Chapter 1

Problems

4. There are 4! possible arrangements. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are 2 · 1 · 2 · 1 = 4 possibilities.

5. The total number of possible codes is $10^3 = 1000$.
The total number of possible codes with no digit repeated is ${}^{10}P_3 = 720$.
The total number of possible codes starting with 1 is $10^2 = 100$.

10.

10.

(a) $8!$

(b) $7! \times 2!$

(c) $4! \times \binom{2}{1} \times 4!$

(d) $4! \times 5!$

(e) $4! \times 2^4$

11. (a) 6!

(b) 3!2!3!

(c) 3!4!

12. $10^3 - 10 \cdot 9 \cdot 8 = 280$ numbers have at least 2 equal values. $280 - 10 = 270$ have exactly 2 equal values.

14. (a) 30^5

(b) $30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$

17. There are $\binom{10}{5}\binom{12}{5}$ possible choices of the 5 men and 5 women. They can then be paired up in 5! ways, since if we arbitrarily order the men then the first man can be paired with any of the 5 women, the next with any of the remaining 4, and so on. Hence, there are $5!\binom{10}{5}\binom{12}{5}$ possible results.

19. The first gift can go to any of the 10 children, the second to any of the remaining 9 children, and so on. Hence, there are $10 \cdot 9 \cdot 8 \cdots 5 \cdot 4 = 604{,}800$ possibilities.

21. (a) There are $\binom{8}{3}\binom{4}{3} + \binom{8}{3}\binom{2}{1}\binom{4}{2} = 896$ possible committees.

There are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the 2 men, and there are $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly 1 of them.

(b) There are $\binom{6}{3}\binom{6}{3} + \binom{2}{1}\binom{6}{2}\binom{6}{3} = 1000$ possible committees.

(c) There are $\binom{7}{3}\binom{5}{3} + \binom{7}{2}\binom{5}{3} + \binom{7}{3}\binom{5}{2} = 910$ possible committees. There are $\binom{7}{3}\binom{5}{3}$ in which neither feuding party serves; $\binom{7}{2}\binom{5}{3}$ in which the feuding women serves; and $\binom{7}{3}\binom{5}{2}$ in which the feuding man serves.

24. There are $\frac{4!}{2!2!}$ paths from A to the circled point; and $\frac{3!}{2!1!}$ paths from the circled point to B. Thus, by the basic principle, there are 18 different paths from A to B that go through the circled point.

29.

$$(x_1 + 2x_2 + 3x_3)^4 = \sum_{\substack{(n_1,n_2,n_3):\\ n_1+n_2+n_3=4}} \binom{4}{n_1, n_2, n_3} x_1^{n_1} (2x_2)^{n_2} (3x_3)^{n_3}$$

31. The total number of ways is $3^{10} = 59{,}049$.

(a) One among the 3 friends will be given 4 presents and the other 2 friends will be given 3 presents each. Therefore, the total number of ways is $\binom{10}{4,3,3} + \binom{10}{3,4,3} + \binom{10}{3,3,4} = 3\left(\frac{10!}{4!3!3!}\right) = 12{,}600$.

34. (a) number of nonnegative integer solutions of $x_1 + x_2 + x_3 + x_4 = 8$.

Hence, answer is $\binom{11}{3} = 165$

(b) here it is the number of positive solutions—hence answer is $\binom{7}{3} = 35$

35. (a) number of nonnegative solutions of $x_1 + \dots + x_6 = 8$

answer = $\binom{13}{5}$

(b) (number of solutions of $x_1 + \ldots + x_6 = 5$) × (number of solutions of $x_1 + \ldots + x_6 = 3$) = $\binom{10}{5}\binom{8}{5}$

36. (a) $x_1 + x_2 + x_3 + x_4 = 20$, $x_1 \geq 2$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$
Let $y_1 = x_1 - 1$, $y_2 = x_2 - 1$, $y_3 = x_3 - 2$, $y_4 = x_4 - 3$

$y_1 + y_2 + y_3 + y_4 = 13$, $y_i > 0$

Hence, there are $\binom{12}{3}$ = 220 possible strategies.

(b) there are $\binom{15}{2}$ investments only in 1, 2, 3

there are $\binom{14}{2}$ investments only in 1, 2, 4

there are $\binom{13}{2}$ investments only in 1, 3, 4

there are $\binom{13}{2}$ investments only in 2, 3, 4

$\binom{15}{2} + \binom{14}{2} + 2\binom{13}{2} + \binom{12}{3}$ = 572 possibilities

37. (a) $\binom{14}{4} = 1001$

(b) $\binom{10}{3} = 120$

(c) There are $\binom{13}{3} = 286$ possible outcomes having 0 trout caught and $\binom{12}{3} = 220$ possible outcomes having 1 trout caught. Hence, using (a), there are $1001 - 286 - 220 = 495$ possible outcomes in which at least 2 of the 10 are trout.

