Appendix A

Computation of localized sets

For each basic block there are 3 bit vectors dedicated to the block-specific properties, namely `Transp`, `Antloc` and `Xcomp`. As mentioned before, a bit vector is a boolean array of value numbers. Each value number is associated with at-least two expressions from the IR. Let the leader expression (as defined in the section on value numbering) associated with the value number v be called $L(v)$.

$$
\begin{array}{lcl}
\texttt{Transp(v,B)} & = & \begin{cases} \mathit{false} & \text{iff } \exists \text{ x} \in \text{operands of L(v) such that } \texttt{Mod(x,B)} = \text{true} \\ \mathit{true} & \text{Otherwise} \end{cases} \\
\texttt{Antloc(v,B)} & = & \texttt{Eval(v,B)} \cap \texttt{Transp(v,B)} \\
\texttt{Xcomp(v,B)} & = & \texttt{Eval(v,B)} \cap \overline{\texttt{Transp(v,B)}} \\
 & & \\
\text{where} & & \\
\quad \texttt{Eval(v,B)} & = & \{\text{v} \mid \text{value number v is computed in B}\} \\
\quad \texttt{Mod(op,B)} & = & \text{operand op modified in B}
\end{array}
\tag{A.1}
$$

Appendix B

Lazy Code motion Transformations

- Down Safety Analysis (Backward data flow analysis)

$$\begin{aligned}
\texttt{Antin(b)} &= \texttt{Antloc(b)} \cup (\texttt{Tranp(b)} \cap \texttt{Antout(b)}) \\
\texttt{Antout(b)} &= \texttt{Xcomp(b)} \cup \begin{cases} \phi & \text{if b = exit} \\ \bigcap\limits_{s \in succ(b)} \texttt{Antin(s)} & \end{cases}
\end{aligned} \tag{B.1}$$

- Up Safety Analysis (Forward data flow analysis)

$$\begin{aligned}
\texttt{Availin(b)} &= \begin{cases} \phi & \text{if b = entry} \\ \bigcap\limits_{p \in pred(b)} (\texttt{Xcomp(p)} \cup \texttt{Availout(p)}) & \end{cases} \\
\texttt{Availout(b)} &= \texttt{Tranp(b)} \cap (\texttt{Antloc(b)} \cup \texttt{Availin(b)})
\end{aligned} \tag{B.2}$$

- Earliest-ness (No data flow analysis)

$$\begin{aligned}
\texttt{Earliestin(b)} &= \texttt{Antin(b)} \cap \bigcap_{p \in pred(b)} (\overline{\texttt{Availout(p)} \cup \texttt{Antout(p)}}) \\
\texttt{Earliestout(b)} &= \texttt{Antout(b)} \cap \overline{\texttt{Tranp(b)}}
\end{aligned} \tag{B.3}$$

- Delayability (Forward data flow analysis)

$$\begin{aligned}
\texttt{Delayin(b)} &= \texttt{Earliestin(b)} \cup \begin{cases} \phi & \text{if b = entry} \\ \bigcap\limits_{p \in pred(b)} (\overline{\texttt{Xcomp(p)}} \cap \texttt{Delayout(p)}) & \end{cases} \\
\texttt{Delayout(b)} &= \texttt{Earliestout(b)} \cup (\texttt{Delayin(b)} \cap \overline{\texttt{Antloc(b)}})
\end{aligned} \tag{B.4}$$

- Latest-ness (No data flow analysis)

$$\begin{aligned}
\texttt{Latestin(b)} &= \texttt{Delayin(b)} \cap \texttt{Antloc(b)} \\
\texttt{Latestout(b)} &= \texttt{Delayout(b)} \cap (\texttt{Xcomp(b)} \cup \bigcup_{s \in succ(b)} \overline{\texttt{Delayin(s)}})
\end{aligned} \tag{B.5}$$

- Isolation Analysis (Backward data flow analysis)

$$\begin{aligned}
\texttt{Isolatedin(b)} &= \texttt{Earliestout(b)} \cup \texttt{Isolatedout(b)} \\
\texttt{Isolatedout(b)} &= \begin{cases} U & \text{if b = exit} \\ \displaystyle\bigcap_{s \in succ(b)} (\texttt{Earliestin(s)} \cup (\overline{\texttt{Antloc(s)}} \cap \texttt{Isolatedin(s)})) \end{cases}
\end{aligned} \tag{B.6}$$

