

第七章

$$7.4 \quad \hat{p}_{sy} = \frac{1}{n} \sum y_i = \frac{132}{200} = .66$$

$$\hat{V}(\hat{p}_{sy}) = \frac{\hat{p}_{sy} \hat{q}_{sy}}{n-1} \left(\frac{N-n}{N} \right) = \frac{(.66)(.34)}{199} \left(\frac{2000-200}{2000} \right)$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = .0637$$

$$7.5 \quad N = 2000, \hat{p}_{sy} = .66, \hat{q}_{sy} = .34, D = B^2 / 4 = (.01)^2 / 4 = (0.005)^2$$

$$n = \frac{Npq}{(N-1)D + pq} \approx \frac{N\hat{p}_{sy}\hat{q}_{sy}}{(N-1)D + \hat{p}_{sy}\hat{q}_{sy}}$$

$$= \frac{2000(.66)(.34)}{1999(.005)^2 + (.66)(.34)} = 1635.72 \approx 1636$$

Note that the sample size nearly equals the population size, so it is not practical to take the sample. One might better measure every employee or, better yet, agree on a larger margin of error for the survey.

7.6-7.7

7.6 $N = 1800, n = 36, \sum y_i = 430.04$

$$S^2 = 0.00581$$

$$\hat{\mu} = \bar{y} = \frac{430.04}{36} = 11.95 \#$$

$$\hat{V}(\hat{\mu}) = \frac{S^2}{n} \left(\frac{N-n}{N} \right) = \frac{0.00581}{36} \left(\frac{1800-36}{1800} \right) = 0.00015816111$$

$$B = 2\sqrt{\hat{V}(\hat{\mu})} = 0.025 \#$$

7.7 $B = 0.03$

$$N = 1800, S^2 = 0.00581, D = \frac{B^2}{4} = \frac{(0.03)^2}{4} = 0.000225$$

$$n = \frac{N\sigma^2}{(N-1)D + \sigma^2} \approx \frac{NS^2}{(N-1)D + S^2} = \frac{1800 \times 0.00581}{(1800-1) \cdot (0.000225) + 0.00581} \approx 25.47$$

$$\approx 26 \#$$

$$\begin{aligned}
 7.9 \quad \hat{p}_{sy} &= \frac{1}{n} \sum y_i = \frac{324}{400} = .81 \\
 \hat{V}(\hat{p}_{sy}) &= \frac{\hat{p}_{sy} \hat{q}_{sy}}{n-1} \left(\frac{N-n}{N} \right) = \frac{.81(.19)}{399} \left(\frac{2800-400}{2800} \right) \\
 B &= 2\sqrt{\hat{V}(\hat{p}_{sy})} = .036
 \end{aligned}$$

7.10

7.10 $n=45$
 $\sum y_i = 90,320$
 $S = 250$
 $\Rightarrow S^2 = 62500$
 $\hat{\mu} = \bar{y} = \frac{90320}{45} = 2007.11$
 $\hat{V}(\hat{\mu}) = \frac{S^2}{n} \left(\frac{N-n}{N} \right) \approx \frac{S^2}{n} = \frac{62500}{45} = 1388.888$
 (當 N 非常大時, 可忽略各 f.p.c (有限母體校正因子))
 $B = 2\sqrt{\hat{V}(\hat{\mu})} = 74.535$ #

$$\begin{aligned}
 7.11 \quad N &= 4500 \quad n = 30 \\
 \sum y_i &= 850 \quad s^2 = 338.64 \\
 \hat{\tau} &= N\bar{y}_{sy} = N\bar{y} = 4500(850/30) = 127500 \\
 \hat{V}(\hat{\tau}) &= N^2 \frac{s^2}{n} \left(\frac{N-n}{N} \right) = (4500)^2 \frac{338.64}{30} \left(\frac{4500-30}{4500} \right) \\
 B &= 2\sqrt{\hat{V}(\hat{\tau})} = 30137.06
 \end{aligned}$$

$$7.16 \quad N = 15200 \quad n = 304 \quad \sum y_i = 88$$

$$\hat{p}_{sy} = \frac{\sum y_i}{n} = \frac{88}{304} = .2895$$

$$\hat{\tau}_{sy} = N\hat{p}_{sy} = N \frac{\sum y_i}{n} = 15200 \left(\frac{88}{304} \right) = 4400$$

$$\begin{aligned} B &= 2\sqrt{\hat{V}(\hat{\tau}_{sy})} = 2N\sqrt{\hat{V}(\hat{p}_{sy})} = 2N\sqrt{\frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1} \left(\frac{N-n}{N} \right)} \\ &= 2(15200)\sqrt{\frac{(.2895)(.7105)}{303} \left(\frac{15200-304}{15200} \right)} = 784.08 \end{aligned}$$

$$7.17 \quad N = 650 \quad n = 65 \quad \sum y_i = 48$$

$$\hat{p}_{sy} = \frac{1}{n} \sum y_i = \frac{48}{65} = .738$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = 2\sqrt{\frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1} \left(\frac{N-n}{N} \right)} = 2\sqrt{\frac{.74(.26)}{64} \left(\frac{650-65}{650} \right)} = .104$$

