第五章

1.

批　號	不良率 (%)	批　號	不良率 (%)	批　號	不良率 (%)
1	$\frac{3}{200}=1.5$	10	4.5	19	2
2	2.5	11	2.5	20	2.5
3	2.0	12	2.0	21	0.5
4	7.5	13	1.5	22	1.5
5	1.0	14	0.5	23	2.0
6	3.0	15	1.0	24	3.5
7	3.5	16	1.5	25	3.0
8	1.5	17	2.5		
9	0	18	0		

(1) $\sum nP = 107$, $\overline{P} = \dfrac{\sum nP}{\sum n} = \dfrac{107}{5000} = 0.0214 = 2.14\%$
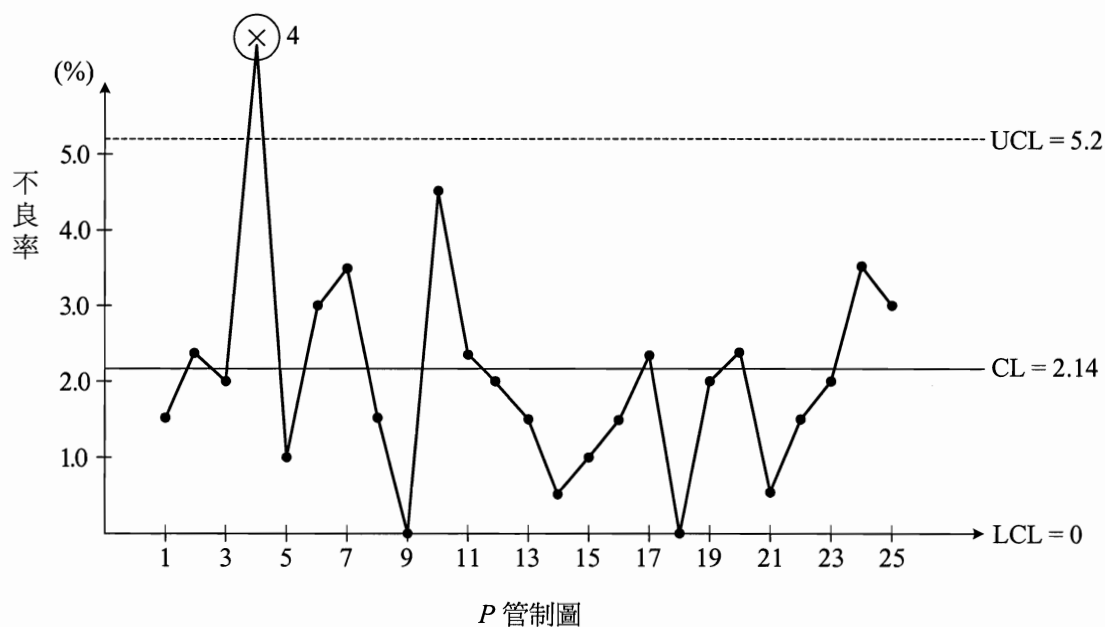
計算試作管制界限，

$$\text{UCL} = \overline{P} + \frac{3}{\sqrt{n}}\sqrt{\overline{P}(1-\overline{P})} = 0.0214 + \frac{3}{\sqrt{200}}\sqrt{0.0214 \times (1-0.0214)} = 0.052$$

$$= 5.2\%$$

$$\text{CL} = \overline{P} = 0.0214 = 2.14\%$$

$$\text{LCL} = -0.0092 = 0 \,(\text{取 } 0)$$

P 管制圖

(2) 製程並沒有處於管制狀況下，因爲第 4 點超出管制上限之外，操作人員理當找出造成製程異常之非機遇原因並且加以排除之後，再進行修正試作管制界限如下，

$$\overline{P} = \frac{\sum nP - nP_d}{\sum n - n_d} = \frac{107 - 15}{5000 - 200} = \frac{92}{4800} = 0.019$$

