3. 已知下列成本資料，將卡車指派到總成本最小的運送路線。總成本為何？

| | | 路線 | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | E |
| | 1 | 4 | 5 | 9 | 8 | 7 |
| | 2 | 6 | 4 | 8 | 3 | 5 |
| 卡車 | 3 | 7 | 3 | 10 | 4 | 6 |
| | 4 | 5 | 2 | 5 | 5 | 8 |
| | 5 | 6 | 5 | 3 | 4 | 9 |

3. Given:
Cost information for assigning trucks to delivery routes:

| | | Route | | | | | Row |
|---|---|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** | **E** | **Min.** |
| | **1** | 4 | 5 | 9 | 8 | 7 | 4 |
| | **2** | 6 | 4 | 8 | 3 | 5 | 3 |
| **Truck** | **3** | 7 | 3 | 10 | 4 | 6 | 3 |
| | **4** | 5 | 2 | 5 | 5 | 8 | 2 |
| | **5** | 6 | 5 | 3 | 4 | 9 | 3 |

Step 1:
Row reduction: Subtract the smallest number in each row from every number in the row.

| | | Route | | | | |
|---|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** | **E** |
| | **1** | 0 | 1 | 5 | 4 | 3 |
| | **2** | 3 | 1 | 5 | 0 | 2 |
| **Truck** | **3** | 4 | 0 | 7 | 1 | 3 |
| | **4** | 3 | 0 | 3 | 3 | 6 |
| | **5** | 3 | 2 | 0 | 1 | 6 |
| **Column Min.** | | 0 | 0 | 0 | 0 | 2 |

Step 2:
Column reduction: Subtract the smallest number in each column of the previous table from every number in the column.

| | | Route | | | | |
|---|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** | **E** |
| | **1** | 0 | 1 | 5 | 4 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | **2** | 3 | 1 | 5 | 0 | 0 |
| **Truck** | **3** | 4 | 0 | 7 | 1 | 1 |
| | **4** | 3 | 0 | 3 | 3 | 4 |
| | **5** | 3 | 2 | 0 | 1 | 4 |

Step 3:
Test whether an optimum assignment can be made. Determine the minimum number of lines (horizontal and/or vertical) needed to cover all zeros. If the number of lines equals the number of rows, an optimum assignment is possible.

**Route**

| | | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|---|
| | **1** | 0 | 1 | 5 | 4 | 1 |
| | **2** | 3 | 1 | 5 | 0 | 0 |
| **Truck** | **3** | 4 | 0 | 7 | 1 | 1 |
| | **4** | 3 | 0 | 3 | 3 | 4 |
| | **5** | 3 | 2 | 0 | 1 | 4 |

Note: A minimum of four lines is needed, which is less than the number of rows (five).

Step 4:
Because the number of lines is less than the number of rows, we must modify the table.
a. Subtract the smallest uncovered number (1) from every uncovered number in the table.
b. Add the smallest uncovered number (1) to the numbers at intersections of cross-out lines.
c. Carry over numbers crossed out but not at intersections to the next table.

**Route**

| | | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|---|
| | **1** | 0 | 2 | 6 | 4 | 1 |
| | **2** | 3 | 2 | 6 | 0 | 0 |
| **Truck** | **3** | 3 | 0 | 7 | 0 | 0 |
| | **4** | 2 | 0 | 3 | 2 | 3 |
| | **5** | 2 | 2 | 0 | 0 | 3 |

Step 5:
Test whether an optimum assignment can be made. Determine the minimum number of lines (horizontal and/or vertical) needed to cover all zeros. If the number of lines equals the number of rows, an optimum assignment is possible.

**Route**

| | | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| | **1** | ~~0~~ | ~~2~~ | ~~6~~ | ~~4~~ | ~~1~~ |
| | **2** | ~~3~~ | ~~2~~ | ~~6~~ | ~~0~~ | ~~0~~ |
| **Truck** | **3** | ~~3~~ | ~~0~~ | ~~7~~ | ~~0~~ | ~~0~~ |
| | **4** | ~~2~~ | ~~0~~ | ~~3~~ | ~~2~~ | ~~3~~ |
| | **5** | ~~2~~ | ~~2~~ | ~~0~~ | ~~0~~ | ~~3~~ |

Note: A minimum of five lines is needed. There are five rows. Because the minimum number of lines equals the number of rows, an optimum assignment can be made.