- 7.18 (a)** $N = \text{number of years in the study (1950 - 1990)} = 41$
 $n = \text{number of births} = 9$

$$\sum y_i = 34153 \quad s^2 = 115959$$

$$\hat{\tau} = N\bar{y}_{sy} = N\bar{y} = 41(34153/9) = 155585.89$$

$$B = 2\sqrt{\hat{V}(\hat{\tau})} = 2\sqrt{N^2 \frac{s^2}{n} \left(\frac{N-n}{N} \right)} = 2\sqrt{(41)^2 \frac{115959}{9} \left(\frac{41-9}{41} \right)} = 8222.95$$

(b) $\sum y_i = 173.60 \quad s^2 = 16.0461$

$$\hat{\mu} = \bar{y} = 173.60/9 = 19.29$$

$$B = 2\sqrt{\hat{V}(\hat{\mu})} = 2\sqrt{\frac{s^2}{n} \left(\frac{N-n}{N} \right)} = 2\sqrt{\frac{16.0461}{9} \left(\frac{41-9}{41} \right)} = 2.36$$

The following plot of year vs. rate shows a definite trend (decreasing trend of 1950-1975, slightly increasing trend of 1975-1990) of birth rate as year advances. The variance of approximation from simple random sampling will overestimate the true variance. Because of trends in the data and the fact that a prediction for 1995 requires extrapolation beyond the range of the data values, the sample mean is not necessarily a good predictor of the birth rate for 1995.



7.27 (a) For Exercise 7.6

Successive Differences (d_i)

.09	.12	.05	.04	.06	.10	.03
.04	.01	.03	.08	.04	.04	.05
.18	.03	.02	.16	.08	.01	.01
.33	.14	.05	.05	.04	.09	.16
.13	.06	.07	.11	.01	.22	.01

$$\sum d_i^2 = .3766$$

$$\hat{V}_d(\bar{y}_{sy}) = \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 = \frac{1800-36}{36(1800)} \frac{1}{2(35)} (.3766) = .000146$$

$$\hat{V}(\bar{y}_{sy}) = \frac{s^2}{n} \left(\frac{N-n}{N} \right) = \frac{.0062}{35} \left(\frac{1800-36}{1800} \right) = .000168$$

(b) For Exercise 7.11

Successive Differences (d_i)

8	9	8	18	6	7
12	11	8	15	9	1
52	12	2	19	87	17
34	34	8	2	72	9
3	22	9	41	1	

$$\sum d_i^2 = 22226$$

$$\begin{aligned} \hat{V}_d(\hat{\tau}_{sy}) &= N^2 \hat{V}_d(\bar{y}_{sy}) = N^2 \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 \\ &= (4500)^2 \frac{4500-30}{30(4500)} \frac{1}{2(29)} (22226) = 256940224.1 \end{aligned}$$

$$\hat{V}(\hat{\tau}_{sy}) = N^2 \frac{s^2}{n} \left(\frac{N-n}{N} \right) = (4500)^2 \frac{338.64}{30} \left(\frac{4500-30}{4500} \right) = 227060586.2$$

(d) For Exercise 7.20 (a),

Successive Differences (d_i)

465 161 498 29 587 468 149 397

$$\sum d_i^2 = 1234,394$$

$$\begin{aligned}\hat{V}_d(\hat{\tau}_{sy}) &= N^2 \hat{V}_d(\bar{y}_{sy}) = N^2 \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 \\ &= (41)^2 \frac{41-9}{9(41)} \frac{1}{2(8)} (1234394) = 11246701\end{aligned}$$

$$\hat{V}(\hat{\tau}_{sy}) = N^2 \frac{s^2}{n} \left(\frac{N-n}{N} \right) = (41)^2 \frac{115959}{9} \left(\frac{41-9}{41} \right) = 16904245$$

For exercise 7.20 (b),

Successive Differences (d_i)

.9 1.3 4.3 1.0 3.8 1.3 .1 .9

$$\sum d_i^2 = 38.94$$

$$\hat{V}_d(\bar{y}_{sy}) = \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 = \frac{41-9}{9(41)} \frac{1}{2(8)} (38.94) = .21$$

$$\hat{V}(\bar{y}_{sy}) = \frac{s^2}{n} \left(\frac{N-n}{N} \right) = \frac{16.0461}{9} \left(\frac{41-9}{41} \right) = 1.39$$