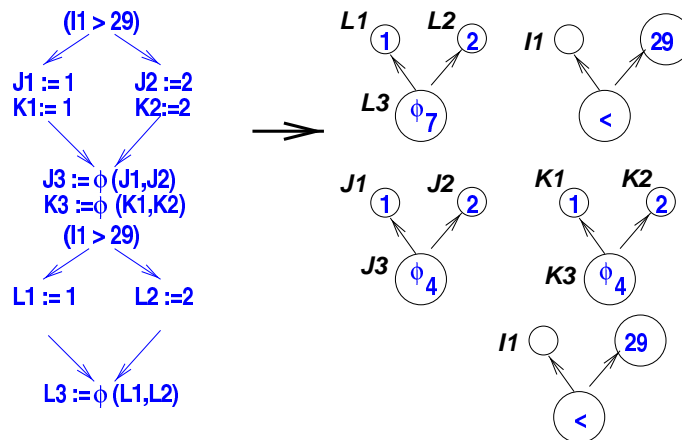**Directed Edges**   *From use to node where value generated*

Labels   *Constant, operator, function symbols*

Ex.

```
        (I1 > 29)

J1 := 1        J2 :=2
K1:= 1         K2:=2



     J3 := φ (J1,J2)
     K3 :=φ (K1,K2)
```

J1 ①   J2 ②   K1 ①   K2 ②

J3 φ4   K3 φ4

I1 ◯   29

BB # for phi function   <

5

Value Graph for SSA (example)

```
        (I1 > 29)

J1 := 1        J2 :=2
K1:= 1         K2:=2



     J3 := φ (J1,J2)
     K3 :=φ (K1,K2)
        (I1 > 29)

L1 := 1        L2 :=2



     L3 := φ (L1,L2)
```

L1 ①   L2 ②   I1 ◯   29

L3 φ7   <

J1 ①   J2 ②   K1 ①   K2 ②

J3 φ4   K3 φ4

I1 ◯   29

<

6

Congruence

A is *congruent* to node B if

1. A is the same node as B, or
2. A and B are constant nodes, with the same constant value, or
3. A and B are operator nodes, with the same operator, and

     their like operands are congruent

Vars X and Y are *equivalent* at P if their nodes are congruent and

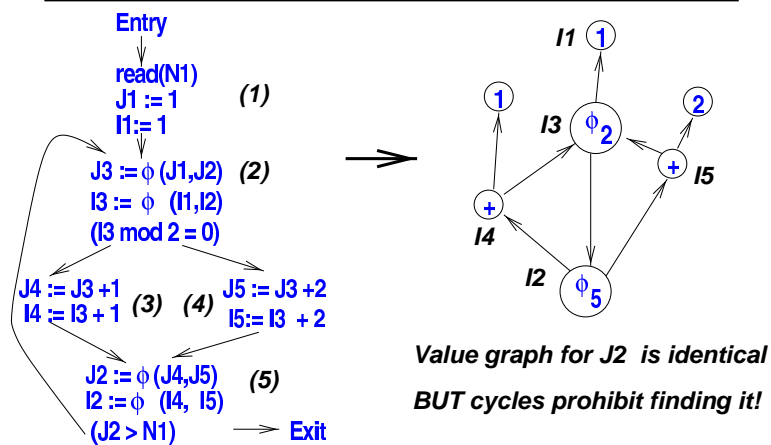defining assignments dominate P.

Ex.
  J1, K1, L1 equivalent
  J2, K2, L2 equivalent
  J3, K3 equivalent, but not with L3

Get equivalence classes of variables

Loop Example

Entry

read(N1)
J1 := 1   (1)
I1:= 1

J3 := φ (J1,J2) (2)
I3 := φ (I1,I2)
(I3 mod 2 = 0)

J4 := J3 +1 (3)   (4) J5 := J3 +2
I4 := I3 + 1   I5:= I3 + 2

J2 := φ (J4,J5)   (5)
I2 := φ (I4, I5)
(J2 > N1) ⟶ Exit

Value graph for J2 is identical

BUT cycles prohibit finding it!

Algorithm Overview

1. Compute SSA.

2. Build value graph for SSA.

3. Optimistically assume all nodes with same label are congruent.
   Determine congruence of nodes by partitioning algorithm.

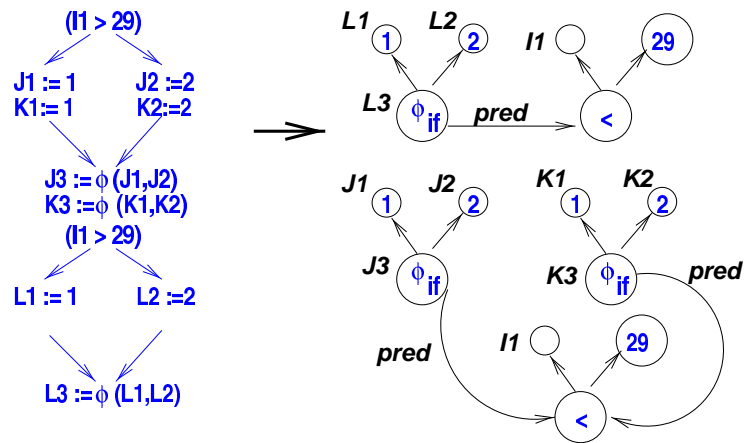3. Check for equivalence.

Partitioning: *(O(E log E))*

1. Put all nodes with same label in same partition.

**i+1: Two nodes are in same partition at step i+1, if at step i,
they are in the same partition and the destination of their
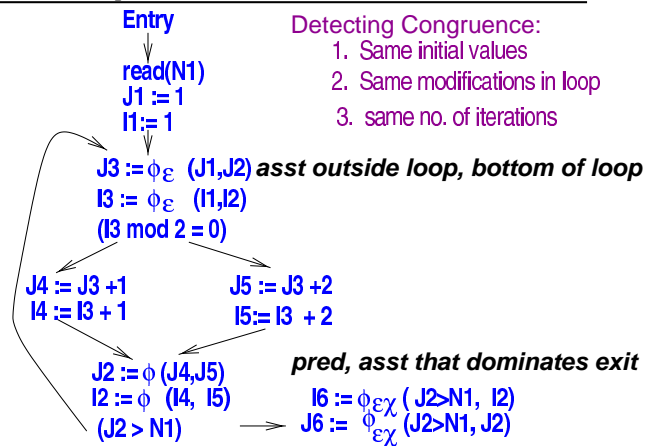edges are in the same partition.**

Taking Control Flow into account

Loop Example

read(N1)
J1 := 1
I1 := 1

Detecting Congruence:
1. Same initial values
2. Same modifications in loop
3. same no. of iterations

J3 := ϕ_ε (J1,J2) *asst outside loop, bottom of loop*
I3 := ϕ_ε (I1,I2)
(I3 mod 2 = 0)

J4 := J3 +1   J5 := J3 +2
I4 := I3 + 1   I5 := I3 + 2

J2 := ϕ (J4,J5)   *pred, asst that dominates exit*
I2 := ϕ (I4, I5)
(J2 > N1)   I6 := $\phi_{\varepsilon\chi}$ (J2>N1, I2)
          J6 := $\phi_{\varepsilon\chi}$ (J2>N1, J2)

11

Other Extensions

Incorporate arrays, pointers

Update, Access functions

Take commutativity into account

Ex. a*b same as b*a

Combine with hash-based approach

(Cooper et. al.)

12