

16. $X_1 = 0.567$, $\mu = ?$, $\sigma = 0.018$ kg

已知 $P(-\infty \leq Z_i \leq Z_1) = 0.015 = \int_{-\infty}^{Z_1} f(Z) dZ$

逆查表 A $\Rightarrow Z_1 = -2.17$

$$Z_1 = \frac{X_1 - \mu}{\sigma} = \mu = X_1 - Z_1 \sigma = 0.567 + 2.17 \times 0.018$$

$$\Rightarrow \mu = 0.606$$

第三章

1. 長生不死的機率 = 0, 龜毛兔角的機率 = 0

2. (1) $P(\text{任何點數}) = 1.0$

(2) $P(5 \text{ 點}) = \frac{1}{6}$

(3) $P(\text{非 } 5 \text{ 點}) = 1 - P(5 \text{ 點}) = 1 - \frac{1}{6} = \frac{5}{6}$

3. (1) $P(\text{珠子}) = 1.0$

(2) $P(\text{粉紅色}) = \frac{35}{35+46+15+4} = 0.35$

(3) $P(\text{黑色}) = \frac{0}{100} = 0$

(4) $P(\text{藍色或綠色}) = \frac{46}{100} + \frac{15}{100} = 0.61$

4. 兩次都抽不中的機率 = 第一次沒抽中之機率 \times
第二次沒抽中之機率

$$P(\text{兩次都不中}) = \left(1 - \frac{1}{10}\right) \left(1 - \frac{1}{10}\right) = \frac{81}{100}$$

$$P(\text{其中至少有一次抽中}) = 1 - P(\text{兩次都不中}) = 1 - \frac{81}{100} = \frac{19}{100}$$

$$\text{只抽一次之抽中率} = \frac{20}{100}$$

當然要選擇只抽一次的方式。

5. 本題屬於超幾何分配。

$$N = 34, M = 5, X = 2, n = 2$$

$$P(X = 2) = \frac{\binom{M}{X} \binom{N-M}{n-x}}{\binom{N}{n}} = \frac{5C_2 \cdot 29C_0}{34C_2} = 0.0178$$

6. $N = 500, M = 2$

$$(1) n = 1, x = 1$$

$$P(X = 1) = \frac{\binom{M}{X} \binom{N-M}{n-x}}{\binom{N}{n}} = \frac{2C_1 \times 498C_0}{500C_1} = \frac{2}{500} = 0.004$$

$$(2) n = 5, x = 1$$

$$P(X = 1) = \frac{2C_1 \times 498C_0}{500C_5} = 0.0198$$

(3) 由於 $\frac{n}{N} = \frac{5}{500} = \frac{1}{100} \leq \frac{1}{20}$ ，可用二項分配取代，

$$P = \frac{2}{500} = 0.004$$

$$P(X = 1) = nC_x P^x q^{n-x} = 5C_1 (0.004)^1 (0.996)^4 = 0.0197$$

7. 運用機率的乘法定理

$$P(\text{引擎發動}) = 0.998 \times 0.997 \times 0.999 \times 0.995 = 0.989$$

8. 共有組數 $= 54C_6 = 2.5827 \times 10^7$

$$\text{中獎機率 } P = \frac{1}{2.5827 \times 10^7}$$

$$n = 2, x = 1$$

$$P(X = 1) = nC_x P^x q^{n-x} = 2C_1 \cdot \left(\frac{1}{2.5827 \times 10^7} \right) \times \left(1 - \frac{1}{2.5827 \times 10^7} \right) = \frac{1}{1.291358 \times 10^7}$$

9. $3C_3 = 9C_9 = 35C_{35} = 1$ 其結果相同：

$$nC_r, \text{ 若 } n = r \Rightarrow nC_r = 1$$

$$10. \quad P(X) = \frac{\binom{M}{X} \binom{N-M}{n-X}}{\binom{N}{n}},$$

$$M = 3, N = 12, n = 4, X = 0, 1, 2, 3, 4$$

$$P(X=0) = \frac{3C_0 9C_4}{12C_4} = 0.2545$$

$$P(X=1) = \frac{3C_1 9C_3}{12C_4} = 0.5091$$

$$P(X=2) = \frac{3C_2 9C_2}{12C_4} = 0.2182$$

$$P(X=3) = \frac{3C_3 9C_1}{12C_4} = 0.0182$$

$$P(X=4) = 0 \quad \text{因爲總共只有 3 個不良品，不可能抽中 4 個不良品。}$$

$$\text{查核 } P(X=0) + P(X=1) + P(X=2) + P(X=3) = 1.0$$

11. (1) 這是蒲松氏分配之問題，

$$n = 30, P = 0.05, \Rightarrow m = nP = 30 \times 0.05 = 1.5$$

$$n \leq 5$$

$$P(X \leq 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$\text{直接分別代入公式 } P(X) = \frac{e^{-m} m^x}{x!}, x = 0, 1, 2, 3, 4, 5$$

