

Chapter 8

Problems

1. $P\{0 \leq X \leq 40\} = 1 - P\{|X - 20| > 20\} \geq 1 - 20/400 = 19/20$

4. (a) $P\left\{\sum_{i=1}^{20} X_i > 15\right\} \leq 20/15$

(b) $P\left\{\sum_{i=1}^{20} X_i > 15\right\} = P\left\{\sum_{i=1}^{20} X_i > 15.5\right\}$

$$\approx P\left\{Z > \frac{15.5 - 20}{\sqrt{20}}\right\}$$

$$= P\{Z > -1.006\}$$

$$\approx .8428$$

6. If X_i is the outcome of the i^{th} roll then $E[X_i] = 7/2$ $\text{Var}(X_i) = 35/12$ and so

$$P\left\{\sum_{i=1}^{79} X_i \leq 300\right\} = P\left\{\sum_{i=1}^{79} X_i \leq 300.5\right\}$$

$$\approx P\left\{N(0,1) \leq \frac{300.5 - 79(7/2)}{(79 \times 35/12)^{1/2}}\right\} = P\{N(0,1) \leq 1.58\} = .9429$$

Self-Test Problems and Exercises

4.

8.4. If X is the number produced at factory A and Y the number produced at factory B , then

$$\begin{aligned} E[Y - X] &= -2, \quad \text{Var}(Y - X) = 36 + 9 = 45 \\ P\{Y - X > 0\} &= P\{Y - X \geq 1\} \\ &= P\{Y - X + 2 \geq 3\} \leq \frac{45}{45 + 9} = 45/54 \end{aligned}$$

4. 若 X 為工廠 A 所生產之數量且 Y 為工廠 B 所生產之數量，則

$$\begin{aligned} E[Y - X] &= -2, \quad \text{Var}(Y - X) = 36 + 9 = 45 \\ P\{Y - X > 0\} &= P\{Y - X \geq 1\} = P\{Y - X + 2 \geq 3\} \leq \frac{45}{45 + 9} = 45/54 \end{aligned}$$

7.

8.7. If X is the time required to service a machine, then

$$E[X] = .2 + .3 = .5$$

Also, since the variance of an exponential random variable is equal to the square of its mean, we have

$$\text{Var}(X) = (.2)^2 + (.3)^2 = .13$$

Therefore, with X_i being the time required to service job i , $i = 1, \dots, 20$, and Z being a standard normal random variable, it follows that

$$\begin{aligned} P\{X_1 + \dots + X_{20} < 8\} &= P\left\{ \frac{X_1 + \dots + X_{20} - 10}{\sqrt{2.6}} < \frac{8 - 10}{\sqrt{2.6}} \right\} \\ &\approx P\{Z < -1.24035\} \\ &\approx .1074 \end{aligned}$$

7. 若 X 表維修一部機器所需時間，則

$$E[X] = 0.2 + 0.3 = 0.5$$

而且，指數隨機變數的變異數等於其平均數的平方，所以

$$\text{Var}(X) = (0.2)^2 + (0.3)^2 = 0.13$$

因此，令 X_i 表服務工作 i 所花時間 $i = 1, \dots, 20$ ，且 Z 為一個標準常態隨機變數，則

$$\begin{aligned} P\{X_1 + \dots + X_{20} < 8\} &= P\left\{ \frac{X_1 + \dots + X_{20} - 10}{\sqrt{2.6}} < \frac{8 - 10}{\sqrt{2.6}} \right\} \\ &\approx P\{Z < -1.24035\} \\ &\approx .1074 \end{aligned}$$