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$M_1 \cap M_2 = \{0\}$

である必要性?

$T: M_1 + M_2 \to M_1 \oplus M_2$

$T(u+v) \mapsto (u, v)$

全射 線型

$e_1 + e_2 + \cdots + e_n \quad f_1 + f_2 + \cdots + f_n$

線型 $a x_1 + a x_2 \cdots a_n x_n$

$T(u+v) = 0 \to (u, v) = 0$

$\hookrightarrow \begin{cases} u = 0 \\ v = 0 \end{cases}, \quad u + v = 0$

単射

kernelで考える

ker

(ii) $(e_i)_{i=1}^{n}$ を M_1 のきてい | M_2 のきていを $\{f_j\}_{j=1}^{m}$

$\xi_i (e_i, 0) \in M_1 \oplus M_2$ $(i = 1, \ldots, n)$ | $\xi_{n+j} = (0, f_j) \in M_1 \oplus M_2$ $(j = 1, \ldots, m)$

$\Sigma f_j = \xi_j$

$= (0, f_1 + f_2 + f_3 + \cdots +)$

また $e_i = \theta_i \ (i = 1, \ldots, n)$, $f_j = \theta_{n+j} \ (j = 1, \ldots, m)$ $\Big\} \ \{\theta_i\}_{i=1}^{n+m}$

$T\theta = \xi$ とすると、

$\theta_{i=1}$

$u + v = \sum u^i \theta_i + \sum v^j \theta_{n+j}$

$\Downarrow$

$T(u+v) = \sum u^i \xi_i + \sum v^j \xi_j$

$\downarrow$

(u, v)

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V : vector space over $\mathbb{K}$

$u_i \in V$,

$\phi^i \in V^* \quad (i = 1, \cdots, p)$

$$(u_1 \wedge \cdots \wedge u_p)(\phi^1, \cdots, \phi^p) \overset{?}{=} \frac{1}{\sqrt{p!}} \det (\phi^i(u_j))_{ij}$$

i, j 成分が $\phi^i(u_j)$ であるような行列 M_{ij}

(右辺)

$M_{1\sigma(1)} \cdots M_{p\sigma(p)}$

$$\underset{\because \text{p58 (1.72)}}{=} \frac{1}{\sqrt{p!}} \sum_{\sigma \in S_p} \mathrm{sgn}(\sigma)\, \phi^1(u_{\sigma(1)}) \cdots \phi^p(u_{\sigma(p)})$$

$\downarrow$

$(u_{\sigma(1)} \otimes \cdots \otimes u_{\sigma(p)})(\phi^1, \cdots, \phi^p)$

$$\begin{array}{ccccc} V^* & & & & \\ \rotatebox{90}{$\in$} & & & & \\ \phi & : & V & \longrightarrow & \mathbb{K} \\ & & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & & u & \longmapsto & \phi(u) \end{array}$$

$$\underset{\because \text{p50 (1.49)}}{=} \frac{1}{\sqrt{p!}} \sum_{\sigma \in S_p} \mathrm{sgn}(\sigma) \left(u_{\sigma(1)} \otimes \cdots \otimes u_{\sigma(p)}\right)(\phi^1, \cdots, \phi^p)$$

$$\underset{\because \text{p53 (1.60)}}{=} \sqrt{p!}\, \mathcal{A}_p (u_1 \otimes \cdots \otimes u_p)(\phi^1, \cdots, \phi^p)$$

$$\underset{\because \text{p55 例1.31}}{=} u_1 \wedge \cdots \wedge u_p (\phi^1, \cdots, \phi^p)$$

$=$ (左辺)

$\boxtimes$.

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\det \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = M_{11} M_{22} - M_{12} M_{21}$$

$$\sum \begin{cases} = (-1)^0 M_{1\sigma(1)} M_{2\sigma(2)} \\ + (-1)^1 M_{1\tau(1)} M_{2\tau(2)} \end{cases}$$

where

$$\begin{array}{ccccc} \sigma & : & \{1, 2\} & \longrightarrow & \{1, 2\} \\ & & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & & 1 & \longmapsto & 1 \\ & & 2 & \longmapsto & 2 \end{array}$$

$$\begin{array}{ccccc} \tau & : & \{1, 2\} & \longrightarrow & \{1, 2\} \\ & & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & & 1 & \longmapsto & 2 \\ & & 2 & \longmapsto & 1 \end{array}$$