Chapter 2

Problems

4. $A = \{1, 0001, 0000001, \ldots\}$ $B = \{01, 00001, 00000001, \ldots\}$

   $(A \cup B)^c = \{00000\ldots, 001, 000001, \ldots\}$

11. E denotes the event "applicant has experience", and Q denotes the event "applicant has qualifications."

    Based on the given information $P(E) = \frac{8}{12}$, $P(Q) = \frac{6}{12}$, and $P(E \cap Q^c) = \frac{4}{12}$, we obtain the following:

    (a) $E = (E \backslash Q) \cup (E \cap Q)$ gives $\frac{2}{3} = P(E) = P(E \cap Q^c) + P(E \cap Q) = \frac{1}{3} + P(E \cap Q)$ such that

    $P(E \cap Q) = \frac{1}{3}$.

    (b) $P(E^c \cap Q^c) = P((E \cup Q)^c) = 1 - P(E \cup Q) = 1 - \frac{2}{3} - \frac{1}{2} + \frac{1}{3} = \frac{1}{6}$.

16. (a) $\dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{6^5}$ (b) $\dfrac{6\binom{5}{2}5 \cdot 4 \cdot 3}{6^5}$ (c) $\dfrac{\binom{6}{2}4\binom{5}{2}\binom{3}{2}}{6^5}$

    (d) $\dfrac{6 \cdot 5 \cdot 4\binom{5}{3}}{6^5}$ (e) $\dfrac{6 \cdot 5\binom{5}{3}}{6^5}$ (f) $\dfrac{6 \cdot 5\binom{5}{4}}{6^5}$

    (g) $\dfrac{6}{6^5}$

17. $\dfrac{\binom{15}{8}\binom{10}{8}\binom{7}{1}}{\binom{25}{16}\binom{9}{1}} = .1102$

27. Imagine that all 10 balls are withdrawn

    $$P(A) = \frac{3 \cdot 9! + 7 \cdot 6 \cdot 3 \cdot 7! + 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 5! + 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 3 \cdot 3!}{10!}$$

28. $P\{\text{same}\} = \dfrac{\binom{5}{3}+\binom{6}{3}+\binom{8}{3}}{\binom{19}{3}}$

$P\{\text{different}\} = \binom{5}{1}\binom{6}{1}\binom{8}{1}\Big/\binom{19}{3}$

If sampling is with replacement

$P\{\text{same}\} = \dfrac{5^3+6^3+8^3}{(19)^3}$

$P\{\text{different}\} = P(RBG) + P\{BRG) + P(\text{RGB}) + \ldots + P(GBR)$

$$= \frac{6\cdot 5\cdot 6\cdot 8}{(19)^3}$$

30. (a) $\dfrac{\binom{7}{3}\binom{8}{3}3!}{\binom{8}{4}\binom{9}{4}4!} = 1/18$

(b) $\dfrac{\binom{7}{3}\binom{8}{3}4!}{\binom{8}{4}\binom{9}{4}4!} - 1/18 = 1/6$

(c) $\dfrac{\binom{7}{3}\binom{8}{4}+\binom{7}{4}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}} = 1/2$

33. $\dfrac{\binom{5}{2}\binom{15}{2}}{\binom{20}{4}} = \dfrac{70}{323}$

35. (a) $$\frac{\binom{12}{3}\binom{16}{2}\binom{18}{2}}{\binom{46}{7}}$$

(b) $$1-\frac{\binom{34}{7}}{\binom{46}{7}}-\frac{\binom{12}{1}\binom{34}{6}}{\binom{46}{7}}$$

(c) $$\frac{\binom{12}{7}+\binom{16}{7}+\binom{18}{7}}{\binom{46}{7}}$$

(d) $$P(R_3 \cup B_3) = P(R_3) + P(B_3) - P(R_3B_3) = \frac{\binom{12}{3}\binom{34}{4}}{\binom{46}{7}} + \frac{\binom{16}{3}\binom{30}{4}}{\binom{46}{7}} - \frac{\binom{12}{3}\binom{16}{3}\binom{18}{1}}{\binom{46}{7}}$$

36. (a) $\binom{4}{2}\Big/\binom{52}{2} \approx .0045$,

(b) $13\binom{4}{2}\Big/\binom{52}{2} = 1/17 \approx .0588$

37. (a) $\binom{7}{5}\Big/\binom{10}{5} = 1/12 \approx .0833$

(b) $\binom{7}{4}\binom{3}{1}\Big/\binom{10}{5} + 1/12 = 1/2$

38. $1/2 = \binom{3}{2}\Big/\binom{n}{2}$ or $n(n-1) = 12$ or $n = 4$.

