第 4 章 線性規劃問題求解法：單行法

題目 **4.4-5 (a) (b)**

4.4-5.

(a) Set $x_1 = x_2 = x_3 = 0$.

(0)　　$Z - 2x_1 - 4x_2 - 3x_3 = 0$

(1)　　$x_1 + 3x_2 + 2x_3 + x_4 = 80 \Rightarrow x_4 = 80$

(2)　　$3x_1 + 4x_2 + 2x_3 + x_5 = 60 \Rightarrow x_5 = 60$

(3)　　$2x_1 + x_2 + 2x_3 + x_6 = 40 \Rightarrow x_6 = 40$

Optimality Test: The coefficients of all nonbasic variables are negative, so the solution $(0, 0, 0, 80, 60, 40)$ is not optimal.

Choose x_2 as the entering basic variable, since it has the largest coefficient.

(1)　　$x_1 + 3x_2 + 2x_3 + x_4 = 80 \Rightarrow x_4 = 80 - 3x_2 \Rightarrow x_2 \leq 26.67$

(2)　　$3x_1 + 4x_2 + 2x_3 + x_5 = 60 \Rightarrow x_5 = 60 - 4x_2 \Rightarrow x_2 \leq 15 \leftarrow \text{minimum}$

(3)　　$2x_1 + x_2 + 2x_3 + x_6 = 40 \Rightarrow x_6 = 40 - x_2 \Rightarrow x_2 \leq 40$

We choose x_5 as the leaving basic variable. Set $x_1 = x_5 = x_3 = 0$.

(0)　　$Z + x_1 - x_3 + x_5 = 60$

(1)　　$-1.25x_1 + 0.5x_3 + x_4 - 0.75x_5 = 35 \Rightarrow x_4 = 35$

(2)　　$0.75x_1 + x_2 + 0.5x_3 - 0.25x_5 = 15 \Rightarrow x_2 = 15$

(3)　　$1.25x_1 + 1.5x_3 - 0.25x_5 + x_6 = 25 \Rightarrow x_6 = 25$

Optimality Test: The coefficient of x_3 is negative, so the solution $(0, 15, 0, 35, 0, 25)$ is not optimal.

Let x_3 be the entering basic variable.

(1)　　$-1.25x_1 + 0.5x_3 + x_4 - 0.75x_5 = 35 \Rightarrow x_4 = 35 - 0.5x_3 \Rightarrow x_3 \leq 70$

(2)　　$0.75x_1 + x_2 + 0.5x_3 + 0.25x_5 = 15 \Rightarrow x_2 = 15 - 0.5x_3 \Rightarrow x_3 \leq 30$

(3)　　$1.25x_1 + 1.5x_3 - 0.25x_5 + x_6 = 25 \Rightarrow x_6 = 25 - 1.5x_3 \Rightarrow x_3 \leq 16.67 \leftarrow \text{min}$

We choose x_6 as the leaving basic variable. Set $x_1 = x_5 = x_6 = 0$.

(0)　　$Z + 1.83x_1 + 0.83x_5 + 0.67x_6 = 76.67$

(1)　　$-1.67x_1 + x_4 - 0.67x_5 - 0.33x_6 = 26.67 \Rightarrow x_4 = 26.67$

(2) $0.33x_1 + x_2 + 0.33x_5 - 0.33x_6 = 6.67 \Rightarrow x_2 = 6.67$

(3) $0.83x_1 + x_3 - 0.17x_5 + 0.67x_6 = 16.67 \Rightarrow x_3 = 16.67$

Optimality Test: All of the coefficients are positive, so the solution $(0, 6.67, 16.67, 26.67, 0, 0)$ is optimal. $Z^* = 76.67$.

