Abs. Alg. #13 19/11/08 金

$\langle s, r^2 \rangle =: \overline{H}$

$\exists 1 \ \pi^{-1}$ $\quad \pi \quad \pi \quad \pi$

$D_8 \geq H := \langle s \rangle \cup \langle r^2 s \rangle \cup \langle r^2 \rangle \geq \langle r^2 \rangle$

$\overline{H} = \{1, r^2, s, r^2 s\} =: H / \langle r^2 \rangle$ $\quad (\exists 1 \ D_8 \geq H \geq \langle r^2 \rangle)$ ∵ Thm 3.20.

$\pi^{-1}(\overline{H})$

$\langle s \rangle / \langle r^2 \rangle = \{1, s\}\{1, r^2\} = \{1, r^2, s, r^2 s\} = \overline{H}$ $\quad (\not\geq \langle r^2 \rangle)$

$\exists 1 \ D_8 \geq \{1, r^2, s, r^2 s\} \geq \langle r^2 \rangle$

$\{1, r^2, s, r^2 s\} / \langle r^2 \rangle = \overline{H}$

★ $\varphi : G/N \longrightarrow H$: homo. を定ぎする方法

$$\begin{tikzcd} G \arrow[r, "\pi"] \arrow[dr, "\Phi"'] & G/N \arrow[d, "\varphi"] \\ & H \end{tikzcd}$$

② $N \leq \ker \Phi$ をチェック.

① $\Phi : G \longrightarrow H$: homo. を定ぎ.

Def.

$1 = N_0 \trianglelefteq \cdots \trianglelefteq N_k = G$

N_{i+1}/N_i : simple group

a composition series

composition factors of G.

*Jordan 標準形.

Thm 3.22. (Jordan-Hölder)

$G \neq 1$. $|G| < \infty$.

(1) G has a composition series.

(2) The comp. factors in a comp. series are unique:

$$\left.\begin{array}{l} 1 = N_0 \trianglelefteq \cdots \trianglelefteq N_r = G \\ 1 = M_0 \trianglelefteq \cdots \trianglelefteq M_s = G \end{array}\right\} \text{comp. ser.}$$

$\Rightarrow r = s$

$\exists \pi$: permutation of $\{1, \cdots, r\}$ s.t.

$M_{\pi(i)}/M_{\pi(i)-1} \cong N_i / N_{i-1}$.

$1 = N_0 \trianglelefteq \cdots \trianglelefteq N_{i-1} \trianglelefteq N_i \trianglelefteq \cdots \trianglelefteq N_r = G$

$N_i/N_{i-1} \cong M_j/M_{j-1}$

$1 = M_0 \trianglelefteq \cdots \trianglelefteq M_{j-1} \trianglelefteq M_j \trianglelefteq \cdots \trianglelefteq M_r = G$.

Thm.

18 の simple group の族と.

26 の simple group があり.

任意の finite sim. g. はそのいずれかに iso.

Thm (Feit-Thompson).

G : simp. g. $|G|$: odd. $\Rightarrow \exists p$: prime s.t. $G \cong \mathbb{Z}_p$

★ Hölder Prog. (2) のイミ.

見つける.

given $A, B \rightsquigarrow B \cong N$, $G/N \cong A$

例.

$A = B = \mathbb{Z}_2$ のとき. $\mathbb{Z}_2 \cong \mathbb{Z}_2$. $G/\mathbb{Z}_2 \cong \mathbb{Z}_2$

なる G として. $\mathbb{Z}_4$, V_4 が知られてる.

が. $\mathbb{Z}_2, \mathbb{Z}_2$ から $\mathbb{Z}_4, V_4$ を構成するような一般的方法は何か?

Def.

G : solvable

$:\Longleftrightarrow \exists$ a chain of subg. : $1 = G_0 \trianglelefteq \cdots \trianglelefteq G_s = G$.

s.t. G_{i+1}/G_i : abelian.

Thm. $|G| < \infty$.

*Sylow's thm. の一般化.

G : solvable

$\Longleftrightarrow \forall n \mid |G|$ with $(n, \frac{|G|}{n}) = 1$.

$\exists H \leq G$ s.t. $|H| = n$.

★

$N, G/N$: solvable $\Rightarrow G$: solvable.

<proof>

$G/N =: \overline{G}$

$1 = N_0 \trianglelefteq \cdots \trianglelefteq N_n = N$: N_{i+1}/N_i abelian.

$\overline{1} = \overline{G_0} \trianglelefteq \cdots \trianglelefteq \overline{G_m} = \overline{G}$: $\overline{G_{i+1}}/\overline{G_i}$ abelian.

Lattice iso. thm. より.

$N \leq \exists G_i \leq G$ s.t. $\overline{G_i} = G_i/N$. , $G_i \trianglelefteq G_{i+1}$

∵ thm. (5) と. $\overline{G_i} \trianglelefteq \overline{G_{i+1}}$

3rd iso. thm. より.

$\overline{G_{i+1}}/\overline{G_i} = (G_{i+1}/N)/(G_i/N)$

$\cong G_{i+1}/G_i$

∵ $N, G_i \trianglelefteq G_{i+1}$

$N \leq G_i$

∴ $1 = N_0 \trianglelefteq \cdots \trianglelefteq N_n = N = G_0 \trianglelefteq \cdots \trianglelefteq G_m = G$

$N \leq G_0$ $\quad$ s.t. $\square/\triangle$: abelian.

$$\left(\begin{array}{l} \psi : G \cong H, \ G : \text{abelian} \\ \quad a, b \mapsto \psi(a), \psi(b). \\ \psi(a)\psi(b) = \psi(ab) = \psi(ba) = \psi(b)\psi(a). \\ \Rightarrow H : \text{abelian} \end{array}\right)$$

∴ G : solvable. $\quad \square$