學號：＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿　姓名：＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿

一、單一選擇題(共 70 分,每題 10 分)

1. (C) 設 $a = \log_3 2$、$b = \log_3 5$，試利用 a、b 表示，則 $\log_4 75 =$　(A) $\dfrac{b+1}{2a}$　(B) $\dfrac{2b+1}{a}$

(C) $\dfrac{2b+1}{2a}$　(D) $\dfrac{b+1}{a}$

解析：$\log_4 75 = \dfrac{\log_3 75}{\log_3 4} = \dfrac{\log_3 3 + \log_3 25}{2\log_3 2} = \dfrac{2b+1}{2a}$

2. (C) 化簡 $\dfrac{(\cos 12° + i\sin 12°)^{12} \times (\cos 2° + i\sin 2°)^5}{(\cos 10° + i\sin 10°)^4 \times (\cos 3° + i\sin 3°)^8} =$　(A)1　(B)-1　(C)i　(D)$-i$

解析：原式 $= \dfrac{(\cos 144° + i\sin 144°) \times (\cos 10° + i\sin 10°)}{(\cos 40° + i\sin 40°) \times (\cos 24° + i\sin 24°)} = \dfrac{\cos 154° + i\sin 154°}{\cos 64° + i\sin 64°}$

$= \cos(154° - 64°) + i\sin(154° - 64°) = \cos 90° + i\sin 90° = i$

3. (A) 若 $\sin\theta + \cos\theta = \dfrac{1}{4}$，則 $\sin 2\theta = ?$　(A)$-\dfrac{15}{16}$　(B)$-\dfrac{1}{16}$　(C)$\dfrac{1}{16}$　(D)$\dfrac{15}{16}$

解析：$\sin\theta + \cos\theta = \dfrac{1}{4} \overset{\text{平方}}{\Rightarrow} (\sin\theta + \cos\theta)^2 = (\dfrac{1}{4})^2 \Rightarrow 1 + 2\sin\theta\cos\theta = \dfrac{1}{16} \quad \therefore \sin 2\theta = -\dfrac{15}{16}$

4. (A) 化簡 $\log 2 + \log\sqrt{15} - \dfrac{1}{2}\log 0.6 =$　(A)1　(B)0　(C)10　(D)2

解析：原式 $= \log 2 + \log\sqrt{15} - \log\sqrt{\dfrac{6}{10}} = \log\dfrac{2 \times \sqrt{15}}{\sqrt{\dfrac{6}{10}}} = \log 10 = 1$

5. (B) $f(x) = \sin\theta - \sqrt{3}\cos\theta - 5$ 之最大值 M，最小值 m，則 $3M - m = ?$　(A)-1　(B)-2

(C)-3　(D)-4

解析：$-\sqrt{1^2 + (\sqrt{3})^2} \le \sin\theta - \sqrt{3}\cos\theta \le \sqrt{1^2 + (\sqrt{3})^2}$

$\Rightarrow -2 \le \sin\theta - \sqrt{3}\cos\theta \le 2 \Rightarrow -7 \le 原式 \le -3 \quad \therefore M = -3, m = -7$

6. (A) 設 $0 < \alpha < \dfrac{\pi}{2}$, $\dfrac{\pi}{2} < \beta < \pi$，且 $\sin\alpha = \dfrac{3}{5}$, $\sin\beta = \dfrac{7}{25}$，則 $\sin(\alpha - \beta)$ 之值　(A)$\dfrac{-4}{5}$

(B)$-\dfrac{3}{5}$　(C)$\dfrac{44}{125}$　(D)$\dfrac{33}{125}$

解析：$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta = \dfrac{3}{5} \cdot \dfrac{-24}{25} - \dfrac{4}{5} \cdot \dfrac{7}{25} = \dfrac{-100}{125} = -\dfrac{4}{5}$　(正弦差角公式)

7. (B) $Z = -\sqrt{3} + 3i$，則 Z 的主幅角為何？　(A)$\dfrac{\pi}{3}$　(B)$\dfrac{2\pi}{3}$　(C)π　(D)$\dfrac{4}{3}\pi$

解析：$|Z| = \sqrt{\left(-\sqrt{3}\right)^2 + 3^2} = 2\sqrt{3}$,　$Z = -\sqrt{3} + 3i$

$= 2\sqrt{3}\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = 2\sqrt{3}\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$　Z 的主幅角為 $\dfrac{2\pi}{3}$

二、計算與證明題(共 30 分,每題 10 分)

1. $2^{2x} - 3 \times 2^{x-1} - 1 = 0$,試求 x 之值。
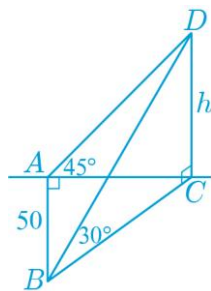
答案：令 $y = 2^x$， $2^{2x} - 3 \times 2^{x-1} - 1 = 0$

$\Rightarrow (2^x)^2 - \dfrac{3}{2} \times 2^x - 1 = 0 \Rightarrow y^2 - \dfrac{3}{2}y - 1 = 0$

$\Rightarrow 2y^2 - 3y - 2 = 0 \Rightarrow (2y+1)(y-2) = 0$

$\Rightarrow y = 2 \text{ 或 } -\dfrac{1}{2} \text{（不合）} \Rightarrow 2^x = 2 \Rightarrow x = 1$

2. 小明在 A 點處觀測熱氣球在其正東方且仰角為 $60°$；小明往正南方前進 100 公尺後到達 B 點,發現熱氣球仰角為 $45°$,則熱氣球的高度為多少公尺？

答案：如圖所示,設熱氣球的高度 $\overline{CD}$ 為 h 公尺

$\triangle DAC(30°-60°-90°) \qquad \dfrac{\overline{AC}}{\overline{CD}} = \cot 60° \Rightarrow \overline{AC} = \dfrac{1}{\sqrt{3}}h$

$\triangle BDC(45°-45°-90°) \qquad \overline{BC} = \overline{CD} = h$

$\because \triangle ABC$ 為直角三角形 $\quad \therefore \overline{BC}^2 = \overline{AC}^2 + \overline{AB}^2$

$\Rightarrow h^2 = \dfrac{1}{3}h^2 + 100^2 \Rightarrow h = 50\sqrt{6}$ 故熱氣球的高度為 $50\sqrt{6}$ 公尺

3. 設 $z_1 = (2+i)^2(1+3i)^2$ 、 $z_2 = (1-i)^4(6-8i)$,試求 $\left|\dfrac{z_1}{z_2}\right|$ 之值。

答案：$\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|} = \dfrac{|(2+i)^2(1+3i)^2|}{|(1-i)^4(6-8i)|} = \dfrac{|2+i|^2|1+3i|^2}{|1-i|^4|6-8i|} = \dfrac{(\sqrt{5})^2 \times (\sqrt{10})^2}{(\sqrt{2})^4 \times \sqrt{100}} = \dfrac{5}{4}$