(b) Optimal solution: $(x_1^*, x_2^*, x_3^*) = (0, 6.67, 16.67)$ and $Z^* = 76.67$

```
Bas|Eq|                Coefficient of              | Right
Var|No| Z|   X1    X2    X3    X4    X5    X6   | side
 __|__|__|_____|_____
   |  |  |                                        |
 Z | 0| 1|   -2    -4    -3    0     0     0   |    0
 X4| 1| 0|    1     3     2    1     0     0   |   80
 X5| 2| 0|    3    4*     2    0     1     0   |   60
 X6| 3| 0|    2     1     2    0     0     1   |   40
```

```
Bas|Eq|                Coefficient of              | Right
Var|No| Z|   X1    X2    X3    X4    X5    X6   | side
 __|__|__|_____|_____
   |  |  |                                        |
 Z | 0| 1|    1     0    -1    0     1     0   |   60
 X4| 1| 0|-1.25     0   0.5    1 -0.75     0   |   35
 X2| 2| 0| 0.75     1   0.5    0  0.25     0   |   15
 X6| 3| 0| 1.25     0  1.5*    0 -0.25     1   |   25
```

```
Bas|Eq|                Coefficient of              | Right
Var|No| Z|   X1    X2    X3    X4    X5    X6   | side
 __|__|__|_____|_____
   |  |  |                                        |
 Z | 0| 1|1.833     0     0    0 0.833 0.667 | 76.67
 X4| 1| 0|-1.67     0     0    1 -0.67 -0.33 | 26.67
 X2| 2| 0|0.333     1     0    0 0.333 -0.33 | 6.667
 X3| 3| 0|0.833     0     1    0 -0.17 0.667 | 16.67
```

題目 4.4-6 (a) (b)

4.4-6.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, \frac{4}{3}, \frac{4}{3})$ and $Z^* = 14\frac{2}{3}$

```
0)  Z-     3 X₁-    5 X₂-    6 X₃+    0 X₄+    0 X₅+    0 X₆+    0 X₇ = 0
1)         2 X₁+    1 X₂+    1 X₃+    1 X₄+    0 X₅+    0 X₆+    0 X₇ = 4
2)         1 X₁+    2 X₂+    1 X₃+    0 X₄+    1 X₅+    0 X₆+    0 X₇ = 4
3)         1 X₁+    1 X₂+    2 X₃+    0 X₄+    0 X₅+    1 X₆+    0 X₇ = 4
4)         1 X₁+    1 X₂+    1 X₃+    0 X₄+    0 X₅+    0 X₆+    1 X₇ = 3
```

$X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $X_4 \geq 0$, $X_5 \geq 0$, $X_6 \geq 0$, $X_7 \geq 0$.

```
0)  Z+     0 X₁-    2 X₂+    0 X₃+    0 X₄+    0 X₅+    3 X₆+    0 X₇ = 12
1)       1.5 X₁+  0.5 X₂+    0 X₃+    1 X₄+    0 X₅-  0.5 X₆+    0 X₇ = 2
2)       0.5 X₁+  1.5 X₂+    0 X₃+    0 X₄+    1 X₅-  0.5 X₆+    0 X₇ = 2
3)       0.5 X₁+  0.5 X₂+    1 X₃+    0 X₄+    0 X₅+  0.5 X₆+    0 X₇ = 2
4)       0.5 X₁+  0.5 X₂+    0 X₃+    0 X₄+    0 X₅-  0.5 X₆+    1 X₇ = 1
```

$X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $X_4 \geq 0$, $X_5 \geq 0$, $X_6 \geq 0$, $X_7 \geq 0$.

```
0)  Z+0.67 X₁+    0 X₂+    0 X₃+     0 X₄+1.33 X₅+2.33 X₆+    0 X₇ = 14.6667
1)    1.333 X₁+   0 X₂+    0 X₃+     1 X₄-0.33 X₅-0.33 X₆+    0 X₇ = 1.33333
2)    0.333 X₁+   1 X₂+    0 X₃+     0 X₄+0.67 X₅-0.33 X₆+    0 X₇ = 1.33333
3)    0.333 X₁+   0 X₂+    1 X₃+     0 X₄-0.33 X₅+0.67 X₆+    0 X₇ = 1.33333
4)    0.333 X₁+   0 X₂+    0 X₃+     0 X₄-0.33 X₅-0.33 X₆+    1 X₇ = 0.33333
```

$X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $X_4 \geq 0$, $X_5 \geq 0$, $X_6 \geq 0$, $X_7 \geq 0$.

