『Abs. Alg. Ⅱ』リモート輪講 #3 $19^{08}28$ 水

Def. G : a set

(1) $\star : G \times G \longrightarrow G$

$(a, b) \longmapsto \star(a, b) =: a \star b$

: binary operation on G.

(2) $\star$: binary operation on G

$\star$ is associative if $\forall a, b, c \in G \quad a \star (b \star c) = (a \star b) \star c$.

Def. G : a set

$\star$: bin. op. on G.

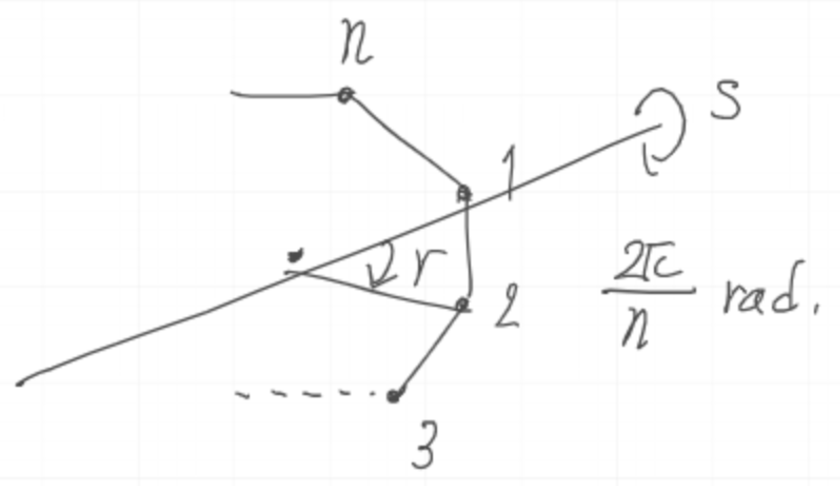
$(G, \star)$ is a group if $\star$ satisfy :

(i) $\star$ is associative.

(ii) $\exists e \in G$ s.t. $\forall a \in G \quad a \star e = e \star a = a$
(e is ~~an~~ the identity of G).

(iii) $\forall a \in G \ \exists a^{-1} \in G$ s.t. $a \star a^{-1} = a^{-1} \star a = e$
(a^{-1} is ~~an~~ the inverse of a).

例1. Dihedral group (二面体群)



操作 r, s が群をなす.

$1 =$ (なにもしない). $= r^n = s^2$

$D_{2n} := \{ \underbrace{1, r, r^2, \cdots, r^{n-1}, s, sr, \cdots, sr^{n-1}}_{2n コ} \}$

$sr = r^{-1}s$ ... ralation

p.46. H : subgroup of G.

$H \overset{?}{:}$ group

(i) $H \overset{?}{:}$ associative

$a, b, c \in H \overset{?}{\Rightarrow} (ab)c = a(bc)$

$a, b, c \in G \Rightarrow (ab)c = a(bc)$.

(ii) $H \overset{?}{:}$ has an identity in H

$\emptyset \neq H \ni {}^{\exists}a, \ a^{-1} \in H$

$a a^{-1} \in H$

$\underset{e_G}{\parallel} = e_H$

(iii) $H \overset{?}{:}$ has inverse in H

p20. Prop 2. cancellation law.

p47. Prop 1. proof.

(1) $H \neq \emptyset$

(2) $x, y \in H. \ xy^{-1} \in H$

$\overset{?}{\Rightarrow} \begin{cases} \text{(a) closed under multi. (products)} \\ \text{(b)} \quad 〃 \quad \text{inverses.} \end{cases}$

$\emptyset \neq H \ni x. \quad x, x \in H. \quad \underset{\underset{1}{\parallel}}{xx^{-1}} \in H$

$1, x \in H \quad \underset{\underset{x^{-1}}{\parallel}}{1x^{-1}} \in H$ (b) o.k.

$x, y \in H. \quad x, y^{-1} \in H. \quad \underset{\underset{xy}{\parallel}}{x(y^{-1})^{-1}} \in H$ (a) o.k. □