

題號 14.4-1

29.4-1.

- (a) P has one recurrent communicating class: $\{0, 1, 2, 3\}$.
 (b) P has 3 communicating classes: $\{0\}$ absorbing, so recurrent; $\{1, 2\}$ recurrent and $\{3\}$ transient.

題號 14.4-5

29.4-5.

- (a) P has two classes: $\{0, 1, 2, 4\}$ transient and $\{3\}$ recurrent.
 (b) The period of $\{0, 1, 2, 4\}$ is 2 and the period of $\{3\}$ is 1.

題號 14.5-5

29.5-5.

- (a) Assuming demand occurs after delivery of orders:

$$P = \begin{pmatrix} 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.3 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0 & 0 \\ 0 & 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0 \\ 0 & 0 & 0 & 0.1 & 0.2 & 0.3 & 0.4 \\ 0 & 0 & 0 & 0 & 0.1 & 0.2 & 0.7 \end{pmatrix}$$

- (b) $\pi P = \pi$ and $\sum_j \pi_j = 1 \Rightarrow \pi = (0.139 \ 0.139 \ 0.139 \ 0.138 \ 0.141 \ 0.130 \ 0.174)$.

- (c) The steady-state probability that a pint of blood is to be discarded is

$$P(D = 0) \cdot P(\text{state} = 7) = 0.4 \times 0.174 = 0.0696.$$

- (d) $P(\text{need for emergency delivery}) = \sum_{i=1}^2 P(\text{state} = i) \cdot P(D > i)$
 $= 0.139 \times (0.2 + 0.1) + 0.139 \times 0.1$
 $= 0.0556$

題號 14.5-6

29.5-6.

For an (s, S) policy with $s = 2$ and $S = 3$:

$$c(x_{t-1}, D_t) = \begin{cases} 10 + 25(3 - x_{t-1}) + 50\max(D_t - 3, 0) & \text{for } x_{t-1} < 2 \\ 50\max(D_t - x_{t-1}, 0) & \text{for } x_{t-1} \geq 2. \end{cases}$$

$$K(0) = E[c(0, D_t)] = 85 + 50[\sum_{j=4}^{\infty}(j - 3) \cdot P(D_t = j)] \simeq 86.2,$$

$$K(1) = E[c(1, D_t)] = 60 + 50[\sum_{j=4}^{\infty}(j - 3) \cdot P(D_t = j)] \simeq 61.2,$$

$$K(2) = E[c(2, D_t)] = 0 + 50[\sum_{j=4}^{\infty}(j - 2) \cdot P(D_t = j)] \simeq 5.2,$$

$$K(3) = E[c(3, D_t)] = 0 + 50[\sum_{j=4}^{\infty}(j - 2) \cdot P(D_t = j)] \simeq 1.2.$$

$$x_{t+1} = \begin{cases} \max(3 - D_{t+1}, 0) & \text{for } x_t < 2 \\ \max(x_t - D_{t+1}, 0) & \text{for } x_t \geq 2 \end{cases}$$

$$P = \begin{pmatrix} 0.080 & 0.184 & 0.368 & 0.368 \\ 0.080 & 0.184 & 0.368 & 0.368 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{pmatrix}$$

Solving the steady-state equations gives $(\pi_0, \pi_1, \pi_2, \pi_3) = (0.148, 0.252, 0.368, 0.232)$.

Then the long-run average cost per week is $\sum_{j=0}^3 K(j) \cdot \pi_j = 30.37$.

題號 14.5-7

29.5-7.

(a)

$$x_{t+1} = \begin{cases} \max(x_t + 2 - D_{t+1}, 0) & \text{for } x_t \leq 1 \\ \max(x_t - D_{t+1}, 0) & \text{for } x_t \geq 2 \end{cases}$$

$$P = \begin{pmatrix} 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{pmatrix}$$

Solving the steady-state equations gives $(\pi_0, \pi_1, \pi_2, \pi_3) = (0.182, 0.285, 0.368, 0.165)$.

$$(b) \lim_{n \rightarrow \infty} E\left(\frac{1}{n} \sum_{t=1}^n c(x_t)\right) = 0 \cdot \pi_0 + 2 \cdot \pi_1 + 8 \cdot \pi_2 + 18 \cdot \pi_3 = 6.48.$$

題號 14.6-4

29.6-4.