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, \frac{4}{3}, \frac{4}{3})$ and $Z^* = 14\frac{2}{3}$

Bas Var	Eq No	z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Right Side
Z	0	1	-3	-5	-6	0	0	0	0	0
X_4	1	0	2	1	1	1	0	0	0	4
X_5	2	0	1	2	1	0	1	0	0	4
X_6	3	0	1	1	2	0	0	1	0	4
X_7	4	0	1	1	1	0	0	0	1	3

Bas Var	Eq No	z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Right Side
Z	0	1	0	-2	0	0	0	3	0	12
X_4	1	0	1.5	0.5	0	1	0	-0.5	0	2
X_5	2	0	0.5	1.5	0	0	1	-0.5	0	2
X_3	3	0	0.5	0.5	1	0	0	0.5	0	2
X_7	4	0	0.5	0.5	0	0	0	-0.5	1	1

Bas Var	Eq No	z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Right Side
Z	0	1	0.6667	0	0	0	1.3333	2.3333	0	14.6667
X_4	1	0	1.3333	0	0	1	-0.333	-0.333	0	1.33333
X_2	2	0	0.3333	1	0	0	0.6667	-0.333	0	1.33333
X_3	3	0	0.3333	0	1	0	-0.333	0.6667	0	1.33333
X_7	4	0	0.3333	0	0	0	-0.333	-0.333	1	0.33333

題目 4.6-2 (a) (b) (c) (d) (e) (f)

4.6-2.

(a) - (b) Initial artificial BF solution: $(0, 0, 0, 0, 300, 300)$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-10M -4	-4M -2	-5M -3	-7M -5	0	0	-600M
X_5	1	0	2	3	4	2	1	0	300
X_6	2	0	8	1	1	5	0	1	300

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	-2.75M -1.5	-3.75M -2.5	-0.75M -2.5	0	1.25M +0.5	-225M +150
X_5	1	0	0	2.75	3.75	0.75	1	-0.25	225
X_1	2	0	1	0.125	0.125	0.625	0	0.125	37.5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	0.3333	0	-2	1M +0.667	1M +0.333	300
X_3	1	0	0	0.7333	1	0.2	0.2667	-0.067	60
X_1	2	0	1	0.0333	0	0.6	-0.033	0.1333	30

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	3.3333	0.4444	0	0	1M +0.556	1M +0.778	400
X_3	1	0	-0.333	0.7222	1	0	0.2778	-0.111	50
X_4	2	0	1.6667	0.0556	0	1	-0.056	0.2222	50

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, 50, 50)$ and $Z^* = 400$

(c) - (d) - (e) - (f) Initial artificial BF solution: $(0, 0, 0, 0, 300, 300)$

Phase 1:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-10	-4	-5	-7	0	0	-600
X_5	1	0	2	3	4	2	1	0	300
X_6	2	0	8	1	1	5	0	1	300

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	-2.75	-3.75	-0.75	0	1.25	-225
X5	1	0	0	2.75	3.75	0.75	1	-0.25	225
X1	2	0	1	0.125	0.125	0.625	0	0.125	37.5

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	0	0	0	1	1	0
X3	1	0	0	0.7333	1	0.2	0.2667	-0.067	60
X1	2	0	1	0.0333	0	0.6	-0.033	0.1333	30

Phase 2:

Bas Var	Eq No	Z	X1	X2	X3	X4	Right Side
Z	0	1	0	0.3333	0	-2	300
X3	1	0	0	0.7333	1	0.2	60
X1	2	0	1	0.0333	0	0.6	30

Bas Var	Eq No	Z	X1	X2	X3	X4	Right Side
Z	0	1	3.3333	0.4444	0	0	400
X3	1	0	-0.333	0.7222	1	0	50
X4	2	0	1.6667	0.0556	0	1	50

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, 50, 50)$ and $Z^* = 400$

題目 4.6-7 (a) (b) (c) (d) (e) (f)

4.6-7.

