物理…原理 #24　20^{03}_{11} *

§7.3.

Lem 7.1

V : 3-dim Euc. vec. sp.

V の向き.

$*$.

$T \in \Lambda^3 V$

$u \in V$

$$*(\underset{\Lambda^3 V}{\underset{\cap}{T}}\ \underset{V}{\underset{\cap}{u}}) = \underbrace{\underset{\mathbb{R}}{\underset{\cap}{T_0}}\ \underset{V}{\underset{\cap}{u}}}_{\underset{V}{\cap}}$$

$\Lambda^3 V$, V 演算積 $\ni$ $\underbrace{*\ \Lambda^2 V}$ $\ni$ $\Lambda^1 V = V$

★ $$\underset{\Lambda^3 V}{\underset{\cap}{\hat{\varepsilon}_0}} = \underset{\mathbb{R}}{\underset{\cap}{\varepsilon_0}}\ \underset{\Lambda^3 V}{\underset{\cap}{\underline{e_1 \wedge e_2 \wedge e_3}}}$$

$$\underset{\mathbb{R}}{\underset{\cap}{c^2}}\ \underset{\Lambda^3 V}{\underset{\cap}{\hat{\varepsilon}_0}}\ \underset{\mathbb{I}\times D \to V}{\underbrace{\delta \mathbb{B}}} = \mathbb{J} + \underset{\Lambda^3 V.}{\underset{\cap}{\hat{\varepsilon}_0}} \underset{:\ \mathbb{I}\times D \to V.}{\underbrace{\frac{\partial \mathbb{E}}{\partial t}}} \qquad (M.4).$$

$$\delta : A^p(D) \longrightarrow A^{p-1}(D)$$

$$\psi_p \longmapsto \delta(\psi_p) := \begin{cases} (-1)^{np+n+1} * d * \psi_p & (p \geq 1 \text{ のとき}) \\ 0 & (\text{otherwise}) \end{cases}$$

$V \supset$ open

$\psi_p : D \longrightarrow \Lambda^p V^*$

o.k. ∵ V : Euc.

?

$\mathbb{B} : \mathbb{I} \times D \longrightarrow \Lambda^2 V$

$\delta\mathbb{B} : \mathbb{I} \times D \longrightarrow V = \Lambda^1 V$

$\mathbb{J} : \underbrace{\mathbb{I} \times D \longrightarrow \Lambda^2 V}$

$\frac{\partial \mathbb{E}}{\partial t} : \mathbb{I} \times D \longrightarrow V.$

(M.4) の右辺.

$$(\mathbb{I}\times D \to \Lambda^2 V) + (\Lambda^3 V)\cdot(\mathbb{I}\times D \to V).$$

given $(t, x) \in \mathbb{I} \times D$

$$\Lambda^2 V + \underbrace{(\Lambda^3 V)\ V}_{\Lambda^2 V.}$$

$$c^2 \hat{\varepsilon}_0\, \delta\mathbb{B} = \mathbb{J} + \hat{\varepsilon}_0 \frac{\partial \mathbb{E}}{\partial t} \qquad (M.4).$$

両辺に $*$ を作用.

$*: \Lambda^p V \longrightarrow \Lambda^{n-p} V$: linear.

$$c^2 (*\hat{\varepsilon}_0)(\delta\mathbb{B}) = *\mathbb{J} + (*\hat{\varepsilon}_0)\frac{\partial \mathbb{E}}{\partial t}.$$

$$c^2\ \varepsilon_0\ \delta\mathbb{B} = *\mathbb{J} + \varepsilon_0 \frac{\partial \mathbb{E}}{\partial t} \qquad (7.9).$$

$$\underset{\Lambda^3 V.}{\underset{\cap}{\hat{\varepsilon}_0}}\ \underset{:\ \mathbb{I}\times D \to \mathbb{R}}{\underbrace{\delta \mathbb{E}}} = -\underset{:\ \mathbb{I}\times D \to \Lambda^3 V}{\underbrace{\hat{\rho}}} \qquad (M.1).$$

両辺に $*$ を作用.

