

110 學年度第二學期五專(日語一甲)數學第一次小考

分數欄

學號：_____ 姓名：_____

一、單一選擇題(共 60 分,每題 12 分)

1. (**C**) 設 θ 為銳角，若 $\tan \theta = 3$ ，則 $\frac{\sin \theta - 2 \cos \theta}{2 \cos \theta + \sin \theta} = ?$ (A) $-\frac{1}{3}$ (B) $-\frac{1}{5}$ (C) $\frac{1}{5}$ (D) $\frac{1}{3}$

解析：所求 =
$$\frac{\frac{\sin \theta - 2 \cos \theta}{\cos \theta}}{\frac{2 \cos \theta + \sin \theta}{\cos \theta}} = \frac{\frac{\sin \theta}{\cos \theta} - 2}{2 + \frac{\sin \theta}{\cos \theta}} = \frac{\tan \theta - 2}{2 + \tan \theta} = \frac{3 - 2}{2 + 3} = \frac{1}{5}$$

2. (**C**) 2005° 的最小正同界角為何？ (A) 5° (B) 200° (C) 205° (D) 25°

解析： $2005^\circ - 5 \times 360^\circ = 205^\circ$

3. (**D**) $\sin 30^\circ + \cos 30^\circ \times \tan 30^\circ - \sin 45^\circ \times \cos 45^\circ = ?$ (A) 0 (B) 1 (C) -1 (D) $\frac{1}{2}$

解析：原式 =
$$\frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

4. (**A**) $\cos^2 80^\circ + \cos^2 10^\circ = ?$ (A) 1 (B) 2 (C) 3 (D) 4

解析：
$$\cos^2 80^\circ + \cos^2 10^\circ = \underbrace{\sin^2 10^\circ}_{\square\square\square\square} + \cos^2 10^\circ = 1$$

5. (**B**) $-\frac{11\pi}{6}$ 等於多少度？ (A) $(\frac{34.54}{6})^\circ$ (B) -330° (C) -30° (D) $-(\frac{11}{6})^\circ$

解析：
$$-\frac{11\pi}{6} = -\frac{11\pi}{6} \times \frac{180^\circ}{\pi} = -330^\circ$$

二、計算與證明題(共 40 分,每題 20 分)

1. 設 θ 為銳角, 若 $\sin \theta - \cos \theta = \frac{1}{5}$, 則:

$$(1) \sin \theta \cos \theta = \underline{\hspace{2cm}}. \quad (2) \tan \theta + \frac{1}{\tan \theta} = \underline{\hspace{2cm}}. \quad (3) \frac{1}{\cos \theta} - \frac{1}{\sin \theta} = \underline{\hspace{2cm}}.$$

答案: (1) $\frac{12}{25}$ (2) $\frac{25}{12}$ (3) $\frac{5}{12}$

解析: (1) $(\sin \theta - \cos \theta)^2 = (\frac{1}{5})^2$

$$\Rightarrow \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{1}{25} \Rightarrow 1 - 2 \sin \theta \cos \theta = \frac{1}{25}$$

$$\Rightarrow 2 \sin \theta \cos \theta = \frac{24}{25} \Rightarrow \sin \theta \cos \theta = \frac{12}{25}$$

$$(2) \tan \theta + \frac{1}{\tan \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} = \frac{1}{\frac{12}{25}} = \frac{25}{12}$$

$$(3) \frac{1}{\cos \theta} - \frac{1}{\sin \theta} = \frac{\sin \theta - \cos \theta}{\sin \theta \cos \theta} = \frac{\frac{1}{5}}{\frac{12}{25}} = \frac{5}{12}$$

2. 一扇形半徑為 4 公分, 圓心角為 135° , 試求此扇形的弧長及面積。

答案: 圓心角 $\theta = 135^\circ = \frac{3\pi}{4}$

(1) 扇形弧長 $S = r\theta = 4 \times \frac{3\pi}{4} = 3\pi$ (公分)

(2) 扇形面積 $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 4^2 \times \frac{3\pi}{4} = 6\pi$ (平方公分)

〈另解〉 $A = \frac{1}{2}rS = \frac{1}{2} \times 4 \times 3\pi = 6\pi$ (平方公分)