# 物理…原理 #27 $20^{04}01^{水}$

## p.275 §6.5. 余微分作用素とL.B.作用素

$V$ : $n$-dim. real met. vec. sp.
$D \subset V$
$\omega_n$ : $\Lambda^n V^*$ の basis
$*$.

**Def 6.9.** $d$ : $A^{p+1}(D)$

$$\delta : A^p(D) \longrightarrow A^{p-1}(D)$$
$$\phi \longmapsto \delta\phi := \begin{cases} (-1)^{np+n+1} * (d(*\phi)) & (p \geq 1) \\ 0 & (p=0) \end{cases}$$

: $\phi$ の余微分

: 余微分作用素

**Thm 6.10.**

(i) $\delta^2 = 0$

(ii) $* \delta d = d \delta *$

(iii) $\delta d * = * d \delta$

## ★ $\delta$の作用の成分表示

$(e_i)$ : $V$ の O.N.B.
$x^i$ : $e_i$ に関する座標関数.
$\omega_n = dx^1 \wedge \cdots \wedge dx^n$　($\Lambda^n V^*$ の basis.)
$\forall \psi \in A^p(D)$

$$\psi(x) = \sum_{i_1 < \cdots < i_p} \psi_{i_1 \cdots i_p}(x)\, dx^{i_1} \wedge \cdots \wedge dx^{i_p} \qquad (6.2)$$

$i = 1, \cdots, n$
pコとる.

$$\delta\psi = -\sum_{j_1 < \cdots < j_{p-1}} \left( \sum_{j=1}^{n} \frac{\partial \psi_{j j_1 \cdots j_{p-1}}}{\partial x^j} \varepsilon(dx^j) \right) \varepsilon(\omega_n)\, dx^{j_1} \wedge \cdots \wedge dx^{j_{p-1}}$$

$p=1$ のとき.

$$\delta\psi = - \sum_{j=1}^{n} \frac{\partial \psi_j}{\partial x^j} \varepsilon(dx^j)\, \varepsilon(\omega_n).$$

さらに $V$ : Euc. vec. sp のとき（例 6.6）.

$$\delta\psi = - \sum_{j=1}^{n} \frac{\partial \psi_j}{\partial x^j}, \quad \psi \in A^1(D).$$

そこで, $D \subset V$ : $n$-dim. Euc. vec. sp.

$$\operatorname{div}\psi := -\delta\psi, \quad \psi \in A^1(D).$$

**Def 6.11.** $V$ : $n$-dim. real met. vec. sp.

$$\Delta_{LB} : A^p(D) \longrightarrow A^p(D)$$
$$\Delta_{LB} := d\delta + \delta d.$$

## ★ $\Delta_{LB}$ の作用の成分表示.

$\psi \in A^p(D)$.

$$\Delta_{LB}\psi = -\sum_{i_1 < \cdots < i_p} \left( \sum_{j=1}^{n} \frac{\partial^2 \psi_{i_1 \cdots i_p}}{(\partial x^j)^2} \varepsilon(dx^j)\, \varepsilon(\omega_n) \right) dx^{i_1} \wedge \cdots \wedge dx^{i_p}.$$

(i) $V$ : Euc. vec. sp.

$$\Delta_{LB}\psi = -\sum_{i_1 < \cdots < i_p} \left( \boxed{\sum_{j=1}^{n} \frac{\partial^2 \psi_{i_1 \cdots i_p}}{(\partial x^j)^2}} \right) dx^{i_1} \wedge \cdots \wedge dx^{i_p}$$

$\Delta$ : $n$次元ラプラシアン

$\Delta\psi$

(ii) $V$ : $(n+1)$ dim ミンコフスキー vec. sp.

$(dx^i)$ : $V^*$ の O.N.B.

$\langle dx^0, dx^0 \rangle = 1$.
$\langle dx^j, dx^j \rangle = -1$ $(j = 1, \cdots, n)$.

$\omega_{n+1} = dx^0 \wedge \cdots \wedge dx^n$

$\square$ : $(n+1)$次元ダランベールシアン.

$$\Delta_{LB}\psi = -\sum_{i_1 < \cdots < i_p} \left( \sum_{j=1}^{\cancel{n}\ n+1} \frac{\partial^2 \psi_{i_1 \cdots i_p}}{(\partial x^j)^2} \underbrace{\varepsilon(dx^j)}_{\begin{cases} 1 & (j=0) \\ -1 & (\text{otherwise}) \end{cases}} \underbrace{\varepsilon(\omega_{\cancel{n}\ n+1})}_{(-1)^n} \right) dx^{i_1} \wedge \cdots \wedge dx^{i_p}.$$