- Insert and Replace points

$$\begin{aligned}
\texttt{Insertin(b)} &= \texttt{Latestin(b)} \cap \overline{\texttt{Isolatedin(b)}} \\
\texttt{Insertout(b)} &= \texttt{Latestout(b)} \cap \overline{\texttt{Isolatedout(b)}} \\
\texttt{Replacein(b)} &= \texttt{Antloc(b)} \cap \overline{\texttt{Latestin(b)} \cap \texttt{Isolatedin(b)}} \\
\texttt{Replaceout(b)} &= \texttt{Xcomp(b)} \cap \overline{\texttt{Latestout(b)} \cap \texttt{Isolatedout(b)}}
\end{aligned} \tag{B.7}$$

Appendix C

Generalized data flow framework

All the equations in Appendix B can be computed using the generic framework defined below.

C.1 Forward Analysis

$$\begin{aligned} \texttt{In(b)} &= \alpha(b) \cup \begin{cases} \perp & \text{if b = entry} \\ \displaystyle\bigwedge_{p \in pred(b)} \beta(p) & \end{cases} \\ \texttt{Out(b)} &= \gamma(b) \end{aligned} \tag{C.1}$$

C.2 Backward Analysis

$$\begin{aligned} \texttt{In(b)} &= \gamma(b) \\ \texttt{Out(b)} &= \alpha(b) \cup \begin{cases} \perp & \text{if b = exit} \\ \displaystyle\bigwedge_{s \in succ(b)} \beta(s) & \end{cases} \end{aligned} \tag{C.2}$$

The following is the function which we call with dataflow equation specific parameters defined subsequently.

$$\texttt{callFramework}(\texttt{Out(b)}, \texttt{In(b)}, \alpha(b), \beta(b), \gamma(b), \bigwedge, \perp, \top, \text{Direction})$$

Following is the list of values that we need to plug-in to α, β and γ for the above generic framework to work.

- Down Safety Analysis (Backward data flow analysis)

$$\begin{aligned} \alpha(x) &= \texttt{Xcomp(x)} \\ \beta(x) &= \texttt{Antin(x)} \\ \gamma(x) &= \texttt{Tranp(x)} \cap \texttt{Antout(x)} \cup \texttt{Antloc(x)} \\ \bigwedge &= \cap \\ \perp &= \phi \\ \top &= V, \text{set of all values} \\ \text{Direction} &= \text{Backward} \end{aligned} \tag{C.3}$$

- Up Safety Analysis (Forward data flow analysis)

$$\begin{array}{lll} \beta(x) & = & \texttt{Xcomp(x)} \cup \texttt{Availout(x)} \\ \gamma(x) & = & \texttt{Antloc(x)} \cup \texttt{Availin(x)} \cap \texttt{Tranp(x)} \\ \bigwedge & = & \cap \\ \bot & = & \phi \\ \top & = & V, \text{set of all values} \\ \text{Direction} & = & \text{Forward} \end{array} \tag{C.4}$$

- Delayability (Forward data flow analysis)

$$\begin{array}{lll} \alpha(x) & = & \texttt{Earliestin(x)} \\ \beta(x) & = & \overline{\texttt{Xcomp(x)}} \cap \texttt{Delayout(x)} \\ \gamma(x) & = & \texttt{Delayin(x)} \cap \overline{\texttt{Antloc(x)}} \cup \texttt{Earliestout(x)} \\ \bigwedge & = & \cap \\ \bot & = & \phi \\ \top & = & V, \text{set of all values} \\ \text{Direction} & = & \text{Forward} \end{array} \tag{C.5}$$

- Isolation Analysis (Backward data flow analysis)

$$\begin{array}{lll} \beta(x) & = & \overline{\texttt{Antloc(x)}} \cap \texttt{Isolatedin(x)} \cup \texttt{Earliestin(x)} \\ \gamma(x) & = & \texttt{Earliestout(x)} \cup \texttt{Isolatedout(x)} \\ \bigwedge & = & \cap \\ \bot & = & V, \text{set of all values} \\ \top & = & V, \text{set of all values} \\ \text{Direction} & = & \text{Backward} \end{array} \tag{C.6}$$