Step 6:
Make the assignments. Begin with rows or columns with only one zero. Match items that have zeros, using only one match for each row and column. Eliminate both the row and the column after the match.

Start with Row 1, Row 4, Column A, & Column C because each has one zero.

| | | **Route** | | | | |
|---|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** | **E** |
| | **1** | [0] | 2 | 6 | 4 | 1 |
| | **2** | 3 | 2 | 6 | [0] | 0 |
| **Truck** | **3** | 3 | 0 | 7 | 0 | [0] |
| | **4** | 2 | [0] | 3 | 2 | 3 |
| | **5** | 2 | 2 | [0] | 0 | 3 |

Note: An alternate solution is possible by flipping Truck 2 & Truck 3: We could assign Truck 2 to Route E & Truck 3 to Route D.

Step 7:
Compute the costs.

| **Assignment** | **Cost** |
|---|---|
| 1-A | $4 |
| 2-D | $3 |
| 3-E | $6 |
| 4-B | $2 |
| 5-C | $3 |
| **Total =** | $18 |

**Alternate Solution:**

| **Assignment** | **Cost** |
|---|---|
| 1-A | $4 |
| 2-E | $5 |
| 3-D | $4 |
| 4-B | $2 |

| | |
|---|---|
| 5-C | $3 |
| **Total =** | $18 |

4. 已知下列資料，建立一個處理成本最小的指派計畫，並解釋你的答案。

| | | 員工 | | |
|---|---|---|---|---|
| | | A | B | C |
| 工作 | 1 | 12 | 8 | 11 |
| | 2 | 13 | 10 | 8 |
| | 3 | 14 | 9 | 14 |
| | 4 | 10 | 7 | 12 |

4. Given:

Processing costs:

| | | **Worker** | | |
|---|---|---|---|---|
| | | **A** | **B** | **C** |
| **Job** | **1** | 12 | 8 | 11 |
| | **2** | 13 | 10 | 8 |
| | **3** | 14 | 9 | 14 |
| | **4** | 10 | 7 | 12 |

Note: The number of jobs exceeds the number of workers. We must add a dummy worker (D) with costs of 0.

| | | **Worker** | | | | **Row** |
|---|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** | **Min.** |
| **Job** | **1** | 12 | 8 | 11 | 0 | 0 |
| | **2** | 13 | 10 | 8 | 0 | 0 |
| | **3** | 14 | 9 | 14 | 0 | 0 |
| | **4** | 10 | 7 | 12 | 0 | 0 |

Step 1:

Row reduction: Subtract the smallest number in each row from every number in the row. Because of the dummy worker, the minimum for each row is 0 and the table will not change.

| | | **Worker** | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Job** | **1** | 12 | 8 | 11 | 0 |
| | **2** | 13 | 10 | 8 | 0 |
| | **3** | 14 | 9 | 14 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| | **4** | 10 | 7 | 12 | 0 |
| **Column Min.** | | 10 | 7 | 8 | 0 |

Step 2:
Column reduction: Subtract the smallest number in each column of the previous table from every number in the column.

| | | **Worker** | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Job** | **1** | 2 | 1 | 3 | 0 |
| | **2** | 3 | 3 | 0 | 0 |
| | **3** | 4 | 2 | 6 | 0 |
| | **4** | 0 | 0 | 4 | 0 |

Step 3:
Test whether an optimum assignment can be made. Determine the minimum number of lines (horizontal and/or vertical) needed to cover all zeros. If the number of lines equals the number of rows, an optimum assignment is possible.

| | | **Worker** | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Job** | **1** | 2 | 1 | 3 | 0 |
| | **2** | 3 | 3 | 0 | 0 |
| | **3** | 4 | 2 | 6 | 0 |
| | **4** | 0 | 0 | 4 | 0 |

Note: A minimum of three lines is needed, which is less than the number of rows (four).