標準的管制界限，

$$\text{UCL} = 0.019 + \frac{3}{\sqrt{200}}\sqrt{0.019 \times (1 - 0.019)} = 0.048$$

$$\text{CL} = 0.019$$

$$\text{LCL} = -0.01 = 0 \text{（取 0）}$$

2. $n = 100$, $\overline{P} = 0.076$

$$\text{UCL} = \overline{P} + \frac{3}{\sqrt{n}}\sqrt{\overline{P}(1 + \overline{P})} = 0.76 + \frac{3}{\sqrt{100}}\sqrt{0.076(1 - 0.076)} = 0.076 + 0.079 = 0.156$$

$$\text{CL} = \overline{P} = 0.076$$

$$\text{LCL} = 0.076 - 0.079 = -0.003 = 0 \text{（取 0）}$$

$$P_1 = \frac{10}{100} = 0.1 \quad \text{尙未超出管制界限，因此，製程應該被判定爲處於管制界限下。}$$

3. (1) $\sum nP = 9.75$, $\sum n = 2000 \times 25 = 50{,}000$
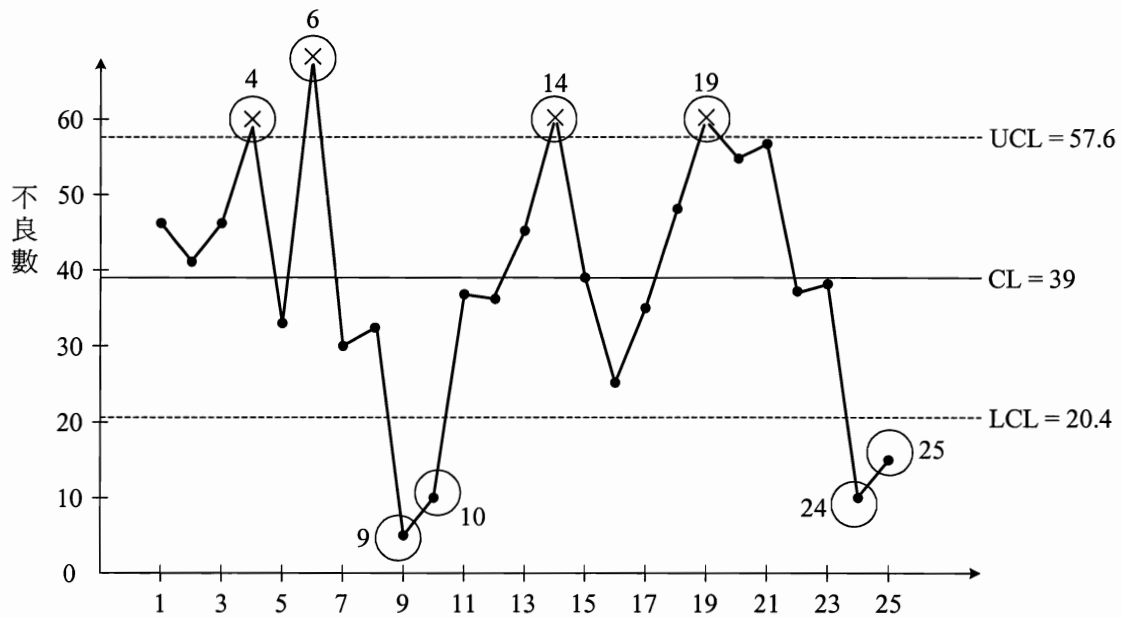
$$\overline{P} = \frac{\sum nP}{\sum n} = \frac{975}{50000} = 0.0195$$

$$\overline{nP} = \frac{\sum nP}{K} = \frac{975}{25} = 39$$

$$\text{UCL} = \overline{nP} + 3\sqrt{\overline{nP}(1 - \overline{P})} = 39 + 3\sqrt{39(1 - 0.0195)} = 39 + 18.6 = 57.6$$

$$\text{CL} = \overline{nP} = 39$$

$$\text{LCL} = \overline{nP} - 3\sqrt{\overline{nP}(1 - \overline{P})} = 39 - 3\sqrt{39(1 - 0.0195)} = 20.4$$

