第七章

7.4 $\hat{p}_{sy} = \frac{1}{n}\sum y_i = \frac{132}{200} = .66$

$$\hat{V}(\hat{p}_{sy}) = \frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1}\left(\frac{N-n}{N}\right) = \frac{(.66)(.34)}{199}\left(\frac{2000-200}{2000}\right)$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = .0637$$

7.5 $N = 2000,\ \hat{p}_{sy} = .66,\ \hat{q}_{sy} = .34,\ D = B^2/4 = (.01)^2/4 = (0.005)^2$

$$n = \frac{Npq}{(N-1)D + pq} \approx \frac{N\hat{p}_{sy}\hat{q}_{sy}}{(N-1)D + \hat{p}_{sy}\hat{q}_{sy}}$$

$$= \frac{2000(.66)(.34)}{1999(.005)^2 + (.66)(.34)} = 1635.72 \approx 1636$$

Note that the sample size nearly equals the population size, so it is not practical to take the sample. One might better measure every employee or, better yet, agree on a larger margin of error for the survey.

7.6-7.7

$\boxed{7.6}$ $N = 1800,\ n = 36,\ \sum y_i = 430.04$

$s^2 = 0.00581$

$\hat{\mu} = \bar{y} = \frac{430.04}{36} = 11.95$ #

$\hat{V}(\hat{\mu}) = \frac{s^2}{n}\left(\frac{N-n}{N}\right) = \frac{0.00581}{36}\left(\frac{1800-36}{1800}\right) = 0.000158161111$

$B = 2\sqrt{\hat{V}(\hat{\mu})} = 0.025$ #

$\boxed{7.7}$ $B = 0.03$

$N = 1800 \quad s^2 = 0.00581,\ D = \frac{B^2}{4} = \frac{(0.03)^2}{4} = 0.000225$

$$n = \frac{N\sigma^2}{(N-1)D + \sigma^2} \approx \frac{Ns^2}{(N-1)D + s^2} = \frac{1800 \times 0.00581}{(1800-1)\cdot(0.000225) + 0.00581} \approx 25.47$$

≈ 26 #

7.9 $\hat{p}_{sy} = \frac{1}{n}\sum y_i = \frac{324}{400} = .81$

$$\hat{V}(\hat{p}_{sy}) = \frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1}\left(\frac{N-n}{N}\right) = \frac{.81(.19)}{399}\left(\frac{2800-400}{2800}\right)$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = .036$$

7.10

7.10 $n = 45$

$\sum y_i = 90,320$

$s = 250$

$\Rightarrow s^2 = 62500$

$$\hat{\mu} = \bar{y} = \frac{90320}{45} = 2007.11$$

$$\hat{V}(\hat{\mu}) = \frac{s^2}{n}\left(\frac{N-n}{N}\right) \approx \frac{s^2}{n} = \frac{62500}{45} = 1388.888$$

(當N非常大時，可忽略 f.p.c (有限母體校正因子))

$$B = 2\sqrt{\hat{V}(\hat{\mu})} = 74.535$$

#

7.11 $N = 4500 \quad n = 30$

$\sum y_i = 850 \quad s^2 = 338.64$

$$\hat{\tau} = N\bar{y}_{sy} = N\bar{y} = 4500(850/30) = 127500$$

$$\hat{V}(\hat{\tau}) = N^2 \frac{s^2}{n}\left(\frac{N-n}{N}\right) = (4500)^2 \frac{338.64}{30}\left(\frac{4500-30}{4500}\right)$$

$$B = 2\sqrt{\hat{V}(\hat{\tau})} = 30137.06$$

7.16 $N = 15200 \quad n = 304 \quad \sum y_i = 88$

$$\hat{p}_{sy} = \frac{\sum y_i}{n} = \frac{88}{304} = .2895$$

$$\hat{\tau}_{sy} = N\hat{p}_{sy} = N\frac{\sum y_i}{n} = 15200\left(\frac{88}{304}\right) = 4400$$

$$B = 2\sqrt{\hat{V}(\hat{\tau}_{sy})} = 2N\sqrt{\hat{V}(\hat{p}_{sy})} = 2N\sqrt{\frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1}\left(\frac{N-n}{N}\right)}$$

$$= 2(15200)\sqrt{\frac{(.2895)(.7105)}{303}\left(\frac{15200-304}{15200}\right)} = 784.08$$

7.17 $N = 650 \quad n = 65 \quad \sum y_i = 48$

$$\hat{p}_{sy} = \frac{1}{n}\sum y_i = \frac{48}{65} = .738$$

$$B = 2\sqrt{\hat{V}(\hat{p}_{sy})} = 2\sqrt{\frac{\hat{p}_{sy}\hat{q}_{sy}}{n-1}\left(\frac{N-n}{N}\right)} = 2\sqrt{\frac{.74(.26)}{64}\left(\frac{650-65}{650}\right)} = .104$$

