

## 第七章

7.4       $\hat{p}_{sy} = \frac{1}{n} \sum y_i = \frac{132}{200} = .66$

$$\hat{V}(\hat{p}_{sy}) = \frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1} \left( \frac{N-n}{N} \right) = \frac{(.66)(.34)}{199} \left( \frac{2000-200}{2000} \right)$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = .0637$$

7.5       $N = 2000, \hat{p}_{sy} = .66, \hat{q}_{sy} = .34, D = B^2 / 4 = (.01)^2 / 4 = (0.005)^2$

$$n = \frac{Npq}{(N-1)D + pq} \approx \frac{N\hat{p}_{sy}\hat{q}_{sy}}{(N-1)D + \hat{p}_{sy}\hat{q}_{sy}}$$

$$= \frac{2000(.66)(.34)}{1999(.005)^2 + (.66)(.34)} = 1635.72 \approx 1636$$

Note that the sample size nearly equals the population size, so it is not practical to take the sample. One might better measure every employee or, better yet, agree on a larger margin of error for the survey.

### 7.6-7.7

7.6       $N = 1800, n = 36, \sum y_i = 430.04$

$$S^2 = 0.00581$$

$$\hat{\mu} = \bar{y} = \frac{430.04}{36} = 11.95 \times$$

$$\hat{V}(\hat{\mu}) = \frac{S^2}{n} \left( \frac{N-\mu}{N} \right) = \frac{0.00581}{36} \left( \frac{1800-36}{1800} \right) = 0.00015816111$$


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7.7       $B = 2\sqrt{\hat{V}(\hat{\mu})} = 0.025 \times$

7.7       $B = 0.03$

$$N = 1800, S^2 = 0.00581, D = \frac{B^2}{4} = \frac{(0.03)^2}{4} = 0.000225$$

$$n = \frac{N\sigma^2}{(N-1)D + \sigma^2} \approx \frac{N\sigma^2}{(N-1)D + S^2} = \frac{1800 \times 0.00581}{(1800-1) \times 0.000225 + 0.00581} \approx 25.47$$

$$\approx 26 \times$$

$$7.9 \quad \hat{p}_{sy} = \frac{1}{n} \sum y_i = \frac{324}{400} = .81$$

$$\hat{V}(\hat{p}_{sy}) = \frac{\hat{p}_{sy} \hat{q}_{sy}}{n-1} \left( \frac{N-n}{N} \right) = \frac{.81(.19)}{399} \left( \frac{2800-400}{2800} \right)$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = .036$$

7.10

$$7.10 \quad n = 45$$

$$\sum y_i = 90,320$$

$$s = 250$$

$$\Rightarrow s^2 = 62500$$

$$\hat{\mu} = \bar{y} = \frac{90320}{45} = 2007.11$$

$$\hat{V}(\hat{\mu}) = \frac{s^2}{n} \left( \frac{N-n}{N} \right) \approx \frac{s^2}{n} = \frac{62500}{45} = 1388.888$$

(當  $N$  非常大時，可忽略 f.p.c (有限母體校正因子)

$$B = 2\sqrt{\hat{V}(\hat{\mu})} = 74.535 \quad \#$$

$$7.11 \quad N = 4500 \quad n = 30$$

$$\sum y_i = 850 \quad s^2 = 338.64$$

$$\hat{\tau} = Ny_{sy} = N\bar{y} = 4500(850/30) = 127500$$

$$\hat{V}(\hat{\tau}) = N^2 \frac{s^2}{n} \left( \frac{N-n}{N} \right) = (4500)^2 \frac{338.64}{30} \left( \frac{4500-30}{4500} \right)$$

$$B = 2\sqrt{\hat{V}(\hat{\tau})} = 30137.06$$

$$7.16 \quad N = 15200 \quad n = 304 \quad \sum y_i = 88$$

$$\hat{p}_{sy} = \frac{\sum y_i}{n} = \frac{88}{304} = .2895$$

$$\hat{\tau}_{sy} = N\hat{p}_{sy} = N \frac{\sum y_i}{n} = 15200 \left( \frac{88}{304} \right) = 4400$$

$$\begin{aligned} B &= 2\sqrt{\hat{V}(\hat{\tau}_{sy})} = 2N\sqrt{\hat{V}(\hat{p}_{sy})} = 2N\sqrt{\frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1} \left( \frac{N-n}{N} \right)} \\ &= 2(15200)\sqrt{\frac{(0.2895)(0.7105)}{303} \left( \frac{15200-304}{15200} \right)} = 784.08 \end{aligned}$$

$$7.17 \quad N = 650 \quad n = 65 \quad \sum y_i = 48$$

$$\hat{p}_{sy} = \frac{1}{n} \sum y_i = \frac{48}{65} = .738$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = 2\sqrt{\frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1} \left( \frac{N-n}{N} \right)} = 2\sqrt{\frac{.74(.26)}{64} \left( \frac{650-65}{650} \right)} = .104$$