(2) 從上圖知，共有，

$nP_4 = 58$ ，$nP_6 = 68$ ，$nP_9 = 5$ ，$nP_{10} = 10$

$nP_{14} = 61$ ，$nP_{19} = 61$ ，$nP_{24} = 10$ ，$nP_{25} = 15$

超出界外，一般而言，可以假定它們都是起因於非機率原因，操作人員應該找出原因並且採取矯正措施之後，理當剔除越界的點，並且修正管制界限。

請注意，點 9，10，24，與 25 雖然超出管制下限，但是屬於低不良數原本是可喜之事，理當不該剔除，唯一要剔除之原因是品檢人員的量測或計數有誤而引起。本題假定點 9，10，24 與 25 並不是特殊原因造成，因此，不必剔除。

$$\overline{P} = \frac{\sum nP - nP_d}{\sum n - n_d} = \frac{975 - 58 - 68 - 61 - 61}{50,000 - 2000 \times 4} = \frac{727}{42,000} = 0.0173$$

$$\overline{nP} = \frac{727}{25 - 4} = 34.6$$

$$UCL = \overline{nP} + 3\sqrt{\overline{nP}(1 - \overline{P})} = 34.6 + 3\sqrt{24.6(1 - 0.0173)} = 52.1$$

$$CL = \overline{nP} = 34.6$$

$$LCL = \overline{nP} - 3\sqrt{\overline{nP} - (1 - \overline{P})} = 34.6 - 3\sqrt{34.6(1 - 0.0173)} = 17.1$$

4. (1) 計算每一批量之不良率，

$$P_1 = \frac{5}{500} = 0.01$$

$$P_2 = \frac{6}{550} = 0.011$$

$$\vdots$$

$$P_{24} = \frac{0}{500} = 0，如下表$$

批　　號	P_i	批　　號	P_i	批　　號	P_i
1	0.010	9	0.012	17	0
2	0.011	10	0.013	18	0
3	0.011	11	0.008	19	0.008
4	0.012	12	0.010	20	0.008
5	0.025	13	0.008	21	0.007
6	0.012	14	0.006	22	0.012
7	0.007	15	0.004	23	0.013
8	0.009	16	0.002	24	0

(2) 計算平均抽檢數

$$\bar{n} = \frac{\sum n}{K} = \frac{13950}{24} = 581.25 \approx 581$$

(3) 計算平均不良率

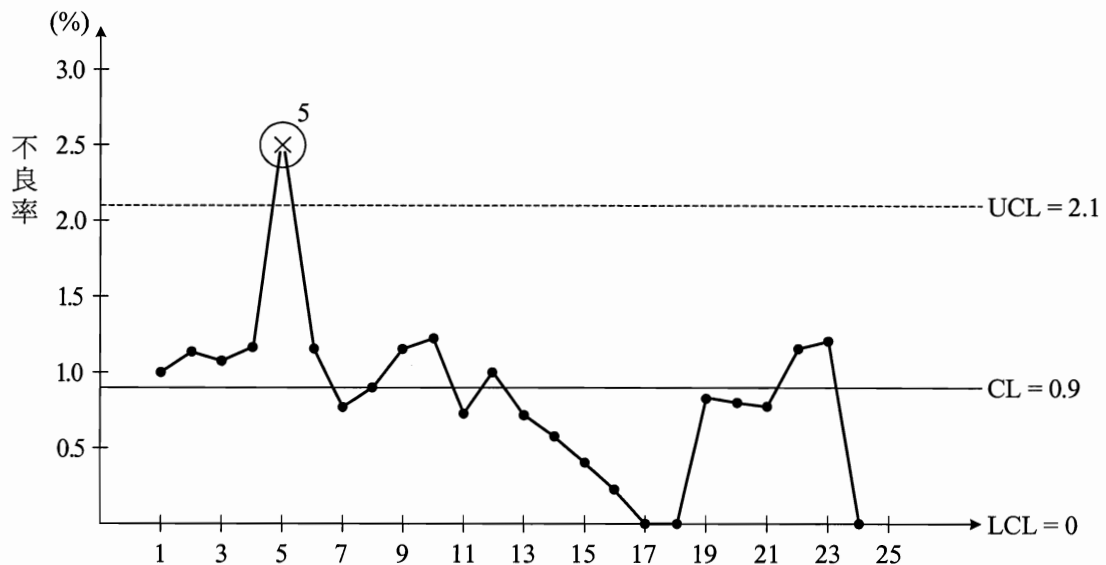$$\bar{P} = \frac{\sum nP}{\sum n} = \frac{124}{13950} = 0.009$$

