# Chapter 1

## Problems

10. (a) $8! = 40{,}320$

(b) $2 \cdot 7! = 10{,}080$

(c) $5!4! = 2{,}880$

(d) $4!2^4 = 384$

14. (a) $30^5$

(b) $30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$

21. (a) There are $\binom{8}{3}\binom{4}{3} + \binom{8}{3}\binom{2}{1}\binom{4}{2} = 896$ possible committees.

There are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the 2 men, and there are $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly 1 of them.

(b) There are $\binom{6}{3}\binom{6}{3} + \binom{2}{1}\binom{6}{2}\binom{6}{3} = 1000$ possible committees.

(c) There are $\binom{7}{3}\binom{5}{3} + \binom{7}{2}\binom{5}{3} + \binom{7}{3}\binom{5}{2} = 910$ possible committees. There are $\binom{7}{3}\binom{5}{3}$ in which neither feuding party serves; $\binom{7}{2}\binom{5}{3}$ in which the feuding women serves; and $\binom{7}{3}\binom{5}{2}$ in which the feuding man serves.

29.

$$(x_1 + 2x_2 + 3x_3)^4 = \sum_{\substack{(n_1,n_2,n_3):\\ n_1+n_2+n_3=4}} \binom{4}{n_1, n_2, n_3} x_1^{n_1} (2x_2)^{n_2} (3x_3)^{n_3}$$

31. The total number of ways is $3^{10} = 59{,}049$.

(a) One among the 3 friends will be given 4 presents and the other 2 friends will be given 3 presents each. Therefore, the total number of ways is $\binom{10}{4,3,3} + \binom{10}{3,4,3} + \binom{10}{3,3,4} = 3\left(\frac{10!}{4!3!3!}\right) = 12{,}600$.

34. (a) number of nonnegative integer solutions of $x_1 + x_2 + x_3 + x_4 = 8$.

Hence, answer is $\binom{11}{3} = 165$

(b) here it is the number of positive solutions—hence answer is $\binom{7}{3} = 35$

35. (a) number of nonnegative solutions of $x_1 + \ldots + x_6 = 8$

answer = $\binom{13}{5}$

(b) (number of solutions of $x_1 + \ldots + x_6 = 5$) × (number of solutions of $x_1 + \ldots + x_6 = 3$) = $\binom{10}{5}\binom{8}{5}$

36. (a) $x_1 + x_2 + x_3 + x_4 = 20$, $x_1 \geq 2$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$

Let $y_1 = x_1 - 1$, $y_2 = x_2 - 1$, $y_3 = x_3 - 2$, $y_4 = x_4 - 3$

$y_1 + y_2 + y_3 + y_4 = 13$, $y_i > 0$

Hence, there are $\binom{12}{3} = 220$ possible strategies.

(b) there are $\binom{15}{2}$ investments only in 1, 2, 3

there are $\binom{14}{2}$ investments only in 1, 2, 4

there are $\binom{13}{2}$ investments only in 1, 3, 4

there are $\binom{13}{2}$ investments only in 2, 3, 4

$\binom{15}{2} + \binom{14}{2} + 2\binom{13}{2} + \binom{12}{3} = 552$ possibilities

37. (a) $\binom{14}{4} = 1001$

(b) $\binom{10}{3} = 120$

(c) There are $\binom{13}{3} = 286$ possible outcomes having 0 trout caught and $\binom{12}{3} = 220$ possible outcomes having 1 trout caught. Hence, using (a), there are $1001 - 286 - 220 = 495$ possible outcomes in which at least 2 of the 10 are trout.

## Self-Test Problems and Exercises

11.

11. (a) 我們可以將此視為 7 個階段的實驗。首先從團體中選出 6 對代表性的夫妻，然後再從每對中選出 1 人。依一般化的基本計數原理，共有 $\binom{10}{6}2^6$ 不同的選擇。

(b) 首先從團體中選出 6 對夫妻，然後再從其中選出 3 對來貢獻 1 個男人。所以共 $\binom{10}{6}\binom{6}{3}=\frac{10!}{4!3!3!}$ 不同的選擇。

19.

19. 首先選擇 3 個位置給數字，然後放字母和數字。所以，共有 $\binom{8}{3}\cdot 26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 10\cdot 9\cdot 8$ 種不同的牌照。若數字必須連續，則共有 6 種可能的位置給數字，造成了共有 $6\cdot 26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 10\cdot 9\cdot 8$ 種不同的牌照。