(a) Initial artificial BF solution: $(0, 0, 0, 0, 20, 50)$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
			$-3M$ -2	$-2M$ -5	$-2M$ -3	$1M$	0	0	
Z	0	1	-2	-5	-3	$1M$	0	0	$-70M$
X_5	1	0	1	-2	1	-1	1	0	20
X_6	2	0	2	4	1	0	0	1	50

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 0, 50)$ and $Z^* = 150$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
				$-8M$ -9	$1M$ -1	$-2M$ -2	$3M$ $+2$	0	$-10M$ $+40$
Z	0	1	0	-9	-1	-2	$+2$	0	$+40$
X_1	1	0	1	-2	1	-1	1	0	20
X_6	2	0	0	8	-1	2	-2	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
							$1M$	$1M$	
Z	0	1	0	0	-2.125	0.25	-0.25	$+1.125$	51.25
X_1	1	0	1	0	0.75	-0.5	0.5	0.25	22.5
X_2	2	0	0	1	-0.125	0.25	-0.25	0.125	1.25

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
							$1M$	$1M$	
Z	0	1	2.8333	0	0	-1.167	$+1.167$	$+1.833$	115
X_3	1	0	1.3333	0	1	-0.667	0.6667	0.3333	30
X_2	2	0	0.1667	1	0	0.1667	-0.167	0.1667	5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
								$1M$	
Z	0	1	4	7	0	0	$1M$	$+3$	150
X_3	1	0	2	4	1	0	0	1	50
X_4	2	0	1	6	0	1	-1	1	30

(c) Initial artificial BF solution: $(0, 0, 0, 0, 20, 50)$

Phase 1:

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	X₅	X₆	Right Side
					Coefficient of				
Z	0	1	-3	-2	-2	1	0	0	-70
X₅	1	0	1	-2	1	-1	1	0	20
X₆	2	0	2	4	1	0	0	1	50

(d)

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	X₅	X₆	Right Side
					Coefficient of				
Z	0	1	0	-8	1	-2	3	0	-10
X₁	1	0	1	-2	1	-1	1	0	20
X₆	2	0	0	8	-1	2	-2	1	10

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	X₅	X₆	Right Side
					Coefficient of				
Z	0	1	0	0	0	0	1	1	0
X₁	1	0	1	0	0.75	-0.5	0.5	0.25	22.5
X₂	2	0	0	1	-0.125	0.25	-0.25	0.125	1.25

(e) - (f) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 0, 50)$ and $Z^* = 150$

Phase 2:

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	Right Side
				Coefficient of			
Z	0	1	0	0	-2.125	0.25	51.25
X₁	1	0	1	0	0.75	-0.5	22.5
X₂	2	0	0	1	-0.125	0.25	1.25

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	Right Side
				Coefficient of			
Z	0	1	2.8333	0	0	-1.167	115
X₃	1	0	1.3333	0	1	-0.667	30
X₂	2	0	0.1667	1	0	0.1667	5

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	Right Side
				Coefficient of			
Z	0	1	4	7	0	0	150
X₃	1	0	2	4	1	0	50
X₄	2	0	1	6	0	1	30

題目 4.6-9 (a) (b)

4.6-9.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 15, 15)$ and $Z^* = 90$

Bas Var	Eq No	Z	\multicolumn{6}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	-5M +3	-4M +2	-8M +4	1M	0	0	-180M
X_5	1	0	2	1	3	0	1	0	60
X_6	2	0	3	3	5	-1	0	1	120

Bas Var	Eq No	Z	\multicolumn{6}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0.333M +0.333	-1.33M +0.667	0	1M	2.667M -1.333	0	-20M -80
X_3	1	0	0.6667	0.3333	1	0	0.3333	0	20
X_6	2	0	-0.333	1.3333	0	-1	-1.667	1	20

