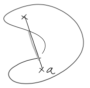
物理...原理 #30　$20^{04}22^{水}$

(M.1)
.2　⟵ (7.37)
.3　⟵ (7.38)　⟵ (7.100)
.4

§7.5.

$\mathcal{D}$: star set.
(O_V 中心).



非 star set.

ポアンカレの Lem. (p.269. Thm 6.8)

「$\mathcal{D}$上の p 次閉形式は完全」.
外ビすると0.　何から外ビしたもの.

$\psi \in A^p(\mathcal{D})$
$\psi : \mathcal{D} \longrightarrow \bigwedge^p V^*$
V : Euc. vec. sp.　§6.2.3.
$\mathbb{B}(t,\cdot) : \mathcal{D} \longrightarrow \bigwedge^2 V$
$\mathbb{B} : I \times \mathcal{D} \longrightarrow \bigwedge^2 V$

と, $d\mathbb{B} = 0$ (M.3) より,

$$\exists \mathbb{A} : I \times \mathcal{D} \longrightarrow \bigwedge^{2-1} V = V$$

$$\text{s.t. } \mathbb{B} = d\mathbb{A} \qquad (7.35).$$

: $\mathbb{B}$ に対する vector potential.

これと

$$d\mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \quad \text{(M.2)}$$

より,

$$d\left(\mathbb{E} + \frac{\partial \mathbb{A}}{\partial t}\right) = 0$$

$\mathbb{E} + \frac{\partial \mathbb{A}}{\partial t} : I \times \mathcal{D} \longrightarrow V = \bigwedge^1 V$

再びポアンカレ.

$$\exists \phi : I \times \mathcal{D} \to \bigwedge^{1-1} V = \mathbb{R}$$

$$\text{s.t. } \mathbb{E} + \frac{\partial \mathbb{A}}{\partial t} = -d\phi$$

$$\therefore\ \mathbb{E} = -d\phi - \frac{\partial \mathbb{A}}{\partial t} \qquad (7.36).$$

: scalar potential

(M.1), (M.4) に代入,

$$\hat{\varepsilon}_0 \delta\left(d\phi + \frac{\partial \mathbb{A}}{\partial t}\right) = \hat{\rho} \qquad (7.37)$$

$$\hat{\varepsilon}_0\left(\frac{\partial^2 \mathbb{A}}{\partial t^2} + c^2 \delta d\mathbb{A}\right) = \mathbb{J} - \hat{\varepsilon}_0 d\frac{\partial \phi}{\partial t} \qquad (7.38).$$

$(\phi, \mathbb{A})$: e.m. potential

§7.6.

$$\begin{array}{ccc} (\phi, \mathbb{A}) & \rightsquigarrow & (\mathbb{E}, \mathbb{B}) \\ \Big\downarrow G_{\Lambda, c} & \nearrow & \\ (\tilde{\phi}, \tilde{\mathbb{A}}) & & \end{array}$$

gauge invariance
〃 symmetry.

$$\mathbb{B} = d\mathbb{A} = d\tilde{\mathbb{A}}, \quad \mathbb{E} = -d\phi - \frac{\partial \mathbb{A}}{\partial t} = -d\tilde{\phi} - \frac{\partial \tilde{\mathbb{A}}}{\partial t}$$

$$d(\tilde{\mathbb{A}} - \mathbb{A}) = 0, \quad \frac{\partial}{\partial t}(\tilde{\mathbb{A}} - \mathbb{A}) + d(\tilde{\phi} - \phi) = 0.$$

ポアンカレ.

$$\exists \Lambda : I \times \mathcal{D} \longrightarrow \mathbb{R}$$

$$\text{s.t. } \tilde{\mathbb{A}} - \mathbb{A} = d\Lambda.$$

$$\therefore\ d\left(\frac{\partial \Lambda}{\partial t} + \tilde{\phi} - \phi\right) = 0.$$

$$\because\ \exists c : I \longrightarrow \mathbb{R}$$

$$\text{s.t. } \frac{\partial \Lambda}{\partial t} + \tilde{\phi} - \phi = c.$$

以上から,

$$\tilde{\mathbb{A}} = \mathbb{A} + d\Lambda.$$

$$\tilde{\phi} = \phi - \frac{\partial \Lambda}{\partial t} + c.$$

そこで,

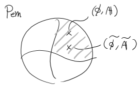
$$P_{em} := \left\{ (\phi, \mathbb{A}) \;\middle|\; \begin{array}{l} \phi : I \times \mathcal{D} \to \mathbb{R},\ \mathbb{A} : I \times \mathcal{D} \to V, \\ (\phi, \mathbb{A}) \text{ follows } (7.37), (7.38) \end{array} \right\}.$$

$$\begin{array}{ccccc} G_{\Lambda, c} & : & P_{em} & \longrightarrow & P_{em} \\ & & \cup & & \cup \\ & & (\phi, \mathbb{A}) & \longmapsto & G_{\Lambda, c}(\phi, \mathbb{A}) \end{array}$$

$$:= \left(\phi - \frac{\partial \mathbb{A}}{\partial t} + c,\ \mathbb{A} + d\mathbb{A}\right),$$

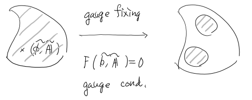
とすると,

$$(\tilde{\phi}, \tilde{\mathbb{A}}) = G_{\Lambda, c}(\phi, \mathbb{A}).$$



P_{em} の同値類 = 電磁場

§7.6.3.



☆ Lorentz gauge.

(7.37), (7.38) より,

$$\boxed{\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta_{LB}\right)}\phi - \frac{\partial}{\partial t}\boxed{\left(-\delta\mathbb{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t}\right)} = \frac{\rho}{\varepsilon_0}$$

$$\boxed{}\,\mathbb{A} + d\,\boxed{} = \frac{*\mathbb{J}}{c^2 \varepsilon_0}$$

もし $\boxed{} = 0$ なら,　—— Lorentz 条件.

$\boxed{}\ \phi = \frac{\rho}{\varepsilon_0}$
$\boxed{}\ \mathbb{A} = \frac{*\mathbb{J}}{c^2 \varepsilon_0}$
} 波動方程式.

☆ Coulomb gauge.

Lem 7.5.　$(\phi', \mathbb{A}')$: e.m. pot.

$$\exists \Lambda \text{ s.t. } \Delta_{LB}\Lambda = -\delta\mathbb{A}'$$

$$\Rightarrow G_{\Lambda, 0}(\phi', \mathbb{A}') =: (\phi, \mathbb{A}) \text{ は } \underline{\delta\mathbb{A} = 0} \text{ をみたす.}$$

Coulomb 条件 ?

Lorentz と同様に,

$$\Delta_{LB}\phi - \frac{\partial}{\partial t}\underbrace{(-\delta\mathbb{A})}_{0} = \frac{\rho}{\varepsilon_0}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta_{LB}\right)\mathbb{A} + d\left(\underbrace{-\delta\mathbb{A}}_{0} + \frac{1}{c^2}\frac{\partial \phi}{\partial t}\right) = \frac{*\mathbb{J}}{c^2 \varepsilon_0}$$