(4) 計算試作管制界限

$$\text{UCL} = \bar{P} + \frac{3}{\sqrt{\bar{n}}}\sqrt{\bar{P}(1-\bar{P})} = 0.009 + \frac{3}{\sqrt{581}}\sqrt{0.009(1-0.009)} = 0.021$$

$$\text{CL} = 0.009$$

$$\text{LCL} = \bar{P} - \frac{3}{\sqrt{\bar{n}}}\sqrt{\bar{P}(1-\bar{P})} = 0.009 - \frac{3}{\sqrt{581}}\sqrt{0.009(1-0.009)} = -0.003$$

$$= 0（取 0）$$

核算 $P_5 = \dfrac{6}{100}, n_5 = 600 \Rightarrow UCL_5 = 0.009 + \dfrac{3}{\sqrt{600}}\sqrt{0.009(1-0.009)}$

$= 0.02056$ 比 0.021 更小。因此，批爲越界。

(5) 計算標準管制界限

  $P_5 = 0.025$ 超過管制上限，故要剔除，再修正管制界限

  $$\overline{P} = \frac{\sum nP - nP_d}{\sum n - n_d} = \frac{124 - 15}{13950 - 600} = \frac{109}{13350} = 0.008$$

  $$\overline{n} = \frac{13950 - 600}{24 - 1} = 580.43 \approx 580$$

  $$UCL = 0.008 + \frac{3}{\sqrt{580}}\sqrt{0.08(1-0.08)} = 0.019$$

  $$CL = 0.008$$

  $$LCL = 0.008 - \frac{3}{\sqrt{580}}\sqrt{0.008(1-0.008)} = -0.03 = 0\ (\text{取}\ 0)$$

5. 不良率的管制界限之公式

  $$\begin{matrix}UCL\\LCL\end{matrix} = \overline{P} \pm \frac{3}{\sqrt{n}}\sqrt{\overline{P}(1-\overline{P})}，UCL 與 LCL \propto \frac{1}{\sqrt{n}}，因此，$$

  當 $n_1 = 1000$

$$\Rightarrow \text{UCL} = 0.08 + \frac{3}{\sqrt{1000}}\sqrt{0.08(1-0.08)} = 0.106$$
$$> 0.052$$
$$\text{LCL} = 0.08 - \frac{3}{\sqrt{1000}}\sqrt{0.08(1-0.08)} = 0.054$$

當　　$n_2 = 1500$

$$\Rightarrow \begin{matrix}\text{UCL}\\\text{LCL}\end{matrix} = 0.08 \pm \frac{3}{\sqrt{1500}}\sqrt{0.08(1-0.08)} = \begin{matrix}0.101\\0.059\end{matrix} > 0.042$$

當　　$n_3 = 2000$

$$\Rightarrow \begin{matrix}\text{UCL}\\\text{LCL}\end{matrix} = 0.08 \pm \frac{3}{\sqrt{2000}}\sqrt{0.08(1-0.08)} = \begin{matrix}0.098\\0.062\end{matrix} > 0.036$$

當 n 愈大，管制界限之範圍愈小。

6. (1) $\overline{C} = \dfrac{\sum C}{K} = \dfrac{186}{25} = 7.44$

　　　$\text{UCL} = \overline{C} + 3\sqrt{\overline{C}} = 7.44 + 3\sqrt{7.44} = 15.6$

　　　$\text{CL} = \overline{C} = 7.44$

　　　$\text{LCL} = \overline{C} - 3\sqrt{\overline{C}} = -0.74 = 0\,(\,取\,0\,)$

(2) $\because \begin{matrix}\text{UCL} = 15.6\\\text{LCL} = 0\end{matrix} \Rightarrow C_{13} = 20$, $C_{16} = 20$，兩點越界，