Bas Var	Eq No	Z	\multicolumn{6}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	X_5	X_6	
							1M	1M	
Z	0	1	0.5	0	0	0.5	-0.5	-0.5	-90
X_3	1	0	0.75	0	1	0.25	0.75	-0.25	15
X_2	2	0	-0.25	1	0	-0.75	-1.25	0.75	15

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 15, 15)$ and $Z^* = 90$

Phase 1:

Bas Var	Eq No	Z	\multicolumn{6}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	-5	-4	-8	1	0	0	-180
X_5	1	0	2	1	3	0	1	0	60
X_6	2	0	3	3	5	-1	0	1	120

Bas Var	Eq No	Z	\multicolumn{6}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0.3333	-1.333	0	1	2.6667	0	-20
X_3	1	0	0.6667	0.3333	1	0	0.3333	0	20
X_6	2	0	-0.333	1.3333	0	-1	-1.667	1	20

Bas Var	Eq No	Z	\multicolumn{6}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	-3e-20	0	0	0	1	1	0
X_3	1	0	0.75	0	1	0.25	0.75	-0.25	15
X_2	2	0	-0.25	1	0	-0.75	-1.25	0.75	15

Phase 2:

Bas Var	Eq No	Z	\multicolumn{4}{c}{Coefficient of}	Right Side
			X_1	X_2	X_3	X_4	
Z	0	1	0.5	0	0	0.5	-90
X_3	1	0	0.75	0	1	0.25	15
X_2	2	0	-0.25	1	0	-0.75	15

題目 4.6-10 (a) (b)

4.6-10.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (20, 30, 0)$ and $Z^* = 120$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-1M +3	2	-1M +7	1M	0	0	-20M
X5	1	0	-1	1	0	0	1	0	10
X6	2	0	2	-1	1	-1	0	1	10

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	-0.5M +3.5	-0.5M +5.5	0.5M +1.5	0	0.5M -1.5	-15M -15
X5	1	0	0	0.5	0.5	-0.5	1	0.5	15
X1	2	0	1	-0.5	0.5	-0.5	0	0.5	5

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	0	2	5	1M -7	1M -5	-120
X2	1	0	0	1	1	-1	2	1	30
X1	2	0	1	0	1	-1	1	1	20

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (20, 30, 0)$ and $Z^* = 120$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-1	0	-1	1	0	0	-20
X5	1	0	-1	1	0	0	1	0	10
X6	2	0	2	-1	1	-1	0	1	10

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	-0.5	-0.5	0.5	0	0.5	-15
X5	1	0	0	0.5	0.5	-0.5	1	0.5	15
X1	2	0	1	-0.5	0.5	-0.5	0	0.5	5

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	0	0	0	1	1	0
X2	1	0	0	1	1	-1	2	1	30
X1	2	0	1	0	1	-1	1	1	20

Phase 2:

Bas Var	Eq No	Z	X1	X2	X3	X4	Right Side
Z	0	1	0	0	2	5	-120
X2	1	0	0	1	1	-1	30
X1	2	0	1	0	1	-1	20

題目 4.7-2 (a) (b) (c) (d)

4.7-2.

