Chapter 2

Problems

4. $A = \{1,0001,0000001, \ldots\}$ $B = \{01, 00001, 00000001, \ldots\}$

   $(A \cup B)^c = \{00000 \ldots, 001, 000001, \ldots\}$

11. E denotes the event "applicant has experience", and Q denotes the event "applicant has qualifications."

    Based on the given information $P(E) = \frac{8}{12}$, $P(Q) = \frac{6}{12}$, and $P(E \cap Q^c) = \frac{4}{12}$, we obtain the following:

    (a) $E = (E \backslash Q) \cup (E \cap Q)$ gives $\frac{2}{3} = P(E) = P(E \cap Q^c) + P(E \cap Q) = \frac{1}{3} + P(E \cap Q)$ such that

    $P(E \cap Q) = \frac{1}{3}$.

    (b) $P(E^c \cap Q^c) = P((E \cup Q)^c) = 1 - P(E \cup Q) = 1 - \frac{2}{3} - \frac{1}{2} + \frac{1}{3} = \frac{1}{6}$.

33. $\dfrac{\binom{5}{2}\binom{15}{2}}{\binom{20}{4}} = \dfrac{70}{323}$

38. $1/2 = \binom{3}{2} \Big/ \binom{n}{2}$ or $n(n-1) = 12$ or $n = 4$.

41. $1 - \dfrac{5^4}{6^4}$

45. $1/n$ if discard, $\dfrac{(n-1)^{k-1}}{n^k}$ if do not discard

47. $\dfrac{\binom{8}{2}\binom{5}{2}}{\binom{14}{5}} = .1399$

52. (a) $$\frac{20 \cdot 18 \cdot 16 \cdot 14 \cdot 12 \cdot 10 \cdot 8 \cdot 6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}$$

    (b) $$\frac{\binom{10}{1}\binom{9}{6}\frac{8!}{2!}2^6}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}$$

53. Let A_i be the event that couple i sit next to each other. Then

$$P(\cup_{i=1}^4 A_i) = 4\frac{2\cdot 7!}{8!} - 6\frac{2^2\cdot 6!}{8!} + 4\frac{2^3\cdot 5!}{8!} - \frac{2^4\cdot 4!}{8!}$$

and the desired probability is 1 minus the preceding.

55. (a) $P(S \cup H \cup D \cup C) = P(S) + \ldots - P(SHDC)$

$$= \frac{4\binom{2}{2}}{\binom{52}{13}} - \frac{6\binom{2}{2}\binom{2}{2}\binom{48}{9}}{\binom{52}{13}} + \frac{4\binom{2}{2}^3\binom{46}{7}}{\binom{52}{13}} - \frac{\binom{2}{2}^4\binom{44}{5}}{\binom{52}{13}}$$

$$= \frac{4\binom{50}{11} - 6\binom{48}{9} + 4\binom{46}{7} - \binom{44}{5}}{\binom{52}{13}}$$

(b) $$P(1 \cup 2 \cup \ldots \cup 13) = \frac{13\binom{48}{9}}{\binom{52}{13}} - \frac{\binom{13}{2}\binom{44}{5}}{\binom{52}{13}} + \frac{\binom{13}{3}\binom{40}{1}}{\binom{52}{13}}$$

56. Player B. If Player A chooses spinner (a) then B can choose spinner (c). If A chooses (b) then B chooses (a). If A chooses (c) then B chooses (b). In each case B wins probability 5/9.

Self-Test Problems and Exercises

3.

3. 依對稱性，第 14 張牌為 52 張牌中任 1 張的機率均等，故機率為 4/52。一個更正式的推算為計算 52! 個結果中第 14 張為 A 的個數，這產生

$$p = \frac{4 \cdot 51 \cdot 50 \cdots 2 \cdot 1}{(52)!} = \frac{4}{52}$$

令 A 表第 1 個 A 出現在第 14 張牌之事件，則

$$P(A) = \frac{48 \cdot 47 \cdots 36 \cdot 4}{52 \cdot 51 \cdots 40 \cdot 39} = .0312$$

20.

20. 在所有藍球被抽出前所有紅球皆被抽出若且唯若最後被抽出的為藍球。因為所有 30 粒球會被抽出之機率均等，故所求機率為 10 / 30。