7.18 **(a)** N = number of years in the study (1950 - 1990) = 41

n = number of births = 9

$$\sum y_i = 34153 \quad s^2 = 115959$$

$$\hat{\tau} = N\bar{y}_{sy} = N\bar{y} = 41(34153/9) = 155585.89$$

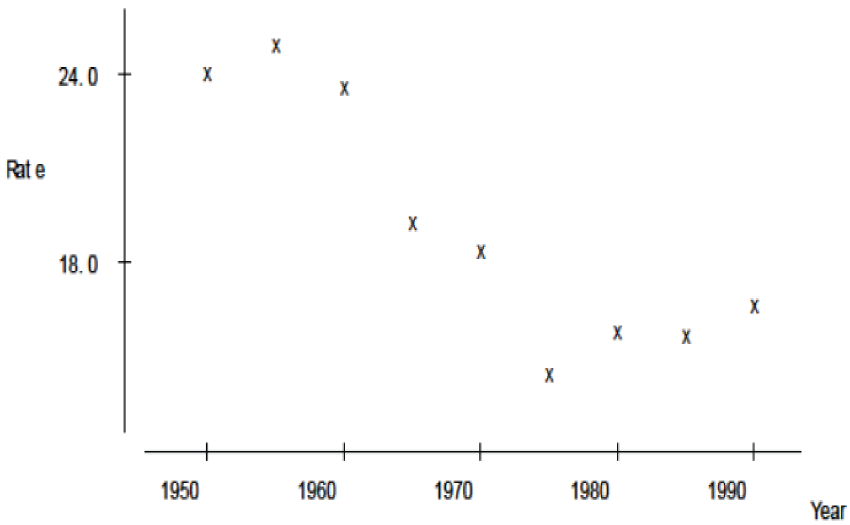
$$B = 2\sqrt{\hat{V}(\hat{\tau})} = 2\sqrt{N^2 \frac{s^2}{n}\left(\frac{N-n}{N}\right)} = 2\sqrt{(41)^2 \frac{115959}{9}\left(\frac{41-9}{41}\right)} = 8222.95$$

(b) $\sum y_i = 173.60 \quad s^2 = 16.0461$

$$\hat{\mu} = \bar{y} = 173.60/9 = 19.29$$

$$B = 2\sqrt{\hat{V}(\hat{\mu})} = 2\sqrt{\frac{s^2}{n}\left(\frac{N-n}{N}\right)} = 2\sqrt{\frac{16.0461}{9}\left(\frac{41-9}{41}\right)}2.36$$

The following plot of year vs. rate shows a definite trend (decreasing trend of 1950-1975, slightly increasing trend of 1975-1990) of birth rate as year advances. The variance of approximation from simple random sampling will overestimate the true variance. Because of trends in the data and the fact that a prediction for 1995 requires extrapolation beyond the range of the data values, the sample mean is not necessarily a good predictor of the birth rate for 1995.



7.27

(a)

7.27

(a) For exercise 7.6

Successive Differences (d_i)

0.09	0.12	0.05	0.04	0.06	0.10	0.03
0.04	0.01	0.03	0.08	0.04	0.04	0.05
0.18	0.03	0.02	0.16	0.08	0.01	0.01
0.3	0.14	0.05	0.05	0.04	0.09	0.16
0.1	0.06	0.07	0.11	0.01	0.22	0.01

$\sum d_i^2 = 0.3508$

$$\hat{V}_d(\bar{y}_{sy}) = \frac{N-n}{nN} \cdot \frac{1}{2(n-1)} \sum d_i^2 = \frac{1800-36}{36(1800)} \cdot \frac{1}{2(35)} (0.3508) = 0.000136$$

$$\hat{V}(\bar{y}_{sy}) = \frac{s^2}{n} \cdot \left(\frac{N-n}{N}\right) = \frac{0.00581}{36} \left(\frac{1800-36}{1800}\right) = 0.000158$$

#

(b) For Exercise 7.11

Successive Differences (d_i)

8	9	8	18	6	7
12	11	8	15	9	1
52	12	2	19	87	17
34	34	8	2	72	9
3	22	9	41	1	

$$\sum d_i^2 = 22226$$

$$\hat{V}_d(\hat{\tau}_{sy}) = N^2 \hat{V}_d(\bar{y}_{sy}) = N^2 \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2$$

$$= (4500)^2 \frac{4500-30}{30(4500)} \frac{1}{2(29)} (22226) = 256940224.1$$

$$\hat{V}(\hat{\tau}_{sy}) = N^2 \frac{s^2}{n} \left(\frac{N-n}{N}\right) = (4500)^2 \frac{338.64}{30} \left(\frac{4500-30}{4500}\right) = 227060586.2$$

(d) For Exercise 7.20 (a),

Successive Differences (d_i)

465	161	498	29	587	468	149	397

$$\sum d_i^2 = 1234{,}394$$

$$\hat{V}_d(\hat{\tau}_{sy}) = N^2 \hat{V}_d(\bar{y}_{sy}) = N^2 \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2$$

$$= (41)^2 \frac{41-9}{9(41)} \frac{1}{2(8)} (1234394) = 11246701$$

$$\hat{V}(\hat{\tau}_{sy}) = N^2 \frac{s^2}{n} \left(\frac{N-n}{N} \right) = (41)^2 \frac{115959}{9} \left(\frac{41-9}{41} \right) = 16904245$$

For exercise 7.20 (b),

Successive Differences (d_i)

.9	1.3	4.3	1.0	3.8	1.3	.1	.9

$$\sum d_i^2 = 38.94$$

$$\hat{V}_d(\bar{y}_{sy}) = \frac{N-n}{nN} \frac{1}{2(n-1)} \sum d_i^2 = \frac{41-9}{9(41)} \frac{1}{2(8)} (38.94) = .21$$

$$\hat{V}(\bar{y}_{sy}) = \frac{s^2}{n} \left(\frac{N-n}{N} \right) = \frac{16.0461}{9} \left(\frac{41-9}{41} \right) = 1.39$$