(a)
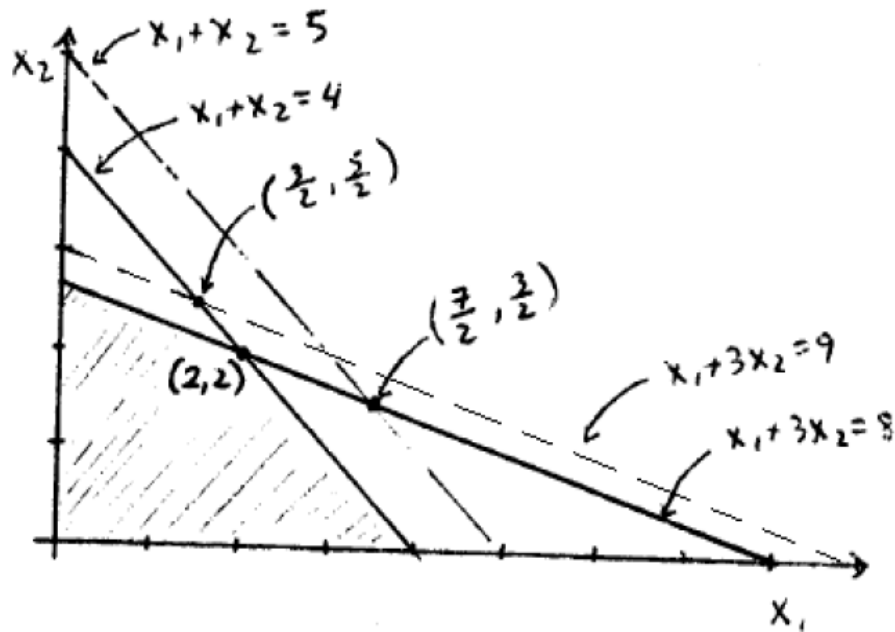
Constraint (1): $x_1 + 3x_2 \leq 8$: $x_1 + 3x_2 = 8 \Rightarrow x_1 = x_2 = 2$ and $Z = 6$

$\qquad\qquad\qquad\qquad x_1 + 3x_2 = 9 \Rightarrow x_1 = 3/2, x_2 = 5/2$ and $Z = 13/2$

$\qquad\qquad\qquad\qquad \Delta Z = 13/2 - 6 = 1/2 = y_1^*$

Constraint (2): $x_1 + x_2 \leq 4$: $x_1 + x_2 = 4 \Rightarrow x_1 = x_2 = 2$ and $Z = 6$

$\qquad\qquad\qquad\qquad x_1 + x_2 = 5 \Rightarrow x_1 = 7/2, x_2 = 3/2$ and $Z = 13/2$

$\qquad\qquad\qquad\qquad \Delta Z = 13/2 - 6 = 1/2 = y_2^*$

(b) From (a), we see that the right-hand sides $b_1 = 8$ and $b_2 = 4$ are sensitive parameters. The graph in part (a) shows that both constraints are active (binding) at the optimal solution, so all the coefficients $a_{11} = 1$, $a_{12} = 3$, $a_{21} = 1$, and $a_{22} = 1$ are sensitive parameters, too. As will be seen in (c), the objective coefficients $c_1 = 1$ and $c_2 = 2$ are not sensitive parameters.

(c) Observe that the optimal solution remains the same for $2/3 \leq c_1 \leq 2$ (with $c_2 = 2$ fixed) and $1 \leq c_2 \leq 3$ (with $c_1 = 1$ fixed)
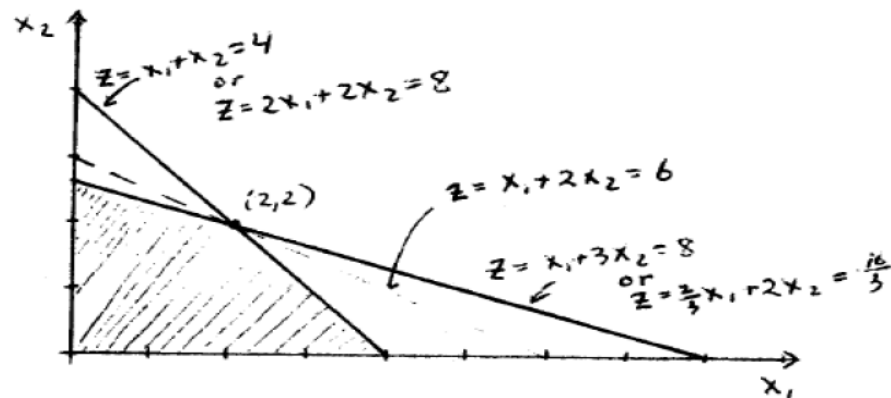

