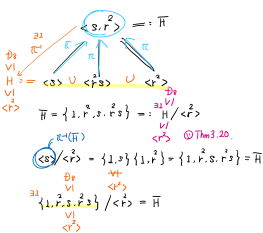
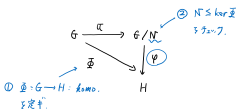


Abs. Alg. #13 10/10/08



* $\varphi: G/N \rightarrow H$: homo. 2 定する方法



Def. $1 = N_0 \triangleleft \dots \triangleleft N_k = G$ * Jordan 標準形.
 N_{i+1}/N_i : simple group
 a composition series
 composition factors of G .

Thm 3.22. (Jordan-Hölder).

$$G \neq 1, |G| < \infty.$$

- G has a composition series.
- The comp. factors in a comp. series are unique:

$$\left. \begin{aligned} 1 = N_0 \triangleleft \dots \triangleleft N_r = G \\ 1 = M_0 \triangleleft \dots \triangleleft M_s = G \end{aligned} \right\} \text{comp. ser.}$$

$$\Rightarrow r = s$$

$\exists \alpha$: permutation of $\{1, \dots, r\}$ s.t.

$$N_{\alpha(i)}/N_{\alpha(i)-1} \cong N_i/N_{i-1}$$

$$1 = N_0 \triangleleft \dots \triangleleft (N_{i-1} \triangleleft N_i) \triangleleft \dots \triangleleft N_r = G$$

$$\cong N_i/N_{i-1}$$

$$1 = M_0 \triangleleft \dots \triangleleft (M_{j-1} \triangleleft M_j) \triangleleft \dots \triangleleft M_r = G$$

$$\cong M_j/M_{j-1}$$

Thm.

- 18 n simple group 家族.
- 26 n simple group 家族.
- 任意の finite sim. g. は その 1 つ の しか 18 iso.

Thm (Feit-Thompson).

$$G: \text{simp. g. } |G|: \text{odd} \Rightarrow \exists p: \text{prime s.t. } G \cong \mathbb{Z}_p$$

* Hölder Prog. (2) の 1 つ. 見つける.

$$\text{given } A, B \rightsquigarrow B \cong N, G/N \cong A$$

例.

$$A = B = \mathbb{Z}_2 \text{ の 1 つ. } \mathbb{Z}_2 \cong \mathbb{Z}_2, G/\mathbb{Z}_2 \cong \mathbb{Z}_2$$

$$\text{13 } G \cong \mathbb{Z}_4, \mathbb{Z}_4, \mathbb{V}_4 \text{ の 1 つ.}$$

例. $\mathbb{Z}_2, \mathbb{Z}_2$ の 1 つ $\mathbb{Z}_4, \mathbb{V}_4$ を 構成 する 1 つ の 一般 的 方法 何 かな?

Def.

G : solvable
 $\Leftrightarrow \exists$ a chain of subg.: $1 = G_0 \triangleleft \dots \triangleleft G_s = G$
 s.t. G_{i+1}/G_i : abelian.

Thm.

$$|G| < \infty.$$

* Sylow's thm.
 の 一般 化.

G : solvable

$$\Leftrightarrow \forall n \mid |G| \text{ with } (n, \frac{|G|}{n}) = 1.$$

$$\exists H \leq G \text{ s.t. } |H| = n.$$

* 1

$$N, G/N: \text{solvable} \Rightarrow G: \text{solvable}.$$

<proof>

$$G/N =: \bar{G}$$

$$1 = N_0 \triangleleft \dots \triangleleft N_n = N: N_{i+1}/N_i \text{ abelian.}$$

$$\bar{1} = \bar{G}_0 \triangleleft \dots \triangleleft \bar{G}_m = \bar{G}: \bar{G}_{i+1}/\bar{G}_i \text{ abelian.}$$

Lattice iso. thm. (4).

$$N \trianglelefteq G_i \leq G \text{ s.t. } \bar{G}_i = G_i/N, G_i \trianglelefteq G_{i+1}$$

$$\textcircled{1} \text{ thm. (5) } \& \bar{G}_i \trianglelefteq \bar{G}_{i+1}$$

3rd iso. thm. (4).

$$\bar{G}_{i+1}/\bar{G}_i = (G_{i+1}/N)/(G_i/N)$$

$$\cong G_{i+1}/G_i$$

$$\textcircled{1} N, G_i \trianglelefteq G_{i+1}$$

$$N \trianglelefteq G_i$$

$$\therefore 1 = N_0 \triangleleft \dots \triangleleft N_n = N \trianglelefteq G_0 \triangleleft \dots \triangleleft G_m = G$$

$$N \leq G_0 \text{ s.t. } G_0/\Delta: \text{abelian.}$$

$$\left(\begin{aligned} \psi: G &\cong H, G: \text{abelian} \\ \psi(a)\psi(b) &= \psi(ab) = \psi(ba) = \psi(b)\psi(a) \\ &\Rightarrow H: \text{abelian} \end{aligned} \right)$$

$$\therefore G: \text{solvable.}$$

□.