(2)	x	$P(X) = \frac{e^{-1.5}(1.5)^x}{x!}$	$P(X) = nC_x P^x q^{n-x}$
	0	0.22313	0.21464
	1	0.33467	0.33890
	2	0.25102	0.25684
	3	0.12551	0.12705
	4	0.04707	0.04514
	5	0.01412	0.01235
	$\Sigma P(X) = 0.99552$		0.99672

$$12. \quad N = 20, M = 20 \times \frac{20}{100} = 4, n = 3$$

$$(1) P(X=2) = \frac{\binom{M}{X} \binom{N-M}{n-x}}{\binom{N}{n}}$$

$$\Rightarrow P(X=2) = \frac{4C_2 \cdot 16C_1}{20C_1} = 0.0842$$

$$(2) P(X \geq 2) = P(X=2) + P(X=3) = 0.0842 + 0.0035 = 0.0877$$

$$(3) P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.4912 + 0.4211 + 0.0842 = 0.9965$$

$$\text{核算 } P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= 0.4912 + 0.4211 + 0.0842 + 0.0035 = 1.0$$

13. 二項分配之公式

$$P(X) = nC_x P^x q^{n-x}, P = \frac{6}{100}, q = \frac{94}{100}, n = 5, X \geq 2$$

$$\text{先求 } P(X \leq 1) = P(X=0) + P(X=1) = 5C_0 \left(\frac{6}{100}\right)^0 \left(\frac{94}{100}\right)^5 + 5C_1 \left(\frac{6}{100}\right)^1 \left(\frac{94}{100}\right)^4$$

$$= 0.7339 + 0.2342 = 0.9681$$

$$\text{再求 } P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.9681 = 0.0319$$

14. 猜中之機率 $P = \frac{1}{2}, n = 9, x = 4$

$$P(X=4) = 9C_4 \cdot \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^5 = 0.2461$$

15. 蒲松氏分配

$$P(X) = \frac{e^{-m} m^x}{x!}, n = 10, P = \frac{5}{100} = 0.05 \Rightarrow m = nP = 0.5$$

$$P(X=2) = \frac{e^{-0.5} (0.5)^2}{2!} = 0.0758$$

$$\text{二項分配 } P(X) = nC_x P^x q^{n-x} = 10C_2 (0.05)^2 \cdot (0.95)^8 = 0.0746$$

兩者已經非常接近。

16. $P = 0.08, n = 20, x = 2, 3, 4, \dots$

$$P(X) = \frac{e^{-m} m^x}{x!}$$

先求 $X \leq 1$ 之機率， $m = nP = 20 \times 0.08 = 1.6$

$$P(X \leq 1) = P(X = 0) + P(X = 1) = 0.2019 + 0.3230 = 0.5249$$

$$\text{再求 } P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.5249 = 0.4751$$

17. 先用蒲松氏分配，

$$P = \frac{3}{15} = 0.2, n = 3, x = 1, m = nP = 3 \times 0.2 = 0.6$$

$$P(X \leq 1) = 0.3293$$

再用二項分配

$$P(X = 1) = {}^nC_x P^x q^{n-x} = {}^3C_1 (0.2)^1 (0.8)^2 = 0.384$$

不合理，因為正確答案 0.384 而蒲松氏近似值 = 0.3293 兩者相差過大。

18. 先用蒲松氏分配

$$P = 0.09, n = 67, X = 3 \Rightarrow m = nP = 67 \times 0.09 = 6.03$$

$$\Rightarrow P(X = 3) = 0.0879$$

再用二項分配

$$P(X = 3) = {}^67C_3 (0.09)^3 (0.91)^{64} = 0.0835 \text{ (兩者接近，合理)}$$

第四章

1. 可量測數據：又名連續數據，如長度，重量，時間等數據。

可計數的數據：又名不連續數據不可量測之數據。如不良品之個數。

不良率屬於不連續數據，因為，如不良率為 $\frac{3}{100} = 0.03$ ，看起來好像是有基本量測單位的數據，事實不然，下一個不良率不一定會出現 0.04 或 0.02 之數據，而且它們也沒有基本量測單位之觀念。

2. 數據之收集應該考慮 (1) 澄清數據之收集目的；(2) 有效率的收集數據；(3) 依照數據所顯示的事實，採取行動。