(d) The dashed lines "- - -" in the graph below suggest that the CP solution ranges from $(4, 0)$ to $(0, 4)$ when $4 \leq b_1 \leq 12$. Outside this range, the CP solution becomes infeasible. The dashed lines "- · -" represent the second constraint for different right-hand side values. They suggest that the CP solution ranges from $(0, 8/3)$ to $(0, 8)$ when $8/3 \leq b_2 \leq 8$. Hence, the allowable ranges are $4 \leq b_1 \leq 12$ and $8/3 \leq b_2 \leq 8$.
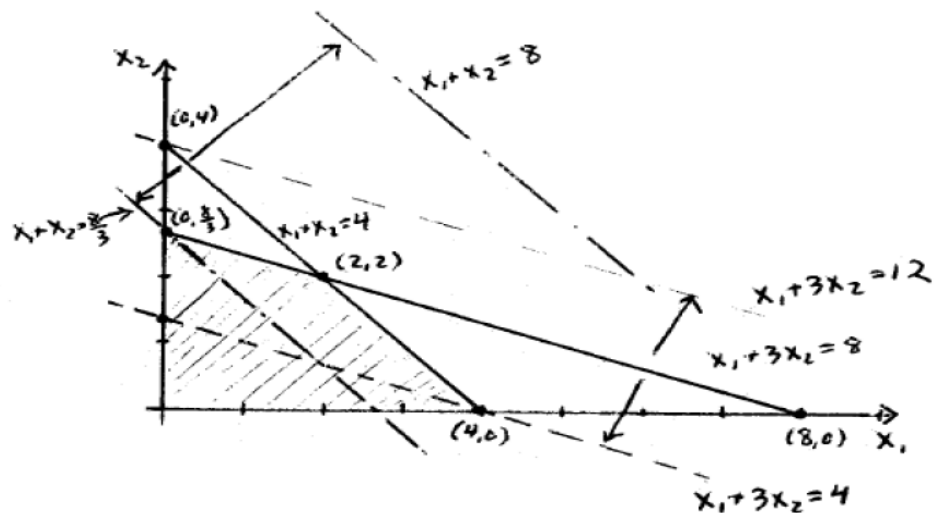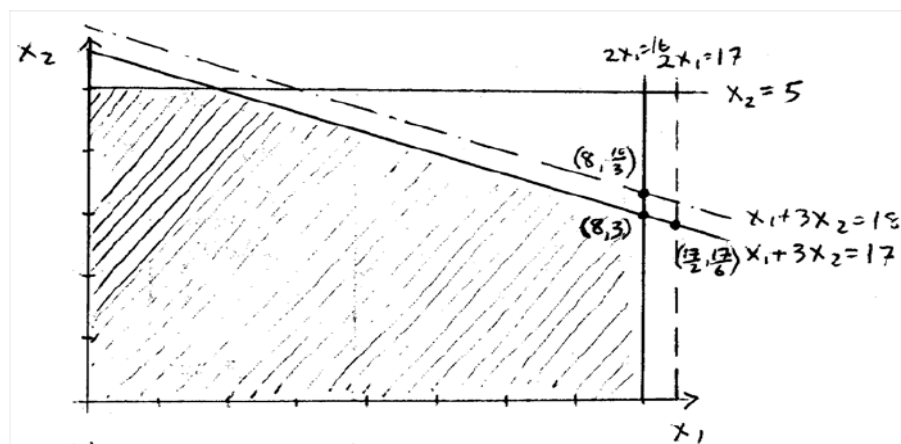
題目 **4.7-3**

4.7-3.