41. $1 - \frac{5^4}{6^4}$

45. $1/n$ if discard, $\frac{(n-1)^{k-1}}{n^k}$ if do not discard

47. $\frac{\binom{8}{2}\binom{5}{2}}{\binom{14}{5}} = .1399$

52.

52

(a) $\frac{\binom{10}{8}\left(\binom{2}{1}\right)^8}{\binom{20}{8}} = \frac{20\times 18\times 16\times 14\times 12\times 10\times 8\times 6}{20\times 19\times 18\times 17\times 16\times 15\times 14\times 13}$ #

(b) (法一) $\frac{\binom{10}{1}\binom{2}{2}\binom{9}{6}\left(\binom{2}{1}\right)^6}{\binom{20}{8}}$ #

(法二) $\frac{\binom{10}{1}\binom{2}{2}\binom{18}{1}\binom{16}{1}\binom{14}{1}\binom{12}{1}\binom{10}{1}\binom{8}{1}\times\frac{1}{6!}}{\binom{20}{8}}$ #

53. Let A_i be the event that couple i sit next to each other. Then

$$P(\cup_{i=1}^{4} A_i) = 4\frac{2\cdot 7!}{8!} - 6\frac{2^2\cdot 6!}{8!} + 4\frac{2^3\cdot 5!}{8!} - \frac{2^4\cdot 4!}{8!}$$

and the desired probability is 1 minus the preceding.

$P(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c)$

$= P(A_1 \cup A_2 \cup A_3 \cup A_4)^c$

$= 1 - P(A_1 \cup A_2 \cup A_3 \cup A_4)$

55.

55. (a) $P(S \cup H \cup D \cup C) = P(S) + \cdots - P(SHDC)$

$$= \frac{4\binom{2}{2}\binom{50}{11}}{\binom{52}{13}} - \frac{6\binom{2}{2}\binom{2}{2}\binom{48}{9}}{\binom{52}{13}} + \frac{4\binom{2}{2}^3\binom{46}{7}}{\binom{52}{13}} - \frac{\binom{2}{2}^4\binom{44}{5}}{\binom{52}{13}}$$

$$= \frac{4\binom{50}{11} - 6\binom{48}{9} + 4\binom{46}{7} - \binom{44}{5}}{\binom{52}{13}} \qquad \#$$

(b) $P(1 \cup 2 \cup \cdots \cup 13)$

$$= \frac{13\binom{48}{9}}{\binom{52}{13}} - \frac{\binom{13}{2}\binom{44}{5}}{\binom{52}{13}} + \frac{\binom{13}{3}\binom{40}{1}}{\binom{52}{13}} \qquad \#$$

56. Player B. If Player A chooses spinner (a) then B can choose spinner (c). If A chooses (b) then B chooses (a). If A chooses (c) then B chooses (b). In each case B wins probability 5/9.

Self-Test Problems and Exercises

3.

3. 依對稱性，第 14 張牌為 52 張牌中任 1 張的機率均等，故機率為 4/52。一個更正式的推算為計算 52! 個結果中第 14 張為 A 的個數，這產生

$$p = \frac{4 \cdot 51 \cdot 50 \cdots 2 \cdot 1}{(52)!} = \frac{4}{52}$$

令 A 表第 1 個 A 出現在第 14 張牌之事件，則

$$P(A) = \frac{48 \cdot 47 \cdots 36 \cdot 4}{52 \cdot 51 \cdots 40 \cdot 39} = .0312$$

5.

5. (a) $\frac{52 \cdot 48 \cdot 44 \cdot 40}{52 \cdot 51 \cdot 50 \cdot 49} = .6761$

(b) $\frac{52 \cdot 39 \cdot 26 \cdot 13}{52 \cdot 51 \cdot 50 \cdot 49} = .1055$

6.

6. 令 R 和 B 分別表 2 粒球皆為紅色和藍色之事件，則

$$P(R \cup B) = P(R) + P(B) = \frac{3 \cdot 4}{6 \cdot 10} + \frac{3 \cdot 6}{6 \cdot 10} = 1/2$$

20.

20. 在所有藍球被抽出前所有紅球皆被抽出若且唯若最後被抽出的為藍球。因為所有 30 粒球會被抽出之機率均等，故所求機率為 10 / 30。