(a)

$$P = \begin{pmatrix} 0.25 & 0.5 & 0.25 \\ 0.75 & 0.25 & 0 \\ 0.25 & 0.5 & 0.25 \end{pmatrix}$$

(b)

$$P^{(2)} = \begin{pmatrix} 0.5 & 0.375 & 0.125 \\ 0.375 & 0.438 & 0.188 \\ 0.5 & 0.375 & 0.125 \end{pmatrix}$$

$$P^{(5)} = \begin{pmatrix} 0.449 & 0.4 & 0.15 \\ 0.451 & 0.399 & 0.149 \\ 0.449 & 0.4 & 0.15 \end{pmatrix}$$

$$P^{(10)} = \begin{pmatrix} 0.45 & 0.4 & 0.15 \\ 0.45 & 0.4 & 0.15 \\ 0.45 & 0.4 & 0.15 \end{pmatrix}$$

$$\begin{aligned} \text{(c)} \quad \mu_{00} &= 1 + 0.5\mu_{10} + 0.25\mu_{20} \\ \mu_{10} &= 1 + 0.25\mu_{10} \\ \mu_{20} &= 1 + 0.5\mu_{10} + 0.25\mu_{20} \end{aligned}$$

$$\Rightarrow \mu_{00} = 20/9, \mu_{10} = 4/3, \mu_{20} = 20/9$$

$$\begin{aligned} \mu_{01} &= 1 + 0.25\mu_{01} + 0.25\mu_{21} \\ \mu_{11} &= 1 + 0.75\mu_{01} \\ \mu_{21} &= 1 + 0.25\mu_{01} + 0.25\mu_{21} \end{aligned}$$

$$\Rightarrow \mu_{01} = 2, \mu_{11} = 2\frac{1}{2}, \mu_{21} = 2$$

$$\begin{aligned} \mu_{02} &= 1 + 0.25\mu_{02} + 0.5\mu_{12} \\ \mu_{12} &= 1 + 0.75\mu_{02} + 0.25\mu_{12} \\ \mu_{22} &= 1 + 0.25\mu_{02} + 0.5\mu_{12} \end{aligned}$$

$$\Rightarrow \mu_{02} = 20/3, \mu_{12} = 8, \mu_{22} = 20/3$$

(d) The steady-state probability vector is (0.45 0.4 0.15).

$$\text{(e)} \quad \pi \cdot C = 0(0.45) + 2(0.4) + 8(0.15) = \$ 2 / \text{week}$$

題號 14.6-5

29.6-5.

(a)

$$P = \begin{pmatrix} 0 & 0.875 & 0.062 & 0.062 \\ 0 & 0.75 & 0.125 & 0.125 \\ 0 & 0 & 0.5 & 0.5 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\pi_0 = 0.154, \pi_1 = 0.538, \pi_2 = 0.154, \text{ and } \pi_3 = 0.154$$

(b) $\pi \cdot C = 1(0.538) + 3(0.154) + 6(0.154) = \1923.08

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(c)
$$\begin{aligned} \mu_{00} &= 1 + 0.875\mu_{10} + 0.0625\mu_{20} + 0.0625\mu_{30} \\ \mu_{10} &= 1 + 0.75\mu_{10} + 0.125\mu_{20} + 0.125\mu_{30} \\ \mu_{20} &= 1 + 0.5\mu_{20} + 0.5\mu_{30} \\ \mu_{30} &= 1 + 0 \end{aligned}$$

So the expected recurrence time for state 0 is $\mu_{00} = 6.5$.

題號 14.7-1

29.7-1.

(a) $P_{00} = P_{TT} = 1$; $P_{i,i-1} = q$; $P_{i,i+1} = p$; $P_{i,k} = 0$ else.