　　應該被剔除，再計算標準管制界限，

　　　$\overline{C} = \dfrac{186 - 20 - 20}{25 - 2} = 6.35$

　　　$\text{UCL} = 6.35 + 3\sqrt{6.35} = 13.9$

　　　$\text{CL} = 6.35$

　　　$\text{LCL} = -1.21 = 0\,(\,取\,0\,)$

7. $\overline{C} = \dfrac{\sum C}{K} = \dfrac{372}{27} = 13.8$

　　$\text{UCL} = \overline{C} + 3\sqrt{\overline{C}} = 13.8 + 3\sqrt{13.8} = 24.9$

　　$\text{CL} = 13.8$

　　$\text{LCL} = \overline{C} - 3\sqrt{\overline{C}} = 13.8 - 3\sqrt{13.8} = 2.7$

　　6 月 17 日之缺點數為 38 超出管制界限，因此，修正

$$\overline{C} = \frac{372 - 38}{27 - 1} = 12.8$$

$$\left.\begin{array}{l} \text{UCL} = 12.8 + 3\sqrt{12.8} = 23.53 \\ \text{CL} = 12.8 \\ \text{LCL} = 12.8 - 3\sqrt{12.8} = 2.1 \end{array}\right\}$$ 八月份的 C 管制圖所需的管制界限

8. (1) $\overline{\mu} = \dfrac{\sum C}{\sum n}$ ， $\mu = \dfrac{C}{n}$

$\Rightarrow \quad \overline{\mu} = \dfrac{793}{223} = 3.56$

$\text{UCL} = \overline{\mu} + 3\sqrt{\dfrac{\overline{\mu}}{n}}$

$\text{UCL} = \overline{\mu}$

$\text{LCL} = \overline{\mu} - 3\sqrt{\dfrac{\overline{\mu}}{n}}$

$\quad n = 10$, $K = 1 \sim 7$

$\Rightarrow \left\{\begin{array}{l} \text{UCL} = 3.56 + 3\sqrt{\dfrac{3.56}{10}} = 5.35 \\ \quad\text{CL} = 2.56 \\ \text{LCL}_0 = 3.56 - 3\sqrt{\dfrac{3.56}{10}} = 1.77 \end{array}\right.$

$\quad n = 9$, $K = 17 \sim 25$

$\Rightarrow \left\{\begin{array}{l} \text{UCL} = 3.56 + 3\sqrt{\dfrac{3.56}{9}} = 5.45 \\ \quad\text{CL} = 3.56 \\ \text{LCL}_0 = 3.56 - 3\sqrt{\dfrac{3.56}{9}} = 1.67 \end{array}\right.$

$\quad n = 8$, $K = 8 \sim 16$

$\Rightarrow \left\{\begin{array}{l} \text{UCL} = 3.56 + 3\sqrt{\dfrac{3.56}{8}} = 5.56 \\ \quad\text{CL} = 3.56 \\ \text{LCL} = 3.56 - 3\sqrt{\dfrac{3.56}{8}} = 1.56 \end{array}\right.$

批 號	$\mu = \dfrac{C}{n}$	批 號	$\mu = \dfrac{C}{n}$	批 號	$\mu = \dfrac{C}{n}$
1	4.4	9	3.8	17	4.4
2	5.0	10	3.9	18	3.3
3	3.6	11	4.8	19	2.8
4	4.0	12	3.6	20	3.1
5	4.0	13	3.8	21	3.2
6	0.5	14	6.0	22	3.3
7	2.7	15	6.0	23	1.1
8	4.4	16	5.6	24	1.3
				25	1.4

(2) 沒有任何點超出管制界限之外。

第六章

1. $\quad\sigma_0 = \dfrac{\overline{S}}{C_4}$ ，$\because n = 4 \Rightarrow$ 查表，$C_4 = 0.9213$

$$\overline{S} = \dfrac{\displaystyle\sum_{i=1}^{20} S_i}{20} = 0.04055$$

$\Rightarrow \sigma_0 = \dfrac{0.04055}{0.9213} = 0.044$

製程能力 $= 6\sigma_0 = 6 \times 0.044 = 0.26$

2. 當 $n = 3 \Rightarrow$ 查表，$C_4 = 0.8862$

$\Rightarrow \sigma_0 = \dfrac{0.04055}{0.8862} = 0.0458$

$\Rightarrow$ 製程能力 $= 6\sigma_0 = 6 \times 0.0458 = 0.275$

當 $n = 8 \Rightarrow$ 查表，$C_4 = 0.9650$

$\Rightarrow \sigma_0 = \dfrac{0.04055}{0.9650} = 0.042$

$\Rightarrow$ 製程能力 $= 6\sigma_0 = 6 \times 0.042 = 0.252$

3. (1) 製程處於管制狀況下，卻生產一些不合格品，這是非常有可能發生。

(2) 比方說，下面這些數據