(a) Optimal Solution: $(x_1^*, x_2^*) = (8, 3)$ and $Z^* = 38$
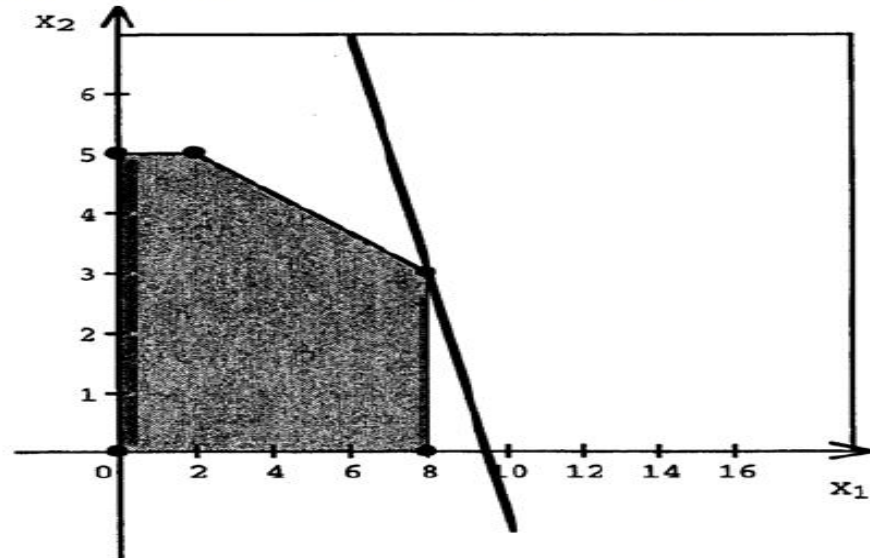
Corner Point	Z
$(8, 3)$	38*
$(8, 0)$	32
$(2, 5)$	18
$(0, 5)$	10
$(0, 0)$	0

(b)

Increasing resource 1 to 17 units increases Z to $4(8.5) + 2(2.83) = 39.67$, so $\Delta Z = y_1^* = 1.67$.

Increasing resource 2 to 18 units increases Z to $4(8) + 2(3.33) = 38.33$, so $\Delta Z = y_2^* = 0.67$.

The third constraint is not binding, so $y_3^* = 0$.

(c) To increase Z by 15, resource 1 should be increased by $\frac{15}{y_1^*} = \frac{15}{1.67} \approx 9$. Solving the LP problem with resource 1 set to $16 + 9 = 25$ returns the result $Z = 53$.

題目 4.7-4 (a) (b)

4.7-4.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0.5, 0, 4.5)$ and $Z^* = 14$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-1	7	-3	0	0	0	0
X4	1	0	2	1	-1	1	0	0	4
X5	2	0	4	-3	0	0	1	0	2
X6	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-10	13	0	0	0	3	9
X4	1	0	-1	3	0	1	0	1	7
X5	2	0	4	-3	0	0	1	0	2
X3	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	5.5	0	0	2.5	3	14
X4	1	0	0	2.25	0	1	0.25	1	7.5
X1	2	0	1	-0.75	0	0	0.25	0	0.5
X3	3	0	0	-0.25	1	0	0.75	1	4.5

(b) The shadow prices for the three resources are given by the reduced costs (in the objective function) for the corresponding slack variables. These values are circled in the table above. The shadow prices for resources 1, 2 and 3 are 0, 2.5 and 3 respectively. They represent the rate at which the objective function value z increases as the corresponding resource is increased. For instance, increasing resource 3 by one unit increases Z by 3, provided that no other constraints cause any trouble.

題目 4.7-5(a) (b)

4.7-5.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 1, 3)$ and $Z^* = 7$

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	X₅	X₆	Right Side
Z	0	1	-2	2	-3	0	0	0	0
X₄	1	0	-1	1	1	1	0	0	4
X₅	2	0	2	-1	1	0	1	0	2
X₆	3	0	1	1	3	0	0	1	12

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	X₅	X₆	Right Side
Z	0	1	4	-1	0	0	3	0	6
X₄	1	0	-3	2	0	1	-1	0	2
X₃	2	0	2	-1	1	0	1	0	2
X₆	3	0	-5	4	0	0	-3	1	6

Bas Var	Eq No	Z	X₁	X₂	X₃	X₄	X₅	X₆	Right Side
Z	0	1	2.5	0	0	0.5	2.5	0	7
X₂	1	0	-1.5	1	0	0.5	-0.5	0	1
X₃	2	0	0.5	0	1	0.5	0.5	0	3
X₆	3	0	1	0	0	-2	-1	1	2

(b) The shadow prices are $y_1^* = 0.5$, $y_2^* = 2.5$ and $y_3^* = 0$. They are the marginal values of resources 1, 2 and 3 respectively.