Step 4:
Because the number of lines is less than the number of rows, we must modify the table.

a. Subtract the smallest uncovered number (1) from every uncovered number in the table.
b. Add the smallest uncovered number (1) to the numbers at intersections of cross-out lines.
c. Carry over numbers crossed out but not at intersections to the next table.

| | | **Worker** | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Job** | **1** | 1 | 0 | 3 | 0 |
| | **2** | 2 | 2 | 0 | 0 |
| | **3** | 3 | 1 | 6 | 0 |
| | **4** | 0 | 0 | 5 | 1 |

Step 5:
Test whether an optimum assignment can be made. Determine the minimum number of lines (horizontal and/or vertical) needed to cover all zeros. If the number of lines equals the number of rows, an optimum assignment is possible.

| | | Worker | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Job** | **1** | ~~1~~ | ~~0~~ | ~~3~~ | ~~0~~ |
| | **2** | 2 | 2 | 0 | 0 |
| | **3** | 3 | 1 | 6 | 0 |
| | **4** | ~~0~~ | ~~0~~ | ~~5~~ | ~~1~~ |

Note: A minimum of four lines is needed. There are four rows. Because the minimum number of lines equals the number of rows, an optimum assignment can be made.

Step 6:
Make the assignments. Begin with rows or columns with only one zero. Match items that have zeros, using only one match for each row and column. Eliminate both the row and the column after the match.

Start with Row 3, Column A, & Column C because each has one zero.

| | | Worker | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Job** | **1** | 1 | [0] | 3 | 0 |
| | **2** | 2 | 2 | [0] | 0 |
| | **3** | 3 | 1 | 6 | [0] |
| | **4** | [0] | 0 | 5 | 1 |

Step 7:
Compute the costs.

| **Assignment** | **Cost** |
|---|---|
| 1-B | $8 |
| 2-C | $8 |
| 3-D | $0 |
| 4-A | $10 |
| **Total =** | $26 |

Note: The implication of scheduling Job 3 to the dummy worker (D) is that Job 3 will not be performed.

6.下表的資料是有關 4 個在工作中心等待處理的工作。

| 工作 | 工作時間（天） | 到期日（天） |
|---|---|---|
| A | 14 | 20 |
| B | 10 | 16 |
| C | 7 | 15 |
| D | 6 | 17 |

a.利用(1)FCFS，(2)SPT，(3)EDD，以及(4)CR 來做工作排序。假設上表是依到達次序排列。

b.針對 *a* 小題中的每種方法，決定(1)平均工作流程時間；(2)平均延遲時間；以及(3)在工作中心的平均工作數。

c.是否有特別好的方法？請解釋。

6. Given:

The following table contains information on four jobs waiting for processing at a work center (the jobs are listed in the order of their arrival):

| Job | Job Time (days) | Due Date (days) |
|---|---|---|
| A | 14 | 20 |
| B | 10 | 16 |
| C | 7 | 15 |
| D | 6 | 17 |

a. Determine job sequence for (1) FCFS, (2) SPT, (3) EDD, and (4) CR:

(1) FCFS: A-B-C-D
(2) SPT: D-C-B-A
(3) EDD: C-B-D-A
(4) CR: A-C-D-B (see below for work)

CR Calculations (round to 2 decimals):

$$CR = \frac{Time\ Remaining\ Until\ Due\ Date}{Processing\ Time\ Remaining}$$

Initial critical ratios at day 0:

| Job | Processing Time (Days) | Due Date (days) | Critical Ratio Calculation |
|---|---|---|---|
| A | 14 | 20 | (20 – 0) / 14 = 1.43 (lowest) |
| B | 10 | 16 | (16 – 0) /10 = 1.60 |
| C | 7 | 15 | (15 – 0) / 7 = 2.14 |
| D | 6 | 17 | (17 – 0) / 6 = 2.83 |

Job A is scheduled first.

At day 0 + 14 = 14 [Job A completed], the critical ratios are:

| Job | Processing Time (Days) | Due Date (days) | Critical Ratio Calculation |
|---|---|---|---|
| A | – | – | – |
| B | 10 | 16 | (16 – 14) /10 = 0.20 |
| C | 7 | 15 | (15 – 14) / 7 = 0.14 (lowest) |
| D | 6 | 17 | (17 – 14) / 6 = 0.50 |

Job C is scheduled second.

At day 14 + 7 = 21 [Job C completed], the critical ratios are:

| Job | Processing Time (Days) | Due Date (days) | Critical Ratio Calculation |
|---|---|---|---|
| A | – | – | – |
| B | 10 | 16 | (16 – 21) /10 = –0.50 |
| C | – | – | – |
| D | 6 | 17 | (17 – 21) / 6 = –0.67 (lowest) |

Job D is scheduled third.
At day 21 + 6 = 27 [Job D completed], only Job B remains.