$$\varepsilon_0\, \delta\mathbb{E} = -\rho \qquad (7.10).$$

$$d\mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \qquad (M.2)$$

両辺に δ を作用,

$$\delta d \mathbb{E} = -\delta \frac{\partial \mathbb{B}}{\partial t}$$

(7.9). より,

$$\delta\mathbb{B} = \frac{1}{c^2 \varepsilon_0} * \mathbb{J} + \frac{1}{c^2}\frac{\partial \mathbb{E}}{\partial t}$$

両辺 $\partial/\partial t$.

$$\delta\frac{\partial \mathbb{B}}{\partial t} = \frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t} * \mathbb{J} + \frac{1}{c^2}\frac{\partial^2 \mathbb{E}}{\partial t^2}$$

$$\therefore\ \delta d\mathbb{E} = -\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t} * \mathbb{J} - \frac{1}{c^2}\frac{\partial^2 \mathbb{E}}{\partial t^2}$$

Δ_{LB} の定義.

$$\delta d + d\delta = \Delta_{LB}$$

$$\therefore\ \delta d\mathbb{E} + d\delta\mathbb{E} = \Delta_{LB}\mathbb{E}$$

(7.10) より.

$$d\delta\mathbb{E} = -\frac{1}{\varepsilon_0} d\rho$$

$$\therefore\ \frac{1}{c^2}\frac{\partial^2 \mathbb{E}}{\partial t^2} + \Delta_{LB}\mathbb{E}$$

$$= \left(\cancel{-\delta d\mathbb{E}} - \frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t} * \mathbb{J}\right) + \left(\cancel{\delta d\mathbb{E}} - \frac{1}{\varepsilon_0} d\rho\right). \qquad (7.13).$$

$$\therefore\ \underbrace{\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta_{LB}\right)}_{\overset{\cdot\cdot}{=} \square}\mathbb{E} = -\frac{1}{c^2\varepsilon_0}\frac{\partial}{\partial t} * \mathbb{J} - \frac{1}{\varepsilon_0} d\rho$$

$$c^2\ \varepsilon_0\, d\delta\mathbb{B} = d * \mathbb{J} + \varepsilon_0 \boxed{d\frac{\partial \mathbb{E}}{\partial t}} \qquad (7.9).$$

$$d\mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \qquad (M.2)$$

$$\therefore\ \boxed{d\frac{\partial \mathbb{E}}{\partial t}} = -\frac{\partial^2 \mathbb{B}}{\partial t^2}$$

より.

$$c^2 \varepsilon_0 \boxed{d\delta\mathbb{B}} = d * \mathbb{J} - \varepsilon_0 \frac{\partial^2 \mathbb{B}}{\partial t^2}$$

Δ_{LB} の定義より.

$$\underbrace{\delta d\mathbb{B}}_{0.} + \boxed{d\delta\mathbb{B}} = \Delta_{LB}\mathbb{B}.$$

$$\therefore\ \frac{1}{c^2}\frac{\partial^2 \mathbb{B}}{\partial t^2}$$

$$= \frac{1}{c^2\varepsilon_0} d * \mathbb{J} - \underbrace{d\delta\mathbb{B}}_{\Delta_{LB}\mathbb{B}.}$$

$$\therefore\ \underbrace{\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta_{LB}\right)}_{\overset{\cdot\cdot}{=} \square}\mathbb{B} = \frac{1}{c^2\varepsilon_0} d * \mathbb{J}$$

§6.4.1. 回転 rot

V : 3-dim Euc. vec. sp.

$D \subset V$: open

V の向き.

$*$.

$$\mathrm{rot} : A^1(D) \longrightarrow A^1(D)$$

$$\psi \longmapsto \mathrm{rot}\,\psi := *(d\psi).$$

: ψ の回転

§6.4.2. 発散 div

$$\mathrm{div} : A^1(D) \longrightarrow A^0(D) := C^\infty_{\mathbb{R}}(D) = \{ f : D \longrightarrow \mathbb{R} \mid f : C^\infty \}$$

$$\psi \longmapsto \mathrm{div}\,\psi := *(d(*\psi)).$$

: ψ の発散.

★ $$*(f^1 \wedge f^2 \wedge f^3) \overset{?}{=} 1$$