物理の原理 #21 $20^{01}22^{水}$

Def 6.6. $\psi \in A^p(D)$

ψ : 閉形式 (closed form)

$:\Leftrightarrow d\psi = 0.$

Def 6.7 $\psi \in A^p(D).$

ψ : 完全形式 (exact form) .

$:\Leftrightarrow \exists \phi \in A^{p-1}(D)$ s.t. $\psi = d\phi$

$d\psi = d(d\phi) = 0.$

★ 完全 $\Rightarrow$ 閉.

閉 $\not\Rightarrow$ 完全. 反例.

例 6.3. V : 2-dim Euc. vec. sp.

(e_1, e_2) : ONB of V.

$D = V \setminus \{0\}$

$$A = \frac{-x^2}{\|x\|^2} dx^1 + \frac{x^1}{\|x\|^2} dx^2$$

$(x = x^1 e_1 + x^2 e_2 \in D).$

例 6.1 で $p = n-1$, $n = 2$ のとき.

$$d\psi = \left(\frac{\partial \psi_2}{\partial x^1} - \frac{\partial \psi_1}{\partial x^2} \right) dx^1 \wedge dx^2$$

$$\therefore \quad dA = \left(\frac{\partial \frac{x^1}{\|x\|^2}}{\partial x^1} - \frac{\partial \frac{-x^2}{\|x\|^2}}{\partial x^2} \right) dx^1 \wedge dx^2 \quad ?$$

$= 0.$

$\therefore$ A は closed form.

A は exact form ではない. 仮に

$\exists \phi \in A^0(D)$ s.t. $A = d\phi$

とすると,

$$d\phi = \frac{\partial \phi}{\partial x^1} dx^1 + \frac{\partial \phi}{\partial x^2} dx^2$$

より,

$$\frac{\partial \phi}{\partial x^1} = \frac{-x^2}{\|x\|^2}, \quad \frac{\partial \phi}{\partial x^2} = \frac{x^1}{\|x\|^2}$$

$x^2 \neq 0$ のもとで

$$\phi(x) = -\tan^{-1}\frac{x^1}{x^2} + C$$

これは $x^1 \neq 0$, $(x^1, 0)$ で不連続. で矛盾. ?