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ q & 0 & p & 0 & \cdots \\ & \vdots & & \ddots & \\ & & q & 0 & p & 0 \\ & & 0 & q & 0 & p \\ & & 0 & 0 & 0 & 1 \end{pmatrix}$$

- (b) Class 1: $\{0\}$ absorbing
 Class 2: $\{T\}$ absorbing
 Class 3: $\{1, 2, \dots, T-1\}$ transient

(c) Let $f_{iK} = P(\text{absorption at } K \text{ starting at } i)$. Then $f_{00} = f_{33} = 1$, $f_{30} = f_{03} = 0$. Since $P_{ij} = 0$ for $|i-j| \neq 1$ and $P_{i,i+1} = p$, $P_{i,i-1} = q$, we get:

$$\begin{aligned} f_{10} &= q + pf_{20} \\ f_{13} &= 1 - f_{10} \\ f_{20} &= qf_{10} \\ f_{23} &= 1 - f_{20} \end{aligned}$$

Solving this system gives

$$f_{10} = \frac{q}{1-pq} = 0.886, f_{13} = 0.114, f_{20} = 0.62, f_{23} = 0.38.$$

(d) Plugging in $p = 0.7$ in the formulas in part (c), we obtain

$$f_{10} = 0.38, f_{13} = 0.62, f_{20} = 0.114, f_{23} = 0.886.$$

Observe that when $p > 1/2$, the drift is towards T and when $p < 1/2$, it is towards 0.

題號 14.7-2

29.7-2.

- (a) 0 = Have to honor warranty
1 = Reorder in 1st year
2 = Reorder in 2nd year
3 = Reorder in 3rd year

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.01 & 0 & 0.99 & 0 \\ 0.05 & 0 & 0 & 0.95 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- (b) The probability that the manufacturer has to honor the warranty is f_{10} .

$$\begin{aligned} f_{10} &= 0.01f_{00} + 0f_{10} + 0.99f_{20} + 0f_{30} \\ f_{20} &= 0.05f_{00} + 0f_{10} + 0f_{20} + 0.95f_{30} \\ f_{00} &= 1 \text{ and } f_{30} = 0 \\ \Rightarrow f_{10} &= 0.01 + 0.99f_{20} \text{ and } f_{20} = 0.05 \\ \Rightarrow f_{10} &= 0.0595 = 5.95\%. \end{aligned}$$

state 0: 兩年內故障 → 必須換貨(保固吸收)

state 1: 第1年使用中(新機出賣後)

state 2: 第2年使用中(第1年沒壞)

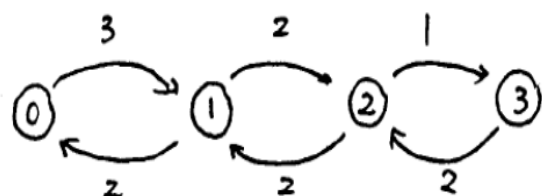
state 3: 兩年都沒壞 → 保固結束(安全吸收)

⇒ f_{10} : 從「第1年開始」最終被吸收在 state 0 的機率
= 「該製造商需要履行保證換貨」的機率

題號 14.8-1

29.8-1.

(a)



(b) Steady-state equations:

$$3\pi_0 = 2\pi_1$$

$$4\pi_1 = 3\pi_0 + 2\pi_2$$

$$3\pi_2 = 2\pi_1 + 2\pi_3$$

$$2\pi_3 = \pi_2$$

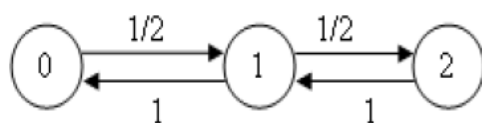
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

(c) Solving the steady-state equations gives $\pi = \left(\frac{4}{19}, \frac{6}{19}, \frac{6}{19}, \frac{3}{19}\right)$.

題號 14.8-2

29.8-2.

(a) Let the state be the number of jobs at the work center.



(b) Steady-state equations:

$$\frac{1}{2}\pi_0 = \pi_1$$

$$\frac{3}{2}\pi_1 = \frac{1}{2}\pi_0 + \pi_2$$

$$\pi_2 = \frac{1}{2}\pi_1$$

$$\pi_0 + \pi_1 + \pi_2 = 1$$

(c) Solving the steady-state equations gives $\pi = \left(\frac{4}{7}, \frac{2}{7}, \frac{1}{7}\right)$.