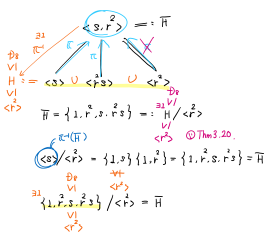
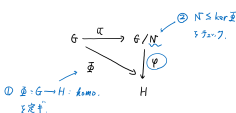


# Abs. Alg. #13 19'08金



★  $\varphi: G/N \rightarrow H$ : homo. 定義方法



Def.  $1 = N_0 \triangleleft \dots \triangleleft N_k = G$  \* Jordan 標準形.  
 $N_{i+1}/N_i$ : simple group  
 a composition series  
 composition factors of  $G$ .

Thm 3.22. (Jordan-Hölder)

$$G \neq 1, |G| < \infty$$

(1)  $G$  has a composition series.

(2) The comp. factors in a comp. series are unique:

$$\left. \begin{aligned} 1 = N_0 \triangleleft \dots \triangleleft N_r = G \\ 1 = M_0 \triangleleft \dots \triangleleft M_s = G \end{aligned} \right\} \text{comp. ser.}$$

$$\Rightarrow r = s$$

$\exists \alpha$ : permutation of  $\{1, \dots, r\}$  s.t.

$$N_{\alpha(i)}/N_{\alpha(i)-1} \cong N_i/N_{i-1}$$

$$1 = N_0 \triangleleft \dots \triangleleft N_{i-1} \triangleleft N_i \triangleleft \dots \triangleleft N_r = G$$

$$1 = M_0 \triangleleft \dots \triangleleft M_{j-1} \triangleleft M_j \triangleleft \dots \triangleleft M_r = G$$

Thm.

18 n simple group 族.

26 n simple group 族.

任意の finite sim. g. はそのうちから iso.

Thm (Feit-Thompson).

$$G: \text{simp. g.}, |G|: \text{odd} \Rightarrow \exists p: \text{prime s.t. } G \cong \mathbb{Z}_p$$

★

Hölder Prop. (2) の  $\Leftarrow$ .

$$\text{given } A, B \rightsquigarrow B \cong N, G/N \cong A$$

例.

$$A = B = \mathbb{Z}_2 \text{ のとき, } \mathbb{Z}_2 \cong \mathbb{Z}_2, G/\mathbb{Z}_2 \cong \mathbb{Z}_2$$

$$\text{つまり } G \cong \mathbb{Z}_4, \mathbb{V}_4 \text{ のどちらか?}$$

例.  $\mathbb{Z}_2, \mathbb{Z}_2$  から  $\mathbb{Z}_4, \mathbb{V}_4$  を構成する一般的な方法は何?

Def.

$G$ : solvable

$$: \Leftrightarrow \exists \text{ a chain of subg. } 1 = G_0 \triangleleft \dots \triangleleft G_s = G$$

$$\text{s.t. } G_{i+1}/G_i: \text{abelian}$$

Thm.

$$|G| < \infty$$

$G$ : solvable

$$\Leftrightarrow \forall n \mid |G| \text{ with } (n, \frac{|G|}{n}) = 1$$

$$\exists H \leq G \text{ s.t. } |H| = n$$

\* Sylow's thm.  
の一般化.

★

$$N, G/N: \text{solvable} \Rightarrow G: \text{solvable}$$

<proof>

$$G/N =: \bar{G}$$

$$1 = N_0 \triangleleft \dots \triangleleft N_n = N: N_{i+1}/N_i \text{ abelian}$$

$$\bar{1} = \bar{G}_0 \triangleleft \dots \triangleleft \bar{G}_m = \bar{G}: \bar{G}_{i+1}/\bar{G}_i \text{ abelian}$$

Lattice iso. thm. (1).

$$N \leq \bar{G}_i \leq G \text{ s.t. } \bar{G}_i = G/N, \bar{G}_i \triangleleft \bar{G}_{i+1}$$

$$\textcircled{1} \text{ thm. (5) } \bar{G}_i \triangleleft \bar{G}_{i+1}$$

3rd iso. thm. (1).

$$\bar{G}_{i+1}/\bar{G}_i = (G_{i+1}/N)/(G_i/N)$$

$$\cong G_{i+1}/G_i$$

$$\textcircled{1} N, G_i \triangleleft G_{i+1}$$

$$N \triangleleft G_i$$

$$\therefore 1 = N_0 \triangleleft \dots \triangleleft N_n = N \triangleleft G_0 \triangleleft \dots \triangleleft G_m = G$$

$$N \triangleleft G_0 \text{ s.t. } G_0/N: \text{abelian}$$

$$\left( \begin{aligned} \psi: G &\cong H, G: \text{abelian} \\ a, b &\mapsto \psi(a), \psi(b) \\ \psi(a)\psi(b) &= \psi(ab) = \psi(ba) = \psi(b)\psi(a) \\ &\Rightarrow H: \text{abelian} \end{aligned} \right)$$

$$\therefore G: \text{solvable}$$

□.