物理…原理 #25 2003 18 pk

§6.4.3. 回転と発散の幾何学的・物理的意味.

V : 3-dim Euc. vec. sp.

(e_i) : V の O.N.B.

$\forall x \in V$

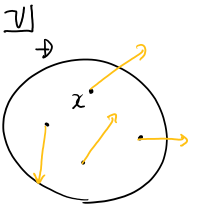
$$x = \sum_{i=1}^{3} x_i e_i$$

$D \subset V$

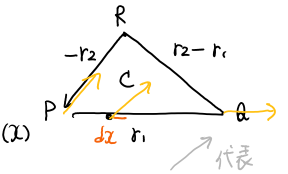
$u : D \to V$

$$x \mapsto u(x) = \sum u_i(x) e_i$$

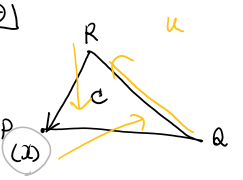
: D 上の「流 u」



☆回転 div



$\int_C \langle u, dx \rangle$?

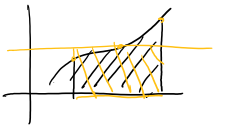


C に沿う u の線積分

$$\int_C \langle u, dx \rangle$$

$$\approx \left\langle \frac{u(P)+u(Q)}{2}, r_1 \right\rangle + \cdots$$

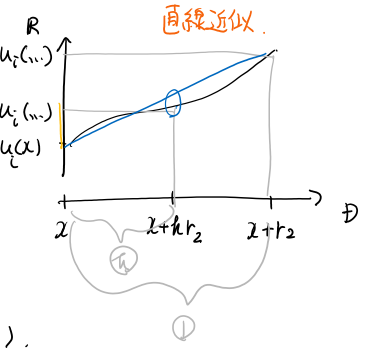
$$= \frac{1}{2} \left\{ \langle u(P)-u(R), r_1 \rangle + \langle u(Q)-u(P), r_2 \rangle \right\}$$



$$= \sum_{i=1}^{3} \left[u_i(x) - u_i(x+r_2) \right] (r_1)_i$$

$u_i : D \to \mathbb{R}$.

直線近似.



$$\xrightarrow{h \to 0} u'(x, r_2) = \sum_j (r_2)_j \, u'(x, e_j)$$

$\sum_j (r_2)_j e_j$

$$= \sum_j (r_2)_j \frac{\partial u_i(x)}{\partial x_j} \quad (r_1)_i$$

☺ linear (Thm 3.23).

$$\int_C \langle u, dx \rangle \approx \frac{1}{2} \sum_{i,j=1}^{3} \frac{\partial u_i}{\partial x_j} \left[(r_2)_i (r_1)_j - (r_1)_i (r_2)_j \right]$$

$$= \frac{1}{2} \sum_{i<j} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \left[(r_2)_i (r_1)_j - (r_1)_i (r_2)_j \right]$$

$$= \left\langle \underbrace{\operatorname{rot} u}_{\| \cdot \| = \eta_c}, \underbrace{\frac{r_2 \times r_1}{\| r_2 \times r_1 \|}} \right\rangle \underbrace{\frac{\| r_2 \times r_1 \|}{2}}_{= |S_C|}$$