- 7.18 (a)  $N$  = number of years in the study ( $1950 - 1990$ ) = 41  
 $n$  = number of births = 9

$$\sum y_i = 34153 \quad s^2 = 115959$$

$$\hat{\tau} = N\bar{y}_{sy} = N\bar{y} = 41(34153 / 9) = 155585.89$$

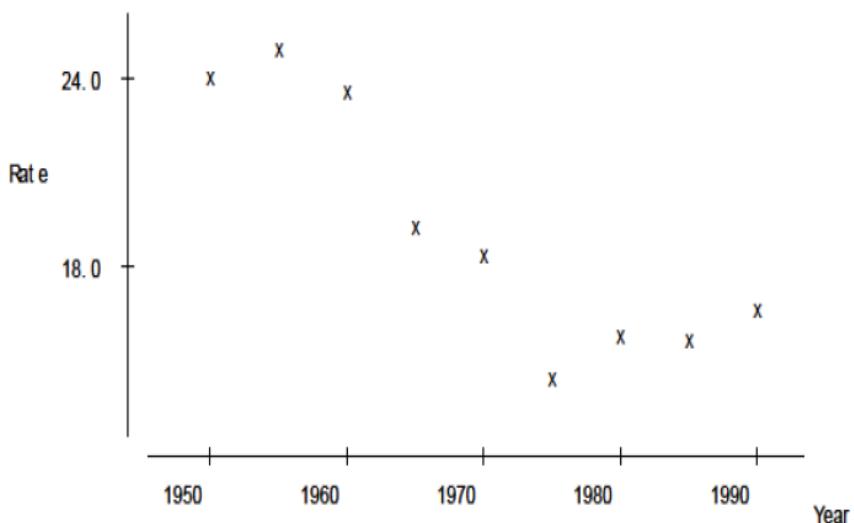
$$B = 2\sqrt{\hat{V}(\hat{\tau})} = 2\sqrt{N^2 \frac{s^2}{n} \left( \frac{N-n}{N} \right)} = 2\sqrt{(41)^2 \frac{115959}{9} \left( \frac{41-9}{41} \right)} = 8222.95$$

(b)  $\sum y_i = 173.60 \quad s^2 = 16.0461$

$$\hat{\mu} = \bar{y} = 173.60 / 9 = 19.29$$

$$B = 2\sqrt{\hat{V}(\hat{\mu})} = 2\sqrt{\frac{s^2}{n} \left( \frac{N-n}{N} \right)} = 2\sqrt{\frac{16.0461}{9} \left( \frac{41-9}{41} \right)} = 2.36$$

The following plot of year vs. rate shows a definite trend (decreasing trend of 1950-1975, slightly increasing trend of 1975-1990) of birth rate as year advances. The variance of approximation from simple random sampling will overestimate the true variance. Because of trends in the data and the fact that a prediction for 1995 requires extrapolation beyond the range of the data values, the sample mean is not necessarily a good predictor of the birth rate for 1995.



**7.27 (a)** For Exercise 7.6

Successive Differences ( $d_i$ )

.09	.12	.05	.04	.06	.10	.03
.04	.01	.03	.08	.04	.04	.05
.18	.03	.02	.16	.08	.01	.01
.33	.14	.05	.05	.04	.09	.16
.13	.06	.07	.11	.01	.22	.01

$$\sum d_i^2 = .3766$$

$$\hat{V}_d(\bar{y}_{sy}) = \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 = \frac{1800-36}{36(1800)} \frac{1}{2(35)} (.3766) = .000146$$

$$\hat{V}(\bar{y}_{sy}) = \frac{s^2}{n} \left( \frac{N-n}{N} \right) = \frac{.0062}{35} \left( \frac{1800-36}{1800} \right) = .000168$$

**(b)** For Exercise 7.11

Successive Differences ( $d_i$ )

8	9	8	18	6	7
12	11	8	15	9	1
52	12	2	19	87	17
34	34	8	2	72	9
3	22	9	41	1	

$$\sum d_i^2 = 22226$$

$$\begin{aligned}\hat{V}_d(\hat{\tau}_{sy}) &= N^2 \hat{V}_d(\bar{y}_{sy}) = N^2 \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 \\ &= (4500)^2 \frac{4500-30}{30(4500)} \frac{1}{2(29)} (22226) = 256940224.1\end{aligned}$$

$$\hat{V}(\hat{\tau}_{sy}) = N^2 \frac{s^2}{n} \left( \frac{N-n}{N} \right) = (4500)^2 \frac{338.64}{30} \left( \frac{4500-30}{4500} \right) = 227060586.2$$

**(d)** For Exercise 7.20 (a),

Successive Differences ( $d_i$ )

$$465 \quad 161 \quad 498 \quad 29 \quad 587 \quad 468 \quad 149 \quad 397$$

$$\sum d_i^2 = 1234,394$$

$$\begin{aligned}\hat{V}_d(\hat{\tau}_{sy}) &= N^2 \hat{V}_d(\bar{y}_{sy}) = N^2 \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 \\ &= (41)^2 \frac{41-9}{9(41)} \frac{1}{2(8)} (1234394) = 11246701\end{aligned}$$

$$\hat{V}(\hat{\tau}_{sy}) = N^2 \frac{s^2}{n} \left( \frac{N-n}{N} \right) = (41)^2 \frac{115959}{9} \left( \frac{41-9}{41} \right) = 16904245$$

For exercise 7.20 (b),

Successive Differences ( $d_i$ )

$$.9 \quad 1.3 \quad 4.3 \quad 1.0 \quad 3.8 \quad 1.3 \quad .1 \quad .9$$

$$\sum d_i^2 = 38.94$$

$$\begin{aligned}\hat{V}_d(\bar{y}_{sy}) &= \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 = \frac{41-9}{9(41)} \frac{1}{2(8)} (38.94) = .21 \\ \hat{V}(\bar{y}_{sy}) &= \frac{s^2}{n} \left( \frac{N-n}{N} \right) = \frac{16.0461}{9} \left( \frac{41-9}{41} \right) = 1.39\end{aligned}$$