(a) 令 x_i 分別代表第 i 種魚的補獲數量, $i=1,2,\ldots,5$

$\Rightarrow \quad x_1 + x_2 + \cdots + x_5 = 10, \quad x_i \geq 0, \ i=1,2,\ldots,5$

$\therefore \quad \binom{10+5-1}{5-1} = \binom{14}{4} = 1001$ #

(b) 令 x_1 代表鱒魚數量 $\Rightarrow x_1 = 3$

因此 $x_2 + x_3 + x_4 + x_5 = 10 - 3 = 7, \quad x_i \geq 0, \ i=2,3,\ldots,5$

$\therefore \quad \binom{7+4-1}{4-1} = \binom{10}{3} = 120$ #

(c) 令 x_1 代表鱒魚數量

10條中至少2條是鱒魚

$\Rightarrow$ 全部 − 0條 − 1條

0條 $\Rightarrow \quad x_2 + x_3 + x_4 + x_5 = 10 \Rightarrow \binom{10+4-1}{4-1} = \binom{13}{3}$

1條 $\Rightarrow \quad x_2 + x_3 + x_4 + x_5 = 10 - 1 = 9 \Rightarrow \binom{9+4-1}{4-1} = \binom{12}{3}$

因此 $\binom{14}{4} - \binom{13}{3} - \binom{12}{3}$

$= 1001 - 286 - 220 = 495$ #

Self-Test Problems and Exercises

1.

1. (a) $2\cdot5\cdot4! = 240$ 個相異的排列法

(b) 共有 $6! = 720$ 個可能的排行法，但其中 A 在 B 之前和 B 在 A 之前的排列法一樣多，所求為 360 個排列法。

(c) 所求為 $720 / 6 = 120$ 個可能的排列法。

(d) 所求為 $360 / 2 = 180$ 個排列具有 A 在 B 之前且 C 在 D 之前。

(e) 所求為 $4\cdot24 = 96$ 個相異的排列法。

(f) E 在最後之排列法有 $5!$ 個。所以有 $6! - 5! = 600$ 個排列使得 E 不是排在最後。

4.

4. (a) $\binom{10}{7}$

(b) $\binom{5}{3}\binom{5}{4}+\binom{5}{4}\binom{5}{3}+\binom{5}{5}\binom{5}{2}$

5.

5. $\binom{7}{3,\,2,\,2} = 210$

11.

11. (a) 我們可以將此視為 7 個階段的實驗。首先從團體中選出 6 對代表性的夫妻，然後再從每對中選出 1 人。依一般化的基本計數原理，共有 $\binom{10}{6}2^6$ 不同的選擇。

(b) 首先從團體中選出 6 對夫妻，然後再從其中選出 3 對來貢獻 1 個男人。所以共 $\binom{10}{6}\binom{6}{3}=\frac{10!}{4!3!3!}$ 不同的選擇。

(b)

$$C^{10對}_{6對} \times C^{6對}_{3對} \times \underbrace{C^{1男}_{1} \times C^{1男} \times C^{1男}}_{來自所挑選3對} \times \underbrace{C^{3對}_{3對}}_{剩餘3對} \times C^{1女}_{1} \times C^{1女}_{1} \times C^{1女}_{1}$$

$\boxed{\text{另3種作法}}$

①

$$C^{10男}_{3男} \times C^{7女}_{3女}$$

②

$$\underbrace{C^{10男}_{1男} \times C^{9男}_{1男} \times C^{8男}_{1男}}_{3男} \times \frac{1}{3!} \times \overbrace{C^{7女}_{1女} \times C^{6女}_{1女} \times C^{5女}_{1女}}^{3女} \times \frac{1}{3!}$$

修正前面3男的順序

修正前面3女的順序

③

$$C^{10對}_{1對} \times C^{1男}_{1男} \times C^{9對}_{1對} \times C^{1男}_{1男} \times C^{8對}_{1對} \times \frac{1}{3!} \times C^{7對}_{1對} \times C^{1女}_{1女} \times C^{6對}_{1對} \times C^{1女}_{1女} \times C^{5對}_{1對} \times C^{1女}_{1女} \times \frac{1}{3!}$$

修正前面3對夫妻的順序

修正後面3對夫妻的順序

13.

13. （方程式 $x_1+\cdots+x_5=4$ 之解的個數）（$x_1+\cdots+x_5=5$ 之解的個數）（$x_1+\cdots+x_5=6$ 之解的個數）$=\binom{8}{4}\binom{9}{4}\binom{10}{4}$

19.

19. 首先選擇 3 個位置給數字，然後放字母和數字。所以，共有 $\binom{8}{3}\cdot 26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 10\cdot 9\cdot 8$ 種不同的牌照。若數字必須連續，則共有 6 種可能的位置給數字，造成了共有 $6\cdot 26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 10\cdot 9\cdot 8$ 種不同的牌照。