物理の原理 #28　$20^{04}08^{水}$

$\psi \in A^p(D)$

$$\Delta_{LB}\psi = -\sum_{i_1<\cdots<i_p}\left(\sum_{j=1}^{n}\frac{\partial^2\psi_{i_1\cdots i_p}}{(\partial x^j)^2}\varepsilon(dx^j)\,\varepsilon(\omega_n)\right)dx^{i_1}\wedge\cdots\wedge dx^{i_p} \quad (6.31).$$

ex.
$\dim D = 3,\ p = 2.$
$(i_1, i_2) = (1,2), (2,3), (3,1),$

p. 280　(ii).

V : $(n+1)$ dim Mink. vec. sp.
$(dx^i)_{i=0}^{n}$: V^* の O.N.B.

$$\langle dx^i, dx^i\rangle = \begin{cases} 1 & (i=0) \\ -1 & (\text{otherwise}) \end{cases}.$$

$$\omega_{n+1} = dx^0\wedge\cdots\wedge dx^n$$

(6.31) より, $\psi \in A^p(D)$.

$$\Delta_{LB}\psi = -\sum_{i_1<\cdots<i_p}\left(\frac{\partial^2\boxed{\psi_{i_1\cdots i_p}}}{(\partial x^0)^2}\cdot 1\cdot(-1)^n + \sum_{j=1}^{n}\frac{\partial^2\boxed{\psi_{i_1\cdots i_p}}}{(\partial x^j)^2}\cdot(-1)\cdot(-1)^n\right)dx^{i_1}\wedge\cdots\wedge dx^{i_p}.$$

$$= (-1)^{n+1}\underbrace{\sum_{i_1<\cdots<i_p}\left(\underbrace{\left[\frac{\partial^2}{(\partial x^0)^2} - \sum_{j=1}^{n}\frac{\partial^2}{(\partial x^j)^2}\right]}_{\square}\psi_{i_1\cdots i_p}\right)dx^{i_1}\wedge\cdots\wedge dx^{i_p}}_{\square\psi}$$

$\square$: $(n+1)$次元ダランベールシアン.

$$\begin{cases} \Delta_{LB} : A^p(D) \longrightarrow A^p(D). \\ \psi : D \longrightarrow \Lambda^p V^* \\ \psi_{i_1\cdots i_p} : D \longrightarrow \mathbb{R} \end{cases}$$

6.6. 微分形式の積分

$\psi \in A^n(D)$. $\psi(x) = f(x^1,\cdots,x^n)\,dx^1\wedge\cdots\wedge dx^n$, $x \in D$.

$f : \tilde{D} \longrightarrow \mathbb{R}$.

$$\int_D \psi := \int\cdots\int_{\tilde{D}} f(x^1,\cdots,x^n)\,dx^1\cdots dx^n$$

i がない.　(e^i) による? … よらない.

Vの向きを固定すれば.

例 6.7.

$\gamma : (a,b) \longrightarrow D$: 曲線

$$\phi = \sum_{i=1}^{n}\phi_i\,dx^i \in A^1(D).$$

$$\int_\gamma \phi = \int_K \phi_{\mathbb{R}}$$

∵ (6.37).

$$= \int_K \sum_{i=1}^{n}\phi_i(\gamma^1(t),\cdots,\gamma^n(t))\,\frac{\partial\gamma^i(t)}{\partial t}\,dt$$

$$\gamma = \left\{\sum_{i=1}^{n}\gamma^i(t)e_i \;\middle|\; t \in K \subset \mathbb{R}\right\}, \quad K = (a,b).$$

$$= \int_a^b \sum_{i=1}^{n}\phi_i(\gamma(t))\,\dot\gamma^i(t)\,dt.$$

$$= \int_a^b \langle\phi(\gamma(t)), \dot\gamma(t)\rangle\,dt$$

☆ $\phi \in A^1(D)$ の $\gamma \subset D$ 上の積分は,
ベクトル場 ϕ の γ に沿う線積分.