Chapter 8

Problems

1. $P\{0 \le X \le 40\} = 1 - P\{|X - 20| > 20\} \ge 1 - 20/400 = 19/20$

4. (a) $P\left\{\sum_{i=1}^{20} X_i > 15\right\} \le 20/15$

(b) $P\left\{\sum_{i=1}^{20} X_i > 15\right\} = P\left\{\sum_{i=1}^{20} X_i > 15.5\right\}$

$$\approx P\left\{Z > \frac{15.5 - 20}{\sqrt{20}}\right\}$$

$$= P\{Z > -1.006\}$$

$$\approx .8428$$

6. If X_i is the outcome of the i^{th} roll then $E[X_i] = 7/2$ $\quad$ $\text{Var}(X_i) = 35/12$ and so

$$P\left\{\sum_{i=1}^{79} X_i \le 300\right\} = P\left\{\sum_{i=1}^{79} X_i \le 300.5\right\}$$

$$\approx P\left\{N(0,1) \le \frac{300.5 - 79(7/2)}{(79 \times 35/12)^{1/2}}\right\} = P\{N(0,1) \le 1.58\} = .9429$$

7. $P\left\{\sum_{i=1}^{100} X_i > 525\right\} \approx P\left\{N(0,1) > \frac{525 - 500}{\sqrt{(100 \times 25)}}\right\} = P\{N(0,1) > .5\} = .3085$

where the above uses that an exponential with mean 5 has variance 25.

8. If we let X_i denote the life of bulb i and let R_i be the time to replace bulb i then the desired probability is $P\left\{\sum_{i=1}^{100} X_i + \sum_{i=1}^{99} R_i \le 550\right\}$. Since $\sum X_i + \sum R_i$ has mean $100 \times 5 + 99 \times .25 = 524.75$ and variance $2500 + 99/48 = 2502$ it follows that the desired probability is approximately equal to $P\{N(0, 1) \le [550 - 524.75]/(2502)^{1/2}\} = P\{N(0, 1) \le .505\} = .693$
It should be noted that the above used that

$$\text{Var}(R_i) = \text{Var}\left(\frac{1}{2}\text{Unif}[0,1]\right) = 1/48$$

Self-Test Problems and Exercises

4.

8.4. If X is the number produced at factory A and Y the number produced at factory B, then

$$E[Y - X] = -2, \quad \mathrm{Var}(Y - X) = 36 + 9 = 45$$

$$P\{Y - X > 0\} = P\{Y - X \geq 1\}$$
$$= P\{Y - X + 2 \geq 3\} \leq \frac{45}{45 + 9} = 45/54$$

4. 若 X 為工廠 A 所生產之數量且 Y 為工廠 B 所生產之數量，則

$$E[Y - X] = -2, \quad \mathrm{Var}(Y - X) = 36 + 9 = 45$$

$$P\{Y - X > 0\} = P\{Y - X \geq 1\} = P\{Y - X + 2 \geq 3\} \leq \frac{45}{45+9} = 45/54$$

7.

8.7. If X is the time required to service a machine, then

$$E[X] = .2 + .3 = .5$$

Also, since the variance of an exponential random variable is equal to the square of its mean, we have

$$\mathrm{Var}(X) = (.2)^2 + (.3)^2 = .13$$

Therefore, with X_i being the time required to service job $i, i = 1, \ldots, 20$, and Z being a standard normal random variable, it follows that

$$P\{X_1 + \cdots + X_{20} < 8\} = P\left\{\frac{X_1 + \cdots + X_{20} - 10}{\sqrt{2.6}} < \frac{8 - 10}{\sqrt{2.6}}\right\}$$
$$\approx P\{Z < -1.24035\}$$
$$\approx .1074$$

7. 若 X 表維修一部機器所需時間，則

$$E[X] = 0.2 + 0.3 = 0.5$$

而且，指數隨機變數的變異數等於其平均數的平方，所以

$$\mathrm{Var}(X) = (0.2)^2 + (0.3)^2 = 0.13$$

因此，令 X_i 表服務工作 i 所花時間 $i = 1, \cdots, 20$，且 Z 為一個標準常態隨機變數，則

$$P\{X_1 + \cdots + X_{20} < 8\} = P\left\{\frac{X_1 + \cdots + X_{20} - 10}{\sqrt{2.6}} < \frac{8-10}{\sqrt{2.6}}\right\}$$
$$\approx P\{Z < -1.24035\}$$
$$\approx .1074$$