Job B is scheduled fourth and will be completed on day 27 + 10 = 37.

b. For each of the methods above, determine the (1) average flow time, (2) average tardiness, and (3) average number of jobs at the work center.

Job flow time = Time it takes from when job arrives (day 0 in this problem) until it is complete.
Tardiness = Flow time – Due date (0 if negative).
Average number of jobs = Total flow time / Makespan.
Makespan = Total of job times.

| FCFS: | | | Job time | | Flow time | | Due date | | Days |
|---|---|---|---|---|---|---|---|---|---|
| | Job | | (days) | | (days) | | (days) | | tardy |
| | A | | 14 | | 14 | | 20 | | 0 |
| | B | | 10 | | 24 | | 16 | | 8 |
| | C | | 7 | | 31 | | 15 | | 16 |
| | D | | 6 | | 37 | | 17 | | 20 |
| | | | 37 | | 106 | | | | 44 |

Average job flow time = Total job flow time / Number of jobs = 106 / 4 = 26.50 days.

Average job tardiness = Total job tardiness / Number of jobs = 44 / 4 = 11.00 days.

Average number of jobs = Total flow time / Makespan = 106 / 37 = 2.86 jobs.

| SPT: | | | Job time | | Flow time | | Due date | | Days |
|---|---|---|---|---|---|---|---|---|---|
| | Job | | (days) | | (days) | | (days) | | tardy |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D | | 6 | | 6 | | 17 | | 0 |
| | C | | 7 | | 13 | | 15 | | 0 |
| | B | | 10 | | 23 | | 16 | | 7 |
| | A | | 14 | | 37 | | 20 | | 17 |
| | | | 37 | | 79 | | | | 24 |

Average job flow time = Total job flow time / Number of jobs = 79 / 4 = 19.75 days.

Average job tardiness = Total job tardiness / Number of jobs = 24 / 4 = 6.00 days.

Average number of jobs = Total flow time / Makespan = 79 / 37 = 2.14 jobs.

| **EDD:** | | | **Job time** | | **Flow time** | | **Due date** | | **Days** |
|---|---|---|---|---|---|---|---|---|---|
| | **Job** | | **(days)** | | **(days)** | | **(days)** | | **tardy** |
| | C | | 7 | | 7 | | 15 | | 0 |
| | B | | 10 | | 17 | | 16 | | 1 |
| | D | | 6 | | 23 | | 17 | | 6 |
| | A | | 14 | | 37 | | 20 | | 17 |
| | | | 37 | | 84 | | | | 24 |

Average job flow time = Total job flow time / Number of jobs = 84 / 4 = 21.00 days.

Average job tardiness = Total job tardiness / Number of jobs = 24 / 4 = 6.00 days.

Average number of jobs = Total flow time / Makespan = 84 / 37 = 2.27 jobs.

| **CR:** | | | **Job time** | | **Flow time** | | **Due date** | | **Days** |
|---|---|---|---|---|---|---|---|---|---|
| | **Job** | | **(days)** | | **(days)** | | **(days)** | | **tardy** |
| | A | | 14 | | 14 | | 20 | | 0 |
| | C | | 7 | | 21 | | 15 | | 6 |
| | D | | 6 | | 27 | | 17 | | 10 |
| | B | | 10 | | 37 | | 16 | | 21 |
| | | | 37 | | 99 | | | | 37 |

Average job flow time = Total job flow time / Number of jobs = 99 / 4 = 24.75 days.

Average job tardiness = Total job tardiness / Number of jobs = 37 / 4 = 9.25 days.

Average number of jobs = Total flow time / Makespan = 99 / 37 = 2.68 jobs.

c. SPT is superior because it provides the lowest (and therefore best) value for average flow and average number of jobs, while tying EDD for lowest average days tardy.

| 法則 | 平均流程時間(天) | 平均延遲時間(天) | 工作中心的平均工作數 |
| --- | --- | --- | --- |
| FCFS | 26.5 | 11 | 2.86 |
| ✓ SPT | 19.75 | 6 | 2.14 |
| EDD | 21 | 6 | 2.27 |
| CR | 24.75 | 9.25 | 2.68 |